



Chulalongkorn University

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template from KACTL

2024-08-05

1	Template
2	Mathematics
3	Numerical
4	Group
5	Data Structures
6	Number Theory
7	Graph
8	Polynomials
9	Strings
10	Dynamic Programming
11	Convolutions
	Template (1)

template.cpp	27 lines
<pre>#pragma once #include <bits/stdc++.h> #define sz(x) (int)(x).size() #define all(x) (x).begin(), (x).end() using namespace std; using ll = long long; using db = long double; using vi = vector<int>; using vl = vector<ll>; using vd = vector<db>; using pii = pair<int, int>; using pll = pair<ll, ll>; using pdd = pair<db, db>; const int INF = 0x3fffffff; // const int MOD=1000000007; const int MOD = 998244353; const ll LINF = 0xffffffffffffffff; const db DINF = numeric_limits<db>::infinity(); const db EPS = 1e-9; const db PI = acos(db(-1)); int main(){ cin.tie(nullptr)->sync_with_stdio(false); }</pre>	
c.sh	2 lines
<pre>g++ -std=gnu++2a -Wall \$1 -o a.out ./a.out</pre>	

1	<u>Mathematics</u> (2)
1	2.1 Goldbatch’s Conjecture
1	• Even number can be written in sum of two primes (Up to 1e12)
1	• Range of N^{th} prime and $N + 1^{th}$ prime will be less than or equal to 300 (Up to 1e12)
1	2.2 Divisibility
6	Number of divisors of N is given by $\prod_{i=1}^k (a_i + 1)$ where $N = \prod_{i=1}^k p_i^{a_i}$ and p_i are prime factors of N .
6	<u>Numerical</u> (3)
7	3.1 Newton’s Method
9	if $F(Q) = 0$, then $Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$
9	$Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}$
10	$Q = \ln P = \int \frac{P'}{P} dx$
	$Q = e^P : Q_{2n} \equiv Q_n(1 + P - \ln Q_n) \pmod{x^{2n}}$
	$Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2}(Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}$
	$Q = P^k = \alpha^k x^{kt} e^{k \ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1$
	<u>Group</u> (4)
	4.1 Monoid
	monoid/MonoidBase.hpp
	Description: Monoid Base class.
	e75b74. 6 lines
	<pre>template<class T,T (*combine)(T,T),T (*identity)()> struct MonoidBase{ using value_type = T; static constexpr T op(const T &x,const T &y){return combine(x,y);} static constexpr T unit(){return identity();} }; 4.2 Action action/MonoidActionBase.hpp Description: Monoid Action Base class. 425d83. 11 lines template<class MInfo,class MTag,typename MInfo::value_type (*combine)(typename MInfo::value_type,typename MTag::value_type)> struct MonoidActionBase{ using InfoMonoid = MInfo; using TagMonoid = MTag; using Info = typename InfoMonoid::value_type; using Tag = typename TagMonoid::value_type; static constexpr Info op(const Info &a,const Tag &b){ return combine(a,b); } };</pre>

action/DefaultAction.hpp	
Description: Default Action class.	e45f00, 10 lines
<pre>template<class Monoid> struct DefaultAction{ using InfoMonoid = Monoid; using TagMonoid = Monoid; using Info = typename Monoid::value_type; using Tag = typename Monoid::value_type; static constexpr Info op(const Info &a,const Tag &b){ return Monoid::op(a,b); } };</pre>	
OrderedSet.hpp	
Description: Ordered Set	1a7f5f, 14 lines
<pre>"../template/Header.hpp", <bits/extc++.h> using namespace __gnu_pbds; template <class T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>; // can be change to less_equal void usage() { ordered_set<int> st, st_2; st.insert(2); st.insert(1); cout << st.order_of_key(2); cout << *st.find_by_order(1); st.join(st_2); // merge }</pre>	
FenwickTree.hpp	
Description: Fenwick / Binary Indexed Tree	43767a, 41 lines
<pre>template<class T> struct Fenwick{ int n,logn; vector<T> t; Fenwick(){} Fenwick(int _n){init(vector<T>(_n,T{}));} template<class U> Fenwick(const vector<U> &a){init(a);} template<class U> void init(const vector<U> &a){ n=(int)a.size(); logn=31-__builtin_clz(n); t.assign(n+1,T{}); for(int i=1;i<=n;i++){ t[i]=t[i]+a[i-1]; int j=i+(i&-i); if(j<=n)t[j]=t[j]+t[i]; } } void update(int x,const T &v){ for(int i=x+1;i<=n;i+=i&-i)t[i]=t[i]+v; } void update(int l,int r,const T &v){ update(l,v),update(r+1,-v); } T query(int x){ T res{}; for(int i=x+1;i>0;i-=i&-i)res=res+t[i]; return res; } };</pre>	

```

T query(int l,int r){
    return query(r)-query(l-1);
}
int find(const T &k){
    int x=0;
    T cur{};
    for(int i=1<<logn;i>0;i>=1)
        if(x+i<=n&&cur+t[x+i]<=k)x+=i,cur=cur+t[x];
    return x;
}
};

```

SegmentTree.hpp

Description: Segment Tree

c51dec, 85 lines

```

template<class Monoid>
struct SegmentTree{
    using T = typename Monoid::value_type;
    int n;
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n,function<T(int)> create){init(n,create);}
    SegmentTree(int n,T v=Monoid::unit()){init(n,[&](int){
        return v;});}
    template<class U>
    SegmentTree(const vector<U> &a){init((int)a.size(), [&](int
        i){return T(a[i]);});}
    void init(int _n,function<T(int)> create){
        n=_n;
        t.assign(4<<(31-__builtin_clz(n)),Monoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
                if(l==r)return void(t[i]=create(l));
                int m=(l+r)/2;
                build(l,m,i*2);
                build(m+1,r,i*2+1);
                pull(i);
            };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=Monoid::op(t[i*2],t[i*2+1]);
    }
    void modify(int l,int r,int i,int x,const T &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const T &v){
        modify(0,n-1,1,x,v);
    }
    template<class U>
    void update(int l,int r,int i,int x,const U &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=Monoid::op(t[i],v));
        int m=(l+r)/2;
        update(l,m,i*2,x,v);
        update(m+1,r,i*2+1,x,v);
        pull(i);
    }
    template<class U>
    void update(int x,const U &v){
        update(0,n-1,1,x,v);
    }
    T query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return Monoid::unit();

```

```

        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        return Monoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,
            x,y));
    }
    T query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};

```

LazySegmentTree.hpp

Description: Segment Tree with Lazy Propagation

91ab0c, 103 lines

```

template<class MonoidAction>
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n;
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n,function<Info(int)> create){init(n,
        create);}
    LazySegmentTree(int n,Info v=InfoMonoid::unit()){init(n
        ,[&](int){return v;});}
    template<class T>
    LazySegmentTree(const vector<T> &a){init((int)a.size(), [&](
        int i){return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        n=_n;
        int m=4<<(31-__builtin_clz(n));
        t.assign(m,InfoMonoid::unit());
        lz.assign(m,TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
                if(l==r)return void(t[i]=create(l));
                int m=(l+r)/2;
                build(l,m,i*2);
                build(m+1,r,i*2+1);
                pull(i);
            };
        build(0,n-1,1);
    }
};

