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template from KACTL

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Template (1)

```
template.cpp                                     27 lines

#pragma once

#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()

using namespace std;

using ll = long long;
using db = long double;
using vi = vector<int>;
using vl = vector<ll>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<ll, ll>;
using pdd = pair<db, db>;
const int INF = 0x3fffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const ll LINF = 0xffffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));
```

```
1 int main(){
   cin.tie(nullptr)->sync_with_stdio(false);
1 }

1 c.sh                                     2 lines
g++ -std=gnu++2a -Wall $1 -o a.out
2 ./a.out
```

Mathematics (2)

2.1    Goldbatch’s Conjecture

- 8        • Even number can be written in sum of two primes (Up to 1e12)
- 9
- 12       • Range of  $N^{th}$  prime and  $N + 1^{th}$  prime will be less than or equal to 300 (Up to 1e12)

2.2    Divisibility

Number of divisors of  $N$  is given by  $\prod_{i=1}^k (a_i + 1)$  where  $N = \prod_{i=1}^k p_i^{a_i}$  and  $p_i$  are prime factors of  $N$ .

Combinatorial (3)

3.1    Permutations

3.1.1    Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

```
IntPerm.h
Description: Permutation -> integer conversion. (Not order preserving.)
Integer -> permutation can use a lookup table.
Time: O(n)                                     044568, 6 lines

int permToInt(vi &v){
    int use = 0, i = 0, r = 0;
    for (int x : v) r = r * ++i + __builtin_popcount(use & -(1 << x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

3.1.2    Cycles

Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

3.1.3    Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.1.4    Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

3.2    Partitions and subsets

3.2.1    Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

3.2.2    Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

3.2.3    Binomials

```
multinomial.h
Description: Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$ .          a0a312, 6 lines

ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i, 1, sz(v)) rep(j, 0, v[i])
        c = c * ++m / (j+1);
    return c;
}
```

3.3    General purpose numbers

3.3.1    Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).  
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x)dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

3.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$\begin{aligned} c(n,k) &= c(n-1,k-1) + (n-1)c(n-1,k), \quad c(0,0) = 1 \\ \sum_{k=0}^n c(n,k)x^k &= x(x+1)\dots(x+n-1) \end{aligned}$$

$$\begin{aligned} c(8,k) &= 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1 \\ c(n,2) &= 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots \end{aligned}$$

3.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k\ j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1\ j$ :s s.t.  $\pi(j) \geq j$ ,  $k\ j$ :s s.t.  $\pi(j) > j$ .

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

3.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

3.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ . For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

3.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$   
# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$   
# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

3.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$\begin{aligned} C_0 &= 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i} \\ C_n &= 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots \end{aligned}$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

Numerical (4)

4.1 Newton’s Method

if  $F(Q) = 0$ , then  $Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}$

$$Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}$$

$$Q = \ln P = \int \frac{P'}{P} dx$$

$$Q = e^P : Q_{2n} \equiv Q_n (1 + P - \ln Q_n) \pmod{x^{2n}}$$

$$Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2} (Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}$$

$$Q = P^k = \alpha^k x^{kt} e^{k \ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1$$

Group (5)

5.1 Monoid

```
monoid/MonoidBase.hpp
Description: Monoid Base class.
e75b74, 6 lines

template<class T,T (*combine)(T,T),T (*identity)()>
struct MonoidBase{
    using value_type = T;
    static constexpr T op(const T &x,const T &y){return combine(x,y);}
    static constexpr T unit(){return identity();}
};
```

5.2 Action

```
action/MonoidActionBase.hpp
Description: Monoid Action Base class.
425d83, 11 lines

template<class MInfo,class MTag,typename MInfo::value_type
(*combine)(typename MInfo::value_type,typename MTag::
value_type)>
struct MonoidActionBase{
    using InfoMonoid = MInfo;
```

```
using TagMonoid = MTag;
using Info = typename InfoMonoid::value_type;
using Tag = typename TagMonoid::value_type;
static constexpr Info op(const Info &a,const Tag &b){
    return combine(a,b);
}
};
```

```
action/DefaultAction.hpp
Description: Default Action class.
e45000, 10 lines

template<class Monoid>
struct DefaultAction{
    using InfoMonoid = Monoid;
    using TagMonoid = Monoid;
    using Info = typename Monoid::value_type;
    using Tag = typename Monoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b){
        return Monoid::op(a,b);
    }
};
```

Data Structures (6)

```
OrderedSet.hpp
Description: Ordered Set
1a7ff5f, 14 lines

using namespace __gnu_pbds;

template <class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
tree_order_statistics_node_update>;
// can be change to less_equal

void usage() {
    ordered_set<int> st, st_2;
    st.insert(2);
    st.insert(1);
    cout << st.order_of_key(2);
    cout << *st.find_by_order(1);
    st.join(st_2); // merge
}
```

```
FenwickTree.hpp
Description: Fenwick / Binary Indexed Tree
43767a, 41 lines

template<class T>
struct Fenwick{
    int n,logn;
    vector<T> t;
    Fenwick(){}
    Fenwick(int _n){init(vector<T>(_n,T{}));}
    template<class U>
    Fenwick(const vector<U> &a){init(a);}
    template<class U>
    void init(const vector<U> &a){
        n=(int)a.size();
        logn=31-__builtin_clz(n);
        t.assign(n+1,T{});
        for(int i=1;i<=n;i++){
            t[i]=t[i]+a[i-1];
            int j=i+(i&-i);
            if(j<=n)t[j]=t[j]+t[i];
        }
    }
    void update(int x,const T &v){
        for(int i=x+1;i<=n;i+=i&-i)t[i]=t[i]+v;
    }
};
```

```

void update(int l,int r,const T &v){
    update(l,v),update(r+1,-v);
}
T query(int x){
    T res{};
    for(int i=x+1;i>0;i-=i&-i)res=res+t[i];
    return res;
}
T query(int l,int r){
    return query(r)-query(l-1);
}
int find(const T &k){
    int x=0;
    T cur{};
    for(int i=1<<logn;i>0;i>=1)
        if(x+i<=n&&cur+t[x+i]<=k)x+=i,cur=cur+t[x];
    return x;
}
};

```

### SmallSegmentTree.hpp

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.

**Time:**  $\mathcal{O}(\log N)$

0f4bdb, 19 lines

```

struct Tree {
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
    T query(int b, int e) { // query [b, e)
        T ra = unit, rb = unit;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb);
        }
        return f(ra, rb);
    }
};

```

### SegmentTree.hpp

**Description:** Segment Tree

c51dec, 85 lines

```

template<class Monoid>
struct SegmentTree{
    using T = typename Monoid::value_type;
    int n;
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n,function<T(int)>> create){init(n,create);}
    SegmentTree(int n,T v=Monoid::unit()){init(n,[&](int){
        return v;});}
    template<class U>
    SegmentTree(const vector<U> &a){init((int)a.size(), [&](int
        i){return T(a[i]);});}
    void init(int _n,function<T(int)>> create){
        n=_n;
        t.assign(4<<(31-__builtin_clz(n)),Monoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            ){
                if(l==r)return void(t[i]=create(l));
                int m=(l+r)/2;
                build(l,m,i*2);
                build(m+1,r,i*2+1);
            };
    }
};

```

```

        pull(i);
    };
    build(0,n-1,1);
}
void pull(int i){
    t[i]=Monoid::op(t[i*2],t[i*2+1]);
}
void modify(int l,int r,int i,int x,const T &v){
    if(x<l||r<x)return;
    if(l==r)return void(t[i]=v);
    int m=(l+r)/2;
    modify(l,m,i*2,x,v);
    modify(m+1,r,i*2+1,x,v);
    pull(i);
}
void modify(int x,const T &v){
    modify(0,n-1,1,x,v);
}
template<class U>
void update(int l,int r,int i,int x,const U &v){
    if(x<l||r<x)return;
    if(l==r)return void(t[i]=Monoid::op(t[i],v));
    int m=(l+r)/2;
    update(l,m,i*2,x,v);
    update(m+1,r,i*2+1,x,v);
    pull(i);
}
template<class U>
void update(int x,const U &v){
    update(0,n-1,1,x,v);
}
T query(int l,int r,int i,int x,int y){
    if(y<l||r<x)return Monoid::unit();
    if(x<=l&&r<=y)return t[i];
    int m=(l+r)/2;
    return Monoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,
        x,y));
}
T query(int x,int y){
    return query(0,n-1,1,x,y);
}
template<class F>
int findfirst(int l,int r,int i,int x,int y,const F &f){
    if(y<l||r<x||!f(t[i]))return n;
    if(l==r)return l;
    int m=(l+r)/2;
    int res=findfirst(l,m,i*2,x,y,f);
    if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
    return res;
}
template<class F>
int findfirst(int x,int y,const F &f){
    return findfirst(0,n-1,1,x,y,f);
}
template<class F>
int findlast(int l,int r,int i,int x,int y,const F &f){
    if(y<l||r<x||!f(t[i]))return -1;
    if(l==r)return l;
    int m=(l+r)/2;
    int res=findlast(m+1,r,i*2+1,x,y,f);
    if(res==-1)res=findlast(l,m,i*2,x,y,f);
    return res;
}
template<class F>
int findlast(int x,int y,const F &f){
    return findlast(0,n-1,1,x,y,f);
}
};

```

### SegmentTreeBeats.hpp

**Description:** Segment Tree Beats

"../template/Header.hpp"

efa2ef, 134 lines

```

const int N = 2e5 + 5;
const int K = 1 << 19;

int n, q;
ll a[N];

struct SegTree{
    struct Node{
        ll sum,add;
        ll mn,mn2,fn;
        ll mx,mx2,fx;
        Node(){
            sum=add=fn=fx=0,mn=mn2=LINF,mx=mx2=-LINF;
        }
        Node(ll v){
            sum=mn=mx=v,add=0,mn2=LINF,mx2=-LINF,fn=fx=1;
        }
        friend Node operator+(const Node &l,const Node &r){
            Node res;
            res.sum=l.sum+r.sum;
            res.add=0;
            if(l.mx>r.mx){
                res.mx=l.mx,res.fx=l.fx;
                res.mx2=max(l.mx2,r.mx);
            }else if(r.mx>l.mx){
                res.mx=r.mx,res.fx=r.fx;
                res.mx2=max(r.mx2,l.mx);
            }else{
                res.mx=l.mx,res.fx=l.fx+r.fx;
                res.mx2=max(l.mx2,r.mx2);
            }
            if(l.mn<r.mn){
                res.mn=l.mn,res.fn=l.fn;
                res.mn2=min(l.mn2,r.mn);
            }else if(r.mn<l.mn){
                res.mn=r.mn,res.fn=r.fn;
                res.mn2=min(r.mn2,l.mn);
            }else{
                res.mn=l.mn,res.fn=l.fn+r.fn;
                res.mn2=min(l.mn2,r.mn2);
            }
            return res;
        }
    }
    void apply(int l,int r,ll v){
        sum+=(r-l+1)*v;
        mx+=v,mx2+=v;
        mn+=v,mn2+=v;
        add+=v;
    }
    void chmin(ll v){
        if(v>=mx)return;
        sum+=(v-mx)*fx;
        if(mn==mx)mn=v;
        if(mn2==mx)mn2=v;
        mx=v;
    }
    void chmax(ll v){
        if(v<=mn)return;
        sum+=(v-mn)*fn;
        if(mx==mn)mx=v;
        if(mx2==mn)mx2=v;
        mn=v;
    }
}t[K];
void pull(int i){
    t[i]=t[i*2]+t[i*2+1];
}

```

```

}
void push(int l,int r,int i){
    int m=(l+r)/2;
    t[i*2].apply(l,m,t[i].add);
    t[i*2+1].apply(m+1,r,t[i].add);
    t[i*2].chmin(t[i].mx);
    t[i*2+1].chmin(t[i].mx);
    t[i*2].chmax(t[i].mn);
    t[i*2+1].chmax(t[i].mn);
    t[i].add=0;
}
void build(int l,int r,int i){
    if(l==r)return void(t[i]=Node(a[l]));
    int m=(l+r)/2;
    build(l,m,i*2);
    build(m+1,r,i*2+1);
    pull(i);
}
void build(){
    build(1,n,1);
}
void range_add(int l,int r,int i,int x,int y,ll v){
    if(y<l||r<x)return;
    if(x<=l&&r<=y)return t[i].apply(l,r,v);
    int m=(l+r)/2;
    push(l,r,i);
    range_add(l,m,i*2,x,y,v);
    range_add(m+1,r,i*2+1,x,y,v);
    pull(i);
}
void range_add(int x,int y,ll v){
    range_add(1,n,1,x,y,v);
}
void range_chmin(int l,int r,int i,int x,int y,ll v){
    if(y<l||r<x||t[i].mx<=v)return;
    if(x<=l&&r<=y&&t[i].mx2<v)return t[i].chmin(v);
    int m=(l+r)/2;
    push(l,r,i);
    range_chmin(l,m,i*2,x,y,v);
    range_chmin(m+1,r,i*2+1,x,y,v);
    pull(i);
}
void range_chmin(int x,int y,ll v){
    range_chmin(1,n,1,x,y,v);
}
void range_chmax(int l,int r,int i,int x,int y,ll v){
    if(y<l||r<x||t[i].mn>=v)return;
    if(x<=l&&r<=y&&t[i].mn2>v)return t[i].chmax(v);
    int m=(l+r)/2;
    push(l,r,i);
    range_chmax(l,m,i*2,x,y,v);
    range_chmax(m+1,r,i*2+1,x,y,v);
    pull(i);
}
void range_chmax(int x,int y,ll v){
    range_chmax(1,n,1,x,y,v);
}
ll query(int l,int r,int i,int x,int y){
    if(y<l||r<x)return 0;
    if(x<=l&&r<=y)return t[i].sum;
    int m=(l+r)/2;
    push(l,r,i);
    return query(l,m,i*2,x,y)+query(m+1,r,i*2+1,x,y);
}
ll query(int x,int y){
    return query(1,n,1,x,y);
}
};

```

## LazySegmentTree.hpp

Description: Segment Tree with Lazy Propagation

91ab0c, 103 lines

