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The `tumthesis` Class

A Tutorial for Theses

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I hereby confirm that this is my own work, and that I used only the cited sources and materials.

München, February 10, 2021

Wolfgang F. Riedl, Michael RitterTranslation by Oliver Cooley

Abstract

Here we give a short summary of the project or thesis of length at most a quarter of a page. This could be e. g. as follows:

This document is an introduction to the use of the LaTeX-package `tumthesis.cls`, with which theses can be written in the TUM style. The basic structure of the example files is explained and some optional components are mentioned briefly. There are also some tips for LaTeXbeginners (and also for more advanced users who want to learn some more) as well as suggested reading for individual study.

Zusammenfassung

Hier schreibt man eine kurze Zusammenfassung der Arbeit im Umfang von maximal einer Viertelseite. Das kann z. B. so aussehen:

Die Arbeit führt in die Verwendung des LaTeX-Pakets `tumthesis.cls` ein, mit dem Abschlussarbeiten im TUM-Stil gesetzt werden können. Die grundlegende Gliederung der Beispieldateien wird erklärt und auf optionale Bestandteile wird kurz eingangen. Außerdem enthält der Text ein paar Tipps für LaTeX-Anfänger (und auch für Fortgeschrittene, die noch etwas dazulernen wollen) sowie Literaturhinweise zum Selbststudium.

Contents

Chapter 1

Introduction

This chapter provides a short example of the use of the `tumthesis` class.

It is impossible to collect the information about the true prior of bidders valuation. In addition, it may impact on bidders' incentive and performance of the auction mechanism. Therefore, it is interesting to design a mechanism with less assumption on the prior distribution/with limited information of this prior. Normally we start to consider the worst case analysis.

1.1 Files

Table 1.1 shows all the files associated with this example, each with a short description.

File name	**Description**
`tumthesis.cls`	class file, defines basic commands and incorporates important packages
`tumcolors.sty`	LaTeXpackage in which the official TUM colours and logos are defined; used by tumthesis.cls.
`thesis.tex`	Main file of this example and starting point for own project. All other `.tex` files are included using this file.
`thesis.pdf`	PDF version of thesis.tex
`preamble.tex`	Preamble loading custom packages of user
`abstract.tex`	Abstract text in English and German
`introduction.tex`	Text for this chapter
`conclusion.tex`	Text for the following chapter
`appendix.tex`	Text for the appendix
`thesis.bib`	BibTeX file for the bibliography
`TopMath-Bildmotiv.jpg`	TopMath Logo for the title page

Table 1.1: Files for this example

The following commands should be used to to compile the final pdf from these source files:

```
pdflatex thesis
biber thesis
makeindex -s myindex.ist
pdflatex thesis
pdflatex thesis
```

The first run-through of `pdflatex` creates various auxiliary files and a (mostly complete) pdf output – some graphics may appear in the wrong place and the references and citations do not yet work correctly. With the `biber` command, the system works through the `thesis.bib` file and creates the bibliography (c.f. Section 1.7.2), while the `makeindex` command creates the index (c.f. Section 1.8.1). The subsequent two `pdflatex` commands set references and place graphics correctly.

Advanced users can also automise the process by using the command

```
latexmk --pdf thesis
```

The `latexmk` tool then ensures each command in the BibTEX runthrough is called the correct number of times.

1.2 Configuration and Options

Custom packages of the user or overwritten settings can be loaded in file `preamble.tex`. The code contained in this file will be included and executed right at the end of the class (just before loading the last packages `hyperref` and `cleveref`).

The `tumthesis` class accepts some options, that help to adjust the titlepage and some behaviour:

- `topmath`: This option places the “TopMath-Bildmotiv” on the titlepage:

  ```
  \documentclass[topmath]{tumthesis}
  ```

Alternatively, you can place your own logo on the titlepage:

- `titlepicture`: Filename of your logo
- `titlepictureX`: Horizontal distance (including unit) between lower right corner of the titlepage and lower right corner of the logo
- `titlepictureY`: Vertical distance (including unit) between lower right corner of the titlepage and lower right corner of the logo

  ```
  \documentclass[titlepicture=MA_CMYK.pdf,titlepictureX=25mm,
  titlepictureY=40mm]{tumthesis}
  ```

would include the math logo once again in the lower right corner of the titlepage.

Furthermore, you can adjust the behaviour of theorems' titles:

- `theoremtitle`: Whether the content of a theorem is typeset next to its name (nobreak) or not (break, standard option)

  ```
  \documentclass[theoremtitle=nobreak]{tumthesis}
  ```

A further option allows the user to specify the BibLaTeX backend to be used:

- `biblatexBackend`: The default is set to biber, every valid option for parameter "backend" of the biblatex package is an alternative:

  ```
  \documentclass[biblatexBackend=bibtex]{tumthesis}
  ```

1.3 Basic Settings

At the very beginning, the `thesis.tex` file fixes some basic settings. The code is as follows:

```
% -------------------------------
% PDF-Information
\hypersetup{
 pdfauthor={Wolfgang Ferdinand Riedl, Michael Ritter},
 pdftitle={The tumthesis Class},
 pdfsubject={A Tutorial for Theses},
 pdfkeywords={Master's Thesis, Bachelor's Thesis},
 colorlinks=true, %coloured links (for the PDF version)
% colorlinks=false, % no coloured links (for the print version)
}

% -------------------------------

% Basisdaten

\author{Wolfgang F. Riedl, Michael Ritter}
\title{The \texttt{tumthesis} Class}
\subtitle{A Tutorial for Theses}
\faculty{Fakultät für Mathematik}
\institute{Lehrstuhl für Angewandte Geometrie und Diskrete Mathematik}
%\subject{master}
%\subject{bachelor}
%\subject{diploma}
```

```
%\subject{project}
%\subject{seminar}
%\subject{idp}
\subject{Short Overview}
\professor{Prof. Dr. Peter Gritzmann} %Themensteller
\advisor{Dr. René Brandenberg} %Betreuer
\date{26.12.2012} %Submission Date
\place{München} %Place where document is signed
```

The inputs in the `hypersetup` command do not appear in the document itself, but are embedded into the pdf file as metadata and can be viewed in Acrobat Reader (and many other pdf-viewers). The remaining commands should be largely self-explanatory. The `subject{}` command accepts any desired text as input (such as “Short Overview” in this example), or you may use one of the pre-defined key words, which automatically create a title of “Master’s Thesis”, “Bachelor’s Thesis” or other suitable output. To do this, simply remove the commentary marks from the appropriate line and comment out the currently active `subject` command. Be careful, of course, to match the selected language (see Section 1.4). (The `\subject{}` can also be omitted entirely, in which case there will be no designation on the title page and only the author name will appear.)

1.4 Language selection und Character set

The class supports English and German as language options. The language is set using the command

```
\selectlanguage{english}
```

at the start of `thesis.tex`. The language can be changed at any point in the document using this command or respectively `\selectlanguage{ngerman}`. This also means that some settings are automatically changed to match, e. g. the commands `\eg` and `\ie` result in the appropriate text (these commands should be used in any case, since they ensure that the spacing is typographically correct) and the headings change, but there are also some more subtle changes such as the rules for automatic hyphenation. An example of such a language switch can be found in the abstract.

For your own files it is important to select the correct “encoding”. Here the default is Unicode (UTF-8). This allows you to type in umlauts and other special characters directly, but may require the correct settings in the editing programme. Some editors, especially in Windows systems, are set to Latin-1 rather than Unicode – this can lead to interesting errors! [KM12a]

1.5 Printing

When printing, be sure to print the thesis double-sided. The side margins, headers and footers are set out for double-sided printing and binding. If you want to change the space left for binding simply adjust the line

```
BCOR =5 mm % Binding correction , ensures sufficient space
    for binding
```

in file `tumthesis.cls`.

1.6 Titlepage

Per default, the titlepage is set to a standard titlepage following the TUM-Styleguide as close as possible. By replacing line

```
\maketitlepage%
```

in the file `thesis.tex` with

```
\maketitlepageDissertation%
```

a titlepage suitable for a dissertation is created.

1.7 Important notes

1.7.1 Math Environments

As this class loads the package `ntheorem`, math environments of the following form lead to errors: `\[ ... \]`. Replace them by `\begin{equation*} ... \end{equation*}` instead.

1.7.2 Biblatex

he package currently uses the biber backend which can handle UTF-8 encoded bibliography files. This default option can be changed by the parameter `biblatexBackend` as described above.

You can find manuals for the setup of your LaTeXeditor with biber for most editors by searching the web.

1.8 Some Packages

The `tumthesis.cls` class includes a range of useful packages, which are listed below.

1.8.1 Index

The tumthesis style file loads the package imakeidx, which allows quick and easy generation of an *index*. To include a word or definition the index, simply append `\index{keyword}`. For example, the keyword "index" is added to the index of this document in the following way:

```
... easy generation of an \emph{index}\index{index}. To ...
```

Adding symbols is just as easy: The symbol ζ is included in the index by the line

```
... the symbol $\zeta$\index{$\zeta$} is included ...
```

To change the position of the symbols (or any other elements) in your index, you can specify an additional keyword (which may also be a formula):

To include the symbol π in the index such that it appears in the place where the word "pi" would appear and also in the place where the symbol "p_i" would appear, you can use the code

```
... symbol $\pi$\index{pi@$\pi$}\index{$p_i$@$\pi$} is included ...
```

Subcategories can also easily be realized, by using `\index{keyword!subkeyword}`: For example, the definition of a metric space

Definition 1.1 (Metric)
Let X be a set and $d : X \times X \longrightarrow \mathbb{R}$. The function d is a metric on X if the following three properties hold for all $x, y, z \in X$

1. $d(x, x) \geq 0$ and $d(x, y) = 0 \iff x = y$ (non-negativity)
2. $d(x, y) = d(y, x)$ (symmetry)
3. $d(x, z) \leq d(x, y) + d(y, z)$ (triangle inequality).

can be properly referenced in the index by the following code:

```
\begin{definition}[Metric]
  \index{metric}
  Let $X$ be a set and $d: X \times X \longrightarrow \mathbb{R}$. The function $d$ is a metric on $X$ if the following three properties hold for all $x,y,z \in X$
  \begin{enumerate}
    \item $d(x,x) \geq 0$ and $d(x,y) = 0 \iff x = y$ (non-negativity)\index{metric!non-negativity}
    \item $d(x,y) = d(y,x)$ (symmetry)\index{metric!symmetry}
    \item $d(x,z) \leq d(x,y) + d(y,z)$ (triangle inequality)\index{metric!triangle inequality}.
  \end{enumerate}
\end{definition}
```

To create the index, the line

```
\makeindex[title=Index,options=-s myindex]
```

has to be added to the file `thesis.tex` *before* the `\begin{document}` command! The option `title=Index` specifies the heading of the index, in this case “Index”; the second option sets a custom style file (does not work in any environment, see compiler options below if it does not work).

The index can then be added to the document by the command

```
%Add the index to the table of contents
\addcontentsline{toc}{chapter}{Index}
%print the index
\printindex
```

The index has to be compiled by the command `makeindex` (which most editors will apply automatically). The layout of the index can be modified by specifing a style file via the option `-s stylefile.ist` (or sometimes the option given to the `\makeindex` command above). This document was compiled using the file `myindex.ist`, which is included in the package.

