

Chulalongkorn University

998244353

Teetat T., Borworntat D., Pakapim E.

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Template (1)		

```
template.cpp
```

27 lines

```
#pragma once
#include <bits/stdc++.h>
#define sz(x) (int)(x).size()
#define all(x) (x).begin(), (x).end()
using namespace std;
using 11 = long long;
using db = long double;
using vi = vector<int>;
using v1 = vector<11>;
using vd = vector<db>;
using pii = pair<int, int>;
using pll = pair<11, 11>;
using pdd = pair<db, db>;
const int INF = 0x3ffffffff;
// const int MOD=1000000007;
const int MOD = 998244353;
const 11 LINF = 0x1ffffffffffffffff;
const db DINF = numeric_limits<db>::infinity();
const db EPS = 1e-9;
const db PI = acos(db(-1));
 cin.tie(nullptr) -> sync_with_stdio(false);
c.sh
```

g++ -std=gnu++2a -Wall \$1 -o a.out

./a.out

Mathematics (2)

2.1 Goldbatch's Conjecture

- Even number can be written in sum of two primes (Up to
- Range of N^{th} prime and $N+1^{th}$ prime will be less than or equal to 300 (Up to 1e12)

2.2 Divisibility

Number of divisors of N is given by $\prod_{i=1}^k (a_i + 1)$ where $N = \prod_{i=1}^k p_i^{a_i}$ and p_i are prime factors of N.

Numerical (3)

3.1 Newton's Method

```
if F(Q) = 0, then Q_{2n} \equiv Q_n - \frac{F(Q_n)}{F'(Q_n)} \pmod{x^{2n}}
                Q = P^{-1} : Q_{2n} \equiv Q_n \cdot (2 - P \cdot Q_n^2) \pmod{x^{2n}}
                                  Q = \ln P = \int \frac{P'}{P} \mathrm{d}x
                Q = e^p : Q_{2n} \equiv Q_n(1 + P - \ln Q_n) \pmod{x^{2n}}
                Q = \sqrt{P} : Q_{2n} \equiv \frac{1}{2} (Q_n + P \cdot Q_n^{-1}) \pmod{x^{2n}}
               Q = P^k = \alpha^k x^{kt} e^{k \ln T}; P = \alpha \cdot x^t \cdot T, T(0) = 1
```

Group (4)

4.1 Monoid

```
monoid/MonoidBase.hpp
Description: Monoid Base class.
```

template < class T, T (*combine) (T, T), T (*identity) () > struct MonoidBase{ using value_type = T; static constexpr T op(const T &x,const T &y) {return combine

static constexpr T unit() {return identity();}

Action

action/MonoidActionBase.hpp

```
Description: Monoid Action Base class.
"../../template/Header.hpp"
template < class MInfo, class MTag, typename MInfo::value_type
    (*combine) (typename MInfo::value_type,typename MTag::
         value_type)>
struct MonoidActionBase{
    using InfoMonoid = MInfo;
    using TagMonoid = MTag;
    using Info = typename InfoMonoid::value_type;
    using Tag = typename TagMonoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b) {
        return combine (a,b);
```

```
action/DefaultAction.hpp
Description: Default Action class.
```

e45000, 10 lines

```
template < class Monoid>
struct DefaultAction{
    using InfoMonoid = Monoid;
    using TagMonoid = Monoid;
    using Info = typename Monoid::value_type;
    using Tag = typename Monoid::value_type;
    static constexpr Info op(const Info &a,const Tag &b) {
        return Monoid::op(a,b);
};
```

Data Structures (5)

```
OrderedSet.hpp
Description: Ordered Set
"../template/Header.hpp", <bits/extc++.h>
using namespace __gnu_pbds;
template <class T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// can be change to less equal
void usage() {
 ordered_set<int> st, st_2;
  st.insert(2);
  st.insert(1);
  cout << st.order_of_key(2);</pre>
 cout << *st.find_by_order(1);</pre>
 st.join(st_2); // merge
```

FenwickTree.hpp Description: Fenwick / Binary Indexed Tree

43767a, 41 lines

```
template<class T>
struct Fenwick{
    int n, logn;
    vector<T> t;
    Fenwick(){}
    Fenwick(int _n) {init(vector<T>(_n,T{}));}
    template<class U>
    Fenwick(const vector<U> &a) {init(a);}
    template<class U>
    void init(const vector<U> &a) {
        n=(int)a.size();
        logn=31-__builtin_clz(n);
        t.assign(n+1,T\{\});
        for (int i=1; i<=n; i++) {</pre>
            t[i]=t[i]+a[i-1];
            int j=i+(i&-i);
            if(j<=n)t[j]=t[j]+t[i];
    void update(int x,const T &v) {
        for (int i=x+1; i<=n; i+=i&-i) t[i] =t[i]+v;</pre>
    void update(int 1,int r,const T &v) {
        update (1, v), update (r+1, -v);
    T query(int x) {
        T res{};
        for(int i=x+1;i>0;i-=i&-i)res=res+t[i];
        return res;
```

T querv(int 1, int r) {

SegmentTree LazySegmentTree

```
return query (r) -query (1-1);
    int find(const T &k){
        int x=0;
        T cur{};
        for(int i=1<<logn; i>0; i>>=1)
             if (x+i<=n&&cur+t[x+i]<=k) x+=i, cur=cur+t[x];</pre>
};
SegmentTree.hpp
Description: Segment Tree
                                                        c51dec, 85 lines
template < class Monoid>
struct SegmentTree{
    using T = typename Monoid::value type;
    int n:
    vector<T> t;
    SegmentTree(){}
    SegmentTree(int n, function<T(int)> create) {init(n, create);}
    SegmentTree(int n,T v=Monoid::unit()) {init(n,[&](int) {
    template<class U>
    SegmentTree(const vector<U> &a) {init((int)a.size(), [&](int
         i) {return T(a[i]);});}
    void init(int n,function<T(int)> create){
        t.assign(4<<(31- builtin clz(n)), Monoid::unit());
        function<void(int,int,int)> build=[&](int 1,int r,int i
            if(l==r)return void(t[i]=create(l));
            int m = (1+r)/2;
            build(l,m,i*2);
            build(m+1, r, i*2+1);
            pull(i);
        build(0, n-1, 1);
    void pull(int i){
        t[i] = Monoid::op(t[i*2],t[i*2+1]);
    void modify(int 1,int r,int i,int x,const T &v) {
        if (x<1 | | r<x) return;
        if(l==r)return void(t[i]=v);
        int m = (1+r)/2;
        modify(1, m, i*2, x, v);
        modify(m+1,r,i*2+1,x,v);
        pull(i);
    void modify(int x,const T &v){
        modify (0, n-1, 1, x, v);
    template < class U>
    void update(int 1, int r, int i, int x, const U &v) {
        if (x<1 | | r<x) return;
        if(l==r)return void(t[i]=Monoid::op(t[i],v));
        int m = (1+r)/2;
        update (1, m, i*2, x, v);
        update (m+1,r,i*2+1,x,v);
        pull(i);
    template<class U>
    void update(int x,const U &v) {
        update (0, n-1, 1, x, v);
    T query(int 1, int r, int i, int x, int y) {
        if (y<1||r<x) return Monoid::unit();</pre>
```

```
if (x<=1&&r<=y) return t[i];</pre>
        int m = (1+r)/2;
        return Monoid::op(query(1, m, i*2, x, y), query(m+1, r, i*2+1,
    T query(int x, int y) {
        return query (0, n-1, 1, x, y);
    template < class F>
    int findfirst(int 1, int r, int i, int x, int y, const F &f) {
        if (y<1||r<x||!f(t[i])) return n;</pre>
        if(l==r)return 1;
        int m = (1+r)/2;
        int res=findfirst(1, m, i*2, x, y, f);
        if (res==n) res=findfirst (m+1, r, i*2+1, x, y, f);
        return res;
    template<class F>
    int findfirst(int x,int y,const F &f) {
        return findfirst(0, n-1, 1, x, y, f);
    template < class F>
    int findlast(int 1,int r,int i,int x,int y,const F &f) {
        if (y<1 | | r<x | | ! f (t [i])) return -1;</pre>
        if(l==r)return 1;
        int m = (1+r)/2;
        int res=findlast (m+1, r, i*2+1, x, y, f);
        if(res==-1) res=findlast(1, m, i*2, x, y, f);
        return res;
    template<class F>
    int findlast(int x,int y,const F &f) {
        return findlast(0, n-1, 1, x, y, f);
};
LazySegmentTree.hpp
Description: Segment Tree with Lazy Propagation
                                                       91ab0c, 103 lines
template < class MonoidAction >
struct LazySegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    int n:
    vector<Info> t;
    vector<Tag> lz;
    LazySegmentTree(){}
    LazySegmentTree(int n, function<Info(int)> create) {init(n,
    LazySegmentTree(int n, Info v=InfoMonoid::unit()) {init(n
         , [&] (int) {return v;});}
    template < class T>
    LazySegmentTree(const vector<T> &a) {init((int)a.size(), [&](
         int i) {return Info(a[i]);});}
    void init(int _n,function<Info(int)> create){
        int m=4<<(31-__builtin_clz(n));</pre>
        t.assign(m, InfoMonoid::unit());
        lz.assign(m, TagMonoid::unit());
        function<void(int,int,int)> build=[&](int l,int r,int i
            if (l==r) return void(t[i]=create(l));
            int m=(1+r)/2;
            build(l, m, i * 2);
            build(m+1, r, i*2+1);
            pull(i);
```

```
build(0, n-1, 1);
void pull(int i){
    t[i]=InfoMonoid::op(t[i*2],t[i*2+1]);
void apply(int i,const Tag &v){
    t[i]=MonoidAction::op(t[i],v);
    lz[i]=TagMonoid::op(lz[i],v);
void push(int i){
    apply(i*2,lz[i]);
    apply(i*2+1,lz[i]);
    lz[i]=TagMonoid::unit();
void modify(int 1,int r,int i,int x,const Info &v){
    if (x<1 | | r<x) return;
    if(l==r)return void(t[i]=v);
    int m = (1+r)/2;
    push(i);
    modify (1, m, i*2, x, v);
    modify (m+1,r,i*2+1,x,v);
    pull(i);
void modify(int x,const Info &v) {
    modify (0, n-1, 1, x, v);
void update(int 1,int r,int i,int x,int y,const Tag &v) {
    if (y<1 | | r<x) return;</pre>
    if (x<=l&&r<=y) return apply(i,v);</pre>
    int m = (1+r)/2;
    update(1, m, i * 2, x, y, v);
    update (m+1,r,i*2+1,x,y,v);
    pull(i);
void update(int x,int y,const Tag &v) {
    update (0, n-1, 1, x, y, v);
Info query(int 1,int r,int i,int x,int y){
    if (y<1||r<x) return InfoMonoid::unit();</pre>
    if (x<=1&&r<=y) return t[i];
    int m = (1+r)/2;
    return InfoMonoid::op(query(1, m, i * 2, x, y), query(m+1, r, i
         *2+1, x, y));
Info query(int x,int y){
    return query (0, n-1, 1, x, y);
template < class F>
int findfirst(int 1, int r, int i, int x, int y, const F &f) {
    if(y<1||r<x||!f(t[i]))return n;
    if(l==r)return 1;
    int m = (1+r)/2;
    int res=findfirst(1, m, i*2, x, y, f);
    if (res==n) res=findfirst (m+1, r, i*2+1, x, y, f);
    return res;
template<class F>
int findfirst(int x,int y,const F &f) {
    return findfirst(0,n-1,1,x,y,f);
template < class F>
int findlast(int 1,int r,int i,int x,int y,const F &f) {
    if (y<1||r<x||!f(t[i])) return -1;</pre>
    if (l==r) return 1;
    int m=(1+r)/2;
    push(i);
```

