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template from KACTL

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Template (1)

```
template.cpp27 lines

#pragma once

#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

using ll = long long;
using db = long double;
using vi = vector<int>;
using vl = vector<ll>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<db, db>;
const int INF = 0x3fffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const ll LINF = 0xffffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));

int main(){
    cin.tie(nullptr)->sync_with_stdio(false);
}
```

```
c.sh2 lines

g++ -std=gnu++2a -Wall $1 -o a.out
./a.out

Mathematics (2)

2.1 Goldbatch’s Conjecture

    • Even number can be written in sum of two primes (Up to 1e12)

    • Range of  $N^{th}$  prime and  $N + 1^{th}$  prime will be less than or equal to 300 (Up to 1e12)

2.2 Divisibility

Number of divisors of  $N$  is given by  $\prod_{i=1}^k (a_i + 1)$  where  $N = \prod_{i=1}^k p_i^{a_i}$  and  $p_i$  are prime factors of  $N$ .

Numerical (3)

3.1 Newton’s Method

if  $F(Q) = 0$ , then  $Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$ 


$$Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}$$



$$Q = \ln P = \int \frac{P'}{P} dx$$



$$Q = e^P : Q_{2n} \equiv Q_n(1 + P - \ln Q_n) \pmod{x^{2n}}$$



$$Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2}(Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}$$



$$Q = P^k = \alpha^k x^{kt} e^{k \ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1$$


Group (4)

4.1 Monoid

monoid/MonoidBase.hpp
Description: Monoid Base class. e75b74, 6 lines

template<class T, T (*combine)(T, T), T (*identity)()>
struct MonoidBase{
    using value_type = T;
    static constexpr T op(const T &x, const T &y){return combine(x, y);}
    static constexpr T unit(){return identity();}
};

4.2 Action

action/MonoidActionBase.hpp
Description: Monoid Action Base class. 425d83, 11 lines

template<class MInfo, class MTag, typename MInfo::value_type
(*combine)(typename MInfo::value_type, typename MTag::value_type)>
struct MonoidActionBase{
```

```
using InfoMonoid = MInfo;
using TagMonoid = MTag;
using Info = typename InfoMonoid::value_type;
using Tag = typename TagMonoid::value_type;
static constexpr Info op(const Info &a, const Tag &b){
    return combine(a, b);
}
};

action/DefaultAction.hpp
Description: Default Action class. e45000, 10 lines

template<class Monoid>
struct DefaultAction{
    using InfoMonoid = Monoid;
    using TagMonoid = Monoid;
    using Info = typename Monoid::value_type;
    using Tag = typename Monoid::value_type;
    static constexpr Info op(const Info &a, const Tag &b){
        return Monoid::op(a, b);
    }
};

Data Structures (5)

OrderedSet.hpp
Description: Ordered Set 1a7f5f, 14 lines

template<Header.hpp> <bits/extc++.h>
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
// can be change to less_equal

void usage() {
    ordered_set<int> st, st_2;
    st.insert(2);
    st.insert(1);
    cout << st.order_of_key(2);
    cout << *st.find_by_order(1);
    st.join(st_2); // merge
}

FenwickTree.hpp
Description: Fenwick / Binary Indexed Tree 43767a, 41 lines

template<class T>
struct Fenwick{
    int n, logn;
    vector<T> t;
    Fenwick(){}
    Fenwick(int _n){init(vector<T>(_n, T{}));}
    template<class U>
    Fenwick(const vector<U> &a){init(a);}
    template<class U>
    void init(const vector<U> &a){
        n=(int)a.size();
        logn=31-__builtin_clz(n);
        t.assign(n+1, T{});
        for(int i=1; i<=n; i++){
            t[i]=t[i]+a[i-1];
            int j=i+(i&-i);
            if(j<=n)t[j]=t[j]+t[i];
        }
    }
    void update(int x, const T &v){
        for(int i=x+1; i<=n; i+=i&-i)t[i]=t[i]+v;
```

```
    }
    void update(int l,int r,const T &v){
        update(l,v),update(r+1,-v);
    }
    T query(int x){
        T res{};
        for(int i=x+1;i>0;i-=i&-i)res=res+t[i];
        return res;
    }
    T query(int l,int r){
        return query(r)-query(l-1);
    }
    int find(const T &k){
        int x=0;
        T cur{};
        for(int i=1<<logn;i>0;i>=1)
            if(x+i<=n&&cur+t[x+i]<=k)x+=i,cur=cur+t[x];
        return x;
    }
};
```

SegmentTree.hpp

Description: Segment Tree c51dec, 85 lines

```
template<class Monoid>
struct SegmentTree{
    using T = typename Monoid::value_type;
    int n;
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n,function<T(int)> create){init(n,create);}
    SegmentTree(int n,T v=Monoid::unit()){init(n,[&](int){
        return v;});}
    template<class U>
    SegmentTree(const vector<U> &a){init((int)a.size(), [&](int
        i){return T(a[i]);});}
    void init(int _n,function<T(int)> create){
        n=_n;
        t.assign(4<<(31-__builtin_clz(n)),Monoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
                if(l==r)return void(t[i]=create(l));
                int m=(l+r)/2;
                build(l,m,i*2);
                build(m+1,r,i*2+1);
                pull(i);
            };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=Monoid::op(t[i*2],t[i*2+1]);
    }
    void modify(int l,int r,int i,int x,const T &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const T &v){
        modify(0,n-1,1,x,v);
    }
    template<class U>
    void update(int l,int r,int i,int x,const U &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=Monoid::op(t[i],v));
        int m=(l+r)/2;
        update(l,m,i*2,x,v);
        update(m+1,r,i*2+1,x,v);
    }
};
```

```
        update(m+1,r,i*2+1,x,v);
        pull(i);
    }
    template<class U>
    void update(int x,const U &v){
        update(0,n-1,1,x,v);
    }
    T query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return Monoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        return Monoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,
            x,y));
    }
    T query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};
```

LazySegmentTree.hpp

Description: Segment Tree with Lazy Propagation 91ab0c, 103 lines

```
template<class MonoidAction>
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n;
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n,function<Info(int)> create){init(n,
        create);}
    LazySegmentTree(int n,Info v=InfoMonoid::unit()){init(n
        ,[&](int){return v;});}
    template<class T>
    LazySegmentTree(const vector<T> &a){init((int)a.size(), [&](
        int i){return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        n=_n;
        int m=4<<(31-__builtin_clz(n));
        t.assign(m,InfoMonoid::unit());
    }
};
```

```
        lz.assign(m,TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
                if(l==r)return void(t[i]=create(l));
                int m=(l+r)/2;
                build(l,m,i*2);
                build(m+1,r,i*2+1);
                pull(i);
            };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
    }
    void apply(int i,const Tag &v){
        t[i]=MonoidAction::op(t[i],v);
        lz[i]=TagMonoid::op(lz[i],v);
    }
    void push(int i){
        apply(i*2,lz[i]);
        apply(i*2+1,lz[i]);
        lz[i]=TagMonoid::unit();
    }
    void modify(int l,int r,int i,int x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        push(i);
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const Info &v){
        modify(0,n-1,1,x,v);
    }
    void update(int l,int r,int i,int x,int y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(i,v);
        int m=(l+r)/2;
        push(i);
        update(l,m,i*2,x,y,v);
        update(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void update(int x,int y,const Tag &v){
        update(0,n-1,1,x,y,v);
    }
    Info query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        push(i);
        return InfoMonoid::op(query(l,m,i*2,x,y),query(m+1,r,i
            *2+1,x,y));
    }
    Info query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
```

```

int findfirst(int x,int y,const F &f){
    return findfirst(0,n-1,l,x,y,f);
}
template<class F>
int findlast(int l,int r,int i,int x,int y,const F &f){
    if(y<l||r<x||!f(t[i]))return -1;
    if(l==r)return l;
    int m=(l+r)/2;
    push(i);
    int res=findlast(m+1,r,i*2+1,x,y,f);
    if(res==-1)res=findlast(l,m,i*2,x,y,f);
    return res;
}
template<class F>
int findlast(int x,int y,const F &f){
    return findlast(0,n-1,l,x,y,f);
}
};

```

DynamicSegmentTree.hpp

Description: Dynamic Segment Tree

e84eeb, 106 lines

```

template<class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node{
        Info val;
        Tag lz;
        Ptr l,r;
        Node(Info v):val(v),lz(TagMonoid::unit()),l(nullptr),r(nullptr){}
        Node(Info v,Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr rt;
    function<Info(ll,ll)> create;
    DynamicSegmentTree() {init(0,0);}
    DynamicSegmentTree(ll n){init(0,n-1);}
    DynamicSegmentTree(ll lb,ll ub){init(lb,ub);}
    DynamicSegmentTree(ll lb,ll ub,function<Info(ll,ll)> create){init(lb,ub,create);}
    void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll l,ll r){return InfoMonoid::unit();}){
        lb=_lb,ub=_ub;
        create=_create;
        rt=new Node(create(lb,ub));
    }
    Info val(Ptr t){
        return t?t->val:InfoMonoid::unit();
    }
    void pull(Ptr &t){
        t->val=InfoMonoid::op(val(t->l),val(t->r));
    }
    void apply(Ptr &t,const Tag &v,ll l,ll r){
        if(!t)t=new Node(create(l,r));
        t->val=MonoidAction::op(t->val,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void push(Ptr &t,ll l,ll m,ll r){
        apply(t->l,t->lz,l,m);
        apply(t->r,t->lz,m+1,r);
        t->lz=TagMonoid::unit();
    }
    void modify(ll l,ll r,Ptr &t,ll x,const Info &v){