1.8.2 scrbook

The `tumthesis.cls` class builds entirely on `scrbook.cls`. In particular, this means that all of the options and commands in `scrbook` are available here. More information can be found in the documentation [KM12a] or in the printed version Kohm and Morawski [KM12b].

1.8.3 csquotes

Among other things, this package makes the `\enquote{}` command available, which automatically ensures that quotation marks are correct. The package takes account of the currently active language: in English text “English quotation marks” appear, wobei deutsche Texte „entsprechende Anführungszeichen“ bekommen.

1.8.4 cleveref

In LaTeX we normally make references using `\ref{}`. This package defines the new commands `\cref` and `\Cref`, which ensure that as well as the correct number, the correct descriptive text also appears (in the currently selected language). The latter command also ensures capitalisation and should therefore be used at the beginning of a sentence (although in German this often makes no difference, since labels are usually nouns, which are capitalised in any case). An example can be found above and also here: The references to Table 1.1 are made using `cleveref`, the word “Table” is included automatically.

1.8.5 ntheorem

This package prepares a range of standard environments for definitions, theorems, proofs etc. The labels are determined by the language. For example:

Definition 1.2
Every element of a vector space is called a *vector*.

Theorem 1.3 (Fundamental theorem of vector space terminology)
For every vector v there is a vector space V with $v \in V$.

Proof. The proof is trivial and is left as an exercise for the reader. It is really not hard, just try it. □

Wir zeigen jetzt eine deutsche Version dieses Beweises:

Beweis. Der Beweis von Theorem 1.3 ist höchst trivial und nur ein Vollidiot würde es nicht selber können. Wenn Sie sich überhaupt die Mühe gemacht haben, diesen Beweis zu lesen, überlegen sie sich vielleicht, ob Sie nicht lieber ein anderes Fach studieren sollten. □

We formulate another theorem in order to demonstrate another feature of `cref` with which one may group several references together. To this end, we reference Theorems 1.3 and 1.4

Theorem 1.4
LaTeX is great!

The environment can of course be extended and customised. For more information, see the examples in `tumthesis.cls` or the documentation for the `ntheorem` package [MS11]. All of the pre-defined environments are listed in Table 1.2.

1.8.6 booktabs

This package allows for nicer tables, such as Table 1.1. There are many tips on setting out tables in the extensive documentation for this package.

1.8.7 tabularx

A tabular modifying the width of certain columns in order to achieve a custom width can be constructed using package `tabularx`.

Environment	Text	
	English	German
`definition`	Definition	Definition
`theorem`	Theorem	Satz
`satz`	Theorem	Satz
`lemma`	Lemma	Lemma
`proposition`	Proposition	Proposition
`corollary`	Corollary	Korollar
`korollar`	Corollary	Korollar
`remark`	Remark	Bemerkung
`bemerkung`	Remark	Bemerkung
`example`	Example	Beispiel
`beispiel`	Example	Beispiel
`proof`	Proof	Beweis
`beweis`	Proof	Beweis
`conjecture`	Vermutung	Conjecture
`vermutung`	Vermutung	Conjecture
`problem`	Problem	problem

Table 1.2: predefined `ntheorem` environments

1.8.8 TikZ

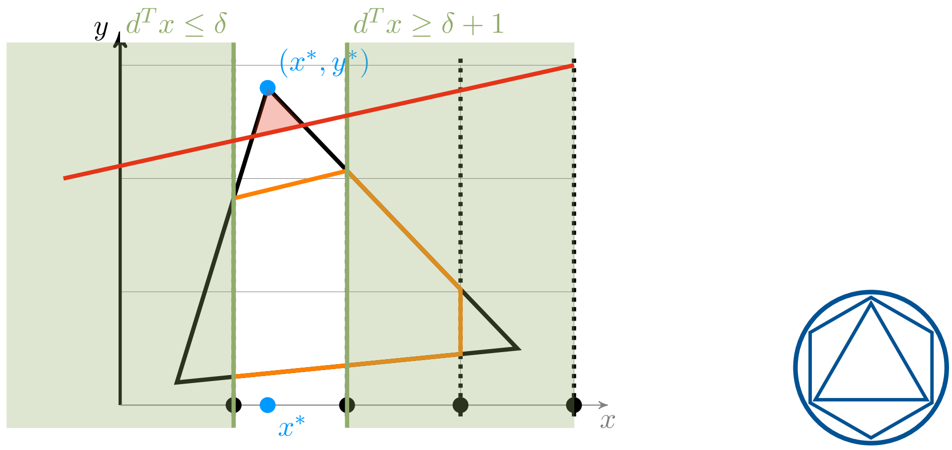
TikZ may not be a graphics program, but you can still create some excellent graphics with it. The documentation [Tan08] is extensive. Online under `http://www.texample.net` are a number of examples which demonstrate what you can do with this package. Lest the list of figures remain empty, we include a TikZ graphic in Fig. 1.1a. Fear not, you do not have to understand the code straight away. There are some simpler and well-described examples in the TikZ manual.

1.8.9 subcaption

To create multiple subfigures within a figure, you can use the package `subcaption` and its environment `\begin{subfigure} ... \end{subfigure}`. It gives the opportunity to create subfigures and subtables using the same syntax as used for figures and tables.

1.8.10 fixme

This package allows you to make notes in your document which mark points where more work is needed. A "List of Corrections" then appears at the very end of the document, FiXme Note! in which all the notes are listed. To demonstrate, this paragraph contains two such



(a) Example of a split disjunction

(b) The Math Logo

Figure 1.1: Two graphics

FixMe notes—these appear as notes at the side, but also in the aforementioned "List of Corrections" at the very end of the document. Here also there are many possible settings and the *documentation* is recommended reading. We mention here one setting which should appear at the beginning of the `thesis.tex` file:

FiXme Note!

```
%FixMe-Status: final (no FixMe notes) or draft (notes visible)
\fxsetup{draft}
%\fxsetup{final}
```

Replacing the "draft" line with the "final" line will result in two things: All `\fxfatal{}` commands will become LaTeXerrors which break off the TeXcompilation of the document (useful for highlighting really bad mistakes which should definitely not be overlooked). All other fixme commands (i. e. `\fxnote{}`, `\fxwarning{}`, `\fxerror{}`) will simply become invisible, the notes in the text and the "List of Corrections" at the end of the document disappear. More information on the available commands and the many possible settings can be found in the documentation [Ver09].

1.8.11 hyperref

The `hyperref` package fixes a number of pdf settings (see Section 1.3). Furthermore, the package ensures that all references, citations and the table of contents become clickable links which allow the reader to jump back and forth in the document. By default these links appear in black and are therefore not immediately visible. Alternatively with the `colorlinks=true` setting in the `\hypersetup{}` command at the beginning of the document, these links become dark blue. This is convenient for the on-screen version; however for the print version you should revert to black, i. e. use the option `colorlinks=false` (after all, you can't click on links in the printed document).

```
\hypersetup{
 pdfauthor={Wolfgang Ferdinand Riedl, Michael Ritter},
 pdftitle={The tumthesis Class},
 pdfsubject={A Tutorial for Theses},
 pdfkeywords={Master's Thesis, Bachelor's Thesis},
 colorlinks=true, %coloured links (for the PDF version)
% colorlinks=false, % no coloured links (for the print version)
}
```

1.8.12 listings

The `listings` package allows you to produce nicely formatted source code listings. In this example it is used to produce LaTeX source code. The standard settings take care of line numbering, line breaks and various other details. Of course many settings can be customised; for details see the documentation [HM07]. One small warning: The package is set up to deal with umlauts and "ß" in the source texts, but other special characters may cause problems (even in comments). It is best to avoid special characters altogether in source texts—if this is not possible, take a look at `tumthesis.cls` to see how to modify the settings to deal with other special characters.

1.8.13 algorithm2e

`algorithm2e` provides the possibility to create floating algorithm environments. In contrast to `listings` it does not produce source code formatted according to a certain programming language, instead you write formatted pseudo-code with a predefined syntax.

1.8.14 tumcolors2

The package `tumcolors2` provides a number of colors according to the TUM styleguide. It also contains commands that draw the TUM logo and the math faculty logo.

```
\mathlogo{width=2cm}
\mathlogo{height=2cm}
\tumlogo{width=2cm}
\tumlogo{height=2cm}
```

example	name	alternatives
	tumblue1	tumblau1, tumblue, tumblau
	tumblue2	tumblau2
	tumblue3	tumblau3
	tumblue4	tumblau4
	dark tumblue1	dunkles tumblau1, dark tumblue, dunkles tumblau
	dark tumblue2	dunkles tumblau2
	dark tumblue3	dunkles tumblau3
	dark tumblue4	dunkles tumblau4
	dark gray	dunkelgrau
	medium gray	mittelgrau
	light gray	hellgrau
	accentuating light blue	Akzent-Hellblau
	accentuating dark blue	Akzent-Dunkelblau
	accentuating ivory	Akzent-Elfenbein
	accentuating orange	Akzent-Orange
	accentuating green	Akzent-Gruen

Table 1.3: TUM colors

Chapter 2

Single-bidder and Single-item Deterministic Auctions

In this chapter, we will evaluate DAPX under single-bidder and single-item deterministic auction with some known probability distribution. We will use Myerson optimal acution as our auction mechanism, that seller make a reserve price for the item and if the bidder bids higher than the reserve price, she gets the item, otherwise the item remains un-sale. It is also called take it or leave it auction. We use the reserve price proposed by the paper. Then we experiment this auction with the reserve price and comparing it with the optimal mean μ and an upper bound σ on the standard deviation of the item's distribution. We experiment a *take-it-or-leave-it* auction with this reserve price p_D and in this case, given the known distribution and a deterministic auction, we can determine DAPX explicitly by experimenting the expected revenue with ρ_D and the optimal reserve price in the experiments, and DAPX $= \frac{\text{OPT}(F)}{\text{REV}(F)}$. Then compare the experimental DAPX against the theoretical upper bound ρ_D.
First, we will introduce the reserve price p_D proposed by the paper. We start with 2.1

Definition 2.1 (Function ρ)
For any $r \geqslant 0$, let $\rho(r) = \rho_D$ be the unique positive solution of equation

$$\frac{(\rho - 1)^3}{(2\rho - 1)^2} = r^2$$

where $r = \frac{\sigma}{\mu}$.

The proposed *take-it-or-leave-it price* is

$$p_D = \mu \cdot \frac{\rho_D}{2\rho_D - 1} \tag{2.1}$$

where ρ_D is given in Definition 2.1.
We know from Myerson [50] that for single-item settings the optimum revenue can be achieved by a deterministic mechanism by setting a price p and it becomes a single take-it-or-leave-it auction with price p, and therefore the optimal revenue is:

$$\text{OPT}(F) = \sup_{p \geqslant 0} \text{REV}(p; F) = \sup_{p \geqslant 0} p \times (1 - F(p-)) \tag{2.2}$$

where $F(p-) = Pr[x < p]$, we will use this equation to compute the optimal revenue in the following experiments. We denote OPT(F) as Myerson optimal operator. Given a valuation probability distribution, we are able to find a reserve price $p = \arg\max_{p \geqslant 0} \ p \times (1 - F(p-))$, and we denote this price as optimal reserve price p_{opt}. where $r = \frac{\sigma}{u}$ is called *coefficient of variation (CV)*. If we look at the left-hand side expression $\frac{(\rho_D - 1)^3}{(2\rho_D - 1)^2}$ is increasing and goes from 0 at $\rho_D = 1$, and to ∞ at $\rho_D \to \infty$, so that for any non-negative r there is a unique solution $\rho_D \in [1, \infty)$ to the above equation. The paper proposed a reserve price in terms of this ratio for the *take-it-or-leave-it* auction in following formula:

Procedure ?? below shows our design for the auction experiment.