return res;

template<class F>

DynamicSegmentTree DSU BinaryTrie

```
return findlast(0, n-1, 1, x, y, f);
};
DynamicSegmentTree.hpp
Description: Dynamic Segment Tree
                                                     e84eeb, 106 lines
template < class MonoidAction>
struct DynamicSegmentTree{
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    struct Node;
    using Ptr = Node*;
    struct Node {
        Info val;
        Tag lz:
        Node (Info v): val(v), lz(TagMonoid::unit()), l(nullptr), r(
        Node(Info v, Tag t):val(v),lz(t),l(nullptr),r(nullptr){}
    11 lb.ub:
    Ptr rt;
    function<Info(11,11)> create;
    DynamicSegmentTree() {init(0,0);}
    DynamicSegmentTree(ll n) {init(0, n-1);}
    DynamicSegmentTree(11 lb, ll ub) {init(lb, ub);}
    DynamicSegmentTree(11 lb,11 ub,function<Info(11,11)> create
         ) {init(lb,ub,create);}
    void init(ll _lb,ll _ub,function<Info(ll,ll)> _create=[](ll
          1,11 r) {return InfoMonoid::unit();}) {
        lb= lb, ub= ub;
        create=_create;
        rt=new Node(create(lb,ub));
    Info val(Ptr t){
        return t?t->val:InfoMonoid::unit();
    void pull(Ptr &t){
        t->val=InfoMonoid::op(val(t->1),val(t->r));
    void apply(Ptr &t,const Tag &v,ll 1,ll r){
        if(!t)t=new Node(create(1,r));
        t->val=MonoidAction::op(t->val,v);
        t->1z=TagMonoid::op(t->1z,v);
    void push(Ptr &t,ll l,ll m,ll r){
        apply (t->1,t->1z,1,m);
        apply (t->r,t->lz,m+1,r);
        t->lz=TagMonoid::unit();
    void modify(ll 1,ll r,Ptr &t,ll x,const Info &v) {
        if (x<1 | | r<x) return;
        if(l==r) return void(t->val=v);
        11 m=1+(r-1)/2;
        push(t,l,m,r);
        modify(1, m, t->1, x, v);
        modify (m+1, r, t->r, x, v);
        pull(t);
    void modify(ll x,const Info &v){
```

int res=findlast(m+1,r,i*2+1,x,v,f);

int findlast(int x,int y,const F &f) {

if (res==-1) res=findlast (1, m, i*2, x, y, f);

```
modify(lb,ub,rt,x,v);
    void update(ll 1,ll r,Ptr &t,ll x,ll y,const Tag &v) {
        if (y<1 | | r<x) return;</pre>
        if (x<=l&&r<=y) return apply(t,v,l,r);</pre>
        11 m=1+(r-1)/2;
        push(t,l,m,r);
        update(l, m, t->l, x, y, v);
        update (m+1, r, t->r, x, y, v);
        pull(t);
    void update(ll x,ll y,const Tag &v) {
        update(lb,ub,rt,x,y,v);
    Info query(ll 1,ll r,Ptr &t,ll x,ll y){
        if (y<1||r<x) return InfoMonoid::unit();</pre>
        if(x<=l&&r<=y)return t->val;
        11 m=1+(r-1)/2;
        push(t,1,m,r);
        return InfoMonoid::op(query(l,m,t->l,x,y),query(m+1,r,t
             ->r,x,y));
    Info query(ll x,ll y){
        return query(lb,ub,rt,x,y);
    template < class F>
    11 findfirst(ll 1, ll r, Ptr t, ll x, ll y, const F &f) {
        if (y<1||r<x||!f(t->val))return -1;
        if (l==r) return 1;
        11 m=1+(r-1)/2;
        push(t,l,m,r);
        ll res=findfirst(l, m, t->l, x, y, f);
        if (res==-1) res=findfirst (m+1, r, t->r, x, y, f);
    template<class F>
    ll findfirst(ll x, ll y, const F &f) {
        return findfirst(lb,ub,rt,x,y,f);
    template<class F>
    11 findlast(ll 1,ll r,Ptr t,ll x,ll y,const F &f){
        if (v<1||r<x||!t||!f(t->val))return -1;
        if(l==r)return 1;
        11 m=1+(r-1)/2;
        push(t,1,m,r);
        11 res=findlast(m+1,r,t->r,x,y,f);
        if (res==-1) res=findlast(1, m, t->1, x, y, f);
        return res;
    template<class F>
    11 findlast(ll x,ll y,const F &f) {
        return findlast(lb,ub,rt,x,y,f);
};
DSU.hpp
Description: Disjoint Set Union.
                                                        0b3cb8, 26 lines
struct DSU{
    vector<int> p,sz;
    DSU() { }
    DSU(int n) {init(n);}
    void init(int n) {
        p.resize(n);
        iota(p.begin(),p.end(),0);
        sz.assign(n,1);
    int find(int u) {
        return p[u] == u?u:p[u] = find(p[u]);
```

```
bool same(int u,int v) {
        return find(u) == find(v);
    bool merge(int u,int v) {
        u=find(u), v=find(v);
        if (u==v) return false;
        sz[u]+=sz[v];
        p[v]=u;
        return true;
    int size(int u) {
        return sz[find(u)];
};
BinaryTrie.hpp
Description: Binary Trie
                                                      ae5b7a, 66 lines
template<int BIT,class T = uint32_t,class S = int>
struct BinaryTrie{
    struct Node {
        array<int,2> ch;
        S cnt:
        Node():ch\{-1,-1\}, cnt(0){}
    vector<Node> t;
    BinaryTrie():t{Node()}{}
    int new node(){
        t.emplace back(Node());
        return t.size()-1;
    S size(){
        return t[0].cnt;
    bool empty(){
        return size() ==0;
    S get cnt(int i) {
        return i!=-1?t[i].cnt:S(0);
    void insert(T x,S k=1){
        int u=0;
        t[u].cnt+=k;
        for(int i=BIT-1; i>=0;i--) {
            int v=x>>i&1;
            if(t[u].ch[v] == -1)t[u].ch[v] = new_node();
            u=t[u].ch[v];
            t[u].cnt+=k;
    void erase(T x,S k=1){
        int u=0;
        assert(t[u].cnt>=k);
        t[u].cnt-=k;
        for(int i=BIT-1;i>=0;i--) {
            int v=x>>i&1;
            u=t[u].ch[v];
            assert (u!=-1&&t[u].cnt>=k);
            t[u].cnt-=k;
    T kth(S k, T x=0) {
        assert(k<size());
        int u=0;
        T res=0;
        for(int i=BIT-1;i>=0;i--) {
            int v=x>>i&1;
            if (k < get_cnt (t[u].ch[v])) {
```

