Seminar: Advanced Topics in Quantum Computing

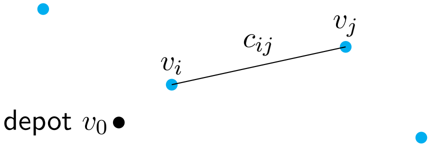
On efficient encodings for QAOA solutions to vehicle routing problems

Eben Jowie Haezer

December 2, 2023

VRP

VRP



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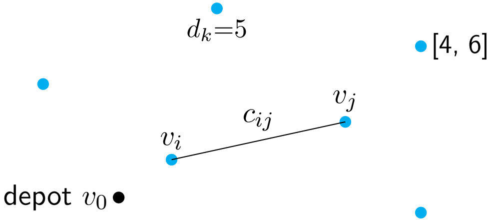

VRP

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Adiabatic computation

A physical system remains in its instantaneous eigenstate if a given perturbation is acting on it slowly enough and if there is a gap between the eigenvalue and the rest of the Hamiltonian's spectrum. [1]

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$$H(t) = \left(1 - \frac{t}{T}\right) \cdot H_0 + \frac{t}{T} \cdot H_p$$

Adiabatic approximation

$$e^{A+B} = \lim_{n\to\infty} \left(e^{\frac{A}{n}} \cdot e^{\frac{B}{n}}\right)^n$$

Eq. Ref. [2] [3]

Adiabatic approximation

$$\forall A, B \in \mathbb{C}^{m \times m}.\ e^{A+B} = \lim_{n \to \infty} \left(e^{\frac{A}{n}} \cdot e^{\frac{B}{n}} \right)^n$$

$$\left(\, [A, B] = 0 \implies e^{A+B} = e^A e^B \, \right)$$

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$$|\psi(\beta, \gamma)\rangle = \prod_k e^{-i\beta_k H_x} e^{-i\gamma_k H_f} \ |+\rangle^{\otimes m}$$

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QAOA

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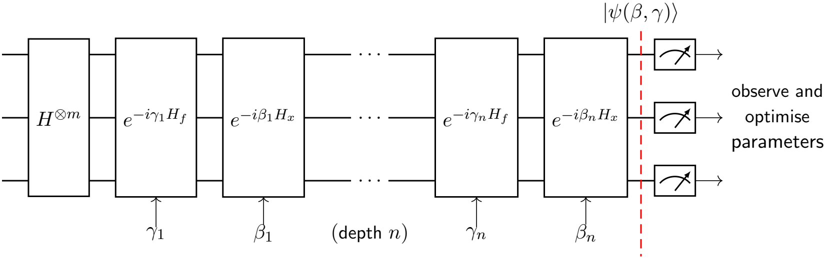

[3]

QUBO

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quadratic unconstrained binary optimisation

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$$|x^*\rangle = \underset{|x\rangle \in \{0,1\}^n}{\operatorname{argmin}} \ \langle x|Q|x\rangle$$

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QUBO

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quadratic unconstrained binary optimisation

$$|x^*\rangle = \underset{|x\rangle \in \{0,1\}^n}{\operatorname{argmin}} \langle x|Q|x\rangle$$

where $Q \in \mathbb{R}^{n \times n}$.

$$f_Q(x) = \langle x|Q|x\rangle = \sum_{i=1}^{n} \sum_{j=i}^{n} Q_{ij} x_i x_j$$

Eq. Ref. [4] [5]

