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template from KACTL

2024-11-27

Template (1)

template.cpp
27 lines

```cpp
#pragma once

#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

using ll = long long;
using db = long double;
using vi = vector<int>;
using vl = vector<ll>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<db, db>;
const int INF = 0x3fffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const ll LINF = 0x1fffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));
```

```cpp
int main(){
    cin.tie(nullptr)->sync_with_stdio(false);
}
```

c.sh
2 lines

```sh
g++ -std=gnu++2a -Wall $1 -o a.out
./a.out
```

Mathematics (2)

2.1 Goldbatch's Conjecture

- Even number can be written in sum of two primes (Up to $1e12$)

- Range of Nth prime and $N+1^{th}$ prime will be less than or equal to 300 (Up to $1e12$)

2.2 Divisibility

Number of divisors of N is given by $\prod_{i=1}^{k}(a_i + 1)$ where $N = \prod_{i=1}^{k} p_i^{a_i}$ and p_i are prime factors of N.

Combinatorial (3)

3.1 Permutations

3.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.

Time: $\mathcal{O}(n)$
044568, 6 lines

```cpp
int permToInt(vi &v){
    int use = 0, i = 0, r = 0;
    for (int x : v) r = r * ++i + __builtin_popcount(use & -(1 <<
        x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

3.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

3.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

3.2 Partitions and subsets

3.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim$2e5	$\sim$2e8

3.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

3.2.3 Binomials

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! ... k_n!}$.
a0a312, 6 lines

```cpp
ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i,1,sz(v)) rep(j,0,v[i])
        c = c * ++m / (j+1);
    return c;
}
```

3.3 General purpose numbers

3.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

3.3.2 Stirling numbers of the first kind
Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0)=1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8,0,5040,13068,13132,6769,1960,322,28,1$
$c(n,2) = 0,0,1,3,11,50,274,1764,13068,109584,\ldots$

3.3.3 Eulerian numbers
Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j}(k+1-j)^n$$

3.3.4 Stirling numbers of the second kind
Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

3.3.5 Bell numbers
Total number of partitions of n distinct elements. $B(n) = 1,1,2,5,15,52,203,877,4140,21147,\ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

3.3.6 Labeled unrooted trees
on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

3.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1,1,2,5,14,42,132,429,1430,4862,16796,58786,\ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Numerical (4)

4.1 Newton's Method
if $F(Q) = 0$, then $Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$

$$Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}$$

$$Q = \ln P = \int \frac{P'}{P} dx$$

$$Q = e^p : Q_{2n} \equiv Q_n(1 + P - \ln Q_n) \pmod{x^{2n}}$$

$$Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2}(Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}$$

$$Q = P^k = \alpha^k x^{kt} e^{k\ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1$$

Group (5)

5.1 Monoid
monoid/MonoidBase.hpp
Description: Monoid Base class.   e75b74, 6 lines

```cpp
template<class T,T (*combine)(T,T),T (*identity)()>
struct MonoidBase{
    using value_type = T;
    static constexpr T op(const T &x,const T &y){return combine
        (x,y);}
    static constexpr T unit(){return identity();}
};
```

5.2 Action
action/MonoidActionBase.hpp
Description: Monoid Action Base class.
"../../template/Header.hpp"   425d83, 11 lines

```cpp
template<class MInfo,class MTag,typename MInfo::value_type
    (*combine)(typename MInfo::value_type,typename MTag::
    value_type)>
struct MonoidActionBase{
    using InfoMonoid = MInfo;
```

```cpp
    using TagMonoid = MTag;
    using Info = typename InfoMonoid::value_type;
    using Tag = typename TagMonoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b){
        return combine(a,b);
    }
};
```

action/DefaultAction.hpp
Description: Default Action class.   e45000, 10 lines

```cpp
template<class Monoid>
struct DefaultAction{
    using InfoMonoid = Monoid;
    using TagMonoid = Monoid;
    using Info = typename Monoid::value_type;
    using Tag = typename Monoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b){
        return Monoid::op(a,b);
    }
};
```

Data Structures (6)

OrderedSet.hpp
Description: Ordered Set
"../template/Header.hpp", <bits/extc++.h>   1a7f5f, 14 lines

```cpp
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// can be change to less_equal

void usage() {
    ordered_set<int> st, st_2;
    st.insert(2);
    st.insert(1);
    cout << st.order_of_key(2);
    cout << *st.find_by_order(1);
    st.join(st_2); // merge
}
```

FenwickTree.hpp
Description: Fenwick / Binary Indexed Tree   43767a, 41 lines

```cpp
template<class T>
struct Fenwick{
    int n,logn;
    vector<T> t;
    Fenwick(){}
    Fenwick(int _n){init(vector<T>(_n,T{}));}
    template<class U>
    Fenwick(const vector<U> &a){init(a);}
    template<class U>
    void init(const vector<U> &a){
        n=(int)a.size();
        logn=31-__builtin_clz(n);
        t.assign(n+1,T{});
        for(int i=1;i<=n;i++){
            t[i]=t[i]+a[i-1];
            int j=i+(i&-i);
            if(j<=n)t[j]=t[j]+t[i];
        }
    }
    void update(int x,const T &v){
        for(int i=x+1;i<=n;i+=i&-i)t[i]=t[i]+v;
    }
```

```
    void update(int l,int r,const T &v){
        update(l,v),update(r+1,-v);
    }
    T query(int x){
        T res{};
        for(int i=x+1;i>0;i-=i&-i)res=res+t[i];
        return res;
    }
    T query(int l,int r){
        return query(r)-query(l-1);
    }
    int find(const T &k){
        int x=0;
        T cur{};
        for(int i=1<<logn;i>0;i>>=1)
            if(x+i<=n&&cur+t[x+i]<=k)x+=i,cur=cur+t[x];
        return x;
    }
};
```

SmallSegmentTree.hpp
Description: Zero-indexed max-tree. Bounds are inclusive to the left and
exclusive to the right. Can be changed by modifying T, f and unit.
Time: $\mathcal{O}(\log N)$
0f4bdb, 19 lines

```
struct Tree {
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any associative fn)
  vector<T> s; int n;
  Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  }
  T query(int b, int e) { // query [b, e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);
  }
};
```

SegmentTree.hpp
Description: Segment Tree
c51dec, 85 lines

```
template<class Monoid>
struct SegmentTree{
    using T = typename Monoid::value_type;
    int n;
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n,function<T(int)> create){init(n,create);}
    SegmentTree(int n,T v=Monoid::unit()){init(n,[&](int){
        return v;});}
    template<class U>
    SegmentTree(const vector<U> &a){init((int)a.size(),[&](int
        i){return T(a[i]);});}
    void init(int _n,function<T(int)> create){
        n=_n;
        t.assign(4<<(31-__builtin_clz(n)),Monoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
            if(l==r)return void(t[i]=create(l));
            int m=(l+r)/2;
            build(l,m,i*2);
            build(m+1,r,i*2+1);
```

```
            pull(i);
        };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=Monoid::op(t[i*2],t[i*2+1]);
    }
    void modify(int l,int r,int i,int x,const T &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const T &v){
        modify(0,n-1,1,x,v);
    }
    template<class U>
    void update(int l,int r,int i,int x,const U &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=Monoid::op(t[i],v));
        int m=(l+r)/2;
        update(l,m,i*2,x,v);
        update(m+1,r,i*2+1,x,v);
        pull(i);
    }
    template<class U>
    void update(int x,const U &v){
        update(0,n-1,1,x,v);
    }
    T query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return Monoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        return Monoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,
            x,y));
    }
    T query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};
```

SegmentTreeBeats.hpp
Description: Segment Tree Beats
"../template/Header.hpp" efa2ef, 134 lines

```
const int N = 2e5 + 5;
const int K = 1 << 19;

int n, q;
ll a[N];

struct SegTree{
    struct Node{
        ll sum,add;
        ll mn,mn2,fn;
        ll mx,mx2,fx;
        Node(){
            sum=add=fn=fx=0,mn=mn2=LINF,mx=mx2=-LINF;
        }
        Node(ll v){
            sum=mn=mx=v,add=0,mn2=LINF,mx2=-LINF,fn=fx=1;
        }
        friend Node operator+(const Node &l,const Node &r){
            Node res;
            res.sum=l.sum+r.sum;
            res.add=0;
            if(l.mx>r.mx){
                res.mx=l.mx,res.fx=l.fx;
                res.mx2=max(l.mx2,r.mx);
            }else if(r.mx>l.mx){
                res.mx=r.mx,res.fx=r.fx;
                res.mx2=max(r.mx2,l.mx);
            }else{
                res.mx=l.mx,res.fx=l.fx+r.fx;
                res.mx2=max(l.mx2,r.mx2);
            }
            if(l.mn<r.mn){
                res.mn=l.mn,res.fn=l.fn;
                res.mn2=min(l.mn2,r.mn);
            }else if(r.mn<l.mn){
                res.mn=r.mn,res.fn=r.fn;
                res.mn2=min(r.mn2,l.mn);
            }else{
                res.mn=l.mn,res.fn=l.fn+r.fn;
                res.mn2=min(l.mn2,r.mn2);
            }
            return res;
        }
        void apply(int l,int r,ll v){
            sum+=(r-l+1)*v;
            mx+=v,mx2+=v;
            mn+=v,mn2+=v;
            add+=v;
        }
        void chmin(ll v){
            if(v>=mx)return;
            sum+=(v-mx)*fx;
            if(mn==mx)mn=v;
            if(mn2==mx)mn2=v;
            mx=v;
        }
        void chmax(ll v){
            if(v<=mn)return;
            sum+=(v-mn)*fn;
            if(mx==mn)mx=v;
            if(mx2==mn)mx2=v;
            mn=v;
        }
    }t[K];
    void pull(int i){
        t[i]=t[i*2]+t[i*2+1];
```

```cpp
    }
    void push(int l,int r,int i){
        int m=(l+r)/2;
        t[i*2].apply(l,m,t[i].add);
        t[i*2+1].apply(m+1,r,t[i].add);
        t[i*2].chmin(t[i].mx);
        t[i*2+1].chmin(t[i].mx);
        t[i*2].chmax(t[i].mn);
        t[i*2+1].chmax(t[i].mn);
        t[i].add=0;
    }
    void build(int l,int r,int i){
        if(l==r)return void(t[i]=Node(a[l]));
        int m=(l+r)/2;
        build(l,m,i*2);
        build(m+1,r,i*2+1);
        pull(i);
    }
    void build(){
        build(1,n,1);
    }
    void range_add(int l,int r,int i,int x,int y,ll v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return t[i].apply(l,r,v);
        int m=(l+r)/2;
        push(l,r,i);
        range_add(l,m,i*2,x,y,v);
        range_add(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void range_add(int x,int y,ll v){
        range_add(1,n,1,x,y,v);
    }
    void range_chmin(int l,int r,int i,int x,int y,ll v){
        if(y<l||r<x||t[i].mx<=v)return;
        if(x<=l&&r<=y&&t[i].mx2<v)return t[i].chmin(v);
        int m=(l+r)/2;
        push(l,r,i);
        range_chmin(l,m,i*2,x,y,v);
        range_chmin(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void range_chmin(int x,int y,ll v){
        range_chmin(1,n,1,x,y,v);
    }
    void range_chmax(int l,int r,int i,int x,int y,ll v){
        if(y<l||r<x||t[i].mn>=v)return;
        if(x<=l&&r<=y&&t[i].mn2>v)return t[i].chmax(v);
        int m=(l+r)/2;
        push(l,r,i);
        range_chmax(l,m,i*2,x,y,v);
        range_chmax(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void range_chmax(int x,int y,ll v){
        range_chmax(1,n,1,x,y,v);
    }
    ll query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return 0;
        if(x<=l&&r<=y)return t[i].sum;
        int m=(l+r)/2;
        push(l,r,i);
        return query(l,m,i*2,x,y)+query(m+1,r,i*2+1,x,y);
    }
    ll query(int x,int y){
        return query(1,n,1,x,y);
    }
};
```

LazySegmentTree.hpp
Description: Segment Tree with Lazy Propagation

