

# Advanced Epidemiological Analysis

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# Chapter 1

## Overview

This is the coursebook for the Colorado State University course ERHS 732, Advanced Epidemiological Analysis. This course provides the opportunity to implement theoretical expertise through designing and conducting advanced epidemiologic research analyses and to gain in-depth experience analyzing datasets from the environmental epidemiology literature.

This book is in development over the Fall 2021 semester.

### 1.1 License

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# Chapter 2

## Course information

This is the coursebook for the Colorado State University course ERHS 732, Advanced Epidemiological Analysis. This course provides the opportunity to implement theoretical expertise through designing and conducting advanced epidemiologic research analyses and to gain in-depth experience analyzing datasets from the environmental epidemiology literature. This course will complement the student's training in advanced epidemiological methods, leveraging regression approaches and statistical programming, providing the opportunity to implement their theoretical expertise through designing and conducting advanced epidemiologic research analyses. Although basic theoretical frameworks behind analysis and statistical modeling approaches will be introduced, this course will not go into depth into statistical and epidemiologic theory and students are expected to be familiar with general epidemiologic concepts such as confounding, selection bias etc. During the course, students will gain in-depth experience analyzing two datasets from the environmental epidemiology literature—(1) time series data with daily measures of weather, air pollution, and cardiorespiratory outcomes in London, England and (2) a dataset with measures from the Framingham Heart Study. Additional datasets and studies will be discussed and explored as a supplement.

This class will utilize a variety of instructional formats, including short lectures, readings, topic specific examples from the substantive literature, discussion and directed group work on in-course coding exercises putting lecture and discussion content into practice. A variety of teaching modalities will be used, including group discussions, student directed discussions, and in-class group exercises. It is expected that before coming to class, students will read the required papers for the week, as well as any associated code included in the papers' supplemental materials. Students should come to class prepared to do statistical programming (i.e., bring a laptop with statistical software, download any datasets needed for the week etc). Participation is based on in-class coding exercises based on each week's topic. If a student misses a class, they will be expected to complete

the in-course exercise outside of class to receive credit for participation in that exercise. Students will be required to do mid-term and final projects which will be presented in class and submitted as a written write-up describing the project.

## 2.1 Course learning objectives

The learning objectives for this proposed course complement core epidemiology and statistics courses required by the program and provide the opportunity for students to implement theoretical skills and knowledge gained in those courses in a more applied setting.

Upon successful completion of this course students will be able to:

1. List several possible statistical approaches to answering an epidemiological research questions. (*Knowledge*)
2. Choose among analytical approaches learned in previous courses to identify one that is reasonable for an epidemiological research question. (*Application*)
3. Design a plan for cleaning and analyzing data to answer an epidemiological research question, drawing on techniques learned in previous and concurrent courses. (*Synthesis*)
4. Justify the methods and code used to answer an epidemiological research question. (*Evaluation*)
5. Explain the advantages and limitations of a chosen methodological approach for evaluating epidemiological data. (*Evaluation*)
6. Apply advanced epidemiological methods to analyze example data, using a regression modeling framework. (*Application*)
7. Apply statistical programming techniques learned in previous courses to prepare epidemiological data for statistical analysis and to conduct the analysis. (*Application*)
8. Interpret the output from statistical analyses of data for an epidemiological research question. (*Evaluation*)
9. Defend conclusions from their analysis. (*Comprehension*)
10. Write a report describing the methods, results, and conclusions from an epidemiological analysis. (*Application*)
11. Construct a reproducible document with embedded code to clean and analyze data to answer an epidemiological research question. (*Application*)

## 2.2 Meeting time and place

The class will meet on Mondays, 2:00–3:40 PM on the Colorado State University campus in MRB 312.

## 2.3 Class Structure and Expectations

- **Homework/preparation:** It is expected that *before* coming to class, students will read the required papers for the week, as well as the online book sections assigned for the week. Reading assignments will be announced the week before each class session. Students should come to class prepared to do statistical programming (i.e., bring in a laptop with statistical software, download any datasets needed for the week).
- **In-class schedule:**
  - Topic overview: Each class will start with a brief overview of the week's topic. This will focus on the material covered in that week's assigned reading in the online book and papers.
  - Discussion of analysis and coding points: Students and faculty will be divided into small groups to discuss the assigned reading and think more deeply about the content. This is a time to bring up questions and relate the chapter concepts to other datasets and/or analysis methods you are familiar with.
  - Group work: In small groups, students will work on designing an epidemiological analysis for the week's topic and developing code to implement that analysis. This will follow the prompts given in the assigned reading from the online book for the week.
  - Wrap-up: We will reconvene as one group at the end to discuss topics that came up in small group work and to outline expectations for students before the next meeting.

## 2.4 Course grading

Assessment Components	Percentage of Grade
Midterm written report	30
Midterm presentation	15
Final written report	30
Final presentation	15
Participation in in-course exercises	10

- **Midterm report (written and presentation):** Students will work in groups to prepare an oral presentation and accompanying written report presenting an epidemiologic analysis using a time series dataset similar to the London dataset used for the first half of the course. The group may pick a research question based on the topics covered in the first half of the course. The presentation should be 15 minutes and should be structured like a conference presentation (Introduction, Methods, Results, and Discussion). The written report should be approximately six pages (single spaced) and should cover the same topics. It should include at least two (up to four) well-designed figures and / or tables. The written report should be created following reproducible research principles and using a

bibliography referencing system (e.g., BibTex if the student uses RMarkdown to write the report). The report should be written to the standard expected for a peer-reviewed publication in terms of clarity, grammar, spelling, and referencing. These Midterm reports will be due (and presented) the eighth week of class (seventh class session, since there will be no class for Labor Day).

- **Final report (written and presentation):** Each student will prepare an oral presentation and accompanying written report presenting an epidemiologic analysis using either a dataset similar to the Framingham dataset used for the second half of the course or their own dataset from their research. The student may pick a research question based on the topics covered in the second half of the course. The presentation should be 10 minutes and should be structured like a conference presentation (Introduction, Methods, Results, and Discussion). The written report should be approximately six pages (single spaced) and should cover the same topics. It should include at least two well-designed figures and / or tables. The written report should be created following reproducible research principles and using a bibliography referencing system (e.g., BibTex if the student uses RMarkdown to write the report). The report should be written to the standard expected for a peer-reviewed publication in terms of clarity, grammar, spelling, and referencing. The final presentations will be given during the assigned time period for finals for our course.
- **Participation:** Attendance is an essential part of participating in the class. We understand things come up, however it is expected you attend every class and come prepared. Further, it is expected that you will actively participate in discussions and group work during the class period.