};

u=t[u].ch[v];

res|=T(1)<<i;

 $u=t[u].ch[v^1];$

k-=get_cnt(t[u].ch[v]);

}else{

LiChaoTree DynamicLiChaoTree SplayTreeBase

```
return res;
    T min(T x) {
        return kth(0,x);
    T max(T x) {
        return kth(size()-1,x);
LiChaoTree.hpp
Description: Li-Chao Tree (minimize).
                                                        4ab713. 52 lines
template<class T>
struct LiChaoTree{
    static const T INF=numeric limits<T>::max()/2;
    struct Line{
        T m,c;
        Line(T _m, T _c):m(_m),c(_c){}
        inline T eval(T x)const{return m*x+c;}
    };
    vector<T> xs;
    vector<Line> t:
    LiChaoTree(){}
    LiChaoTree (const vector<T> &x):xs(x) {init(x);}
    LiChaoTree(int n):xs(n){
        vector<T> x(n);
        iota(x.begin(), x.end(), 0);
        init(x);
    void init(const vector<T> &x) {
        xs=x:
        sort(xs.begin(), xs.end());
        xs.erase(unique(xs.begin(),xs.end()),xs.end());
        t.assign(4<<(31-__builtin_clz(xs.size())),Line(0,INF));
    void insert(int l,int r,int i,Line v) {
        int m = (1+r)/2;
        if(v.eval(xs[m]) <t[i].eval(xs[m])) swap(t[i],v);</pre>
        if(v.eval(xs[1]) <t[i].eval(xs[1]))insert(1, m, i * 2, v);</pre>
        if(v.eval(xs[r]) < t[i].eval(xs[r]))insert(m+1,r,i*2+1,v)
    inline void insert(T m,T c){
        insert (0, (int) xs.size()-1,1, Line(m,c));
    void insert_range(int 1, int r, int i, T x, T y, Line v) {
        if (y<xs[1] | |xs[r]<x) return;
        if (x<=xs[1]&&xs[r]<=y) return insert(1,r,i,v);</pre>
        int m = (1+r)/2;
        insert_range(1, m, i \star 2, x, y, v);
        insert_range(m+1, r, i*2+1, x, y, v);
    inline void insert_range(T m, T c, T x, T y) {
        insert_range(0, (int)xs.size()-1,1,x,y,Line(m,c));
    T query(int l, int r, int i, T x) {
        if(l==r)return t[i].eval(x);
        int m = (1+r)/2;
        if (x<=xs[m]) return min(t[i].eval(x), query(1, m, i*2, x));</pre>
        return min(t[i].eval(x), query(m+1, r, i*2+1, x));
```

```
inline T query(T x){
        return query(0,(int)xs.size()-1,1,x);
};
DynamicLiChaoTree.hpp
Description: Dynamic Li-Chao Tree (minimize)
                                                        b8af36, 50 lines
template < class T>
struct DynamicLiChaoTree{
    static const T INF=numeric_limits<T>::max()/2;
    struct Line{
        T m,c;
        Line (T _m, T _c) : m(_m), c(_c) {}
        inline T eval(T x)const{return m*x+c;}
    struct Node;
    using Ptr = Node*;
    struct Node{
        Line v;
        Ptr l,r;
        Node():v(0,INF),l(nullptr),r(nullptr){}
        Node(Line _v):v(_v),l(nullptr),r(nullptr){}
    11 1b, ub;
    Ptr root;
    DynamicLiChaoTree(ll _lb,ll _ub):lb(_lb),ub(_ub),root(
         nullptr) {}
    void insert(T 1, T r, Ptr &t, Line v) {
        if(!t)return void(t=new Node(v));
        T m=1+(r-1)/2;
        if (v.eval(m) <t->v.eval(m)) swap(t->v,v);
        if (v.eval(1) <t->v.eval(1)) insert(1, m, t->1, v);
        if (v.eval(r) <t->v.eval(r)) insert(m+1, r, t->r, v);
    inline void insert(T m, T c){
        insert(lb, ub, root, Line(m, c));
    void insert_range(T 1, T r, Ptr &t, T x, T y, Line v) {
        if (y<1 | |r<x) return;
        if(!t)t=new Node();
        if (x<=1&&r<=y) return insert(1, r, t, v);</pre>
        T m=1+(r-1)/2;
        insert_range(1, m, t->1, x, y, v);
        insert_range(m+1, r, t->r, x, y, v);
    inline void insert range(T m, T c, T x, T y) {
        insert_range(lb, ub, root, x, y, Line(m, c));
    T query(T 1,T r,Ptr t,T x){
        if(!t)return INF;
        T m=1+(r-1)/2;
        if(x \le m) return min(t > v.eval(x), query(1, m, t > 1, x));
        return min(t->v.eval(x), query(m+1, r, t->r, x));
    inline T query(T x){
        return query(lb,ub,root,x);
};
SplayTreeBase.hpp
Description: Splay Tree. splay(u) will make node u be the root of the tree
in amortized O(log n) time.
template < class Node >
struct SplayTreeBase{
    using Ptr = Node*;
    bool is_root(Ptr t){
        return ! (t->p) | | (t->p->l!=t&&t->p->r!=t);
```

```
} // The parent of the root stores the path parant in link
     cut tree.
int size(Ptr t){
    return t?t->size:0;
virtual void push(Ptr t){};
virtual void pull(Ptr t){};
int pos(Ptr t){
    if(t->p){
        if (t->p->l==t) return -1;
        if(t->p->r==t)return 1;
    return 0;
void rotate(Ptr t) {
    Ptr x=t->p, y=x->p;
    if (pos(t) == -1) {
        if ((x->1=t->r))t->r->p=x;
        t->r=x, x->p=t;
    }else{
        if((x->r=t->1))t->1->p=x;
        t->l=x,x->p=t;
    pull(x),pull(t);
    if((t->p=y)){
        if (y->1==x) y->1=t;
        if (y->r==x) y->r=t;
void splay(Ptr t) {
    if(!t)return;
    push(t);
    while(!is_root(t)){
        Ptr x=t->p;
        if(is_root(x)){
            push(x), push(t);
             rotate(t);
        }else{
            Ptr y=x->p;
            push(y), push(x), push(t);
            if(pos(x) == pos(t)) rotate(x), rotate(t);
            else rotate(t), rotate(t);
Ptr get_first(Ptr t){
    while (t->1) push (t), t=t->1;
    splay(t);
    return t;
Ptr get_last(Ptr t){
    while (t->r) push (t), t=t->r;
    splav(t);
    return t:
Ptr merge(Ptr 1,Ptr r) {
    splay(1), splay(r);
    if(!1)return r;
    if(!r)return 1;
    l=get_last(l);
    1->r=r;
    r->p=1;
    pull(1);
    return 1;
pair<Ptr, Ptr> split (Ptr t, int k) {
    if(!t)return {nullptr,nullptr};
    if (k==0) return {nullptr,t};
    if(k==size(t))return {t,nullptr};
```

```
push(t);
        if(k<=size(t->1)){
            auto x=split(t->1,k);
            t->1=x.second;
            t->p=nullptr;
            if(x.second) x.second->p=t;
            pull(t);
            return {x.first,t};
        }else{
            auto x=split(t->r, k-size(t->1)-1);
            t->r=x.first;
            t->p=nullptr;
            if(x.first)x.first->p=t;
            pull(t);
            return {t,x.second};
    void insert(Ptr &t,int k,Ptr v) {
       splay(t);
        auto x=split(t,k);
        t=merge(merge(x.first,v),x.second);
    void erase(Ptr &t,int k){
       splay(t);
        auto x=split(t,k);
       auto y=split(x.second,1);
        // delete y.first;
        t=merge(x.first,y.second);
    template < class T>
   Ptr build(const vector<T> &v) {
        if(v.empty())return nullptr;
        function<Ptr(int,int)> build=[&](int 1,int r){
            if(l==r) return new Node(v[1]);
            int m = (1+r)/2;
            return merge(build(1,m),build(m+1,r));
        return build(0, v.size()-1);
};
LazyReversibleBBST.hpp
Description: Lazy Reversible BBST Base.
                                                     904708, 81 lines
template < class Tree, class Node, class MonoidAction >
struct LazyReversibleBBST:Tree{
    using Tree::merge;
    using Tree::split;
    using typename Tree::Ptr;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    LazyReversibleBBST() = default;
                                                                   };
    Info sum(Ptr t){
        return t?t->sum:InfoMonoid::unit();
   void pull(Ptr t) {
       if(!t)return;
       push(t);
        t->size=1;
        t->sum=t->val;
```

