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template from KACTL

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Template (1)

template.cpp

27 lines

```
#pragma once

#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

using ll = long long;
using db = long double;
using vi = vector<int>;
using vl = vector<ll>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<db, db>;
const int INF = 0x3fffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const ll LINF = 0x1fffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));

int main(){
  cin.tie(nullptr)->sync_with_stdio(false);
}
```

c.sh

2 lines

```
g++ -std=gnu++2a -Wall $1 -o a.out
./a.out
```

Mathematics (2)

2.1 Goldbatch's Conjecture

- Even number can be written in sum of two primes (Up to $1e12$)
- Range of N^{th} prime and $N+1^{th}$ prime will be less than or equal to 300 (Up to $1e12$)

2.2 Divisibility

Number of divisors of N is given by $\prod_{i=1}^{k}(a_i+1)$ where $N=\prod_{i=1}^{k}p_i^{a_i}$ and p_i are prime factors of N.

Numerical (3)

3.1 Newton's Method

if $F(Q)=0$, then $Q_{2n}\equiv Q_n-\frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$

$$Q=P^{-1}: Q_{2n}\equiv Q_n\cdot(2-P\cdot Q_n^2) \pmod{x^{2n}}$$

$$Q=\ln P=\int \frac{P'}{P}\mathrm{d}x$$

$$Q=e^p: Q_{2n}\equiv Q_n(1+P-\ln Q_n) \pmod{x^{2n}}$$

$$Q=\sqrt{P}: Q_{2n}\equiv \frac{1}{2}(Q_n+P\cdot Q_n^{-1}) \pmod{x^{2n}}$$

$$Q=P^k=\alpha^k x^{kt} e^{k\ln T}; P=\alpha\cdot x^t\cdot T, T(0)=1$$

Group (4)

4.1 Monoid

monoid/MonoidBase.hpp

Description: Monoid Base class.

e75b74, 6 lines

```
template<class T,T (*combine)(T,T),T (*identity)()>
struct MonoidBase{
    using value_type = T;
    static constexpr T op(const T &x,const T &y){return combine
        (x,y);}
    static constexpr T unit(){return identity();}
};
```

4.2 Action

action/MonoidActionBase.hpp

Description: Monoid Action Base class.

"../../template/Header.hpp" 425d83, 11 lines

```
template<class MInfo,class MTag,typename MInfo::value_type
    (*combine)(typename MInfo::value_type,typename MTag::
        value_type)>
struct MonoidActionBase{
    using InfoMonoid = MInfo;
    using TagMonoid = MTag;
    using Info = typename InfoMonoid::value_type;
    using Tag = typename TagMonoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b){
        return combine(a,b);
    }
};
```

action/DefaultAction.hpp

Description: Default Action class.

e45000, 10 lines

```
template<class Monoid>
struct DefaultAction{
    using InfoMonoid = Monoid;
    using TagMonoid = Monoid;
    using Info = typename Monoid::value_type;
    using Tag = typename Monoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b){
        return Monoid::op(a,b);
    }
};
```

Data Structures (5)

OrderedSet.hpp

Description: Ordered Set

"../template/Header.hpp", <bits/extc++.h> 1a7f5f, 14 lines

```
using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
     tree_order_statistics_node_update>;
// can be change to less_equal

void usage() {
  ordered_set<int> st, st_2;
  st.insert(2);
  st.insert(1);
  cout << st.order_of_key(2);
  cout << *st.find_by_order(1);
  st.join(st_2); // merge
}
```

FenwickTree.hpp

Description: Fenwick / Binary Indexed Tree

43767a, 41 lines

```
template<class T>
struct Fenwick{
    int n,logn;
    vector<T> t;
    Fenwick(){}
    Fenwick(int _n){init(vector<T>(_n,T{}));}
    template<class U>
    Fenwick(const vector<U> &a){init(a);}
    template<class U>
    void init(const vector<U> &a){
        n=(int)a.size();
        logn=31-__builtin_clz(n);
        t.assign(n+1,T{});
        for(int i=1;i<=n;i++){
            t[i]=t[i]+a[i-1];
            int j=i+(i&-i);
            if(j<=n)t[j]=t[j]+t[i];
        }
    }
    void update(int x,const T &v){
        for(int i=x+1;i<=n;i+=i&-i)t[i]=t[i]+v;
    }
    void update(int l,int r,const T &v){
        update(l,v),update(r+1,-v);
    }
    T query(int x){
        T res{};
        for(int i=x+1;i>0;i-=i&-i)res=res+t[i];
        return res;
    }
```

```
    T query(int l,int r){
        return query(r)-query(l-1);
    }
    int find(const T &k){
        int x=0;
        T cur{};
        for(int i=1<<logn;i>0;i>>=1)
            if(x+i<=n&&cur+t[x+i]<=k)x+=i,cur=cur+t[x];
        return x;
    }
};
```

SegmentTree.hpp

Description: Segment Tree

c51dec, 85 lines

```
template<class Monoid>
struct SegmentTree{
    using T = typename Monoid::value_type;
    int n;
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n,function<T(int)> create){init(n,create);}
    SegmentTree(int n,T v=Monoid::unit()){init(n,[&](int){
        return v;});}
    template<class U>
    SegmentTree(const vector<U> &a){init((int)a.size(),[&](int
        i){return T(a[i]);});}
    void init(int _n,function<T(int)> create){
        n=_n;
        t.assign(4<<(31-__builtin_clz(n)),Monoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
            if(l==r)return void(t[i]=create(l));
            int m=(l+r)/2;
            build(l,m,i*2);
            build(m+1,r,i*2+1);
            pull(i);
        };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=Monoid::op(t[i*2],t[i*2+1]);
    }
    void modify(int l,int r,int i,int x,const T &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const T &v){
        modify(0,n-1,1,x,v);
    }
    template<class U>
    void update(int l,int r,int i,int x,const U &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=Monoid::op(t[i],v));
        int m=(l+r)/2;
        update(l,m,i*2,x,v);
        update(m+1,r,i*2+1,x,v);
        pull(i);
    }
    template<class U>
    void update(int x,const U &v){
        update(0,n-1,1,x,v);
    }
    T query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return Monoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        return Monoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,
            x,y));
    }
    T query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};
```