## 2.5 Course Schedule

Class	Date	Study type	Topic	Book sections
1	August 30	Time series	Time series / case-crossover study designs	3.1–3.4
2	September 3	Time series	Time series / case-crossover study designs	3.5
3	September 13	Time series	Generalized linear models	4.1
4	September 20	Time series	Generalized linear models	TBD
5	September 27	Time series	Natural experiments	TBD
6	October 4	Time series	Risk assessment	TBD
7	October 11	Time series	Group midterm reports	None
8	October 18	Cohort	Longitudinal cohort study designs	TBD
9	October 25	Cohort	Longitudinal cohort study designs	TBD
10	November 1	Cohort	Inverse probability weighting, Propensity scores	TBD
11	November 8	Cohort	Mixed models	TBD
12	November 15	Cohort	Instrumental variables	TBD
13	November 29	Cohort	Counterfactuals / Causal inference	TBD
14	December 6	Cohort	Finals preparation	None
15	December 13	Cohort	Finals presentation	None

## 2.6 Textbooks and Course Materials

Readings for this course will focus on peer-reviewed literature that will be posted for the students in the class, as well as assigned reading from this online book.

Additional general references that will be useful to students throughout the semester include:

- Garrett Grolemund and Hadley Wickham, *R for Data Science*, O'Reilly, 2017. (Available for free online at <https://r4ds.had.co.nz/> and in print through most large book sellers.)
- Miguel A. Hernán and James M. Robins, *Causal Inference: What If*, Boca Raton: Chapman & Hall/CRC, 2020. (Available for free online at [https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/2021/01/ciwhatif\\_hernanrobins\\_31jan21.pdf](https://cdn1.sph.harvard.edu/wp-content/uploads/sites/1268/2021/01/ciwhatif_hernanrobins_31jan21.pdf) with a print version anticipated in 2021.)
- Francesca Dominici and Roger D. Peng, *Statistical Methods for Environmental Epidemiology with R*, Springer, 2008. (Available online through the CSU library or in print through Springer.)

## 2.7 Prerequisites and Preparation

This course assumes experience in epidemiology and some experience programming in statistical programming (e.g., R, SAS) and statistics. Students should have taken (1) Epidemiologic Methods (ERHS 532/PBHL 570), (2) Advanced Epidemiology (ERHS 640), (3) R Programming for Research (ERHS 535) or SAS and Epidemiologic Data Management (ERHS 534), and (4) Design and Data Analysis for Researchers I (STAT 511), or equivalent courses or experience, prior to taking this course. While previous SAS experience is acceptable for the course, example code will be in R, so you will find it helpful to review the basics of R prior to class.

If you would like to prepare for this course, the best way is by reviewing R programming, epidemiology, and regression modeling. In each chapter of the book, we provide the required and supplemental reading for the week. If you need a review in any of these topics, we recommend starting with the papers and book chapters listed as general reviews of these topics in the chapters' supplemental reading sections.

## 2.8 Academic Honesty

Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. For more on Colorado State University's policies on Academic Integrity / Misconduct.



# Chapter 3

## Time series / case-crossover studies

We'll start by exploring common characteristics in time series data for environmental epidemiology. In the first half of the class, we're focusing on a very specific type of study—one that leverages large-scale vital statistics data, collected at a regular time scale (e.g., daily), combined with large-scale measurements of a climate-related exposure, with the goal of estimating the typical relationship between the level of the exposure and risk of a health outcome. For example, we may have daily measurements of particulate matter pollution for a city, measured daily at a set of Environmental Protection Agency (EPA) monitors. We want to investigate how risk of cardiovascular mortality changes in the city from day to day in association with these pollution levels. If we have daily counts of the number of cardiovascular deaths in the city, we can create a statistical model that fits the exposure-response association between particulate matter concentration and daily risk of cardiovascular mortality. These statistical models—and the type of data used to fit them—will be the focus of the first part of this course.

### 3.1 Readings

The required readings for this chapter are:

- Bhaskaran et al. (2013) Provides an overview of time series regression in environmental epidemiology.
- Vicedo-Cabrera et al. (2019) Provides a tutorial of all the steps for a projecting of health impacts of temperature extremes under climate change. One of the steps is to fit the exposure-response association using present-day data (the section on “Estimation of Exposure-Response Associations” in the paper). In this chapter, we will go into details on that step, and

that section of the paper is the only required reading for this chapter. Later in the class, we'll look at other steps covered in this paper. Supplemental material for this paper is available to download by clicking <http://links.lww.com/EDE/B504>. You will need the data in this supplement for the exercises for class.

The following are supplemental readings (i.e., not required, but may be of interest) associated with the material in this chapter:

- Armstrong et al. (2012) Commentary that provides context on how epidemiological research on temperature and health can help inform climate change policy.
- Dominici and Peng (2008c) Overview of study designs for studying climate-related exposures (air pollution in this case) and human health. Chapter in a book that is available online through the CSU library.
- Armstrong (2006) Covers similar material as Bhaskaran et al. (2013), but with more focus on the statistical modeling framework
- Gasparini and Armstrong (2010) Describes some of the advances made to time series study designs and statistical analysis, specifically in the context of temperature
- Basu et al. (2005) Compares time series and case-crossover study designs in the context of exploring temperature and health. Includes a nice illustration of different referent periods, including time-stratified.
- Armstrong et al. (2014) This paper describes different data structures for case-crossover data, as well as how conditional Poisson regression can be used in some cases to fit a statistical model to these data. Supplemental material for this paper is available at <https://bmcmedresmethodol.biomedcentral.com/articles/10.1186/1471-2288-14-122#Sec13>.
- Imai et al. (2015) Typically, the time series study design covered in this chapter is used to study non-communicable health outcomes. This paper discusses opportunities and limitations in applying a similar framework for infectious disease.
- Dominici and Peng (2008b) Heavier on statistics. Describes some of the statistical challenges of working with time series data for air pollution epidemiology. Chapter in a book that is available online through the CSU library.
- Lu and Zeger (2007) Heavier on statistics. This paper shows how, under conditions often common for environmental epidemiology studies, case-crossover and time series methods are equivalent.
- Gasparini (2014) Heavier on statistics. This provides the statistical framework for the distributed lag model for environmental epidemiology time series studies.
- Dunn and Smyth (2018) Introduction to statistical models, moving into regression models and generalized linear models. Chapter in a book that is available online through the CSU library.
- James et al. (2013) General overview of linear regression, with an R coding “lab” at the end to provide coding examples. Covers model fit, continuous,

binary, and categorical covariates, and interaction terms. Chapter in a book that is available online through the CSU library.