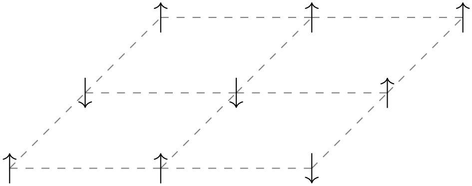
QUBO

Problem	QUBO formulation
Number Partitioning (NP)	Glover et al.[2]
Maximum Cut (MC)	Glover et al.[2]
Minimum Vertex Cover (MVC)	Glover et al.[2]
Set Packing (SP)	Glover et al.[2]
Set Partitioning (SPP)	Glover et al.[2]
Maximum 2-SAT (M2SAT)	Glover et al.[2]
Maximum 3-SAT (M3SAT)	Dinneen et al.[9]
Graph Coloring (GC)	Glover et al.[2]
General 0/1 Programming (G01P)	Glover et al.[2]
Quadratic Assignment (QA)	Glover et al.[2]
Quadratic Knapsack (QK)	Glover et al.[2]
Graph Partitioning	Lucas[7]
Decisional Clique Problem	Lucas[7]
Maximum Clique Problem	Chapuis[19]
Binary Integer Linear Programming	Lucas[7]
Exact Cover	Lucas[7]
3SAT	Lucas[7]
Maximal Independent Set	Djidjev et al.[8]
Minimal Maximal Matching	Lucas[7]
Set Cover	Lucas[7]
Knapsack with Integer Weights	Lucas[7]
Clique Cover	Lucas[7]
Job Sequencing Problem	Lucas[7]
Hamiltonian Cycles Problem	Lucas[7]
Minimal Spanning Tree	Lucas[7]
Steiner Trees	Lucas[7]
Directed Feedback Vertex Set	Lucas[7]
Undirected Feedback Vertex Set	Lucas[7]
Feedback Edge Set	Lucas[7]
Traveling Salesman (TSP)	Lucas[7]
Traveling Salesman with Time Windows (TSPTW)	Papalitsas et al.[1], Salehi et al.[67]
Graph Isomorphism	Calude et al.[4]
Subgraph Isomorphism	Calude et al.[4]
Induced Subgraph	Calude et al.[4]
Capacitated Vehicle Routing (CVRP)	Irie et al.[5], Feld et al.[31]
Multi-Depot Capacitated Vehicle Routing (MDCVRP)	Harikrishnakumar et al.[6]
L1 norm	Yokota et al.[3]
k-Medoids	Bauckhage1 et al.[10]
Contact Map Overlap Problem	Oliveira et al.[11]
Minimum Multicut Problem	Cruz-Santos et al.[12]
Broadcast Time Problem	Calude et al.[13]
Maximum Common Subgraph Isomorphism	Huang et al.[14]
Densest k-subgraph	Calude et al.[15]
Longest Path Problem	McCollum et al.[16]
Airport Gateway Assignment	Stollenwerk et al.[18]
Linear regression	Date et al.[17]
Support Vector Machine	Date et al.[17]
k-means clustering	Date et al.[17]
Eigencentrality	Prosper et al.[20]
Container Assignment Problem	Phillipson et al.[21]
k-colorable subgraph problem	Rodolfo et al.[22]
Routing and Wavelength Assignment with Protection	Oylum et al.[23]
Aircraft Loading Optimization	Giovanni et al.[24]
Linear least squares / system of linear equations	Ajinkya et al.[25]
Traffic Flow Optimization	Neukart et al.[26]
Permutation Synchronization	Tolga et al.[27]
Max-Flow Problem	Krauss et al.[28]
Network Shortest Path Problem	Krauss et al.[29]
Structural Isomer Search Problem	Terry et al.[30]