Auction Experiment

procedure AUCTION EXPERIMENT($\mu, \sigma, Probabilitydistribution$)
    $r = \frac{\sigma}{\mu}$
    determine p_{opt} using Myerson optimal operator
    solve ρ_D from function $\rho(r)$ in Definition 2.1
    $p_D = \mu \cdot \frac{\rho_D}{2\rho_D - 1}$
    $n = 100000$ ▷ n can be any large number
    bid_data ← n random numbers generated from given distribution
    REV = 0 and OPT = 0
    for each *bid* in bid_data **do**   ▷ stop until the last bid in bid_data
        if $bid \geqslant p_{opt}$ **then**
            OPT = OPT + p_{opt} ▷ bidders wins and pay with p_{opt} for the item
        if $bid \geqslant p_D$ **then**
            REV = REV + p_D ▷ bidders wins and pay with p_D for the item
    Experimental_DAPX = $\frac{\text{OPT}}{\text{REV}}$
    return Experimental_DAPX ,ρ_D

Find DAPX tentatively

We first pick up an arbitrary uniform distribution, for example a = 2 and b = 10 and then (detailed code can be found in appendix ... or link github)

- we can compute ρ_D numerically using scientific package in python fsolve**(explain what method this function used to solve nonlinear functions) function from scipy. Then substitude it into above equation to compute reserve price p_r.
- Run an auction n times (n can be a large number, i.e.10000000), for each auction, generate a random number x (bid) from U[a,b], compare it against p_r. If the bid bits price p_r, then the bidder wins this auction and the revenue of this auction is p_r

- We sum up the revenue for all the winning auctions and then take average ($expected\_revenue = \frac{total\_revenue}{n}$)

Although we know the exact optimal revenue for this auction, to have a fair comparison to the expected revenue, we should run the above experiment on $p_{opt} = \frac{b}{2}$. In this case, we can find our experiment result DAPX $= \frac{\text{OPT}}{\text{REV}} = \frac{\text{REV}(p=max\{a, \frac{b}{2}\})}{\text{REV}(p=\frac{\rho_D}{2\rho_D - 1} \cdot \mu)}$ for uniform distribution.

Example:show one Experiment

Experiment on r

Now we want to see when r changes how DAPX differs. One idea is that we can fix mean mu and gradually increase $b - a$, because mu if middle point between a and b, at each iteration i, a_i and b_i take a same "step" away this mu, therefore $sigma$ increases meanwhile r increases. Within each step, we have a new uniform distribution, we repeat above experiment for this distribution and dertermine the experiment DAPX. We save the results of DAPX and also ρ_d for each step and plot them in the same graph for comparison. As you can see Figure 0.0 below (we start $a_0 =$ and $b_0 =$, so we have $mu =$ and $sigma =$, and step size is 0.1, and we take 50 iterations, then $a_n =$ and $b_n =$,)

2.1 Uniform distribution

For an uniform distribution $U[a, b], 0 \leqslant a \leqslant b$, we know:

- mean $\mu = \frac{b+a}{2}$ and $\sigma^2 = \frac{(b-a)^2}{12}$
- CDF $F(x) = \frac{x-a}{b-a}$
- PDE $f(x) = \frac{1}{b-a}$

Using Myerson optimal operator, we can write:

$$\text{OPT}(F) = \sup_{p \geqslant 0} \text{REV}(p; F) = \sup_{p \geqslant 0} p \times (1 - \frac{p-a}{b-a})$$

$p \times (1 - \frac{p-a}{b-a})$ is a concave function, therefore, there exists a maximum point. Then we take its first derivative

$$(p \times (1 - \frac{p-a}{b-a}))' = (\frac{pb - p^2}{b-a})' = \frac{b-2p}{b-a}$$

Because $p \in [a, b]$, we need to divide it into two cases.

- Case 1: if $\frac{b}{2} < a$, $\frac{b-2p}{b-a}$ is negative then it means this function is monotone decreasing on $[a, b]$. Thus when $p = a$, we get $\text{OPT}(F)$.

- Case 2: if $\frac{b}{2} \geqslant a$, then the maximum expected revenue can be achieved when $p = \frac{b}{2}$

Combine two cases above, when $p_{opt} = max\{a, \frac{b}{2}\}$, we have $\text{OPT}(F) = p_{opt} \times (1 - \frac{p_{opt}-a}{b-a})$.
For a given uniform distribution, i.e. a and b are known, therefore r^2 is also known: $r^2 = (\frac{\sigma}{u})^2 = \frac{(b-a)^2}{3(b+a)^2}$, then we can determine ρ_D by solving equation in Definition 2.1 and compute reserve price p_D using Equation 2.1, Then our expected revenue

$$\text{REV}(F) = \frac{\rho_D}{2\rho_D - 1} \cdot \mu \cdot (1 - \frac{\frac{\rho_D}{2\rho_D-1} \cdot \mu - a}{b-a})$$

However, we do not determine $\text{OPT}(F)$ and $\text{REV}(F)$ using above expressions during the experiments. We only use their reserve price p_{opt} and p_D during the experiments.

2.1.1 The Bound of r and DAPX

Based on our experiments, we notice that no matter how we change a and b, the *coefficience of variation* r of uniform distribution is way smaller than 1. Which is expected, as uniform distribution is well defined distribution and actually indeed we can write r in terms of a and b explicitly:

$$r = \frac{\mu}{\sigma} = \frac{b-a}{\sqrt{3}(a+b)} = \frac{a+b-2a}{\sqrt{3}(a+b)} = \frac{1}{\sqrt{3}}(1 - \frac{2}{1+\frac{b}{a}})$$

when a = 0. then $r = \frac{1}{\sqrt{3}}$, otherwise, when $b \to \infty$ and $a \leqslant b$, we can have:

$$\sup r = \lim_{\frac{b}{a} \to \infty} (\frac{1}{\sqrt{3}}(1 - \frac{2}{1+\frac{b}{a}})) = \frac{1}{\sqrt{3}}$$

as we can see the CV of uniform distribution is at most $\frac{1}{\sqrt{3}}$.

From equation 2.1, ρ_D is monotonically increasing with r, then ρ_D is also upper bounded. we can compute its upper bound using numerical solver by setting $r = \frac{1}{\sqrt{3}}$ in equation 2.1. We know the equation of computing the reserve price, $p_r = (\frac{a+b}{2}) \cdot \frac{\rho_D}{2\rho_D - 1} = \frac{1}{2 - \frac{1}{\rho_D}} \cdot (\frac{a+b}{2})$.
Now let's write DAPX explicitly, when $\frac{b}{2} \geqslant a$ so $p_{opt} = \frac{b}{2}$:

$$\text{DAPX} = \frac{\text{OPT}(F)}{\text{REV}(F)} = \frac{\frac{b^2}{4(b-a)}}{\frac{\rho_D}{2\rho_D-1} \cdot \mu \cdot (1 - \frac{\frac{\rho_D}{2\rho_D-1} \cdot \mu - a}{b-a})}$$

simplified we get

$$\text{DAPX} = \frac{b^2}{\frac{2\rho_D}{2\rho_D-1} \cdot 2\mu \cdot (b - \frac{\rho_D}{2\rho_D-1} \cdot \mu)}$$

subsititute $b = \sqrt{3}\sigma + \mu$

$$\begin{aligned}
\text{DAPX} &= \frac{(\sqrt{3}\sigma + \mu)^2}{\frac{2\rho_D}{2\rho_D - 1} \cdot 2\mu \cdot (\sqrt{3}\sigma + \mu - \frac{\rho_D}{2\rho_D - 1} \cdot \mu)} \\
&= \frac{\mu^2 \cdot (\sqrt{3}r + 1)^2}{\frac{2\rho_D}{2\rho_D - 1} \cdot 2\mu^2 \cdot (\sqrt{3}r + 1 - \frac{\rho_D}{2\rho_D - 1})} \\
&= \frac{(\sqrt{3}r + 1)^2}{4 \cdot \frac{\rho_D}{2\rho_D - 1} \cdot (\sqrt{3}r + 1 - \frac{\rho_D}{2\rho_D - 1})}
\end{aligned}$$

when $\frac{b}{2} \leqslant a$ so $p_{opt} = a$

$$\text{DAPX} = \frac{\text{OPT}}{\text{REV}} = \frac{a}{\frac{\rho_D}{2\rho_D - 1} \cdot \mu \cdot (1 - \frac{\frac{\rho_D}{2\rho_D - 1} \cdot \mu - a}{b - a})}$$

simplified we get

$$\text{DAPX} = \frac{a(b-a)}{\frac{\rho_D}{2\rho_D - 1} \cdot \mu \cdot (b - \frac{\rho_D}{2\rho_D - 1} \cdot \mu)}$$

subsititute $b = \mu + \sqrt{3}\sigma$ and $a = \mu - \sqrt{3}\sigma$

$$\begin{aligned}
\text{DAPX} &= \frac{2\sqrt{3}\sigma(\mu - \sqrt{3}\sigma)}{\frac{\rho_D}{2\rho_D - 1} \cdot \mu \cdot (\sqrt{3}\sigma + \mu - \frac{\rho_D}{2\rho_D - 1} \cdot \mu)} \\
&= \frac{\mu^2 \cdot r(1 - \sqrt{3}r)}{\mu^2 \cdot \frac{\rho_D}{2\rho_D - 1} \cdot (\sqrt{3}r + 1 - \frac{\rho_D}{2\rho_D - 1})} \\
&= \frac{r(1 - \sqrt{3}r)}{\frac{\rho_D}{2\rho_D - 1} \cdot (\sqrt{3}r + 1 - \frac{\rho_D}{2\rho_D - 1})}
\end{aligned}$$

Below we represent the result in Fig. 2.1, which compares DAPX of uniform distribution against theoretical value ρ_D from the paper with different r values.(show example, set a =1, change r value, since for each r there is a corresponding value of b, which is a valid uniform distribution, then we can get following plot
$\frac{\rho_D}{2\rho_D - 1} = \frac{1}{2 - \frac{1}{\rho_D}}$ as $\rho_D \geqslant 1$ because optimal revenue is always greater than the expected revenue, then $\frac{\rho_D}{2\rho_D - 1} \leqslant 1$. ρ_d is also a function of r, it seems that DAPX is a quadratic equation in terms of r????? not too sure, since I cannot derive the pd using r **note**: another interesting things when a uniform ditribution has $\frac{b}{a} = 2.44224957$, then DAPX $= 1$, by setting $p = p_{opt}$ which is equavlent to $\frac{\rho_D}{2\rho_D - 1} \cdot (\frac{a+b}{2}) = \frac{b}{2}$ then solving equation 2.1.