```

```

        if(x<l||r<x)return;
        if(l==r)return void(t->val=v);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        modify(l,m,t->l,x,v);
        modify(m+1,r,t->r,x,v);
        pull(t);
    }
    void modify(ll x,const Info &v){
        modify(lb,ub,rt,x,v);
    }
    void update(ll l,ll r,Ptr &t,ll x,ll y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(t,v,l,r);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        update(l,m,t->l,x,y,v);
        update(m+1,r,t->r,x,y,v);
        pull(t);
    }
    void update(ll x,ll y,const Tag &v){
        update(lb,ub,rt,x,y,v);
    }
    Info query(ll l,ll r,Ptr &t,ll x,ll y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t->val;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t->r,x,y));
    }
    Info query(ll x,ll y){
        return query(lb,ub,rt,x,y);
    }
    template<class F>
    ll findfirst(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findfirst(l,m,t->l,x,y,f);
        if(res==-1)res=findfirst(m+1,r,t->r,x,y,f);
        return res;
    }
    template<class F>
    ll findfirst(ll x,ll y,const F &f){
        return findfirst(lb,ub,rt,x,y,f);
    }
    template<class F>
    ll findlast(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!t||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findlast(m+1,r,t->r,x,y,f);
        if(res==-1)res=findlast(l,m,t->l,x,y,f);
        return res;
    }
    template<class F>
    ll findlast(ll x,ll y,const F &f){
        return findlast(lb,ub,rt,x,y,f);
    }
};

```

DSU.hpp

Description: Disjoint Set Union.

0b3eb8, 26 lines

```

struct DSU{
    vector<int> p,sz;

```

```

    DSU(){}
    DSU(int n){init(n);}
    void init(int n){
        p.resize(n);
        iota(p.begin(),p.end(),0);
        sz.assign(n,1);
    }
    int find(int u){
        return p[u]==u?p[u]=find(p[u]);
    }
    bool same(int u,int v){
        return find(u)==find(v);
    }
    bool merge(int u,int v){
        u=find(u),v=find(v);
        if(u==v)return false;
        sz[u]+=sz[v];
        p[v]=u;
        return true;
    }
    int size(int u){
        return sz[find(u)];
    }
};

```

BinaryTrie.hpp

Description: Binary Trie

ae5b7a, 66 lines

```

template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
    struct Node{
        array<int,2> ch;
        S cnt;
        Node():ch{-1,-1},cnt(0){}
    };
    vector<Node> t;
    BinaryTrie():t{Node{}}{}
    int new_node(){
        t.emplace_back(Node());
        return t.size()-1;
    }
    S size(){
        return t[0].cnt;
    }
    bool empty(){
        return size()==0;
    }
    S get_cnt(int i){
        return i!=-1?t[i].cnt:S(0);
    }
    void insert(T x,S k=1){
        int u=0;
        t[u].cnt+=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(t[u].ch[v]==-1)t[u].ch[v]=new_node();
            u=t[u].ch[v];
            t[u].cnt+=k;
        }
    }
    void erase(T x,S k=1){
        int u=0;
        assert(t[u].cnt>=k);
        t[u].cnt-=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            u=t[u].ch[v];
            assert(u!=-1&&t[u].cnt>=k);
            t[u].cnt-=k;
        }
    }
};

```

```

    }
}
T kth(S k, T x=0) {
    assert(k<size());
    int u=0;
    T res=0;
    for(int i=BIT-1; i>=0; i--){
        int v=x>>i&1;
        if(k<get_cnt(t[u].ch[v])){
            u=t[u].ch[v];
        }else{
            res|=T(1)<<i;
            k-=get_cnt(t[u].ch[v]);
            u=t[u].ch[v^1];
        }
    }
    return res;
}
T min(T x){
    return kth(0,x);
}
T max(T x){
    return kth(size()-1,x);
}
};

```

LiChaoTree.hpp

Description: Li-Chao Tree (minimize).

4ab713, 52 lines

```

template<class T>
struct LiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x) const{return m*x+c;}
    };
    vector<T> xs;
    vector<Line> t;
    LiChaoTree(){}
    LiChaoTree(const vector<T> &x):xs(x){init(x);}
    LiChaoTree(int n):xs(n){
        vector<T> x(n);
        iota(x.begin(),x.end(),0);
        init(x);
    }
    void init(const vector<T> &x){
        xs=x;
        sort(xs.begin(),xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    }
    void insert(int l,int r,int i,Line v){
        int m=(l+r)/2;
        if(v.eval(xs[m])<t[i].eval(xs[m]))swap(t[i],v);
        if(v.eval(xs[l])<t[i].eval(xs[l]))insert(l,m,i*2,v);
        if(v.eval(xs[r])<t[i].eval(xs[r]))insert(m+1,r,i*2+1,v);
    }
    inline void insert(T m,T c){
        insert(0,(int)xs.size()-1,l,Line(m,c));
    }
    void insert_range(int l,int r,int i,T x,T y,Line v){
        if(y<xs[l]||xs[r]<x)return;
        if(x<=xs[l]&&xs[r]<=y)return insert(l,r,i,v);
        int m=(l+r)/2;
        insert_range(l,m,i*2,x,y,v);
        insert_range(m+1,r,i*2+1,x,y,v);
    }
};

```

```

    inline void insert_range(T m,T c,T x,T y){
        insert_range(0,(int)xs.size()-1,l,x,y,Line(m,c));
    }
    T query(int l,int r,int i,T x){
        if(l==r)return t[i].eval(x);
        int m=(l+r)/2;
        if(x<=xs[m])return min(t[i].eval(x),query(l,m,i*2,x));
        return min(t[i].eval(x),query(m+1,r,i*2+1,x));
    }
    inline T query(T x){
        return query(0,(int)xs.size()-1,l,x);
    }
};

```

DynamicLiChaoTree.hpp

Description: Dynamic Li-Chao Tree (minimize).

b8af36, 50 lines

```

template<class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x) const{return m*x+c;}
    };
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(nullptr){}
    void insert(T l,T r,Ptr &t,Line v){
        if(!t)return void(t=new Node(v));
        T m=l+(r-l)/2;
        if(v.eval(m)<t->v.eval(m))swap(t->v,v);
        if(v.eval(l)<t->v.eval(l))insert(l,m,t->l,v);
        if(v.eval(r)<t->v.eval(r))insert(m+1,r,t->r,v);
    }
    inline void insert(T m,T c){
        insert(lb,ub,root,Line(m,c));
    }
    void insert_range(T l,T r,Ptr &t,T x,T y,Line v){
        if(y<l||r<x)return;
        if(!t)t=new Node();
        if(x<=l&&r<=y)return insert(l,r,t,v);
        T m=l+(r-l)/2;
        insert_range(l,m,t->l,x,y,v);
        insert_range(m+1,r,t->r,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(lb,ub,root,x,y,Line(m,c));
    }
    T query(T l,T r,Ptr t,T x){
        if(!t)return INF;
        T m=l+(r-l)/2;
        if(x<=m)return min(t->v.eval(x),query(l,m,t->l,x));
        return min(t->v.eval(x),query(m+1,r,t->r,x));
    }
    inline T query(T x){
        return query(lb,ub,root,x);
    }
};

```

SplayTreeBase.hpp

Description: Splay Tree. splay(u) will make node u be the root of the tree in amortized O(log n) time.

cc90a9, 113 lines

```

template<class Node>
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return !(t->p)|| (t->p->l!=t&&t->p->r!=t);
    } // The parent of the root stores the path parant in link cut tree.
    int size(Ptr t){
        return t?t->size:0;
    }
    virtual void push(Ptr t){};
    virtual void pull(Ptr t){};
    int pos(Ptr t){
        if(t->p){
            if(t->p->l==t)return -1;
            if(t->p->r==t)return 1;
        }
        return 0;
    }
    void rotate(Ptr t){
        Ptr x=t->p,y=x->p;
        if(pos(t)==-1){
            if((x->l=t->r))t->r->p=x;
            t->r=x,x->p=t;
        }else{
            if((x->r=t->l))t->l->p=x;
            t->l=x,x->p=t;
        }
        pull(x),pull(t);
        if((t->p==y)){
            if(y->l==x)y->l=t;
            if(y->r==x)y->r=t;
        }
    }
    void splay(Ptr t){
        if(!t)return;
        push(t);
        while(!is_root(t)){
            Ptr x=t->p;
            if(is_root(x)){
                push(x),push(t);
                rotate(t);
            }else{
                Ptr y=x->p;
                push(y),push(x),push(t);
                if(pos(x)==pos(t))rotate(x),rotate(t);
                else rotate(t),rotate(t);
            }
        }
    }
    Ptr get_first(Ptr t){
        while(t->l)push(t),t=t->l;
        splay(t);
        return t;
    }
    Ptr get_last(Ptr t){
        while(t->r)push(t),t=t->r;
        splay(t);
        return t;
    }
    Ptr merge(Ptr l,Ptr r){
        splay(l),splay(r);
        if(!l)return r;
        if(!r)return l;
        l=get_last(l);
    }
};