k-densest Common Sub-Graph Isomorphism	Huang et al.[32]
Community Detection	Negre et al.[33]
Nurse Scheduling problem	Ikeda et al.[34]
Aircraft Loading Optimization	Pilon et al.[35]
PageRank	Garnerone et al.[36]
Ramsey numbers	Gaitan et al.[37]
Generalized Ramsey numbers	Ranjbar et al.[38]
Transaction Settlement	Braine et al.[39]
Sensor placement problem in water distribution networks	Speziali et al.[40]
Fault Detection and Diagnosis of Graph-Based Systems	Perdomo-Ortiz et al.[41]
Bounded-Depth Steiner Trees	Liu et al.[42]
Graph Matching with Permutation Matrix Constraints	Benkner et al.[43]
Gaussian Process Variance Reduction	Bottarelli et al.[44]
Quantum Permutation Synchronization	Birdal et al.[45]
Unit Commitment Problem	Ajagekar et al.[46]
Heat Exchanger Network Synthesis	Ajagekar et al.[46]
Garden Optimization Problem	Calaza et al.[47]
Two-Dimensional Cutting Stock Problem	Arai et al.[48]
Labelled Maximum Weighted Common Subgraph	Hernandez et al.[49]
Maximum Weighted Co-k-plex	Hernandez et al.[49]
Molecular Similarity based on Graphs	Hernandez et al.[49] (*)
Portfolio Optimization (Modern Portfolio Theory)	Palmer et al.[51], Phillipson et al.[52]
Weighted Maximum Cut	Pelofske et al.[53]
Weighted Maximum Clique	Pelofske et al.[53]
Satellite Scheduling	Stollenwerk et al.[54]
Refinery Scheduling	Ossorio-Castillo et al. [55]
Job Shop Scheduling	Venturelli et al.[56]
Extended Job Shop Scheduling with Autonomous Ground Vehicles	Geitz et al.[73]
Parallel Flexible Job Shop Scheduling	Denkena et al.[57]
Bin Packing with Integer Weights	Lodewijks[58] (alternative: ref)
Number Partitioning with m sets	Lodewijks[58]
Graph Partitioning with m sets	Lodewijks[58]
Subset Sum Problem	Lodewijks[58]
Numerical Three-Dimensional Matching	Lodewijks[58]
Social Workers Problem	Adelomou et al.[59]
EV-Bus Charging Scheduling Problem	Yu et al.[61]
Vehicle Routing Problem	Borowski et al.[60]
Robot Path Planning	Finžgar et al.[62]
Scheduling on Undirected Hamiltonian Paths	Rieffel et al.[63]
Market Graph Clustering	Hong et al.[64]
Balanced k-Means Clustering	Arthur et al.[65]
Distance-based Clustering in general	Matsumoto et al.[66]
Credit Scoring and Classification	Milne et al.[68]
Dynamic Portfolio Optimization	Mugel et al.[69]
Railway Dispatching and Conflict Management Optimization	Domino et al.[70]
Workflow Application Scheduling	Tomasiewicz et al.[71]
Mirrored Double Round-robin Tournament	Kuramata et al.[72]
Transaction Scheduling	Bittner et al.[74]
Transformation Estimation	Golyanik et al.[75]
Point Set Registration	Golyanik et al.[75]
Maximum Cardinality Matching	Vert et al.[76]
Multi-car Paint Shop Optimization	Yarkoni et al.[77]
Binary Paint Shop Problem	Streif et al.[78]

[6]

Ising model



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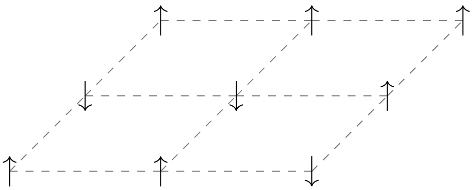

$$H = -\sum_{\langle i\ j \rangle} J_{ij}\sigma_i^z\sigma_j^z - \mu\sum_i h_i^z\sigma_i^z - \mu\sum_i h_i^x\sigma_i^x$$

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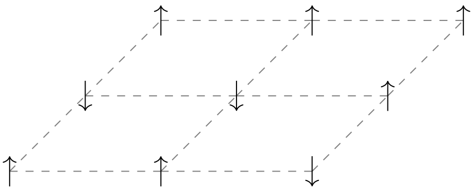

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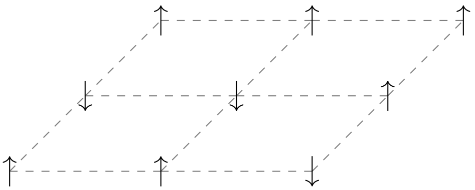

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$$\sigma \mapsto 2x-1,\ x\in\{0,1\}$$

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Problem Constraints

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$$\Delta = \sum_{b=0}^{\lceil \log_2 1+m \rceil - 1} 2^b \; a^r_b, \;\; a^r_b \in \{0,1\}$$

Two phase procedure

$$H_p = H_c + p_1 H_{\mathsf{once}} + p_2 H_{\mathsf{step}} + p_3 H_{\mathsf{cap}}$$