t->revsum=t->val;

t->size+=t->l->size;

t->sum=InfoMonoid::op(t->l->sum,t->sum);

t->revsum=InfoMonoid::op(t->revsum,t->l->revsum);

if(t->1){

```
if(t->r){
            t->size+=t->r->size;
            t->sum=InfoMonoid::op(t->sum,t->r->sum);
            t->revsum=InfoMonoid::op(t->r->revsum,t->revsum);
    void push (Ptr t) {
       if(!t)return;
        if (t->rev) {
            toggle(t->1);
            toggle(t->r);
            t->rev=false:
        if(t->lz!=TagMonoid::unit()){
            propagate(t->1,t->1z);
            propagate(t->r,t->lz);
            t->1z=TagMonoid::unit();
    void toggle(Ptr t) {
       if(!t)return;
        swap (t->1,t->r);
        swap (t->sum, t->revsum);
        t->rev^=true;
    void propagate (Ptr t, const Tag &v) {
        if(!t)return;
        t->val=MonoidAction::op(t->val,v);
        t->sum=MonoidAction::op(t->sum, v);
        t->revsum=MonoidAction::op(t->revsum, v);
        t->1z=TagMonoid::op(t->1z,v);
    void apply(Ptr &t,int 1,int r,const Tag &v) {
        if(1>r)return;
        auto x=split(t,1);
        auto y=split(x.second, r-1+1);
        propagate(y.first,v);
        t=merge(x.first,merge(y.first,y.second));
    Info query(Ptr &t,int 1,int r){
        if(l>r)return InfoMonoid::unit();
        auto x=split(t,1);
        auto y=split(x.second,r-l+1);
        Info res=sum(y.first);
        t=merge(x.first,merge(y.first,y.second));
        return res;
    void reverse(Ptr &t,int 1,int r) {
        if (1>r) return;
        auto x=split(t,1);
        auto y=split(x.second, r-l+1);
        toggle(v.first);
        t=merge(x.first, merge(y.first, y.second));
LazyReversibleSplayTree.hpp
Description: Lazy Reversible Splay Tree.
"SplayTreeBase.hpp", "LazyReversibleBBST.hpp"
                                                      b8455b, 23 lines
template < class MonoidAction>
struct LazyReversibleSplayTreeNode{
    using Ptr = LazyReversibleSplayTreeNode*;
    using InfoMonoid = typename MonoidAction::InfoMonoid;
    using TagMonoid = typename MonoidAction::TagMonoid;
    using Info = typename MonoidAction::Info;
    using Tag = typename MonoidAction::Tag;
    using value_type = Info;
   Ptr 1, r, p;
```

```
Info val, sum, revsum;
    Tag lz:
    int size;
    bool rev;
    LazyReversibleSplayTreeNode(const Info &_val=InfoMonoid::
         unit(),const Tag &_lz=TagMonoid::unit())
        :1(),r(),p(),val(_val),sum(_val),revsum(_val),lz(_lz),
             size(1), rev(false) {}
};
template < class MonoidAction >
struct LazyReversibleSplayTree
    : LazyReversibleBBST<SplayTreeBase<
         LazyReversibleSplayTreeNode<MonoidAction>>,
      LazyReversibleSplayTreeNode<MonoidAction>, MonoidAction>{
    using Node = LazyReversibleSplayTreeNode<MonoidAction>;
};
LinkCutTreeBase.hpp
Description: Link Cut Tree Base.
Usage: evert(u): make u be the root of the tree.
link(u,v): attach u to v.
cut(u,v): remove edge between u and v.
get_root(u): get the root of the tree containing u.
lca(u,v): get the lowest common ancestor of u and v.
fold(u,v): get the value of the path from u to v. _{\rm b432c3,\ 59\ lines}
template < class Splay>
struct LinkCutTreeBase:Splay{
    using Node = typename Splay::Node;
    using Ptr = Node*;
    using T = typename Node::value_type;
    Ptr expose(Ptr t){
        Ptr pc=nullptr; // preferred child
        for (Ptr cur=t; cur; cur=cur->p) {
            this->splay(cur);
            cur->r=pc;
            this->pull(cur);
            pc=cur;
        this->splay(t);
        return pc;
    void evert(Ptr t) { // make t be the root of the tree
        expose(t);
        this->toggle(t);
        this->push(t);
    void link(Ptr u,Ptr v) { // attach u to v
        evert (u):
        expose (v);
        u->p=v;
    {f void} cut (Ptr u, Ptr v) { // cut edge between u and v
        evert (u):
        expose(v);
        assert (u->p==v);
        v->l=u->p=nullptr;
        this->pull(v);
    Ptr get_root(Ptr t){
        expose(t);
        while (t->1) this->push (t), t=t->1;
        this->splay(t);
        return t;
    Ptr lca(Ptr u, Ptr v) {
        if(get_root(u)!=get_root(v))return nullptr;
        expose(u);
```

swap(x,x1);

```
return expose(v);
    void set val(Ptr t,const T &val){
        this->evert(t);
        t->val=val;
        this->pull(t);
    T get_val(Ptr t){
        this->evert(t);
        return t->val:
    T fold(Ptr u,Ptr v) {
        evert(u);
        expose(v);
        return v->sum;
LazvLinkCutTree.hpp
Description: Lazy Link Cut Tree.
Usage: using Lct = LazyLinkCutTree<Action>;
using Ptr = Lct::Ptr;
using Node = Lct:: Node;
vector<Ptr> ptr(n);
for(int i=0;i<n;i++)ptr[i]=new Node(val[i]);</pre>
auto link=[&](int u,int v){
Lct::link(ptr[u],ptr[v]);
auto cut=[&](int u,int v){
Lct::cut(ptr[u],ptr[v]);
auto update=[&](int u,int v,const Action:: Tag &val){
Lct: : apply(ptr[u],ptr[v],val);
auto query=[&](int u,int v){
return Lct::fold(ptr[u],ptr[v]);
                                                       ead3da, 12 lines
"LazyReversibleSplayTree.hpp", "LinkCutTreeBase.hpp"
template < class MonoidAction>
struct LazyLinkCutTree:LinkCutTreeBase<LazyReversibleSplayTree</pre>
     MonoidAction>>{
    using base = LinkCutTreeBase<LazyReversibleSplayTree<</pre>
         MonoidAction>>;
    using Ptr = typename base::Ptr;
    using Tag = typename MonoidAction::Tag;
    void apply (Ptr u, Ptr v, const Tag &val) {
        this->evert(u);
        this->expose(v);
        this->propagate(v, val);
Number Theory (6)
ExtendedEuclid.hpp
Description: Extended Euclid algorithm for solving diophantine equation
(ax + by = gcd(a, b)).
Time: \mathcal{O}(\log \max\{a, b\})
"../template/Header.hpp"
                                                       229e7c, 13 lines
pair<11,11> euclid(11 a,11 b) {
    11 x=1, y=0, x1=0, y1=1;
    while(b!=0){
        11 q=a/b;
        x = q * x1;
        y = q * y1;
        a-=q*b;
```

```
swap(y,y1);
        swap(a,b);
    return {x,y};
6.1 Prime Numbers
LinearSieve.hpp
Description: Prime Number Generator in Linear Time
Time: \mathcal{O}(N)
"../template/Header.hpp"
                                                        194fb1, 15 lines
vi linear sieve(int n) {
 vi prime, composite(n + 1);
 for(int i=2; i<=n; ++i) {
    if(!composite[i]) {
      prime.emplace_back(i);
    for(int j=0; j<(int) prime.size() && i*prime[j]<=n; ++j) {</pre>
      composite[i * prime[j]] = true;
      if(i % prime[j] == 0) {
        break:
 return prime;
FastEratosthenes.hpp
Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 \approx 1.5s
"../template/Header.hpp"
                                                       295b58, 33 lines
const int LIM = 1e6;
bitset<LIM> isPrime:
vi eratosthenes() {
 const int S = (int) round(sqrt(LIM)), R = LIM / 2;
 vi pr = \{2\}, sieve(S + 1);
 pr.reserve(int(LIM/log(LIM) * 1.1));
 vector<pii> cp;
 for(int i=3; i<=S; i+=2) {
    if(!sieve[i]) {
      cp.emplace_back(i, i * i / 2);
      for(int j=i*i; j<=S; j+=2*i) {</pre>
        sieve[i] = 1;
 for(int L=1; L<=R; L+=S) {</pre>
    array<bool, S> block{};
    for(auto &[p, idx]: cp) {
      for(int i=idx; i<S+L; idx=(i+=p)) {</pre>
        block[i - L] = 1;
    for(int i=0; i<min(S, R-L); ++i) {</pre>
      if(!block[i]) {
        pr.emplace_back((L + i) * 2 + 1);
 for(int i: pr) {
    isPrime[i] = 1;
 return pr;
```

```
"FastEratosthenes.hpp"
                                                        88fb23, 18 lines
pair<int, int> goldbatchConjecture(int s, vi pr = {}) {
  if (s <= 2 || s % 2 != 0) {
    return make_pair(-1, -1);
  if (pr.size() == 0) {
    pr = eratosthenes();
  for (auto x : pr) {
    if (x > s / 2) {
      break;
    int d = s - x;
    if (binary_search(pr.begin(), pr.end(), d)) {
      return make_pair(min(x, d), max(x, d));
  return make_pair(-1, -1);
Graph (7)
7.1 Matching
HopcroftKarp.hpp
Description: Fast bipartite matching algorithm.
Time: \mathcal{O}\left(E\sqrt{V}\right)
"../template/Header.hpp
                                                        0bd56f, 52 lines
struct HopcroftKarp{
    int n.m:
    vi l,r,lv,ptr;
    vector<vi> adi;
    HopcroftKarp(){}
    HopcroftKarp(int _n,int _m) {init(_n,_m);}
    void init(int _n,int _m) {
        n=_n, m=_m;
        adj.assign(n+m, vi{});
    void addEdge(int u,int v) {
        adj[u].emplace_back(v+n);
    void bfs(){
        lv=vi(n,-1);
        queue<int> q;
        for (int i=0; i<n; i++) if (1[i] ==-1) {</pre>
             lv[i]=0;
             q.emplace(i);
        while(!q.empty()){
            int u=q.front();
             for(int v:adj[u])if(r[v]!=-1&&lv[r[v]]==-1){
                 lv[r[v]]=lv[u]+1;
                 q.emplace(r[v]);
    bool dfs(int u) {
        for(int &i=ptr[u];i<sz(adj[u]);i++) {</pre>
             int v=adj[u][i];
             if (r[v] ==-1||(lv[r[v]] == lv[u] +1&&dfs(r[v]))) {
                 l[u]=v,r[v]=u;
                 return true;
```