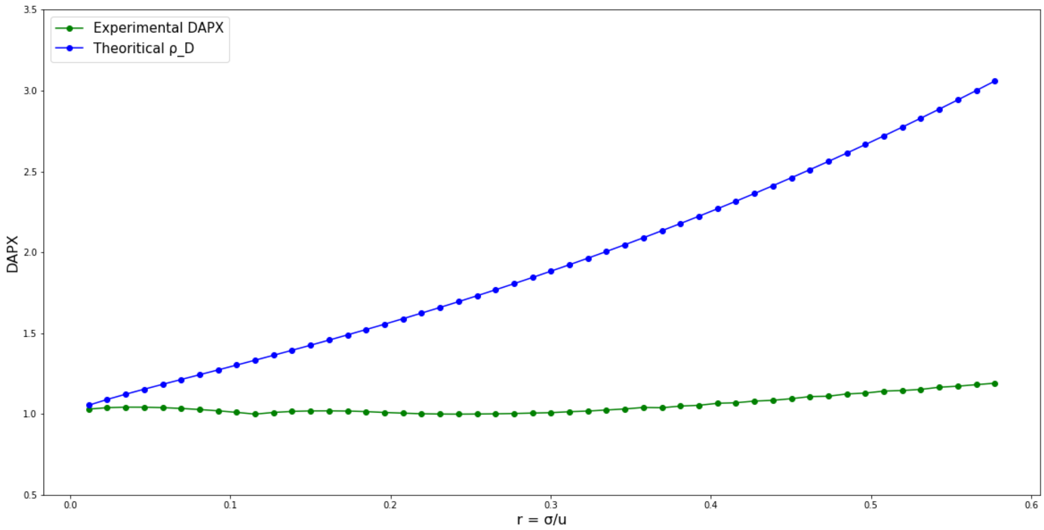


Figure 2.1: DAPX of uniform distribution versus ρ_D

2.2 Exponential and Poisson distribution

These two distributions have very special property that the mean and the standard deviation are the same, which results in constant CV, for exponential distribution $r = \frac{\sigma}{\mu} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}} = 1$ and for poisson distribution $r = \frac{\sigma}{\mu} = \frac{\lambda}{\lambda} = 1$, therefore ρ_D is also constant by solving equation 2.1. Thus from these two distributions we cannot find a useful relation between DAPX and r.

We can still perform some insight of this type distribution, and let us look at exponential distribution for example. For any exponential distribution, we denote as $exp(\lambda)$, its mean and standard deviation are $\frac{1}{\lambda}$, then using Meyerson optimal operator, we can determine the optimal reserve price is $\frac{1}{\lambda}$ and the optimal revenue is $\frac{1}{\lambda e}$. Let's denote $\hat{\rho_D}$ as the the value of ρ_D when $r = 1$, and our expected revenue is $\frac{\hat{\rho_D}}{\lambda(2\hat{\rho_D}-1)}e^{-\frac{\hat{\rho_D}}{(2\hat{\rho_D}-1)}}$, then:

$$
\begin{aligned}
\text{DAPX} &= \frac{\text{OPT}(F)}{\text{REV}(F)} \\
&= \frac{\frac{1}{\lambda \cdot e}}{\frac{\hat{\rho_D}}{\lambda(2\hat{\rho_D}-1)} e^{-\frac{\hat{\rho_D}}{(2\hat{\rho_D}-1)}}} \\
&= \frac{1}{\frac{\hat{\rho_D}}{(2\hat{\rho_D}-1)} e^{1-\frac{\hat{\rho_D}}{(2\hat{\rho_D}-1)}}}
\end{aligned}
$$

From above expression we can see DAPX is indepedent of λ, and it is a constant as well. we will not explore any futher on these two distributions.

2.3 Truncated Normal distribution

Another interesting probability distribution to explore is the truncated normal distribution. The definition of truncated normal distribution is: suppose X is from a normal distribution with mean $\hat{\mu}$ and variance $\hat{\sigma}^2$ and lies within the interval $[a, b]$, then X conditional on $a \leqslant X \leqslant b$ has a truncated normal distribution $\text{TN}(\hat{\mu}, \hat{\sigma^2}, a, b)$. Here we assume $\hat{\mu} \geqslant 0$, because if $\hat{\mu} < 0$, that means the density function of truncated normal distribution is monotone decreasing, which means we have high probability of low valuation bidders and low probability of high valuation bidders. This kind of characteristics can be captured by another probability distribution which we introduce in the next section called Pareto distribution. To notice here $\hat{\mu}$ and $\hat{\sigma}$ are the mean and standard deviation of the normal distribution before the truncation, and we denote μ and σ are the mean and standard deviation of the truncated normal distribution. We also assume the truncated range is $[0, \infty)$, thus set $a = 0$ and $b = \infty$. The corresponding PDF and CDF of $\text{TN}(\hat{\mu}, \hat{\sigma}^2, 0, \infty)$: (using the notation from [KJB94]):

$$
f_t(x) = \begin{cases} \frac{1}{\hat{\sigma}} \frac{\phi(\frac{x-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} & \text{if } x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}
$$

$$
F_t(x) = \begin{cases} \frac{\Phi(\frac{x-\hat{\mu}}{\hat{\sigma}})-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} & \text{if } x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}
$$

where $\Phi(\cdot)$ the cumulative distribution function and $\phi(\cdot)$ the probability density function of the standard normal distribution, and the mean and variance of $\mathrm{TN}(\hat{\mu}, \hat{\sigma}^2, 0, \infty)$

$$E(X|X \geqslant 0) = \mu = \hat{\mu} + \frac{\hat{\sigma}\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1 - \Phi(\frac{-\mu}{\hat{\sigma}})}$$

$$Var(X|X \geqslant 0) = \sigma^2 = \hat{\sigma}^2 (1 + \frac{\frac{-\hat{\mu}}{\hat{\sigma}}\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1 - \Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} - (\frac{\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1 - \Phi(\frac{-\hat{\mu}}{\hat{\sigma}})})^2)$$

then our r for a given $\mathrm{TN}(\hat{\mu}, \hat{\sigma}^2, 0, \infty)$

$$r = \frac{\sigma}{\mu} = \frac{\hat{\sigma}\sqrt{1 + \frac{\frac{-\hat{\mu}}{\hat{\sigma}}\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} - (\frac{\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})})^2}}{\hat{\mu} + \frac{\hat{\sigma}\phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\mu}{\hat{\sigma}})}}$$

To simplify the expression, we denote $\hat{h}(x) = \frac{\phi(x)}{1-\Phi(x)}$ Then we get:

$$\begin{aligned} r &= \frac{\hat{\sigma}\sqrt{1 - \frac{\hat{\mu}}{\hat{\sigma}}\hat{h}(-\frac{\hat{\mu}}{\hat{\sigma}}) - \hat{h}(-\frac{\hat{\mu}}{\hat{\sigma}})^2}}{\hat{\mu} + \hat{\sigma}\hat{h}(-\frac{\hat{\mu}}{\hat{\sigma}})} \\ &= \frac{\sqrt{1 - \frac{\hat{\mu}}{\hat{\sigma}}\hat{h}(-\frac{\hat{\mu}}{\hat{\sigma}}) - \hat{h}(-\frac{\hat{\mu}}{\sigma})^2}}{\frac{\hat{\mu}}{\hat{\sigma}} + \hat{h}(-\frac{\hat{\mu}}{\hat{\sigma}})} \end{aligned}$$

Let's set $y = -\frac{\hat{\mu}}{\hat{\sigma}} \in (-\infty, 0]$, then

$$r = \frac{\sqrt{1 + y\hat{h}(y) - \hat{h}(y)^2}}{-y + \hat{h}(y)}$$

If y increases then $\phi(y)$ increases, and $1 - \Phi(y)$ decreases, thus $\hat{h}(y)$ is monotone increasing on $(-\infty, 0]$. We can investigate the upper bound and lower bound for $\hat{h}(y)$ on $(-\infty, 0]$: when $y = 0$, $\hat{h}(0) = \frac{\phi(0)}{1-\Phi(0)} = 2\phi(0)$ $(2\phi(0) \approx)$; when $y \to -\infty$, $\hat{h}(-\infty) = \frac{0}{1-0} = 0$. We find out that $0 \leqslant \hat{h}(y) \leqslant 2\phi(0)$. Now we consider two extreme cases

- Case 1: when $y \to -\infty$ then:

$$
\begin{aligned}
r &= \lim_{y\to-\infty} \frac{\sqrt{1 + y\hat{h}(y) - \hat{h}(y)^2}}{-y + \hat{h}(y)} \\
&= \lim_{y\to-\infty} \sqrt{\frac{1 + y\hat{h}(y) - \hat{h}(y)^2}{\left(-y + \hat{h}(y)\right)^2}} \\
&= \lim_{y\to-\infty} \sqrt{\frac{1 - \hat{h}(y)(\hat{h}(y) - y)}{\left(\hat{h}(y) - y\right)^2}} \\
&= \lim_{y\to-\infty} \sqrt{\frac{1 - \hat{h}(y)}{\hat{h}(y) - y}} \\
&= \sqrt{\frac{1 - \hat{h}(-\infty)}{\hat{h}(-\infty) - (-\infty)}} \\
&= \sqrt{\frac{1 - 0}{0 + \infty}} \\
&= 0
\end{aligned}
$$

- Case 2: when $y \to 0$ then:

$$
\begin{aligned}
r &= \lim_{y\to 0} \frac{\sqrt{1 + y\hat{h}(y) - \hat{h}(y)^2}}{-y + \hat{h}(y)} \\
&= \frac{\sqrt{1 + 0 \cdot \hat{h}(0) - \hat{h}(0)^2}}{-0 + \hat{h}(0)} \\
&= \frac{\sqrt{1 - 4\phi(0)^2}}{2\phi(0)}
\end{aligned}
$$

If we plot r in terms of y, as we can see from Fig. 2.2, r is monotone increasing. Then r is upper bounded when $y = 0$. Since we know $\phi(0) \approx 0.3989$, substitute it into Case 2 equation, the upper bound of $r \approx 0.7555$.
To find optimal revenue for truncated normal distribution, we consider an alternative way by using *virtual valuation*. Previously we compute the optimal reserve price by maximize the expected revenue formula, because of the simplicity of uniform distribution, the equation of expected revenue is concave and simple to compute the derivative. However for truncated normal distribution, it is not easy to programme this way, therefore based on Myerson optimal auction the optimal reserve price is the

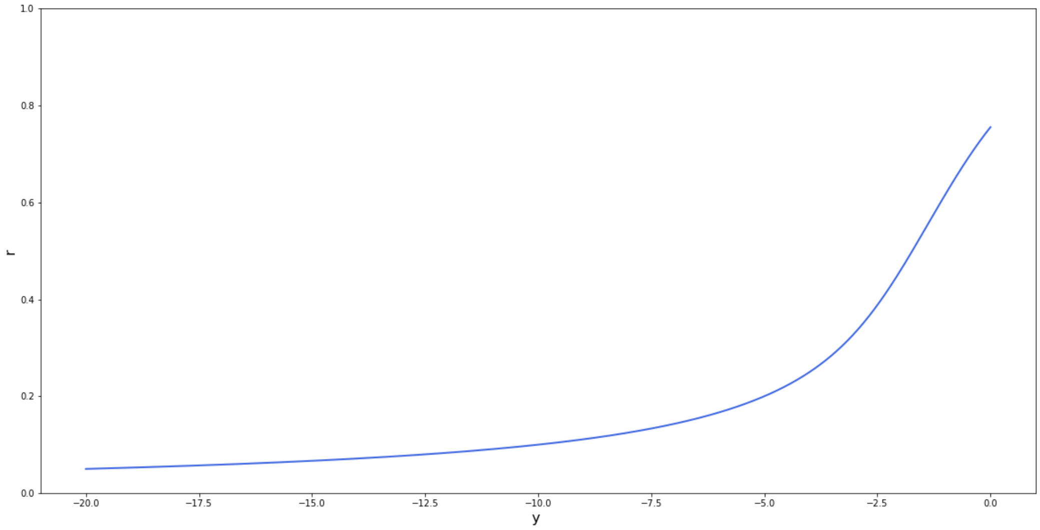


Figure 2.2: The relation between r and y

price which makes the *virtual valuation* equal 0. The *virtual valuation* is defined as
Definition 2.2 The *virtual valuation* of bidder i with valuation v_i is

$$\psi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$$

In our case, we only have one bidder, so just use v instead of v_i, then we can write $\psi(v) = v - \frac{1-F_t(v)}{f_t(v)}$, and the optimal reserve price $p_{opt} = \psi^{-1}(0)$. During the experiments, we use numerical solver to solve this equation $\psi(v) = 0$ to get p_{opt}.