```

template<class MonoidAction>
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n;
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n,function<Info(int)> create){init(n,create);}
    LazySegmentTree(int n,Info v=InfoMonoid::unit()){init(n,[&](int){return v;});}
    template<class T>
    LazySegmentTree(const vector<T> &a){init((int)a.size(),[&](int i){return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        n=_n;
        int m=4<<(31-__builtin_clz(n));
        t.assign(m,InfoMonoid::unit());
        lz.assign(m,TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i){
            if(l==r)return void(t[i]=create(l));
            int m=(l+r)/2;
            build(l,m,i*2);
            build(m+1,r,i*2+1);
            pull(i);
        };
        build(0,n-1,1);
    }
    void pull(int i){
        t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
    }
    void apply(int i,const Tag &v){
        t[i]=MonoidAction::op(t[i],v);
        lz[i]=TagMonoid::op(lz[i],v);
    }
    void push(int i){
        apply(i*2,lz[i]);
        apply(i*2+1,lz[i]);
        lz[i]=TagMonoid::unit();
    }
    void modify(int l,int r,int i,int x,const Info &v){
        if(x<l||r<x)return;
        if(l==r)return void(t[i]=v);
        int m=(l+r)/2;
        push(i);
        modify(l,m,i*2,x,v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    }
    void modify(int x,const Info &v){
        modify(0,n-1,1,x,v);
    }
    void update(int l,int r,int i,int x,int y,const Tag &v){
        if(y<l||r<x)return;
        if(x<=l&&r<=y)return apply(i,v);
        int m=(l+r)/2;
        push(i);
        update(l,m,i*2,x,y,v);
        update(m+1,r,i*2+1,x,y,v);
        pull(i);
    }
    void update(int x,int y,const Tag &v){

```

```

        update(0,n-1,1,x,y,v);
    }
    Info query(int l,int r,int i,int x,int y){
        if(y<l||r<x)return InfoMonoid::unit();
        if(x<=l&&r<=y)return t[i];
        int m=(l+r)/2;
        push(i);
        return InfoMonoid::op(query(l,m,i*2,x,y),query(m+1,r,i*2+1,x,y));
    }
    Info query(int x,int y){
        return query(0,n-1,1,x,y);
    }
    template<class F>
    int findfirst(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return n;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findfirst(l,m,i*2,x,y,f);
        if(res==n)res=findfirst(m+1,r,i*2+1,x,y,f);
        return res;
    }
    template<class F>
    int findfirst(int x,int y,const F &f){
        return findfirst(0,n-1,1,x,y,f);
    }
    template<class F>
    int findlast(int l,int r,int i,int x,int y,const F &f){
        if(y<l||r<x||!f(t[i]))return -1;
        if(l==r)return l;
        int m=(l+r)/2;
        push(i);
        int res=findlast(m+1,r,i*2+1,x,y,f);
        if(res==-1)res=findlast(l,m,i*2,x,y,f);
        return res;
    }
    template<class F>
    int findlast(int x,int y,const F &f){
        return findlast(0,n-1,1,x,y,f);
    }
};

```

## DynamicSegmentTree.hpp

Description: Dynamic Segment Tree

e84eeb, 106 lines

```

template<class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node{
        Info val;
        Tag lz;
        Ptr l,r;
        Node(Info v):val(v),lz(TagMonoid::unit()),l(nullptr),r(nullptr){}
        Node(Info v,Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr rt;
    function<Info(ll,ll)> create;
    DynamicSegmentTree(){}init(0,0);
    DynamicSegmentTree(ll n){init(0,n-1);}
    DynamicSegmentTree(ll lb,ll ub){init(lb,ub);}

```

```
DynamicSegmentTree(ll lb,ll ub,function<Info(ll,ll)> create
){init(lb,ub,create);}
void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll
l,ll r){return InfoMonoid::unit();}){
lb=_lb,ub=_ub;
create=_create;
rt=new Node(create(lb,ub));
}
Info val(Ptr t){
return t?t->val:InfoMonoid::unit();
}
void pull(Ptr &t){
t->val=InfoMonoid::op(val(t->l),val(t->r));
}
void apply(Ptr &t,const Tag &v,ll l,ll r){
if(!t)t=new Node(create(l,r));
t->val=MonoidAction::op(t->val,v);
t->lz=TagMonoid::op(t->lz,v);
}
void push(Ptr &t,ll l,ll m,ll r){
apply(t->l,t->lz,l,m);
apply(t->r,t->lz,m+1,r);
t->lz=TagMonoid::unit();
}
void modify(ll l,ll r,Ptr &t,ll x,const Info &v){
if(x<l||r<x)return;
if(l==r)return void(t->val=v);
ll m=l+(r-l)/2;
push(t,l,m,r);
modify(l,m,t->l,x,v);
modify(m+1,r,t->r,x,v);
pull(t);
}
void modify(ll x,const Info &v){
modify(lb,ub,rt,x,v);
}
void update(ll l,ll r,Ptr &t,ll x,ll y,const Tag &v){
if(y<l||r<x)return;
if(x<=l&&r<=y)return apply(t,v,l,r);
ll m=l+(r-l)/2;
push(t,l,m,r);
update(l,m,t->l,x,y,v);
update(m+1,r,t->r,x,y,v);
pull(t);
}
void update(ll x,ll y,const Tag &v){
update(lb,ub,rt,x,y,v);
}
Info query(ll l,ll r,Ptr &t,ll x,ll y){
if(y<l||r<x)return InfoMonoid::unit();
if(x<=l&&r<=y)return t->val;
ll m=l+(r-l)/2;
push(t,l,m,r);
return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t
->r,x,y));
}
Info query(ll x,ll y){
return query(lb,ub,rt,x,y);
}
template<class F>
ll findfirst(ll l,ll r,Ptr t,ll x,ll y,const F &f){
if(y<l||r<x||!f(t->val))return -1;
if(l==r)return l;
ll m=l+(r-l)/2;
push(t,l,m,r);
ll res=findfirst(l,m,t->l,x,y,f);
if(res!=-1)res=findfirst(m+1,r,t->r,x,y,f);
return res;
}

```

```
template<class F>
ll findfirst(ll x,ll y,const F &f){
return findfirst(lb,ub,rt,x,y,f);
}
template<class F>
ll findlast(ll l,ll r,Ptr t,ll x,ll y,const F &f){
if(y<l||r<x||!t||!f(t->val))return -1;
if(l==r)return l;
ll m=l+(r-l)/2;
push(t,l,m,r);
ll res=findlast(m+1,r,t->r,x,y,f);
if(res!=-1)res=findlast(l,m,t->l,x,y,f);
return res;
}
template<class F>
ll findlast(ll x,ll y,const F &f){
return findlast(lb,ub,rt,x,y,f);
}
};

```

DSU.hpp

Description: Disjoint Set Union.

0b3cb8, 26 lines

```
struct DSU{
vector<int> p,sz;
DSU(){}
DSU(int n){init(n);}
void init(int n){
p.resize(n);
iota(p.begin(),p.end(),0);
sz.assign(n,1);
}
int find(int u){
return p[u]==u?p[u]=find(p[u]);
}
bool same(int u,int v){
return find(u)==find(v);
}
bool merge(int u,int v){
u=find(u),v=find(v);
if(u==v)return false;
sz[u]+=sz[v];
p[v]=u;
return true;
}
int size(int u){
return sz[find(u)];
}
};

```

BinaryTrie.hpp

Description: Binary Trie

ae5b7a, 66 lines

```
template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
struct Node{
array<int,2> ch;
S cnt;
Node():ch{-1,-1},cnt(0){}
};
vector<Node> t;
BinaryTrie():t{Node()}{}
int new_node(){
t.emplace_back(Node());
return t.size()-1;
}
S size(){
return t[0].cnt;
}
}

```

```
bool empty(){
return size()==0;
}
S get_cnt(int i){
return i!=-1?t[i].cnt:S(0);
}
void insert(T x,S k=1){
int u=0;
t[u].cnt+=k;
for(int i=BIT-1;i>=0;i--){
int v=x>>i&1;
if(t[u].ch[v]==-1)t[u].ch[v]=new_node();
u=t[u].ch[v];
t[u].cnt+=k;
}
}
void erase(T x,S k=1){
int u=0;
assert(t[u].cnt==k);
t[u].cnt-=k;
for(int i=BIT-1;i>=0;i--){
int v=x>>i&1;
u=t[u].ch[v];
assert(u!=-1&&t[u].cnt>=k);
t[u].cnt-=k;
}
}
T kth(S k,T x=0){
assert(k<size());
int u=0;
T res=0;
for(int i=BIT-1;i>=0;i--){
int v=x>>i&1;
if(k<get_cnt(t[u].ch[v])){
u=t[u].ch[v];
}else{
res|=T(1)<<i;
k-=get_cnt(t[u].ch[v]);
u=t[u].ch[v^1];
}
}
return res;
}
T min(T x){
return kth(0,x);
}
T max(T x){
return kth(size()-1,x);
}
};

```

LiChaoTree.hpp

Description: Li-Chao Tree (minimize).

4ab713, 52 lines

```
template<class T>
struct LiChaoTree{
static const T INF=numeric_limits<T>::max()/2;
struct Line{
T m,c;
Line(T _m,T _c):m(_m),c(_c){}
inline T eval(T x)const{return m*x+c;}
};
vector<T> xs;
vector<Line> t;
LiChaoTree(){}
LiChaoTree(const vector<T> &x):xs(x){init(x);}
LiChaoTree(int n):xs(n){
vector<T> x(n);
iota(x.begin(),x.end(),0);
}
}

```

```

        init(x);
    }
    void init(const vector<T> &x){
        xs=x;
        sort(xs.begin(),xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    }
    void insert(int l,int r,int i,Line v){
        int m=(l+r)/2;
        if(v.eval(xs[m])<t[i].eval(xs[m]))swap(t[i],v);
        if(v.eval(xs[l])<t[i].eval(xs[l]))insert(l,m,i*2,v);
        if(v.eval(xs[r])<t[i].eval(xs[r]))insert(m+1,r,i*2+1,v);
    }
    inline void insert(T m,T c){
        insert(0,(int)xs.size()-1,1,Line(m,c));
    }
    void insert_range(int l,int r,int i,T x,T y,Line v){
        if(y<xs[l]||xs[r]<x)return;
        if(x<=xs[l]&&xs[r]<=y)return insert(l,r,i,v);
        int m=(l+r)/2;
        insert_range(l,m,i*2,x,y,v);
        insert_range(m+1,r,i*2+1,x,y,v);
    }
    inline void insert_range(T m,T c,T x,T y){
        insert_range(0,(int)xs.size()-1,1,x,y,Line(m,c));
    }
    T query(int l,int r,int i,T x){
        if(l==r)return t[i].eval(x);
        int m=(l+r)/2;
        if(x<=xs[m])return min(t[i].eval(x),query(l,m,i*2,x));
        return min(t[i].eval(x),query(m+1,r,i*2+1,x));
    }
    inline T query(T x){
        return query(0,(int)xs.size()-1,1,x);
    }
};

```

## DynamicLiChaoTree.hpp

**Description:** Dynamic Li-Chao Tree (minimize).

b8af36, 50 lines

```

template<class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m,T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    };
    ll lb,ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(nullptr){}
    void insert(T l,T r,Ptr &t,Line v){
        if(!t)return void(t=new Node(v));
        T m=l+(r-l)/2;
        if(v.eval(m)<t->v.eval(m))swap(t->v,v);
        if(v.eval(l)<t->v.eval(l))insert(l,m,t->l,v);
        if(v.eval(r)<t->v.eval(r))insert(m+1,r,t->r,v);
    }
};

```

```

inline void insert(T m,T c){
    insert(lb,ub,root,Line(m,c));
}
void insert_range(T l,T r,Ptr &t,T x,T y,Line v){
    if(y<l||r<x)return;
    if(!t)t=new Node();
    if(x<=l&&r<=y)return insert(l,r,t,v);
    T m=l+(r-l)/2;
    insert_range(l,m,t->l,x,y,v);
    insert_range(m+1,r,t->r,x,y,v);
}
inline void insert_range(T m,T c,T x,T y){
    insert_range(lb,ub,root,x,y,Line(m,c));
}
T query(T l,T r,Ptr t,T x){
    if(!t)return INF;
    T m=l+(r-l)/2;
    if(x<=m)return min(t->v.eval(x),query(l,m,t->l,x));
    return min(t->v.eval(x),query(m+1,r,t->r,x));
}
inline T query(T x){
    return query(lb,ub,root,x);
}
};

```

## SplayTreeBase.hpp

**Description:** Splay Tree. splay(u) will make node u be the root of the tree in amortized  $O(\log n)$  time.

cc90a9, 113 lines