GolbatchConjecture.hpp

Time: $\mathcal{O}(N \log N)$

Description: Find two prime numbers which sum equals s

```
}
    return false;
}
int maxMatching() {
    int match=0, cnt=0;
    l=r=vi(n+m,-1);
    do{
        ptr=vi(n);
        bfs();
        cnt=0;
        for(int i=0;i<n;i++)if(l[i]==-1&&dfs(i))cnt++;
        match+=cnt;
    }while(cnt);
    return match;
}
};</pre>
```

Kuhn.hpp

Description: Kuhn Algorithm to find maximum bipartite matching or find augmenting path in bipartite graph.

Time: $\mathcal{O}(VE)$

```
"../template/Header.hpp" fc7d17, 15 lines
vi adj[1010], match(1010, -1);
bitset<1010> visited;
bool kuhn(int u) {
   if(visited[u]) {
      return false;
   }
   visited[u] = true;
   for(auto x: adj[u]) {
      if(match[x] == -1 || kuhn(match[x])) {
       match[x] = u;
      return true;
   }
   }
   return false;
}
```

7.2 Network Flow

Dinic.hpp

Description: Fast max-flow algorithm. Time: $\mathcal{O}(VE \log U)$ where $U = \max |\text{cap}|$

7409c7, 68 lines

```
template<class T>
struct Dinic{
    struct Edge {
        int to:
        11 flow, cap;
        Edge(int _to,ll _cap):to(_to),flow(0),cap(_cap){}
       ll getcap(){
            return cap-flow;
    };
   int n;
    11 U;
    vector<Edge> e;
   vector<vi> adj;
   vi ptr, lvl;
   Dinic(){}
   Dinic(int _n) {
        init(_n);
    void init(int _n){
       n=_n, U=0;
        e.clear();
        adj.assign(n,{});
   void addEdge(int u,int v,ll cap){
```

```
U=max(U,cap);
        adj[u].emplace_back(sz(e));
        e.emplace_back(v,cap);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u,0); // change 0 to cap for undirected
             flow
   bool bfs(int s,int t,ll scale) {
       lvl.assign(n,0);
        vi a{s}:
        lv1[s]=1;
        for (int i=0; i < sz(q); i++) {</pre>
            int u=q[i];
            for(auto j:adj[u])if(!lvl[e[j].to]&&e[j].getcap()>=
                 q.emplace_back(e[j].to);
                lvl[e[j].to]=lvl[u]+1;
        return lvl[t];
    11 dfs(int u,int t,ll f){
        if (u==t||!f) return f;
        for(int &i=ptr[u];i<sz(adj[u]);i++) {</pre>
            int j=adj[u][i];
            if(lvl[e[j].to]==lvl[u]+1){
                if(ll p=dfs(e[j].to,t,min(f,e[j].getcap()))){
                     e[i].flow+=p;
                     e[j^1].flow-=p;
                     return p;
        return 0;
    11 flow(int s,int t){
        11 flow=0;
        for (11 L=111<< (63-_builtin_clz11(U)); L>0; L>>=1) //L =
              1 may be faster but it's O(V^2 E)
        while(bfs(s,t,L)){
            ptr.assign(n,0);
             while(ll p=dfs(s,t,LINF))flow+=p;
        };
        return flow;
};
MinCostFlow.hpp
Description: minimum-cost flow algorithm.
Time: \mathcal{O}(FE \log V) where F is max flow.
"../template/Header.hpp"
                                                      8eald2, 83 lines
template<class F,class C>
struct MinCostFlow{
    struct Edge{
        int to;
       F flow, cap;
        Edge(int _to,F _cap,C _cost):to(_to),flow(0),cap(_cap),
             cost(cost){}
        F getcap(){
            return cap-flow;
    };
    int n;
   vector<Edge> e;
   vector<vi> adj;
   vector<C> pot, dist;
   vi pre;
   bool neg;
```

```
const C CINF=numeric_limits<C>::max()/2;
    MinCostFlow(){}
    MinCostFlow(int _n) {
        init(_n);
    void init(int n){
        n=n;
        e.clear();
        adj.assign(n,{});
        neg=false;
    void addEdge(int u,int v,F cap,C cost){
        adj[u].emplace_back(sz(e));
        e.emplace_back(v,cap,cost);
        adj[v].emplace_back(sz(e));
        e.emplace_back(u,0,-cost);
        if (cost<0) neg=true;</pre>
    bool dijkstra(int s,int t){
        using P = pair<C,int>;
        dist.assign(n,CINF);
        pre.assign(n,-1);
        priority_queue<P, vector<P>, greater<P>> pg;
        dist[s]=0;
        pq.emplace(0,s);
        while(!pq.empty()){
            auto [d,u]=pq.top();
            pq.pop();
            if (dist[u] < d) continue;</pre>
            for(int i:adj[u]){
                int v=e[i].to;
                C ndist=d+pot[u]-pot[v]+e[i].cost;
                if(e[i].getcap()>0&&dist[v]>ndist){
                     pre[v]=i;
                     dist[v]=ndist;
                     pq.emplace(ndist, v);
        return dist[t] < CINF;
    pair<F,C> flow(int s,int t) {
        F flow=0;
        C cost=0;
        pot.assign(n,0);
        if (neg) for (int t=0; t< n; t++) for (int i=0; i< sz(e); i++) if (e
             [i].getcap()>0){
            int u=e[i^1].to, v=e[i].to;
            pot[v]=min(pot[v],pot[u]+e[i].cost);
        } // Bellman-Ford
        while(dijkstra(s,t)){
            for(int i=0;i<n;i++)pot[i]+=dist[i];</pre>
            for(int u=t;u!=s;u=e[pre[u]^1].to){
                 aug=min(aug,e[pre[u]].getcap());
            } // find bottleneck
            for(int u=t;u!=s;u=e[pre[u]^1].to){
                e[pre[u]].flow+=aug;
                 e[pre[u]^1].flow-=aug;
            } // push flow
            flow+=aug;
            cost+=aug*pot[t];
        return {flow,cost};
};
```

const F FINF=numeric limits<F>::max()/2;