2.3.1 Truncated Normal distribution is MHR

One assumption of Myerson optimal auction is that the *virtual valuation* needs to be regular, which means the *virtual valuation* is non-decreasing in values. Thus we need to prove the regularity of the *virtual valuation* for truncated normal distribution and to prove the regularity we can also prove that the truncated normal distribution is a MHR distribution, because MHR implies regularity. Below we show our proof for $\mathrm{TN}(\hat{\mu}, \hat{\sigma}^2, 0, \infty)$ is MHR distribution,

Proof. Hazard rate for this truncated normal distribution is $h(v) = \frac{f_t(v)}{1-F_t(v)}$, to prove it is non-decreasing, we can take its first derivative and see if the first derivative is nonnagative or not (here we assume $v \geqslant 0$):

$$\begin{aligned} h^{'}(v) &= \frac{f_t^{'}(v)}{1-F_t(v)} + \frac{f_t(v) \cdot F_t^{'}(v)}{(1-F_t(v))^2} \\ &= \frac{f_t^{'}(v)}{1-F_t(v)} + \frac{f_t^2(v)}{(1-F_t(v))^2} \\ &= \frac{f_t^{'}(v)(1-F_t(v)) + f_t^2(v)}{(1-F_t(v))^2} \end{aligned}$$

Clearly the denominator is nonnegative, so we only need to check the nominator, also $f_t(v) = \frac{1}{\hat{\sigma}} \frac{\phi(\frac{v-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})}$ and $F_t(v) = \frac{\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})}$,then:

$$\begin{aligned} f_t^{'}(v)(1-F_t(v)) + f_t^2(v) &= \frac{1}{\hat{\sigma}} \frac{-\frac{v-\hat{\mu}}{\hat{\sigma}^2}\phi(\frac{v-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} \cdot \frac{1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})}{1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}})} + \frac{\phi^2(\frac{v-\hat{\mu}}{\hat{\sigma}})}{\hat{\sigma}^2(1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}}))^2} \\ &= \frac{-\frac{v-\hat{\mu}}{\hat{\sigma}} \cdot \phi(\frac{v-\hat{\mu}}{\hat{\sigma}}) \left(1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})\right) + \phi^2(\frac{v-\hat{\mu}}{\hat{\sigma}})}{\hat{\sigma}^2(1-\Phi(\frac{-\hat{\mu}}{\hat{\sigma}}))^2} \end{aligned}$$

Now again we only need to prove the nominator is nonnegative or not. We consider it into two cases:

$$-\frac{v-\hat{\mu}}{\hat{\sigma}} \cdot \phi(\frac{v-\hat{\mu}}{\hat{\sigma}}) \left(1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})\right) + \phi^2(\frac{v-\hat{\mu}}{\hat{\sigma}})$$

- Case 1: if $v \leqslant \hat{\mu}$, then $-\frac{v-\hat{\mu}}{\hat{\sigma}} \geqslant 0$,
  we know $\phi(\frac{v-\hat{\mu}}{\hat{\sigma}}), 1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})$ is nonnegative, then $-\frac{v-\hat{\mu}}{\hat{\sigma}} \cdot \phi(\frac{v-\hat{\mu}}{\hat{\sigma}})(1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})) + \phi^2(\frac{-\hat{\mu}}{\hat{\sigma}})$ is nonnegative

- Case 2: if $v > \hat{\mu}$, then $\frac{v-\hat{\mu}}{\hat{\sigma}} > 0$ and set $x = \frac{v-\hat{\mu}}{\hat{\sigma}}$,
  we know that Q-funtion is defined as $Q(x) = 1 - \Phi(x)$, and it is bounded when $x > 0$,

  $$\left(\frac{x}{1+x^2}\right)\phi(x) < Q(x) < \frac{\phi(x)}{x}$$

  thus
  $-\frac{v-\hat{\mu}}{\sigma} \cdot \phi(\frac{v-\hat{\mu}}{\hat{\sigma}})(1-\Phi(\frac{v-\hat{\mu}}{\hat{\sigma}})) + \phi^2(\frac{v-\hat{\mu}}{\hat{\sigma}})$
  $= -x\phi(x)(1-\Phi(x)) + \phi^2(x)$
  $= -x\phi(x)Q(x) + \phi^2(x) > -x\phi(x) \cdot \frac{\phi(x)}{x} + \phi^2(x) = -\phi^2(x) + \phi^2(x) = 0$
  the expression is nonnegative.

In both case, we prove that the nominator is nonnegative, thus the first derivative of truncated normal hazard rate is nonnegative, which means its hazard rate is monotone non-decreasing, and truncated normal distribution is a MHR distribution. Regularity assumption satisfied. □

2.3.2 Result of experiment: TBC

We experiment different truncated normal distributions with different r values, and evaluate the corresponding DAPXs. In the implementation, we need to assign values to $\hat{\mu}, \hat{\sigma}$, in order to have different distributions, first we can set either of them to a fix value, in our case, we set $\hat{\mu}$ fixed and gradually increase $\hat{\sigma}$. As we derive in previous section, r is a function of $\frac{\hat{\sigma}}{\hat{\mu}}$, only related to the ratio of $\hat{\mu}$ and $\hat{\sigma}$, so without loss we can set $\hat{\mu} = 1$. We plot the DAPX of truncated normal distributions and the theoretical ρ_D in two plots: the left plot with small values of r and the right with lager values of r. Fig. 2.3 shows how these two values increase with r increases. ρ_D increases really fast while the experimental DAPX remains small. Since we gradually increase $\hat{\sigma}$, we can see the increment in Fig. 2.3 is not uniform. Thus we also plot the results with parameter $\hat{\sigma}$. The left figure in Fig. 2.4 shows the results with small $\hat{\sigma}$, while the right one is with larger $\hat{\sigma}$. Previously we find out r is upper bounded, therefore ρ_D is also upper bounded, and we can observe a clear convergence in Fig. 2.3 and Fig. 2.4. We can also notice that the DAPX of truncated normal is also bounded. Our experiment shows the upper bound of DAPX of truncated normal is approximate 1.1522 for $r = 0.7555$.

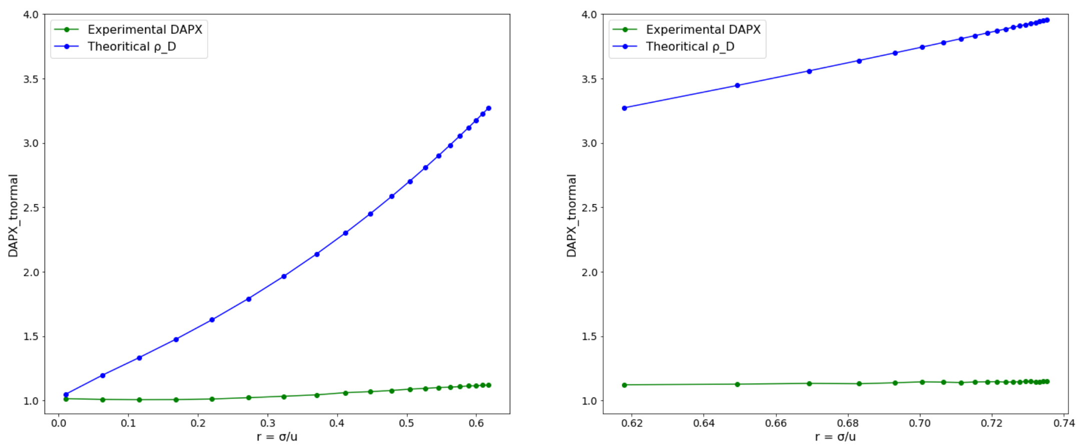


Figure 2.3: DAPX of truncated normal distribution versus ρ_D with different r values

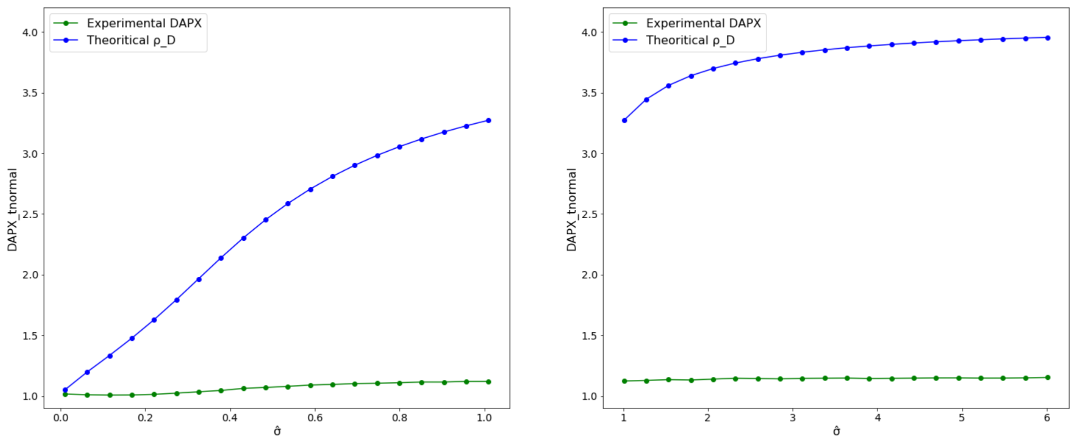


Figure 2.4: DAPX of truncated normal distribution versus ρ_D with different $\hat{\sigma}$ values

2.4 Pareto distribution

Here we also want to consider Pareto distribution. This distribution can possible be used in description of auction data when most bids are consetrated at lower values while high bid is hardly to occur.
Pareto distribution is defined by two parameters $x_m > 0, c > 0$ and $x \in [x_m, \infty]$. To simplify, we choose the scale parameter $x_m = 1$ and denote this Pareto distribution as Pareto(1) due to its support range. Next we can write corresponding PDF and CDF as following

$$f(x) = \frac{c}{x^{c+1}}$$

and

$$F(x) = 1 - \frac{1}{x^c}$$

Another special property of Pareto distribution is that when parameter $c \leqslant 1$, it does not have a mean (the mean is infinite), in addition, when $c \leqslant 2$ its variance is infinite too. Thus in order to have a valid r, we assume the parameter $c > 2$. Then the mean and standard deviation of this Pareto distribution are

$$\mu = \tfrac{c}{c-1} \qquad\qquad \sigma = \sqrt{\tfrac{c}{(c-1)^2(c-2))}}$$

We first determine the optimal reserve price for this Pareto distribution. We use Myersons optimal operator again

$$\text{OPT}\,(F) = \sup_{p \geqslant 1} \text{REV}(p; F) = \sup_{p \geqslant 1} p \cdot \left(1 - \left(1 - \frac{1}{p^c}\right)\right) = \sup_{p \geqslant 1} \frac{1}{p^{c-1}}$$

since $c > 2$, then $\frac{1}{p^{c-1}}$ is monotone decreasing. When reserve price $p = 1$, we can achieve the maximum expected revenue, thus $\text{OPT}(F) = 1$. Now let's check whether r is bounded or not

$$r = \frac{\sigma}{\mu} = \frac{\sqrt{\frac{c}{(c-1)^2(c-2))}}}{\frac{c}{c-1}} = \frac{1}{\sqrt{c(c-2)}}$$