```

template<class Node>
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return !(t->p)|| (t->p->l!=t&&t->p->r!=t);
    } // The parent of the root stores the path parant in link cut tree.
    int size(Ptr t){
        return t?t->size:0;
    }
    virtual void push(Ptr t){};
    virtual void pull(Ptr t){};
    int pos(Ptr t){
        if(t->p){
            if(t->p->l==t)return -1;
            if(t->p->r==t)return 1;
        }
        return 0;
    }
    void rotate(Ptr t){
        Ptr x=t->p,y=x->p;
        if(pos(t)==-1){
            if((x->l==t->r))t->r->p=x;
            t->r=x,x->p=t;
        }else{
            if((x->r==t->l))t->l->p=x;
            t->l=x,x->p=t;
        }
        pull(x),pull(t);
        if((t->p==y)){
            if(y->l==x)y->l=t;
            if(y->r==x)y->r=t;
        }
    }
    void splay(Ptr t){
        if(!t)return;
        push(t);
        while(!is_root(t)){
            Ptr x=t->p;
            if(is_root(x)){

```

```

                push(x),push(t);
                rotate(t);
            }else{
                Ptr y=x->p;
                push(y),push(x),push(t);
                if(pos(x)==pos(t))rotate(x),rotate(t);
                else rotate(t),rotate(t);
            }
        }
        Ptr get_first(Ptr t){
            while(t->l)push(t),t=t->l;
            splay(t);
            return t;
        }
        Ptr get_last(Ptr t){
            while(t->r)push(t),t=t->r;
            splay(t);
            return t;
        }
        Ptr merge(Ptr l,Ptr r){
            splay(l),splay(r);
            if(!l)return r;
            if(!r)return l;
            l=get_last(l);
            l->r=r;
            r->p=l;
            pull(l);
            return l;
        }
        pair<Ptr,Ptr> split(Ptr t,int k){
            if(!t)return {nullptr,nullptr};
            if(k==0)return {nullptr,t};
            if(k==size(t))return {t,nullptr};
            push(t);
            if(k<=size(t->l)){
                auto x=split(t->l,k);
                t->l=x.second;
                t->p=nullptr;
                if(x.second)x.second->p=t;
                pull(t);
                return {x.first,t};
            }else{
                auto x=split(t->r,k-size(t->l)-1);
                t->r=x.first;
                t->p=nullptr;
                if(x.first)x.first->p=t;
                pull(t);
                return {t,x.second};
            }
        }
        void insert(Ptr &t,int k,Ptr v){
            splay(t);
            auto x=split(t,k);
            t=merge(merge(x.first,v),x.second);
        }
        void erase(Ptr &t,int k){
            splay(t);
            auto x=split(t,k);
            auto y=split(x.second,1);
            // delete y.first;
            t=merge(x.first,y.second);
        }
    };
    template<class T>
    Ptr build(const vector<T> &v){
        if(v.empty())return nullptr;
        function<Ptr(int,int)> build=[&](int l,int r){
            if(l==r)return new Node(v[l]);
            int m=(l+r)/2;

```



```
        return merge(build(l,m),build(m+1,r));
    };
    return build(0,v.size()-1);
}
};
```

### LazyReversibleBBST.hpp

**Description:** Lazy Reversible BBST Base.

904708, 81 lines

```
template<class Tree,class Node,class MonoidAction>
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;

    LazyReversibleBBST()=default;

    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
    }
    void pull(Ptr t){
        if(!t) return;
        push(t);
        t->size=1;
        t->sum=t->val;
        t->revsum=t->val;
        if(t->l){
            t->size+=t->l->size;
            t->sum=InfoMonoid::op(t->l->sum,t->sum);
            t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);
        }
        if(t->r){
            t->size+=t->r->size;
            t->sum=InfoMonoid::op(t->sum,t->r->sum);
            t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
        }
    }
    void push(Ptr t){
        if(!t) return;
        if(t->rev){
            toggle(t->l);
            toggle(t->r);
            t->rev=false;
        }
        if(t->lz!=TagMonoid::unit()){
            propagate(t->l,t->lz);
            propagate(t->r,t->lz);
            t->lz=TagMonoid::unit();
        }
    }
    void toggle(Ptr t){
        if(!t) return;
        swap(t->l,t->r);
        swap(t->sum,t->revsum);
        t->rev^=true;
    }
    void propagate(Ptr t,const Tag &v){
        if(!t) return;
        t->val=MonoidAction::op(t->val,v);
        t->sum=MonoidAction::op(t->sum,v);
        t->revsum=MonoidAction::op(t->revsum,v);
        t->lz=TagMonoid::op(t->lz,v);
    }
    void apply(Ptr &t,int l,int r,const Tag &v){
        if(l>r) return;
```

```
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        propagate(y.first,v);
        t=merge(x.first,merge(y.first,y.second));
    }
    Info query(Ptr &t,int l,int r){
        if(l>r) return InfoMonoid::unit();
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        Info res=sum(y.first);
        t=merge(x.first,merge(y.first,y.second));
        return res;
    }
    void reverse(Ptr &t,int l,int r){
        if(l>r) return;
        auto x=split(t,l);
        auto y=split(x.second,r-l+1);
        toggle(y.first);
        t=merge(x.first,merge(y.first,y.second));
    }
};
```

### LazyReversibleSplayTree.hpp

**Description:** Lazy Reversible Splay Tree.

"SplayTreeBase.hpp", "LazyReversibleBBST.hpp"b8455b, 23 lines

```
template<class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
    Ptr l,r,p;
    Info val,sum,revsum;
    Tag lz;
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::
        unit(),const Tag &_lz=TagMonoid::unit())
        :l(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),
        size(1),rev(false){}
};

template<class MonoidAction>
struct LazyReversibleSplayTree
: LazyReversibleBBST<SplayTreeBase<
    LazyReversibleSplayTreeNode<MonoidAction>>,
    LazyReversibleSplayTreeNode<MonoidAction>,>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};
```

### LinkCutTreeBase.hpp

**Description:** Link Cut Tree Base.  
**Usage:** evert(u): make u be the root of the tree.  
link(u,v): attach u to v.  
cut(u,v): remove edge between u and v.  
get\_root(u): get the root of the tree containing u.  
lca(u,v): get the lowest common ancestor of u and v.  
fold(u,v): get the value of the path from u to v.

b432c3, 59 lines

```
template<class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for(Ptr cur=t;cur;cur=cur->p){
```

```
        this->splay(cur);
        cur->r=pc;
        this->pull(cur);
        pc=cur;
    }
    this->splay(t);
    return pc;
}
void evert(Ptr t){ // make t be the root of the tree
    expose(t);
    this->toggle(t);
    this->push(t);
}
void link(Ptr u,Ptr v){ // attach u to v
    evert(u);
    expose(v);
    u->p=v;
}
void cut(Ptr u,Ptr v){ // cut edge between u and v
    evert(u);
    expose(v);
    assert(u->p==v);
    v->l=u->p=nullptr;
    this->pull(v);
}
Ptr get_root(Ptr t){
    expose(t);
    while(t->l) this->push(t),t=t->l;
    this->splay(t);
    return t;
}
Ptr lca(Ptr u,Ptr v){
    if(get_root(u)!=get_root(v)) return nullptr;
    expose(u);
    return expose(v);
}
void set_val(Ptr t,const T &val){
    this->evert(t);
    t->val=val;
    this->pull(t);
}
T get_val(Ptr t){
    this->evert(t);
    return t->val;
}
T fold(Ptr u,Ptr v){
    evert(u);
    expose(v);
    return v->sum;
}
};
```

### LazyLinkCutTree.hpp

**Description:** Lazy Link Cut Tree.



**Usage:** using Lct = LazyLinkCutTree<Action>;  
using Ptr = Lct::Ptr;  
using Node = Lct::Node;  
vector<Ptr> ptr(n);  
for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);  
auto link=[&](int u,int v){  
Lct::link(ptr[u],ptr[v]);  
};  
auto cut=[&](int u,int v){  
Lct::cut(ptr[u],ptr[v]);  
};  
auto update=[&](int u,int v,const Action::Tag &val){  
Lct::apply(ptr[u],ptr[v],val);  
};  
auto query=[&](int u,int v){  
return Lct::fold(ptr[u],ptr[v]);  
};

"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp" ead3da, 12 lines

```
template<class MonoidAction>
struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree<
    MonoidAction>>{
    using base = LinkCutTreeBase<LazyReversibleSplayTree<
        MonoidAction>>;
    using Ptr = typename base::Ptr;
    using Tag = typename MonoidAction::Tag;

    void apply(Ptr u,Ptr v,const Tag &val){
        this->evert(u);
        this->expose(v);
        this->propagate(v,val);
    }
};
```

## StaticTopTree.hpp

**Description:** Static Top Tree

"../template/Header.hpp" 15bea8, 148 lines

```
enum Type{Compress,Rake,AddEdge,AddVertex,Vertex};

struct StaticTopTree{
    int n,node_id;
    int root,stt_root;
    vector<vector<int>> &adj;
    vector<int> hv;
    vector<int> lch,rch,par;
    vector<Type> type;
    StaticTopTree(vector<vector<int>> &_adj,int _root):adj(_adj)
    ){
        n=adj.size();
        root=_root;
        node_id=n;
        hv.assign(n,-1);
        lch.assign(4*n,-1);
        rch.assign(4*n,-1);
        par.assign(4*n,-1);
        type.assign(4*n,Type::Vertex);
        dfs(root);
        stt_root=compress(root).first;
    }
    int dfs(int u){
        int mx=0,sz=1;
        for(auto v:adj[u]){
            int x=dfs(v);
            sz+=x;
            if(x>mx){
                hv[u]=v;
                mx=x;
            }
        }
    }
```

```
        return sz;
    }
    int add(int i,int l,int r,Type t){
        if(i==-1)i=node_id++;
        lch[i]=l,rch[i]=r,par[i]=-1,type[i]=t;
        if(l!=-1)par[l]=i;
        if(r!=-1)par[r]=i;
        return i;
    }
    pair<int,int> merge(vector<pair<int,int>> a,Type t){
        if(a.size()==1)return a[0];
        int tot=0;
        vector<pair<int,int>> l,r;
        for(auto [i,s]:a)tot+=s;
        for(auto [i,s]:a){
            if(tot>s)l.emplace_back(i,s);
            else r.emplace_back(i,s);
            tot-=s*2;
        }
        auto [i,si]=merge(l,t);
        auto [j,sj]=merge(r,t);
        return {add(-1,i,j,t),si+sj};
    }
    pair<int,int> compress(int i){
        vector<pair<int,int>> a{add_vertex(i)};
        while(hv[i]!=-1)a.emplace_back(add_vertex(i=hv[i]));
        return merge(a,Type::Compress);
    }
    pair<int,int> rake(int i){
        vector<pair<int,int>> a;
        for(auto j:adj[i])if(j!=hv[i])a.emplace_back(add_edge(j));
        return a.empty()?make_pair(-1,0):merge(a,Type::Rake);
    }
    pair<int,int> add_edge(int i){
        auto [j,s]=compress(i);
        return {add(-1,j,-1,Type::AddEdge),s};
    }
    pair<int,int> add_vertex(int i){
        auto [j,s]=rake(i);
        return {add(i,j,-1,j==-1?Type::Vertex:Type::AddVertex),
            s+1};
    }
};

const int N=2e5+5;

int n,q;
ll a[N];
vector<vector<int>> adj;

using Path = pair<ll,ll>;
using Point = ll;

Path compress(Path p,Path ch){
    auto [a,b]=ch;
    auto [c,d]=p;
    return {a*c%MOD,(b*c+d)%MOD};
}

Point rake(Point l,Point r){
    return l*r%MOD;
}

Path add_vertex(Point x,int i){
    return {x,a[i]};
}

Point add_edge(Path x){
```

```
    return x.second;
}

Path vertex(int i){
    return {1LL,a[i]};
}

Path path[4*N];
Point point[4*N];

void example(){
    cin.tie(nullptr)->sync_with_stdio(false);
    cin >> n >> q;
    adj.resize(n);
    for(int i=1;i<n;i++){
        int p;
        cin >> p;
        adj[p-1].emplace_back(i);
    }
    for(int i=0;i<n;i++)cin >> a[i];
    StaticTopTree stt(adj,0);
    auto update=[&](int i){
        if(stt.type[i]==Type::Compress){
            path[i]=compress(path[stt.lch[i]],path[stt.rch[i]]);
        }
        else if(stt.type[i]==Type::Rake){
            point[i]=rake(point[stt.lch[i]],point[stt.rch[i]]);
        }
        else if(stt.type[i]==Type::AddVertex){
            path[i]=add_vertex(point[stt.lch[i]],i);
        }
        else if(stt.type[i]==Type::AddEdge){
            point[i]=add_edge(path[stt.lch[i]]);
        }
        else{
            path[i]=vertex(i);
        }
    };
    function<void(int)> dfs=[&](int u){
        if(u==-1)return;
        dfs(stt.lch[u]);
        dfs(stt.rch[u]);
        update(u);
    };
    dfs(stt.stt_root);
    while(q--){
        int v,x;
        cin >> v >> x;
        v--;
        a[v]=x;
        for(;v!=-1;v=stt.par[v])update(v);
        cout << path[stt.stt_root].second << "\n";
    }
}
```

## Number Theory (7)

### ExtendedEuclid.hpp

**Description:** Extended Euclid algorithm for solving diophantine equation ( $ax + by = \gcd(a, b)$ ).

**Time:**  $\mathcal{O}(\log \max\{a, b\})$

"../template/Header.hpp" 229e7c, 13 lines

```
pair<ll,ll> euclid(ll a,ll b){
    ll x=1,y=0,x1=0,y1=1;
    while(b!=0){
        ll q=a/b;
        x-=q*x1;
        y-=q*y1;
        a-=q*b;
        swap(x,x1);
```