Polynomials (8)

```
FormalPowerSeries.hpp
```

```
Description: basic operations of formal power series
                                                      416433, 136 lines
template < class mint >
struct FormalPowerSeries:vector<mint>{
    using vector<mint>::vector;
    using FPS = FormalPowerSeries;
    FPS & operator += (const FPS &rhs) {
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]+=rhs[i];</pre>
        return *this;
    FPS & operator += (const mint &rhs) {
        if(this->empty())this->resize(1);
        (*this) [0]+=rhs;
        return *this;
    FPS & operator -= (const FPS &rhs) {
        if(rhs.size()>this->size())this->resize(rhs.size());
        for(int i=0;i<rhs.size();i++)(*this)[i]-=rhs[i];</pre>
        return *this;
    FPS & operator -= (const mint &rhs) {
        if(this->empty())this->resize(1);
        (*this) [0] -= rhs;
        return *this;
    FPS & operator *= (const FPS & rhs) {
        auto res=NTT<mint>()(*this,rhs);
        return *this=FPS(res.begin(),res.end());
    FPS & operator *= (const mint &rhs) {
        for(auto &a:*this)a*=rhs;
        return *this:
    friend FPS operator+(FPS lhs,const FPS &rhs) {return lhs+=
         rhs: 1
    friend FPS operator+(FPS lhs,const mint &rhs) {return lhs+=
         rhs: }
    friend FPS operator+ (const mint &lhs, FPS &rhs) {return rhs+=
         lhs; }
    friend FPS operator-(FPS lhs,const FPS &rhs) {return lhs-=
         rhs; }
    friend FPS operator-(FPS lhs,const mint &rhs) {return lhs-=
    friend FPS operator-(const mint &lhs, FPS rhs) {return -(rhs-
    friend FPS operator*(FPS lhs,const FPS &rhs) {return lhs*=
    friend FPS operator*(FPS lhs,const mint &rhs){return lhs*=
    friend FPS operator* (const mint &lhs, FPS rhs) {return rhs*=
         lhs: }
    FPS operator-(){return (*this)*-1;}
    FPS rev() {
        FPS res(*this);
        reverse (res.beign(), res.end());
        return res;
   FPS pre(int sz) {
        FPS res(this->begin(),this->begin()+min((int)this->size
```

if(res.size() < sz) res.resize(sz);</pre>

```
return res:
FPS shrink(){
    FPS res(*this);
    while(!res.empty() &&res.back() ==mint{}) res.pop_back();
    return res;
FPS operator>>(int sz){
    if (this->size()<=sz) return {};</pre>
    FPS res(*this);
    res.erase(res.begin(),res.begin()+sz);
    return res;
FPS operator<<(int sz) {</pre>
    FPS res(*this);
    res.insert(res.begin(),sz,mint{});
    return res;
FPS diff(){
    const int n=this->size();
    FPS res(max(0,n-1));
    for (int i=1; i < n; i++) res[i-1] = (*this)[i] *mint(i);</pre>
    return res;
FPS integral(){
    const int n=this->size();
    FPS res(n+1);
    res[0]=0;
    if(n>0)res[1]=1;
    11 mod=mint::get_mod();
    for (int i=2; i<=n; i++) res[i] = (-res[mod%i]) * (mod/i);</pre>
    for (int i=0; i<n; i++) res[i+1] *=(*this)[i];</pre>
    return res;
mint eval(const mint &x) {
    mint res=0, w=1;
    for(auto &a:*this)res+=a*w,w*=x;
    return res;
FPS inv(int deg=-1) {
    assert(!this->empty()&&(*this)[0]!=mint(0));
    if (deg==-1) deg=this->size();
    FPS res{mint(1)/(*this)[0]};
    for(int i=2;i>>1<deq;i<<=1){</pre>
         res=(res*(mint(2)-res*pre(i))).pre(i);
    return res.pre(deg);
FPS log(int deg=-1) {
    assert(!this->empty()&&(*this)[0] ==mint(1));
    if (deg==-1) deg=this->size();
    return (pre(deg).diff()*inv(deg)).pre(deg-1).integral()
FPS exp(int deg=-1) {
    assert (this->empty() | | (*this) [0] == mint(0));
    if (deg==-1) deg=this->size();
    FPS res{mint(1)};
    for(int i=2;i>>1<deq;i<<=1){
         res=(res*(pre(i)-res.log(i)+mint(1))).pre(i);
    return res.pre(deg);
FPS pow(ll k,int deg=-1) {
    const int n=this->size();
    if (deg==-1) deg=n;
    if(k==0){
         FPS res(deg);
```

```
if (deg) res[0]=mint(1);
             return res;
         for (int i=0; i<n; i++) {</pre>
             if (__int128_t(i) *k>=deg) return FPS(deg, mint(0));
             if((*this)[i]==mint(0))continue;
             mint rev=mint(1)/(*this)[i];
             FPS res=(((*this*rev)>>i).log(deg)*k).exp(deg);
             res=((res*binpow((*this)[i],k))<<(i*k)).pre(deg);
             return res;
         return FPS(deg,mint(0));
};
using FPS=FormalPowerSeries<mint>;
FFT.hpp
Description: Fast Fourier transform
Time: \mathcal{O}(N \log N)
                                                         5d476b, 73 lines
"../template/Header.hpp"
template < class T=11, int mod=0>
struct FFT{
  using vt = vector<T>;
  using cd = complex<db>;
  using vc = vector<cd> ;
  static const bool INT=true;
  static void fft(vc &a){
    int n=a.size(),L=31- builtin clz(n);
    vc rt(n);
    rt[1]=1;
    for(int k=2; k<n; k*=2) {</pre>
      cd z=polar(db(1),PI/k);
      for(int i=k;i<2*k;i++)rt[i]=i&1?rt[i/2]*z:rt[i/2];</pre>
    vi rev(n);
    for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;</pre>
    for (int i=1; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
    for (int k=1; k < n; k \neq 2) for (int i=0; i < n; i+2 \neq k) for (int j=0; j < k
         ;j++){
      cd z=rt[j+k]*a[i+j+k];
      a[i+j+k] = a[i+j]-z;
      a[i+j]+=z;
  template < class U>
  static db norm(const U &x) {
    return INT?round(x):x;
  static vt conv(const vt &a, const vt &b) {
    if(a.empty()||b.empty())return {};
    vt res(a.size()+b.size()-1);
    int L=32-__builtin_clz(res.size()), n=1<<L;</pre>
    vc in(n), out(n);
    copy(a.begin(),a.end(),in.begin());
    for(int i=0;i<b.size();i++)in[i].imag(b[i]);</pre>
    fft(in);
    for(auto &x:in)x*=x;
    for (int i=0; i<n; i++) out [i] = in [-i&(n-1)] - conj(in[i]);</pre>
    for(int i=0;i<res.size();i++)res[i]=norm(imag(out[i])/(4*n)</pre>
    return res;
  static vl convMod(const vl &a,const vl &b) {
    assert (mod>0);
    if(a.empty()||b.empty())return {};
    vl res(a.size()+b.size()-1);
```