```

```

        l->r=r;
        r->p=l;
        pull(l);
        return l;
    }
    pair<Ptr,Ptr> split(Ptr t,int k){
        if(!t) return {nullptr,nullptr};
        if(k==0) return {nullptr,t};
        if(k==size(t)) return {t,nullptr};
        push(t);
        if(k<=size(t->l)){
            auto x=split(t->l,k);
            t->l=x.second;
            t->p=nullptr;
            if(x.second)x.second->p=t;
            pull(t);
            return {x.first,t};
        }else{
            auto x=split(t->r,k-size(t->l)-1);
            t->r=x.first;
            t->p=nullptr;
            if(x.first)x.first->p=t;
            pull(t);
            return {t,x.second};
        }
    }
    void insert(Ptr &t,int k,Ptr v){
        splay(t);
        auto x=split(t,k);
        t=merge(merge(x.first,v),x.second);
    }
    void erase(Ptr &t,int k){
        splay(t);
        auto x=split(t,k);
        auto y=split(x.second,1);
        // delete y.first;
        t=merge(x.first,y.second);
    }
    template<class T>
    Ptr build(const vector<T> &v){
        if(v.empty()) return nullptr;
        function<Ptr(int,int)> build=[&](int l,int r){
            if(l==r) return new Node(v[l]);
            int m=(l+r)/2;
            return merge(build(l,m),build(m+1,r));
        };
        return build(0,v.size()-1);
    }
};

```

LazyReversibleBBST.hpp

Description: Lazy Reversible BBST Base.

904708, 81 lines

```

template<class Tree,class Node,class MonoidAction>
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;

    LazyReversibleBBST()=default;

    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
    }
    void pull(Ptr t){

```

```

        if(!t) return;
        push(t);
        t->size=1;
        t->sum=t->val;
        t->revsum=t->val;
        if(t->l){
            t->size+=t->l->size;
            t->sum=InfoMonoid::op(t->l->sum,t->sum);
            t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);
        }
        if(t->r){
            t->size+=t->r->size;
            t->sum=InfoMonoid::op(t->sum,t->r->sum);
            t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
        }
    }
    void push(Ptr t){
        if(!t) return;
        if(t->rev){
            toggle(t->l);
            toggle(t->r);
            t->rev=false;
        }
        if(t->lz!=TagMonoid::unit()){
            propagate(t->l,t->lz);
            propagate(t->r,t->lz);
            t->lz=TagMonoid::unit();
        }
    }
    void toggle(Ptr t){
        if(!t) return;
        swap(t->l,t->r);
        swap(t->sum,t->revsum);
        t->rev^=true;
    }
    void propagate(Ptr t,const Tag &v){
        if(!t) return;
        t->val=MonoidAction::op(t->val,v);
        t->sum=MonoidAction::op(t->sum,v);
        t->revsum=MonoidAction::op(t->revsum,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void apply(Ptr &t,int l,int r,const Tag &v){
        if(l>r) return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        propagate(y.first,v);
        t=merge(x.first,merge(y.first,y.second));
    }
    Info query(Ptr &t,int l,int r){
        if(l>r) return InfoMonoid::unit();
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        Info res=sum(y.first);
        t=merge(x.first,merge(y.first,y.second));
        return res;
    }
    void reverse(Ptr &t,int l,int r){
        if(l>r) return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        toggle(y.first);
        t=merge(x.first,merge(y.first,y.second));
    }
};

```

LazyReversibleSplayTree.hpp

Description: Lazy Reversible Splay Tree.

"SplayTreeBase.hpp", "LazyReversibleBBST.hpp" b8455b, 23 lines

```

template<class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
    Ptr l,r,p;
    Info val,sum,revsum;
    Tag lz;
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::unit(),const Tag &_lz=TagMonoid::unit())
        :l(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),size(1),rev(false){}
};

template<class MonoidAction>
struct LazyReversibleSplayTree
: LazyReversibleBBST<SplayTreeBase<
    LazyReversibleSplayTreeNode<MonoidAction>>,
    LazyReversibleSplayTreeNode<MonoidAction>,MonoidAction>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};

```

LinkCutTreeBase.hpp

Description: Link Cut Tree Base.

Usage: evert(u): make u be the root of the tree.

link(u,v): attach u to v.

cut(u,v): remove edge between u and v.

get_root(u): get the root of the tree containing u.

lca(u,v): get the lowest common ancestor of u and v.

fold(u,v): get the value of the path from u to v.

b432c3, 59 lines

```

template<class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for(Ptr cur=t;cur;cur=cur->p){
            this->splay(cur);
            cur->r=pc;
            this->pull(cur);
            pc=cur;
        }
        this->splay(t);
        return pc;
    }
    void evert(Ptr t){ // make t be the root of the tree
        expose(t);
        this->toggle(t);
        this->push(t);
    }
    void link(Ptr u,Ptr v){ // attach u to v
        evert(u);
        expose(v);
        u->p=v;
    }
    void cut(Ptr u,Ptr v){ // cut edge between u and v
        evert(u);
        expose(v);
        assert(u->p==v);
    }
};

```

```

        v->l=u->p=nullptr;
        this->pull(v);
    }
    Ptr get_root(Ptr t){
        expose(t);
        while(t->l)this->push(t),t=t->l;
        this->splay(t);
        return t;
    }
    Ptr lca(Ptr u,Ptr v){
        if(get_root(u)!=get_root(v))return nullptr;
        expose(u);
        return expose(v);
    }
    void set_val(Ptr t,const T &val){
        this->evert(t);
        t->val=val;
        this->pull(t);
    }
    T get_val(Ptr t){
        this->evert(t);
        return t->val;
    }
    T fold(Ptr u,Ptr v){
        evert(u);
        expose(v);
        return v->sum;
    }
};

```

LazyLinkCutTree.hpp

Description: Lazy Link Cut Tree.

Usage: using Lct = LazyLinkCutTree<Action>;

using Ptr = Lct::Ptr;

using Node = Lct::Node;

vector<Ptr> ptr(n);

for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);

auto link=[&](int u,int v){

Lct::link(ptr[u],ptr[v]);

};

auto cut=[&](int u,int v){

Lct::cut(ptr[u],ptr[v]);

};

auto update=[&](int u,int v,const Action::Tag &val){

Lct::apply(ptr[u],ptr[v],val);

};

auto query=[&](int u,int v){

return Lct::fold(ptr[u],ptr[v]);

};

"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp" ead3da, 12 lines

template<class MonoidAction>

struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree<

MonoidAction>>{

using base = LinkCutTreeBase<LazyReversibleSplayTree<

MonoidAction>>;

using Ptr = typename base::Ptr;

using Tag = typename MonoidAction::Tag;

void apply(Ptr u,Ptr v,const Tag &val){

this->evert(u);

this->expose(v);

this->propagate(v,val);

}

};

Number Theory (6)

ExtendedEuclid.hpp

Description: Extended Euclid algorithm for solving diophantine equation (ax + by = gcd(a, b)).

Time: $\mathcal{O}(\log \max\{a, b\})$

"../template/Header.hpp" 229e7c, 13 lines

pair<ll,ll> euclid(ll a,ll b){

ll x=1,y=0,x1=0,y1=1;

while(b!=0){

ll q=a/b;

x-=q*x1;

y-=q*y1;

a-=q*b;

swap(x,x1);

swap(y,y1);

swap(a,b);

}

return {x,y};

}

6.1 Prime Numbers

LinearSieve.hpp

Description: Prime Number Generator in Linear Time

Time: $\mathcal{O}(N)$

"../template/Header.hpp" 194fb1, 15 lines

vi linear_sieve(int n) {

vi prime, composite(n + 1);

for(int i=2; i<=n; ++i) {

if(!composite[i]) {

prime.emplace_back(i);

}

for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {

composite[i * prime[j]] = true;

if(i % prime[j] == 0) {

break;

}

}

}

return prime;

}

FastEratosthenes.hpp

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 ≈ 1.5s

"../template/Header.hpp" 295b58, 33 lines

const int LIM = 1e6;

bitset<LIM> isPrime;

vi eratosthenes() {

const int S = (int) round(sqrt(LIM)), R = LIM / 2;

vi pr = {2}, sieve(S + 1);

pr.reserve((int)(LIM/log(LIM) * 1.1));

vector<pii> cp;

for(int i=3; i<=S; i+=2) {

if(!sieve[i]) {

cp.emplace_back(i, i * i / 2);

for(int j=i*i; j<=S; j+=2*i) {

sieve[j] = 1;