```

```

        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
    }
    void apply(int i,const Tag &v){
        t[i]=MonoidAction::op(t[i],v);
        lz[i]=TagMonoid::op(lz[i],v);
    }
    void push(int i){
        apply(i*2,lz[i]);
        apply(i*2+1,lz[i]);
        lz[i]=TagMonoid::unit();
    }
    void modify(int l,int r,int i,int x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        push(i);
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const Info &v){
        modify(0,n-1,1,x,v);
    }
    void update(int l,int r,int i,int x,int y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(i,v);
        int m=(l+r)/2;
        push(i);
        update(l,m,i*2,x,y,v);
        update(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void update(int x,int y,const Tag &v){
        update(0,n-1,1,x,y,v);
    }
    Info query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        push(i);
        return InfoMonoid::op(query(l,m,i*2,x,y),query(m+1,r,i
            *2+1,x,y));
    }
    Info query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);

```

```
int res=findlast(m+1,r,i*2+1,x,y,f);
if(res==-1)res=findlast(l,m,i*2,x,y,f);
return res;
}
template<class F>
int findlast(int x,int y,const F &f){
    return findlast(0,n-1,l,x,y,f);
}
};
```

DynamicSegmentTree.hpp

Description: Dynamic Segment Tree

e84eeb, 106 lines

```
template<class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node{
        Info val;
        Tag lz;
        Ptr l,r;
        Node(Info v):val(v),lz(TagMonoid::unit()),l(nullptr),r(nullptr){}
        Node(Info v,Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr rt;
    function<Info(ll,ll)> create;
    DynamicSegmentTree() {init(0,0);}
    DynamicSegmentTree(ll n){init(0,n-1);}
    DynamicSegmentTree(ll lb,ll ub){init(lb,ub);}
    DynamicSegmentTree(ll lb,ll ub,function<Info(ll,ll)> create)
        {init(lb,ub,create);}
    void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll l,ll r){return InfoMonoid::unit();}){
        lb=_lb,ub=_ub;
        create=_create;
        rt=new Node(create(lb,ub));
    }
    Info val(Ptr t){
        return t?t->val:InfoMonoid::unit();
    }
    void pull(Ptr &t){
        t->val=InfoMonoid::op(val(t->l),val(t->r));
    }
    void apply(Ptr &t,const Tag &v,ll l,ll r){
        if(!t)t=new Node(create(l,r));
        t->val=MonoidAction::op(t->val,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void push(Ptr &t,ll l,ll m,ll r){
        apply(t->l,t->lz,l,m);
        apply(t->r,t->lz,m+1,r);
        t->lz=TagMonoid::unit();
    }
    void modify(ll l,ll r,Ptr &t,ll x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t->val=v);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        modify(l,m,t->l,x,v);
        modify(m+1,r,t->r,x,v);
        pull(t);
    }
    void modify(ll x,const Info &v){
```

```
        modify(lb,ub,rt,x,v);
    }
    void update(ll l,ll r,Ptr &t,ll x,ll y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(t,v,l,r);
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        update(l,m,t->l,x,y,v);
        update(m+1,r,t->r,x,y,v);
        pull(t);
    }
    void update(ll x,ll y,const Tag &v){
        update(lb,ub,rt,x,y,v);
    }
    Info query(ll l,ll r,Ptr &t,ll x,ll y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t->val;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t->r,x,y));
    }
    Info query(ll x,ll y){
        return query(lb,ub,rt,x,y);
    }
    template<class F>
    ll findfirst(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findfirst(l,m,t->l,x,y,f);
        if(res!=-1)res=findfirst(m+1,r,t->r,x,y,f);
        return res;
    }
    template<class F>
    ll findfirst(ll x,ll y,const F &f){
        return findfirst(lb,ub,rt,x,y,f);
    }
    template<class F>
    ll findlast(ll l,ll r,Ptr t,ll x,ll y,const F &f){
        if(y<l||r<x||!t||!f(t->val))return -1;
        if(l==r)return l;
        ll m=l+(r-l)/2;
        push(t,l,m,r);
        ll res=findlast(m+1,r,t->r,x,y,f);
        if(res!=-1)res=findlast(l,m,t->l,x,y,f);
        return res;
    }
    template<class F>
    ll findlast(ll x,ll y,const F &f){
        return findlast(lb,ub,rt,x,y,f);
    }
};
```

DSU.hpp

Description: Disjoint Set Union.

0b3eb8, 26 lines

```
struct DSU{
    vector<int> p,sz;
    DSU(){}
    DSU(int n){init(n);}
    void init(int n){
        p.resize(n);
        iota(p.begin(),p.end(),0);
        sz.assign(n,1);
    }
    int find(int u){
        return p[u]==u?p[u]=find(p[u]);
```

```
    }
    bool same(int u,int v){
        return find(u)==find(v);
    }
    bool merge(int u,int v){
        u=find(u),v=find(v);
        if(u==v)return false;
        sz[u]+=sz[v];
        p[v]=u;
        return true;
    }
    int size(int u){
        return sz[find(u)];
    }
};
```

BinaryTrie.hpp

Description: Binary Trie

ae5b7a, 66 lines

```
template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
    struct Node{
        array<int,2> ch;
        S cnt;
        Node():ch{-1,-1},cnt(0){}
    };
    vector<Node> t;
    BinaryTrie():t{Node()}{}
    int new_node(){
        t.emplace_back(Node());
        return t.size()-1;
    }
    S size(){
        return t[0].cnt;
    }
    bool empty(){
        return size()==0;
    }
    S get_cnt(int i){
        return i!=-1?t[i].cnt:S(0);
    }
    void insert(T x,S k=1){
        int u=0;
        t[u].cnt+=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(t[u].ch[v]==-1)t[u].ch[v]=new_node();
            u=t[u].ch[v];
            t[u].cnt+=k;
        }
    }
    void erase(T x,S k=1){
        int u=0;
        assert(t[u].cnt>=k);
        t[u].cnt-=k;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            u=t[u].ch[v];
            assert(u!=-1&&t[u].cnt>=k);
            t[u].cnt-=k;
        }
    }
    T kth(S k,T x=0){
        assert(k<size());
        int u=0;
        T res=0;
        for(int i=BIT-1;i>=0;i--){
            int v=x>>i&1;
            if(k<get_cnt(t[u].ch[v])){
```

```
        u=t[u].ch[v];
    }else{
        res|=T(1)<<i;
        k-=get_cnt(t[u].ch[v]);
        u=t[u].ch[v^1];
    }
}
return res;
}
T min(T x){
    return kth(0,x);
}
T max(T x){
    return kth(size()-1,x);
}
};
```

LiChaoTree.hpp

Description: Li-Chao Tree (minimize). 4ab713, 52 lines

```
template<class T>
struct LiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    vector<T> xs;
    vector<Line> t;
    LiChaoTree(){}
    LiChaoTree(const vector<T> &x):xs(x){init(x);}
    LiChaoTree(int n):xs(n){
        vector<T> x(n);
        iota(x.begin(),x.end(),0);
        init(x);
    }
    void init(const vector<T> &x){
        xs=x;
        sort(xs.begin(),xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    }
    void insert(int l,int r,int i,Line v){
        int m=(l+r)/2;
        if(v.eval(xs[m])<t[i].eval(xs[m]))swap(t[i],v);
        if(v.eval(xs[l])<t[i].eval(xs[l]))insert(l,m,i*2,v);
        if(v.eval(xs[r])<t[i].eval(xs[r]))insert(m+1,r,i*2+1,v);
    }
    inline void insert(T m,T c){
        insert(0,(int)xs.size()-1,1,Line(m,c));
    }
    void insert_range(int l,int r,int i,T x,T y,Line v){
        if(y<xs[l]||xs[r]<x)return;
        if(x<=xs[l]&&xs[r]<=y)return insert(l,r,i,v);
        int m=(l+r)/2;
        insert_range(l,m,i*2,x,y,v);
        insert_range(m+1,r,i*2+1,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(0,(int)xs.size()-1,l,x,y,Line(m,c));
    }
    T query(int l,int r,int i,T x){
        if(l==r)return t[i].eval(x);
        int m=(l+r)/2;
        if(x<=xs[m])return min(t[i].eval(x),query(l,m,i*2,x));
        return min(t[i].eval(x),query(m+1,r,i*2+1,x));
    }
};
```