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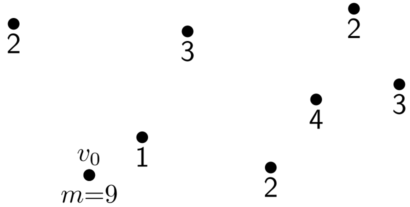
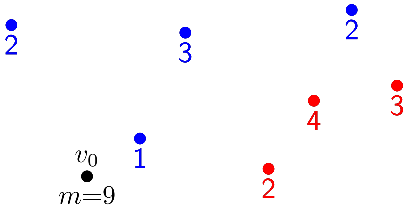
divide and conquer [8] [9]

- clustering phase
- routing phase

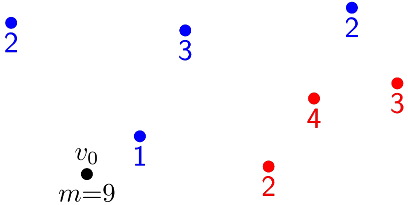
Clustering



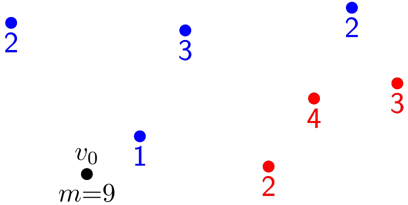
Clustering



Clustering

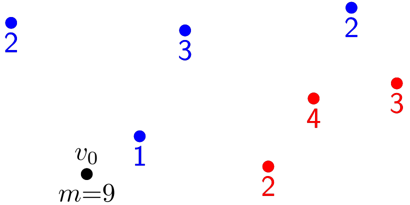


subject to constraints:



subject to constraints: **once**, **cap**

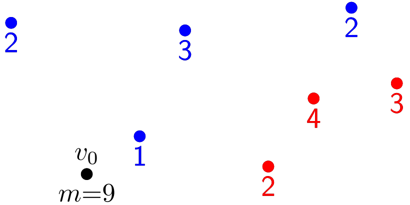
Clustering



subject to constraints: **once**, **cap**

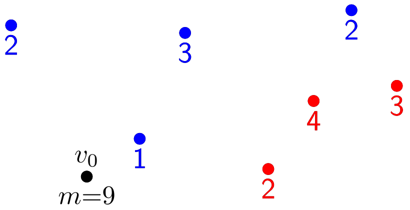
additional constraint: **cluster**

Clustering



subject to constraints: **once**, **cap**

additional constraint: **cluster**
The cumulative edge cost c_{ij} between all pairs of nodes (i, j) within a cluster should be minimal.



subject to constraints: **once**, **cap**

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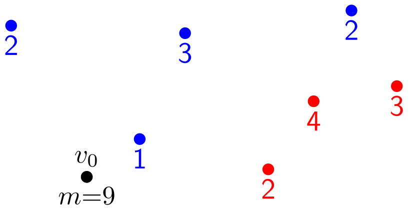
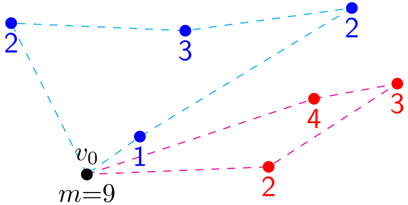
The cumulative edge cost c_{ij} between all pairs of nodes (i, j) within a cluster should be minimal.

$$H_{\text{cluster}} = \sum_r \sum_{i,\, j} c_{ij}\ x_i^r\ x_j^r$$

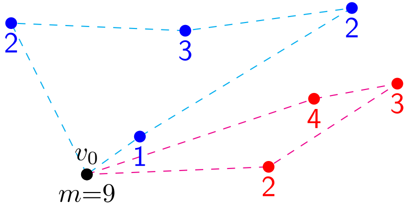
where $x_i^r = \sum_t x_{i,t}^k$


2
3
2
3
4
1
v_0
$m{=}9$
2

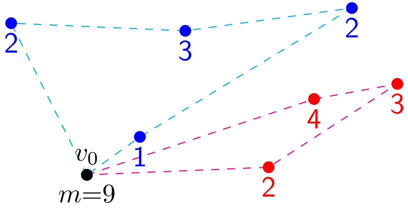
Routing



Routing



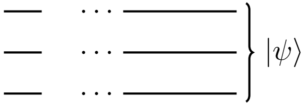
subject to constraints: **once**, **step**, edge costs