```cpp
template<class MonoidAction>
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n;
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n,function<Info(int)> create){init(n,
        create);}
    LazySegmentTree(int n,Info v=InfoMonoid::unit()){init(n
        ,[&](int){return v;});}
    template<class T>
    LazySegmentTree(const vector<T> &a){init((int)a.size(),[&](
        int i){return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        n=_n;
        int m=4<<(31-__builtin_clz(n));
        t.assign(m,InfoMonoid::unit());
        lz.assign(m,TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
            if(l==r)return void(t[i]=create(l));
            int m=(l+r)/2;
            build(l,m,i*2);
            build(m+1,r,i*2+1);
            pull(i);
        };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
    }
    void apply(int i,const Tag &v){
        t[i]=MonoidAction::op(t[i],v);
        lz[i]=TagMonoid::op(lz[i],v);
    }
    void push(int i){
        apply(i*2,lz[i]);
        apply(i*2+1,lz[i]);
        lz[i]=TagMonoid::unit();
    }
    void modify(int l,int r,int i,int x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        push(i);
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const Info &v){
        modify(0,n-1,1,x,v);
    }
    void update(int l,int r,int i,int x,int y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(i,v);
        int m=(l+r)/2;
        push(i);
        update(l,m,i*2,x,y,v);
        update(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void update(int x,int y,const Tag &v){
```

```cpp
        update(0,n-1,1,x,y,v);
    }
    Info query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        push(i);
        return InfoMonoid::op(query(l,m,i*2,x,y),query(m+1,r,i
            *2+1,x,y));
    }
    Info query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};
```

DynamicSegmentTree.hpp
Description: Dynamic Segment Tree

```cpp
template<class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node{
        Info val;
        Tag lz;
        Ptr l,r;
        Node(Info v):val(v),lz(TagMonoid::unit()),l(nullptr),r(
            nullptr){}
        Node(Info v,Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr rt;
    function<Info(ll,ll)> create;
    DynamicSegmentTree(){init(0,0);}
    DynamicSegmentTree(ll n){init(0,n-1);}
    DynamicSegmentTree(ll lb,ll ub){init(lb,ub);}
```

```cpp
    DynamicSegmentTree(ll lb,ll ub,function<Info(ll,ll)> create
        ){init(lb,ub,create);}
    void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll
        l,ll r){return InfoMonoid::unit();}){
        lb=_lb,ub=_ub;
        create=_create;
        rt=new Node(create(lb,ub));
    }
    Info val(Ptr t){
        return t?t->val:InfoMonoid::unit();
    }
    void pull(Ptr &t){
        t->val=InfoMonoid::op(val(t->l),val(t->r));
    }
    void apply(Ptr &t,const Tag &v,ll l,ll r){
        if(!t)t=new Node(create(l,r));
        t->val=MonoidAction::op(t->val,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void push(Ptr &t,ll l,ll m,ll r){
        apply(t->l,t->lz,l,m);
        apply(t->r,t->lz,m+1,r);
        t->lz=TagMonoid::unit();
    }
    void modify(ll l,ll r,Ptr &t,ll x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t->val=v);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        modify(l,m,t->l,x,v);
        modify(m+1,r,t->r,x,v);
        pull(t);
    }
    void modify(ll x,const Info &v){
        modify(lb,ub,rt,x,v);
    }
    void update(ll l,ll r,Ptr &t,ll x,ll y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(t,v,l,r);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        update(l,m,t->l,x,y,v);
        update(m+1,r,t->r,x,y,v);
        pull(t);
    }
    void update(ll x,ll y,const Tag &v){
        update(lb,ub,rt,x,y,v);
    }
    Info query(ll l,ll r,Ptr &t,ll x,ll y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t->val;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t
            ->r,x,y));
    }
    Info query(ll x,ll y){
        return query(lb,ub,rt,x,y);
    }
    template<class F>
    ll findfirst(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findfirst(l,m,t->l,x,y,f);
        if(res==-1)res=findfirst(m+1,r,t->r,x,y,f);
        return res;
    }
```

```cpp
    template<class F>
    ll findfirst(ll x,ll y,const F &f){
        return findfirst(lb,ub,rt,x,y,f);
    }
    template<class F>
    ll findlast(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!t||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findlast(m+1,r,t->r,x,y,f);
        if(res==-1)res=findlast(l,m,t->l,x,y,f);
        return res;
    }
    template<class F>
    ll findlast(ll x,ll y,const F &f){
        return findlast(lb,ub,rt,x,y,f);
    }
};
```

DSU.hpp
Description: Disjoint Set Union.   0b3cb8, 26 lines

```cpp
struct DSU{
    vector<int> p,sz;
    DSU(){}
    DSU(int n){init(n);}
    void init(int n){
        p.resize(n);
        iota(p.begin(),p.end(),0);
        sz.assign(n,1);
    }
    int find(int u){
        return p[u]==u?u:p[u]=find(p[u]);
    }
    bool same(int u,int v){
        return find(u)==find(v);
    }
    bool merge(int u,int v){
        u=find(u),v=find(v);
        if(u==v)return false;
        sz[u]+=sz[v];
        p[v]=u;
        return true;
    }
    int size(int u){
        return sz[find(u)];
    }
};
```

BinaryTrie.hpp
Description: Binary Trie   ae5b7a, 66 lines

```cpp
template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
    struct Node{
        array<int,2> ch;
        S cnt;
        Node():ch{-1,-1},cnt(0){}
    };
    vector<Node> t;
    BinaryTrie():t{Node()}{}
    int new_node(){
        t.emplace_back(Node());
        return t.size()-1;
    }
    S size(){
        return t[0].cnt;
    }
```

```cpp
    bool empty(){
        return size()==0;
    }
    S get_cnt(int i){
        return i!=-1?t[i].cnt:S(0);
    }
    void insert(T x,S k=1){
        int u=0;
        t[u].cnt+=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(t[u].ch[v]==-1)t[u].ch[v]=new_node();
            u=t[u].ch[v];
            t[u].cnt+=k;
        }
    }
    void erase(T x,S k=1){
        int u=0;
        assert(t[u].cnt>=k);
        t[u].cnt-=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            u=t[u].ch[v];
            assert(u!=-1&&t[u].cnt>=k);
            t[u].cnt-=k;
        }
    }
    T kth(S k,T x=0){
        assert(k<size());
        int u=0;
        T res=0;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(k<get_cnt(t[u].ch[v])){
                u=t[u].ch[v];
            }else{
                res|=T(1)<<i;
                k-=get_cnt(t[u].ch[v]);
                u=t[u].ch[v^1];
            }
        }
        return res;
    }
    T min(T x){
        return kth(0,x);
    }
    T max(T x){
        return kth(size()-1,x);
    }
};
```

LiChaoTree.hpp
Description: Li-Chao Tree (minimize).   4ab713, 52 lines

```cpp
template<class T>
struct LiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    vector<T> xs;
    vector<Line> t;
    LiChaoTree(){}
    LiChaoTree(const vector<T> &x):xs(x){init(x);}
    LiChaoTree(int n):xs(n){
        vector<T> x(n);
        iota(x.begin(),x.end(),0);
```

```cpp
        init(x);
    }
    void init(const vector<T> &x){
        xs=x;
        sort(xs.begin(),xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    }
    void insert(int l,int r,int i,Line v){
        int m=(l+r)/2;
        if(v.eval(xs[m])<t[i].eval(xs[m]))swap(t[i],v);
        if(v.eval(xs[l])<t[i].eval(xs[l]))insert(l,m,i*2,v);
        if(v.eval(xs[r])<t[i].eval(xs[r]))insert(m+1,r,i*2+1,v)
            ;
    }
    inline void insert(T m,T c){
        insert(0,(int)xs.size()-1,1,Line(m,c));
    }
    void insert_range(int l,int r,int i,T x,T y,Line v){
        if(y<xs[l]||xs[r]<x)return;
        if(x<=xs[l]&&xs[r]<=y)return insert(l,r,i,v);
        int m=(l+r)/2;
        insert_range(l,m,i*2,x,y,v);
        insert_range(m+1,r,i*2+1,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(0,(int)xs.size()-1,1,x,y,Line(m,c));
    }
    T query(int l,int r,int i,T x){
        if(l==r)return t[i].eval(x);
        int m=(l+r)/2;
        if(x<=xs[m])return min(t[i].eval(x),query(l,m,i*2,x));
        return min(t[i].eval(x),query(m+1,r,i*2+1,x));
    }
    inline T query(T x){
        return query(0,(int)xs.size()-1,1,x);
    }
};
```

DynamicLiChaoTree.hpp
Description: Dynamic Li-Chao Tree (minimize).

```cpp
template<class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(
        nullptr){}
    void insert(T l,T r,Ptr &t,Line v){
        if(!t)return void(t=new Node(v));
        T m=l+(r-l)/2;
        if(v.eval(m)<t->v.eval(m))swap(t->v,v);
        if(v.eval(l)<t->v.eval(l))insert(l,m,t->l,v);
        if(v.eval(r)<t->v.eval(r))insert(m+1,r,t->r,v);
    }
```

```cpp
    inline void insert(T m,T c){
        insert(lb,ub,root,Line(m,c));
    }
    void insert_range(T l,T r,Ptr &t,T x,T y,Line v){
        if(y<l||r<x)return;
        if(!t)t=new Node();
        if(x<=l&&r<=y)return insert(l,r,t,v);
        T m=l+(r-l)/2;
        insert_range(l,m,t->l,x,y,v);
        insert_range(m+1,r,t->r,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(lb,ub,root,x,y,Line(m,c));
    }
    T query(T l,T r,Ptr t,T x){
        if(!t)return INF;
        T m=l+(r-l)/2;
        if(x<=m)return min(t->v.eval(x),query(l,m,t->l,x));
        return min(t->v.eval(x),query(m+1,r,t->r,x));
    }
    inline T query(T x){
        return query(lb,ub,root,x);
    }
};
```

SplayTreeBase.hpp
Description: Splay Tree. splay(u) will make node u be the root of the tree in amortized O(log n) time.