}

}

for(int L=1; L<=R; L+=S) {

array<bool, S> block{};

for(auto &[p, idx]: cp) {

for(int i=idx; i<S+L; idx=(i+p)) {

block[i - L] = 1;

}

}

for(int i=0; i<min(S, R-L); ++i) {

```

        if(!block[i]) {
            pr.emplace_back((L + i) * 2 + 1);
        }
    }
}
for(int i: pr) {
    isPrime[i] = 1;
}
return pr;
}

```

GolbatchConjecture.hpp

Description: Find two prime numbers which sum equals s

Time: $\mathcal{O}(N \log N)$

"FastEratosthenes.hpp" 88fb23, 18 lines

pair<int, int> golbatchConjecture(int s, vi pr = {}){

if (s <= 2 || s % 2 != 0) {

return make_pair(-1, -1);

}

if (pr.size() == 0) {

pr = eratosthenes();

}

for (auto x : pr) {

if (x > s / 2) {

break;

}

int d = s - x;

if (binary_search(pr.begin(), pr.end(), d)) {

return make_pair(min(x, d), max(x, d));

}

}

return make_pair(-1, -1);

}

Graph (7)

7.1 Matching

HopcroftKarp.hpp

Description: Fast bipartite matching algorithm.

Time: $\mathcal{O}(E\sqrt{V})$

"../template/Header.hpp" 0bd56f, 52 lines

struct HopcroftKarp{

int n,m;

vi l,r,lv,ptr;

vector<vi> adj;

HopcroftKarp(){};

HopcroftKarp(int _n,int _m){init(_n,_m);}

void init(int _n,int _m){

n=_n,m=_m;

adj.assign(n+m,vi{});

}

void addEdge(int u,int v){

adj[u].emplace_back(v+n);

}

void bfs(){

lv=vi(n,-1);

queue<int> q;

for(int i=0;i<n;i++)if(l[i]==-1){

lv[i]=0;

q.emplace(i);

}

while(!q.empty()){

int u=q.front();

q.pop();

for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){

lv[r[v]]=lv[u]+1;


```

        q.emplace(r[v]);
    }
}
bool dfs(int u){
    for(int &i=ptr[u];i<sz(adj[u]);i++){
        int v=adj[u][i];
        if(r[v]==-1 || (lv[r[v]]==lv[u]+1&&dfs(r[v]))){
            l[u]=v,r[v]=u;
            return true;
        }
    }
    return false;
}
int maxMatching(){
    int match=0,cnt=0;
    l=r=vi(n+m,-1);
    do{
        ptr=vi(n);
        bfs();
        cnt=0;
        for(int i=0;i<n;i++)if(l[i]==-1&&dfs(i))cnt++;
        match+=cnt;
    }while(cnt);
    return match;
}
};

```

Kuhn.hpp

Description: Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.

Time: $\mathcal{O}(VE)$

../template/Header.hpp fc7d17, 15 lines

```

vi adj[1010], match(1010, -1);
bitset<1010> visited;
bool kuhn(int u) {
    if(visited[u]) {
        return false;
    }
    visited[u] = true;
    for(auto x: adj[u]) {
        if(match[x] == -1 || kuhn(match[x])) {
            match[x] = u;
            return true;
        }
    }
    return false;
}

```

7.2 Network Flow

Dinic.hpp

Description: Fast max-flow algorithm.

Time: $\mathcal{O}(VE \log U)$ where $U = \max|cap|$

../template/Header.hpp 7409c7, 68 lines

```

template<class T>
struct Dinic{
    struct Edge{
        int to;
        ll flow,cap;
        Edge(int _to,ll _cap):to(_to),flow(0),cap(_cap){}
        ll getcap(){
            return cap-flow;
        }
    };
    int n;
    ll U;
    vector<Edge> e;
    vector<vi> adj;

```

```

    vi ptr,lvl;
    Dinic(){
        Dinic(int _n){
            init(_n);
        }
        void init(int _n){
            n=_n,U=0;
            e.clear();
            adj.assign(n,{});
        }
        void addEdge(int u,int v,ll cap){
            U=max(U,cap);
            adj[u].emplace_back(sz(e));
            e.emplace_back(v,cap);
            adj[v].emplace_back(sz(e));
            e.emplace_back(u,0); // change 0 to cap for undirected flow
        }
        bool bfs(int s,int t,ll scale){
            lvl.assign(n,0);
            vi q{s};
            lvl[s]=1;
            for(int i=0;i<sz(q);i++){
                int u=q[i];
                for(auto j:adj[u])if(!lvl[e[j].to]&&e[j].getcap()>=scale){
                    q.emplace_back(e[j].to);
                    lvl[e[j].to]=lvl[u]+1;
                }
            }
            return lvl[t];
        }
        ll dfs(int u,int t,ll f){
            if(u==t||!f)return f;
            for(int &i=ptr[u];i<sz(adj[u]);i++){
                int j=adj[u][i];
                if(lvl[e[j].to]==lvl[u]+1){
                    if(ll p=dfs(e[j].to,t,min(f,e[j].getcap()))){
                        e[j].flow+=p;
                        e[j^1].flow-=p;
                        return p;
                    }
                }
            }
            return 0;
        }
        ll flow(int s,int t){
            ll flow=0;
            for(ll L=1ll<<(63-__builtin_clzll(U));L>0;L>=1) // L = 1 may be faster but it's  $\mathcal{O}(V^2 E)$ 
                while(bfs(s,t,L)){
                    ptr.assign(n,0);
                    while(ll p=dfs(s,t,LINF))flow+=p;
                }
            };
            return flow;
        }
    };
};

```

MinCostFlow.hpp

Description: minimum-cost flow algorithm.

Time: $\mathcal{O}(FE \log V)$ where F is max flow.

../template/Header.hpp 8ea1d2, 83 lines

```

template<class F,class C>
struct MinCostFlow{
    struct Edge{
        int to;
        F flow,cap;
        C cost;

```

```

        Edge(int _to,F _cap,C _cost):to(_to),flow(0),cap(_cap),
            cost(_cost){}
        F getcap(){
            return cap-flow;
        }
    };
    int n;
    vector<Edge> e;
    vector<vi> adj;
    vector<C> pot,dist;
    vi pre;
    bool neg;
    const F FINF=numeric_limits<F>::max()/2;
    const C CINF=numeric_limits<C>::max()/2;
    MinCostFlow(){
        MinCostFlow(int _n){
            init(_n);
        }
        void init(int _n){
            n=_n;
            e.clear();
            adj.assign(n,{});
            neg=false;
        }
        void addEdge(int u,int v,F cap,C cost){
            adj[u].emplace_back(sz(e));
            e.emplace_back(v,cap,cost);
            adj[v].emplace_back(sz(e));
            e.emplace_back(u,0,-cost);
            if(cost<0)neg=true;
        }
        bool dijkstra(int s,int t){
            using P = pair<C,int>;
            dist.assign(n,CINF);
            pre.assign(n,-1);
            priority_queue<P,vector<P>,greater<P>> pq;
            dist[s]=0;
            pq.emplace(0,s);
            while(!pq.empty()){
                auto [d,u]=pq.top();
                pq.pop();
                if(dist[u]<d)continue;
                for(int i:adj[u]){
                    int v=e[i].to;
                    C ndist=d+pot[u]-pot[v]+e[i].cost;
                    if(e[i].getcap()>0&&dist[v]>ndist){
                        pre[v]=i;
                        dist[v]=ndist;
                        pq.emplace(ndist,v);
                    }
                }
            }
            return dist[t]<CINF;
        }
        pair<F,C> flow(int s,int t){
            F flow=0;
            C cost=0;
            pot.assign(n,0);
            if(neg)for(int t=0;t<n;t++)for(int i=0;i<sz(e);i++)if(e[i].getcap()>0){
                int u=e[i^1].to,v=e[i].to;
                pot[v]=min(pot[v],pot[u]+e[i].cost);
            } // Bellman-Ford
            while(dijkstra(s,t)){
                for(int i=0;i<n;i++)pot[i]+=dist[i];
                F aug=FINF;
                for(int u=t;u!=s;u=pre[u])pot[u]^1){
                    aug=min(aug,e[pre[u]].getcap());
                } // find bottleneck
            }

```



```
for(int u=t;u!=s;u=e[pre[u]^1].to){
    e[pre[u]].flow+=aug;
    e[pre[u]^1].flow-=aug;
} // push flow
flow+=aug;
cost+=aug*pot[t];
}
return {flow,cost};
}
};
```

Tree (8)