subject to constraints: **once**, **step**, edge costs

$$H_{\mathsf{tsp}} = q_1 H_{\mathsf{once}} + q_2 H_{\mathsf{step}} + q_3 H_c$$

Complete encoding



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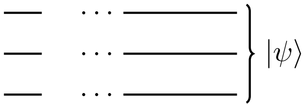
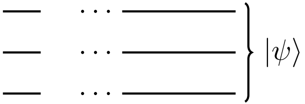


$$|\psi\rangle = \sum_{z \in \{0,1\}^n} \beta_z \, |z\rangle$$

Complete encoding



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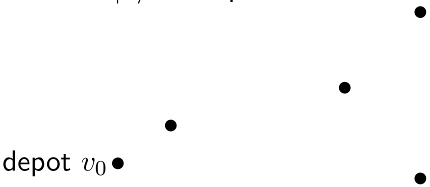
$|z\rangle$ corresponds to



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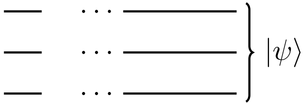

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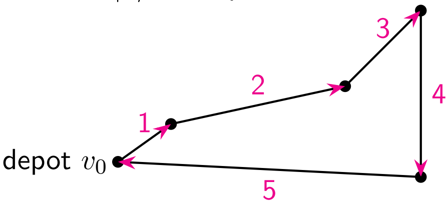

Minimal encoding

Eq. Ref. [10]

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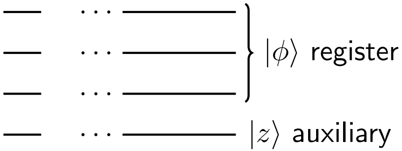

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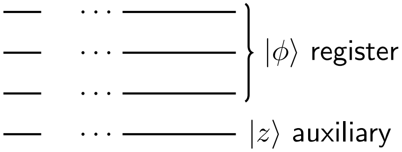
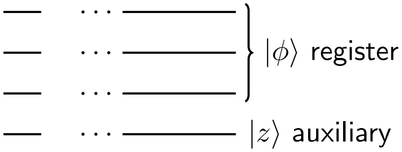


register width n_r, aux width n_a, $n_q = n_r + n_a$

Eq. Ref. [10]

Minimal encoding



register width n_r, aux width n_a, $n_q = n_r + n_a$

$$|\psi(\theta)\rangle = \sum_{i=1}^{n_c} \beta_k(a_i|0\rangle + b_i|1\rangle)\,|\phi_i\rangle$$

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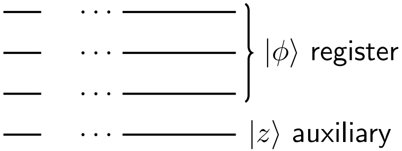
Minimal encoding



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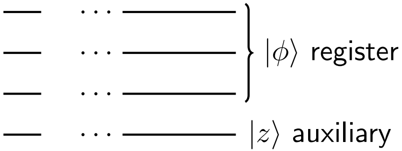
$|\phi_i\rangle$ corresponds to a single classical variable $x_{i,t}^r$.

Eq. Ref. [10]

Minimal encoding



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$$n_q = 1 + \log_2 n_c$$

Eq. Ref. [10]

Example

let $n_c = 4$, $n_r = 2$.

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Example

let $n_c = 4$, $n_r = 2$.

$$x^* = (1, 1, 1, 0)^\dagger$$

$$|\psi^*\rangle = \frac{1}{2}(|1\rangle|00\rangle + |1\rangle|01\rangle + |1\rangle|10\rangle + |0\rangle|11\rangle)$$

Solution probability

$$|\psi(\theta)\rangle = \sum_{i=1}^{n_c} \beta_k(a_i|0\rangle + b_i|1\rangle)\,|\phi_i\rangle$$

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$$\Pr x = \prod_{i=1}^{n_c} \Pr x_i = \prod_{i=1}^{n_c} \|b_i\|^2$$

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$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