```cpp
template<class Node>
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return !(t->p)||(t->p->l!=t&&t->p->r!=t);
    } // The parent of the root stores the path parant in link
        cut tree.
    int size(Ptr t){
        return t?t->size:0;
    }
    virtual void push(Ptr t){};
    virtual void pull(Ptr t){};
    int pos(Ptr t){
        if(t->p){
            if(t->p->l==t)return -1;
            if(t->p->r==t)return 1;
        }
        return 0;
    }
    void rotate(Ptr t){
        Ptr x=t->p,y=x->p;
        if(pos(t)==-1){
            if((x->l=t->r))t->r->p=x;
            t->r=x,x->p=t;
        }else{
            if((x->r=t->l))t->l->p=x;
            t->l=x,x->p=t;
        }
        pull(x),pull(t);
        if((t->p=y)){
            if(y->l==x)y->l=t;
            if(y->r==x)y->r=t;
        }
    }
    void splay(Ptr t){
        if(!t)return;
        push(t);
        while(!is_root(t)){
            Ptr x=t->p;
            if(is_root(x)){
```

```cpp
                push(x),push(t);
                rotate(t);
            }else{
                Ptr y=x->p;
                push(y),push(x),push(t);
                if(pos(x)==pos(t))rotate(x),rotate(t);
                else rotate(t),rotate(t);
            }
        }
    }
    Ptr get_first(Ptr t){
        while(t->l)push(t),t=t->l;
        splay(t);
        return t;
    }
    Ptr get_last(Ptr t){
        while(t->r)push(t),t=t->r;
        splay(t);
        return t;
    }
    Ptr merge(Ptr l,Ptr r){
        splay(l),splay(r);
        if(!l)return r;
        if(!r)return l;
        l=get_last(l);
        l->r=r;
        r->p=l;
        pull(l);
        return l;
    }
    pair<Ptr,Ptr> split(Ptr t,int k){
        if(!t)return {nullptr,nullptr};
        if(k==0)return {nullptr,t};
        if(k==size(t))return {t,nullptr};
        push(t);
        if(k<=size(t->l)){
            auto x=split(t->l,k);
            t->l=x.second;
            t->p=nullptr;
            if(x.second)x.second->p=t;
            pull(t);
            return {x.first,t};
        }else{
            auto x=split(t->r,k-size(t->l)-1);
            t->r=x.first;
            t->p=nullptr;
            if(x.first)x.first->p=t;
            pull(t);
            return {t,x.second};
        }
    }
    void insert(Ptr &t,int k,Ptr v){
        splay(t);
        auto x=split(t,k);
        t=merge(merge(x.first,v),x.second);
    }
    void erase(Ptr &t,int k){
        splay(t);
        auto x=split(t,k);
        auto y=split(x.second,1);
        // delete y.first;
        t=merge(x.first,y.second);
    }
    template<class T>
    Ptr build(const vector<T> &v){
        if(v.empty())return nullptr;
        function<Ptr(int,int)> build=[&](int l,int r){
            if(l==r)return new Node(v[l]);
            int m=(l+r)/2;
```

```
        return merge(build(l,m),build(m+1,r));
    };
    return build(0,v.size()-1);
}
};
```

LazyReversibleBBST.hpp
Description: Lazy Reversible BBST Base.   904708, 81 lines

```
template<class Tree,class Node,class MonoidAction>
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;

    LazyReversibleBBST()=default;

    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
    }
    void pull(Ptr t){
        if(!t)return;
        push(t);
        t->size=1;
        t->sum=t->val;
        t->revsum=t->val;
        if(t->l){
            t->size+=t->l->size;
            t->sum=InfoMonoid::op(t->l->sum,t->sum);
            t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);
        }
        if(t->r){
            t->size+=t->r->size;
            t->sum=InfoMonoid::op(t->sum,t->r->sum);
            t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
        }
    }
    void push(Ptr t){
        if(!t)return;
        if(t->rev){
            toggle(t->l);
            toggle(t->r);
            t->rev=false;
        }
        if(t->lz!=TagMonoid::unit()){
            propagate(t->l,t->lz);
            propagate(t->r,t->lz);
            t->lz=TagMonoid::unit();
        }
    }
    void toggle(Ptr t){
        if(!t)return;
        swap(t->l,t->r);
        swap(t->sum,t->revsum);
        t->rev^=true;
    }
    void propagate(Ptr t,const Tag &v){
        if(!t)return;
        t->val=MonoidAction::op(t->val,v);
        t->sum=MonoidAction::op(t->sum,v);
        t->revsum=MonoidAction::op(t->revsum,v);
        t->lz=MonoidAction::op(t->lz,v);
    }
    void apply(Ptr &t,int l,int r,const Tag &v){
        if(l>r)return;
```

```
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        propagate(y.first,v);
        t=merge(x.first,merge(y.first,y.second));
    }
    Info query(Ptr &t,int l,int r){
        if(l>r)return InfoMonoid::unit();
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        Info res=sum(y.first);
        t=merge(x.first,merge(y.first,y.second));
        return res;
    }
    void reverse(Ptr &t,int l,int r){
        if(l>r)return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        toggle(y.first);
        t=merge(x.first,merge(y.first,y.second));
    }
};
```

LazyReversibleSplayTree.hpp
Description: Lazy Reversible Splay Tree.
"SplayTreeBase.hpp", "LazyReversibleBBST.hpp"   b8455b, 23 lines

```
template<class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
    Ptr l,r,p;
    Info val,sum,revsum;
    Tag lz;
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::
        unit(),const Tag &_lz=TagMonoid::unit())
        :l(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),
            size(1),rev(false){}
};

template<class MonoidAction>
struct LazyReversibleSplayTree
    : LazyReversibleBBST<SplayTreeBase<
        LazyReversibleSplayTreeNode<MonoidAction>>,
      LazyReversibleSplayTreeNode<MonoidAction>,MonoidAction>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};
```

LinkCutTreeBase.hpp
Description: Link Cut Tree Base.
Usage: evert(u): make u be the root of the tree.
link(u,v): attach u to v.
cut(u,v): remove edge between u and v.
get_root(u): get the root of the tree containing u.
lca(u,v): get the lowest common ancestor of u and v.
fold(u,v): get the value of the path from u to v.   b432c3, 59 lines

```
template<class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for(Ptr cur=t;cur;cur=cur->p){
```

```
            this->splay(cur);
            cur->r=pc;
            this->pull(cur);
            pc=cur;
        }
        this->splay(t);
        return pc;
    }
    void evert(Ptr t){ // make t be the root of the tree
        expose(t);
        this->toggle(t);
        this->push(t);
    }
    void link(Ptr u,Ptr v){ // attach u to v
        evert(u);
        expose(v);
        u->p=v;
    }
    void cut(Ptr u,Ptr v){ // cut edge between u and v
        evert(u);
        expose(v);
        assert(u->p==v);
        v->l=u->p=nullptr;
        this->pull(v);
    }
    Ptr get_root(Ptr t){
        expose(t);
        while(t->l)this->push(t),t=t->l;
        this->splay(t);
        return t;
    }
    Ptr lca(Ptr u,Ptr v){
        if(get_root(u)!=get_root(v))return nullptr;
        expose(u);
        return expose(v);
    }
    void set_val(Ptr t,const T &val){
        this->evert(t);
        t->val=val;
        this->pull(t);
    }
    T get_val(Ptr t){
        this->evert(t);
        return t->val;
    }
    T fold(Ptr u,Ptr v){
        evert(u);
        expose(v);
        return v->sum;
    }
};
```

LazyLinkCutTree.hpp
Description: Lazy Link Cut Tree.

Usage: using Lct = LazyLinkCutTree<Action>;
```cpp
using Ptr = Lct::Ptr;
using Node = Lct::Node;
vector<Ptr> ptr(n);
for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);
auto link=[&](int u,int v){
Lct::link(ptr[u],ptr[v]);
};
auto cut=[&](int u,int v){
Lct::cut(ptr[u],ptr[v]);
};
auto update=[&](int u,int v,const Action::Tag &val){
Lct::apply(ptr[u],ptr[v],val);
};
auto query=[&](int u,int v){
return Lct::fold(ptr[u],ptr[v]);
};
```
"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp"   ead3da, 12 lines
```cpp
template<class MonoidAction>
struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree<
    MonoidAction>>{
    using base = LinkCutTreeBase<LazyReversibleSplayTree<
        MonoidAction>>;
    using Ptr = typename base::Ptr;
    using Tag = typename MonoidAction::Tag;

    void apply(Ptr u,Ptr v,const Tag &val){
        this->evert(u);
        this->expose(v);
        this->propagate(v,val);
    }
};
```

StaticTopTree.hpp
Description: Static Top Tree.
                                                              7e10be, 186 lines
```cpp
template<class HLD>
struct StaticTopTree{
    using P = pair<int,int>;
    enum Type{Compress,Rake,AddEdge,AddVertex,Vertex};
    int n,root;
    HLD &hld;
    vector<int> lch,rch,par;
    vector<Type> type;
    StaticTopTree(HLD &_hld):hld(_hld){build();}
    void build(){
        n=hld.n;
        lch=rch=par=vector<int>(n,-1);
        type.assign(n,Compress);
        root=compress(hld.root).second;
    }
    int add(int i,int l,int r,Type t){
        if(i==-1){
            i=n++;
            lch.emplace_back(l);
            rch.emplace_back(r);
            par.emplace_back(-1);
            type.emplace_back(t);
        }else{
            lch[i]=l,rch[i]=r,type[i]=t;
        }
        if(l!=-1)par[l]=i;
        if(r!=-1)par[r]=i;
        return i;
    }
    /*
    pair<int,int> merge(vector<pair<int,int>> a,Type t){
        if(a.size()==1)return a[0];
        int tot=0;
```
```
        vector<pair<int,int>> l,r;
        for(auto [i,s]:a)tot+=s;
        for(auto [i,s]:a){
            (tot>s?l:r).emplace_back(i,s);
            tot-=s*2;
        }
        auto [i,si]=merge(l,t);
        auto [j,sj]=merge(r,t);
        return {add(-1,i,j,t),si+sj};
    }
    */
```
```cpp
    P compress(int i){
        vector<P> a{add_vertex(i)};
        auto work=[&](){
            auto [sj,j]=a.back();
            a.pop_back();
            auto [si,i]=a.back();
            a.back()={max(si,sj)+1,add(-1,i,j,Compress)};
        };
        while(hld.hv[i]!=-1){
            a.emplace_back(add_vertex(i=hld.hv[i]));
            while(true){
                if(a.size()>=3&&(a.end()[-3].first==a.end()
                    [-2].first||a.end()[-3].first<=a.back().
                    first)){
                    P tmp=a.back();
                    a.pop_back();
                    work();
                    a.emplace_back(tmp);
                }else if(a.size()>=2&&a.end()[-2].first<=a.back
                    ().first){
                    work();
                }else break;
            }
        }
        while(a.size()>=2)work();
        return a[0];
    }
    P rake(int i){
        priority_queue<P,vector<P>,greater<P>> pq;
        for(int j:hld.g[i])if(j!=hld.par[i]&&j!=hld.hv[i])pq.
            emplace(add_edge(j));
        while(pq.size()>=2){
            auto [si,i]=pq.top();pq.pop();
            auto [sj,j]=pq.top();pq.pop();
            pq.emplace(max(si,sj)+1,add(-1,i,j,Rake));
        }
        return pq.empty()?make_pair(0,-1):pq.top();
    }
    P add_edge(int i){
        auto [sj,j]=compress(i);
        return {sj+1,add(-1,j,-1,AddEdge)};
    }
    P add_vertex(int i){
        auto [sj,j]=rake(i);
        return {sj+1,add(i,j,-1,j==-1?Vertex:AddVertex)};
    }
};
```
```
/*
struct TreeDP{
    struct Path{
        static Path unit();
    };
    struct Point{
        static Point unit();
    };
    static Path compress(Path l,Path r);
    static Point rake(Point l,Point r);
```
```
    static Point add_edge(Path p);
    static Path add_vertex(Point p,int u);
    static Path vertex(int u);
};
*/
```
```cpp
template<class HLD,class TreeDP>
struct StaticTopTreeRerootingDP{
    using Path = typename TreeDP::Path;
    using Point = typename TreeDP::Point;
    StaticTopTree<HLD> stt;
    vector<Path> path,rpath;
    vector<Point> point;
    StaticTopTreeRerootingDP(HLD &hld):stt(hld){
        int n=stt.n;
        path.resize(n);
        point.resize(n);
        rpath.resize(n);
        dfs(stt.root);
    }
    void _update(int u){
        if(stt.type[u]==stt.Vertex){
            path[u]=rpath[u]=TreeDP::vertex(u);
        }else if(stt.type[u]==stt.Compress){
            path[u]=TreeDP::compress(path[stt.lch[u]],path[stt.
                rch[u]]);
            rpath[u]=TreeDP::compress(rpath[stt.rch[u]],rpath[
                stt.lch[u]]);
        }else if(stt.type[u]==stt.Rake){
            point[u]=TreeDP::rake(point[stt.lch[u]],point[stt.
                rch[u]]);
        }else if(stt.type[u]==stt.AddEdge){
            point[u]=TreeDP::add_edge(path[stt.lch[u]]);
        }else{
            path[u]=rpath[u]=TreeDP::add_vertex(point[stt.lch[u
                ]],u);
        }
    }
    void dfs(int u){
        if(u==-1)return;
        dfs(stt.lch[u]);
        dfs(stt.rch[u]);
        _update(u);
    }
    void update(int u){
        for(;u!=-1;u=stt.par[u])_update(u);
    }
    Path query_all(){
        return path[stt.root];
    }
    Path query_subtree(int u){
        Path res=path[u];
        while(true){
            int p=stt.par[u];
            if(p==-1||stt.type[p]!=stt.Compress)break;
            if(stt.lch[p]==u)res=TreeDP::compress(path[stt.rch[
                p]],res);
            u=p;
        }
        return res;
    }
    Path query_reroot(int u){
        auto rec=[&](auto &&rec,int u)->Point{
            int p=stt.par[u];
            Path below=Path::unit(),above=Path::unit();
            while(p!=-1&&stt.type[p]==stt.Compress){
                int l=stt.lch[p],r=stt.rch[p];
                if(l==u)below=TreeDP::compress(below,path[r]);
                else above=TreeDP::compress(above,rpath[l]);
                u=p;
            }
```