```
        swap(y,y1);
        swap(a,b);
    }
    return {x,y};
}
```

euclid.h  
**Description:** Finds two integers  $x$  and  $y$ , such that  $ax + by = \gcd(a, b)$ . If you just need gcd, use the built in `__gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a \pmod b$ .  $x = x_0 + k * (b/g)$   $y = y_0 - k * (a/g)$   
33ba5f, 8 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

CRT.hpp  
**Description:** Chinese Remainder Theorem.  
`crt(a, m, b, n)` computes  $x$  such that  $x \equiv a \pmod m$ ,  $x \equiv b \pmod n$ . If  $|a| < m$  and  $|b| < n$ ,  $x$  will obey  $0 \leq x < \text{lcm}(m, n)$ . Assumes  $mn < 2^{62}$ .  
**Time:**  $\log(n)$   
"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
}
```

phiFunction.hpp  
**Description:** Euler's  $\phi$  function is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1$ ,  $p$  prime  $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2}...p_r^{k_r}$  then  $\phi(n) = (p_1 - 1)p_1^{k_1-1}...(p_r - 1)p_r^{k_r-1}$ .  $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$ .  
 $\sum_{d|n} \phi(d) = n$ ,  $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$   
**Euler's thm:**  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$ .  
**Fermat's little thm:**  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod p \forall a$ .  
efae90, 10 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
    for(int i=0; i<LIM; ++i) phi[i] = i & 1 ? i : i / 2;
    for (int i = 3; i < LIM; i += 2)
        if (phi[i] == i)
            for (int j = i; j < LIM; j += i)
                phi[j] -= phi[j] / i;
}
```

## 7.1 Prime Numbers

LinearSieve.hpp  
**Description:** Prime Number Generator in Linear Time  
**Time:**  $\mathcal{O}(N)$   
"../template/Header.hpp" 194fb1, 15 lines

```
vi linear_sieve(int n) {
    vi prime, composite(n + 1);
    for(int i=2; i<=n; ++i) {
        if(!composite[i]) {
            prime.emplace_back(i);
        }
        for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {
            composite[i * prime[j]] = true;
            if(i % prime[j] == 0) {
                break;
            }
        }
    }
}
```

```
    }
}

return prime;
}
```

FastEratosthenes.hpp  
**Description:** Prime sieve for generating all primes smaller than LIM.  
**Time:** LIM=1e9  $\approx 1.5s$   
"../template/Header.hpp" 295b58, 33 lines

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int) round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S + 1);
    pr.reserve((int)(LIM/log(LIM) * 1.1));
    vector<pii> cp;
    for(int i=3; i<=S; i+=2) {
        if(!sieve[i]) {
            cp.emplace_back(i, i * i / 2);
            for(int j=i*i; j<=S; j+=2*i) {
                sieve[j] = 1;
            }
        }
    }
    for(int L=1; L<=R; L+=S) {
        array<bool, S> block{};
        for(auto &[p, idx]: cp) {
            for(int i=idx; i<S+L; idx=(i+=p)) {
                block[i - L] = 1;
            }
        }
        for(int i=0; i<min(S, R-L); ++i) {
            if(!block[i]) {
                pr.emplace_back((L + i) * 2 + 1);
            }
        }
    }
    for(int i: pr) {
        isPrime[i] = 1;
    }
    return pr;
}
```

GolbatchConjecture.hpp  
**Description:** Find two prime numbers which sum equals  $s$   
**Time:**  $\mathcal{O}(N \log N)$   
"FastEratosthenes.hpp" 88fb23, 18 lines

```
pair<int, int> goldbatchConjecture(int s, vi pr = {}){
    if (s <= 2 || s % 2 != 0) {
        return make_pair(-1, -1);
    }
    if (pr.size() == 0) {
        pr = eratosthenes();
    }
    for (auto x : pr) {
        if (x > s / 2) {
            break;
        }
        int d = s - x;
        if (binary_search(pr.begin(), pr.end(), d)) {
            return make_pair(min(x, d), max(x, d));
        }
    }
    return make_pair(-1, -1);
}
```

## Graph (8)

### 8.1 Matching

HopcroftKarp.hpp  
**Description:** Fast bipartite matching algorithm.  
**Time:**  $\mathcal{O}(E\sqrt{V})$   
"../template/Header.hpp" 0bd56f, 52 lines

```
struct HopcroftKarp{
    int n,m;
    vi l,r,lv,ptr;
    vector<vi> adj;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m){init(_n,_m);}
    void init(int _n,int _m){
        n=_n,m=_m;
        adj.assign(n+m,vi{});
    }
    void addEdge(int u,int v){
        adj[u].emplace_back(v+n);
    }
    void bfs(){
        lv=vi(n,-1);
        queue<int> q;
        for(int i=0;i<n;i++){if(l[i]==-1){
            lv[i]=0;
            q.emplace(i);
        }}
        while(!q.empty()){
            int u=q.front();
            q.pop();
            for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){
                lv[r[v]]=lv[u]+1;
                q.emplace(r[v]);
            }
        }
    }
    bool dfs(int u){
        for(int &i=ptr[u];i<sz(adj[u]);i++){
            int v=adj[u][i];
            if(r[v]==-1|| (lv[r[v]]==lv[u]+1&&dfs(r[v]))){
                l[u]=v,r[v]=u;
                return true;
            }
        }
        return false;
    }
    int maxMatching(){
        int match=0,cnt=0;
        l=r=vi(n+m,-1);
        do{
            ptr=vi(n);
            bfs();
            cnt=0;
            for(int i=0;i<n;i++){if(l[i]==-1&&dfs(i))cnt++;
            match+=cnt;
        }while(cnt);
        return match;
    }
};
```

Kuhn.hpp  
**Description:** Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.  
**Time:**  $\mathcal{O}(VE)$   
"../template/Header.hpp" fc7d17, 15 lines  

```
vi adj[1010], match(1010, -1);
bitset<1010> visited;
```

```
bool kuhn(int u) {
    if(visited[u]) {
        return false;
    }
    visited[u] = true;
    for(auto x: adj[u]) {
        if(match[x] == -1 || kuhn(match[x])) {
            match[x] = u;
            return true;
        }
    }
    return false;
}
```

## 8.2 Network Flow

### Dinic.hpp

**Description:** Dinic's Algorithm for finding the maximum flow.

**Time:**  $O(VE \log U)$  where  $U$  is the maximum flow.

2b9ab1, 88 lines

```
template<class T, bool directed=true, bool scaling=true>
struct Dinic{
    static constexpr T INF=numeric_limits<T>::max()/2;
    struct Edge{
        int to;
        T flow, cap;
        Edge(int _to, T _cap):to(_to), flow(0), cap(_cap){}
        T remain() {return cap-flow;};
    };
    int n, s, t;
    T U;
    vector<Edge> e;
    vector<vector<int>> g;
    vector<int> ptr, lv;
    bool calculated;
    T max_flow;
    Dinic(){}
    Dinic(int n, int s, int t){init(n, s, t);}
    void init(int _n, int _s, int _t){
        n=_n, s=_s, t=_t;
        U=0;
        e.clear();
        g.assign(n, {});
        calculated=false;
    }
    void add_edge(int from, int to, T cap){
        assert(0<=from&&from<n&&0<=to&&to<n);
        g[from].emplace_back(e.size());
        e.emplace_back(to, cap);
        g[to].emplace_back(e.size());
        e.emplace_back(from, directed?0:cap);
        U=max(U, cap);
    }
    bool bfs(T scale){
        lv.assign(n, -1);
        vector<int> q{s};
        lv[s]=0;
        for(int i=0; i<(int)q.size(); i++){
            int u=q[i];
            for(int j:g[u]){
                int v=e[j].to;
                if(lv[v]==-1&&e[j].remain()>=scale){
                    q.emplace_back(v);
                    lv[v]=lv[u]+1;
                }
            }
        }
        return lv[t]!=-1;
    }
    T dfs(int u, int t, T f){
        if(u==t||f==0) return f;
        for(int &i=ptr[u]; i<(int)g[u].size(); i++){
            int j=g[u][i];
            int v=e[j].to;
            if(lv[v]==lv[u]+1){
                T res=dfs(v, t, min(f, e[j].remain()));
                if(res>0){
                    e[j].flow+=res;
                    e[j^1].flow-=res;
                    return res;
                }
            }
        }
        return 0;
    }
    T flow(){
        if(calculated) return max_flow;
        calculated=true;
        max_flow=0;
        for(T scale=scaling?1LL<<(63-__builtin_clzll(U)):1LL;
            scale>0; scale>>=1){
            while(bfs(scale)){
                ptr.assign(n, 0);
                while(true){
                    T f=dfs(s, t, INF);
                    if(f==0) break;
                    max_flow+=f;
                }
            }
        }
        return max_flow;
    }
    pair<T, vector<int>> cut(){
        flow();
        vector<int> res(n);
        for(int i=0; i<n; i++) res[i]=(lv[i]==-1);
        return {max_flow, res};
    }
};
```

```
if(u==t||f==0) return f;
for(int &i=ptr[u]; i<(int)g[u].size(); i++){
    int j=g[u][i];
    int v=e[j].to;
    if(lv[v]==lv[u]+1){
        T res=dfs(v, t, min(f, e[j].remain()));
        if(res>0){
            e[j].flow+=res;
            e[j^1].flow-=res;
            return res;
        }
    }
}
return 0;
}
}
T flow(){
    if(calculated) return max_flow;
    calculated=true;
    max_flow=0;
    for(T scale=scaling?1LL<<(63-__builtin_clzll(U)):1LL;
        scale>0; scale>>=1){
        while(bfs(scale)){
            ptr.assign(n, 0);
            while(true){
                T f=dfs(s, t, INF);
                if(f==0) break;
                max_flow+=f;
            }
        }
    }
    return max_flow;
}
pair<T, vector<int>> cut(){
    flow();
    vector<int> res(n);
    for(int i=0; i<n; i++) res[i]=(lv[i]==-1);
    return {max_flow, res};
}
};
```

### MinCostFlow.hpp

**Description:** minimum-cost flow algorithm.

**Time:**  $O(FE \log V)$  where  $F$  is max flow.

.../template/Header.hpp 8eald2, 83 lines

```
template<class F, class C>
struct MinCostFlow{
    struct Edge{
        int to;
        F flow, cap;
        C cost;
        Edge(int _to, F _cap, C _cost):to(_to), flow(0), cap(_cap),
            cost(_cost){}
        F getcap(){
            return cap-flow;
        }
    };
    int n;
    vector<Edge> e;
    vector<vi> adj;
    vector<C> pot, dist;
    vi pre;
    bool neg;
    const F FINF=numeric_limits<F>::max()/2;
    const C CINF=numeric_limits<C>::max()/2;
    MinCostFlow(){}
    MinCostFlow(int _n){
        init(_n);
    }
    void init(int _n){
```

```
n=_n;
e.clear();
adj.assign(n, {});
neg=false;
}
void addEdge(int u, int v, F cap, C cost){
    adj[u].emplace_back(sz(e));
    e.emplace_back(v, cap, cost);
    adj[v].emplace_back(sz(e));
    e.emplace_back(u, 0, -cost);
    if(cost<0) neg=true;
}
bool dijkstra(int s, int t){
    using P = pair<C, int>;
    dist.assign(n, CINF);
    pre.assign(n, -1);
    priority_queue<P, vector<P>, greater<P>> pq;
    dist[s]=0;
    pq.emplace(0, s);
    while(!pq.empty()){
        auto [d, u]=pq.top();
        pq.pop();
        if(dist[u]<d) continue;
        for(int i:adj[u]){
            int v=e[i].to;
            C ndist=d+pot[u]-pot[v]+e[i].cost;
            if(e[i].getcap()>0&&dist[v]>ndist){
                pre[v]=i;
                dist[v]=ndist;
                pq.emplace(ndist, v);
            }
        }
    }
    return dist[t]<CINF;
}
pair<F, C> flow(int s, int t){
    F flow=0;
    C cost=0;
    pot.assign(n, 0);
    if(neg) for(int t=0; t<n; t++) for(int i=0; i<sz(e); i++) if(e[i].getcap()>0){
        int u=e[i^1].to, v=e[i].to;
        pot[v]=min(pot[v], pot[u]+e[i].cost);
    } // Bellman-Ford
    while(dijkstra(s, t)){
        for(int i=0; i<n; i++) pot[i]+=dist[i];
        F aug=FINF;
        for(int u=t; u!=s; u=e[pre[u]^1].to){
            aug=min(aug, e[pre[u]].getcap());
        } // find bottleneck
        for(int u=t; u!=s; u=e[pre[u]^1].to){
            e[pre[u]].flow+=aug;
            e[pre[u]^1].flow-=aug;
        } // push flow
        flow+=aug;
        cost+=aug*pot[t];
    }
    return {flow, cost};
}
};
```

### BinaryOptimization.hpp

**Description:** Binary Optimization. minimize  $\kappa + \sum_i \theta_i(x_i) + \sum_{i<j} \phi_{ij}(x_i, x_j) + \sum_{i<j<k} \psi_{ijk}(x_i, x_j, x_k)$  where  $x_i \in \{0, 1\}$  and  $\phi_{ij}, \psi_{ijk}$  are submodular functions. a set function  $f$  is submodular if  $f(S) + f(T) \geq f(S \cap T) + f(S \cup T)$  for all  $S, T$ .  $\phi_{ij}(0, 1) + \phi_{ij}(1, 0) \geq \phi_{ij}(1, 1) + \phi_{ij}(0, 0)$ .