LazySegmentTree.hpp

Description: Segment Tree with Lazy Propagation

91ab0c, 103 lines

```
template<class MonoidAction>
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n;
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n,function<Info(int)> create){init(n,
        create);}
    LazySegmentTree(int n,Info v=InfoMonoid::unit()){init(n
        ,[&](int){return v;});}
    template<class T>
    LazySegmentTree(const vector<T> &a){init((int)a.size(),[&](
        int i){return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        n=_n;
        int m=4<<(31-__builtin_clz(n));
        t.assign(m,InfoMonoid::unit());
        lz.assign(m,TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
            if(l==r)return void(t[i]=create(l));
            int m=(l+r)/2;
            build(l,m,i*2);
            build(m+1,r,i*2+1);
            pull(i);
        };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
    }
    void apply(int i,const Tag &v){
        t[i]=MonoidAction::op(t[i],v);
        lz[i]=TagMonoid::op(lz[i],v);
    }
    void push(int i){
        apply(i*2,lz[i]);
        apply(i*2+1,lz[i]);
        lz[i]=TagMonoid::unit();
    }
    void modify(int l,int r,int i,int x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        push(i);
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const Info &v){
        modify(0,n-1,1,x,v);
    }
    void update(int l,int r,int i,int x,int y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(i,v);
        int m=(l+r)/2;
        push(i);
        update(l,m,i*2,x,y,v);
        update(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void update(int x,int y,const Tag &v){
        update(0,n-1,1,x,y,v);
    }
    Info query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        push(i);
        return InfoMonoid::op(query(l,m,i*2,x,y),query(m+1,r,i
            *2+1,x,y));
    }
    Info query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
```

```
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};
```

DynamicSegmentTree.hpp

Description: Dynamic Segment Tree

e84eeb, 106 lines

```
template<class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node{
        Info val;
        Tag lz;
        Ptr l,r;
        Node(Info v):val(v),lz(TagMonoid::unit()),l(nullptr),r(
            nullptr){}
        Node(Info v,Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr rt;
    function<Info(ll,ll)> create;
    DynamicSegmentTree(){init(0,0);}
    DynamicSegmentTree(ll n){init(0,n-1);}
    DynamicSegmentTree(ll lb,ll ub){init(lb,ub);}
    DynamicSegmentTree(ll lb,ll ub,function<Info(ll,ll)> create
        ){init(lb,ub,create);}
    void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll
        l,ll r){return InfoMonoid::unit();}){
        lb=_lb,ub=_ub;
        create=_create;
        rt=new Node(create(lb,ub));
    }
    Info val(Ptr t){
        return t?t->val:InfoMonoid::unit();
    }
    void pull(Ptr &t){
        t->val=InfoMonoid::op(val(t->l),val(t->r));
    }
    void apply(Ptr &t,const Tag &v,ll l,ll r){
        if(!t)t=new Node(create(l,r));
        t->val=MonoidAction::op(t->val,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void push(Ptr &t,ll l,ll m,ll r){
        apply(t->l,t->lz,l,m);
        apply(t->r,t->lz,m+1,r);
        t->lz=TagMonoid::unit();
    }
    void modify(ll l,ll r,Ptr &t,ll x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t->val=v);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        modify(l,m,t->l,x,v);
        modify(m+1,r,t->r,x,v);
        pull(t);
    }
    void modify(ll x,const Info &v){
        modify(lb,ub,rt,x,v);
    }
    void update(ll l,ll r,Ptr &t,ll x,ll y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(t,v,l,r);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        update(l,m,t->l,x,y,v);
        update(m+1,r,t->r,x,y,v);
        pull(t);
    }
    void update(ll x,ll y,const Tag &v){
        update(lb,ub,rt,x,y,v);
    }
    Info query(ll l,ll r,Ptr &t,ll x,ll y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t->val;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t
            ->r,x,y));
    }
    Info query(ll x,ll y){
        return query(lb,ub,rt,x,y);
    }
    template<class F>
    ll findfirst(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findfirst(l,m,t->l,x,y,f);
        if(res==-1)res=findfirst(m+1,r,t->r,x,y,f);
        return res;
    }
    template<class F>
    ll findfirst(ll x,ll y,const F &f){
        return findfirst(lb,ub,rt,x,y,f);
    }
    template<class F>
    ll findlast(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!t||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findlast(m+1,r,t->r,x,y,f);
        if(res==-1)res=findlast(l,m,t->l,x,y,f);
        return res;
    }
    template<class F>
    ll findlast(ll x,ll y,const F &f){
        return findlast(lb,ub,rt,x,y,f);
    }
};
```

DSU.hpp

Description: Disjoint Set Union.

0b3cb8, 26 lines

```
struct DSU{
    vector<int> p,sz;
    DSU(){}
    DSU(int n){init(n);}
    void init(int n){
        p.resize(n);
        iota(p.begin(),p.end(),0);
        sz.assign(n,1);
    }
    int find(int u){
        return p[u]==u?u:p[u]=find(p[u]);
    }
    bool same(int u,int v){
        return find(u)==find(v);
    }
    bool merge(int u,int v){
        u=find(u),v=find(v);
        if(u==v)return false;
        sz[u]+=sz[v];
        p[v]=u;
        return true;
    }
    int size(int u){
        return sz[find(u)];
    }
};
```

BinaryTrie.hpp

Description: Binary Trie

ae5b7a, 66 lines

```
template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
    struct Node{
        array<int,2> ch;
        S cnt;
        Node():ch{-1,-1},cnt(0){}
    };
    vector<Node> t;
    BinaryTrie():t{Node()}{}
    int new_node(){
        t.emplace_back(Node());
        return t.size()-1;
    }
    S size(){
        return t[0].cnt;
    }
    bool empty(){
        return size()==0;
    }
    S get_cnt(int i){
        return i!=-1?t[i].cnt:S(0);
    }
    void insert(T x,S k=1){
        int u=0;
        t[u].cnt+=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(t[u].ch[v]==-1)t[u].ch[v]=new_node();
            u=t[u].ch[v];
            t[u].cnt+=k;
        }
    }
    void erase(T x,S k=1){
        int u=0;
        assert(t[u].cnt>=k);
        t[u].cnt-=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            u=t[u].ch[v];
            assert(u!=-1&&t[u].cnt>=k);
            t[u].cnt-=k;
        }
    }
    T kth(S k,T x=0){
        assert(k<size());
        int u=0;
        T res=0;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(k<get_cnt(t[u].ch[v])){
```

```
            u=t[u].ch[v];
        }else{
            res|=T(1)<<i;
            k-=get_cnt(t[u].ch[v]);
            u=t[u].ch[v^1];
        }
    }
    return res;
}
T min(T x){
    return kth(0,x);
}
T max(T x){
    return kth(size()-1,x);
}
};
```

LiChaoTree.hpp

Description: Li-Chao Tree (minimize).