LiChaoTree DynamicLiChaoTree SplayTreeBase

```
    inline T query(T x){
        return query(0,(int)xs.size()-1,1,x);
    }
};

DynamicLiChaoTree.hpp
Description: Dynamic Li-Chao Tree (minimize). b8af36, 50 lines

template<class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(nullptr){}
    void insert(T l,T r,Ptr &t,Line v){
        if(!t)return void(t=new Node(v));
        T m=l+(r-l)/2;
        if(v.eval(m)<t->v.eval(m))swap(t->v,v);
        if(v.eval(l)<t->v.eval(l))insert(l,m,t->l,v);
        if(v.eval(r)<t->v.eval(r))insert(m+1,r,t->r,v);
    }
    inline void insert(T m,T c){
        insert(lb,ub,root,Line(m,c));
    }
    void insert_range(T l,T r,Ptr &t,T x,T y,Line v){
        if(y<l||r<x)return;
        if(!t)t=new Node();
        if(x<=l&&r<=y)return insert(l,r,t,v);
        T m=l+(r-l)/2;
        insert_range(l,m,t->l,x,y,v);
        insert_range(m+1,r,t->r,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(lb,ub,root,x,y,Line(m,c));
    }
    T query(T l,T r,Ptr t,T x){
        if(!t)return INF;
        T m=l+(r-l)/2;
        if(x<=m)return min(t->v.eval(x),query(l,m,t->l,x));
        return min(t->v.eval(x),query(m+1,r,t->r,x));
    }
    inline T query(T x){
        return query(lb,ub,root,x);
    }
};

SplayTreeBase.hpp
Description: Splay Tree. splay(u) will make node u be the root of the tree in amortized O(log n) time. cc90a9, 113 lines

template<class Node>
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return !(t->p)|| (t->p->l!=t&&t->p->r!=t);
    }
};
```

```
} // The parent of the root stores the path parant in link cut tree.
int size(Ptr t){
    return t?t->size:0;
}
virtual void push(Ptr t){};
virtual void pull(Ptr t){};
int pos(Ptr t){
    if(t->p){
        if(t->p->l==t)return -1;
        if(t->p->r==t)return 1;
    }
    return 0;
}
void rotate(Ptr t){
    Ptr x=t->p,y=x->p;
    if(pos(t)==-1){
        if((x->l==t->r))t->r->p=x;
        t->r=x,x->p=t;
    }else{
        if((x->r==t->l))t->l->p=x;
        t->l=x,x->p=t;
    }
    pull(x),pull(t);
    if((t->p==y)){
        if(y->l==x)y->l=t;
        if(y->r==x)y->r=t;
    }
}
void splay(Ptr t){
    if(!t)return;
    push(t);
    while(!is_root(t)){
        Ptr x=t->p;
        if(is_root(x)){
            push(x),push(t);
            rotate(t);
        }else{
            Ptr y=x->p;
            push(y),push(x),push(t);
            if(pos(x)==pos(t))rotate(x),rotate(t);
            else rotate(t),rotate(t);
        }
    }
}
Ptr get_first(Ptr t){
    while(t->l)push(t),t=t->l;
    splay(t);
    return t;
}
Ptr get_last(Ptr t){
    while(t->r)push(t),t=t->r;
    splay(t);
    return t;
}
Ptr merge(Ptr l,Ptr r){
    splay(l),splay(r);
    if(!l)return r;
    if(!r)return l;
    l=get_last(l);
    l->r=r;
    r->p=l;
    pull(l);
    return l;
}
pair<Ptr,Ptr> split(Ptr t,int k){
    if(!t)return {nullptr,nullptr};
    if(k==0)return {nullptr,t};
    if(k==size(t))return {t,nullptr};
};
```

```
push(t);
if(k<=size(t->l)){
    auto x=split(t->l,k);
    t->l=x.second;
    t->p=nullptr;
    if(x.second)x.second->p=t;
    pull(t);
    return {x.first,t};
}else{
    auto x=split(t->r,k-size(t->l)-1);
    t->r=x.first;
    t->p=nullptr;
    if(x.first)x.first->p=t;
    pull(t);
    return {t,x.second};
}
}

void insert(Ptr &t,int k,Ptr v){
    splay(t);
    auto x=split(t,k);
    t=merge(merge(x.first,v),x.second);
}

void erase(Ptr &t,int k){
    splay(t);
    auto x=split(t,k);
    auto y=split(x.second,1);
    // delete y.first;
    t=merge(x.first,y.second);
}

template<class T>
Ptr build(const vector<T> &v){
    if(v.empty())return nullptr;
    function<Ptr(int,int)> build=[&](int l,int r){
        if(l==r)return new Node(v[l]);
        int m=(l+r)/2;
        return merge(build(l,m),build(m+1,r));
    };
    return build(0,v.size()-1);
}

};
```

LazyReversibleBBST.hpp

Description: Lazy Reversible BBST Base. 904708, 81 lines

```
template<class Tree,class Node,class MonoidAction>
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;

    LazyReversibleBBST()=default;

    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
    }

    void pull(Ptr t){
        if(!t)return;
        push(t);
        t->size=1;
        t->sum=t->val;
        t->revsum=t->val;
        if(t->l){
            t->size+=t->l->size;
            t->sum=InfoMonoid::op(t->l->sum,t->sum);
            t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);
        }
    }
};
```

```
    }
    if(t->r){
        t->size+=t->r->size;
        t->sum=InfoMonoid::op(t->sum,t->r->sum);
        t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
    }
}

void push(Ptr t){
    if(!t)return;
    if(t->rev){
        toggle(t->l);
        toggle(t->r);
        t->rev=false;
    }
    if(t->lz!=TagMonoid::unit()){
        propagate(t->l,t->lz);
        propagate(t->r,t->lz);
        t->lz=TagMonoid::unit();
    }
}

void toggle(Ptr t){
    if(!t)return;
    swap(t->l,t->r);
    swap(t->sum,t->revsum);
    t->rev^=true;
}

void propagate(Ptr t,const Tag &v){
    if(!t)return;
    t->val=MonoidAction::op(t->val,v);
    t->sum=MonoidAction::op(t->sum,v);
    t->revsum=MonoidAction::op(t->revsum,v);
    t->lz=TagMonoid::op(t->lz,v);
}

void apply(Ptr &t,int l,int r,const Tag &v){
    if(l>r)return;
    auto x=split(t,l);
    auto y=split(x.second,r-l+1);
    propagate(y.first,v);
    t=merge(x.first,merge(y.first,y.second));
}

Info query(Ptr &t,int l,int r){
    if(l>r)return InfoMonoid::unit();
    auto x=split(t,l);
    auto y=split(x.second,r-l+1);
    Info res=sum(y.first);
    t=merge(x.first,merge(y.first,y.second));
    return res;
}

void reverse(Ptr &t,int l,int r){
    if(l>r)return;
    auto x=split(t,l);
    auto y=split(x.second,r-l+1);
    toggle(y.first);
    t=merge(x.first,merge(y.first,y.second));
}
};
```