As we can see from the expression, when $c \to 2$, $r \to \infty$, thus r is unbounded and our theoretical bound ρ_D is also unbounded. However our experiment shows the DAPX of the Pareto distribution is not unbounded, as we can see figure....., so we write down the expression for DAPX explicitly:

$$\begin{aligned}
\text{DAPX} = \frac{\text{OPT}(F)}{\text{REV}(F)} &= \frac{1}{p_D(1-F(p_D))} = \frac{1}{p_D\left(1-\left(1-\frac{1}{p_D^c}\right)\right)} \\
&= p_D^{c-1} = (\mu\frac{\rho_D}{2\rho_D - 1})^{c-1} = \left(\frac{c}{c-1}\cdot\frac{1}{2-\frac{1}{\rho_D}}\right)^{c-1} \\
&= \left((1+\frac{1}{c-1})\cdot\frac{1}{2-\frac{1}{\rho_D}}\right)^{c-1}
\end{aligned}$$

From the expression, when $c \to 2$, we know $r \to \infty$ and $\rho_D \to \infty$ also, then $\text{DAPX} \to 1$; when $c \to \infty$, and $r \to 0$ and $\rho_D \to 1$, so $\text{DAPX} \to 1$ also. This matches the results in both Fig. 2.6 and Fig. 2.7. Fig. 2.5 shows the experimental DAPX comparing to ρ_D, as we can see, the DAPX of Pareto(1) remains small while ρ_D increases exponentially. Fig. 2.6 shows the experimental DAPX by itself, as we can see, the DAPX remains around 1 and we observe a clear upper bound. To see more details how DAPX values under small r values, we run the experiment with 50 steps at range (0, 2) of r. Results are represented in Fig. 2.7, and from the experiment the upper bound of Pareto(1) $\text{DAPX} \approx 1.1416$ to 4 decimals for $r = 0.2700$.

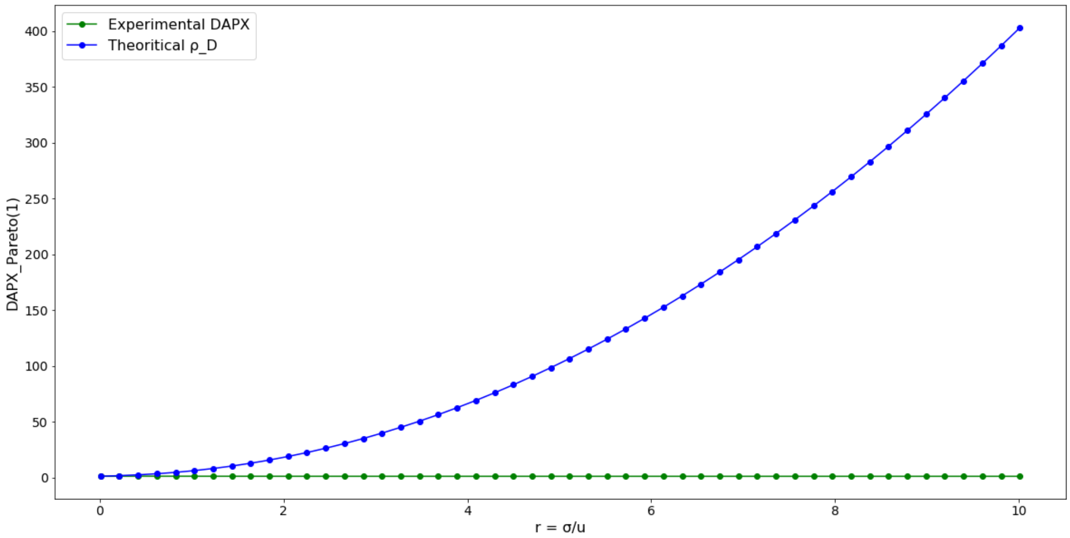


Figure 2.5: DAPX of Pareto(1) distribution versus ρ_D

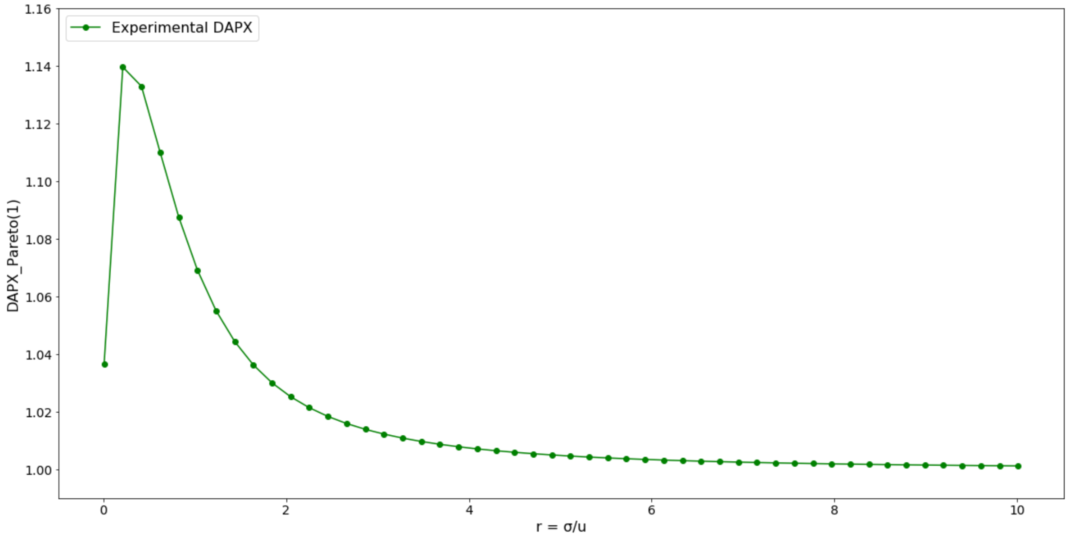


Figure 2.6: DAPX of Pareto(1) distribution with r

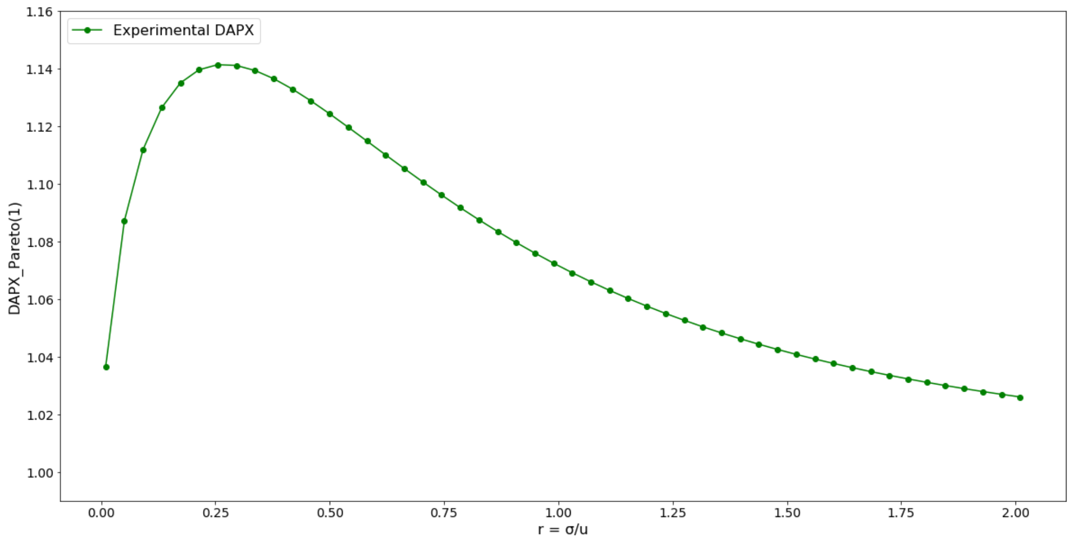


Figure 2.7: DAPX of Pareto(1) distribution when r remains small

2.4.1 Pareto Extension

We usually consider all the valuation is non-negative, therefore we would like to modify the previous Pareto distribution to have support $[0, \infty)$, which means shifting the distribution to the left by 1. Let's denote this distribution as Pareto(0) and its PDF and CDF are

$$f(x) = \frac{c}{(x+1)^{c+1}}$$

and

$$F(x) = 1 - \frac{1}{(x+1)^c}$$

Below we make sure it is a valid distribution:

$$\int_0^\infty f(x)dx = 1 - \frac{1}{(x+1)^c}\bigg|_0^\infty = 1 - 0 - (1 - \frac{1}{1}) = 1$$

$$F(0) = 1 - \frac{1}{(0+1)^c} = 0$$

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} 1 - \frac{1}{(x+1)^c} = 1$$

We can derive mean and standard deviation of Pareto(0) from Pareto(1). We denote $E[X], Var[X]$ are mean and variance of Pareto(1) , then $E[X-1], Var[X-1]$ is the mean and variance for Pareto(0). We get $E[X-1] = E[X]-1$ and $Var[X-1] = Var[X]$, thus from this relation, we can write down the formula for μ, σ

$$\mu = \tfrac{1}{c-1} \qquad \text{and} \qquad \sigma = \sqrt{\tfrac{c}{(c-1)^2(c-2))}}$$

This time $p_{opt} = \underset{p \geqslant 0}{\arg\max}\ p(1-(1-\dfrac{1}{(p+1)^c})) = \underset{p \geqslant 0}{\arg\max}\ \dfrac{p}{(p+1)^c}$, to find p_{opt}, we take the first derivative of $\frac{p}{(p+1)^c}$, and check if the maximum exists or not.

$$\frac{p}{(p+1)^c}' = \frac{1}{(p+1)^c} - \frac{pc}{(p+1)^{c+1}} = \frac{1-p(c-1)}{(p+1)^{c+1}}$$

Since $p \geqslant 0, c > 2$, when $p < \frac{1}{c-1}$, then $\frac{p}{(p+1)^c}' > 0$, which means $\frac{p}{(p+1)^c}$ monotonously increases at $[0,\frac{1}{c-1})$, while $p > \frac{1}{c-1}$, then $\frac{p}{(p+1)^c}' < 0$ means $\frac{p}{(p+1)^c}$ monotonously decreases at $(\frac{1}{c-1},\infty)$. Therefore the maximum value can be achieved when $p = \frac{1}{c-1}$. Then $p_{opt} = \frac{1}{c-1}$, we can also write down the optimal revenue:

$$\mathrm{OPT}(F) = \frac{1}{c-1} \cdot \frac{1}{(\frac{1}{c-1}+1)^c} = \frac{(c-1)^{c-1}}{c^c}$$

and r

$$r = \frac{\sigma}{\mu} = \frac{\sqrt{\frac{c}{(c-1)^2(c-2)}}}{\frac{1}{c-1}} = \sqrt{\frac{c}{c-2}} = \sqrt{\frac{1}{1-\frac{2}{c}}}$$

We can get similar conclusion as before: if $c \to 2$, $r \to \infty$; if $c \to \infty$, $r \to 1$. Therefore r is also unbounded in this case. Then we write down DAPX for Pareto(0) explicitly

$$\mathrm{DAPX} = \frac{\mathrm{OPT}(F)}{\mathrm{REV}(F)} = \frac{\frac{(c-1)^{c-1}}{c^c}}{p_D(1-F(p_D))}$$

$$= \frac{(c-1)^{c-1}}{c^c} \cdot \frac{(p_D+1)^c}{p_D}$$

where $p_D = \frac{1}{c-1} \cdot \frac{\rho_D}{2\rho_D - 1}$, from above expression, it is hard to see if an upper bound for DAPX of Pareto(0) exists or not. If we plot above expression, from Fig. 2.8, we can see DAPX is a function of r which is monotone decreasing, so it is bounded when $r \to 1$. We can find this upper bound using the experiment by setting parameter c to a very large number, so $r \to 1$, and our result shows the Pareto(0) DAPX is 1.1638 to 4 decimal places for $c = 10^{10}$.