"Dinic.hpp"

cd8c59, 92 lines

```
template<class T, bool minimize=true>
```

```

struct BinaryOptimization{
    static constexpr T INF=numeric_limits<T>::max()/2;
    int n,s,t,node_id;
    T base;
    map<pair<int,int>,T> edges;
    BinaryOptimization(int _n:n(_n),s(n),t(n+1),node_id(n+2),
        base(0){}
    void add_edge(int u,int v,T w){
        assert(w>=0);
        if(u==v||w==0)return;
        auto &e=edges[{u,v}];
        e=min(e+w,INF);
    }
    void add0(T w){
        base+=w;
    }
    void _addl(int i,T a,T b){
        if(a<=b){
            add0(a);
            add_edge(s,i,b-a);
        }else{
            add0(b);
            add_edge(i,t,a-b);
        }
    }
    void addl(int i,T x0,T x1){
        assert(0<=i&&i<n);
        if(!minimize)x0=-x0,x1=-x1;
        _addl(i,x0,x1);
    }
    void _add2(int i,int j,T a,T b,T c,T d){
        assert(b+c>=a+d);
        add0(a);
        _addl(i,0,c-a);
        _addl(j,0,d-c);
        add_edge(i,j,b+c-a-d);
    }
    void add2(int i,int j,T x00,T x01,T x10,T x11){
        assert(i!=j&&0<=i&&i<n&&0<=j&&j<n);
        if(!minimize)x00=-x00,x01=-x01,x10=-x10,x11=-x11;
        _add2(i,j,x00,x01,x10,x11);
    }
    void _add3(int i,int j,int k,T a,T b,T c,T d,T e,T f,T g,T
        h){
        T p=a+d+f+g-b-c-e-h;
        if(p>=0){
            add0(a);
            _addl(i,0,f-b);
            _addl(j,0,g-e);
            _addl(k,0,d-c);
            _add2(i,j,0,c+e-a-g,0,0);
            _add2(i,k,0,0,b+e-a-f,0);
            _add2(j,k,0,0,b+c-a-d,0,0);
            int u=node_id++;
            add0(-p);
            add_edge(i,u,p);
            add_edge(j,u,p);
            add_edge(k,u,p);
            add_edge(u,t,p);
        }else{
            add0(h);
            _addl(i,c-g,0);
            _addl(j,b-d,0);
            _addl(k,e-f,0);
            _add2(i,j,0,0,d+f-b-h,0);
            _add2(i,k,0,d+g-c-h,0,0);
            _add2(j,k,0,0,f+g-e-h,0);
            int u=node_id++;
            add0(p);

```

```

            add_edge(s,u,-p);
            add_edge(u,i,-p);
            add_edge(u,j,-p);
            add_edge(u,k,-p);
        }
    }
    void add3(int i,int j,int k,T x000,T x001,T x010,T x011,T
        x100,T x101,T x110,T x111){
        assert(i!=j&&j!=k&&k!=i&&0<=i&&i<n&&0<=j&&j<n&&0<=k&&k<
            n);
        if(!minimize){
            x000=-x000,x001=-x001,x010=-x010,x011=-x011;
            x100=-x100,x101=-x101,x110=-x110,x111=-x111;
        }
        _add3(i,j,k,x000,x001,x010,x011,x100,x101,x110,x111);
    }
    T solve(){
        Dinic<T> dinic(node_id,s,t);
        for(auto &[p,w]:edges){
            auto [u,v]=p;
            dinic.add_edge(u,v,w);
        }
        T ans=dinic.flow()+base;
        return minimize?ans:-ans;
    }
};

```

## KaryOptimization.hpp

**Description:** k-ary Optimization. minimize  $\kappa + \sum_i \theta_i(x_i) + \sum_{i < j} \phi_{ij}(x_i, x_j)$  where  $x_i \in \{0, 1, \dots, k-1\}$  and  $\phi_{i,j}$  is monge. A function  $f$  is monge if  $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$  for all  $a < b$  and  $c < d$ .  $\phi_{ij}(x-1, y) + \phi_{ij}(x, y+1) \leq \phi_{ij}(x-1, y+1) + \phi_{ij}(x, y)$ .  $\phi_{ij}(x, y) + \phi_{ij}(x-1, y+1) - \phi_{ij}(x-1, y) - \phi_{ij}(x, y+1) \geq 0$ .

"Dinic.hpp" 422f8a, 88 lines

```

template<class T,bool minimize=true>
struct K_aryOptimization{
    static constexpr T INF=numeric_limits<T>::max()/2;
    int n,s,t,node_id;
    T base;
    vector<int> ks;
    vector<vector<int>> id;
    map<pair<int,int>,T> edges;
    K_aryOptimization(int n,int k){init(vector<int>(n,k));}
    K_aryOptimization(const vector<int> &_ks){init(_ks);}
    void init(const vector<int> &_ks){
        ks=_ks;
        n=ks.size();
        s=0,t=1,node_id=2;
        base=0;
        id.clear();
        edges.clear();
        for(auto &k:ks){
            assert(k>=1);
            vector<int> a(k+1);
            a[0]=s,a[k]=t;
            for(int i=1;i<k;i++)a[i]=node_id++;
            id.emplace_back(a);
            for(int i=2;i<k;i++)add_edge(a[i],a[i-1],INF);
        }
    }
    void add_edge(int u,int v,T w){
        assert(w>=0);
        if(u==v||w==0)return;
        auto &e=edges[{u,v}];
        e=min(e+w,INF);
    }
    void add0(T w){
        base+=w;
    }
};

```

```

    }
    void _addl(int i,vector<T> cost){
        add0(cost[0]);
        for(int j=1;j<ks[i];j++){
            T x=cost[j]-cost[j-1];
            if(x>0)add_edge(id[i][j],t,x);
            if(x<0)add0(x),add_edge(s,id[i][j],-x);
        }
    }
    void addl(int i,vector<T> cost){
        assert(0<=i&&i<n&&(int)cost.size()==ks[i]);
        if(!minimize)for(auto &x:cost)x=-x;
        _addl(i,cost);
    }
    void _add2(int i,int j,vector<vector<T>> cost){
        int h=ks[i],w=ks[j];
        _addl(j,cost[0]);
        for(int x=h-1;x>=0;x--)for(int y=0;y<w;y++)cost[x][y]=-
            cost[0][y];
        vector<T> a(h);
        for(int x=0;x<h;x++)a[x]=cost[x][w-1];
        _addl(i,a);
        for(int x=0;x<h;x++)for(int y=0;y<w;y++)cost[x][y]=-a[x
            ];
        for(int x=1;x<h;x++){
            for(int y=0;y<w-1;y++){
                T w=cost[x][y]+cost[x-1][y+1]-cost[x-1][y]-cost
                    [x][y+1];
                assert(w>=0); // monge
                add_edge(id[i][x],id[j][y+1],w);
            }
        }
    }
    void add2(int i,int j,vector<vector<T>> cost){
        assert(0<=i&&i<n&&0<=j&&j<n&&i!=j);
        assert((int)cost.size()==ks[i]);
        for(auto &v:cost)assert((int)v.size()==ks[j]);
        if(!minimize)for(auto &v:cost)for(auto &x:v)x=-x;
        _add2(i,j,cost);
    }
    pair<T,vector<int>> solve(){
        Dinic<T> dinic(node_id,s,t);
        for(auto &[p,w]:edges){
            auto [u,v]=p;
            dinic.add_edge(u,v,w);
        }
        auto [val,cut]=dinic.cut();
        val+=base;
        if(!minimize)val=-val;
        vector<int> ans(n);
        for(int i=0;i<n;i++){
            ans[i]=ks[i]-1;
            for(int j=1;j<ks[i];j++)ans[i]-=cut[id[i][j]];
        }
        return {val,ans};
    }
};

```

## 8.3 Connectivity

### SCC.hpp

**Description:** Strongly Connected Component.

"../template/Header.hpp" 82a9d1, 34 lines

```

template<class G>
pair<int,vector<int>> strongly_connected_component(G &g){
    static_assert(G::is_directed);
    int n=g.n;
    vector<int> disc(n,-1),low(n),scc(n,-1);
    stack<int> st;

```

```
vector<bool> in_st(n);
int t=0,scc_cnt=0;
function<void(int,int)> dfs=[&](int u,int p){
    disc[u]=low[u]=t++;
    st.emplace(u);
    in_st[u]=true;
    for(int v:g[u]){
        if(disc[v]==-1){
            dfs(v,u);
            low[u]=min(low[u],low[v]);
        }else if(in_st[v]){
            low[u]=min(low[u],disc[v]);
        }
    }
    if(disc[u]==low[u]){
        while(true){
            int v=st.top();
            st.pop();
            in_st[v]=false;
            scc[v]=scc_cnt;
            if(v==u)break;
        }
        scc_cnt++;
    }
};
for(int i=0;i<n;i++)if(disc[i]==-1)dfs(i,-1);
return {scc_cnt,scc};
}
```

LowLink.hpp  
Description: Low Link. f4ad2f, 33 lines

```
template<class G>
struct LowLink{
    G &g;
    int n;
    vector<int> disc,low,par,ord;
    vector<pair<int,int>> bridge;
    vector<int> articulation;
    int t=0;
    LowLink(G &g):g(_g),n(g.n),disc(n,-1),low(n),par(n,-1){
        for(int i=0;i<n;i++)if(disc[i]==-1)dfs(i);
    }
    void dfs(int u){
        disc[u]=low[u]=t++;
        ord.emplace_back(u);
        int child=0;
        bool found_par=false;
        for(int v:g[u]){
            if(disc[v]==-1){
                par[v]=u;
                dfs(v);
                low[u]=min(low[u],low[v]);
                if(low[v]>disc[u])bridge.emplace_back(u,v);
                if(par[u]!=-1&&low[v]>=disc[u])articulation.
                    emplace_back(u);
                child++;
            }else if(v!=par[u]||found_par){
                low[u]=min(low[u],disc[v]);
            }else{
                found_par=true;
            }
        }
        if(par[u]==-1&&child>1)articulation.emplace_back(u);
    }
};
```

Tree (9)

HLD.hpp  
Description: HLD cf6882, 45 lines

```
vector<vi> adj;
vector<int> sz, lvl, hv, hd, p, disc;
int t;

void dfs(int u, int parent) {
    sz[u] = 1;
    lvl[u] = lvl[parent] + 1;
    p[u] = parent;
    int c_hv=0, c_max=0;
    for(auto v: adj[u]) {
        if(v == parent) continue;
        dfs(v, u);
        sz[u] += sz[v];
        if(c_max < sz[v]) {
            c_hv = v;
            c_max = sz[v];
        }
    }
    hv[u] = c_hv;

    void hld(int u, int parent) {
        if(hd[u] == 0) {
            hd[u] = u;
        }
        disc[u] = ++t;
        if(hv[u] != 0) {
            hd[hv[u]] = hd[u];
            hld(hv[u], u);
        }
        for(auto v: adj[u]) {
            if(v == parent || v == hv[u]) {
                continue;
            }
            hld(v, u);
        }
    }

    int lca(int u, int v) {
        while(hd[u] != hd[v]) {
            if(lvl[hd[u]] > lvl[hd[v]]) swap(u, v);
            v=p[hd[v]];
        }
        return lvl[u] < lvl[v] ? u: v;
    }
}
```

CentroidDecom.hpp  
Description: Centroid e46d44, 32 lines

```
vector<vi> adj;
vi sz;
vector<bool> used;

int find_size(int u, int p) {
    sz[u] = 1;
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
        sz[u] += find_size(v, u);
    }
    return sz[u];
}

int find_cen(int u, int p, int t) {
    for(auto v: adj[u]) {
        if(v == p || used[v]) continue;
```

```
        if(sz[v] * 2 > t) find_cen(v, u, t);
    }
    return u;
}

void decom(int u) {
    u = find_cen(u, 0, find_size(u, 0));
    used[u] = true;
    for(auto v: adj[u]) {
        // dfs do something
    }
    for(auto v: adj[u]) {
        if(used[v]) continue;
        decom(v);
    }
}
```

Polynomials (10)

FormalPowerSeries.hpp  
Description: basic operations of formal power series 416433, 136 lines

```
template<class mint>
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;

    FPS &operator+=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]+=rhs[i];
        return *this;
    }
    FPS &operator+=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]+=rhs;
        return *this;
    }
    FPS &operator-=(const FPS &rhs){
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]-=rhs[i];
        return *this;
    }
    FPS &operator-=(const mint &rhs){
        if(this->empty())this->resize(1);
        (*this)[0]-=rhs;
        return *this;
    }
    FPS &operator*=(const FPS &rhs){
        auto res=NTT<mint>()(*this, rhs);
        return *this=FPS(res.begin(),res.end());
    }
    FPS &operator*=(const mint &rhs){
        for(auto &a:*this)a*=rhs;
        return *this;
    }
    friend FPS operator+(FPS lhs,const FPS &rhs){return lhs+=
        rhs;}
    friend FPS operator+(FPS lhs,const mint &rhs){return lhs+=
        rhs;}
    friend FPS operator+(const mint &lhs,FPS &rhs){return rhs+=
        lhs;}
    friend FPS operator-(FPS lhs,const FPS &rhs){return lhs-=
        rhs;}
    friend FPS operator-(FPS lhs,const mint &rhs){return lhs-=
        rhs;}
    friend FPS operator-(const mint &lhs,FPS rhs){return -(rhs-
        lhs);}
```

```

friend FPS operator*(FPS lhs,const FPS &rhs){return lhs*=
    rhs;}
friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=
    rhs;}
friend FPS operator*(const mint &lhs,FPS rhs){return rhs*=
    lhs;}

FPS operator-() {return (*this)*-1;}

FPS rev() {
    FPS res(*this);
    reverse(res.begin(),res.end());
    return res;
}

FPS pre(int sz){
    FPS res(this->begin(),this->begin()+min((int)this->size
        (),sz));
    if(res.size()<sz)res.resize(sz);
    return res;
}

FPS shrink(){
    FPS res(*this);
    while(!res.empty() && res.back()==mint{}) res.pop_back();
    return res;
}

FPS operator>>(int sz){
    if(this->size()<=sz) return {};
    FPS res(*this);
    res.erase(res.begin(),res.begin()+sz);
    return res;
}

FPS operator<<(int sz){
    FPS res(*this);
    res.insert(res.begin(),sz,mint{});
    return res;
}

FPS diff(){
    const int n=this->size();
    FPS res(max(0,n-1));
    for(int i=1;i<n;i++)res[i-1]=(*this)[i]*mint(i);
    return res;
}

FPS integral(){
    const int n=this->size();
    FPS res(n+1);
    res[0]=0;
    if(n>0)res[1]=1;
    ll mod=mint::get_mod();
    for(int i=2;i<=n;i++)res[i]=(-res[mod%i])*(mod/i);
    for(int i=0;i<n;i++)res[i+1]=(*this)[i];
    return res;
}

mint eval(const mint &x){
    mint res=0,w=1;
    for(auto &a:*this)res+=a*w,w*=x;
    return res;
}

FPS inv(int deg=-1){
    assert(!this->empty() && (*this)[0]!=mint(0));
    if(deg==-1)deg=this->size();
    FPS res(mint(1)/(*this)[0]);
    for(int i=2;i>>1<deg;i<=1){
        res=(res*(mint(2)-res*pre(i))).pre(i);
    }
    return res.pre(deg);
}

FPS log(int deg=-1){
    assert(!this->empty() && (*this)[0]==mint(1));