```
            p=stt.par[u];
        }
        if(p!=-1){
            u=p;
            p=stt.par[u];
            Point sum=Point::unit();
            while(stt.type[p]==stt.Rake){
                int l=stt.lch[p],r=stt.rch[p];
                sum=TreeDP::rake(sum,u==r?point[l]:point[r
                    ]);
                u=p;
                p=stt.par[u];
            }
            sum=TreeDP::rake(sum,rec(rec,p));
            above=TreeDP::compress(above,TreeDP::add_vertex
                (sum,p));
        }
        return TreeDP::rake(TreeDP::add_edge(below),TreeDP
            ::add_edge(above));
    };
    Point res=rec(rec,u);
    if(stt.type[u]==stt.AddVertex){
        res=TreeDP::rake(res,point[stt.lch[u]]);
    }
    return TreeDP::add_vertex(res,u);
};
```

Number Theory (7)

ExtendedEuclid.hpp
Description: Extended Euclid algorithm for solving diophantine equation $(ax + by = \gcd(a, b))$.
Time: $\mathcal{O}(\log \max\{a, b\})$
"../template/Header.hpp" 229e7c, 13 lines

```
pair<ll,ll> euclid(ll a,ll b){
    ll x=1,y=0,x1=0,y1=1;
    while(b!=0){
        ll q=a/b;
        x-=q*x1;
        y-=q*y1;
        a-=q*b;
        swap(x,x1);
        swap(y,y1);
        swap(a,b);
    }
    return {x,y};
}
```

euclid.h
Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$. $x = x_0 + k*(b/g)$ $y = y_0 - k*(a/g)$
33ba8f, 9 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

CRT.hpp
Description: Chinese Remainder Theorem.
crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$
"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
```

```
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
}
```

phiFunction.hpp
Description: *Euler's ϕ function* is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1}...(p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.
efae90, 10 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    for(int i=0; i<LIM; ++i) phi[i] = i & 1 ? i : i / 2;
    for (int i = 3; i < LIM; i += 2)
        if (phi[i] == i)
            for (int j = i; j < LIM; j += i)
                phi[j] -= phi[j] / i;
}
```

FloorSum.hpp
Description: Floor sum function. $f(a, b, c, n) = \sum_{x=0}^{n} \lfloor \frac{ax+b}{c} \rfloor$ becareful when a,b,c are negetive (use custom floor division and mod instead)
Time: $\mathcal{O}(\log a)$
d088d2, 7 lines

```
ll floor_sum(ll a,ll b,ll c,ll n){
    ll res=n*(n+1)/2*(a/c)+(n+1)*(b/c);
    a%=c,b%=c;
    if(a==0)return res;
    ll m=(a*n+b)/c;
    return res+n*m-floor_sum(c,c-b-1,a,m-1);
}
```

7.1 Prime Numbers

MillerRabin.hpp
Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.
be7e00, 25 lines

```
using ull = uint64_t;

ull modmul(ull a, ull b, ull M) {
    ll ret = a * b - M * ull(1.L / M * a * b);
    return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
    ull ans = 1;
    for (; e; b = modmul(b, b, mod), e /= 2)
        if (e & 1) ans = modmul(ans, b, mod);
    return ans;
}

bool isPrime(ull n) {
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) {    // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
```

```
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
```

LinearSieve.hpp
Description: Prime Number Generator in Linear Time
Time: $\mathcal{O}(N)$
"../template/Header.hpp" 194fb1, 15 lines

```
vi linear_sieve(int n) {
    vi prime, composite(n + 1);
    for(int i=2; i<=n; ++i) {
        if(!composite[i]) {
            prime.emplace_back(i);
        }
        for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {
            composite[i * prime[j]] = true;
            if(i % prime[j] == 0) {
                break;
            }
        }
    }
    return prime;
}
```

FastEratosthenes.hpp
Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 ≈ 1.5s
"../template/Header.hpp" 295b58, 33 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int) round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S + 1);
    pr.reserve((int)(LIM/log(LIM) * 1.1));
    vector<pii> cp;
    for(int i=3; i<=S; i+=2) {
        if(!sieve[i]) {
            cp.emplace_back(i, i * i / 2);
            for(int j=i*i; j<=S; j+=2*i) {
                sieve[j] = 1;
            }
        }
    }
    for(int L=1; L<=R; L+=S) {
        array<bool, S> block{};
        for(auto &[p, idx]: cp) {
            for(int i=idx; i<S+L; idx=(i+=p)) {
                block[i - L] = 1;
            }
        }
        for(int i=0; i<min(S, R-L); ++i) {
            if(!block[i]) {
                pr.emplace_back((L + i) * 2 + 1);
            }
        }
    }
    for(int i: pr) {
        isPrime[i] = 1;
    }
    return pr;
}
```

GolbatchConjecture.hpp
Description: Find two prime numbers which sum equals s
Time: $\mathcal{O}(N \log N)$
"FastEratosthenes.hpp" 88fb23, 18 lines

```cpp
pair<int, int> goldbatchConjecture(int s, vi pr = {}){
    if (s <= 2 || s % 2 != 0) {
        return make_pair(-1, -1);
    }
    if (pr.size() == 0) {
        pr = eratosthenes();
    }
    for (auto x : pr) {
        if (x > s / 2) {
            break;
        }
        int d = s - x;
        if (binary_search(pr.begin(), pr.end(), d)) {
            return make_pair(min(x, d), max(x, d));
        }
    }
    return make_pair(-1, -1);
}
```

Graph (8)

8.1 Matching

HopcroftKarp.hpp
Description: Fast bipartite matching algorithm.
Time: $\mathcal{O}\left(E\sqrt{V}\right)$

"../template/Header.hpp"   0bd56f, 52 lines

```cpp
struct HopcroftKarp{
    int n,m;
    vi l,r,lv,ptr;
    vector<vi> adj;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m){init(_n,_m);}
    void init(int _n,int _m){
        n=_n,m=_m;
        adj.assign(n+m,vi{});
    }
    void addEdge(int u,int v){
        adj[u].emplace_back(v+n);
    }
    void bfs(){
        lv=vi(n,-1);
        queue<int> q;
        for(int i=0;i<n;i++)if(l[i]==-1){
            lv[i]=0;
            q.emplace(i);
        }
        while(!q.empty()){
            int u=q.front();
            q.pop();
            for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){
                lv[r[v]]=lv[u]+1;
                q.emplace(r[v]);
            }
        }
    }
    bool dfs(int u){
        for(int &i=ptr[u];i<sz(adj[u]);i++){
            int v=adj[u][i];
            if(r[v]==-1||(lv[r[v]]==lv[u]+1&&dfs(r[v]))){
                l[u]=v,r[v]=u;
                return true;
            }
        }
        return false;
    }
    int maxMatching(){
```

```cpp
        int match=0,cnt=0;
        l=r=vi(n+m,-1);
        do{
            ptr=vi(n);
            bfs();
            cnt=0;
            for(int i=0;i<n;i++)if(l[i]==-1&&dfs(i))cnt++;
            match+=cnt;
        }while(cnt);
        return match;
    }
};
```

Kuhn.hpp
Description: Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.
Time: $\mathcal{O}\left(VE\right)$

"../template/Header.hpp"   fc7d17, 15 lines

```cpp
vi adj[1010], match(1010, -1);
bitset<1010> visited;
bool kuhn(int u) {
    if(visited[u]) {
        return false;
    }
    visited[u] = true;
    for(auto x: adj[u]) {
        if(match[x] == -1 || kuhn(match[x])) {
            match[x] = u;
            return true;
        }
    }
    return false;
}
```

8.2 Network Flow

Dinic.hpp
Description: Dinic's Algorithm for finding the maximum flow.
Time: $\mathcal{O}\left(VE\log U\right)$ where U is the maximum flow.

2b9ab1, 88 lines

```cpp
template<class T,bool directed=true,bool scaling=true>
struct Dinic{
    static constexpr T INF=numeric_limits<T>::max()/2;
    struct Edge{
        int to;
        T flow,cap;
        Edge(int _to,T _cap):to(_to),flow(0),cap(_cap){}
        T remain(){return cap-flow;}
    };
    int n,s,t;
    T U;
    vector<Edge> e;
    vector<vector<int>> g;
    vector<int> ptr,lv;
    bool calculated;
    T max_flow;
    Dinic(){}
    Dinic(int n,int s,int t){init(n,s,t);}
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        U=0;
        e.clear();
        g.assign(n,{});
        calculated=false;
    }
    void add_edge(int from,int to,T cap){
        assert(0<=from&&from<n&&0<=to&&to<n);
        g[from].emplace_back(e.size());
        e.emplace_back(to,cap);
```

```cpp
        g[to].emplace_back(e.size());
        e.emplace_back(from,directed?0:cap);
        U=max(U,cap);
    }
    bool bfs(T scale){
        lv.assign(n,-1);
        vector<int> q{s};
        lv[s]=0;
        for(int i=0;i<(int)q.size();i++){
            int u=q[i];
            for(int j:g[u]){
                int v=e[j].to;
                if(lv[v]==-1&&e[j].remain()>=scale){
                    q.emplace_back(v);
                    lv[v]=lv[u]+1;
                }
            }
        }
        return lv[t]!=-1;
    }
    T dfs(int u,int t,T f){
        if(u==t||f==0)return f;
        for(int &i=ptr[u];i<(int)g[u].size();i++){
            int j=g[u][i];
            int v=e[j].to;
            if(lv[v]==lv[u]+1){
                T res=dfs(v,t,min(f,e[j].remain()));
                if(res>0){
                    e[j].flow+=res;
                    e[j^1].flow-=res;
                    return res;
                }
            }
        }
        return 0;
    }
    T flow(){
        if(calculated)return max_flow;
        calculated=true;
        max_flow=0;
        for(T scale=scaling?1LL<<(63-__builtin_clzll(U)):1LL;
            scale>0;scale>>=1){
            while(bfs(scale)){
                ptr.assign(n,0);
                while(true){
                    T f=dfs(s,t,INF);
                    if(f==0)break;
                    max_flow+=f;
                }
            }
        }
        return max_flow;
    }
    pair<T,vector<int>> cut(){
        flow();
        vector<int> res(n);
        for(int i=0;i<n;i++)res[i]=(lv[i]==-1);
        return {max_flow,res};
    }
};
```

MinCostFlow.hpp
Description: minimum-cost flow algorithm.
Time: $\mathcal{O}\left(FE\log V\right)$ where F is max flow.