NTT Manacher SuffixArray ZAlgo DVC SlopeTrick

for (int i=0; i<n; i++) out [-i&(n-1)]=in1[i]*in2[i]*inv;</pre>

ntt(in1),ntt(in2);

```
int L=32- builtin clz(res.size()),n=1<<L;</pre>
    11 cut=int(sgrt(mod));
    vc in1(n),in2(n),out1(n),out2(n);
    for(int i=0;i<a.size();i++)in1[i]=cd(ll(a[i])/cut,ll(a[i])%</pre>
         cut); // a1 + i * a2
    for(int i=0;i<b.size();i++)in2[i]=cd(ll(b[i])/cut,ll(b[i])%</pre>
         cut); // b1 + i * b2
    fft(in1).fft(in2):
    for (int i=0; i<n; i++) {</pre>
      int i=-i&(n-1);
      out1[j]=(in1[i]+conj(in1[j]))*in2[i]/(2.1*n); // f1 * (q1)
            + i * g2) = f1 * g1 + i f1 * g2
      out2[j]=(in1[i]-conj(in1[j]))*in2[i]/cd(0.1,2.1*n); // f2
             * (g1 + i * g2) = f2 * g1 + i f2 * g2
    fft (out1), fft (out2);
    for(int i=0;i<res.size();i++){</pre>
      11 x=round(real(out1[i])), y=round(imag(out1[i]))+round(
           real(out2[i])), z=round(imag(out2[i]));
      res[i]=((xmod*cut+y)mod*cut+z)mod; // a1*b1*cut^2
           + (a1 * b2 + a2 * b1) * cut + a2 * b2
    return res;
  vt operator()(const vt &a,const vt &b){
    return mod>0?conv(a,b):convMod(a,b);
};
template<>
struct FFT < db > {
  static const bool INT=false;
NTT.hpp
Description: Number theoretic transform
Time: \mathcal{O}(N \log N)
"../template/Header.hpp", "../modular-arithmetic/BinPow.hpp",
"../modular-arithmetic/MontgomeryModInt.hpp"
                                                        2b2392, 39 lines
template < class mint = mint >
struct NTT{
  using vm = vector<mint>;
  static constexpr mint root=mint::get_root();
    static assert(root!=0);
  static void ntt(vm &a) {
    int n=a.size(),L=31-__builtin_clz(n);
    vm rt(n);
    rt[1]=1;
    for(int k=2, s=2; k<n; k*=2, s++) {</pre>
     mint z[]={1,binpow(root,MOD>>s)};
      for(int i=k;i<2*k;i++)rt[i]=rt[i/2]*z[i&1];</pre>
    for(int i=1;i<n;i++)rev[i]=(rev[i/2]|(i&1)<<L)/2;</pre>
    for (int i=1; i < n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
    for (int k=1; k < n; k \ne 2) for (int i=0; i < n; i+2 \ne k) for (int j=0; j < k
      mint z=rt[j+k]*a[i+j+k];
      a[i+j+k]=a[i+j]-z;
      a[i+j]+=z;
  static vm conv(const vm &a,const vm &b) {
    if(a.empty()||b.empty())return {};
    int s=a.size()+b.size()-1, n=1<<(32-__builtin_clz(s));</pre>
   mint inv=mint(n).inv();
   vm in1(a), in2(b), out(n);
    in1.resize(n),in2.resize(n);
```

```
ntt (out):
    return vm(out.begin(),out.begin()+s);
  vm operator()(const vm &a,const vm &b){
    return conv(a,b);
};
Strings (9)
Manacher.hpp
Description: Manacher's Algorithm. pal[i] := the length of the longest
palindrome centered at i/2.
                                                         53856e, 15 lines
"../template/Header.hpp"
template<class STR>
vector<int> manacher(const STR &s) {
    int n=(int)s.size();
    if (n==0) return {};
    vector<int> pal(2*n-1);
    for (int p=0, 1=-1, r=-1; p<2*n-1; p++) {
        int i=(p+1)>>1, j=p>>1;
        int k=(i>=r?0:min(r-i,pal[2*(l+r)-p]));
        while (j+k+1 < n \& \& i-k-1 > = 0 \& \& s [j+k+1] == s [i-k-1]) k++;
        pal[p]=k;
        if(j+k>r)l=i-k,r=j+k;
    for(int i=0;i<2*n-1;i++)pal[i]=pal[i]<<1|(i&1^1);</pre>
    return pal;
SuffixArray.hpp
Description: Suffix Automaton.
"../data-structure/SparseTable.hpp", "../group/monoid/Min.hpp"
                                                         b9cfb1, 39 lines
template<class STR>
struct SuffixArray{
    int n:
    vector<int> sa,isa,lcp;
    SparseTable<MinMonoid<int>> st;
    SuffixArrav(){}
    SuffixArray(const STR &s) {init(s);}
    void init(const STR &s){
        n=(int)s.size();
        sa=isa=lcp=vector<int>(n+1);
        iota(sa.begin()+1, sa.end(), 0);
        sort(sa.begin()+1,sa.end(),[&](int i,int j){return s[i
              ]<s[j];});
        for (int i=1; i<=n; i++) {</pre>
             int x=sa[i-1],y=sa[i];
             isa[y]=i>1&&s[x]==s[y]?isa[x]:i;
        for(int len=1;len<=n;len<<=1) {</pre>
             vector<int> ps(sa),pi(isa),pos(n+1);
             iota(pos.begin(),pos.end(),0);
             for(auto i:ps) if((i-=len)>=0) sa[pos[isa[i]]++]=i;
             for (int i=1; i<=n; i++) {</pre>
                 int x=sa[i-1],y=sa[i];
                 isa[y]=pi[x]==pi[y]&&pi[x+len]==pi[y+len]?isa[x
        for (int i=0, k=0; i<n; i++) {</pre>
             for(int j=sa[isa[i]-1]; j+k<n&&s[j+k]==s[i+k]; k++);</pre>
             lcp[isa[i]]=k;
             if(k)k--;
```

```
st.init(lcp);
    int get_lcp(int i,int j){
        if (i== j) return n-i;
        auto [1,r]=minmax(isa[i],isa[j]);
        return st.query(1+1,r);
};
ZAlgo.hpp
Description: Z Algorithm. z[i] := the length of the longest common prefix
between s and s[i:].
"../template/Header.hpp"
template<class STR>
vector<int> z_algorithm(const STR &s) {
    int n=(int)s.size();
    vector<int> z(n);
    z[0]=n;
    for (int i=1, 1=0, r=1; i < n; i++) {</pre>
        if(i<r)z[i]=min(r-i,z[i-l]);
        while (i+z[i] <n&&s[z[i]] ==s[i+z[i]]) z[i]++;
        if(i+z[i]>r)l=i,r=i+z[i];
    return z;
Dynamic Programming (10)
DVC.hpp
Description: Optimize O(N^2K) to O(NK \log N)
"../template/Header.hpp"
                                                      aa5ddf. 19 lines
vector<vl> cst, dp;
11 cost(int 1, int r) {
  return cst[l][r];
void divide(int 1, int r, int opt_1, int opt_r, int c) {
  if(l > r) return ;
  int mid = (1 + r) / 2;
  pair<11, int> best = make_pair(INF, -1);
  for(int k=opt_l; k<=min(mid, opt_r); ++k) {</pre>
    best = min(best, make_pair(dp[c - 1][k] + cost(k + 1, mid),
  dp[c][mid] = best.first;
  divide(1, mid - 1, opt_1, best.second, c);
  divide(mid + 1, r, best.second, opt_r, c);
// for(int c=1; c \le K; ++c) divide(1, N, 1, N, c);
SlopeTrick.hpp
Description: Absolute Smth
"../template/Header.hpp"
                                                       f62f9a, 36 lines
11 extending_value;
struct slope_trick {
  multiset<11> ms_1, ms_r;
  11 min y = 011, 1z 1 = 011, 1z r = 011;
  bool extending = false;
  void add_line(ll v) {
    if(extending) {
```

lz_l -= extending_value;

lz r -= extending value;

10

```
extending = true;
    if(ms_1.empty() && ms_r.empty()) {
      ms_l.emplace(v);
      ms_r.emplace(v);
    else if(v <= *ms_l.rbegin() + lz_l) {</pre>
      min_y += (*ms_l.rbegin() + lz_l) - v;
      ms_r.emplace(*ms_l.rbegin() + lz_l - lz_r);
      ms_l.erase(--ms_l.end());
      ms_l.emplace(v - lz_l);
      ms_l.emplace(v - lz_l);
    else if(v \ge *ms_r.begin() + lz_r) {
      min_y += v - (*ms_r.begin() + lz_r);
      ms_l.emplace(*ms_r.begin() + lz_r - lz_l);
      ms_r.erase(ms_r.begin());
      ms_r.emplace(v - lz_r);
      ms_r.emplace(v - lz_r);
    else {
      ms_l.emplace(v - lz_l);
      ms_r.emplace(v - lz_r);
};
Convolutions (11)
And Convolution. hpp
Description: Bitwise AND Convolution. Superset Zeta Transform: A'[S] =
\sum_{T \supset S} A[T]. Superset Mobius Transform: A[T] = \sum_{S \supset T} (-1)^{|S-T|} A'[S].
Time: \mathcal{O}(N \log N).
"../template/Header.hpp"
template<class T>
void superset_zeta(vector<T> &a){
    int n=(int)a.size();
    assert (n==(n\&-n));
    for (int i=1; i<n; i<<=1) {</pre>
        for (int j=0; j<n; j++) {</pre>
             if (j&i) {
                 a[j^i] += a[j];
template<class T>
void superset_mobius(vector<T> &a){
    int n=(int)a.size();
    assert (n==(n\&-n));
    for(int i=n;i>>=1;) {
        for (int j=0; j<n; j++) {</pre>
             if(j&i){
                 a[j^i] -= a[j];
template < class T>
vector<T> and_convolution(vector<T> a, vector<T> b) {
    superset_zeta(a);
    superset_zeta(b);
    for(int i=0;i<(int)a.size();i++)a[i] *=b[i];</pre>
    superset_mobius(a);
    return a;
```

```
GCDConvolution.hpp
Description: GCD Convolution. Multiple Zeta Transform: A'[n] =
\sum_{n|m} A[m]. Multiple Mobius Transform: A[n] = \sum_{n|m} \mu(m/n) A'[m].
Time: \mathcal{O}(N \log \log N).
"../template/Header.hpp"
                                                           7f6c2d, 34 lines
template < class T>
void multiple_zeta(vector<T> &a){
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++) {
        if(!is_prime[p])continue;
         for (int i= (n-1)/p; i>=1; i--) {
             is_prime[i*p]=false;
             a[i] += a[i * p];
template < class T>
void multiple_mobius(vector<T> &a) {
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for(int p=2;p<n;p++) {</pre>
        if(!is_prime[p])continue;
         for (int i=1; i*p<n; i++) {</pre>
             is_prime[i*p]=false;
             a[i] -= a[i * p];
template<class T>
vector<T> gcd_convolution(vector<T> a, vector<T> b) {
    multiple_zeta(a);
    multiple zeta(b);
    for (int i=0; i < (int) a.size(); i++) a[i] *=b[i];</pre>
    multiple mobius(a):
    return a;
LCMConvolution.hpp
Description: LCM Convolution. Divisor Zeta Transform: A'[n] =
\sum_{d|n} A[d]. Divisor Mobius Transform: A[n] = \sum_{d|n} \mu(n/d)A'[d].
Time: \mathcal{O}(N \log \log N).
"../template/Header.hpp"
                                                           41fe9d, 34 lines
template<class T>
void divisor_zeta(vector<T> &a) {
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for (int p=2; p<n; p++) {</pre>
        if(!is_prime[p])continue;
        for(int i=1;i*p<n;i++) {</pre>
             is_prime[i*p]=false;
             a[i*p]+=a[i];
template<class T>
void divisor_mobius(vector<T> &a) {
    int n=(int)a.size();
    vector<bool> is_prime(n,true);
    for (int p=2; p<n; p++) {</pre>
        if(!is_prime[p])continue;
        for (int i= (n-1)/p; i>=1; i--) {
```