HLD.hpp

Description: HLD

cf6882, 45 lines

```
vector<vi> adj;
vector<int> sz, lvl, hv, hd, p, disc;
int t;

void dfs(int u, int parent) {
    sz[u] = 1;
    lvl[u] = lvl[parent] + 1;
    p[u] = parent;
    int c_hv=0, c_max=0;
    for(auto v: adj[u]) {
        if(v == parent) continue;
        dfs(v, u);
        sz[u] += sz[v];
        if(c_max < sz[v]) {
            c_hv = v;
            c_max = sz[v];
        }
    }
    hv[u] = c_hv;
}
```

```
void hld(int u, int parent) {
    if(hd[u] == 0) {
        hd[u] = u;
    }
    disc[u] = ++t;
    if(hv[u] != 0) {
        hd[hv[u]] = hd[u];
        hld(hv[u], u);
    }
    for(auto v: adj[u]) {
        if(v == parent || v == hv[u]) {
            continue;
        }
        hld(v, u);
    }
}
```

```
int lca(int u, int v) {
    while(hd[u] != hd[v]) {
        if(lvl[hd[u]] > lvl[hd[v]]) swap(u, v);
        v=p[hd[v]];
    }
    return lvl[u] < lvl[v] ? u: v;
}
```

CentroidDecom.hpp

Description: Centroid

e46d44, 32 lines

```
vector<vi> adj;
vi sz;
```

```
vector<bool> used;

int find_size(int u, int p) {
    sz[u] = 1;
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
        sz[u] += find_size(v, u);
    }
    return sz[u];
}

int find_cen(int u, int p, int t) {
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
        if(sz[v] * 2 > t) find_cen(v, u, t);
    }
    return u;
}

void decom(int u) {
    u = find_cen(u, 0, find_size(u, 0));
    used[u] = true;
    for(auto v: adj[u]) {
        // dfs do something
    }
    for(auto v: adj[u]) {
        if(used[v]) continue;
        decom(v);
    }
}
```

Polynomials (9)

FormalPowerSeries.hpp

Description: basic operations of formal power series

"NTT.hpp" 416433, 136 lines

```
template<class mint>
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;

    FPS &operator+=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]+=rhs[i];
        return *this;
    }
    FPS &operator+=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]+=rhs;
        return *this;
    }
    FPS &operator-=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]-=rhs[i];
        return *this;
    }
    FPS &operator-=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]-=rhs;
        return *this;
    }
    FPS &operator*=(const FPS &rhs){
        auto res=NTT<mint>()(*this,rhs);
        return *this=FPS(res.begin(),res.end());
    }
    FPS &operator*=(const mint &rhs){
        for(auto &a:*this)a*=rhs;
```

```
        return *this;
    }
    friend FPS operator+(FPS lhs,const FPS &rhs){return lhs+=rhs;}
    friend FPS operator+(FPS lhs,const mint &rhs){return lhs+=rhs;}
    friend FPS operator+(const mint &lhs,FPS &rhs){return rhs+=lhs;}
    friend FPS operator-(FPS lhs,const FPS &rhs){return lhs-=rhs;}
    friend FPS operator-(FPS lhs,const mint &rhs){return lhs-=rhs;}
    friend FPS operator-(const mint &lhs,FPS rhs){return -(rhs-lhs);}
    friend FPS operator*(FPS lhs,const FPS &rhs){return lhs*=rhs;}
    friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=rhs;}
    friend FPS operator*(const mint &lhs,FPS rhs){return rhs*=lhs;}

    FPS operator-(){return (*this)*-1;}

    FPS rev(){
        FPS res(*this);
        reverse(res.begin(),res.end());
        return res;
    }
    FPS pre(int sz){
        FPS res(this->begin(),this->begin()+min((int)this->size(),sz));
        if(res.size()<sz)res.resize(sz);
        return res;
    }
    FPS shrink(){
        FPS res(*this);
        while(!res.empty()&&res.back()==mint{})res.pop_back();
        return res;
    }
    FPS operator>>(int sz){
        if(this->size()<=sz)return {};
        FPS res(*this);
        res.erase(res.begin(),res.begin()+sz);
        return res;
    }
    FPS operator<<(int sz){
        FPS res(*this);
        res.insert(res.begin(),sz,mint{});
        return res;
    }
    FPS diff(){
        const int n=this->size();
        FPS res(max(0,n-1));
        for(int i=1;i<n;i++)res[i-1]=(*this)[i]*mint(i);
        return res;
    }
    FPS integral(){
        const int n=this->size();
        FPS res(n+1);
        res[0]=0;
        if(n>0)res[1]=1;
        ll mod=mint::get_mod();
        for(int i=2;i<=n;i++)res[i]=(-res[mod%i])*(mod/i);
        for(int i=0;i<n;i++)res[i+1]=(*this)[i];
        return res;
    }
    mint eval(const mint &x){
        mint res=0,w=1;
        for(auto &a:*this)res+=a*w,w*=x;
```

```

    return res;
}

FPS inv(int deg=-1){
    assert(!this->empty() && (*this)[0] != mint(0));
    if(deg== -1) deg=this->size();
    FPS res(mint(1)/(*this)[0]);
    for(int i=2; i>>1<deg; i<=1){
        res=(res*(mint(2)-res*pre(i))).pre(i);
    }
    return res.pre(deg);
}

FPS log(int deg=-1){
    assert(!this->empty() && (*this)[0] == mint(1));
    if(deg== -1) deg=this->size();
    return (pre(deg).diff()*inv(deg)).pre(deg-1).integral();
}

FPS exp(int deg=-1){
    assert(this->empty() || (*this)[0] == mint(0));
    if(deg== -1) deg=this->size();
    FPS res(mint(1));
    for(int i=2; i>>1<deg; i<=1){
        res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
    }
    return res.pre(deg);
}

FPS pow(ll k, int deg=-1){
    const int n=this->size();
    if(deg== -1) deg=n;
    if(k==0){
        FPS res(deg);
        if(deg) res[0]=mint(1);
        return res;
    }
    for(int i=0; i<n; i++){
        if(__int128_t(i)*k>=deg) return FPS(deg, mint(0));
        if((*this)[i] == mint(0)) continue;
        mint rev=mint(1)/(*this)[i];
        FPS res=(((*this)*rev)>>i).log(deg)*k).exp(deg);
        res=((res*binpow((*this)[i], k))<<(i*k)).pre(deg);
        return res;
    }
    return FPS(deg, mint(0));
}

};
using FPS=FormalPowerSeries<mint>;

```

FFT.hpp

Description: Fast Fourier transform

Time: $\mathcal{O}(N \log N)$

../template/Header.hpp 5d476b, 73 lines

```

template<class T=ll, int mod=0>
struct FFT{
    using vt = vector<T>;
    using cd = complex<db>;
    using vc = vector<cd>;

    static const bool INT=true;

    static void fft(vc &a){
        int n=a.size(), L=31-__builtin_clz(n);
        vc rt(n);
        rt[1]=1;
        for(int k=2; k<n; k*=2){
            cd z=polar(db(1), PI/k);
            for(int i=k; i<2*k; i++) rt[i]=i&1?rt[i/2]*z:rt[i/2];
        }
        vi rev(n);

```

```

        for(int i=1; i<n; i++) rev[i]=(rev[i/2] | (i&1)<<L)/2;
        for(int i=1; i<n; i++) if(i<rev[i]) swap(a[i], a[rev[i]]);
        for(int k=1; k<n; k*=2) for(int i=0; i<n; i+=2*k) for(int j=0; j<k; j++){
            cd z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    template<class U>
    static db norm(const U &x){
        return INT?round(x):x;
    }
    static vt conv(const vt &a, const vt &b){
        if(a.empty() || b.empty()) return {};
        vt res(a.size()+b.size()-1);
        int L=32-__builtin_clz(res.size()), n=1<<L;
        vc in(n), out(n);
        copy(a.begin(), a.end(), in.begin());
        for(int i=0; i<b.size(); i++) in[i].imag(b[i]);
        fft(in);
        for(auto &x:in) x*=x;
        for(int i=0; i<n; i++) out[i]=in[-i&(n-1)]-conj(in[i]);
        fft(out);
        for(int i=0; i<res.size(); i++) res[i]=norm(imag(out[i])/(4*n));
        return res;
    }
    static vl convMod(const vl &a, const vl &b){
        assert(mod>0);
        if(a.empty() || b.empty()) return {};
        vl res(a.size()+b.size()-1);
        int L=32-__builtin_clz(res.size()), n=1<<L;
        ll cut=int(sqrt(mod));
        vc in1(n), in2(n), out1(n), out2(n);
        for(int i=0; i<a.size(); i++) in1[i]=cd(ll(a[i])/cut, ll(a[i])%cut); // a1 + i * a2
        for(int i=0; i<b.size(); i++) in2[i]=cd(ll(b[i])/cut, ll(b[i])%cut); // b1 + i * b2
        fft(in1), fft(in2);
        for(int i=0; i<n; i++){
            int j=-i&(n-1);
            out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.1*n); // f1 * (g1 + i * g2) = f1 * g1 + i f1 * g2
            out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.1, 2.1*n); // f2 * (g1 + i * g2) = f2 * g1 + i f2 * g2
        }
        fft(out1), fft(out2);
        for(int i=0; i<res.size(); i++){
            ll x=round(real(out1[i])), y=round(imag(out1[i]))+round(real(out2[i])), z=round(imag(out2[i]));
            res[i]=((x%mod*cut+y)%mod*cut+z)%mod; // a1 * b1 * cut^2 + (a1 * b2 + a2 * b1) * cut + a2 * b2
        }
        return res;
    }
    vt operator()(const vt &a, const vt &b){
        return mod>0?conv(a, b):convMod(a, b);
    }
};
template<>
struct FFT<db>{
    static const bool INT=false;
};