"../template/Header.hpp"   8ea1d2, 83 lines

```cpp
template<class F,class C>
struct MinCostFlow{
    struct Edge{
        int to;
```

```cpp
    F flow,cap;
    C cost;
    Edge(int _to,F _cap,C _cost):to(_to),flow(0),cap(_cap),
        cost(_cost){}
    F getcap(){
        return cap-flow;
    }
};
int n;
vector<Edge> e;
vector<vi> adj;
vector<C> pot,dist;
vi pre;
bool neg;
const F FINF=numeric_limits<F>::max()/2;
const C CINF=numeric_limits<C>::max()/2;
MinCostFlow(){}
MinCostFlow(int _n){
    init(_n);
}
void init(int _n){
    n=_n;
    e.clear();
    adj.assign(n,{});
    neg=false;
}
void addEdge(int u,int v,F cap,C cost){
    adj[u].emplace_back(sz(e));
    e.emplace_back(v,cap,cost);
    adj[v].emplace_back(sz(e));
    e.emplace_back(u,0,-cost);
    if(cost<0)neg=true;
}
bool dijkstra(int s,int t){
    using P = pair<C,int>;
    dist.assign(n,CINF);
    pre.assign(n,-1);
    priority_queue<P,vector<P>,greater<P>> pq;
    dist[s]=0;
    pq.emplace(0,s);
    while(!pq.empty()){
        auto [d,u]=pq.top();
        pq.pop();
        if(dist[u]<d)continue;
        for(int i:adj[u]){
            int v=e[i].to;
            C ndist=d+pot[u]-pot[v]+e[i].cost;
            if(e[i].getcap()>0&&dist[v]>ndist){
                pre[v]=i;
                dist[v]=ndist;
                pq.emplace(ndist,v);
            }
        }
    }
    return dist[t]<CINF;
}
pair<F,C> flow(int s,int t){
    F flow=0;
    C cost=0;
    pot.assign(n,0);
    if(neg)for(int t=0;t<n;t++)for(int i=0;i<sz(e);i++)if(e
        [i].getcap()>0){
        int u=e[i^1].to,v=e[i].to;
        pot[v]=min(pot[v],pot[u]+e[i].cost);
    } // Bellman−Ford
    while(dijkstra(s,t)){
        for(int i=0;i<n;i++)pot[i]+=dist[i];
        F aug=FINF;
        for(int u=t;u!=s;u=e[pre[u]^1].to){
```

```cpp
            aug=min(aug,e[pre[u]].getcap());
        } // find bottleneck
        for(int u=t;u!=s;u=e[pre[u]^1].to){
            e[pre[u]].flow+=aug;
            e[pre[u]^1].flow-=aug;
        } // push flow
        flow+=aug;
        cost+=aug*pot[t];
    }
    return {flow,cost};
}
};
```

BinaryOptimization.hpp

Description: Binary Optimization. minimize $\kappa + \sum_i \theta_i(x_i) + \sum_{i<j} \phi_{ij}(x_i,x_j) + \sum_{i<j<k} \psi_{ijk}(x_i,x_j,x_k)$ where $x_i \in \{0,1\}$ and ϕ_{ij}, ψ_{ijk} are submodular functions. a set function f is submodular if $f(S) + f(T) \geq f(S \cap T) + f(S \cup T)$ for all S, T. $\phi_{ij}(0,1) + \phi_{ij}(1,0) \geq \phi_{ij}(1,1) + \phi_{ij}(0,0)$.
"Dinic.hpp"   cd8c59, 92 lines

```cpp
template<class T,bool minimize=true>
struct BinaryOptimization{
    static constexpr T INF=numeric_limits<T>::max()/2;
    int n,s,t,node_id;
    T base;
    map<pair<int,int>,T> edges;
    BinaryOptimization(int _n):n(_n),s(n),t(n+1),node_id(n+2),
        base(0){}
    void add_edge(int u,int v,T w){
        assert(w>=0);
        if(u==v||w==0)return;
        auto &e=edges[{u,v}];
        e=min(e+w,INF);
    }
    void add0(T w){
        base+=w;
    }
    void _add1(int i,T a,T b){
        if(a<=b){
            add0(a);
            add_edge(s,i,b-a);
        }else{
            add0(b);
            add_edge(i,t,a-b);
        }
    }
    void add1(int i,T x0,T x1){
        assert(0<=i&&i<n);
        if(!minimize)x0=-x0,x1=-x1;
        _add1(i,x0,x1);
    }
    void _add2(int i,int j,T a,T b,T c,T d){
        assert(b+c>=a+d);
        add0(a);
        _add1(i,0,c-a);
        _add1(j,0,d-c);
        add_edge(i,j,b+c-a-d);
    }
    void add2(int i,int j,T x00,T x01,T x10,T x11){
        assert(i!=j&&0<=i&&i<n&&0<=j&&j<n);
        if(!minimize)x00=-x00,x01=-x01,x10=-x10,x11=-x11;
        _add2(i,j,x00,x01,x10,x11);
    }
    void _add3(int i,int j,int k,T a,T b,T c,T d,T e,T f,T g,T
        h){
        T p=a+d+f+g-b-c-e-h;
        if(p>=0){
            add0(a);
            _add1(i,0,f-b);
```

```cpp
            _add1(j,0,g-e);
            _add1(k,0,d-c);
            _add2(i,j,0,c+e-a-g,0,0);
            _add2(i,k,0,0,b+e-a-f,0);
            _add2(j,k,0,b+c-a-d,0,0);
            int u=node_id++;
            add0(-p);
            add_edge(i,u,p);
            add_edge(j,u,p);
            add_edge(k,u,p);
            add_edge(u,t,p);
        }else{
            add0(h);
            _add1(i,c-g,0);
            _add1(j,b-d,0);
            _add1(k,e-f,0);
            _add2(i,j,0,0,d+f-b-h,0);
            _add2(i,k,0,d+g-c-h,0,0);
            _add2(j,k,0,0,f+g-e-h,0);
            int u=node_id++;
            add0(p);
            add_edge(s,u,-p);
            add_edge(u,i,-p);
            add_edge(u,j,-p);
            add_edge(u,k,-p);
        }
    }
    void add3(int i,int j,int k,T x000,T x001,T x010,T x011,T
        x100,T x101,T x110,T x111){
        assert(i!=j&&j!=k&&k!=i&&0<=i&&i<n&&0<=j&&j<n&&0<=k&&k<
            n);
        if(!minimize){
            x000=-x000,x001=-x001,x010=-x010,x011=-x011;
            x100=-x100,x101=-x101,x110=-x110,x111=-x111;
        }
        _add3(i,j,k,x000,x001,x010,x011,x100,x101,x110,x111);
    }
    T solve(){
        Dinic<T> dinic(node_id,s,t);
        for(auto &[p,w]:edges){
            auto [u,v]=p;
            dinic.add_edge(u,v,w);
        }
        T ans=dinic.flow()+base;
        return minimize?ans:-ans;
    }
};
```

KaryOptimization.hpp

Description: k-ary Optimization. minimize $\kappa + \sum_i \theta_i(x_i) + \sum_{i<j} \phi_{ij}(x_i,x_j)$ where $x_i \in \{0,1,\ldots,k-1\}$ and $\phi_{i,j}$ is monge. A function f is monge if $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a < b$ and $c < d$. $\phi_{ij}(x-1,y) + \phi_{ij}(x,y+1) \leq \phi_{ij}(x-1,y+1) + \phi_{ij}(x,y)$. $\phi_{ij}(x,y) + \phi_{ij}(x-1,y+1) - \phi_{ij}(x-1,y) - \phi_{ij}(x,y+1) \geq 0$.
"Dinic.hpp"   422f8a, 88 lines

```cpp
template<class T,bool minimize=true>
struct K_aryOptimization{
    static constexpr T INF=numeric_limits<T>::max()/2;
    int n,s,t,node_id;
    T base;
    vector<int> ks;
    vector<vector<int>> id;
    map<pair<int,int>,T> edges;
    K_aryOptimization(int n,int k){init(vector<int>(n,k));}
    K_aryOptimization(const vector<int> &_ks){init(_ks);}
    void init(const vector<int> &_ks){
        ks=_ks;
        n=ks.size();
```

```
        s=0,t=1,node_id=2;
        base=0;
        id.clear();
        edges.clear();
        for(auto &k:ks){
            assert(k>=1);
            vector<int> a(k+1);
            a[0]=s,a[k]=t;
            for(int i=1;i<k;i++)a[i]=node_id++;
            id.emplace_back(a);
            for(int i=2;i<k;i++)add_edge(a[i],a[i-1],INF);
        }
    }
    void add_edge(int u,int v,T w){
        assert(w>=0);
        if(u==v||w==0)return;
        auto &e=edges[{u,v}];
        e=min(e+w,INF);
    }
    void add0(T w){
        base+=w;
    }
    void _add1(int i,vector<T> cost){
        add0(cost[0]);
        for(int j=1;j<ks[i];j++){
            T x=cost[j]-cost[j-1];
            if(x>0)add_edge(id[i][j],t,x);
            if(x<0)add0(x),add_edge(s,id[i][j],-x);
        }
    }
    void add1(int i,vector<T> cost){
        assert(0<=i&&i<n&&(int)cost.size()==ks[i]);
        if(!minimize)for(auto &x:cost)x=-x;
        _add1(i,cost);
    }
    void _add2(int i,int j,vector<vector<T>> cost){
        int h=ks[i],w=ks[j];
        _add1(j,cost[0]);
        for(int x=h-1;x>=0;x--)for(int y=0;y<w;y++)cost[x][y]-=
            cost[0][y];
        vector<T> a(h);
        for(int x=0;x<h;x++)a[x]=cost[x][w-1];
        _add1(i,a);
        for(int x=0;x<h;x++)for(int y=0;y<w;y++)cost[x][y]-=a[x
            ];
        for(int x=1;x<h;x++){
            for(int y=0;y<w-1;y++){
                T w=cost[x][y]+cost[x-1][y+1]-cost[x-1][y]-cost
                    [x][y+1];
                assert(w>=0); // monge
                add_edge(id[i][x],id[j][y+1],w);
            }
        }
    }
    void add2(int i,int j,vector<vector<T>> cost){
        assert(0<=i&&i<n&&0<=j&&j<n&&i!=j);
        assert((int)cost.size()==ks[i]);
        for(auto &v:cost)assert((int)v.size()==ks[j]);
        if(!minimize)for(auto &v:cost)for(auto &x:v)x=-x;
        _add2(i,j,cost);
    }
    pair<T,vector<int>> solve(){
        Dinic<T> dinic(node_id,s,t);
        for(auto &[p,w]:edges){
            auto [u,v]=p;
            dinic.add_edge(u,v,w);
        }
        auto [val,cut]=dinic.cut();
        val+=base;
```

```
        if(!minimize)val=-val;
        vector<int> ans(n);
        for(int i=0;i<n;i++){
            ans[i]=ks[i]-1;
            for(int j=1;j<ks[i];j++)ans[i]-=cut[id[i][j]];
        }
        return {val,ans};
    }
};
```

8.3   Connectivity

SCC.hpp
Description: Strongly Connected Component.
"../template/Header.hpp"   82a9d1, 34 lines

```
template<class G>
pair<int,vector<int>> strongly_connected_component(G &g){
    static_assert(G::is_directed);
    int n=g.n;
    vector<int> disc(n,-1),low(n),scc(n,-1);
    stack<int> st;
    vector<bool> in_st(n);
    int t=0,scc_cnt=0;
    function<void(int,int)> dfs=[&](int u,int p){
        disc[u]=low[u]=t++;
        st.emplace(u);
        in_st[u]=true;
        for(int v:g[u]){
            if(disc[v]==-1){
                dfs(v,u);
                low[u]=min(low[u],low[v]);
            }else if(in_st[v]){
                low[u]=min(low[u],disc[v]);
            }
        }
        if(disc[u]==low[u]){
            while(true){
                int v=st.top();
                st.pop();
                in_st[v]=false;
                scc[v]=scc_cnt;
                if(v==u)break;
            }
            scc_cnt++;
        }
    };
    for(int i=0;i<n;i++)if(disc[i]==-1)dfs(i,-1);
    return {scc_cnt,scc};
}
```