4ab713, 52 lines

```
template<class T>
struct LiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    vector<T> xs;
    vector<Line> t;
    LiChaoTree(){}
    LiChaoTree(const vector<T> &x):xs(x){init(x);}
    LiChaoTree(int n):xs(n){
        vector<T> x(n);
        iota(x.begin(),x.end(),0);
        init(x);
    }
    void init(const vector<T> &x){
        xs=x;
        sort(xs.begin(),xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    }
    void insert(int l,int r,int i,Line v){
        int m=(l+r)/2;
        if(v.eval(xs[m])<t[i].eval(xs[m]))swap(t[i],v);
        if(v.eval(xs[l])<t[i].eval(xs[l]))insert(l,m,i*2,v);
        if(v.eval(xs[r])<t[i].eval(xs[r]))insert(m+1,r,i*2+1,v)
            ;
    }
    inline void insert(T m,T c){
        insert(0,(int)xs.size()-1,1,Line(m,c));
    }
    void insert_range(int l,int r,int i,T x,T y,Line v){
        if(y<xs[l]||xs[r]<x)return;
        if(x<=xs[l]&&xs[r]<=y)return insert(l,r,i,v);
        int m=(l+r)/2;
        insert_range(l,m,i*2,x,y,v);
        insert_range(m+1,r,i*2+1,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(0,(int)xs.size()-1,1,x,y,Line(m,c));
    }
    T query(int l,int r,int i,T x){
        if(l==r)return t[i].eval(x);
        int m=(l+r)/2;
        if(x<=xs[m])return min(t[i].eval(x),query(l,m,i*2,x));
        return min(t[i].eval(x),query(m+1,r,i*2+1,x));
    }
    inline T query(T x){
        return query(0,(int)xs.size()-1,1,x);
    }
};
```

DynamicLiChaoTree.hpp

Description: Dynamic Li-Chao Tree (minimize).

b8af36, 50 lines

```
template<class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(
        nullptr){}
    void insert(T l,T r,Ptr &t,Line v){
        if(!t)return void(t=new Node(v));
        T m=l+(r-l)/2;
        if(v.eval(m)<t->v.eval(m))swap(t->v,v);
        if(v.eval(l)<t->v.eval(l))insert(l,m,t->l,v);
        if(v.eval(r)<t->v.eval(r))insert(m+1,r,t->r,v);
    }
    inline void insert(T m,T c){
        insert(lb,ub,root,Line(m,c));
    }
    void insert_range(T l,T r,Ptr &t,T x,T y,Line v){
        if(y<l||r<x)return;
        if(!t)t=new Node();
        if(x<=l&&r<=y)return insert(l,r,t,v);
        T m=l+(r-l)/2;
        insert_range(l,m,t->l,x,y,v);
        insert_range(m+1,r,t->r,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(lb,ub,root,x,y,Line(m,c));
    }
    T query(T l,T r,Ptr t,T x){
        if(!t)return INF;
        T m=l+(r-l)/2;
        if(x<=m)return min(t->v.eval(x),query(l,m,t->l,x));
        return min(t->v.eval(x),query(m+1,r,t->r,x));
    }
    inline T query(T x){
        return query(lb,ub,root,x);
    }
};
```

SplayTreeBase.hpp

Description: Splay Tree. splay(u) will make node u be the root of the tree in amortized O(log n) time.

cc90a9, 113 lines

```
template<class Node>
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return !(t->p)||(t->p->l!=t&&t->p->r!=t);
    } // The parent of the root stores the path parant in link
        cut tree.
    int size(Ptr t){
        return t?t->size:0;
    }
    virtual void push(Ptr t){};
    virtual void pull(Ptr t){};
    int pos(Ptr t){
        if(t->p){
            if(t->p->l==t)return -1;
            if(t->p->r==t)return 1;
        }
        return 0;
    }
    void rotate(Ptr t){
        Ptr x=t->p,y=x->p;
        if(pos(t)==-1){
            if((x->l=t->r))t->r->p=x;
            t->r=x,x->p=t;
        }else{
            if((x->r=t->l))t->l->p=x;
            t->l=x,x->p=t;
        }
        pull(x),pull(t);
        if((t->p=y)){
            if(y->l==x)y->l=t;
            if(y->r==x)y->r=t;
        }
    }
    void splay(Ptr t){
        if(!t)return;
        push(t);
        while(!is_root(t)){
            Ptr x=t->p;
            if(is_root(x)){
                push(x),push(t);
                rotate(t);
            }else{
                Ptr y=x->p;
                push(y),push(x),push(t);
                if(pos(x)==pos(t))rotate(x),rotate(t);
                else rotate(t),rotate(t);
            }
        }
    }
    Ptr get_first(Ptr t){
        while(t->l)push(t),t=t->l;
        splay(t);
        return t;
    }
    Ptr get_last(Ptr t){
        while(t->r)push(t),t=t->r;
        splay(t);
        return t;
    }
    Ptr merge(Ptr l,Ptr r){
        splay(l),splay(r);
        if(!l)return r;
        if(!r)return l;
        l=get_last(l);
        l->r=r;
        r->p=l;
        pull(l);
        return l;
    }
    pair<Ptr,Ptr> split(Ptr t,int k){
        if(!t)return {nullptr,nullptr};
        if(k==0)return {nullptr,t};
        if(k==size(t))return {t,nullptr};
```

```
        push(t);
        if(k<=size(t->l)){
            auto x=split(t->l,k);
            t->l=x.second;
            t->p=nullptr;
            if(x.second)x.second->p=t;
            pull(t);
            return {x.first,t};
        }else{
            auto x=split(t->r,k-size(t->l)-1);
            t->r=x.first;
            t->p=nullptr;
            if(x.first)x.first->p=t;
            pull(t);
            return {t,x.second};
        }
    }
    void insert(Ptr &t,int k,Ptr v){
        splay(t);
        auto x=split(t,k);
        t=merge(merge(x.first,v),x.second);
    }
    void erase(Ptr &t,int k){
        splay(t);
        auto x=split(t,k);
        auto y=split(x.second,1);
        // delete y.first;
        t=merge(x.first,y.second);
    }
    template<class T>
    Ptr build(const vector<T> &v){
        if(v.empty())return nullptr;
        function<Ptr(int,int)> build=[&](int l,int r){
            if(l==r)return new Node(v[l]);
            int m=(l+r)/2;
            return merge(build(l,m),build(m+1,r));
        };
        return build(0,v.size()-1);
    }
};
```

LazyReversibleBBST.hpp

Description: Lazy Reversible BBST Base.

904708, 81 lines

```
template<class Tree,class Node,class MonoidAction>
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;

    LazyReversibleBBST()=default;

    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
    }
    void pull(Ptr t){
        if(!t)return;
        push(t);
        t->size=1;
        t->sum=t->val;
        t->revsum=t->val;
        if(t->l){
            t->size+=t->l->size;
            t->sum=InfoMonoid::op(t->l->sum,t->sum);
            t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);
        }
        if(t->r){
            t->size+=t->r->size;
            t->sum=InfoMonoid::op(t->sum,t->r->sum);
            t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
        }
    }
    void push(Ptr t){
        if(!t)return;
        if(t->rev){
            toggle(t->l);
            toggle(t->r);
            t->rev=false;
        }
        if(t->lz!=TagMonoid::unit()){
            propagate(t->l,t->lz);
            propagate(t->r,t->lz);
            t->lz=TagMonoid::unit();
        }
    }
    void toggle(Ptr t){
        if(!t)return;
        swap(t->l,t->r);
        swap(t->sum,t->revsum);
        t->rev^=true;
    }
    void propagate(Ptr t,const Tag &v){
        if(!t)return;
        t->val=MonoidAction::op(t->val,v);
        t->sum=MonoidAction::op(t->sum,v);
        t->revsum=MonoidAction::op(t->revsum,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void apply(Ptr &t,int l,int r,const Tag &v){
        if(l>r)return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        propagate(y.first,v);
        t=merge(x.first,merge(y.first,y.second));
    }
    Info query(Ptr &t,int l,int r){
        if(l>r)return InfoMonoid::unit();
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        Info res=sum(y.first);
        t=merge(x.first,merge(y.first,y.second));
        return res;
    }
    void reverse(Ptr &t,int l,int r){
        if(l>r)return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        toggle(y.first);
        t=merge(x.first,merge(y.first,y.second));
    }
};
```

LazyReversibleSplayTree.hpp

Description: Lazy Reversible Splay Tree.