is_prime[i*p]=false;

```
a[i*p]-=a[i];
template<class T>
vector<T> lcm_convolution(vector<T> a, vector<T> b) {
     divisor_zeta(a);
     divisor_zeta(b);
     for (int i=0; i < (int) a.size(); i++) a[i] *=b[i];</pre>
     divisor_mobius(a);
     return a;
ORConvolution.hpp
Description: Bitwise OR Convolution. Subset Zeta Transform: A'[S] =
\sum_{T \subset S} A[T]. Subset Mobius Transform: A[T] = \sum_{S \subset T} (-1)^{|T-S|} A'[S].
Time: \mathcal{O}(N \log N).
"../template/Header.hpp'
                                                            c58b77, 34 lines
template<class T>
void subset_zeta(vector<T> &a) {
    int n=(int)a.size();
    assert (n==(n\&-n));
     for (int i=1; i<n; i<<=1) {</pre>
         for (int j=0; j<n; j++) {</pre>
              if(j&i){
                  a[j] += a[j^i];
template < class T>
void subset_mobius(vector<T> &a) {
     int n=(int)a.size();
     assert (n==(n\&-n));
     for(int i=n;i>>=1;) {
         for (int j=0; j<n; j++) {</pre>
              if(j&i){
                  a[j]-=a[j^i];
template < class T>
vector<T> or convolution(vector<T> a, vector<T> b) {
     subset_zeta(a);
     subset_zeta(b);
     for(int i=0;i<(int)a.size();i++)a[i]*=b[i];</pre>
     subset_mobius(a);
     return a;
XORConvolution.hpp
Description: Bitwise XOR Convolution. Fast Walsh-Hadamard Transform:
A'[S] = \sum_{T} (-1)^{|S\&T|} A[T].
Time: \mathcal{O}(N \log N).
"../template/Header.hpp"
                                                            05848d, 29 lines
template < class T>
void fwht(vector<T> &a) {
     int n=(int)a.size();
     assert (n==(n\&-n));
     for (int i=1; i<n; i<<=1) {</pre>
         for(int j=0; j<n; j++) {
              if(j&i){
                  T &u=a[j^i],&v=a[j];
```

```
tie (u, v) = make_pair(u+v, u-v);
template<class T>
vector<T> xor_convolution(vector<T> a, vector<T> b) {
    int n=(int)a.size();
    fwht(a);
    fwht(b);
    for(int i=0;i<n;i++)a[i]*=b[i];</pre>
    fwht(a);
    T \text{ div=}T(1)/T(n);
    if(div==T(0)){
        for(auto &x:a)x/=n;
    }else{
        for(auto &x:a) x*=div;
    return a;
11.1 Various
Gaussian Elimination. hpp
Description: Gaussian Elimination
"../template/Header.hpp"
                                                        e89ecb, 34 lines
struct Gauss {
  int n, sz;
  vector<ll> basis;
  Gauss(int n = 0) {
    init(n);
  void init(int _n) {
   n = _n, sz = 0;
   basis.assign(n, 0);
  void insert(ll x) {
    for (int i = n - 1; i >= 0; i--)
      if (x >> i & 1) {
        if (!basis[i]) {
          basis[i] = x;
          sz++;
          return;
        x ^= basis[i];
  11 \text{ getmax}(11 \text{ k} = 0)  {
    11 tot = 111 << sz, res = 0;
    for (int i = n - 1; i >= 0; i--)
      if (basis[i]) {
        tot >>= 1;
        if ((k >= tot && res >> i & 1) || (k < tot && res >> i
             & 1 ^ 1))
          res ^= basis[i];
        if (k >= tot)
          k -= tot;
    return res;
Binary Trie.hpp
Description: Binary Trie
"../template/Header.hpp"
                                                        525bf4, 59 lines
using node_t = array<int, 2>;
template<size_t S>
struct binary_trie {
```

```
vector<node_t> t = {node_t()};
 vector<int> cnt = {0};
 int cnt nodes = 0;
 void insert(int v) {
   int cur = 0;
   cnt[0]++;
   for(int i=S-1; i>=0; --i) {
     int b = (v & (1 << i)) ? 1: 0;</pre>
     if(!t[cur][b]) {
       t[cur][b] = ++cnt_nodes;
       t.emplace_back(node_t());
       cnt.emplace_back(0);
     cnt[t[cur][b]]++;
     cur = t[cur][b];
 void remove(int v) {
   int cur = 0;
   cnt[0]--;
   for(int i=S-1; i>=0; --i) {
     int b = (v & (1 << i)) ? 1: 0;</pre>
     cnt[t[cur][b]]--;
     cur = t[cur][b];
 int get_min(int v) {
   int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
     int b = (v \& (1 << i)) ? 1 : 0;
     if(t[cur][b] && cnt[t[cur][b]]) {
       cur = t[cur][b];
      else {
       res |= (1 << i);
       cur = t[cur][!b];
    return res;
 int get max(int v) {
   int cur = 0, res = 0;
    for(int i=(int) S-1; i>=0; --i) {
     int b = (v \& (1 << i)) ? 1 : 0;
     if(t[cur][!b] && cnt[t[cur][!b]]) {
       res |= (1 << i);
       cur = t[cur][!b];
      else {
       cur = t[cur][b];
   return res;
};
```

Competitive Programming Topics



topics.txt

159 lines

Recursion Divide and conquer Finding interesting points in N log N Algorithm analysis Master theorem Amortized time complexity Greedy algorithm Scheduling Max contiguous subvector sum Invariants Huffman encoding Graph theory Dynamic graphs (extra book-keeping) Breadth first search Depth first search * Normal trees / DFS trees Dijkstra's algorithm MST: Prim's algorithm Bellman-Ford Konig's theorem and vertex cover Min-cost max flow Lovasz toggle Matrix tree theorem Maximal matching, general graphs Hopcroft-Karp Hall's marriage theorem Graphical sequences Flovd-Warshall Euler cycles Flow networks * Augmenting paths * Edmonds-Karp Bipartite matching Min. path cover Topological sorting Strongly connected components Cut vertices, cut-edges and biconnected components Edge coloring * Trees Vertex coloring * Bipartite graphs (=> trees) * 3^n (special case of set cover) Diameter and centroid K'th shortest path Shortest cycle Dynamic programming Knapsack Coin change Longest common subsequence Longest increasing subsequence Number of paths in a dag Shortest path in a dag Dynprog over intervals Dynprog over subsets Dynprog over probabilities Dynprog over trees 3^n set cover Divide and conquer Knuth optimization Convex hull optimizations

RMQ (sparse table a.k.a 2^k-jumps)

Bitonic cycle

Log partitioning (loop over most restricted) Combinatorics Computation of binomial coefficients Pigeon-hole principle Inclusion/exclusion Catalan number Pick's theorem Number theory Integer parts Divisibility Euclidean algorithm Modular arithmetic * Modular multiplication * Modular inverses * Modular exponentiation by squaring Chinese remainder theorem Fermat's little theorem Euler's theorem Phi function Frobenius number Quadratic reciprocity Pollard-Rho Miller-Rabin Hensel lifting Vieta root jumping Game theory Combinatorial games Game trees Mini-max Nim Games on graphs Games on graphs with loops Grundy numbers Bipartite games without repetition General games without repetition Alpha-beta pruning Probability theory Optimization Binary search Ternary search Unimodality and convex functions Binary search on derivative Numerical methods Numeric integration Newton's method Root-finding with binary/ternary search Golden section search Matrices Gaussian elimination Exponentiation by squaring Sorting Radix sort Geometry Coordinates and vectors * Cross product * Scalar product Convex hull Polygon cut Closest pair Coordinate-compression Quadtrees KD-trees All segment-segment intersection Discretization (convert to events and sweep) Angle sweeping Line sweeping Discrete second derivatives Strings

Longest common substring Palindrome subsequences Knuth-Morris-Pratt Tries Rolling polynomial hashes Suffix array Suffix tree Aho-Corasick Manacher's algorithm Letter position lists Combinatorial search Meet in the middle Brute-force with pruning Best-first (A*) Bidirectional search Iterative deepening DFS / A* Data structures LCA (2^k-jumps in trees in general) Pull/push-technique on trees Heavy-light decomposition Centroid decomposition Lazy propagation Self-balancing trees Convex hull trick (wcipeg.com/wiki/Convex_hull_trick) Monotone queues / monotone stacks / sliding queues Sliding queue using 2 stacks Persistent segment tree

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