LowLink.hpp
Description: Low Link.
"../template/Header.hpp"   f4ad2f, 33 lines

```
template<class G>
struct LowLink{
    G &g;
    int n;
    vector<int> disc,low,par,ord;
    vector<pair<int,int>> bridge;
    vector<int> articulation;
    int t=0;
    LowLink(G &_g):g(_g),n(g.n),disc(n,-1),low(n),par(n,-1){
        for(int i=0;i<n;i++)if(disc[i]==-1)dfs(i);
    }
    void dfs(int u){
        disc[u]=low[u]=t++;
        ord.emplace_back(u);
        int child=0;
        bool found_par=false;
```

```
        for(int v:g[u]){
            if(disc[v]==-1){
                par[v]=u;
                dfs(v);
                low[u]=min(low[u],low[v]);
                if(low[v]>disc[u])bridge.emplace_back(u,v);
                if(par[u]!=-1&&low[v]>=disc[u])articulation.
                    emplace_back(u);
                child++;
            }else if(v!=par[u]||found_par){
                low[u]=min(low[u],disc[v]);
            }else{
                found_par=true;
            }
        }
        if(par[u]==-1&&child>1)articulation.emplace_back(u);
    }
};
```

<u>Tree</u> (9)

HLD.hpp
Description: HLD
"../template/Header.hpp"   cf6882, 45 lines

```
vector<vi> adj;
vector<int> sz, lvl, hv, hd, p, disc;
int t;

void dfs(int u, int parent) {
    sz[u] = 1;
    lvl[u] = lvl[parent] + 1;
    p[u] = parent;
    int c_hv=0, c_max=0;
    for(auto v: adj[u]) {
        if(v == parent) continue;
        dfs(v, u);
        sz[u] += sz[v];
        if(c_max < sz[v]) {
            c_hv = v;
            c_max = sz[v];
        }
    }
    hv[u] = c_hv;
}

void hld(int u, int parent) {
    if(hd[u] == 0) {
        hd[u] = u;
    }
    disc[u] = ++t;
    if(hv[u] != 0) {
        hd[hv[u]] = hd[u];
        hld(hv[u], u);
    }
    for(auto v: adj[u]) {
        if(v == parent || v == hv[u]) {
            continue;
        }
        hld(v, u);
    }
}

int lca(int u, int v) {
    while(hd[u] != hd[v]) {
        if(lvl[hd[u]] > lvl[hd[v]]) swap(u, v);
        v=p[hd[v]];
    }
    return lvl[u] < lvl[v] ? u: v;
```

```
}
```

CentroidDecom.hpp
Description: Centroid
`"../template/Header.hpp"`   e46d44, 32 lines

```cpp
vector<vi> adj;
vi sz;
vector<bool> used;

int find_size(int u, int p) {
    sz[u] = 1;
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
        sz[u] += find_size(v, u);
    }
    return sz[u];
}

int find_cen(int u, int p, int t) {
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
        if(sz[v] * 2 > t) find_cen(v, u, t);
    }
    return u;
}

void decom(int u) {
    u = find_cen(u, 0, find_size(u, 0));
    used[u] = true;
    for(auto v: adj[u]) {
        // dfs do something
    }
    for(auto v: adj[u]) {
        if(used[v]) continue;
        decom(v);
    }
}
```

Polynomials (10)

FormalPowerSeries.hpp
Description: basic operations of formal power series
`"NTT.hpp"`   416433, 136 lines

```cpp
template<class mint>
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;

    FPS &operator+=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]+=rhs[i];
        return *this;
    }
    FPS &operator+=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]+=rhs;
        return *this;
    }
    FPS &operator-=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]-=rhs[i];
        return *this;
    }
    FPS &operator-=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]-=rhs;
        return *this;
```

```cpp
}
    FPS &operator*=(const FPS &rhs){
        auto res=NTT<mint>()(*this,rhs);
        return *this=FPS(res.begin(),res.end());
    }
    FPS &operator*=(const mint &rhs){
        for(auto &a:*this)a*=rhs;
        return *this;
    }
    friend FPS operator+(FPS lhs,const FPS &rhs){return lhs+=
        rhs;}
    friend FPS operator+(FPS lhs,const mint &rhs){return lhs+=
        rhs;}
    friend FPS operator+(const mint &lhs,FPS &rhs){return rhs+=
        lhs;}
    friend FPS operator-(FPS lhs,const FPS &rhs){return lhs-=
        rhs;}
    friend FPS operator-(FPS lhs,const mint &rhs){return lhs-=
        rhs;}
    friend FPS operator-(const mint &lhs,FPS rhs){return -(rhs-
        lhs);}
    friend FPS operator*(FPS lhs,const FPS &rhs){return lhs*=
        rhs;}
    friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=
        rhs;}
    friend FPS operator*(const mint &lhs,FPS rhs){return rhs*=
        lhs;}

    FPS operator-(){return (*this)*-1;}

    FPS rev(){
        FPS res(*this);
        reverse(res.beign(),res.end());
        return res;
    }
    FPS pre(int sz){
        FPS res(this->begin(),this->begin()+min((int)this->size
            (),sz));
        if(res.size()<sz)res.resize(sz);
        return res;
    }
    FPS shrink(){
        FPS res(*this);
        while(!res.empty()&&res.back()==mint{})res.pop_back();
        return res;
    }
    FPS operator>>(int sz){
        if(this->size()<=sz)return {};
        FPS res(*this);
        res.erase(res.begin(),res.begin()+sz);
        return res;
    }
    FPS operator<<(int sz){
        FPS res(*this);
        res.insert(res.begin(),sz,mint{});
        return res;
    }
    FPS diff(){
        const int n=this->size();
        FPS res(max(0,n-1));
        for(int i=1;i<n;i++)res[i-1]=(*this)[i]*mint(i);
        return res;
    }
    FPS integral(){
        const int n=this->size();
        FPS res(n+1);
        res[0]=0;
        if(n>0)res[1]=1;
        ll mod=mint::get_mod();
```

```cpp
        for(int i=2;i<=n;i++)res[i]=(-res[mod%i])*(mod/i);
        for(int i=0;i<n;i++)res[i+1]*=(*this)[i];
        return res;
    }
    mint eval(const mint &x){
        mint res=0,w=1;
        for(auto &a:*this)res+=a*w,w*=x;
        return res;
    }

    FPS inv(int deg=-1){
        assert(!this->empty()&&(*this)[0]!=mint(0));
        if(deg==-1)deg=this->size();
        FPS res{mint(1)/(*this)[0]};
        for(int i=2;i>>1<deg;i<<=1){
            res=(res*(mint(2)-res*pre(i))).pre(i);
        }
        return res.pre(deg);
    }
    FPS log(int deg=-1){
        assert(!this->empty()&&(*this)[0]==mint(1));
        if(deg==-1)deg=this->size();
        return (pre(deg).diff()*inv(deg)).pre(deg-1).integral()
            ;
    }
    FPS exp(int deg=-1){
        assert(this->empty()||(*this)[0]==mint(0));
        if(deg==-1)deg=this->size();
        FPS res{mint(1)};
        for(int i=2;i>>1<deg;i<<=1){
            res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
        }
        return res.pre(deg);
    }
    FPS pow(ll k,int deg=-1){
        const int n=this->size();
        if(deg==-1)deg=n;
        if(k==0){
            FPS res(deg);
            if(deg)res[0]=mint(1);
            return res;
        }
        for(int i=0;i<n;i++){
            if(__int128_t(i)*k>=deg)return FPS(deg,mint(0));
            if((*this)[i]==mint(0))continue;
            mint rev=mint(1)/(*this)[i];
            FPS res=(((*this*rev)>>i).log(deg)*k).exp(deg);
            res=((res*binpow((*this)[i],k))<<(i*k)).pre(deg);
            return res;
        }
        return FPS(deg,mint(0));
    }
};
using FPS=FormalPowerSeries<mint>;
```

FFT.hpp
Description: Fast Fourier transform
Time: $\mathcal{O}(N \log N)$
`"../template/Header.hpp"`   5d476b, 73 lines

```cpp
template<class T=ll,int mod=0>
struct FFT{
    using vt = vector<T>;
    using cd = complex<db>;
    using vc = vector<cd> ;

    static const bool INT=true;

    static void fft(vc &a){
        int n=a.size(),L=31-__builtin_clz(n);
```

```cpp
        vc rt(n);
        rt[1]=1;
        for(int k=2;k<n;k*=2){
            cd z=polar(db(1),PI/k);
            for(int i=k;i<2*k;i++)rt[i]=i&1?rt[i/2]*z:rt[i/2];
        }
        vi rev(n);
        for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
        for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
        for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k
            ;j++){
            cd z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    template<class U>
    static db norm(const U &x){
        return INT?round(x):x;
    }
    static vt conv(const vt &a,const vt &b){
        if(a.empty()||b.empty())return {};
        vt res(a.size()+b.size()-1);
        int L=32-__builtin_clz(res.size()),n=1<<L;
        vc in(n),out(n);
        copy(a.begin(),a.end(),in.begin());
        for(int i=0;i<b.size();i++)in[i].imag(b[i]);
        fft(in);
        for(auto &x:in)x*=x;
        for(int i=0;i<n;i++)out[i]=in[-i&(n-1)]-conj(in[i]);
        fft(out);
        for(int i=0;i<res.size();i++)res[i]=norm(imag(out[i])/(4*n)
            );
        return res;
    }
    static vl convMod(const vl &a,const vl &b){
        assert(mod>0);
        if(a.empty()||b.empty())return {};
        vl res(a.size()+b.size()-1);
        int L=32-__builtin_clz(res.size()),n=1<<L;
        ll cut=int(sqrt(mod));
        vc in1(n),in2(n),out1(n),out2(n);
        for(int i=0;i<a.size();i++)in1[i]=cd(ll(a[i])/cut,ll(a[i])%
            cut); // a1 + i * a2
        for(int i=0;i<b.size();i++)in2[i]=cd(ll(b[i])/cut,ll(b[i])%
            cut); // b1 + i * b2
        fft(in1),fft(in2);
        for(int i=0;i<n;i++){
            int j=-i&(n-1);
            out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.l*n); // f1 * (g1
                + i * g2) = f1 * g1 + i f1 * g2
            out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.l,2.l*n); // f2
                * (g1 + i * g2) = f2 * g1 + i f2 * g2
        }
        fft(out1),fft(out2);
        for(int i=0;i<res.size();i++){
            ll x=round(real(out1[i])),y=round(imag(out1[i]))+round(
                real(out2[i])),z=round(imag(out2[i]));
            res[i]=((x%mod*cut+y)%mod*cut+z)%mod; // a1 * b1 * cut^2
                + (a1 * b2 + a2 * b1) * cut + a2 * b2
        }
        return res;
    }
    vt operator()(const vt &a,const vt &b){
        return mod>0?conv(a,b):convMod(a,b);
    }
};
template<>
struct FFT<db>{
```

NTT.hpp
Description: Number theoretic transform
Time: $\mathcal{O}(N \log N)$
"../template/Header.hpp", "../modular-arithmetic/BinPow.hpp",
"../modular-arithmetic/MontgomeryModInt.hpp"   2b2392, 39 lines

```cpp
template<class mint=mint>
struct NTT{
    using vm = vector<mint>;

    static constexpr mint root=mint::get_root();
    static_assert(root!=0);

    static void ntt(vm &a){
        int n=a.size(),L=31-__builtin_clz(n);
        vm rt(n);
        rt[1]=1;
        for(int k=2,s=2;k<n;k*=2,s++){
            mint z[]={1,binpow(root,MOD>>s)};
            for(int i=k;i<2*k;i++)rt[i]=rt[i/2]*z[i&1];
        }
        vi rev(n);
        for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
        for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
        for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k
            ;j++){
            mint z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    static vm conv(const vm &a,const vm &b){
        if(a.empty()||b.empty())return {};
        int s=a.size()+b.size()-1,n=1<<(32-__builtin_clz(s));
        mint inv=mint(n).inv();
        vm in1(a),in2(b),out(n);
        in1.resize(n),in2.resize(n);
        ntt(in1),ntt(in2);
        for(int i=0;i<n;i++)out[-i&(n-1)]=in1[i]*in2[i]*inv;
        ntt(out);
        return vm(out.begin(),out.begin()+s);
    }
    vm operator()(const vm &a,const vm &b){
        return conv(a,b);
    }
};
```