LazyReversibleSplayTree.hpp

Description: Lazy Reversible Splay Tree. "SplayTreeBase.hpp", "LazyReversibleBBST.hpp" b8455b, 23 lines

```
template<class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
    Ptr l,r,p;
```

```
    Info val,sum,revsum;
    Tag lz;
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::unit(),const Tag &_lz=TagMonoid::unit())
        :l(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),size(1),rev(false){}
};

template<class MonoidAction>
struct LazyReversibleSplayTree
    : LazyReversibleBBST<SplayTreeBase<
        LazyReversibleSplayTreeNode<MonoidAction>>,
        LazyReversibleSplayTreeNode<MonoidAction>,MonoidAction>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};
```

LinkCutTreeBase.hpp

Description: LinkCut Tree Base.
Usage: evert(u): make u be the root of the tree.
link(u,v): attach u to v.
cut(u,v): remove edge between u and v.
get_root(u): get the root of the tree containing u.
lca(u,v): get the lowest common ancestor of u and v.
fold(u,v): get the value of the path from u to v. b432c3, 59 lines

```
template<class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for(Ptr cur=t;cur;cur=cur->p){
            this->splay(cur);
            cur->r=pc;
            this->pull(cur);
            pc=cur;
        }
        this->splay(t);
        return pc;
    }

    void evert(Ptr t){ // make t be the root of the tree
        expose(t);
        this->toggle(t);
        this->push(t);
    }

    void link(Ptr u,Ptr v){ // attach u to v
        evert(u);
        expose(v);
        u->p=v;
    }

    void cut(Ptr u,Ptr v){ // cut edge between u and v
        evert(u);
        expose(v);
        assert(u->p==v);
        v->l=u->p=nullptr;
        this->pull(v);
    }

    Ptr get_root(Ptr t){
        expose(t);
        while(t->l)this->push(t),t=t->l;
        this->splay(t);
        return t;
    }

    Ptr lca(Ptr u,Ptr v){
        if(get_root(u)!=get_root(v))return nullptr;
        expose(u);
```

```
        return expose(v);
    }
    void set_val(Ptr t,const T &val){
        this->evert(t);
        t->val=val;
        this->pull(t);
    }
    T get_val(Ptr t){
        this->evert(t);
        return t->val;
    }
    T fold(Ptr u,Ptr v){
        evert(u);
        expose(v);
        return v->sum;
    }
};
```

LazyLinkCutTree.hpp

Description: Lazy Link Cut Tree.

Usage: using Lct = LazyLinkCutTree<Action>;
using Ptr = Lct::Ptr;
using Node = Lct::Node;
vector<Ptr> ptr(n);
for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);
auto link=[&](int u,int v){
Lct::link(ptr[u],ptr[v]);
};
auto cut=[&](int u,int v){
Lct::cut(ptr[u],ptr[v]);
};
auto update=[&](int u,int v,const Action::Tag &val){
Lct::apply(ptr[u],ptr[v],val);
};
auto query=[&](int u,int v){
return Lct::fold(ptr[u],ptr[v]);
};

"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp"	ead3da, 12 lines
--	------------------

```
template<class MonoidAction>
struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree<
    MonoidAction>>{
    using base = LinkCutTreeBase<LazyReversibleSplayTree<
        MonoidAction>>;
    using Ptr = typename base::Ptr;
    using Tag = typename MonoidAction::Tag;

    void apply(Ptr u,Ptr v,const Tag &val){
        this->evert(u);
        this->expose(v);
        this->propagate(v,val);
    }
};
```

Number Theory (6)

ExtendedEuclid.hpp

Description: Extended Euclid algorithm for solving diophantine equation (ax + by = gcd(a, b)).

Time: $\mathcal{O}(\log \max\{a,b\})$

"../template/Header.hpp"	229e7c, 13 lines
--------------------------	------------------

```
pair<ll,ll> euclid(ll a,ll b){
    ll x=1,y=0,x1=0,y1=1;
    while(b!=0){
        ll q=a/b;
        x-=q*x1;
        y-=q*y1;
        a-=q*b;
```

```
        swap(x,x1);
        swap(y,y1);
        swap(a,b);
    }
    return {x,y};
}
```

6.1 Prime Numbers

LinearSieve.hpp

Description: Prime Number Generator in Linear Time

Time: $\mathcal{O}(N)$

"../template/Header.hpp"	194fb1, 15 lines
--------------------------	------------------

```
vi linear_sieve(int n) {
    vi prime, composite(n + 1);
    for(int i=2; i<=n; ++i) {
        if(!composite[i]) {
            prime.emplace_back(i);
        }
        for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {
            composite[i * prime[j]] = true;
            if(i % prime[j] == 0) {
                break;
            }
        }
    }
    return prime;
}
```

FastEratosthenes.hpp

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 \approx 1.5s

"../template/Header.hpp"	295b58, 33 lines
--------------------------	------------------

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int) round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S + 1);
    pr.reserve((int) (LIM/log(LIM) * 1.1));
    vector<pii> cp;
    for(int i=3; i<=S; i+=2) {
        if(!sieve[i]) {
            cp.emplace_back(i, i * i / 2);
            for(int j=i*i; j<=S; j+=2*i) {
                sieve[j] = 1;
            }
        }
    }
    for(int L=1; L<=R; L+=S) {
        array<bool, S> block{};
        for(auto &[p, idx]: cp) {
            for(int i=idx; i<S+L; idx=(i+p)) {
                block[i - L] = 1;
            }
        }
        for(int i=0; i<min(S, R-L); ++i) {
            if(!block[i]) {
                pr.emplace_back((L + i) * 2 + 1);
            }
        }
    }
    for(int i: pr) {
        isPrime[i] = 1;
    }
    return pr;
}
```

GolbatchConjecture.hpp

Description: Find two prime numbers which sum equals s

Time: $\mathcal{O}(N \log N)$

"FastEratosthenes.hpp"	88fb23, 18 lines
------------------------	------------------

```
pair<int, int> golbatchConjecture(int s, vi pr = {}){
    if (s <= 2 || s % 2 != 0) {
        return make_pair(-1, -1);
    }
    if (pr.size() == 0) {
        pr = eratosthenes();
    }
    for (auto x : pr) {
        if (x > s / 2) {
            break;
        }
        int d = s - x;
        if (binary_search(pr.begin(), pr.end(), d)) {
            return make_pair(min(x, d), max(x, d));
        }
    }
    return make_pair(-1, -1);
}
```

Graph (7)

7.1 Matching

HopcroftKarp.hpp

Description: Fast bipartite matching algorithm.

Time: $\mathcal{O}(E\sqrt{V})$

"../template/Header.hpp"	0bd56f, 52 lines
--------------------------	------------------

```
struct HopcroftKarp{
    int n,m;
    vi l,r,lv,ptr;
    vector<vi> adj;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m){init(_n,_m);}
    void init(int _n,int _m){
        n=_n,m=_m;
        adj.assign(n+m,vi{});
    }
    void addEdge(int u,int v){
        adj[u].emplace_back(v+n);
    }
    void bfs(){
        lv=vi(n,-1);
        queue<int> q;
        for(int i=0;i<n;i++)if(l[i]==-1){
            lv[i]=0;
            q.emplace(i);
        }
        while(!q.empty()){
            int u=q.front();
            q.pop();
            for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){
                lv[r[v]]=lv[u]+1;
                q.emplace(r[v]);
            }
        }
    }
    bool dfs(int u){
        for(int &i=ptr[u];i<sz(adj[u]);i++){
            int v=adj[u][i];
            if(r[v]==-1||(lv[r[v]]==lv[u]+1&&dfs(r[v]))){
                l[u]=v,r[v]=u;
                return true;
            }
        }
    }
}
```