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&= 0
\end{aligned}$$

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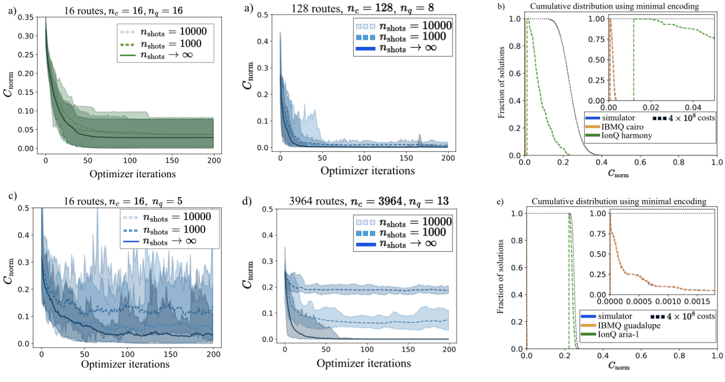
Experimental results

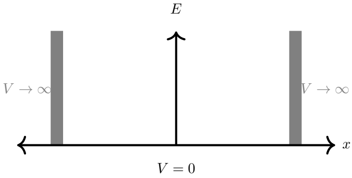


[11]

Thank you for your attention.

`github.com/ge42faz/qsem-23w`

Backup slides

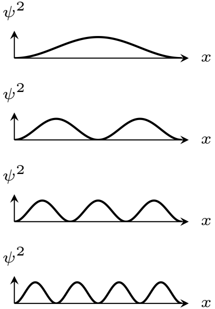


$$i\hbar \, \frac{\partial}{\partial t}\psi(x,t) = \frac{-\hbar^2}{2m} \; \frac{\partial}{\partial x^2}\psi(x,t) \; + \; V(x) \; \psi(x,t)$$

$\psi(x,t) = e^{-i\omega t}(A \sin kx + B \sin kx)$ for x within the box.

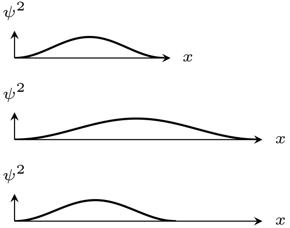
Backup slides

Adiabatic computation



$$H|\psi\rangle = E|\psi\rangle$$

$$E_k = \hbar\omega_k = \frac{n^2\pi^2}{L^2}\frac{\hbar^2}{2m}$$



$$H(t) = \left(1 - \tfrac{t}{T}\right) \cdot H_0 + \tfrac{t}{T} \cdot H_p$$

$$0 \leq t \leq 1,\ T \in \mathcal{O}(\tfrac{1}{g^2})$$

$$[H_0, H_p] \neq 0$$

Backup slides

Ising model

$$\mathbf{B} = \frac{\mu_0}{4\pi} \left(\frac{3\mathbf{r}(\mathbf{m}\cdot\mathbf{r})}{r^5} - \frac{\mathbf{m}}{r^3}\right)$$

$$H = -\sum_{\langle i\ j\rangle} J_{ij}\sigma_i\sigma_j + \mu \sum_i h_i\sigma_i$$

$$\mu = \tfrac{-e}{2m_e} L$$

$J_{ij} > 0$ ferromagnetic, $J_{ij} < 0$ antiferromagnetic.

Backup slides

Cost function

$$C_{\mathsf{cpl}}(\theta) = \langle H \rangle = \langle \psi(\theta) | H | \psi(\theta) \rangle$$

$$\begin{aligned} C_{\mathsf{min}}(\theta) &= \sum_{i \neq j}^{n_c} Q_{ij} \|b_i\|^2 \|b_j\|^2 + \sum_{i=1}^{n_c} Q_{ii} \|b_i\|^2 \\ &= \sum_{i \neq j}^{n_c} Q_{ij} \frac{\langle P_i^1 \rangle}{\langle P_i \rangle} \frac{\langle P_j^1 \rangle}{\langle P_j \rangle} + \sum_{i=1}^{n_c} Q_{ii} \frac{\langle P_i^1 \rangle}{P_i} \end{aligned}$$

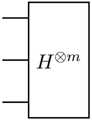
expressed as projectors $P_i = |\phi_i\rangle\langle\phi_i|$ and $P_i^1 = |1\rangle\langle 1| \otimes P_i$.