Strings (11)

Manacher.hpp
Description: Manacher's Algorithm. pal[i] := the length of the longest
palindrome centered at i/2.
"../template/Header.hpp"   53856e, 15 lines

```cpp
template<class STR>
vector<int> manacher(const STR &s){
    int n=(int)s.size();
    if(n==0)return {};
    vector<int> pal(2*n-1);
    for(int p=0,l=-1,r=-1;p<2*n-1;p++){
        int i=(p+1)>>1,j=p>>1;
        int k=(i>=r?0:min(r-i,pal[2*(1+r)-p]));
        while(j+k+1<n&&i-k-1>=0&&s[j+k+1]==s[i-k-1])k++;
        pal[p]=k;
        if(j+k>r)l=i-k,r=j+k;
    }
```

```cpp
    for(int i=0;i<2*n-1;i++)pal[i]=pal[i]<<1|(i&1^1);
    return pal;
}
```

SuffixArray.hpp
Description: Suffix Automaton.
"../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp"   b9cfb1, 39 lines

```cpp
template<class STR>
struct SuffixArray{
    int n;
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArray(){}
    SuffixArray(const STR &s){init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        sa[0]=n;
        iota(sa.begin()+1,sa.end(),0);
        sort(sa.begin()+1,sa.end(),[&](int i,int j){return s[i
            ]<s[j];});
        for(int i=1;i<=n;i++){
            int x=sa[i-1],y=sa[i];
            isa[y]=i>1&&s[x]==s[y]?isa[x]:i;
        }
        for(int len=1;len<=n;len<<=1){
            vector<int> ps(sa),pi(isa),pos(n+1);
            iota(pos.begin(),pos.end(),0);
            for(auto i:ps)if((i-=len)>=0)sa[pos[isa[i]]++]=i;
            for(int i=1;i<=n;i++){
                int x=sa[i-1],y=sa[i];
                isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x
                    ]:i;
            }
        }
        for(int i=0,k=0;i<n;i++){
            for(int j=sa[isa[i]-1];j+k<n&&s[j+k]==s[i+k];k++);
            lcp[isa[i]]=k;
            if(k)k--;
        }
        st.init(lcp);
    }
    int get_lcp(int i,int j){
        if(i==j)return n-i;
        auto [l,r]=minmax(isa[i],isa[j]);
        return st.query(l+1,r);
    }
};
```

ZAlgo.hpp
Description: Z Algorithm. z[i] := the length of the longest common prefix
between s and s[i:].
"../template/Header.hpp"   b93726, 12 lines

```cpp
template<class STR>
vector<int> z_algorithm(const STR &s){
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for(int i=1,l=0,r=1;i<n;i++){
        if(i<r)z[i]=min(r-i,z[i-l]);
        while(i+z[i]<n&&s[z[i]]==s[i+z[i]])z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    }
    return z;
}
```

PrefixFunction.hpp
Description: Prefix function. pi[i] := the length of the longest proper prefix of s[0:i] which is also a suffix of s[0:i].   3d65fe, 11 lines

```cpp
template<class STR>
vector<int> prefix_function(const STR &s){
    int n=(int)s.size();
    vector<int> pi(n);
    for(int i=1,j=0;i<n;i++){
        while(j>0&&s[i]!=s[j])j=pi[j-1];
        if(s[i]==s[j])j++;
        pi[i]=j;
    }
    return pi;
}
```

SuffixAutomaton.hpp
Description: Suffix Automaton.
Find whether a string t is a substring of a string s by traversing the automaton.
Find whether a string t is a suffix of a string s by checking whether the last node is a terminal node.
Find the number of distinct substrings of a string s by calculating the number of distinc path using DP.
Count the number of occurrences of string t in string s. Let p be the node we end up at after traversing t in the automaton. The answer is the number of paths from p to terminal nodes.
Find first occurrence of string t in string s by calculating the longest path in the automaton after reaching node p.   a50940, 49 lines

```cpp
template<class STR>
struct SuffixAutomaton{
    using T = typename STR::value_type;
    struct Node{
        map<T,int> nxt;
        int link,len;
        Node(int link,int len):link(link),len(len){}
    };
    vector<Node> nodes;
    int last;
    SuffixAutomaton():nodes{Node(-1,0)},last(0){}
    SuffixAutomaton(const STR &s):SuffixAutomaton(){
        for(auto c:s)extend(c);
    }
    int new_node(int link,int len){
        nodes.emplace_back(Node(link,len));
        return (int)nodes.size()-1;
    }
    void extend(T c){
        int cur=new_node(0,nodes[last].len+1);
        int p=last;
        while(p!=-1&&!nodes[p].nxt.count(c)){
            nodes[p].nxt[c]=cur;
            p=nodes[p].link;
        }
        if(p!=-1){
            int q=nodes[p].nxt[c];
            if(nodes[p].len+1==nodes[q].len){
                nodes[cur].link=q;
            }else{
                int r=new_node(nodes[q].link,nodes[p].len+1);
                nodes[r].nxt=nodes[q].nxt;
                while(p!=-1&&nodes[p].nxt[c]==q){
                    nodes[p].nxt[c]=r;
                    p=nodes[p].link;
                }
                nodes[q].link=nodes[cur].link=r;
            }
        }
```

```cpp
        last=cur;
    }
    ll distinct_substrings(){
        ll res=0;
        for(int i=1;i<(int)nodes.size();i++){
            res+=nodes[i].len-nodes[nodes[i].link].len;
        }
        return res;
    }
};
```

Geometry (12)

12.1   Geometric primitives

Point.h
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)   47ec0a, 28 lines

```cpp
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()=1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h
Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.   f6bf6b, 4 lines
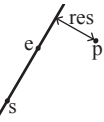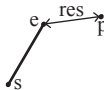
```cpp
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h
Description:
Returns the shortest distance between point p and the line segment from point s to e.

Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;   5c88f4, 6 lines

```cpp
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h
Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.
Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
"Point.h", "OnSegment.h"   9d57f2, 13 lines
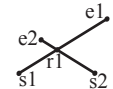
```cpp
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
         oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}
```

lineIntersection.h
Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.
Usage: auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
"Point.h"   a01f81, 8 lines
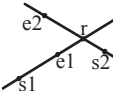
```cpp
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h
Description: Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage: `bool left = sideOf(p1,p2,q)==1;`

`"Point.h"`   3af81c, 9 lines
```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h
Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

`"Point.h"`   c597e8, 3 lines
```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h
Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

`"Point.h"`   03a306, 6 lines
```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h
Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
Usage: `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively`
`oriented triangles with vertices at 0 and i`   0f0602, 35 lines
```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
```

```
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

12.2 Circles
CircleIntersection.h
Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

`"Point.h"`   84d6d3, 11 lines
```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h
Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

`"Point.h"`   b0153d, 13 lines
```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0) return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CirclePolygonIntersection.h
Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

`"../../content/geometry/Point.h"`   a1ee63, 19 lines
```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
```

```
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```
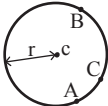
circumcircle.h
Description:
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

`"Point.h"`   1caa3a, 9 lines
```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
         abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h
Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

`"circumcircle.h"`   09dd0a, 17 lines
```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

12.3 Polygons
InsidePolygon.h
Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

`"Point.h"`, `"OnSegment.h"`, `"SegmentDistance.h"`   2bf504, 11 lines
```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

<div align="right">f12300, 6 lines</div>

"Point.h"

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

<div align="right">9706dc, 9 lines</div>

"Point.h"

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    }
    return res / A / 3;
}
```
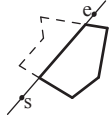
PolygonCut.h

Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

<div align="right">f2b7d4, 13 lines</div>

"Point.h", "lineIntersection.h"

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0;
        if (side != (s.cross(e, prev) < 0))
            res.push_back(lineInter(s, e, cur, prev).second);
        if (side)
            res.push_back(cur);
    }
    return res;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

<div align="right">310954, 13 lines</div>

"Point.h"

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0;
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(n)$

<div align="right">c571b8, 12 lines</div>

"Point.h"

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (;; j = (j + 1) % n) {
            res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

<div align="right">71446b, 14 lines</div>

"Point.h", "sideOf.h", "OnSegment.h"

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet$ $(-1, -1)$ if no collision, $\bullet$ $(i, -1)$ if touching the corner i, $\bullet$ (i, i) if along side $(i, i+1)$, $\bullet$ (i, j) if crossing sides $(i, i+1)$ and $(j, j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i, i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

<div align="right">7cf45b, 39 lines</div>

"Point.h"

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
```

```
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    }
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]};
        }
    return res;
}
```

12.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

<div align="right">ac41a6, 17 lines</div>

"Point.h"

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
        S.insert(p);
    }
    return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

<div align="right">bac5b0, 63 lines</div>

"Point.h"

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
```

```
        x0 = min(x0, p.x); x1 = max(x1, p.x);
        y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
        // split on x if width >= height (not ideal...)
        sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
        // divide by taking half the array for each child (not
        // best performance with many duplicates in the middle)
        int half = sz(vp)/2;
        first = new Node({vp.begin(), vp.begin() + half});
        second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h
Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], t[1][1], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$
"Point.h"   eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
```

```
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
      while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
      }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
```

```
  return pts;
}
```

12.5 3D

PolyhedronVolume.h
Description: Magic formula for the volume of a polyhedron. Faces should point outwards.
3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h
Description: Class to handle points in 3D space. T can be e.g. double or long long.
8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h
Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$
"Point3D.h"   5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };
```

```cpp
vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h
Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.
611f07, 8 lines

```cpp
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Dynamic Programming (13)

DVC.hpp
Description: Optimize $O(N^2K)$ to $O(NK \log N)$
"../template/Header.hpp"   aa5ddf, 19 lines

```cpp
vector<vl> cst, dp;

ll cost(int l, int r) {
```

```cpp
  return cst[l][r];
}

void divide(int l, int r, int opt_l, int opt_r, int c) {
  if(l > r) return ;
  int mid = (l + r) / 2;
  pair<ll, int> best = make_pair(INF, -1);
  for(int k=opt_l; k<=min(mid, opt_r); ++k) {
    best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid),
        k));
  }
  dp[c][mid] = best.first;
  divide(l, mid - 1, opt_l, best.second, c);
  divide(mid + 1, r, best.second, opt_r, c);
}

// for(int c=1; c<=K; ++c) divide(1, N, 1, N, c);
```

SlopeTrick.hpp
Description: Absolute Smth
"../template/Header.hpp"   f62f9a, 36 lines

```cpp
ll extending_value;

struct slope_trick {
  multiset<ll> ms_l, ms_r;
  ll min_y = 0ll, lz_l = 0ll, lz_r = 0ll;
  bool extending = false;
  void add_line(ll v) {
    if(extending) {
      lz_l -= extending_value;
      lz_r -= extending_value;
    }
    extending = true;
    if(ms_l.empty() && ms_r.empty()) {
      ms_l.emplace(v);
      ms_r.emplace(v);
    }
    else if(v <= *ms_l.rbegin() + lz_l) {
      min_y += (*ms_l.rbegin() + lz_l) - v;
      ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
      ms_l.erase(--ms_l.end());
      ms_l.emplace(v - lz_l);
      ms_l.emplace(v - lz_l);
    }
    else if(v >= *ms_r.begin() + lz_r) {
      min_y += v - (*ms_r.begin() + lz_r);
      ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
      ms_r.erase(ms_r.begin());
      ms_r.emplace(v - lz_r);
      ms_r.emplace(v - lz_r);
    }
    else {
      ms_l.emplace(v - lz_l);
      ms_r.emplace(v - lz_r);
    }
  }
};
```

Convolutions (14)