## 3.2 Time series and case-crossover study designs

In the first half of this course, we'll take a deep look at how researchers can study how environmental exposures and health risk are linked using *time series studies*. Let's start by exploring the study design for this type of study, as well as a closely linked study design, that of *case-crossover studies*.

It's important to clarify the vocabulary we're using here. We'll use the terms *time series study* and *case-crossover study* to refer specifically to a type of study common for studying air pollution and other climate-related exposures. However, both terms have broader definitions, particularly in fields outside environmental epidemiology. For example, a *time series study* more generally refers to a study where data is available for the same unit (e.g., a city) for multiple time points, typically at regularly-spaced times (e.g., daily). A variety of statistical methods have been developed to apply to gain insight from this type of data, some of which are currently rarely used in the specific fields of air pollution and climate epidemiology that we'll explore here. For example, there are methods to address autocorrelation over time in measurements—that is, that measurements taken at closer time points are likely somewhat correlated—that we won't cover here and that you won't see applied often in environmental epidemiology studies, but that might be the focus of a "Time Series" course in a statistics or economics department.

In air pollution and climate epidemiology, time series studies typically begin with study data collected for an aggregated area (e.g., city, county, ZIP code) and with a daily resolution. These data are usually secondary data, originally collected by the government or other organizations through vital statistics or other medical records (for the health data) and networks of monitors for the exposure data. In the next section of this chapter, we'll explore common characteristics of these data. These data are used in a time series study to investigate how changes in the daily level of the exposure is associated with risk of a health outcome, focusing on the short-term period. For example, a study might investigate how risk of respiratory hospitalization in a city changes in relationship with the concentration of particulate matter during the week or two following exposure. The study period for these studies is often very long (often a decade or longer), and while single-community time series studies can be conducted, many time series studies for environmental epidemiology now include a large set of communities of national or international scope.

The study design essentially compares a community with itself at different time points—asking if health risk tends to be higher on days when exposure is higher. By comparing the community to itself, the design removes many challenges that would come up when comparing one community to another (e.g., is respiratory

hospitalization risk higher in city A than city B because particulate matter concentrations are typically higher in city A?). Communities differ in demographics and other factors that influence health risk, and it can be hard to properly control for these when exploring the role of environmental exposures. By comparison, demographics tend to change slowly over time (at least, compared to a daily scale) within a community.

One limitation, however, is that the study design is often best-suited to study acute effects, but more limited in studying chronic health effects. This is tied to the design and traditional ways of statistically modeling the resulting data. Since a community is compared with itself, the design removes challenges in comparing across communities, but it introduces new ones in comparing across time. Both environmental exposures and rates of health outcomes can have strong patterns over time, both across the year (e.g., mortality rates tend to follow a strong seasonal pattern, with higher rates in winter) and across longer periods (e.g., over the decade or longer of a study period). These patterns must be addressed through the statistical model fit to the time series data, and they make it hard to disentangle chronic effects of the exposure from unrelated temporal patterns in the exposure and outcome, and so most time series studies will focus on the short-term (or acute) association between exposure and outcome, typically looking at a period of at most about a month following exposure.

The term *case-crossover study* is a bit more specific than *time series study*, although there has been a strong movement in environmental epidemiology towards applying a specific version of the design, and so in this field the term often now implies this more specific version of the design. Broadly, a case-crossover study is one in which the conditions at the time of a health outcome are compared to conditions at other times that should otherwise (i.e., outside of the exposure of interest) be comparable. A case-crossover study could, for example, investigate the association between weather and car accidents by taking a set of car accidents and investigating how weather during the car accident compared to weather in the same location the week before.

One choice in a case-crossover study design is how to select the control time periods. Early studies tended to use a simple method for this—for example, taking the day before, or a day the week before, or some similar period somewhat close to the day of the outcome. As researchers applied the study design to large sets of data (e.g., all deaths in a community over multiple years), they noticed that some choices could create bias in estimates. As a result, most environmental epidemiology case-crossover studies now use a *time-stratified* approach to selecting control days. This selects a set of control days that typically include days both before and after the day of the health outcome, and are a defined set of days within a “stratum” that should be comparable in terms of temporal trends. For daily-resolved data, this stratum typically will include all the days within a month, year, and day of week. For example, one stratum of comparable days might be all the Mondays in January of 2010. These strata are created throughout the study period, and then days are only compared to

other days within their stratum (although, fortunately, there are ways you can apply a single statistical model to fit all the data for this approach rather than having to fit code stratum-by-stratum over many years).

When this is applied to data at an aggregated level (e.g., city, county, or ZIP code), it is in spirit very similar to a time series study design, in that you are comparing a community to itself at different time points. The main difference is that a time series study uses statistical modeling to control from potential confounding from temporal patterns, while a case-crossover study of this type instead controls for this potential confounding by only comparing days that should be “comparable” in terms of temporal trends, for example, comparing a day only to other days in the same month, year, and day of week. You will often hear that case-crossover studies therefore address potential confounding for temporal patterns “by design” rather than “statistically” (as in time series studies). However, in practice (and as we’ll explore in this class), in environmental epidemiology, case-crossover studies often are applied to aggregated community-level data, rather than individual-level data, with exposure assumed to be the same for everyone in the community on a given day. Under these assumptions, time series and case-crossover studies have been determined to be essentially equivalent (and, in fact, can use the same study data), only with slightly different terms used to control for temporal patterns in the statistical model fit to the data. Several interesting papers have been written to explore differences and similarities in these two study designs as applied in environmental epidemiology (Basu et al., 2005; Armstrong et al., 2014; Lu and Zeger, 2007).