```

NTT.hpp

Description: Number theoretic transform

Time: $\mathcal{O}(N \log N)$

../template/Header.hpp, ../modular-arithmetic/BinPow.hpp, ../modular-arithmetic/MontgomeryModInt.hpp 2b2392, 39 lines

```

template<class mint=mint>
struct NTT{
    using vm = vector<mint>;

    static constexpr mint root=mint::get_root();
    static_assert(root!=0);

    static void ntt(vm &a){
        int n=a.size(), L=31-__builtin_clz(n);
        vm rt(n);
        rt[1]=1;
        for(int k=2, s=2; k<n; k*=2, s++){
            mint z[]={1, binpow(root, MOD>>s)};
            for(int i=k; i<2*k; i++) rt[i]=rt[i/2]*z[i&1];
        }
        vi rev(n);
        for(int i=1; i<n; i++) rev[i]=(rev[i/2] | (i&1)<<L)/2;
        for(int i=1; i<n; i++) if(i<rev[i]) swap(a[i], a[rev[i]]);
        for(int k=1; k<n; k*=2) for(int i=0; i<n; i+=2*k) for(int j=0; j<k; j++){
            mint z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    static vm conv(const vm &a, const vm &b){
        if(a.empty() || b.empty()) return {};
        int s=a.size()+b.size()-1, n=1<<(32-__builtin_clz(s));
        mint inv=mint(n).inv();
        vm in1(a), in2(b), out(n);
        in1.resize(n), in2.resize(n);
        ntt(in1), ntt(in2);
        for(int i=0; i<n; i++) out[-i&(n-1)]=in1[i]*in2[i]*inv;
        ntt(out);
        return vm(out.begin(), out.begin()+s);
    }
    vm operator()(const vm &a, const vm &b){
        return conv(a, b);
    }
};

```

Strings (10)

Manacher.hpp

Description: Manacher's Algorithm. $pal[i]$:= the length of the longest palindrome centered at $i/2$.

../template/Header.hpp 53856e, 15 lines

```

template<class STR>
vector<int> manacher(const STR &s){
    int n=(int)s.size();
    if(n==0) return {};
    vector<int> pal(2*n-1);
    for(int p=0, l=-1, r=-1; p<2*n-1; p++){
        int i=(p+1)>>1, j=p>>1;
        int k=(i>=r?0:min(r-i, pal[2*(l+r)-p]));
        while(j+k+1<n&&i-k-1>=0&&s[j+k+1]==s[i-k-1]) k++;
        pal[p]=k;
        if(j+k>r) l=i-k, r=j+k;
    }
    for(int i=0; i<2*n-1; i++) pal[i]=pal[i]<<1 | (i&1^1);
    return pal;
}

```

SuffixArray.hpp

Description: Suffix Automaton.

"../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp" b9cfb1, 39 lines

```
template<class STR>
struct SuffixArray{
    int n;
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArray(){}
    SuffixArray(const STR &s){init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        sa[0]=n;
        iota(sa.begin()+1,sa.end(),0);
        sort(sa.begin()+1,sa.end(),[&](int i,int j){return s[i]<s[j];});
        for(int i=1;i<=n;i++){
            int x=sa[i-1],y=sa[i];
            isa[y]=i+1&&s[x]==s[y]?isa[x]:i;
        }
        for(int len=1;len<=n;len<=1){
            vector<int> ps(sa),pi(isa),pos(n+1);
            iota(pos.begin(),pos.end(),0);
            for(auto i:ps)if((i==len)>=0)sa[pos[isa[i]]++]=i;
            for(int i=1;i<=n;i++){
                int x=sa[i-1],y=sa[i];
                isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x]:i;
            }
        }
        for(int i=0,k=0;i<n;i++){
            for(int j=sa[isa[i]-1];j+k<n&&s[j+k]==s[i+k];k++){
                lcp[isa[i]]=k;
                if(k)k--;
            }
            st.init(lcp);
        }
        int get_lcp(int i,int j){
            if(i==j)return n-1;
            auto [l,r]=minmax(isa[i],isa[j]);
            return st.query(l+1,r);
        }
    }
};
```

ZAlgo.hpp

Description: Z Algorithm. $z[i] :=$ the length of the longest common prefix between s and $s[i:]$.

"../template/Header.hpp" b93726, 12 lines

```
template<class STR>
vector<int> z_algorithm(const STR &s){
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for(int i=1,l=0,r=1;i<n;i++){
        if(i<r)z[i]=min(r-i,z[i-1]);
        while(i+z[i]<n&&s[z[i]]==s[i+z[i]])z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    }
    return z;
}
```

Geometry (11)

11.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()==1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << ", " << p.y << ")"; }
};
```

lineDistance.h

Description:Returns the signed distance between point p and the line containing points a and b . Positive value on left side and negative on right as seen from a towards b . $a=b$ gives nan. P is supposed to be $\text{Point}<T>$ or $\text{Point3D}<T>$ where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D , call `.dist` on the result of the cross product.

"Point.h"

f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:Returns the shortest distance between point p and the line segment from point s to e .**Usage:** $\text{Point}<\text{double}>$ $a, b(2,2), p(1,1);$
bool onSegment = segDist(a,b,p) < $1e-10;$

"Point.h"

5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:If a unique intersection point between the line segments going from s_1 to e_1 and from s_2 to e_2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is $\text{Point}<\text{ll}>$ and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.**Usage:** $\text{vector}<P>$ inter = segInter(s_1,e_1,s_2,e_2);

if (sz(inter)==1)

cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h"

9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}
```

lineIntersection.h

Description:If a unique intersection point of the lines going through s_1,e_1 and s_2,e_2 exists $\{1, \text{point}\}$ is returned. If no intersection point exists $\{0, (0,0)\}$ is returned and if infinitely many exists $\{-1, (0,0)\}$ is returned. The wrong position will be returned if P is $\text{Point}<\text{ll}>$ and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.**Usage:** auto res = lineInter(s_1,e_1,s_2,e_2);

if (res.first == 1)

cout << "intersection point at " << res.second << endl;

"Point.h"

a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

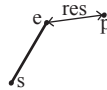
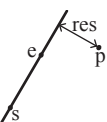
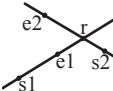
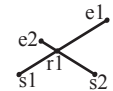
Description: Returns where p is as seen from s towards e . $1/0/-1 \Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be $\text{Point}<T>$ where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.**Usage:** bool left = sideOf(p_1,p_2,q)==1;

"Point.h"

3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
```

```
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```



OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

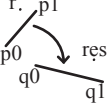
```
"Point.h"
c597e8, 3 lines

template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



```
"Point.h"
03a306, 6 lines

typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage: vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

```
0f0602, 35 lines

struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()}; }
    Angle t360() const { return {x, y, t + 1}; }
};

bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (1l)b.x) <
        make_tuple(b.t, b.half(), a.x * (1l)b.y);
}
```

```
0f0602, 35 lines

struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()}; }
    Angle t360() const { return {x, y, t + 1}; }
};

bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (1l)b.x) <
        make_tuple(b.t, b.half(), a.x * (1l)b.y);
}
```

```
0f0602, 35 lines

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180() ?
        make_pair(a, b) : make_pair(b, a.t360()));
}

Angle operator+(Angle a, Angle b) { // point a + vector b
    Angle r(a.x + b.x, a.y + b.y, a.t);
    if (a.t180() < r) r.t--;
    return r.t180() < a ? r.t360() : r;
}

Angle angleDiff(Angle a, Angle b) { // angle b - angle a
    int tu = b.t - a.t; a.t = b.t;
    return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

11.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

```
"Point.h"
84d6d3, 11 lines

typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
    if (a == b) { assert(r1 != r2); return false; }
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
        p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

```
"Point.h"
b0153d, 13 lines

template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector<pair<P, P>> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back();
    return out;
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

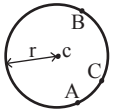
```
"../content/geometry/Point.h"
a1ee63, 19 lines

typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p;
        auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        P u = p + d * s, v = p + d * t;
        return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
    };
    auto sum = 0.0;
    rep(i,0,sz(ps))
        sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
    return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



```
"Point.h"
1caa3a, 9 lines

typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist() /
        abs((B-A).cross(C-A))/2;
}

P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

```
"circumcircle.h"
09dd0a, 17 lines

pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
            rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).dist();
            }
        }
    }
    return {o, r};
}
```

11.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);

Time: $\mathcal{O}(n)$

```
2bf504, 11 lines

"Point.h", "OnSegment.h", "SegmentDistance.h"

template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
    int cnt = 0, n = sz(p);
    rep(i,0,n) {
        P q = p[(i + 1) % n];
        if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
        cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
    }
    return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int T!

```
"Point.h"
f12300, 6 lines

template<class T>
T polygonArea2(vector<Point<T>>& v) {
```

```
T a = v.back().cross(v[0]);
rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    }
    return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

Usage: vector<P> p = ...;

p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0;
        if (side != (s.cross(e, prev) < 0))
            res.push_back(lineInter(s, e, cur, prev).second);
        if (side)
            res.push_back(cur);
    }
    return res;
}
```

ConvexHull.h

Description:

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

Time: $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
```

```
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (; j = (j + 1) % n) {
            res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;
```

```
bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        if (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet(-1, -1)$ if no collision, $\bullet(i, -1)$ if touching the corner i , $\bullet(i, i)$ if along side $(i, i+1)$, $\bullet(i, j)$ if crossing sides $(i, i+1)$ and $(j, j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.

Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}
```

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
```

```
while ((lo + 1) % n != hi) {
    int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
    (cmpL(m) == cmpL(endB) ? lo : hi) = m;
}
res[i] = (lo + !cmpL(hi)) % n;
swap(endA, endB);
}
if (res[0] == res[1]) return {res[0], -1};
if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
        case 0: return {res[0], res[0]};
        case 2: return {res[1], res[1]};
    }
return res;
}
```

11.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.

Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(lo - p).dist2(), {lo, p}});
        S.insert(p);
    }
    return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();
```

```
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
```

```
struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;
};
```

```
T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
}
```

```
Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
        x0 = min(x0, p.x); x1 = max(x1, p.x);
        y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
        // split on x if width >= height (not ideal...)
        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
```

```
// divide by taking half the array for each child (not
// best performance with many duplicates in the middle)
int half = sz(vp)/2;
first = new Node({vp.begin(), vp.begin() + half});
second = new Node({vp.begin() + half, vp.end()});
}
};

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }

        Node *f = node->first, *s = node->second;
        T bfirst = f->distance(p), bsec = s->distance(p);
        if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

        // search closest side first, other side if needed
        auto best = search(f, p);
        if (bsec < best.first)
            best = min(best, search(s, p));
        return best;
    }

    // find nearest point to a point, and its squared distance
    // (requires an arbitrary operator< for Point)
    pair<T, P> nearest(const P& p) {
        return search(root, p);
    }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.

Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
```

FastDelaunay PolyhedronVolume Point3D 3dHull

```
return r;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}
```

11.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y),z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
```



```
vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
vector<F> FS;
auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
        q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
};
rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
        F f = FS[j];
        if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
            E(a,b).rem(f.c);
            E(a,c).rem(f.b);
            E(b,c).rem(f.a);
            swap(FS[j--], FS.back());
            FS.pop_back();
        }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
        F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
        C(a, b, c); C(a, c, b); C(b, c, a);
    }
    for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
        A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
    return FS;
};
```

sphericalDistance.h
Description: Returns the shortest distance on the sphere with radius r between the points with azimuthal angles (longitude) f_1 (ϕ_1) and f_2 (ϕ_2) from x axis and zenith angles (latitude) t_1 (θ_1) and t_2 (θ_2) from z axis ($0 =$ north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. $dx \cdot radius$ is then the difference between the two points in the x direction and $d \cdot radius$ is the total distance between the points.

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

Dynamic Programming (12)

```
DVC.hpp
Description: Optimize  $O(N^2K)$  to  $O(NK \log N)$ 
"../template/Header.hpp"
aa5ddf, 19 lines

vector<vl> cst, dp;

ll cost(int l, int r) {
    return cst[l][r];
}
```

```
void divide(int l, int r, int opt_l, int opt_r, int c) {
    if(l > r) return ;
    int mid = (l + r) / 2;
    pair<ll, int> best = make_pair(INF, -1);
    for(int k=opt_l; k<=min(mid, opt_r); ++k) {
        best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid),
            k));
    }
    dp[c][mid] = best.first;
    divide(l, mid - 1, opt_l, best.second, c);
    divide(mid + 1, r, best.second, opt_r, c);
}

// for(int c=1; c<=K; ++c) divide(1, N, 1, N, c);
```

```
SlopeTrick.hpp
Description: Absolute Smth
"../template/Header.hpp"
f62f9a, 36 lines

ll extending_value;

struct slope_trick {
    multiset<ll> ms_l, ms_r;
    ll min_y = 0ll, lz_l = 0ll, lz_r = 0ll;
    bool extending = false;
    void add_line(ll v) {
        if(extending) {
            lz_l -= extending_value;
            lz_r -= extending_value;
        }
        extending = true;
        if(ms_l.empty() && ms_r.empty()) {
            ms_l.emplace(v);
            ms_r.emplace(v);
        }
        else if(v <= *ms_l.rbegin() + lz_l) {
            min_y += (*ms_l.rbegin() + lz_l) - v;
            ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
            ms_l.erase(--ms_l.end());
            ms_l.emplace(v - lz_l);
            ms_l.emplace(v - lz_l);
        }
        else if(v >= *ms_r.begin() + lz_r) {
            min_y += v - (*ms_r.begin() + lz_r);
            ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
            ms_r.erase(ms_r.begin());
            ms_r.emplace(v - lz_r);
            ms_r.emplace(v - lz_r);
        }
        else {
            ms_l.emplace(v - lz_l);
            ms_r.emplace(v - lz_r);
        }
    }
};
```

Convolutions (13)

```
AndConvolution.hpp
Description: Bitwise AND Convolution. Superset Zeta Transform:  $A'[S] = \sum_{T \supseteq S} A[T]$ . Superset Mobius Transform:  $A[T] = \sum_{S \supseteq T} (-1)^{|S-T|} A'[S]$ .
Time:  $\mathcal{O}(N \log N)$ .
"../template/Header.hpp"
7916f8, 34 lines

template<class T>
void superset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&n));
    for(int i=1;i<n;i<=1){
```

```
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]+=a[j];
            }
        }
    }
}

template<class T>
void superset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&n));
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]-=a[j];
            }
        }
    }
}

template<class T>
vector<T> and_convolution(vector<T> a,vector<T> b){
    superset_zeta(a);
    superset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    superset_mobius(a);
    return a;
}
```

GCDConvolution.hpp
Description: GCD Convolution. Multiple Zeta Transform: $A'[n] = \sum_{n|m} A[m]$. Multiple Mobius Transform: $A[n] = \sum_{n|m} \mu(m/n) A'[m]$.
Time: $\mathcal{O}(N \log \log N)$.

```
"../template/Header.hpp"
7f6c2d, 34 lines

template<class T>
void multiple_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i]+=a[i*p];
        }
    }
}

template<class T>
void multiple_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i]-=a[i*p];
        }
    }
}

template<class T>
vector<T> gcd_convolution(vector<T> a,vector<T> b){
    multiple_zeta(a);
    multiple_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    multiple_mobius(a);
    return a;
}
```



```

}

LCMConvolution.hpp
Description: LCM Convolution. Divisor Zeta Transform:  $A'[n] = \sum_{d|n} A[d]$ . Divisor Mobius Transform:  $A[n] = \sum_{d|n} \mu(n/d) A'[d]$ .
Time:  $\mathcal{O}(N \log \log N)$ .
"../template/Header.hpp" 41fe9d, 34 lines
template<class T>
void divisor_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i*p]+=a[i];
        }
    }

    template<class T>
    void divisor_mobius(vector<T> &a){
        int n=(int)a.size();
        vector<bool> is_prime(n,true);
        for(int p=2;p<n;p++){
            if(!is_prime[p])continue;
            for(int i=(n-1)/p;i>=1;i--){
                is_prime[i*p]=false;
                a[i*p]-=a[i];
            }
        }

        template<class T>
        vector<T> lcm_convolution(vector<T> a,vector<T> b){
            divisor_zeta(a);
            divisor_zeta(b);
            for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
            divisor_mobius(a);
            return a;
        }
    }
}

ORConvolution.hpp
Description: Bitwise OR Convolution. Subset Zeta Transform:  $A'[S] = \sum_{T \subseteq S} A[T]$ . Subset Mobius Transform:  $A[T] = \sum_{S \subseteq T} (-1)^{|T-S|} A'[S]$ .
Time:  $\mathcal{O}(N \log N)$ .
"../template/Header.hpp" c58b77, 34 lines
template<class T>
void subset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]+=a[j^i];
            }
        }
    }

    template<class T>
    void subset_mobius(vector<T> &a){
        int n=(int)a.size();
        assert(n==(n&-n));
        for(int i=n;i>=1;){
            for(int j=0;j<n;j++){
                if(j&i){