"SplayTreeBase.hpp", "LazyReversibleBBST.hpp" b8455b, 23 lines

```
template<class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
    Ptr l,r,p;
    Info val,sum,revsum;
    Tag lz;
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::
        unit(),const Tag &_lz=TagMonoid::unit())
        :l(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),
            size(1),rev(false){}
};

template<class MonoidAction>
struct LazyReversibleSplayTree
    : LazyReversibleBBST<SplayTreeBase<
        LazyReversibleSplayTreeNode<MonoidAction>>,
      LazyReversibleSplayTreeNode<MonoidAction>,MonoidAction>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};
```

LinkCutTreeBase.hpp

Description: Link Cut Tree Base.
Usage: `evert(u)`: make u be the root of the tree.
`link(u,v)`: attach u to v.
`cut(u,v)`: remove edge between u and v.
`get_root(u)`: get the root of the tree containing u.
`lca(u,v)`: get the lowest common ancestor of u and v.
`fold(u,v)`: get the value of the path from u to v.

b432c3, 59 lines

```
template<class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for(Ptr cur=t;cur;cur=cur->p){
            this->splay(cur);
            cur->r=pc;
            this->pull(cur);
            pc=cur;
        }
        this->splay(t);
        return pc;
    }
    void evert(Ptr t){ // make t be the root of the tree
        expose(t);
        this->toggle(t);
        this->push(t);
    }
    void link(Ptr u,Ptr v){ // attach u to v
        evert(u);
        expose(v);
        u->p=v;
    }
    void cut(Ptr u,Ptr v){ // cut edge between u and v
        evert(u);
        expose(v);
        assert(u->p==v);
        v->l=u->p=nullptr;
        this->pull(v);
    }
    Ptr get_root(Ptr t){
        expose(t);
        while(t->l)this->push(t),t=t->l;
        this->splay(t);
        return t;
    }
    Ptr lca(Ptr u,Ptr v){
        if(get_root(u)!=get_root(v))return nullptr;
        expose(u);
```

```
        return expose(v);
    }
    void set_val(Ptr t,const T &val){
        this->evert(t);
        t->val=val;
        this->pull(t);
    }
    T get_val(Ptr t){
        this->evert(t);
        return t->val;
    }
    T fold(Ptr u,Ptr v){
        evert(u);
        expose(v);
        return v->sum;
    }
};
```

LazyLinkCutTree.hpp

Description: Lazy Link Cut Tree.

Usage:

```
using Lct = LazyLinkCutTree<Action>;
using Ptr = Lct::Ptr;
using Node = Lct::Node;
vector<Ptr> ptr(n);
for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);
auto link=[&](int u,int v){
Lct::link(ptr[u],ptr[v]);
};
auto cut=[&](int u,int v){
Lct::cut(ptr[u],ptr[v]);
};
auto update=[&](int u,int v,const Action::Tag &val){
Lct::apply(ptr[u],ptr[v],val);
};
auto query=[&](int u,int v){
return Lct::fold(ptr[u],ptr[v]);
};
```

"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp" ead3da, 12 lines

```
template<class MonoidAction>
struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree<
     MonoidAction>>{
    using base = LinkCutTreeBase<LazyReversibleSplayTree<
         MonoidAction>>;
    using Ptr = typename base::Ptr;
    using Tag = typename MonoidAction::Tag;

    void apply(Ptr u,Ptr v,const Tag &val){
        this->evert(u);
        this->expose(v);
        this->propagate(v,val);
    }
};
```

Number Theory (6)

ExtendedEuclid.hpp

Description: Extended Euclid algorithm for solving diophantine equation (ax + by = gcd(a, b)).

Time: $\mathcal{O}(\log \max\{a, b\})$

"../template/Header.hpp" 229e7c, 13 lines

```
pair<ll,ll> euclid(ll a,ll b){
    ll x=1,y=0,x1=0,y1=1;
    while(b!=0){
        ll q=a/b;
        x-=q*x1;
        y-=q*y1;
        a-=q*b;
        swap(x,x1);
        swap(y,y1);
        swap(a,b);
    }
    return {x,y};
}
```

6.1 Prime Numbers

LinearSieve.hpp

Description: Prime Number Generator in Linear Time

Time: $\mathcal{O}(N)$

"../template/Header.hpp" 194fb1, 15 lines

```
vi linear_sieve(int n) {
  vi prime, composite(n + 1);
  for(int i=2; i<=n; ++i) {
    if(!composite[i]) {
      prime.emplace_back(i);
    }
    for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {
      composite[i * prime[j]] = true;
      if(i % prime[j] == 0) {
        break;
      }
    }
  }
  return prime;
}
```

FastEratosthenes.hpp

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 $\approx$ 1.5s

"../template/Header.hpp" 295b58, 33 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int) round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S + 1);
  pr.reserve(int(LIM/log(LIM) * 1.1));
  vector<pii> cp;
  for(int i=3; i<=S; i+=2) {
    if(!sieve[i]) {
      cp.emplace_back(i, i * i / 2);
      for(int j=i*i; j<=S; j+=2*i) {
        sieve[j] = 1;
      }
    }
  }
  for(int L=1; L<=R; L+=S) {
    array<bool, S> block{};
    for(auto &[p, idx]: cp) {
      for(int i=idx; i<S+L; idx=(i+=p)) {
        block[i - L] = 1;
      }
    }
    for(int i=0; i<min(S, R-L); ++i) {
      if(!block[i]) {
        pr.emplace_back((L + i) * 2 + 1);
      }
    }
  }
  for(int i: pr) {
    isPrime[i] = 1;
  }
  return pr;
}
```

GolbatchConjecture.hpp

Description: Find two prime numbers which sum equals s

Time: $\mathcal{O}(N \log N)$

"FastEratosthenes.hpp" 88fb23, 18 lines

```
pair<int, int> goldbatchConjecture(int s, vi pr = {}){
  if (s <= 2 || s % 2 != 0) {
    return make_pair(-1, -1);
  }
  if (pr.size() == 0) {
    pr = eratosthenes();
  }
  for (auto x : pr) {
    if (x > s / 2) {
      break;
    }
    int d = s - x;
    if (binary_search(pr.begin(), pr.end(), d)) {
      return make_pair(min(x, d), max(x, d));
    }
  }
  return make_pair(-1, -1);
}
```

Graph (7)

7.1 Matching

HopcroftKarp.hpp

Description: Fast bipartite matching algorithm.

Time: $\mathcal{O}\left(E\sqrt{V}\right)$

"../template/Header.hpp" 0bd56f, 52 lines