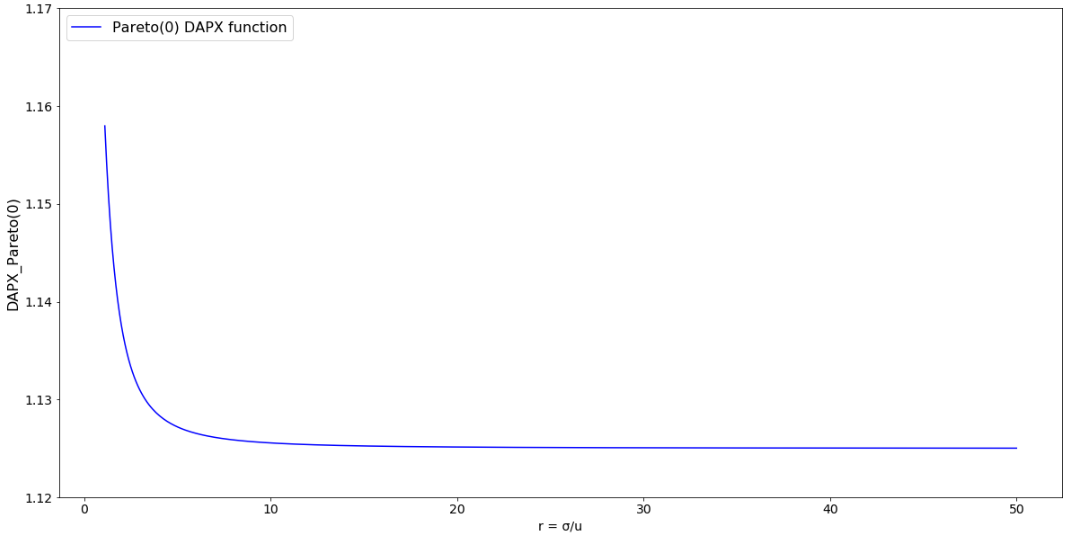


Figure 2.8: Pareto(0) DAPX function

2.4.2 Result

First we show the results of the experimental DAPX and ρ_D with $r \in (0, 10)$. From Fig. 2.9, as we can see ρ_D increase exponentially with r, while the experimental DAPX remain small(around 1). In the second Fig. 2.10, we can see more details of these two values when $r \to 1$.

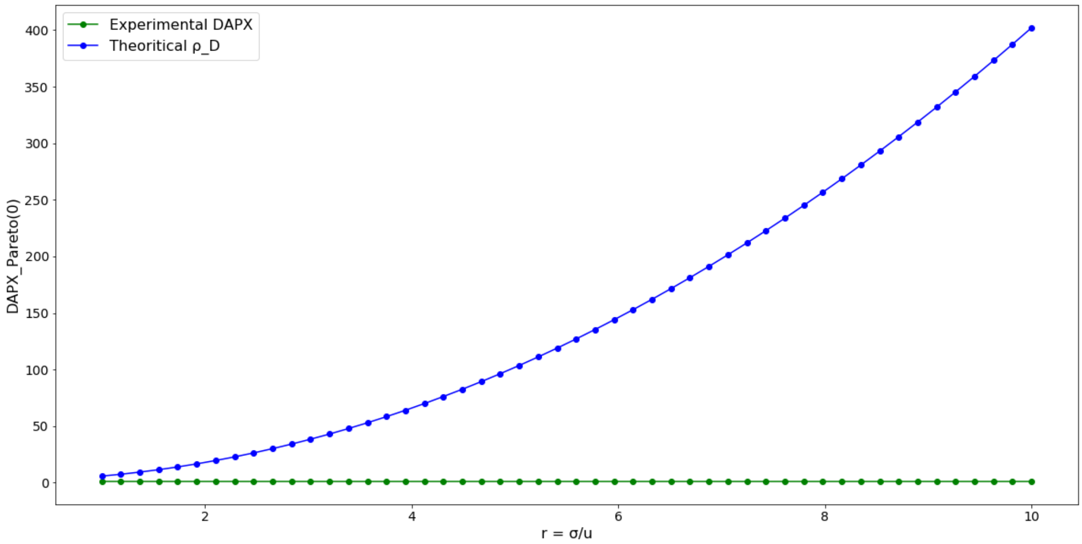


Figure 2.9: Pareto(0) DAPX when $r \in (0, 10)$

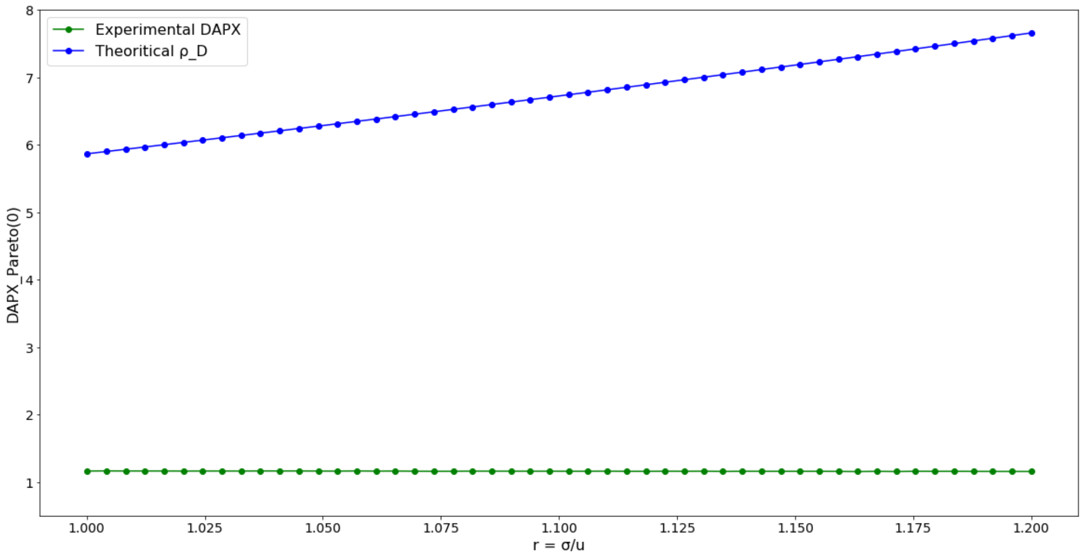


Figure 2.10: Pareto(0) DAPX when $r \in (1, 1.2)$

2.5 Summary of Deterministic SBSI Auction

From above evaluation on different distributions, we notice that for most of these distributions, r is upper bounded, which also means, our theoretical ρ_D and our experimental DAPX have also an upper bound. Table 2.1 represent our foundlings from our evaluation(keep 4 decimals):

Distribution	**r upper bound**	**Theoritical ρ_D upper bound**	**Experimental DAPX upper bound**
Uniform distribution	$\frac{1}{\sqrt{3}}$	3.0593	1.1919
Truncated normal distribution	$\frac{\sqrt{1-4\phi(0)^2}}{2\phi(0)}$	4.0836	1.1511
Pareto distribution ($x \in [1, \infty)$)	∞	∞	1.1416
Pareto distribution ($x \in [0, \infty)$)	∞	∞	1.1638

Table 2.1: Upper bound table

For uniform distribution, we discuss its DAPX in Section 2.1.1. There are two cases, since here we are discuss the upper bound of r, r can be $\frac{1}{\sqrt{3}}$ when $\frac{b}{a} \to \infty$, therefore in this case, $\frac{b}{2} \geqslant a$, then DAPX $= \frac{(\sqrt{3}r+1)^2}{4 \cdot \frac{\rho_D}{2\rho_D - 1} \cdot (\sqrt{3}r + 1 - \frac{\rho_D}{2\rho_D - 1})}$. We subsititute $r = \frac{1}{\sqrt{3}}$ and $\rho_D = 3.5937594$ into this equation, we can compute the upper bound for DAPX of uniform distribution, which is 1.19311999. Our experimental number in Table 2.1 above matches this number. We can also explicitly compute the DAPX upper bound for truncated normal distribution using following equation:

$$\text{DAPX} = \frac{\text{OPT}}{\text{REV}} = \frac{p_{opt}(1 - F_t(p_{opt}))}{p_r(1 - F_t(p_r))}$$

where p_{opt} and p_r can be determined explicitly with $r = 0.7555106397628669$. Then $\sup \text{DAPX} = 1.15149945$, our experimental values matches this number.

Chapter 3

Single-bidder and Single-item Randomization Auction

In this chapter, we evaluate ratio under different probability distribution under single-bidder and single-item randomization mechanism. The paper considers a log-lottery randomization on reserve price which is proposed by Carrasco et al. [paper reference]:

Definition 3.1 (Log-Lottery)
Fix any $\mu > 0$ and $\sigma \geqslant 0$. A log-lottery is a randomized mechanism that sells at a price $P^{log}_{\mu,\sigma}$, which is distributed over the nonnegative interval support $[\pi_1, \pi_2]$ according to the cdf

$$F^{log}_{\mu,\sigma}(x) = \frac{\pi_2 \ln \frac{x}{\pi_1} - (x - \pi_1)}{\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1)}$$

where parameters π_1, π_2 are the (unique) solutions of the system

$$\begin{cases} \pi_1(1 + \ln \frac{\pi_2}{\pi_1}) = \mu \\ \pi_1(2\pi_2 - \pi_1) = \mu^2 + \sigma^2 \end{cases} \tag{3.1}$$

We still perform a take-it-or-leave-it auction, but this time with randomised reserve price. Our experiment steps as follow:

stepssssss

3.1 Log-Lottery Randomization

In order to draw a random price from log lottery distribution defined in Definition 3.1, we need to use a rejection sampling technique. We shortly introduce here. First we need to find a valid upper bound, usually also called envelop of this distribution. Therefore we derive the PDF

$$f^{log}_{\mu,\sigma}(x) = \frac{\pi_2 \frac{1}{x} - 1}{\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1)}$$

$f^{log}_{\mu,\sigma}(x)$ is monotone decreasing on $[\pi_1, \pi_2]$, so $max(f^{log}_{\mu,\sigma}(x)) = f^{log}_{\mu,\sigma}(\pi_1)$, then we can propose an envelope density for rejection sampling algorithm

$$q(x) = \begin{cases} \frac{1}{\pi_2 - \pi_1} & \text{if } x \in [\pi_1, \pi_2] \\ 0 & \text{otherwise} \end{cases}$$

Then a valid envelope constant can be: $A = f^{log}_{\mu,\sigma}(\pi_1) \cdot (\pi_2 - \pi_1)$, since $A \cdot q(x) \geqslant f^{log}_{\mu,\sigma}(x), \forall x \in [\pi_1, \pi_2]$.