These types of study designs in practice use similar datasets. In earlier presentations of the case-crossover design, these data would be set up a bit differently for statistical modeling. More recent work, however, has clarified how they can be modeled similarly to when using a time series study design, allowing the data to be set up in a similar way (Armstrong et al., 2014).

Several excellent commentaries or reviews are available that provide more details on these two study designs and how they have been used specifically investigate the relationship between climate-related exposures and health (Bhaskaran et al., 2013; Armstrong, 2006; Gasparrini and Armstrong, 2010). Further, these designs are just two tools in a wider collection of study designs that can be used to explore the health effects of climate-related exposures. Dominici and Peng (2008c) provides a nice overview of this broader set of designs.

### 3.3 Time series data

Let’s explore the type of dataset that can be used for these time series-style studies in environmental epidemiology. In the examples in this chapter, we’ll be using data that comes as part of the Supplemental Material in one of this chapter’s required readings, (Vicedo-Cabrera et al., 2019). Follow the link for the supplement for this article and then look for the file “lndn\_obs.csv”. This

is the file we'll use as the example data in this chapter.

These data are saved in a csv format (that is, a plain text file, with commas used as the delimiter), and so they can be read into R using the `read_csv` function from the `readr` package (part of the tidyverse). For example, you can use the following code to read in these data, assuming you have saved them in a “data” subdirectory of your current working directory:

```
library(tidyverse) # Loads all the tidyverse packages, including readr
obs <- read_csv("data/lndn_obs.csv")
obs

## # A tibble: 8,279 x 14
##   date      year month   day   doy dow     all all_0_64 all_65_74 all_75_84
##   <date>    <dbl> <dbl> <dbl> <dbl> <chr> <dbl>    <dbl>    <dbl>    <dbl>
## 1 1990-01-01 1990     1     1     1 Mon    220     38      38      82
## 2 1990-01-02 1990     1     2     2 Tue    257     50      67      87
## 3 1990-01-03 1990     1     3     3 Wed    245     39      59      86
## 4 1990-01-04 1990     1     4     4 Thu    226     41      45      77
## 5 1990-01-05 1990     1     5     5 Fri    236     45      54      85
## 6 1990-01-06 1990     1     6     6 Sat    235     48      48      84
## 7 1990-01-07 1990     1     7     7 Sun    231     38      49      96
## 8 1990-01-08 1990     1     8     8 Mon    235     46      57      76
## 9 1990-01-09 1990     1     9     9 Tue    250     48      54      96
## 10 1990-01-10 1990    1    10    10 Wed    214     44      46      62
## # ... with 8,269 more rows, and 4 more variables: all_85plus <dbl>,
## #   tmean <dbl>, tmin <dbl>, tmax <dbl>
```

This example dataset shows many characteristics that are common for datasets for time series studies in environmental epidemiology. Time series data are essentially a sequence of data points repeatedly taken over a certain time interval (e.g., day, week, month etc). General characteristics of time series data for environmental epidemiology studies are:

- Observations are given at an aggregated level. For example, instead of individual observations for each person in London, the `obs` data give counts of deaths throughout London. The level of aggregation is often determined by geopolitical boundaries, for example, counties or ZIP codes in the US.
- Observations are given at regularly spaced time steps over a period. In the `obs` dataset, the time interval is day. Typically, values will be provided continuously over that time period, with observations for each time interval. Occasionally, however, the time series data may only be available for particular seasons (e.g., only warm season dates for an ozone study), or there may be some missing data on either the exposure or health outcome over the course of the study period.
- Observations are available at the same time step (e.g., daily) for (1) the health outcome, (2) the environmental exposure of interest, and (3) potential time-varying confounders. In the `obs` dataset, the health outcome

is mortality (from all causes; sometimes, the health outcome will focus on a specific cause of mortality or other health outcomes such as hospitalizations or emergency room visits). Counts are given for everyone in the city for each day (`all` column), as well as for specific age categories (`all_0_64` for all deaths among those up to 64 years old, and so on). The exposure of interest in the `obs` dataset is temperature, and three metrics of this are included (`tmean`, `tmin`, and `tmax`). Day of the week is one time-varying factor that could be a confounder, or at least help explain variation in the outcome (mortality). This is included through the `dow` variable in the `obs` data. Sometimes, you will also see a marker for holidays included as a potential time-varying confounder, or other exposure variables (temperature is a potential confounder, for example, when investigating the relationship between air pollution and mortality risk).

- Multiple metrics of an exposure and / or multiple health outcome counts may be included for each time step. In the `obs` example, three metrics of temperature are included (minimum daily temperature, maximum daily temperature, and mean daily temperature). Several counts of mortality are included, providing information for specific age categories in the population. The different metrics of exposure will typically be fit in separate models, either as a sensitivity analysis or to explore how exposure measurement affects epidemiological results. If different health outcome counts are available, these can be modeled in separate statistical models to determine an exposure-response function for each outcome.

## 3.4 Exploratory data analysis

When working with time series data, it is helpful to start with some exploratory data analysis. This type of time series data will often be secondary data—it is data that was previously collected, as you are re-using it. Exploratory data analysis is particularly important with secondary data like this. For primary data that you collected yourself, following protocols that you designed yourself, you will often be very familiar with the structure of the data and any quirks in it by the time you are ready to fit a statistical model. With secondary data, however, you will typically start with much less familiarity about the data, how it was collected, and any potential issues with it, like missing data and outliers.

Exploratory data analysis can help you become familiar with your data. You can use summaries and plots to explore the parameters of the data, and also to identify trends and patterns that may be useful in designing an appropriate statistical model. For example, you can explore how values of the health outcome are distributed, which can help you determine what type of regression model would be appropriate, and to see if there are potential confounders that have regular relationships with both the health outcome and the exposure of interest. You can see how many observations have missing data for the outcome, the exposure, or confounders of interest, and you can see if there are any measurements

that look unusual. This can help in identifying quirks in how the data were recorded—for example, in some cases ground-based weather monitors use -99 or -999 to represent missing values, definitely something you want to catch and clean-up in your data (replacing with R’s `NA` for missing values) before fitting a statistical model!