```

    }
    return false;
}
int maxMatching() {
    int match=0, cnt=0;
    l=r=vi(n+m, -1);
    do {
        ptr=vi(n);
        bfs();
        cnt=0;
        for(int i=0; i<n; i++) if(l[i]==-1 && dfs(i)) cnt++;
        match+=cnt;
    } while(cnt);
    return match;
}
};

```

Kuhn.hpp

Description: Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.

Time: $\mathcal{O}(VE)$

../template/Header.hpp fc7d17, 15 lines

```

vi adj[1010], match(1010, -1);
bitset<1010> visited;
bool kuhn(int u) {
    if(visited[u]) {
        return false;
    }
    visited[u] = true;
    for(auto x: adj[u]) {
        if(match[x] == -1 || kuhn(match[x])) {
            match[x] = u;
            return true;
        }
    }
    return false;
}
};

```

7.2 Network Flow

Dinic.hpp

Description: Fast max-flow algorithm.

Time: $\mathcal{O}(VE \log U)$ where $U = \max |cap|$

../template/Header.hpp 7409c7, 68 lines

```

template<class T>
struct Dinic {
    struct Edge {
        int to;
        ll flow, cap;
        Edge(int _to, ll _cap): to(_to), flow(0), cap(_cap) {}
        ll getcap() {
            return cap-flow;
        }
    };
    int n;
    ll U;
    vector<Edge> e;
    vector<vi> adj;
    vi ptr, lvl;
    Dinic() {}
    Dinic(int _n) {
        init(_n);
    }
    void init(int _n) {
        n=_n, U=0;
        e.clear();
        adj.assign(n, {});
    }
    void addEdge(int u, int v, ll cap) {

```

```

        U=max(U, cap);
        adj[u].emplace_back(sz(e));
        e.emplace_back(v, cap);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u, 0); // change 0 to cap for undirected flow
    }
    bool bfs(int s, int t, ll scale) {
        lvl.assign(n, 0);
        vi q{s};
        lvl[s]=1;
        for(int i=0; i<sz(q); i++) {
            int u=q[i];
            for(auto j: adj[u]) if(!lvl[e[j].to] && e[j].getcap() >= scale) {
                q.emplace_back(e[j].to);
                lvl[e[j].to]=lvl[u]+1;
            }
        }
        return lvl[t];
    }
    ll dfs(int u, int t, ll f) {
        if(u==t || !f) return f;
        for(int &i=ptr[u]; i<sz(adj[u]); i++) {
            int j=adj[u][i];
            if(lvl[e[j].to]==lvl[u]+1) {
                if(ll p=dfs(e[j].to, t, min(f, e[j].getcap()))) {
                    e[j].flow+=p;
                    e[j^1].flow-=p;
                    return p;
                }
            }
        }
        return 0;
    }
    ll flow(int s, int t) {
        ll flow=0;
        for(ll L=1; L<<(63-__builtin_clzll(U)); L>0; L>=1) // L = 1 may be faster but it's O(V^2 E)
            while(bfs(s, t, L)) {
                ptr.assign(n, 0);
                while(ll p=dfs(s, t, LINF)) flow+=p;
            };
        return flow;
    }
};

```

MinCostFlow.hpp

Description: minimum-cost flow algorithm.

Time: $\mathcal{O}(FE \log V)$ where F is max flow.

../template/Header.hpp Seald2, 83 lines

```

template<class F, class C>
struct MinCostFlow {
    struct Edge {
        int to;
        F flow, cap;
        C cost;
        Edge(int _to, F _cap, C _cost): to(_to), flow(0), cap(_cap), cost(_cost) {}
        F getcap() {
            return cap-flow;
        }
    };
    int n;
    vector<Edge> e;
    vector<vi> adj;
    vector<C> pot, dist;
    vi pre;
    bool neg;

```

```

    const F FINF=numeric_limits<F>::max()/2;
    const C CINF=numeric_limits<C>::max()/2;
    MinCostFlow() {}
    MinCostFlow(int _n) {
        init(_n);
    }
    void init(int _n) {
        n=_n;
        e.clear();
        adj.assign(n, {});
        neg=false;
    }
    void addEdge(int u, int v, F cap, C cost) {
        adj[u].emplace_back(sz(e));
        e.emplace_back(v, cap, cost);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u, 0, -cost);
        if(cost<0) neg=true;
    }
    bool dijkstra(int s, int t) {
        using P = pair<C, int>;
        dist.assign(n, CINF);
        pre.assign(n, -1);
        priority_queue<P, vector<P>, greater<P>> pq;
        dist[s]=0;
        pq.emplace(0, s);
        while(!pq.empty()) {
            auto [d, u]=pq.top();
            pq.pop();
            if(dist[u]<d) continue;
            for(int i: adj[u]) {
                int v=e[i].to;
                C ndist=d+pot[u]-pot[v]+e[i].cost;
                if(e[i].getcap()>0 && dist[v]>ndist) {
                    pre[v]=i;
                    dist[v]=ndist;
                    pq.emplace(ndist, v);
                }
            }
        }
        return dist[t]<CINF;
    }
    pair<F, C> flow(int s, int t) {
        F flow=0;
        C cost=0;
        pot.assign(n, 0);
        if(neg) for(int t=0; t<n; t++) for(int i=0; i<sz(e); i++) if(e[i].getcap()>0) {
            int u=e[i^1].to, v=e[i].to;
            pot[v]=min(pot[v], pot[u]+e[i].cost);
        } // Bellman-Ford
        while(dijkstra(s, t)) {
            for(int i=0; i<n; i++) pot[i]+=dist[i];
            F aug=FINF;
            for(int u=t; u!=s; u=pre[u]^1] {
                aug=min(aug, e[pre[u]].getcap());
            } // find bottleneck
            for(int u=t; u!=s; u=pre[u]^1] {
                e[pre[u]].flow+=aug;
                e[pre[u]^1].flow-=aug;
            } // push flow
            flow+=aug;
            cost+=aug*pot[t];
        }
        return {flow, cost};
    }
};

```


Polynomials (8)

FormalPowerSeries.hpp

Description: basic operations of formal power series

"NTT.hpp" 416433, 136 lines

```
template<class mint>
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;

    FPS &operator+=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++) (*this)[i]+=rhs[i];
        return *this;
    }
    FPS &operator+=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]+=rhs;
        return *this;
    }
    FPS &operator-=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++) (*this)[i]-=rhs[i];
        return *this;
    }
    FPS &operator-=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]-=rhs;
        return *this;
    }
    FPS &operator*=(const FPS &rhs){
        auto res=NTT<mint>() (*this, rhs);
        return *this=FPS(res.begin(), res.end());
    }
    FPS &operator*=(const mint &rhs){
        for(auto &a:*this) a*=rhs;
        return *this;
    }
    friend FPS operator+(FPS lhs,const FPS &rhs){return lhs+=rhs;}
    friend FPS operator+(FPS lhs,const mint &rhs){return lhs+=rhs;}
    friend FPS operator+(const mint &lhs,FPS &rhs){return rhs+=lhs;}
    friend FPS operator-(FPS lhs,const FPS &rhs){return lhs-=rhs;}
    friend FPS operator-(FPS lhs,const mint &rhs){return lhs-=rhs;}
    friend FPS operator-(const mint &lhs,FPS rhs){return -(rhs-lhs);}
    friend FPS operator*(FPS lhs,const FPS &rhs){return lhs*=rhs;}
    friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=rhs;}
    friend FPS operator*(const mint &lhs,FPS rhs){return rhs*=lhs;}