```

```

    if(deg==-1)deg=this->size();
    return (pre(deg).diff()*inv(deg)).pre(deg-1).integral()
        ;
}

FPS exp(int deg=-1){
    assert(this->empty() || (*this)[0]==mint(0));
    if(deg==-1)deg=this->size();
    FPS res(mint(1));
    for(int i=2;i>>1<deg;i<=1){
        res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
    }
    return res.pre(deg);
}

FPS pow(ll k,int deg=-1){
    const int n=this->size();
    if(deg==-1)deg=n;
    if(k==0){
        FPS res(deg);
        if(deg)res[0]=mint(1);
        return res;
    }
    for(int i=0;i<n;i++){
        if(__int128_t(i)*k>=deg) return FPS(deg,mint(0));
        if((*this)[i]==mint(0)) continue;
        mint rev=mint(1)/(*this)[i];
        FPS res=(((*this)*rev)>>i).log(deg)*k).exp(deg);
        res=((res*binpow((*this)[i],k)<<(i*k)).pre(deg);
        return res;
    }
    return FPS(deg,mint(0));
}

};
using FPS=FormalPowerSeries<mint>;

```

## FFT.hpp

**Description:** Fast Fourier transform

**Time:**  $\mathcal{O}(N \log N)$

```

"../template/Header.hpp" 5d476b, 73 lines

template<class T=ll, int mod=0>
struct FFT{
    using vt = vector<T>;
    using cd = complex<db>;
    using vc = vector<cd>;

    static const bool INT=true;

    static void fft(vc &a){
        int n=a.size(),L=31-__builtin_clz(n);
        vc rt(n);
        rt[1]=1;
        for(int k=2;k<n;k*=2){
            cd z=polar(db(1),PI/k);
            for(int i=k;i<2*k;i++)rt[i]=i&1?rt[i/2]*z:rt[i/2];
        }
        vi rev(n);
        for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
        for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
        for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k
            ;j++){
            cd z=rt[j+k]*a[i+j+k];
            a[i+j+k]=a[i+j]-z;
            a[i+j]+=z;
        }
    }

    template<class U>
    static db norm(const U &x){
        return INT?round(x):x;
    }

    static vt conv(const vt &a,const vt &b){

```

```

    if(a.empty() || b.empty()) return {};
    vt res(a.size()+b.size()-1);
    int L=32-__builtin_clz(res.size()),n=1<<L;
    vc in(n),out(n);
    copy(a.begin(),a.end(),in.begin());
    for(int i=0;i<b.size();i++)in[i].imag(b[i]);
    fft(in);
    for(auto &x:in)x*=x;
    for(int i=0;i<n;i++)out[i]=in[-i&(n-1)]-conj(in[i]);
    fft(out);
    for(int i=0;i<res.size();i++)res[i]=norm(imag(out[i])/(4*n)
        );
    return res;
}

static vl convMod(const vl &a,const vl &b){
    assert(mod>0);
    if(a.empty() || b.empty()) return {};
    vl res(a.size()+b.size()-1);
    int L=32-__builtin_clz(res.size()),n=1<<L;
    ll cut=int(sqrt(mod));
    vc in1(n),in2(n),out1(n),out2(n);
    for(int i=0;i<a.size();i++)in1[i]=cd(ll(a[i])/cut,ll(a[i])%
        cut); // a1 + i * a2
    for(int i=0;i<b.size();i++)in2[i]=cd(ll(b[i])/cut,ll(b[i])%
        cut); // b1 + i * b2
    fft(in1),fft(in2);
    for(int i=0;i<n;i++){
        int j=-i&(n-1);
        out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.1*n); // f1 * (g1
            + i * g2) = f1 * g1 + i f1 * g2
        out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.1,2.1*n); // f2
            * (g1 + i * g2) = f2 * g1 + i f2 * g2
    }
    fft(out1),fft(out2);
    for(int i=0;i<res.size();i++){
        ll x=round(real(out1[i])),y=round(imag(out1[i]))+round(
            real(out2[i])),z=round(imag(out2[i]));
        res[i]=((x%mod*cut+y)%mod*cut+z)%mod; // a1 * b1 * cut^2
            + (a1 * b2 + a2 * b1) * cut + a2 * b2
    }
    return res;
}

vt operator()(const vt &a,const vt &b){
    return mod>0?conv(a,b):convMod(a,b);
}

};
template<>
struct FFT<db>{
    static const bool INT=false;
};

```

## NTT.hpp

**Description:** Number theoretic transform

**Time:**  $\mathcal{O}(N \log N)$

```

"../template/Header.hpp", "../modular-arithmetic/BinPow.hpp",
"../modular-arithmetic/MontgomeryModInt.hpp" 2b2392, 39 lines

template<class mint=mint>
struct NTT{
    using vm = vector<mint>;

    static constexpr mint root=mint::get_root();
    static_assert(root!=0);

    static void ntt(vm &a){
        int n=a.size(),L=31-__builtin_clz(n);
        vm rt(n);
        rt[1]=1;
        for(int k=2,s=2;k<n;k*=2,s++){
            mint z[]={1,binpow(root,MOD>>s)};

```

```
        for(int i=k;i<2*k;i++)rt[i]=rt[i/2]*z[i&1];
    }
    vi rev(n);
    for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;
    for(int i=1;i<n;i++)if(i<rev[i])swap(a[i],a[rev[i]]);
    for(int k=1;k<n;k*=2)for(int i=0;i<n;i+=2*k)for(int j=0;j<k;j++){
        mint z=rt[j+k]*a[i+j+k];
        a[i+j+k]=a[i+j]-z;
        a[i+j]+=z;
    }
}
static vm conv(const vm &a,const vm &b){
    if(a.empty()||b.empty())return {};
    int s=a.size()+b.size()-1,n=1<<(32-__builtin_clz(s));
    mint inv=mint(n).inv();
    vm in1(a),in2(b),out(n);
    in1.resize(n),in2.resize(n);
    ntt(in1),ntt(in2);
    for(int i=0;i<n;i++)out[-i&(n-1)]=in1[i]*in2[i]*inv;
    ntt(out);
    return vm(out.begin(),out.begin()+s);
}
vm operator()(const vm &a,const vm &b){
    return conv(a,b);
}
};
```

## Strings (11)

### Manacher.hpp

**Description:** Manacher's Algorithm.  $pal[i]$  := the length of the longest palindrome centered at  $i/2$ .

```

"../template/Header.hpp"53856e, 15 lines
template<class STR>
vector<int> manacher(const STR &s){
    int n=(int)s.size();
    if(n==0)return {};
    vector<int> pal(2*n-1);
    for(int p=0,l=-1,r=-1;p<2*n-1;p++){
        int i=(p+1)>>1,j=p>>1;
        int k=(i>=r?0:min(r-i,pal[2*(1+r)-p]));
        while(j+k+1<n&&i-k-1>=0&&s[j+k+1]==s[i-k-1])k++;
        pal[p]=k;
        if(j+k>r)l=i-k,r=j+k;
    }
    for(int i=0;i<2*n-1;i++)pal[i]=pal[i]<<1|(i&1^1);
    return pal;
}
```

### SuffixArray.hpp

**Description:** Suffix Automaton.

```

"../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp"b9cfb1, 39 lines
template<class STR>
struct SuffixArray{
    int n;
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArray(){}
    SuffixArray(const STR &s){init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        sa[0]=n;
        iota(sa.begin()+1,sa.end(),0);
        sort(sa.begin()+1,sa.end(),[&(int i,int j){return s[i]<s[j];}]);
    }
```

```
        for(int i=1;i<=n;i++){
            int x=sa[i-1],y=sa[i];
            isa[y]=i>1&&s[x]==s[y]?isa[x]:i;
        }
        for(int len=1;len<=n;len<=<=1){
            vector<int> ps(sa),pi(isa),pos(n+1);
            iota(pos.begin(),pos.end(),0);
            for(auto i:ps)if((i--len)>=0)sa[pos[isa[i]]++]=i;
            for(int i=1;i<=n;i++){
                int x=sa[i-1],y=sa[i];
                isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x]:i;
            }
        }
        for(int i=0,k=0;i<n;i++){
            for(int j=sa[isa[i]-1];j+k<n&&s[j+k]==s[i+k];k++);
            lcp[isa[i]]=k;
            if(k)k--;
        }
        st.init(lcp);
    }
    int get_lcp(int i,int j){
        if(i==j)return n-i;
        auto [l,r]=minmax(isa[i],isa[j]);
        return st.query(l+1,r);
    }
};
```

### ZAlgo.hpp

**Description:** Z Algorithm.  $z[i]$  := the length of the longest common prefix between  $s$  and  $s[i]$ .

```

"../template/Header.hpp"b93726, 12 lines
template<class STR>
vector<int> z_algorithm(const STR &s){
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for(int i=1,l=0,r=1;i<n;i++){
        if(i<r)z[i]=min(r-i,z[i-1]);
        while(i+z[i]<n&&s[z[i]]==s[i+z[i]])z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    }
    return z;
}
```

### PrefixFunction.hpp

**Description:** Prefix function.  $pi[i]$  := the length of the longest proper prefix of  $s[0:i]$  which is also a suffix of  $s[0:i]$ .

```

3d65fe, 11 lines
template<class STR>
vector<int> prefix_function(const STR &s){
    int n=(int)s.size();
    vector<int> pi(n);
    for(int i=1,j=0;i<n;i++){
        while(j>0&&s[i]!=s[j])j=pi[j-1];
        if(s[i]==s[j])j++;
        pi[i]=j;
    }
    return pi;
}
```

### SuffixAutomaton.hpp

**Description:** Suffix Automaton.

Find whether a string  $t$  is a substring of a string  $s$  by traversing the automaton.

Find whether a string  $t$  is a suffix of a string  $s$  by checking whether the last node is a terminal node.

Find the number of distinct substrings of a string  $s$  by calculating the number of distinc path using DP.

Count the number of occurrences of string  $t$  in string  $s$ . Let  $p$  be the node we end up at after traversing  $t$  in the automaton. The answer is the number of paths from  $p$  to terminal nodes.

Find first occurrence of string  $t$  in string  $s$  by calculating the longest path in the automaton after reaching node  $p$ .

```

a50940, 49 lines
template<class STR>
struct SuffixAutomaton{
    using T = typename STR::value_type;
    struct Node{
        map<T,int> nxt;
        int link,len;
        Node(int link,int len):link(link),len(len){}
    };
    vector<Node> nodes;
    int last;
    SuffixAutomaton():nodes{Node(-1,0)},last(0){}
    SuffixAutomaton(const STR &s):SuffixAutomaton(){}
        for(auto c:s)extend(c);
    }
    int new_node(int link,int len){
        nodes.emplace_back(Node(link,len));
        return (int)nodes.size()-1;
    }
    void extend(T c){
        int cur=new_node(0,nodes[last].len+1);
        int p=last;
        while(p!=-1&&!nodes[p].nxt.count(c)){
            nodes[p].nxt[c]=cur;
            p=nodes[p].link;
        }
        if(p!=-1){
            int q=nodes[p].nxt[c];
            if(nodes[p].len+1==nodes[q].len){
                nodes[cur].link=q;
            }else{
                int r=new_node(nodes[q].link,nodes[p].len+1);
                nodes[r].nxt=nodes[q].nxt;
                while(p!=-1&&nodes[p].nxt[c]==q){
                    nodes[p].nxt[c]=r;
                    p=nodes[p].link;
                }
                nodes[q].link=nodes[cur].link=r;
            }
        }
        last=cur;
    }
    11 distinct_substrings(){
        11 res=0;
        for(int i=1;i<(int)nodes.size();i++){
            res+=nodes[i].len-nodes[nodes[i].link].len;
        }
        return res;
    }
};
```



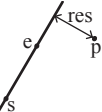
# Geometry (12)

## 12.1 Geometric primitives

**Point.h**  
**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()==1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << ", " << p.y << ")"; }
};
```

**lineDistance.h**  
**Description:** Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



```
"Point.h"
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a)/(b-a).dist();
}
```

**SegmentDistance.h**  
**Description:** Returns the shortest distance between point p and the line segment from point s to e.

**Usage:** Point<double> a, b(2,2), p(1,1);  
bool onSegment = segDist(a,b,p) < 1e-10;

```
"Point.h"
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

**SegmentIntersection.h**  
**Description:** If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

**Usage:** vector<P> inter = segInter(s1,e1,s2,e2);  
if (sz(inter)==1)  
cout << "segments intersect at " << inter[0] << endl;

```
"Point.h", "OnSegment.h"
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
}
```

**lineIntersection.h**  
**Description:** If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.

**Usage:** auto res = lineInter(s1,e1,s2,e2);  
if (res.first == 1)  
cout << "intersection point at " << res.second << endl;

```
"Point.h"
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};
}
```

**sideOf.h**  
**Description:** Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** bool left = sideOf(p1,p2,q)==1;

```
"Point.h"
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```

**OnSegment.h**  
**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p) <=epsilon) instead when using Point<double>.