The following applied exercise will take you through some of the questions you might want to answer through this type of exploratory analysis. In general, the `tidyverse` suite of R packages has loads of tools for exploring and visualizing data in R. The `lubridate` package from the `tidyverse`, for example, is an excellent tool for working with date-time data in R, and time series data will typically have at least one column with the timestamp of the observation (e.g., the date for daily data). You may find it worthwhile to explore this package some more. There is a helpful chapter in Wickham and Grolemund (2016), <https://r4ds.had.co.nz/dates-and-times.html>, as well as a cheatsheet at [https://evoldyn.gitlab.io/evomics-2018/ref-sheets/R\\_lubridate.pdf](https://evoldyn.gitlab.io/evomics-2018/ref-sheets/R_lubridate.pdf). For visualizations, if you are still learning techniques in R, two books you may find useful are Healy (2018) (available online at <https://socviz.co/>) and Chang (2018) (available online at <http://www.cookbook-r.com/Graphs/>).

#### *Applied: Exploring time series data*

Read the example time series data into R and explore it to answer the following questions:

1. What is the study period for the example `obs` dataset? (i.e., what dates / years are covered by the time series data?)
2. Are there any missing dates (i.e., dates with nothing recorded) within this time period? Are there any recorded dates where health outcome measurements are missing? Any where exposure measurements are missing?
3. Are there seasonal trends in the exposure? In the outcome?
4. Are there long-term trends in the exposure? In the outcome?
5. Is the outcome associated with day of week? Is the exposure associated with day of week?

Based on your exploratory analysis in this section, talk about the potential for confounding when these data are analyzed to estimate the association between daily temperature and city-wide mortality. Is confounding by seasonal trends a concern? How about confounding by long-term trends in exposure and mortality? How about confounding by day of week?

#### *Applied exercise: Example code*

1. **What is the study period for the example `obs` dataset? (i.e., what dates / years are covered by the time series data?)**

In the `obs` dataset, the date of each observation is included in a column called `date`. The data type of this column is “Date”—you can check this by using the `class` function from base R:

```
class(obs$date)
```

```
## [1] "Date"
```

Since this column has a “Date” data type, you can run some mathematical function calls on it. For example, you can use the `min` function from base R to get the earliest date in the dataset and the `max` function to get the latest.

```
min(obs$date)
```

```
## [1] "1990-01-01"
```

```
max(obs$date)
```

```
## [1] "2012-08-31"
```

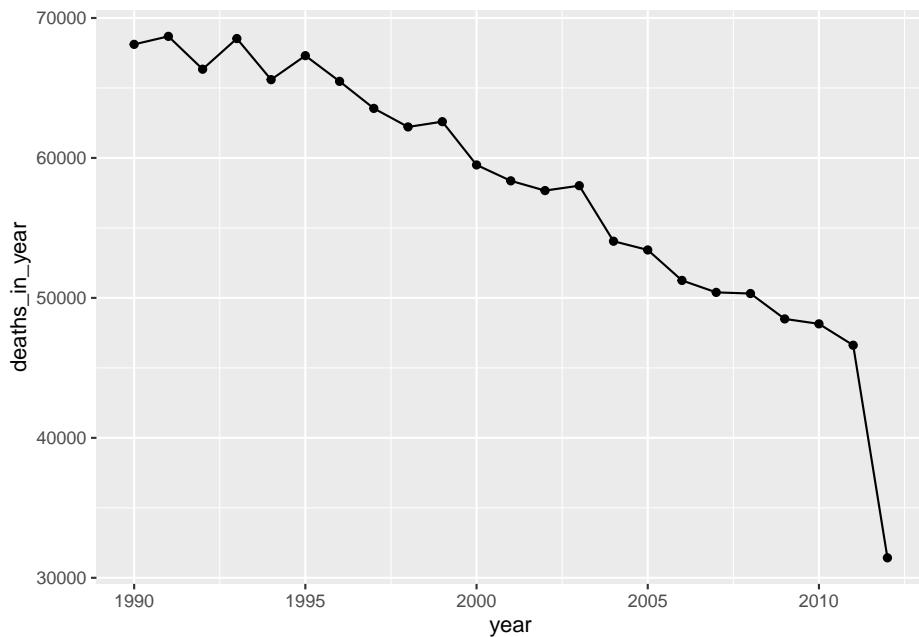
You can also run the `range` function to get both the earliest and latest dates with a single call:

```
range(obs$date)
```

```
## [1] "1990-01-01" "2012-08-31"
```

This provides the range of the study period for these data. One interesting point is that it’s not a round set of years—instead, the data ends during the summer of the last study year. This doesn’t present a big problem, but is certainly something to keep in mind if you’re trying to calculate yearly averages of any values for the dataset. If you’re getting the average of something that varies by season (e.g., temperature), it could be slightly weighted by the months that are included versus excluded in the partial final year of the dataset. Similarly, if you group by year and then count totals by year, the number will be smaller for the last year, since only part of the year’s included. For example, if you wanted to count the total deaths in each year of the study period, it will look like they go down a lot the last year, when really it’s only because only about half of the last year is included in the study period:

```
obs %>%
  group_by(year) %>%
  summarize(deaths_in_year = sum(all)) %>%
  ggplot(aes(x = year, y = deaths_in_year)) +
  geom_line() +
  geom_point()
```



2. Are there any missing dates within this time period? Are there any recorded dates where health outcome measurements are missing? Any where exposure measurements are missing?