    FPS operator-() {return (*this)*-1;}

    FPS rev(){
        FPS res(*this);
        reverse(res.beign(), res.end());
        return res;
    }
    FPS pre(int sz){
        FPS res(this->begin(), this->begin()+min((int) this->size(),sz));
        if(res.size()<sz) res.resize(sz);
```

```
        return res;
    }
    FPS shrink(){
        FPS res(*this);
        while(!res.empty() && res.back()==mint{}) res.pop_back();
        return res;
    }
    FPS operator>>(int sz){
        if(this->size()<=sz) return {};
        FPS res(*this);
        res.erase(res.begin(), res.begin()+sz);
        return res;
    }
    FPS operator<<(int sz){
        FPS res(*this);
        res.insert(res.begin(), sz, mint{});
        return res;
    }
    FPS diff(){
        const int n=this->size();
        FPS res(max(0,n-1));
        for(int i=1;i<n;i++) res[i-1]=(*this)[i]*mint(i);
        return res;
    }
    FPS integral(){
        const int n=this->size();
        FPS res(n+1);
        res[0]=0;
        if(n>0) res[1]=1;
        ll mod=mint::get_mod();
        for(int i=2;i<n;i++) res[i]=(-res[mod%i])*(mod/i);
        for(int i=0;i<n;i++) res[i+1]*=(*this)[i];
        return res;
    }
    mint eval(const mint &x){
        mint res=0,w=1;
        for(auto &a:*this) res+=a*w,w*=x;
        return res;
    }

    FPS inv(int deg=-1){
        assert(!this->empty() && (*this)[0]!=mint(0));
        if(deg===-1) deg=this->size();
        FPS res{mint(1)/(*this)[0]};
        for(int i=2;i>>1<deg;i<=1){
            res=(res*(mint(2)-res*pre(i))).pre(i);
        }
        return res.pre(deg);
    }
    FPS log(int deg=-1){
        assert(!this->empty() && (*this)[0]==mint(1));
        if(deg===-1) deg=this->size();
        return (pre(deg).diff()*inv(deg)).pre(deg-1).integral();
    }
    FPS exp(int deg=-1){
        assert(this->empty() || (*this)[0]==mint(0));
        if(deg===-1) deg=this->size();
        FPS res{mint(1)};
        for(int i=2;i>>1<deg;i<=1){
            res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
        }
        return res.pre(deg);
    }
    FPS pow(ll k,int deg=-1){
        const int n=this->size();
        if(deg===-1) deg=n;
        if(k==0){
            FPS res(deg);
```

```
            if(deg) res[0]=mint(1);
            return res;
        }
        for(int i=0;i<n;i++){
            if(__int128_t(i)*k>=deg) return FPS(deg,mint(0));
            if((*this)[i]==mint(0)) continue;
            mint rev=mint(1)/(*this)[i];
            FPS res=(((*this*rev)>>i).log(deg)*k).exp(deg);
            res=((res*binpow((*this)[i],k)<<(i*k)).pre(deg);
            return res;
        }
        return FPS(deg,mint(0));
    }
};
using FPS=FormalPowerSeries<mint>;

FFT.hpp
Description: Fast Fourier transform
Time:  $\mathcal{O}(N \log N)$ 
"./template/Header.hpp" 5d476b, 73 lines

template<class T=ll,int mod=0>
struct FFT{
    using vt = vector<T>;
    using cd = complex<db>;
    using vc = vector<cd>;

    static const bool INT=true;

    static void fft(vc &a){
        int n=a.size(),L=31-__builtin_clz(n);
        vc rt(n);
        rt[1]=1;
        for(int k=2;k<n;k*=2){
            cd z=polar(db(1),PI/k);
            for(int i=k;i<2*k;i++) rt[i]=i&1?rt[i/2]*z:rt[i/2];
        }
        vi rev(n);
        for(int i=1;i<n;i++) rev[i]=(rev[i/2] | (i&1)<<L)/2;
        for(int i=1;i<n;i++) if(i<rev[i]) swap(a[i],a[rev[i]]);
        for(int k=1;k<n;k*=2) for(int i=0;i<n;i+=2*k) for(int j=0;j<k;j++){
            cd z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    template<class U>
    static db norm(const U &x){
        return INT?round(x):x;
    }
    static vt conv(const vt &a,const vt &b){
        if(a.empty() || b.empty()) return {};
        vt res(a.size()+b.size()-1);
        int L=32-__builtin_clz(res.size()),n=1<<L;
        vc in(n),out(n);
        copy(a.begin(),a.end(),in.begin());
        for(int i=0;i<b.size();i++) in[i].imag(b[i]);
        fft(in);
        for(auto &x:in) x*=x;
        for(int i=0;i<n;i++) out[i]=in[-i&(n-1)]-conj(in[i]);
        fft(out);
        for(int i=0;i<res.size();i++) res[i]=norm(imag(out[i])/(4*n));
        return res;
    }
    static vl convMod(const vl &a,const vl &b){
        assert(mod>0);
        if(a.empty() || b.empty()) return {};
        vl res(a.size()+b.size()-1);
```

```
int L=32-__builtin_clz(res.size()),n=1<<L;
ll cut=int(sqrt(mod));
vc inl(n),in2(n),out1(n),out2(n);
for(int i=0;i<a.size();i++)in1[i]=cd(ll(a[i])/cut,ll(a[i])%cut); // a1 + i * a2
for(int i=0;i<b.size();i++)in2[i]=cd(ll(b[i])/cut,ll(b[i])%cut); // b1 + i * b2
fft(in1),fft(in2);
for(int i=0;i<n;i++){
    int j=-i&(n-1);
    out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.1*n); // f1 * (g1 + i * g2) = f1 * g1 + i f1 * g2
    out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.1,2.1*n); // f2 * (g1 + i * g2) = f2 * g1 + i f2 * g2
}
fft(out1),fft(out2);
for(int i=0;i<res.size();i++){
    ll x=round(real(out1[i])),y=round(imag(out1[i]))+round(real(out2[i])),z=round(imag(out2[i]));
    res[i]=(x%mod*cut+y)%mod*cut+z)%mod; // a1 * b1 * cut^2 + (a1 * b2 + a2 * b1) * cut + a2 * b2
}
return res;
}
vt operator()(const vt &a,const vt &b){
    return mod>0?conv(a,b):convMod(a,b);
}
};
template<>
struct FFT<db>{
    static const bool INT=false;
};
```

NTT.hpp
Description: Number theoretic transform
Time: $\mathcal{O}(N \log N)$

../template/Header.hpp", "../modular-arithmetic/BinPow.hpp", ../modular-arithmetic/MontgomeryModInt.hpp"	2b2392, 39 lines
---	------------------

```
template<class mint=mint>
struct NTT{
    using vm = vector<mint>;

    static constexpr mint root=mint::get_root();
    static_assert(root!=0);

    static void ntt(vm &a){
        int n=a.size(),L=31-__builtin_clz(n);
        vm rt(n);
        rt[1]=1;
        for(int k=2,s=2;k<n;k*=2,s++){
            mint z[]={1,binpow(root,MOD>>s)};
            for(int i=k;i<2*k;i++)rt[i]=rt[i/2]*z[i&1];
        }
        vi rev(n);
        for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
        for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
        for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k;j++){
            mint z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }
    static vm conv(const vm &a,const vm &b){
        if(a.empty()||b.empty())return {};
        int s=a.size()+b.size()-1,n=1<<(32-__builtin_clz(s));
        mint inv=mint(n).inv();
        vm inl(a),in2(b),out(n);
        in1.resize(n),in2.resize(n);
```

NTT Manacher SuffixArray ZAlgo DVC SlopeTrick