```
struct HopcroftKarp{
    int n,m;
    vi l,r,lv,ptr;
    vector<vi> adj;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m){init(_n,_m);}
    void init(int _n,int _m){
        n=_n,m=_m;
        adj.assign(n+m,vi{});
    }
    void addEdge(int u,int v){
        adj[u].emplace_back(v+n);
    }
    void bfs(){
        lv=vi(n,-1);
        queue<int> q;
        for(int i=0;i<n;i++)if(l[i]==-1){
            lv[i]=0;
            q.emplace(i);
        }
        while(!q.empty()){
            int u=q.front();
            q.pop();
            for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){
                lv[r[v]]=lv[u]+1;
                q.emplace(r[v]);
            }
        }
    }
    bool dfs(int u){
        for(int &i=ptr[u];i<sz(adj[u]);i++){
            int v=adj[u][i];
            if(r[v]==-1||(lv[r[v]]==lv[u]+1&&dfs(r[v]))){
                l[u]=v,r[v]=u;
                return true;
            }
```

```
        }
        return false;
    }
    int maxMatching(){
        int match=0,cnt=0;
        l=r=vi(n+m,-1);
        do{
            ptr=vi(n);
            bfs();
            cnt=0;
            for(int i=0;i<n;i++)if(l[i]==-1&&dfs(i))cnt++;
            match+=cnt;
        }while(cnt);
        return match;
    }
};
```

Kuhn.hpp

Description: Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.

Time: $\mathcal{O}(VE)$

"../template/Header.hpp" fc7d17, 15 lines

```
vi adj[1010], match(1010, -1);
bitset<1010> visited;
bool kuhn(int u) {
  if(visited[u]) {
    return false;
  }
  visited[u] = true;
  for(auto x: adj[u]) {
    if(match[x] == -1 || kuhn(match[x])) {
      match[x] = u;
      return true;
    }
  }
  return false;
}
```

7.2 Network Flow

Dinic.hpp

Description: Fast max-flow algorithm.

Time: $\mathcal{O}(VE\log U)$ where $U = \max|\text{cap}|$

"../template/Header.hpp" 7409c7, 68 lines

```
template<class T>
struct Dinic{
    struct Edge{
        int to;
        ll flow,cap;
        Edge(int _to,ll _cap):to(_to),flow(0),cap(_cap){}
        ll getcap(){
            return cap-flow;
        }
    };
    int n;
    ll U;
    vector<Edge> e;
    vector<vi> adj;
    vi ptr,lvl;
    Dinic(){}
    Dinic(int _n){
        init(_n);
    }
    void init(int _n){
        n=_n,U=0;
        e.clear();
        adj.assign(n,{});
    }
    void addEdge(int u,int v,ll cap){
        U=max(U,cap);
        adj[u].emplace_back(sz(e));
        e.emplace_back(v,cap);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u,0); // change 0 to cap for undirected
            flow
    }
    bool bfs(int s,int t,ll scale){
        lvl.assign(n,0);
        vi q{s};
        lvl[s]=1;
        for(int i=0;i<sz(q);i++){
            int u=q[i];
            for(auto j:adj[u])if(!lvl[e[j].to]&&e[j].getcap()>=
                scale){
                q.emplace_back(e[j].to);
                lvl[e[j].to]=lvl[u]+1;
            }
        }
        return lvl[t];
    }
    ll dfs(int u,int t,ll f){
        if(u==t||!f)return f;
        for(int &i=ptr[u];i<sz(adj[u]);i++){
            int j=adj[u][i];
            if(lvl[e[j].to]==lvl[u]+1){
                if(ll p=dfs(e[j].to,t,min(f,e[j].getcap()))){
                    e[j].flow+=p;
                    e[j^1].flow-=p;
                    return p;
                }
            }
        }
        return 0;
    }
    ll flow(int s,int t){
        ll flow=0;
        for(ll L=1ll<<(63-__builtin_clzll(U));L>0;L>>=1) // L =
            1 may be faster but it's O(V^2 E)
        while(bfs(s,t,L)){
            ptr.assign(n,0);
            while(ll p=dfs(s,t,LINF))flow+=p;
        };
        return flow;
    }
};
```

MinCostFlow.hpp

Description: minimum-cost flow algorithm.

Time: $\mathcal{O}(FE\log V)$ where F is max flow.

"../template/Header.hpp" 8ea1d2, 83 lines

```
template<class F,class C>
struct MinCostFlow{
    struct Edge{
        int to;
        F flow,cap;
        C cost;
        Edge(int _to,F _cap,C _cost):to(_to),flow(0),cap(_cap),
            cost(_cost){}
        F getcap(){
            return cap-flow;
        }
    };
    int n;
    vector<Edge> e;
    vector<vi> adj;
    vector<C> pot,dist;
    vi pre;
    bool neg;
    const F FINF=numeric_limits<F>::max()/2;
    const C CINF=numeric_limits<C>::max()/2;
    MinCostFlow(){}
    MinCostFlow(int _n){
        init(_n);
    }
    void init(int _n){
        n=_n;
        e.clear();
        adj.assign(n,{});
        neg=false;
    }
    void addEdge(int u,int v,F cap,C cost){
        adj[u].emplace_back(sz(e));
        e.emplace_back(v,cap,cost);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u,0,-cost);
        if(cost<0)neg=true;
    }
    bool dijkstra(int s,int t){
        using P = pair<C,int>;
        dist.assign(n,CINF);
        pre.assign(n,-1);
        priority_queue<P,vector<P>,greater<P>> pq;
        dist[s]=0;
        pq.emplace(0,s);
        while(!pq.empty()){
            auto [d,u]=pq.top();
            pq.pop();
            if(dist[u]<d)continue;
            for(int i:adj[u]){
                int v=e[i].to;
                C ndist=d+pot[u]-pot[v]+e[i].cost;
                if(e[i].getcap()>0&&dist[v]>ndist){
                    pre[v]=i;
                    dist[v]=ndist;
                    pq.emplace(ndist,v);
                }
            }
        }
        return dist[t]<CINF;
    }
    pair<F,C> flow(int s,int t){
        F flow=0;
        C cost=0;
        pot.assign(n,0);
        if(neg)for(int t=0;t<n;t++)for(int i=0;i<sz(e);i++)if(e
            [i].getcap()>0){
            int u=e[i^1].to,v=e[i].to;
            pot[v]=min(pot[v],pot[u]+e[i].cost);
        } // Bellman-Ford
        while(dijkstra(s,t)){
            for(int i=0;i<n;i++)pot[i]+=dist[i];
            F aug=FINF;
            for(int u=t;u!=s;u=e[pre[u]^1].to){
                aug=min(aug,e[pre[u]].getcap());
            } // find bottleneck
            for(int u=t;u!=s;u=e[pre[u]^1].to){
                e[pre[u]].flow+=aug;
                e[pre[u]^1].flow-=aug;
            } // push flow
            flow+=aug;
            cost+=aug*pot[t];
        }
        return {flow,cost};
    }
};
```

Polynomials (8)

FormalPowerSeries.hpp

Description: basic operations of formal power series

"NTT.hpp" 416433, 136 lines