There are a few things you should check to answer this question. First (and easiest), you can check to see if there are any NA values within any of the observations in the dataset. This helps answer the second and third parts of the question. The `summary` function will provide a summary of the values in each column of the dataset, including the count of missing values (NAs) if there are any:

```
summary(obs)
```

```
##      date          year        month       day
## Min. :1990-01-01  Min. :1990  Min. : 1.000  Min. : 1.00
## 1st Qu.:1995-09-01  1st Qu.:1995  1st Qu.: 3.000  1st Qu.: 8.00
## Median :2001-05-02  Median :2001  Median : 6.000  Median :16.00
## Mean   :2001-05-02  Mean   :2001  Mean   : 6.464  Mean   :15.73
## 3rd Qu.:2006-12-31  3rd Qu.:2006  3rd Qu.: 9.000  3rd Qu.:23.00
## Max.  :2012-08-31  Max.  :2012  Max.  :12.000  Max.  :31.00
##      doy          dow        all      all_0_64
## Min. : 1.0  Length:8279    Min. : 81.0  Min. : 9.0
## 1st Qu.: 90.5 Class :character  1st Qu.:138.0  1st Qu.:27.0
## Median :180.0 Mode :character  Median :157.0  Median :32.0
## Mean   :181.3                    Mean   :160.2  Mean   :32.4
## 3rd Qu.:272.0                    3rd Qu.:178.0 3rd Qu.:37.0
```

```

##   Max.    :366.0                  Max.    :363.0    Max.    :64.0
##   all_65_74      all_75_84      all_85plus     tmean
##   Min.    : 6.00      Min.    :17.00      Min.    :17.00    Min.    :-5.503
##   1st Qu.:23.00      1st Qu.:41.00      1st Qu.:39.00    1st Qu.: 7.470
##   Median  :29.00      Median :49.00      Median :45.00    Median :11.465
##   Mean    :30.45      Mean   :50.65      Mean   :46.68    Mean   :11.614
##   3rd Qu.:37.00      3rd Qu.:58.00      3rd Qu.:53.00    3rd Qu.:15.931
##   Max.    :70.00      Max.    :138.00     Max.    :128.00    Max.    :29.143
##   tmin        tmax
##   Min.    :-8.940     Min.    :-3.785
##   1st Qu.: 3.674     1st Qu.:10.300
##   Median  : 7.638     Median :14.782
##   Mean    : 7.468     Mean   :15.058
##   3rd Qu.:11.438     3rd Qu.:19.830
##   Max.    :20.438     Max.    :37.087

```

Based on this analysis, all observations are complete for all dates included in the dataset. There are no listings for NAs for any of the columns, and this indicates no missing values in the dates for which there's a row in the data.

However, this does not guarantee that every date between the start date and end date of the study period are included in the recorded data. Sometimes, some dates might not get recorded at all in the dataset, and the `summary` function won't help you determine when this is the case. One common example in environmental epidemiology is with ozone pollution data. These are sometimes only measured in the warm season, and so may be shared in a dataset with all dates outside of the warm season excluded.

There are a few alternative explorations you can do to check this. Perhaps the easiest is to check the number of days between the start and end date of the study period, and then see if the number of observations in the dataset is the same:

```

# Calculate number of days in study period
obs %>%
  pull(date) %>%
  range() %>%
  diff()           # Using piping (%>%) throughout to keep code clear
                  # Extract the `date` column as a vector
                  # Take the range of dates (earliest and latest)
                  # Calculate time difference from start to finish of study

## Time difference of 8278 days
# Get number of observations in dataset---should be 1 more than time difference
obs %>%
  nrow()

## [1] 8279

```

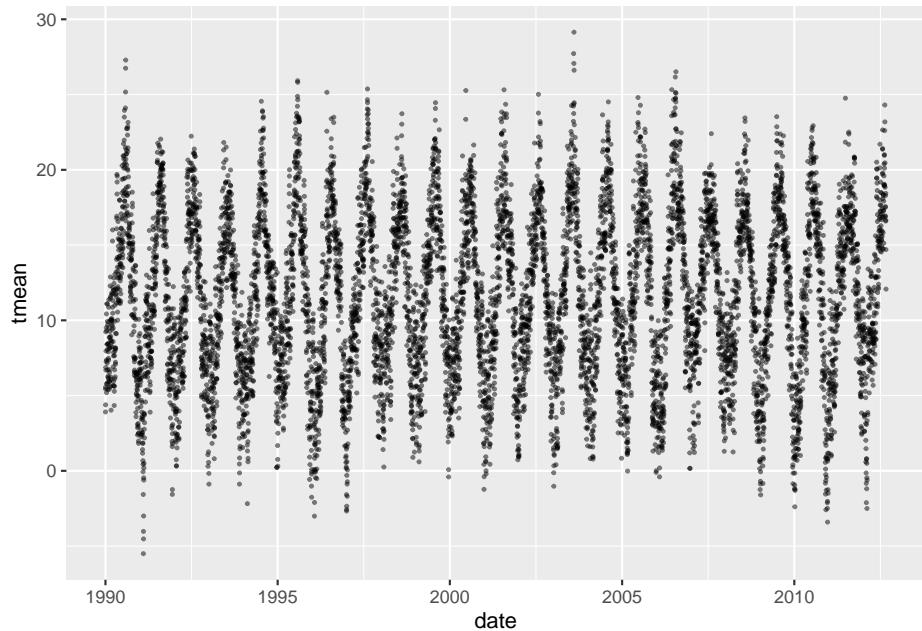
This indicates that there is an observation for every date over the study period, since the number of observations should be one more than the time difference.

In the next question, we'll be plotting observations by time, and typically this will also help you see if there are large chunks of missing dates in the data.

### 3. Are there seasonal trends in the exposure? In the outcome?

You can use a simple plot to visualize patterns over time in both the exposure and the outcome. For example, the following code plots a dot for each daily temperature observation over the study period. The points are set to a smaller size (`size = 0.5`) and plotted with some transparency (`alpha = 0.5`) since there are so many observations.