```
ntt(in1),ntt(in2);
for(int i=0;i<n;i++)out[-i&(n-1)]=in1[i]*in2[i]*inv;
ntt(out);
return vm(out.begin(),out.begin()+s);
}
vm operator()(const vm &a,const vm &b){
    return conv(a,b);
}
};
```

Strings (9)

Manacher.hpp
Description: Manacher's Algorithm. pal[i] := the length of the longest palindrome centered at i/2.

../template/Header.hpp"	53856e, 15 lines
-------------------------	------------------

```
template<class STR>
vector<int> manacher(const STR &s){
    int n=(int)s.size();
    if(n==0)return {};
    vector<int> pal(2*n-1);
    for(int p=0,l=-1,r=-1;p<2*n-1;p++){
        int i=(p+1)>>1,j=p>>1;
        int k=(i>=r?0:min(r-i,pal[2*(l+r)-p]));
        while(j+k+1<n&&i-k-1>=0&&s[j+k+1]==s[i-k-1])k++;
        pal[p]=k;
        if(j+k>r)l=i-k,r=j+k;
    }
    for(int i=0;i<2*n-1;i++)pal[i]=pal[i]<<1|(i&1^1);
    return pal;
}
```

SuffixArray.hpp
Description: Suffix Automaton.

../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp"	b9cfb1, 39 lines
---	------------------

```
template<class STR>
struct SuffixArray{
    int n;
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArray(){}
    SuffixArray(const STR &s){init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        sa[0]=n;
        iota(sa.begin()+1,sa.end(),0);
        sort(sa.begin()+1,sa.end(),[&](int i,int j){return s[i]<s[j]});
        for(int i=1;i<=n;i++){
            int x=sa[i-1],y=sa[i];
            isa[y]=i>1&&s[x]==s[y]?isa[x]:i;
        }
        for(int len=1;len<=n;len<=<=1){
            vector<int> ps(sa),pi(isa),pos(n+1);
            iota(pos.begin(),pos.end(),0);
            for(auto i:ps)if((i=len)>=0)sa[pos[isa[i]]++]=i;
            for(int i=1;i<=n;i++){
                int x=sa[i-1],y=sa[i];
                isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x]:i;
            }
        }
        for(int i=0,k=0;i<n;i++){
            for(int j=sa[isa[i]-1];j+k<n&&s[j+k]==s[i+k];k++);
            lcp[isa[i]]=k;
            if(k)--;
```

```
}
    st.init(lcp);
}
int get_lcp(int i,int j){
    if(i==j)return n-i;
    auto [l,r]=minmax(isa[i],isa[j]);
    return st.query(l+1,r);
}
};

ZAlgo.hpp
Description: Z Algorithm. z[i] := the length of the longest common prefix between s and s[i].
../template/Header.hpp" b93726, 12 lines
template<class STR>
vector<int> z_algorithm(const STR &s){
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for(int i=1,l=0,r=1;i<n;i++){
        if(i<r)z[i]=min(r-i,z[i-1]);
        while(i+z[i]<n&&s[z[i]]==s[i+z[i]])z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    }
    return z;
}
```

Dynamic Programming (10)

DVC.hpp
Description: Optimize $\mathcal{O}(N^2K)$ to $\mathcal{O}(NK \log N)$

../template/Header.hpp"	aa5ddf, 19 lines
-------------------------	------------------

```
vector<vl> cst, dp;

ll cost(int l, int r) {
    return cst[l][r];
}

void divide(int l, int r, int opt_l, int opt_r, int c) {
    if(l > r) return ;
    int mid = (l + r) / 2;
    pair<ll, int> best = make_pair(INF, -1);
    for(int k=opt_l; k<=min(mid, opt_r); ++k) {
        best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid), k));
    }
    dp[c][mid] = best.first;
    divide(l, mid - 1, opt_l, best.second, c);
    divide(mid + 1, r, best.second, opt_r, c);
}

// for(int c=1; c<=K; ++c) divide(1, N, 1, N, c);
```

SlopeTrick.hpp
Description: Absolute Smth

../template/Header.hpp"	f62f9a, 36 lines
-------------------------	------------------

```
ll extending_value;

struct slope_trick {
    multiset<ll> ms_l, ms_r;
    ll min_y = 0ll, lz_l = 0ll, lz_r = 0ll;
    bool extending = false;
    void add_line(ll v) {
        if(extending) {
            lz_l -= extending_value;
            lz_r -= extending_value;
```

```
    }
    extending = true;
    if(ms_l.empty() && ms_r.empty()) {
        ms_l.emplace(v);
        ms_r.emplace(v);
    }
    else if(v <= *ms_l.rbegin() + lz_l) {
        min_y += (*ms_l.rbegin() + lz_l) - v;
        ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
        ms_l.erase(--ms_l.end());
        ms_l.emplace(v - lz_l);
        ms_l.emplace(v - lz_l);
    }
    else if(v >= *ms_r.begin() + lz_r) {
        min_y += v - (*ms_r.begin() + lz_r);
        ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
        ms_r.erase(ms_r.begin());
        ms_r.emplace(v - lz_r);
        ms_r.emplace(v - lz_r);
    }
    else {
        ms_l.emplace(v - lz_l);
        ms_r.emplace(v - lz_r);
    }
}
};
```

Convolutions (11)

AndConvolution.hpp
Description: Bitwise AND Convolution. Superset Zeta Transform: $A'[S] = \sum_{T \supseteq S} A[T]$. Superset Mobius Transform: $A[T] = \sum_{S \supseteq T} (-1)^{|S-T|} A'[S]$.
Time: $\mathcal{O}(N \log N)$.

```
"/template/Header.hpp"7916f8, 34 lines
template<class T>
void superset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]+=a[j];
            }
        }
    }
}

template<class T>
void superset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]-=a[j];
            }
        }
    }
}

template<class T>
vector<T> and_convolution(vector<T> a,vector<T> b){
    superset_zeta(a);
    superset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    superset_mobius(a);
    return a;
}
```

```
    }
}

GCDConvolution.hpp
Description: GCD Convolution. Multiple Zeta Transform:  $A'[n] = \sum_{n|m} A[m]$ . Multiple Mobius Transform:  $A[n] = \sum_{n|m} \mu(m/n) A'[m]$ .
Time:  $\mathcal{O}(N \log \log N)$ .
"/template/Header.hpp"7f6c2d, 34 lines
template<class T>
void multiple_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i]+=a[i*p];
        }
    }
}

template<class T>
void multiple_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i]-=a[i*p];
        }
    }
}

template<class T>
vector<T> gcd_convolution(vector<T> a,vector<T> b){
    multiple_zeta(a);
    multiple_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    multiple_mobius(a);
    return a;
}

LCMConvolution.hpp
Description: LCM Convolution. Divisor Zeta Transform:  $A'[n] = \sum_{d|n} A[d]$ . Divisor Mobius Transform:  $A[n] = \sum_{d|n} \mu(n/d) A'[d]$ .
Time:  $\mathcal{O}(N \log \log N)$ .
"/template/Header.hpp"41fe9d, 34 lines
template<class T>
void divisor_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i*p]+=a[i];
        }
    }
}

template<class T>
void divisor_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
        }
    }
}

template<class T>
vector<T> lcm_convolution(vector<T> a,vector<T> b){
    divisor_zeta(a);
    divisor_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    divisor_mobius(a);
    return a;
}
```