```
"Point.h"
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

**linearTransformation.h**  
**Description:** Apply the linear transformation (translation, rotation and scaling) which takes line p0-pl to line q0-q1 to point r.

```
"Point.h"
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

**Angle.h**  
**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

**Usage:** vector<Angle> v = {w[0], w[0].t360() ...}; // sorted  
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }  
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

```
"Point.h"
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);
        return y < 0 || (y == 0 && x < 0);
    }
    Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
    Angle t180() const { return {-x, -y, t + half()}; }
    Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
        make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
```

```
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180() ?
        make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
    Angle r(a.x + b.x, a.y + b.y, a.t);
    if (a.t180() < r) r.t--;
    return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
    int tu = b.t - a.t; a.t = b.t;
    return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

## 12.2 Circles

### CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	84d6d3, 11 lines
<pre>typedef Point&lt;double&gt; P; bool circleInter(P a,P b,double r1,double r2,pair&lt;P, P&gt;* out) {     if (a == b) { assert(r1 != r2); return false; }     P vec = b - a;     double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;     if (sum*sum &lt; d2    dif*dif &gt; d2) return false;     P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);     *out = {mid + per, mid - per};     return true; }</pre>	

### CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	b0153d, 13 lines
<pre>template&lt;class P&gt; vector&lt;pair&lt;P, P&gt;&gt; tangents(P c1, double r1, P c2, double r2) {     P d = c2 - c1;     double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;     if (d2 == 0    h2 &lt; 0) return {};     vector&lt;pair&lt;P, P&gt;&gt; out;     for (double sign : {-1, 1}) {         P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;         out.push_back({c1 + v * r1, c2 + v * r2});     }     if (h2 == 0) out.pop_back();     return out; }</pre>	

### CirclePolygonIntersection.h

**Description:** Returns the area of the intersection of a circle with a ccw polygon.  
**Time:**  $\mathcal{O}(n)$

".././content/geometry/Point.h"	a1ee63, 19 lines
<pre>typedef Point&lt;double&gt; P; #define arg(p, q) atan2(p.cross(q), p.dot(q)) double circlePoly(P c, double r, vector&lt;P&gt; ps) {     auto tri = [&amp;](P p, P q) {         auto r2 = r * r / 2;         P d = q - p;         auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();         auto det = a * a - b;         if (det &lt;= 0) return arg(p, q) * r2;         auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));         if (t &lt; 0    1 &lt;= s) return arg(p, q) * r2;         P u = p + d * s, v = p + d * t;         return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;     };     auto sum = 0.0;     rep(i,0,sz(ps))         sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);     return sum; }</pre>	

### circumcircle.h

**Description:**

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h"	1caa3a, 9 lines
<pre>typedef Point&lt;double&gt; P; double ccRadius(const P&amp; A, const P&amp; B, const P&amp; C) {     return (B-A).dist()*(C-B).dist()*(A-C).dist()/         abs((B-A).cross(C-A))/2; } P ccCenter(const P&amp; A, const P&amp; B, const P&amp; C) {     P b = C-A, c = B-A;     return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2; }</pre>	

### MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.  
**Time:** expected  $\mathcal{O}(n)$

"circumcircle.h"	09dd0a, 17 lines
<pre>pair&lt;P, double&gt; mec(vector&lt;P&gt; ps) {     shuffle(all(ps), mt19937(time(0)));     P o = ps[0];     double r = 0, EPS = 1 + 1e-8;     rep(i,0,sz(ps)) if ((o - ps[i]).dist() &gt; r * EPS) {         o = ps[i], r = 0;         rep(j,0,i) if ((o - ps[j]).dist() &gt; r * EPS) {             o = (ps[i] + ps[j]) / 2;             r = (o - ps[i]).dist();             rep(k,0,j) if ((o - ps[k]).dist() &gt; r * EPS) {                 o = ccCenter(ps[i], ps[j], ps[k]);                 r = (o - ps[i]).dist();             }         }     }     return {o, r}; }</pre>	

## 12.3 Polygons

### InsidePolygon.h

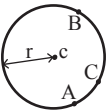
**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.  
**Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}};  
bool in = inPolygon(v, P{3, 3}, false);  
**Time:**  $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h"	2bf504, 11 lines
<pre>template&lt;class P&gt; bool inPolygon(vector&lt;P&gt; &amp;p, P a, bool strict = true) {     int cnt = 0, n = sz(p);     rep(i,0,n) {         P q = p[(i + 1) % n];         if (onSegment(p[i], q, a)) return !strict;         //or: if (segDist(p[i], q, a) &lt;= eps) return !strict;         cnt ^= ((a.y&lt;p[i].y) - (a.y&lt;q.y)) * a.cross(p[i], q) &gt; 0;     }     return cnt; }</pre>	

### PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int T!

"Point.h"	f12300, 6 lines
<pre>template&lt;class T&gt; T polygonArea2(vector&lt;Point&lt;T&gt;&gt;&amp; v) {</pre>	



<pre>T a = v.back().cross(v[0]); rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]); return a; }</pre>	
--	--

### PolygonCenter.h

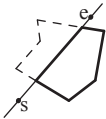
**Description:** Returns the center of mass for a polygon.  
**Time:**  $\mathcal{O}(n)$

"Point.h"	9706dc, 9 lines
<pre>typedef Point&lt;double&gt; P; P polygonCenter(const vector&lt;P&gt;&amp; v) {     P res(0, 0); double A = 0;     for (int i = 0, j = sz(v) - 1; i &lt; sz(v); j = i++) {         res = res + (v[i] + v[j]) * v[j].cross(v[i]);         A += v[j].cross(v[i]);     }     return res / A / 3; }</pre>	

### PolygonCut.h

**Description:** Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

<p><b>Usage:</b> vector&lt;P&gt; p = ...; p = polygonCut(p, P(0,0), P(1,0));</p>	
"Point.h", "LineIntersection.h"	f2b7d4, 13 lines
<pre>typedef Point&lt;double&gt; P; vector&lt;P&gt; polygonCut(const vector&lt;P&gt;&amp; poly, P s, P e) {     vector&lt;P&gt; res;     rep(i,0,sz(poly)) {         P cur = poly[i], prev = i ? poly[i-1] : poly.back();         bool side = s.cross(e, cur) &lt; 0;         if (side != (s.cross(e, prev) &lt; 0))             res.push_back(lineInter(s, e, cur, prev).second);         if (side)             res.push_back(cur);     }     return res; }</pre>	



### ConvexHull.h

**Description:** Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.  
**Time:**  $\mathcal{O}(n \log n)$

"Point.h"	310954, 13 lines
<pre>typedef Point&lt;ll&gt; P; vector&lt;P&gt; convexHull(vector&lt;P&gt; pts) {     if (sz(pts) &lt;= 1) return pts;     sort(all(pts));     vector&lt;P&gt; h(sz(pts)+1);     int s = 0, t = 0;     for (int it = 2; it--; s = --t, reverse(all(pts)))         for (P p : pts) {             while (t &gt;= s + 2 &amp;&amp; h[t-2].cross(h[t-1], p) &lt;= 0) t--;             h[t++] = p;         }     return {h.begin(), h.begin() + t - (t == 2 &amp;&amp; h[0] == h[1])}; }</pre>	

### HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).  
**Time:**  $\mathcal{O}(n)$

"Point.h"	c571b8, 12 lines
<pre>typedef Point&lt;ll&gt; P;</pre>	



```
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (; j = (j + 1) % n) {
            res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

**PointInsideHull.h**  
**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.  
**Time:**  $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"	71446b, 14 lines
--------------------------------------	------------------

typedef Point<ll> P;

```
bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

**LineHullIntersection.h**  
**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:  $\bullet(-1, -1)$  if no collision,  $\bullet(i, -1)$  if touching the corner  $i$ ,  $\bullet(i, i)$  if along side  $(i, i + 1)$ ,  $\bullet(i, j)$  if crossing sides  $(i, i + 1)$  and  $(j, j + 1)$ . In the last case, if a corner  $i$  is crossed, this is treated as happening on side  $(i, i + 1)$ . The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.  
**Time:**  $\mathcal{O}(\log n)$

```
"Point.h" 7cf45b, 39 lines

#define cmp(i, j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    }
    return lo;
}
```

```
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
```

```
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    }
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]};
        }
    return res;
}
```

## 12.4 Misc. Point Set Problems

```
ClosestPair.h
Description: Finds the closest pair of points.
Time:  $\mathcal{O}(n \log n)$ 

"Point.h" ac41a6, 17 lines

typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(lo - p).dist2(), {lo, p}});
        S.insert(p);
    }
    return ret.second;
}
```

**kdTree.h**  
**Description:** KD-tree (2d, can be extended to 3d)

```
"Point.h" bac5b0, 63 lines

typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if width >= height (not ideal...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
```

```
            // divide by taking half the array for each child (not
            // best performance with many duplicates in the middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()});
        }
    };

    struct KDTree {
        Node* root;
        KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

        pair<T, P> search(Node *node, const P& p) {
            if (!node->first) {
                // uncomment if we should not find the point itself:
                // if (p == node->pt) return {INF, P()};
                return make_pair((p - node->pt).dist2(), node->pt);
            }

            Node *f = node->first, *s = node->second;
            T bfirst = f->distance(p), bsec = s->distance(p);
            if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

            // search closest side first, other side if needed
            auto best = search(f, p);
            if (bsec < best.first)
                best = min(best, search(s, p));
            return best;
        }

        // find nearest point to a point, and its squared distance
        // (requires an arbitrary operator< for Point)
        pair<T, P> nearest(const P& p) {
            return search(root, p);
        }
    };
};
```

**FastDelaunay.h**  
**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.  
**Time:**  $\mathcal{O}(n \log n)$

```
"Point.h" eefdf5, 88 lines

typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    ll1 p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
```

```
    return r;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)});
    while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); \
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    }
    return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}
```

## 12.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

```
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt((double)dist2()); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
    double theta() const { return atan2(sqrt(x*x+y*y),z); }
    P unit() const { return *this/(T)dist(); } //makes dist()==1
    //returns unit vector normal to *this and p
    P normal(P p) const { return cross(p).unit(); }
    //returns point rotated 'angle' radians ccw around axis
    P rotate(double angle, P axis) const {
        double s = sin(angle), c = cos(angle); P u = axis.unit();
        return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    }
};
```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards.

**Time:**  $\mathcal{O}(n^2)$

```
"Point3D.h"
5b45fc, 49 lines

typedef Point3D<double> P3;

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
```

```
vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
vector<F> FS;
auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
        q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
};
rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
        F f = FS[j];
        if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
            E(a,b).rem(f.c);
            E(a,c).rem(f.b);
            E(b,c).rem(f.a);
            swap(FS[j-], FS.back());
            FS.pop_back();
        }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
        F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
        C(a, b, c); C(a, c, b); C(b, c, a);
    }
    for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
        A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
    return FS;
};
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius  $r$  between the points with azimuthal angles (longitude)  $f_1$  ( $\phi_1$ ) and  $f_2$  ( $\phi_2$ ) from  $x$  axis and zenith angles (latitude)  $t_1$  ( $\theta_1$ ) and  $t_2$  ( $\theta_2$ ) from  $z$  axis ( $0 =$  north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows.  $dx * radius$  is then the difference between the two points in the  $x$  direction and  $d * radius$  is the total distance between the points.

