Problem 1

Question

Describe quantum circuits computing the following Boolean functions, i.e., quantum circuits U satisfying

$$U\left|x\right\rangle\left|y\right\rangle\left|0\right\rangle^{\otimes w} = \left|x\right\rangle\left|y \oplus f(x)\right\rangle\left|0\right\rangle^{\otimes w}$$

for some w.

(A) $f(x_1, x_2, x_3, x_4) := x_1 \oplus x_2 \oplus x_3 \oplus x_4$

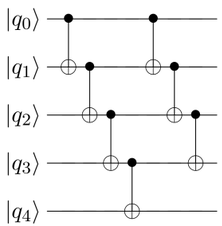
(B) $f(x_1, x_2, x_3, x_4) := x_1 \wedge x_2 \wedge x_3 \wedge x_4$

(C) $f(x_1, x_2, x_3, x_4) := (x_1 \vee x_2) \wedge (x_3 \vee x_4)$

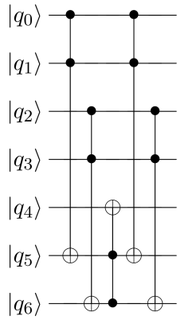
(D) $f(x_1, x_2, x_3) :=$ Majority of x_1, x_2 and x_3

Answer

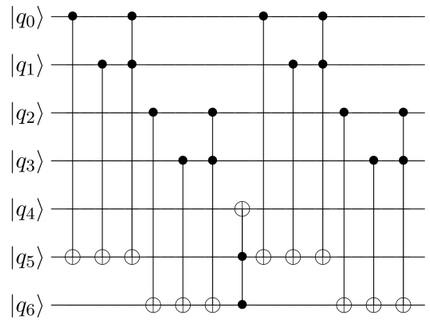
(A)



(B)



(C)



(D)