```
        a[i*p]-=a[i];
    }
}

template<class T>
vector<T> lcm_convolution(vector<T> a,vector<T> b){
    divisor_zeta(a);
    divisor_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    divisor_mobius(a);
    return a;
}

ORConvolution.hpp
Description: Bitwise OR Convolution. Subset Zeta Transform:  $A'[S] = \sum_{T \subseteq S} A[T]$ . Subset Mobius Transform:  $A[T] = \sum_{S \subseteq T} (-1)^{|T-S|} A'[S]$ .
Time:  $\mathcal{O}(N \log N)$ .
"/template/Header.hpp"c58b77, 34 lines
template<class T>
void subset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]+=a[j^i];
            }
        }
    }
}

template<class T>
void subset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]-=a[j^i];
            }
        }
    }
}

template<class T>
vector<T> or_convolution(vector<T> a,vector<T> b){
    subset_zeta(a);
    subset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    subset_mobius(a);
    return a;
}
```

11.1 Various

GaussianElimination.hpp

Description: Gaussian Elimination

```
"/template/Header.hpp"e89ecb, 34 lines
struct Gauss {
    int n, sz;
    vector<ll> basis;
    Gauss(int n = 0) {
        init(n);
    }
    void init(int _n) {
        n = _n, sz = 0;
        basis.assign(n, 0);
    }
};
```

```

    }
    void insert(ll x) {
        for (int i = n - 1; i >= 0; i--)
            if (x >> i & 1) {
                if (!basis[i]) {
                    basis[i] = x;
                    sz++;
                    return;
                }
                x ^= basis[i];
            }
    }
    ll getmax(ll k = 0) {
        ll tot = 1ll << sz, res = 0;
        for (int i = n - 1; i >= 0; i--)
            if (basis[i]) {
                tot >>= 1;
                if ((k >= tot && res >> i & 1) || (k < tot && res >> i
                    & 1 ^ 1))
                    res ^= basis[i];
                if (k >= tot)
                    k -= tot;
            }
        return res;
    }
};
```

BinaryTrie.hpp

Description: Binary Trie

"/template/Header.hpp" 525bf4, 59 lines

```

using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
    vector<node_t> t = {node_t{}};
    vector<int> cnt = {0};
    int cnt_nodes = 0;
    void insert(int v) {
        int cur = 0;
        cnt[0]++;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            if(!t[cur][b]) {
                t[cur][b] = ++cnt_nodes;
                t.emplace_back(node_t{});
                cnt.emplace_back(0);
            }
            cnt[t[cur][b]]++;
            cur = t[cur][b];
        }
    }
    void remove(int v) {
        int cur = 0;
        cnt[0]--;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1: 0;
            cnt[t[cur][b]]--;
            cur = t[cur][b];
        }
    }
    int get_min(int v) {
        int cur = 0, res = 0;
        for(int i=(int) S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(t[cur][b] && cnt[t[cur][b]]) {
                cur = t[cur][b];
            }
            else {
                res |= (1 << i);
                cur = t[cur][!b];
            }
        }
    }
};
```

```

    }
    }
    return res;
}

int get_max(int v) {
    int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
        int b = (v & (1 << i)) ? 1 : 0;
        if(t[cur][!b] && cnt[t[cur][!b]]) {
            res |= (1 << i);
            cur = t[cur][!b];
        }
        else {
            cur = t[cur][b];
        }
    }
    return res;
}
};
```

Competitive Programming Topics

(A)

topics.txt

159 lines

Recursion
Divide and conquer
 Finding interesting points in $N \log N$
Algorithm analysis
 Master theorem
 Amortized time complexity
Greedy algorithm
 Scheduling
 Max contiguous subvector sum
 Invariants
 Huffman encoding
Graph theory
 Dynamic graphs (extra book-keeping)
 Breadth first search
 Depth first search
 * Normal trees / DFS trees
 Dijkstra's algorithm
 MST: Prim's algorithm
 Bellman-Ford
 Konig's theorem and vertex cover
 Min-cost max flow
 Lovasz toggle
 Matrix tree theorem
 Maximal matching, general graphs
 Hopcroft-Karp
 Hall's marriage theorem
 Graphical sequences
 Floyd-Warshall
 Euler cycles
 Flow networks
 * Augmenting paths
 * Edmonds-Karp
 Bipartite matching
 Min. path cover
 Topological sorting
 Strongly connected components
 2-SAT
 Cut vertices, cut-edges and biconnected components
 Edge coloring
 * Trees
 Vertex coloring
 * Bipartite graphs (\Rightarrow trees)
 * 3^n (special case of set cover)
 Diameter and centroid
 K'th shortest path
 Shortest cycle
Dynamic programming
 Knapsack
 Coin change
 Longest common subsequence
 Longest increasing subsequence
 Number of paths in a dag
 Shortest path in a dag
 Dynprog over intervals
 Dynprog over subsets
 Dynprog over probabilities
 Dynprog over trees
 3^n set cover
 Divide and conquer
 Knuth optimization
 Convex hull optimizations
 RMQ (sparse table a.k.a 2^k -jumps)
 Bitonic cycle

 Log partitioning (loop over most restricted)
Combinatorics
 Computation of binomial coefficients
 Pigeon-hole principle
 Inclusion/exclusion
 Catalan number
 Pick's theorem
Number theory
 Integer parts
 Divisibility
 Euclidean algorithm
 Modular arithmetic
 * Modular multiplication
 * Modular inverses
 * Modular exponentiation by squaring
 Chinese remainder theorem
 Fermat's little theorem
 Euler's theorem
 Phi function
 Frobenius number
 Quadratic reciprocity
 Pollard-Rho
 Miller-Rabin
 Hensel lifting
 Vieta root jumping
Game theory
 Combinatorial games
 Game trees
 Mini-max
 Nim
 Games on graphs
 Games on graphs with loops
 Grundy numbers
 Bipartite games without repetition
 General games without repetition
 Alpha-beta pruning
Probability theory
Optimization
 Binary search
 Ternary search
 Unimodality and convex functions
 Binary search on derivative
Numerical methods
 Numeric integration
 Newton's method
 Root-finding with binary/ternary search
 Golden section search
Matrices
 Gaussian elimination
 Exponentiation by squaring
Sorting
 Radix sort
Geometry
 Coordinates and vectors
 * Cross product
 * Scalar product
 Convex hull
 Polygon cut
 Closest pair
 Coordinate-compression
 Quadtrees
 KD-trees
 All segment-segment intersection
Sweeping
 Discretization (convert to events and sweep)
 Angle sweeping
 Line sweeping
 Discrete second derivatives
Strings

 Longest common substring
 Palindrome subsequences
 Knuth-Morris-Pratt
 Tries
 Rolling polynomial hashes
 Suffix array
 Suffix tree
 Aho-Corasick
 Manacher's algorithm
 Letter position lists
Combinatorial search
 Meet in the middle
 Brute-force with pruning
 Best-first (A*)
 Bidirectional search
 Iterative deepening DFS / A*
Data structures
 LCA (2^k -jumps in trees in general)
 Pull/push-technique on trees
 Heavy-light decomposition
 Centroid decomposition
 Lazy propagation
 Self-balancing trees
 Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
 Monotone queues / monotone stacks / sliding queues
 Sliding queue using 2 stacks
 Persistent segment tree