$$\overline{C} = \frac{C(\theta) - C^*}{\Delta C}$$

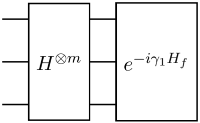
$$|\psi(\beta, \gamma)\rangle =$$

Backup slides
QAOA

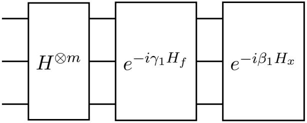
$$|\psi(\beta,\gamma)\rangle = \qquad\qquad\qquad |+\rangle^{\otimes m}$$



$$|\psi(\beta, \gamma)\rangle = \qquad e^{-i\gamma_k H_f} \; |+\rangle^{\otimes m}$$

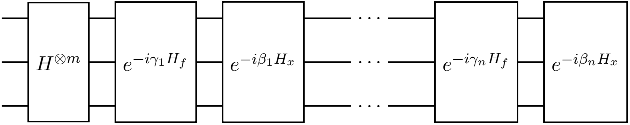


$$|\psi(\beta,\gamma)\rangle = \quad e^{-i\beta_k H_x} e^{-i\gamma_k H_f} \; |+\rangle^{\otimes m}$$



Backup slides
QAOA

$$|\psi(\beta,\gamma)\rangle = \prod_k e^{-i\beta_k H_x} e^{-i\gamma_k H_f} \; |+\rangle^{\otimes m}$$



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Backup slides
QAOA

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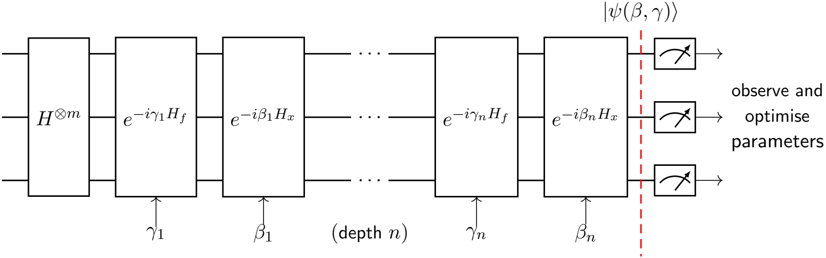

References I

- M. Born and V. Fock, "Beweis des Adiabatensatzes," *Zeitschrift für Physik*, 1928.
- J. E. Cohen, S. Friedland, T. Kato, and F. P. Kelly, "Eigenvalue inequalities for products of matrix exponentials," *Linear Algebra and its Applications*, 1982.
- E. Farhi, J. Goldstone, and S. Gutmann, "A quantum approximate optimization algorithm," 2014.
- A. Lucas, "Ising formulations of many NP problems," *Frontiers in Physics*, 2014.
- F. Glover, G. Kochenberger, R. Hennig, and Y. Du, "A tutorial on formulating and using QUBO models," *Ann Oper Res*, 2022.

References II

- D. Ratke, "List of QUBO formulations," 2021, https://blog.xa0.de/post/List-of-QUBO-formulations/.
- E. Ising, "Beitrag zur Theorie des Ferromagnetismus," *Zeitschrift für Physik*, 1925.
- S. Feld, C. Roch, T. Gabor, C. Seidel, F. Neukart, I. Galter, W. Mauerer, and C. Linnhoff-Popien, "A hybrid solution method for the capacitated vehicle routing problem using a quantum annealer," *Frontiers in ICT*, 2019.
- L. Palackal, B. Poggel, M. Wulff, H. Ehm, J. M. Lorenz, and C. B. Mendl, "Quantum-assisted solution paths for the capacitated vehicle routing problem," 2023.

References III

B. Tan, M.-A. Lemonde, S. Thanasilp, J. Tangpanitanon, and D. G. Angelakis, "Qubit-efficient encoding schemes for binary optimisation problems," *Quantum*, 2021.

I. D. Leonidas, A. Dukakis, B. Tan, and D. G. Angelakis, "Qubit efficient quantum algorithms for the vehicle routing problem on NISQ processors," 2023.