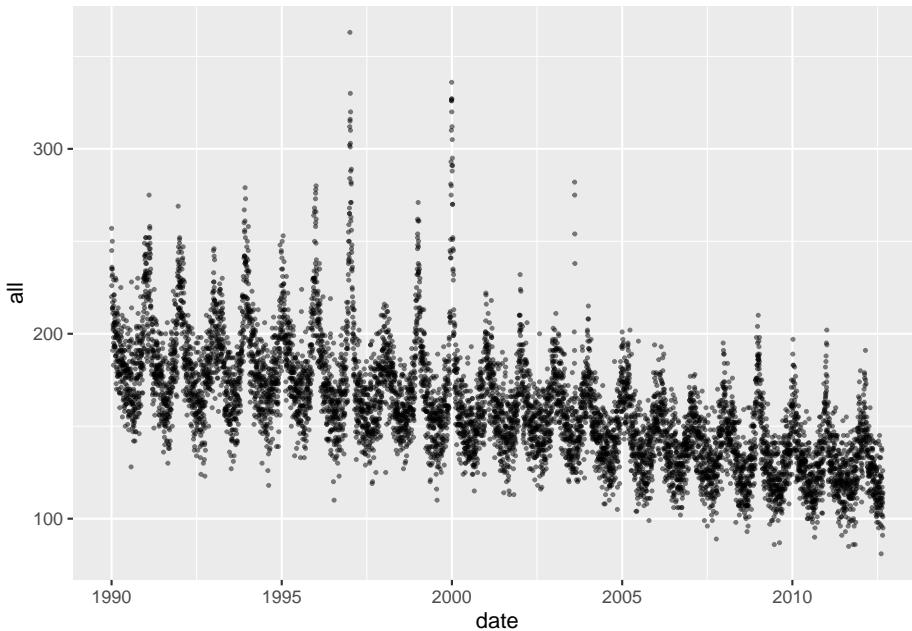
```
ggplot(obs, aes(x = date, y = tmean)) +
  geom_point(alpha = 0.5, size = 0.5)
```



There is (unsurprisingly) clear evidence here of a strong seasonal trend in mean temperature, with values typically lowest in the winter and highest in the summer.

You can plot the outcome variable in the same way:

```
ggplot(obs, aes(x = date, y = all)) +
  geom_point(alpha = 0.5, size = 0.5)
```

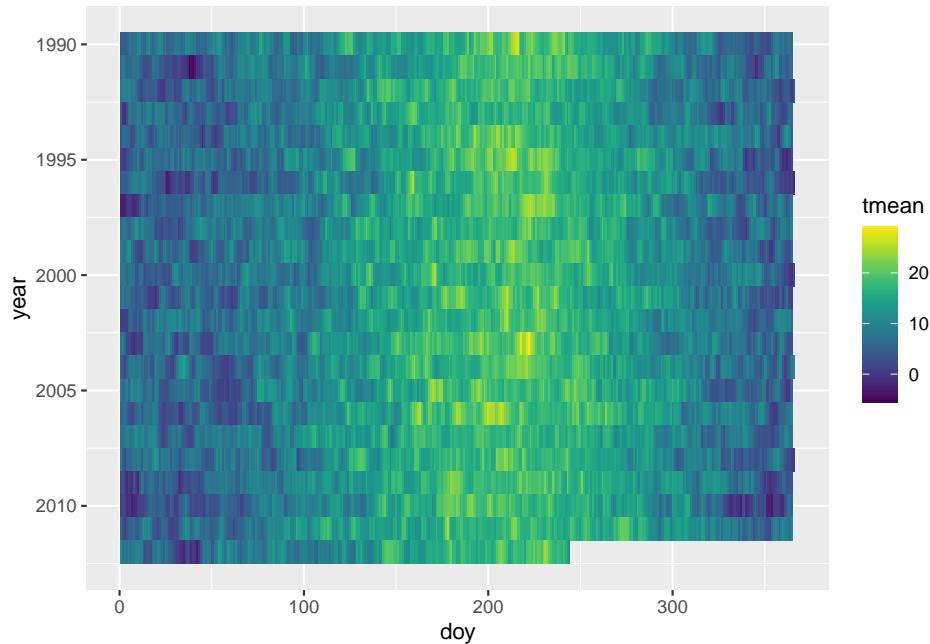


Again, there are seasonal trends, although in this case they are inverted. Mortality tends to be highest in the winter and lowest in the summer. Further, the seasonal pattern is not equally strong in all years—some years it has a much higher winter peak, probably in conjunction with severe influenza seasons.

Another way to look for seasonal trends is with a heatmap-style visualization, with day of year along the x-axis and year along the y-axis. This allows you to see patterns that repeat around the same time of the year each year (and also unusual deviations from normal seasonal patterns).

For example, here's a plot showing temperature in each year, where the observations are aligned on the x-axis by time in year. We're using the `doy`—which stands for “day of year” (i.e., Jan 1 = 1; Jan 2 = 2; ... Dec 31 = 365 as long as it's not a leap year) as the measure of time in the year. We've reversed the y-axis so that the earliest years in the study period start at the top of the visual, then later study years come later—this is a personal style, and it would be no problem to leave the y-axis as-is. We've used the `viridis` color scale for the fill, since that has a number of features that make it preferable to the default R color scale, including that it is perceptible for most types of color blindness and be printed out in grayscale and still be correctly interpreted.

```
library(viridis)
ggplot(obs, aes(x = doy, y = year, fill = tmean)) +
  geom_tile() +
  scale_y_reverse() +
  scale_fill_viridis()
```



From this visualization, you can see that temperatures tend to be higher in the summer months and lower in the winter months. “Spells” of extreme heat or cold are visible—where extreme temperatures tend to persist over a period, rather than randomly fluctuating within a season. You can also see unusual events, like the extreme heat wave in the summer of 2003, indicated with the brightest yellow in the plot.

We created the same style of plot for the health outcome. In this case, we focused on mortality among the oldest age group, as temperature sensitivity tends to increase with age, so this might be where the strongest patterns are evident.

```
ggplot(obs, aes(x = doy, y = year, fill = all_85plus)) +
  geom_tile() +
  scale_y_reverse() +
  scale_fill_viridis()
```