```
template<class mint>
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;

    FPS &operator+=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]+=rhs[i];
        return *this;
    }
    FPS &operator+=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]+=rhs;
        return *this;
    }
    FPS &operator-=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]-=rhs[i];
        return *this;
    }
    FPS &operator-=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]-=rhs;
        return *this;
    }
    FPS &operator*=(const FPS &rhs){
        auto res=NTT<mint>()(*this,rhs);
        return *this=FPS(res.begin(),res.end());
    }
    FPS &operator*=(const mint &rhs){
        for(auto &a:*this)a*=rhs;
        return *this;
    }
    friend FPS operator+(FPS lhs,const FPS &rhs){return lhs+=
        rhs;}
    friend FPS operator+(FPS lhs,const mint &rhs){return lhs+=
        rhs;}
    friend FPS operator+(const mint &lhs,FPS &rhs){return rhs+=
        lhs;}
    friend FPS operator-(FPS lhs,const FPS &rhs){return lhs-=
        rhs;}
    friend FPS operator-(FPS lhs,const mint &rhs){return lhs-=
        rhs;}
    friend FPS operator-(const mint &lhs,FPS rhs){return -(rhs-
        lhs);}
    friend FPS operator*(FPS lhs,const FPS &rhs){return lhs*=
        rhs;}
    friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=
        rhs;}
    friend FPS operator*(const mint &lhs,FPS rhs){return rhs*=
        lhs;}

    FPS operator-(){return (*this)*-1;}

    FPS rev(){
        FPS res(*this);
        reverse(res.beign(),res.end());
        return res;
    }
    FPS pre(int sz){
        FPS res(this->begin(),this->begin()+min((int)this->size
            (),sz));
        if(res.size()<sz)res.resize(sz);
        return res;
    }
    FPS shrink(){
        FPS res(*this);
        while(!res.empty()&&res.back()==mint{})res.pop_back();
        return res;
    }
    FPS operator>>(int sz){
        if(this->size()<=sz)return {};
        FPS res(*this);
        res.erase(res.begin(),res.begin()+sz);
        return res;
    }
    FPS operator<<(int sz){
        FPS res(*this);
        res.insert(res.begin(),sz,mint{});
        return res;
    }
    FPS diff(){
        const int n=this->size();
        FPS res(max(0,n-1));
        for(int i=1;i<n;i++)res[i-1]=(*this)[i]*mint(i);
        return res;
    }
    FPS integral(){
        const int n=this->size();
        FPS res(n+1);
        res[0]=0;
        if(n>0)res[1]=1;
        ll mod=mint::get_mod();
        for(int i=2;i<=n;i++)res[i]=(-res[mod%i])*(mod/i);
        for(int i=0;i<n;i++)res[i+1]*=(*this)[i];
        return res;
    }
    mint eval(const mint &x){
        mint res=0,w=1;
        for(auto &a:*this)res+=a*w,w*=x;
        return res;
    }

    FPS inv(int deg=-1){
        assert(!this->empty()&&(*this)[0]!=mint(0));
        if(deg==-1)deg=this->size();
        FPS res{mint(1)/(*this)[0]};
        for(int i=2;i>>1<deg;i<<=1){
            res=(res*(mint(2)-res*pre(i))).pre(i);
        }
        return res.pre(deg);
    }
    FPS log(int deg=-1){
        assert(!this->empty()&&(*this)[0]==mint(1));
        if(deg==-1)deg=this->size();
        return (pre(deg).diff()*inv(deg)).pre(deg-1).integral()
            ;
    }
    FPS exp(int deg=-1){
        assert(this->empty()||(*this)[0]==mint(0));
        if(deg==-1)deg=this->size();
        FPS res{mint(1)};
        for(int i=2;i>>1<deg;i<<=1){
            res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
        }
        return res.pre(deg);
    }
    FPS pow(ll k,int deg=-1){
        const int n=this->size();
        if(deg==-1)deg=n;
        if(k==0){
            FPS res(deg);
            if(deg)res[0]=mint(1);
            return res;
        }
        for(int i=0;i<n;i++){
            if(__int128_t(i)*k>=deg)return FPS(deg,mint(0));
            if((*this)[i]==mint(0))continue;
            mint rev=mint(1)/(*this)[i];
            FPS res=(((*this*rev)>>i).log(deg)*k).exp(deg);
            res=((res*binpow((*this)[i],k))<<(i*k)).pre(deg);
            return res;
        }
        return FPS(deg,mint(0));
    }
};
using FPS=FormalPowerSeries<mint>;
```

FFT.hpp

Description: Fast Fourier transform

Time: $\mathcal{O}(N \log N)$

"../template/Header.hpp" 5d476b, 73 lines

```
template<class T=ll,int mod=0>
struct FFT{
  using vt = vector<T>;
  using cd = complex<db>;
  using vc = vector<cd> ;

  static const bool INT=true;

  static void fft(vc &a){
    int n=a.size(),L=31-__builtin_clz(n);
    vc rt(n);
    rt[1]=1;
    for(int k=2;k<n;k*=2){
      cd z=polar(db(1),PI/k);
      for(int i=k;i<2*k;i++)rt[i]=i&1?rt[i/2]*z:rt[i/2];
    }
    vi rev(n);
    for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
    for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
    for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k
        ;j++){
      cd z=rt[j+k]*a[i+j+k];
      a[i+j+k]=a[i+j]-z;
      a[i+j]+=z;
    }
  }
  template<class U>
  static db norm(const U &x){
    return INT?round(x):x;
  }
  static vt conv(const vt &a,const vt &b){
    if(a.empty()||b.empty())return {};
    vt res(a.size()+b.size()-1);
    int L=32-__builtin_clz(res.size()),n=1<<L;
    vc in(n),out(n);
    copy(a.begin(),a.end(),in.begin());
    for(int i=0;i<b.size();i++)in[i].imag(b[i]);
    fft(in);
    for(auto &x:in)x*=x;
    for(int i=0;i<n;i++)out[i]=in[-i&(n-1)]-conj(in[i]);
    fft(out);
    for(int i=0;i<res.size();i++)res[i]=norm(imag(out[i])/(4*n)
        );
    return res;
  }
  static vl convMod(const vl &a,const vl &b){
    assert(mod>0);
    if(a.empty()||b.empty())return {};
    vl res(a.size()+b.size()-1);
```