```
611f07, 8 lines

double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}
```

## Dynamic Programming (13)

### DVC.hpp

**Description:** Optimize  $\mathcal{O}(N^2K)$  to  $\mathcal{O}(NK \log N)$

```
"../template/Header.hpp"
aa5ddf, 19 lines

vector<vl> cst, dp;

ll cost(int l, int r) {
    return cst[l][r];
}
```

```
void divide(int l, int r, int opt_l, int opt_r, int c) {
    if(l > r) return ;
    int mid = (l + r) / 2;
    pair<ll, int> best = make_pair(INF, -1);
    for(int k=opt_l; k<=min(mid, opt_r); ++k) {
        best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid),
                                   k));
    }
    dp[c][mid] = best.first;
    divide(l, mid - 1, opt_l, best.second, c);
    divide(mid + 1, r, best.second, opt_r, c);
}

// for(int c=1; c<=K; ++c) divide(1, N, 1, N, c);
```

SlopeTrick.hpp

**Description:** Absolute Smth

```
"../template/Header.hpp"  f62f9a, 36 lines

ll extending_value;

struct slope_trick {
    multiset<ll> ms_l, ms_r;
    ll min_y = 0ll, lz_l = 0ll, lz_r = 0ll;
    bool extending = false;
    void add_line(ll v) {
        if(extending) {
            lz_l -= extending_value;
            lz_r -= extending_value;
        }
        extending = true;
        if(ms_l.empty() && ms_r.empty()) {
            ms_l.emplace(v);
            ms_r.emplace(v);
        }
        else if(v <= *ms_l.rbegin() + lz_l) {
            min_y += (*ms_l.rbegin() + lz_l) - v;
            ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
            ms_l.erase(--ms_l.end());
            ms_l.emplace(v - lz_l);
            ms_l.emplace(v - lz_l);
        }
        else if(v >= *ms_r.begin() + lz_r) {
            min_y += v - (*ms_r.begin() + lz_r);
            ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
            ms_r.erase(ms_r.begin());
            ms_r.emplace(v - lz_r);
            ms_r.emplace(v - lz_r);
        }
        else {
            ms_l.emplace(v - lz_l);
            ms_r.emplace(v - lz_r);
        }
    }
};
```

Convolutions (14)

AndConvolution.hpp

**Description:** Bitwise AND Convolution. Superset Zeta Transform:  $A'[S] = \sum_{T \supseteq S} A[T]$ . Superset Mobius Transform:  $A[T] = \sum_{S \supseteq T} (-1)^{|S-T|} A'[S]$ .  
**Time:**  $\mathcal{O}(N \log N)$ .

```
"../template/Header.hpp"  7916f8, 34 lines

template<class T>
void superset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
```

```
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]+=a[j];
            }
        }
    }
}

template<class T>
void superset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j^i]-=a[j];
            }
        }
    }
}

template<class T>
vector<T> and_convolution(vector<T> a,vector<T> b){
    superset_zeta(a);
    superset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    superset_mobius(a);
    return a;
}
```

GCDConvolution.hpp

**Description:** GCD Convolution. Multiple Zeta Transform:  $A'[n] = \sum_{n|m} A[m]$ . Multiple Mobius Transform:  $A[n] = \sum_{n|m} \mu(m/n)A'[m]$ .  
**Time:**  $\mathcal{O}(N \log \log N)$ .

```
"../template/Header.hpp"  7f6e2d, 34 lines

template<class T>
void multiple_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i]+=a[i*p];
        }
    }
}

template<class T>
void multiple_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i]-=a[i*p];
        }
    }
}

template<class T>
vector<T> gcd_convolution(vector<T> a,vector<T> b){
    multiple_zeta(a);
    multiple_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    multiple_mobius(a);
    return a;
}
```

LCMConvolution.hpp

**Description:** LCM Convolution. Divisor Zeta Transform:  $A'[n] = \sum_{d|n} A[d]$ . Divisor Mobius Transform:  $A[n] = \sum_{d|n} \mu(n/d)A'[d]$ .  
**Time:**  $\mathcal{O}(N \log \log N)$ .

```
"../template/Header.hpp"  41fe9d, 34 lines

template<class T>
void divisor_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++){
            is_prime[i*p]=false;
            a[i*p]+=a[i];
        }
    }
}

template<class T>
void divisor_mobius(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++){
        if(!is_prime[p])continue;
        for(int i=(n-1)/p;i>=1;i--){
            is_prime[i*p]=false;
            a[i*p]-=a[i];
        }
    }
}

template<class T>
vector<T> lcm_convolution(vector<T> a,vector<T> b){
    divisor_zeta(a);
    divisor_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    divisor_mobius(a);
    return a;
}
```

ORConvolution.hpp

**Description:** Bitwise OR Convolution. Subset Zeta Transform:  $A'[S] = \sum_{T \subseteq S} A[T]$ . Subset Mobius Transform:  $A[T] = \sum_{S \subseteq T} (-1)^{|T-S|} A'[S]$ .  
**Time:**  $\mathcal{O}(N \log N)$ .

```
"../template/Header.hpp"  c58b77, 34 lines

template<class T>
void subset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                a[j]+=a[j^i];
            }
        }
    }
}

template<class T>
void subset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=n;i>=1;){
        for(int j=0;j<n;j++){
            if(j&i){
```

```
        a[j]-=a[j^i];
    }
}

template<class T>
vector<T> or_convolution(vector<T> a,vector<T> b){
    subset_zeta(a);
    subset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i]*=b[i];
    subset_mobius(a);
    return a;
}
```

XORConvolution.hpp  
**Description:** Bitwise XOR Convolution. Fast Walsh-Hadamard Transform:  
 $A'[S] = \sum_T (-1)^{|S \& T|} A[T]$ .  
**Time:**  $\mathcal{O}(N \log N)$ .  
"../template/Header.hpp" 05848d, 29 lines

```
template<class T>
void fwht(vector<T> &a){
    int n=(int)a.size();
    assert(n==(n&-n));
    for(int i=1;i<n;i<=1){
        for(int j=0;j<n;j++){
            if(j&i){
                T &u=a[j^i],&v=a[j];
                tie(u,v)=make_pair(u+v,u-v);
            }
        }
    }

template<class T>
vector<T> xor_convolution(vector<T> a,vector<T> b){
    int n=(int)a.size();
    fwht(a);
    fwht(b);
    for(int i=0;i<n;i++)a[i]*=b[i];
    fwht(a);
    T div=T(1)/T(n);
    if(div==T(0)){
        for(auto &x:a)x/=n;
    }else{
        for(auto &x:a)x*=div;
    }
    return a;
}
```

Various (15)

GaussianElimination.hpp  
**Description:** Gaussian Elimination  
"../template/Header.hpp" e89ecb, 34 lines

```
struct Gauss {
    int n, sz;
    vector<ll> basis;
    Gauss(int n = 0) {
        init(n);
    }
    void init(int _n) {
        n = _n, sz = 0;
        basis.assign(n, 0);
    }
    void insert(ll x) {
        for (int i = n - 1; i >= 0; i--)
```

```
        if (x >> i & 1) {
            if (!basis[i]) {
                basis[i] = x;
                sz++;
                return;
            }
            x ^= basis[i];
        }
    }

    ll getMax(ll k = 0) {
        ll tot = 1ll << sz, res = 0;
        for (int i = n - 1; i >= 0; i--)
            if (basis[i]) {
                tot >>= 1;
                if ((k >= tot && res >> i & 1) || (k < tot && res >> i
                    & 1 ^ 1))
                    res ^= basis[i];
                if (k >= tot)
                    k -= tot;
            }
        return res;
    }
};
```

BinaryTrie.hpp  
**Description:** Binary Trie  
"../template/Header.hpp" 525bf4, 59 lines

```
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
    vector<node_t> t = {node_t()};
    vector<int> cnt = {0};
    int cnt_nodes = 0;
    void insert(int v) {
        int cur = 0;
        cnt[0]++;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(!t[cur][b]) {
                t[cur][b] = ++cnt_nodes;
                t.emplace_back(node_t());
                cnt.emplace_back(0);
            }
            cnt[t[cur][b]]++;
            cur = t[cur][b];
        }
    }
    void remove(int v) {
        int cur = 0;
        cnt[0]--;
        for(int i=S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            cnt[t[cur][b]]--;
            cur = t[cur][b];
        }
    }
    int get_min(int v) {
        int cur = 0, res = 0;
        for(int i=(int) S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(t[cur][b] && cnt[t[cur][b]]) {
                cur = t[cur][b];
            }
            else {
                res |= (1 << i);
                cur = t[cur][!b];
            }
        }
        return res;
    }
```

```
    }

    int get_max(int v) {
        int cur = 0, res = 0;
        for(int i=(int) S-1; i>=0; --i) {
            int b = (v & (1 << i)) ? 1 : 0;
            if(t[cur][!b] && cnt[t[cur][!b]]) {
                res |= (1 << i);
                cur = t[cur][!b];
            }
            else {
                cur = t[cur][b];
            }
        }
        return res;
    }
};
```

InfixPropostfix.hpp  
**Description:** Infix to Pro-Postfix  
"../template/Header.hpp" 517f57, 47 lines

```
stack<char> opr;
stack<int> val;

bool isOpr(char x){
    return x == '+' || x == '*';
}

int prio(char x) {
    if(x == '(') return -1;
    if(x == '+') return 1;
    if(x == '*') return 2;
    return 0;
}

int do_opr(int l, int r, char o) {
    if(o == '+') {
        return l + r;
    }
    return l * r;
}

void pop_stack() {
    int rhs = val.top(); val.pop();
    int lhs = val.top(); val.pop();
    int new_val = do_opr(lhs, rhs, opr.top());
    val.emplace(new_val);
    opr.pop();
}

int cal(string s) {
    for(auto x: s) {
        if(isdigit(x)) val.emplace(x - '0');
        else if(x == '(') opr.emplace('(');
        else if(x == ')') {
            while(!opr.empty() && opr.top() != '(')
                pop_stack();
            opr.pop();
        }
        else {
            while(!opr.empty() && prio(opr.top()) >= prio(x))
                pop_stack();
            opr.emplace(x);
        }
    }
    while(!opr.empty()) pop_stack();
    return val.top();
}
```

## 15.1 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

### 15.1.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; ((r^x) >> 2)/c` | `r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`  
if `(i & 1 << b) D[i] += D[i^(1 << b)];`  
computes all sums of subsets.

### 15.1.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).



# Competitive Programming Topics

## (A)

topics.txt

159 lines

Recursion  
Divide and conquer  
    Finding interesting points in  $N \log N$   
Algorithm analysis  
    Master theorem  
    Amortized time complexity  
Greedy algorithm  
    Scheduling  
    Max contiguous subvector sum  
    Invariants  
    Huffman encoding  
Graph theory  
    Dynamic graphs (extra book-keeping)  
    Breadth first search  
    Depth first search  
        \* Normal trees / DFS trees  
    Dijkstra's algorithm  
    MST: Prim's algorithm  
    Bellman-Ford  
    Konig's theorem and vertex cover  
    Min-cost max flow  
    Lovasz toggle  
    Matrix tree theorem  
    Maximal matching, general graphs  
    Hopcroft-Karp  
    Hall's marriage theorem  
    Graphical sequences  
    Floyd-Warshall  
    Euler cycles  
    Flow networks  
        \* Augmenting paths  
        \* Edmonds-Karp  
    Bipartite matching  
    Min. path cover  
    Topological sorting  
    Strongly connected components  
    2-SAT  
    Cut vertices, cut-edges and biconnected components  
    Edge coloring  
        \* Trees  
    Vertex coloring  
        \* Bipartite graphs ( $\Rightarrow$  trees)  
        \*  $3^n$  (special case of set cover)  
    Diameter and centroid  
    K'th shortest path  
    Shortest cycle  
Dynamic programming  
    Knapsack  
    Coin change  
    Longest common subsequence  
    Longest increasing subsequence  
    Number of paths in a dag  
    Shortest path in a dag  
    Dynprog over intervals  
    Dynprog over subsets  
    Dynprog over probabilities  
    Dynprog over trees  
     $3^n$  set cover  
    Divide and conquer  
    Knuth optimization  
    Convex hull optimizations  
    RMQ (sparse table a.k.a  $2^k$ -jumps)  
    Bitonic cycle

    Log partitioning (loop over most restricted)  
Combinatorics  
    Computation of binomial coefficients  
    Pigeon-hole principle  
    Inclusion/exclusion  
    Catalan number  
    Pick's theorem  
Number theory  
    Integer parts  
    Divisibility  
    Euclidean algorithm  
    Modular arithmetic  
        \* Modular multiplication  
        \* Modular inverses  
        \* Modular exponentiation by squaring  
    Chinese remainder theorem  
    Fermat's little theorem  
    Euler's theorem  
    Phi function  
    Frobenius number  
    Quadratic reciprocity  
    Pollard-Rho  
    Miller-Rabin  
    Hensel lifting  
    Vieta root jumping  
Game theory  
    Combinatorial games  
    Game trees  
    Mini-max  
    Nim  
    Games on graphs  
    Games on graphs with loops  
    Grundy numbers  
    Bipartite games without repetition  
    General games without repetition  
    Alpha-beta pruning  
Probability theory  
Optimization  
    Binary search  
    Ternary search  
    Unimodality and convex functions  
    Binary search on derivative  
Numerical methods  
    Numeric integration  
    Newton's method  
    Root-finding with binary/ternary search  
    Golden section search  
Matrices  
    Gaussian elimination  
    Exponentiation by squaring  
Sorting  
    Radix sort  
Geometry  
    Coordinates and vectors  
        \* Cross product  
        \* Scalar product  
    Convex hull  
    Polygon cut  
    Closest pair  
    Coordinate-compression  
    Quadtrees  
    KD-trees  
    All segment-segment intersection  
Sweeping  
    Discretization (convert to events and sweep)  
    Angle sweeping  
    Line sweeping  
    Discrete second derivatives  
Strings

    Longest common substring  
    Palindrome subsequences  
    Knuth-Morris-Pratt  
    Tries  
    Rolling polynomial hashes  
    Suffix array  
    Suffix tree  
    Aho-Corasick  
    Manacher's algorithm  
    Letter position lists  
Combinatorial search  
    Meet in the middle  
    Brute-force with pruning  
    Best-first (A\*)  
    Bidirectional search  
    Iterative deepening DFS / A\*  
Data structures  
    LCA ( $2^k$ -jumps in trees in general)  
    Pull/push-technique on trees  
    Heavy-light decomposition  
    Centroid decomposition  
    Lazy propagation  
    Self-balancing trees  
    Convex hull trick (wcipeg.com/wiki/Convex\_hull\_trick)  
    Monotone queues / monotone stacks / sliding queues  
    Sliding queue using 2 stacks  
    Persistent segment tree