```

```

                a[j]-=a[j^i];
            }
        }
    }
}

XORConvolution.hpp
Description: Bitwise XOR Convolution. Fast Walsh-Hadamard Transform:  $A'[S] = \sum_T (-1)^{|S \& T|} A[T]$ .
Time:  $\mathcal{O}(N \log N)$ .
"../template/Header.hpp" 05848d, 29 lines
template<class T>
vector<T> or_convolution(vector<T> a,vector<T> b){
    subset_zeta(a);
    subset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    subset_mobius(a);
    return a;
}

XORConvolution.hpp
Description: Bitwise XOR Convolution. Fast Walsh-Hadamard Transform:  $A'[S] = \sum_T (-1)^{|S \& T|} A[T]$ .
Time:  $\mathcal{O}(N \log N)$ .
"../template/Header.hpp" 05848d, 29 lines
template<class T>
void fwht(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                T &u=a[j^i],&v=a[j];
                tie(u,v)=make_pair(u+v,u-v);
            }
        }
    }

    template<class T>
    vector<T> xor_convolution(vector<T> a,vector<T> b){
        int n=(int)a.size();
        fwht(a);
        fwht(b);
        for(int i=0;i<n;i++)a[i]*=b[i];
        fwht(a);
        T div=T(1)/T(n);
        if(div==T(0)){
            for(auto &x:a)x/=n;
        }else{
            for(auto &x:a)x*=div;
        }
        return a;
    }
}

GaussianElimination.hpp
Description: Gaussian Elimination
"../template/Header.hpp" e89ecb, 34 lines
struct Gauss {
    int n, sz;
    vector<ll> basis;
    Gauss(int n = 0) {
        init(n);
    }
    void init(int _n) {
        n = _n, sz = 0;
        basis.assign(n, 0);
    }
    void insert(ll x) {
        for (int i = n - 1; i >= 0; i--)

```

```

            if (x >> i & 1) {
                if (!basis[i]) {
                    basis[i] = x;
                    sz++;
                    return;
                }
                x ^= basis[i];
            }
        }
        ll getMax(ll k = 0) {
            ll tot = 1ll << sz, res = 0;
            for (int i = n - 1; i >= 0; i--)
                if (basis[i]) {
                    tot >=> 1;
                    if ((k >= tot && res >> i & 1) || (k < tot && res >> i & 1 ^ 1))
                        res ^= basis[i];
                    if (k >= tot)
                        k -= tot;
                }
            return res;
        }
    };
}

BinaryTrie.hpp
Description: Binary Trie
"../template/Header.hpp" 525bf4, 59 lines
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
    vector<node_t> t = {node_t()};
    vector<int> cnt = {0};
    int cnt_nodes = 0;
    void insert(int v) {
        int cur = 0;
        cnt[0]++;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            if(!t[cur][b]) {
                t[cur][b] = ++cnt_nodes;
                t.emplace_back(node_t());
                cnt.emplace_back(0);
            }
            cnt[t[cur][b]]++;
            cur = t[cur][b];
        }
    }
    void remove(int v) {
        int cur = 0;
        cnt[0]--;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            cnt[t[cur][b]]--;
            cur = t[cur][b];
        }
    }
    int get_min(int v) {
        int cur = 0, res = 0;
        for(int i=(int) S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(t[cur][b] && cnt[t[cur][b]]) {
                cur = t[cur][b];
            }
            else {
                res |= (1 << i);
                cur = t[cur][!b];
            }
        }
        return res;
    }
};

BinaryTrie.hpp
Description: Binary Trie
"../template/Header.hpp" 525bf4, 59 lines
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
    vector<node_t> t = {node_t()};
    vector<int> cnt = {0};
    int cnt_nodes = 0;
    void insert(int v) {
        int cur = 0;
        cnt[0]++;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            if(!t[cur][b]) {
                t[cur][b] = ++cnt_nodes;
                t.emplace_back(node_t());
                cnt.emplace_back(0);
            }
            cnt[t[cur][b]]++;
            cur = t[cur][b];
        }
    }
    void remove(int v) {
        int cur = 0;
        cnt[0]--;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            cnt[t[cur][b]]--;
            cur = t[cur][b];
        }
    }
    int get_min(int v) {
        int cur = 0, res = 0;
        for(int i=(int) S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(t[cur][b] && cnt[t[cur][b]]) {
                cur = t[cur][b];
            }
            else {
                res |= (1 << i);
                cur = t[cur][!b];
            }
        }
        return res;
    }
};
```

Various (14)

```

GaussianElimination.hpp
Description: Gaussian Elimination
"../template/Header.hpp" e89ecb, 34 lines
struct Gauss {
    int n, sz;
    vector<ll> basis;
    Gauss(int n = 0) {
        init(n);
    }
    void init(int _n) {
        n = _n, sz = 0;
        basis.assign(n, 0);
    }
    void insert(ll x) {
        for (int i = n - 1; i >= 0; i--)

```

```

}

int get_max(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
        int b = (v & (1 << i)) ? 1 : 0;
        if(t[cur][!b] && cnt[t[cur][!b]]) {
            res |= (1 << i);
            cur = t[cur][!b];
        }
        else {
            cur = t[cur][b];
        }
    }
    return res;
}
};

```

14.1 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

14.1.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x;) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; ((r^x) >> 2)/c` | `r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
`if (i & 1 << b) D[i] += D[i^(1 << b)];`
 computes all sums of subsets.

14.1.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

Competitive Programming Topics

(A)

topics.txt

159 lines

Recursion
Divide and conquer
 Finding interesting points in $N \log N$
Algorithm analysis
 Master theorem
 Amortized time complexity
Greedy algorithm
 Scheduling
 Max contiguous subvector sum
 Invariants
 Huffman encoding
Graph theory
 Dynamic graphs (extra book-keeping)
 Breadth first search
 Depth first search
 * Normal trees / DFS trees
 Dijkstra's algorithm
 MST: Prim's algorithm
 Bellman-Ford
 Konig's theorem and vertex cover
 Min-cost max flow
 Lovasz toggle
 Matrix tree theorem
 Maximal matching, general graphs
 Hopcroft-Karp
 Hall's marriage theorem
 Graphical sequences
 Floyd-Warshall
 Euler cycles
 Flow networks
 * Augmenting paths
 * Edmonds-Karp
 Bipartite matching
 Min. path cover
 Topological sorting
 Strongly connected components
 2-SAT
 Cut vertices, cut-edges and biconnected components
 Edge coloring
 * Trees
 Vertex coloring
 * Bipartite graphs (\Rightarrow trees)
 * 3^n (special case of set cover)
 Diameter and centroid
 K'th shortest path
 Shortest cycle
Dynamic programming
 Knapsack
 Coin change
 Longest common subsequence
 Longest increasing subsequence
 Number of paths in a dag
 Shortest path in a dag
 Dynprog over intervals
 Dynprog over subsets
 Dynprog over probabilities
 Dynprog over trees
 3^n set cover
 Divide and conquer
 Knuth optimization
 Convex hull optimizations
 RMQ (sparse table a.k.a 2^k -jumps)
 Bitonic cycle

 Log partitioning (loop over most restricted)
Combinatorics
 Computation of binomial coefficients
 Pigeon-hole principle
 Inclusion/exclusion
 Catalan number
 Pick's theorem
Number theory
 Integer parts
 Divisibility
 Euclidean algorithm
 Modular arithmetic
 * Modular multiplication
 * Modular inverses
 * Modular exponentiation by squaring
 Chinese remainder theorem
 Fermat's little theorem
 Euler's theorem
 Phi function
 Frobenius number
 Quadratic reciprocity
 Pollard-Rho
 Miller-Rabin
 Hensel lifting
 Vieta root jumping
Game theory
 Combinatorial games
 Game trees
 Mini-max
 Nim
 Games on graphs
 Games on graphs with loops
 Grundy numbers
 Bipartite games without repetition
 General games without repetition
 Alpha-beta pruning
Probability theory
Optimization
 Binary search
 Ternary search
 Unimodality and convex functions
 Binary search on derivative
Numerical methods
 Numeric integration
 Newton's method
 Root-finding with binary/ternary search
 Golden section search
Matrices
 Gaussian elimination
 Exponentiation by squaring
Sorting
 Radix sort
Geometry
 Coordinates and vectors
 * Cross product
 * Scalar product
 Convex hull
 Polygon cut
 Closest pair
 Coordinate-compression
 Quadtrees
 KD-trees
 All segment-segment intersection
Sweeping
 Discretization (convert to events and sweep)
 Angle sweeping
 Line sweeping
 Discrete second derivatives
Strings

 Longest common substring
 Palindrome subsequences
 Knuth-Morris-Pratt
 Tries
 Rolling polynomial hashes
 Suffix array
 Suffix tree
 Aho-Corasick
 Manacher's algorithm
 Letter position lists
Combinatorial search
 Meet in the middle
 Brute-force with pruning
 Best-first (A*)
 Bidirectional search
 Iterative deepening DFS / A*
Data structures
 LCA (2^k -jumps in trees in general)
 Pull/push-technique on trees
 Heavy-light decomposition
 Centroid decomposition
 Lazy propagation
 Self-balancing trees
 Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
 Monotone queues / monotone stacks / sliding queues
 Sliding queue using 2 stacks
 Persistent segment tree