```
  int L=32-__builtin_clz(res.size()),n=1<<L;
  ll cut=int(sqrt(mod));
  vc in1(n),in2(n),out1(n),out2(n);
  for(int i=0;i<a.size();i++)in1[i]=cd(ll(a[i])/cut,ll(a[i])%
      cut); // a1 + i * a2
  for(int i=0;i<b.size();i++)in2[i]=cd(ll(b[i])/cut,ll(b[i])%
      cut); // b1 + i * b2
  fft(in1),fft(in2);
  for(int i=0;i<n;i++){
    int j=-i&(n-1);
    out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.l*n); // f1 * (g1
        + i * g2) = f1 * g1 + i f1 * g2
    out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.l,2.l*n); // f2
        * (g1 + i * g2) = f2 * g1 + i f2 * g2
  }
  fft(out1),fft(out2);
  for(int i=0;i<res.size();i++){
    ll x=round(real(out1[i])),y=round(imag(out1[i]))+round(
        real(out2[i])),z=round(imag(out2[i]));
    res[i]=((x%mod*cut+y)%mod*cut+z)%mod; // a1 * b1 * cut^2
        + (a1 * b2 + a2 * b1) * cut + a2 * b2
  }
  return res;
 }
 vt operator()(const vt &a,const vt &b){
  return mod>0?conv(a,b):convMod(a,b);
 }
};
template<>
struct FFT<db>{
 static const bool INT=false;
};
```

NTT.hpp

Description: Number theoretic transform
Time: $\mathcal{O}(N \log N)$

"../template/Header.hpp", "../modular-arithmetic/BinPow.hpp",
"../modular-arithmetic/MontgomeryModInt.hpp" 2b2392, 39 lines

```
template<class mint=mint>
struct NTT{
 using vm = vector<mint>;

 static constexpr mint root=mint::get_root();
  static_assert(root!=0);

 static void ntt(vm &a){
  int n=a.size(),L=31-__builtin_clz(n);
  vm rt(n);
  rt[1]=1;
  for(int k=2,s=2;k<n;k*=2,s++){
   mint z[]={1,binpow(root,MOD>>s)};
   for(int i=k;i<2*k;i++)rt[i]=rt[i/2]*z[i&1];
  }
  vi rev(n);
  for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
  for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
  for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k
      ;j++){
   mint z=rt[j+k]*a[i+j+k];
   a[i+j+k]=a[i+j]-z;
   a[i+j]+=z;
  }
 }
 static vm conv(const vm &a,const vm &b){
  if(a.empty()||b.empty())return {};
  int s=a.size()+b.size()-1,n=1<<(32-__builtin_clz(s));
  mint inv=mint(n).inv();
  vm in1(a),in2(b),out(n);
  in1.resize(n),in2.resize(n);
  ntt(in1),ntt(in2);
  for(int i=0;i<n;i++)out[-i&(n-1)]=in1[i]*in2[i]*inv;
  ntt(out);
  return vm(out.begin(),out.begin()+s);
 }
 vm operator()(const vm &a,const vm &b){
  return conv(a,b);
 }
};
```

Strings (9)

Manacher.hpp

Description: Manacher's Algorithm. pal[i] := the length of the longest palindrome centered at i/2.

"../template/Header.hpp" 53856e, 15 lines

```
template<class STR>
vector<int> manacher(const STR &s){
    int n=(int)s.size();
    if(n==0)return {};
    vector<int> pal(2*n-1);
    for(int p=0,l=-1,r=-1;p<2*n-1;p++){
        int i=(p+1)>>1,j=p>>1;
        int k=(i>=r?0:min(r-i,pal[2*(l+r)-p]));
        while(j+k+1<n&&i-k-1>=0&&s[j+k+1]==s[i-k-1])k++;
        pal[p]=k;
        if(j+k>r)l=i-k,r=j+k;
    }
    for(int i=0;i<2*n-1;i++)pal[i]=pal[i]<<1|(i&1^1);
    return pal;
}
```

SuffixArray.hpp

Description: Suffix Automaton.

"../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp" b9cfb1, 39 lines

```
template<class STR>
struct SuffixArray{
    int n;
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArray(){}
    SuffixArray(const STR &s){init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        sa[0]=n;
        iota(sa.begin()+1,sa.end(),0);
        sort(sa.begin()+1,sa.end(),[&](int i,int j){return s[i
            ]<s[j];});
        for(int i=1;i<=n;i++){
            int x=sa[i-1],y=sa[i];
            isa[y]=i>1&&s[x]==s[y]?isa[x]:i;
        }
        for(int len=1;len<=n;len<<=1){
            vector<int> ps(sa),pi(isa),pos(n+1);
            iota(pos.begin(),pos.end(),0);
            for(auto i:ps)if((i-=len)>=0)sa[pos[isa[i]]++]=i;
            for(int i=1;i<=n;i++){
                int x=sa[i-1],y=sa[i];
                isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x
                    ]:i;
            }
        }
        for(int i=0,k=0;i<n;i++){
            for(int j=sa[isa[i]-1];j+k<n&&s[j+k]==s[i+k];k++);
            lcp[isa[i]]=k;
            if(k)k--;
        }
        st.init(lcp);
    }
    int get_lcp(int i,int j){
        if(i==j)return n-i;
        auto [l,r]=minmax(isa[i],isa[j]);
        return st.query(l+1,r);
    }
};
```

ZAlgo.hpp

Description: Z Algorithm. z[i] := the length of the longest common prefix between s and s[i:].

"../template/Header.hpp" b93726, 12 lines

```
template<class STR>
vector<int> z_algorithm(const STR &s){
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for(int i=1,l=0,r=1;i<n;i++){
        if(i<r)z[i]=min(r-i,z[i-l]);
        while(i+z[i]<n&&s[z[i]]==s[i+z[i]])z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    }
    return z;
}
```

Dynamic Programming (10)

DVC.hpp

Description: Optimize $O(N^2K)$ to $O(NK \log N)$

"../template/Header.hpp" aa5ddf, 19 lines

```
vector<vl> cst, dp;

ll cost(int l, int r) {
  return cst[l][r];
}

void divide(int l, int r, int opt_l, int opt_r, int c) {
  if(l > r) return ;
  int mid = (l + r) / 2;
  pair<ll, int> best = make_pair(INF, -1);
  for(int k=opt_l; k<=min(mid, opt_r); ++k) {
    best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid),
        k));
  }
  dp[c][mid] = best.first;
  divide(l, mid - 1, opt_l, best.second, c);
  divide(mid + 1, r, best.second, opt_r, c);
}

// for(int c=1; c<=K; ++c) divide(1, N, 1, N, c);
```

SlopeTrick.hpp

Description: Absolute Smth

"../template/Header.hpp" f62f9a, 36 lines

```
ll extending_value;

struct slope_trick {
  multiset<ll> ms_l, ms_r;
  ll min_y = 0ll, lz_l = 0ll, lz_r = 0ll;
  bool extending = false;
  void add_line(ll v) {
    if(extending) {
      lz_l -= extending_value;
      lz_r -= extending_value;
```