AndConvolution.hpp
Description: Bitwise AND Convolution. Superset Zeta Transform: $A'[S] = \sum_{T \supseteq S} A[T]$. Superset Mobius Transform: $A[T] = \sum_{S \supseteq T} (-1)^{|S-T|} A'[S]$.
Time: $\mathcal{O}(N \log N)$.
"../template/Header.hpp"   7916f8, 34 lines

```cpp
template<class T>
void superset_zeta(vector<T> &a){
```

```cpp
  int n=(int)a.size();
  assert(n==(n&-n));
  for(int i=1;i<n;i<<=1){
    for(int j=0;j<n;j++){
      if(j&i){
        a[j^i]+=a[j];
      }
    }
  }
}

template<class T>
void superset_mobius(vector<T> &a){
  int n=(int)a.size();
  assert(n==(n&-n));
  for(int i=n;i>>=1;){
    for(int j=0;j<n;j++){
      if(j&i){
        a[j^i]-=a[j];
      }
    }
  }
}

template<class T>
vector<T> and_convolution(vector<T> a,vector<T> b){
  superset_zeta(a);
  superset_zeta(b);
  for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
  superset_mobius(a);
  return a;
}
```

GCDConvolution.hpp
Description: GCD Convolution. Multiple Zeta Transform: $A'[n] = \sum_{n|m} A[m]$. Multiple Mobius Transform: $A[n] = \sum_{n|m} \mu(m/n) A'[m]$.
Time: $\mathcal{O}(N \log \log N)$.
"../template/Header.hpp"   7f6c2d, 34 lines

```cpp
template<class T>
void multiple_zeta(vector<T> &a){
  int n=(int)a.size();
  vector<bool> is_prime(n,true);
  for(int p=2;p<n;p++){
    if(!is_prime[p])continue;
    for(int i=(n-1)/p;i>=1;i--){
      is_prime[i*p]=false;
      a[i]+=a[i*p];
    }
  }
}

template<class T>
void multiple_mobius(vector<T> &a){
  int n=(int)a.size();
  vector<bool> is_prime(n,true);
  for(int p=2;p<n;p++){
    if(!is_prime[p])continue;
    for(int i=1;i*p<n;i++){
      is_prime[i*p]=false;
      a[i]-=a[i*p];
    }
  }
}

template<class T>
vector<T> gcd_convolution(vector<T> a,vector<T> b){
  multiple_zeta(a);
  multiple_zeta(b);
```

```
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    multiple_mobius(a);
    return a;
}
```

LCMConvolution.hpp
Description: LCM Convolution. Divisor Zeta Transform: $A'[n] = \sum_{d|n} A[d]$. Divisor Mobius Transform: $A[n] = \sum_{d|n} \mu(n/d) A'[d]$.
Time: $\mathcal{O}(N \log \log N)$.

`"../template/Header.hpp"`   41fe9d, 34 lines

```
template<class T>
void divisor_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i*p]+=a[i];
        }
    }
}

template<class T>
void divisor_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i*p]-=a[i];
        }
    }
}

template<class T>
vector<T> lcm_convolution(vector<T> a,vector<T> b){
    divisor_zeta(a);
    divisor_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    divisor_mobius(a);
    return a;
}
```

ORConvolution.hpp
Description: Bitwise OR Convolution. Subset Zeta Transform: $A'[S] = \sum_{T \subseteq S} A[T]$. Subset Mobius Transform: $A[T] = \sum_{S \subseteq T} (-1)^{|T-S|} A'[S]$.
Time: $\mathcal{O}(N \log N)$.

`"../template/Header.hpp"`   c58b77, 34 lines

```
template<class T>
void subset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]+=a[j^i];
            }
        }
    }
}

template<class T>
void subset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
```

```
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]-=a[j^i];
            }
        }
    }
}

template<class T>
vector<T> or_convolution(vector<T> a,vector<T> b){
    subset_zeta(a);
    subset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    subset_mobius(a);
    return a;
}
```

XORConvolution.hpp
Description: Bitwise XOR Convolution. Fast Walsh-Hadamard Transform: $A'[S] = \sum_T (-1)^{|S\&T|} A[T]$.
Time: $\mathcal{O}(N \log N)$.

`"../template/Header.hpp"`   05848d, 29 lines

```
template<class T>
void fwht(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                T &u=a[j^i],&v=a[j];
                tie(u,v)=make_pair(u+v,u-v);
            }
        }
    }
}

template<class T>
vector<T> xor_convolution(vector<T> a,vector<T> b){
    int n=(int)a.size();
    fwht(a);
    fwht(b);
    for(int i=0;i<n;i++)a[i]*=b[i];
    fwht(a);
    T div=T(1)/T(n);
    if(div==T(0)){
        for(auto &x:a)x/=n;
    }else{
        for(auto &x:a)x*=div;
    }
    return a;
}
```

MaxPlusConvolution.hpp
Description: Max Plus Convolution. Find $C[k] = max_{i+j=k}\{A[i] + B[j]\}$.
Time: $\mathcal{O}(N)$.

7176a2, 94 lines

```
// SMAWK algorithm for finding row-wise maxima.
// f(i,j,k) checks if M[i][j] <= M[i][k].
// f(i,j,k) checks if M[i][k] is at least as good as M[i][j].
// higher is better.
template<class F>
vector<int> smawck(const F &f,const vector<int> &rows,const
    vector<int> &cols){
    int n=(int)rows.size(),m=(int)cols.size();
    if(max(n,m)<=2){
        vector<int> ans(n,-1);
        for(int i=0;i<n;i++){
```

```
        for(int j:cols){
            if(ans[i]==-1||f(rows[i],ans[i],j)){
                ans[i]=j;
            }
        }
    }
    return ans;
}
if(n<m){
    // reduce
    vector<int> st;
    for(int j:cols){
        while(true){
            if(st.empty()){
                st.emplace_back(j);
                break;
            }else if(f(rows[(int)st.size()-1],st.back(),j))
                {
                st.pop_back();
            }else if(st.size()<n){
                st.emplace_back(j);
                break;
            }else{
                break;
            }
        }
    }
    return smawck(f,rows,st);
}
vector<int> ans(n,-1);
vector<int> new_rows;
for(int i=1;i<n;i+=2){
    new_rows.emplace_back(rows[i]);
}
auto res=smawck(f,new_rows,cols);
for(int i=0;i<new_rows.size();i++){
    ans[2*i+1]=res[i];
}
for(int i=0,l=0,r=0;i<n;i+=2){
    if(i+1==n)r=m;
    while(r<m&&cols[r]<=ans[i+1])r++;
    ans[i]=cols[l++];
    for(;l<r;l++){
        if(f(rows[i],ans[i],cols[l])){
            ans[i]=cols[l];
        }
    }
    l--;
}
return ans;
}

template<class F>
vector<int> smawck(const F &f,int n,int m){
    vector<int> rows(n),cols(m);
    iota(rows.begin(),rows.end(),0);
    iota(cols.begin(),cols.end(),0);
    return smawck(f,rows,cols);
}

// Max Plus Convolution.
// b must be convex, i.e. b[i]-b[i-1]>=b[i+1]-b[i].
template<class T>
vector<T> max_plus_convolution_arbitary_convex(vector<T> a,
    const vector<T> &b){
    if(a.empty()||b.empty())return {};
    if((int)b.size()==1){
        for(auto &x:a)x+=b[0];
        return a;
```

```
    }
    int n=(int)a.size(),m=(int)b.size();
    auto f=[&](int i,int j){
        return a[j]+b[i-j];
    };
    auto cmp=[&](int i,int j,int k){
        if(i<k)return false;
        if(i-j>=m)return true;
        return f(i,j)<=f(i,k);
    };
    auto best=smawck(cmp,n+m-1,n);
    vector<T> ans(n+m-1);
    for(int i=0;i<n+m-1;i++){
        ans[i]=f(i,best[i]);
    }
    return ans;
}
```

Various (15)

GaussianElimination.hpp
Description: Gaussian Elimination
"../template/Header.hpp"   e89ecb, 34 lines

```
struct Gauss {
  int n, sz;
  vector<ll> basis;
  Gauss(int n = 0) {
    init(n);
  }
  void init(int _n) {
    n = _n, sz = 0;
    basis.assign(n, 0);
  }
  void insert(ll x) {
    for (int i = n - 1; i >= 0; i--)
      if (x >> i & 1) {
        if (!basis[i]) {
          basis[i] = x;
          sz++;
          return;
        }
        x ^= basis[i];
      }
  }
  ll getmax(ll k = 0) {
    ll tot = 1ll << sz, res = 0;
    for (int i = n - 1; i >= 0; i--)
      if (basis[i]) {
        tot >>= 1;
        if ((k >= tot && res >> i & 1) || (k < tot && res >> i
             & 1 ^ 1))
          res ^= basis[i];
        if (k >= tot)
          k -= tot;
      }
    return res;
  }
};
```

BinaryTrie.hpp
Description: Binary Trie
"../template/Header.hpp"   525bf4, 59 lines

```
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
  vector<node_t> t = {node_t()};
  vector<int> cnt = {0};
```

```
  int cnt_nodes = 0;
  void insert(int v) {
    int cur = 0;
    cnt[0]++;
    for(int i=S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1: 0;
      if(!t[cur][b]) {
        t[cur][b] = ++cnt_nodes;
        t.emplace_back(node_t());
        cnt.emplace_back(0);
      }
      cnt[t[cur][b]]++;
      cur = t[cur][b];
    }
  }
  void remove(int v) {
    int cur = 0;
    cnt[0]--;
    for(int i=S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1: 0;
      cnt[t[cur][b]]--;
      cur = t[cur][b];
    }
  }
  int get_min(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1 : 0;
      if(t[cur][b] && cnt[t[cur][b]]) {
        cur = t[cur][b];
      }
      else {
        res |= (1 << i);
        cur = t[cur][!b];
      }
    }
    return res;
  }

  int get_max(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1 : 0;
      if(t[cur][!b] && cnt[t[cur][!b]]) {
        res |= (1 << i);
        cur = t[cur][!b];
      }
      else {
        cur = t[cur][b];
      }
    }
    return res;
  }
};
```

InfixPropostfix.hpp
Description: Infix to Pro-Postfix
"../template/Header.hpp"   517f57, 47 lines

```
stack<char> opr;
stack<int> val;

bool isOpr(char x){
  return x == '+' || x == '*';
}

int prio(char x) {
  if(x == '(') return -1;
  if(x == '+') return 1;
  if(x == '*') return 2;
```

```
  return 0;
}

int do_opr(int l, int r, char o) {
  if(o == '+') {
    return l + r;
  }
  return l * r;
}

void pop_stack() {
  int rhs = val.top(); val.pop();
  int lhs = val.top(); val.pop();
  int new_val = do_opr(lhs, rhs, opr.top());
  val.emplace(new_val);
  opr.pop();
}

int cal(string s) {
  for(auto x: s) {
    if(isdigit(x)) val.emplace(x - '0');
    else if(x == '(') opr.emplace('(');
    else if(x == ')') {
      while(!opr.empty() && opr.top() != '(')
        pop_stack();
      opr.pop();
    }
    else {
      while(!opr.empty() && prio(opr.top()) >= prio(x))
        pop_stack();
      opr.emplace(x);
    }
  }
  while(!opr.empty()) pop_stack();
  return val.top();
}
```

15.1 Optimization tricks
`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

15.1.1 Bit hacks

- `x & -x` is the least bit in x.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of m (except m itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after x with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

15.1.2 Pragmas

- **#pragma** GCC optimize ("Ofast") will make GCC auto-vectorize loops and optimizes floating points better.
- **#pragma** GCC target ("avx2") can double performance of vectorized code, but causes crashes on old machines.
- **#pragma** GCC optimize ("trapv") kills the program on integer overflows (but is really slow).

Competitive Programming Topics

(A)

topics.txt
_____ 159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
```

```
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
```

```
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```