Algorithm 1 rejection sampling algorithm

procedure REJECTION SAMPLING(for bounded density on bounded support)
  $n \leftarrow 0$
  while $n \leqslant N$ **do** ▷ N is sample size
    draw $x \sim q(x)$
    compute acception probability $a := \frac{f^{log}_{\mu,\sigma}(x)}{Aq(x)}$
    draw a random variable $u \sim U[0, 1]$
    if $u \leqslant a$ **then**
      accept x, add it to x_{list}
      $n \leftarrow n + 1$
  return x_{list}

3.2 Uniform distribution

Now the robust reserve price $P^{log}_{\mu,\sigma}$ is distributed over the nonnegative interval support $[\pi_1, \pi 2]$ and we assume bidder valuation is follow a uniform distribution $U[a, b]$. Then using a randomized mechanism, our expected revenue

$$\begin{aligned}
\text{REV}(P^{log}_{\mu,\sigma}; U[a, b]) &= \mathop{\mathbb{E}}_{p \sim F^{log}_{\mu,\sigma}} [p(1 - F_{uniform}(p))] \\
&= \int_{\pi_1}^{\pi_2} p(1 - \frac{p-a}{b-a}) f^{log}_{\mu,\sigma}(p) dp \\
&= \int_{\pi_1}^{\pi_2} p(1 - \frac{p-a}{b-a}) \cdot \frac{\pi_2 \frac{1}{p} - 1}{\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1)} dp \\
&= \int_{\pi_1}^{\pi_2} \frac{p(b-p)}{b-a} \cdot \frac{\pi_2 - p}{p(\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1))} dp \\
&= \frac{1}{(b-a)(\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1))} \int_{\pi_1}^{\pi_2} (b-p)(\pi_2 - p) dp \\
&= \frac{1}{(b-a)(\pi_2 \ln \frac{\pi_2}{\pi_1} - (\pi_2 - \pi_1))} \cdot \left(\frac{p^3}{3} - \frac{bp^2}{2} - \frac{\pi_2 p^2}{2} + b\pi_2 p \Big|_{\pi_2}^{\pi_1} \right)
\end{aligned}$$

As we can see, the expected revenue can be determined explicitely, we also know that the optimal revenue for a given uniform distribution will be $\mathrm{OPT}(F_{uniform}) = \frac{b^2}{4(b-a)}$. Then $\mathrm{APX} = \frac{\mathrm{OPT}(F_{uniform})}{\mathrm{REV}(P^{log}_{\mu,\sigma};F_{uniform})}$

3.3 Normal distribution

Similarly the expected revenue of a truncated normal distribution $N_t(\mu, \sigma^2)$

$$
\begin{aligned}
\mathrm{REV}(P^{log}_{\mu,\sigma}; N_t(\mu,\sigma^2)) &= \mathop{\mathbb{E}}_{p\sim F^{log}_{\mu,\sigma}}[p(1-F_{N_t}(p))] \\
&= \int_{\pi_1}^{\pi_2} p(1-k(F_N(p)-F_N(0)))f^{log}_{\mu,\sigma}(p)dp \\
&= \int_{\pi_1}^{\pi_2} p(1-k(F_N(p)-F_N(0)))\cdot \frac{\pi_2-p}{p(\pi_2\ln\frac{\pi_2}{\pi_1}-(\pi_2-\pi_1))}dp \\
&= \frac{1}{\pi_2\ln\frac{\pi_2}{\pi_1}-(\pi_2-\pi_1)}\int_{\pi_1}^{\pi_2}(1-k(F_N(p)-F_N(0)))(\pi_2-p)dp
\end{aligned}
$$

π_1, π_2 are the (unique) solutions of the system:

$$
\begin{cases}
\pi_1(1+\ln\frac{\pi_2}{\pi_1}) = \mu \\
\pi_1(2\pi_2-\pi_1) = \mu^2+\sigma^2
\end{cases}
$$

however $\exists \mu, \sigma$ that the above system has no solutions. For example we can write:

$$
\begin{cases}
\pi_2 = \pi_1 e^{\frac{\mu}{\pi_1}-1} \\
\pi_2 = \frac{\frac{\mu^2+\sigma^2}{\pi_1}+\pi_1}{2}
\end{cases}
$$

If there is a solution for π_2 then by ploting above two equations, there should be an intersection, but it seems when r increases, there is no solution for the system.

3.4 Observation from experiment

As we can see from the plots above for both uniform distribution and truncated normal distribution, we can see DAPX is always smaller than APX, which means for these two distribution and for one bidder and one items, running deterministic robust auction mechanism is better than running log lottery randomization.

However here also comes our questions: in general, randomization is better than deterministic mechanism. Here we compared one deterministic mechanism with one randomization mechanism given a certain valuation distribution. In the paper, APX is the ratio under worst case (worst probability distribution) with log lottery randomization.

1.Randomization is better than deterministic, but given a valuation distribution, using one specific randomization may not be better. Thus whether log lottery is the best randomization distribution. How about using uniform or truncated normal distribution for price randomization? Especially is log lottery is the best for uniform and truncated normal distribution? 2.During the experiment, we use rejection sampling to generate random log lottery price, is the sample size sufficient enough? 3.A better deterministic mechanism can result a better approximate robust ratio than a specific randomization mechanism, i.e. better than log lottery randomization. However in worse case, APX is always better than DAPX.

3.5 Uniform Randomization

3.6 Truncated Normal Randomization

Chapter 4

Additional notes

4.1 truncated normal distribution part

$$f_t(x) = \begin{cases} kf(x) & \text{if } x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

where k is a normalizing constant. We can determine k by using the knowledge of the sum of the probablity equals to 1. Then we have:

$$1 = \int_R f_t(x)dx = \int_0^\infty kf(x)dx = k \cdot \int_0^\infty f(x)dx$$
$$\implies k = \frac{1}{\int_0^\infty f(x)dx} = \frac{1}{1-F(0)}$$

Here $x \geqslant 0$, then the coresponding cdf is:

$$F_t(x) = \int_0^x kf(t)dt = k \cdot \int_0^x f(t)dt = k(F(x) - F(0))$$

question to answer:
1.two random variables, X,Y, let Y = c*X where c>0 constant. let F_X, f_X be the cdf, pdf of X what is the relation of F_Y, f_y to F_X, f_X
$F_Y(y) = P(Y \geqslant y) = P(cX \leqslant y) = P(X \leqslant \frac{y}{c}) = F_X(\frac{y}{c})$
we know $f_Y(y) = f_X(g^{-1}(y))|\frac{dg^{-1}(y)}{dy}|$ thus:
$f_Y(y) = f_X(\frac{y}{c}) \cdot |\frac{1}{c}| = \frac{1}{c}f_X(\frac{y}{c})$
2. what is the relation of myerson(Y) and myerson(X)
Myerson(X) optimal revenue can be achieved by a deterministic mechanism, let denot v_x of optimal reverse price for X, then OPT(X) = $v_x(1 - F_X(v_x))$, what if optimal reserve price for Y? we can compute optimla reserve price by setting virtual valuation for Y equal to 0:

$$v - \frac{1 - F_X(\frac{v}{c})}{\frac{1}{c}f_X(\frac{v}{c})} = 0$$
$$\frac{v}{c} - \frac{1 - F_X(\frac{v}{c})}{f_X(\frac{v}{c})} = 0$$

set $v^{'} = \frac{v}{c}$, above equation has solution v_x. Then denote optimal reserve price for Y: $v_y = cv_x$ then OPT(Y) $= v_y(1 - F_Y(v_y)) = cv_x(1 - F_X(\frac{cv_x}{c})) = cv_x(1 - F_X(v_x))$. Therefore OPT(Y) = cOPT(X) 3. what is the relation of REV(Y) and REV(x), given v as reserve price
REV(Y) $= v(1 - F_Y(v)) = v(1 - F_X(\frac{v}{c}))$
REV(X) $= v(1 - F_X(v))$

$$v(1 - F_X(\frac{v}{c})) \overset{0<c<1}{<} v(1 - F_X(v))$$
$$\overset{c=1}{=}$$
$$\overset{c>1}{>}$$

Experiment Evaluation of Robust Revenue-Maximizing Auctions

n = 1000000
outside for loop Normal distribution APX = [1.12655521] the total runtime: 123.03474499999993 seconds

np.average($opt/exp_norm_revenue_list$) 1.1516288151898313
within the for loop $np.average(APX_norm_list)$ Normal distribution APX = 1.1515851528631347 the total runtime: 2193.8029290000004 seconds sum$(opt_norm_list)/sum(exp_norm_revenue_list)$ array([1.12662361])
n = 100000 Normal distribution APX = 1.1518479638482406 the total runtime: 220.4375460000001 seconds array([1.12673402])

Chapter 5

One hot encoding

one hot encoding is convert a categorical variable into n*k (n is number of instances, k is number of categories). Dummy variable trap leads to the problem known as multicolinearity.

- The features were preprocessed with dummy coding for the categorical variables
- and min-max-scaling for numerical ones
- use poisson deviation as loss function
- use relu as activation function and one hidden layer
- out put final layer logit identically and feed it into GLM

$$L(\hat{y}, y, i) = \begin{cases} \hat{y} - yln(\hat{y}) + ln(\Gamma(y)) & \text{if } i = 1 \\ \hat{y} + \frac{y}{\hat{y}} - \sqrt{y} & \text{if } i = 2 \\ \frac{(\hat{y})^{2-i}}{2-i} - y\frac{(\hat{y})^{1-i}}{1-i} - (\frac{y^{2-i}}{2-i} - y\frac{y^{1-i}}{1-i}) & \text{otherwise} \end{cases}$$

source for possible tweedie loss function:https://towardsdatascience.com/tweedie-loss-function-for-right-skewed-data-2c5ca470678f
source:https://www.analyticsvidhya.com/blog/2020/03/one-hot-encoding-vs-label-encoding-using-scikit-learn/

Chapter 6

Further Reading

In this chapter we collect some suggested literature for LaTeX matters, which may be of use for beginners and for more advanced users and may provide some useful tips.

lshort: "The Not So Short Introduction to LaTeX" (see [Oet+11]) is an up-to-date introduction which can be worked through in a moderate amount of time (the authors give an estimated time of 157 minutes for version 5.01, the most recent at the time of writing.) An up-to-date version can be found at `http://tobi.oetiker.ch/lshort/lshort.pdf`.

LaTeX and Friends: The book [Don12] is a recommended and up-to-date introduction to LaTeX which addresses many current packages. Worth a look for both beginners and advanced users.

l2tabu: There are many tips for older packages and LaTeX commands in [ET11], which are particularly recommended for advanced LaTeX users. Here you can learn why certain commands are best avoided and what the alternatives are. Note: The `tumthesis.cls` class automatically calls upon the `nag` package, which immediately rings alarm bells with many of the mistakes listed in l2tabu.

Appendix A

Remarks on Implementation

In the appendix you can include e. g. computer codes or further remarks which would disturb the flow of the main text. If you do not need an appendix, you can simply leave out this file (in which case you should also delete the `\include` command in `thesis.tex`).

List of Figures

All figures in this document were created by the author using TikZ, the excellent TEX-package by Till Tantau, see [Tan08].

List of Tables

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List of Corrections