```
    }
    extending = true;
    if(ms_l.empty() && ms_r.empty()) {
      ms_l.emplace(v);
      ms_r.emplace(v);
    }
    else if(v <= *ms_l.rbegin() + lz_l) {
      min_y += (*ms_l.rbegin() + lz_l) - v;
      ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
      ms_l.erase(--ms_l.end());
      ms_l.emplace(v - lz_l);
      ms_l.emplace(v - lz_l);
    }
    else if(v >= *ms_r.begin() + lz_r) {
      min_y += v - (*ms_r.begin() + lz_r);
      ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
      ms_r.erase(ms_r.begin());
      ms_r.emplace(v - lz_r);
      ms_r.emplace(v - lz_r);
    }
    else {
      ms_l.emplace(v - lz_l);
      ms_r.emplace(v - lz_r);
    }
  }
};
```

Convolutions (11)

AndConvolution.hpp

Description: Bitwise AND Convolution. Superset Zeta Transform: $A'[S] = \sum_{T \supseteq S} A[T]$. Superset Mobius Transform: $A[T] = \sum_{S \supseteq T} (-1)^{|S-T|} A'[S]$.
Time: $\mathcal{O}(N \log N)$.

"../template/Header.hpp" 7916f8, 34 lines

```
template<class T>
void superset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]+=a[j];
            }
        }
    }
}

template<class T>
void superset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]-=a[j];
            }
        }
    }
}

template<class T>
vector<T> and_convolution(vector<T> a,vector<T> b){
    superset_zeta(a);
    superset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    superset_mobius(a);
    return a;
}
```

GCDConvolution.hpp

Description: GCD Convolution. Multiple Zeta Transform: $A'[n] = \sum_{n|m} A[m]$. Multiple Mobius Transform: $A[n] = \sum_{n|m} \mu(m/n)A'[m]$.
Time: $\mathcal{O}(N \log\log N)$.

"../template/Header.hpp" 7f6c2d, 34 lines

```
template<class T>
void multiple_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i]+=a[i*p];
        }
    }
}

template<class T>
void multiple_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i]-=a[i*p];
        }
    }
}

template<class T>
vector<T> gcd_convolution(vector<T> a,vector<T> b){
    multiple_zeta(a);
    multiple_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    multiple_mobius(a);
    return a;
}
```

LCMConvolution.hpp

Description: LCM Convolution. Divisor Zeta Transform: $A'[n] = \sum_{d|n} A[d]$. Divisor Mobius Transform: $A[n] = \sum_{d|n} \mu(n/d)A'[d]$.
Time: $\mathcal{O}(N \log\log N)$.

"../template/Header.hpp" 41fe9d, 34 lines

```
template<class T>
void divisor_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i*p]+=a[i];
        }
    }
}

template<class T>
void divisor_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i*p]-=a[i];
        }
    }
}

template<class T>
vector<T> lcm_convolution(vector<T> a,vector<T> b){
    divisor_zeta(a);
    divisor_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    divisor_mobius(a);
    return a;
}
```

ORConvolution.hpp

Description: Bitwise OR Convolution. Subset Zeta Transform: $A'[S] = \sum_{T \subseteq S} A[T]$. Subset Mobius Transform: $A[T] = \sum_{S \subseteq T} (-1)^{|T-S|} A'[S]$.
Time: $\mathcal{O}(N \log N)$.

"../template/Header.hpp" c58b77, 34 lines

```
template<class T>
void subset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]+=a[j^i];
            }
        }
    }
}

template<class T>
void subset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]-=a[j^i];
            }
        }
    }
}

template<class T>
vector<T> or_convolution(vector<T> a,vector<T> b){
    subset_zeta(a);
    subset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    subset_mobius(a);
    return a;
}
```

XORConvolution.hpp

Description: Bitwise XOR Convolution. Fast Walsh-Hadamard Transform: $A'[S] = \sum_T (-1)^{|S\&T|} A[T]$.
Time: $\mathcal{O}(N \log N)$.

"../template/Header.hpp" 05848d, 29 lines

```
template<class T>
void fwht(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                T &u=a[j^i],&v=a[j];
```

```
                tie(u,v)=make_pair(u+v,u-v);
            }
        }
    }
}

template<class T>
vector<T> xor_convolution(vector<T> a,vector<T> b){
    int n=(int)a.size();
    fwht(a);
    fwht(b);
    for(int i=0;i<n;i++)a[i]*=b[i];
    fwht(a);
    T div=T(1)/T(n);
    if(div==T(0)){
        for(auto &x:a)x/=n;
    }else{
        for(auto &x:a)x*=div;
    }
    return a;
}
```

11.1 Various

GaussianElimination.hpp

Description: Gaussian Elimination

"../template/Header.hpp" e89ecb, 34 lines

```
struct Gauss {
  int n, sz;
  vector<ll> basis;
  Gauss(int n = 0) {
    init(n);
  }
  void init(int _n) {
    n = _n, sz = 0;
    basis.assign(n, 0);
  }
  void insert(ll x) {
    for (int i = n - 1; i >= 0; i--)
      if (x >> i & 1) {
        if (!basis[i]) {
          basis[i] = x;
          sz++;
          return;
        }
        x ^= basis[i];
      }
  }
  ll getmax(ll k = 0) {
    ll tot = 1ll << sz, res = 0;
    for (int i = n - 1; i >= 0; i--)
      if (basis[i]) {
        tot >>= 1;
        if ((k >= tot && res >> i & 1) || (k < tot && res >> i
            & 1 ^ 1))
          res ^= basis[i];
        if (k >= tot)
          k -= tot;
      }
    return res;
  }
};
```

BinaryTrie.hpp

Description: Binary Trie

"../template/Header.hpp" 525bf4, 59 lines

```
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
  vector<node_t> t = {node_t()};
  vector<int> cnt = {0};
  int cnt_nodes = 0;
  void insert(int v) {
    int cur = 0;
    cnt[0]++;
    for(int i=S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1: 0;
      if(!t[cur][b]) {
        t[cur][b] = ++cnt_nodes;
        t.emplace_back(node_t());
        cnt.emplace_back(0);
      }
      cnt[t[cur][b]]++;
      cur = t[cur][b];
    }
  }
  void remove(int v) {
    int cur = 0;
    cnt[0]--;
    for(int i=S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1: 0;
      cnt[t[cur][b]]--;
      cur = t[cur][b];
    }
  }
  int get_min(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1 : 0;
      if(t[cur][b] && cnt[t[cur][b]]) {
        cur = t[cur][b];
      }
      else {
        res |= (1 << i);
        cur = t[cur][!b];
      }
    }
    return res;
  }

  int get_max(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
      int b = (v & (1 << i)) ? 1 : 0;
      if(t[cur][!b] && cnt[t[cur][!b]]) {
        res |= (1 << i);
        cur = t[cur][!b];
      }
      else {
        cur = t[cur][b];
      }
    }
    return res;
  }
};
```

Competitive Programming Topics (A)

topics.txt

159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```