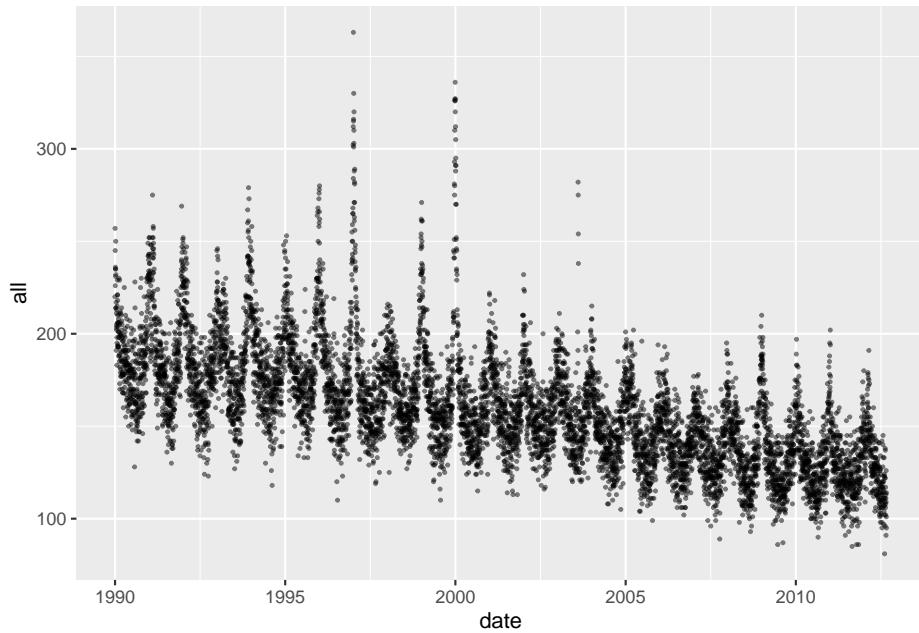


For mortality, there tends to be an increase in the winter compared to the summer. Some winters have stretches with particularly high mortality—these are likely a result of seasons with strong influenza outbreaks. You can also see on this plot the impact of the 2003 heat wave on mortality among this oldest age group—an unusual spot of light green in the summer.

#### 4. Are there long-term trends in the exposure? In the outcome?

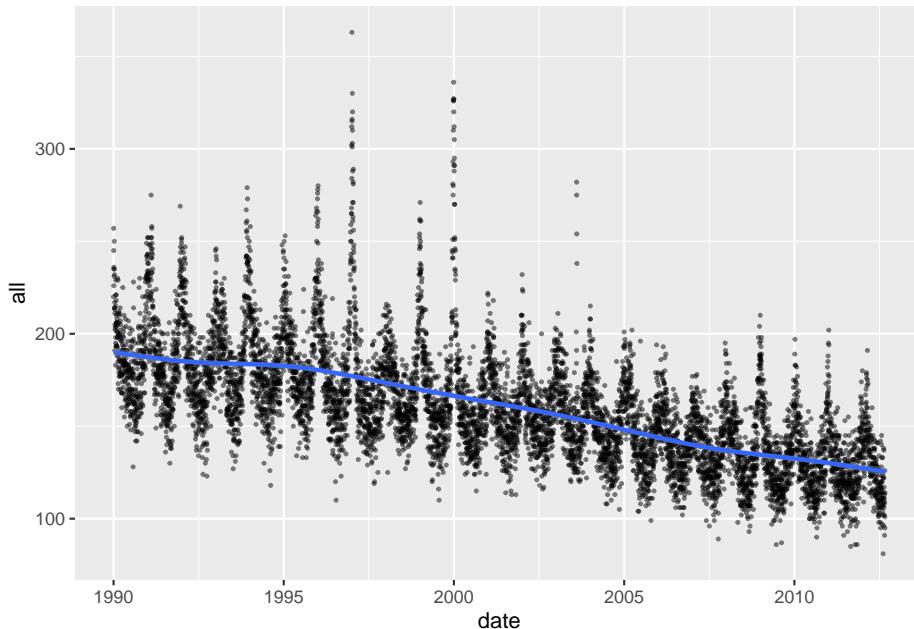
Some of the plots we created in the last section help in exploring this question. For example, the following plot shows a clear pattern of decreasing daily mortality counts, on average, over the course of the study period:

```
ggplot(obs, aes(x = date, y = all)) +
  geom_point(alpha = 0.5, size = 0.5)
```



It can be helpful to add a smooth line to help detect these longer-term patterns, which you can do with `geom_smooth`:

```
ggplot(obs, aes(x = date, y = all)) +  
  geom_point(alpha = 0.5, size = 0.5) +  
  geom_smooth()
```



You could also take the median mortality count across each year in the study period, although you should take out any years without a full year's worth of data before you do this, since there are seasonal trends in the outcome:

```
obs %>%
  group_by(year) %>%
  filter(year != 2012) %>% # Take out the last year
  summarize(median_mort = median(all)) %>%
  ggplot(aes(x = year, y = median_mort)) +
  geom_line()
```



Again, we see a clear pattern of decreasing mortality rates in this city over time. This means we need to think carefully about long-term time patterns as a potential confounder. It will be particularly important to think about this if the exposure also has a strong pattern over time. For example, air pollution regulations have meant that, in many cities, there may be long-term decreases in pollution concentrations over a study period.

##### 5. Is the outcome associated with day of week? Is the exposure associated with day of week?

The data already has day of week as a column in the data (`dow`). However, this is in a character data type, so it doesn't have the order of weekdays encoded (e.g., Monday comes before Tuesday). This makes it hard to look for patterns related to things like weekend / weekday.

```
class(obs$dow)
```

```
## [1] "character"
```

We could convert this to a factor and encode the weekday order when we do it, but it's even easier to just recreate the column from the `date` column. We used the `wday` function from the `lubridate` package to do this—it extracts weekday as a factor, with the order of weekdays encoded (using a special “ordered” factor type):

```
library(lubridate)
obs <- obs %>%
  mutate(dow = wday(date, label = TRUE))
```

```
class(obs$dow)

## [1] "ordered" "factor"

levels(obs$dow)

## [1] "Sun" "Mon" "Tue" "Wed" "Thu" "Fri" "Sat"
```

We looked at the mean, median, and 25th and 75th quantiles of the mortality counts by day of week:

```
obs %>%
  group_by(dow) %>%
  summarize(mean(all),
            median(all),
            quantile(all, 0.25),
            quantile(all, 0.75))

## # A tibble: 7 x 5
##   dow    `mean(all)` `median(all)` `quantile(all, 0.25)` `quantile(all, 0.75)`
## * <ord>     <dbl>        <dbl>             <dbl>             <dbl>
## 1 Sun      156.         154              136              173
## 2 Mon      161.         159              138              179
## 3 Tue      161.         158              139              179
## 4 Wed      160.         157              138.             179
## 5 Thu      161.         158              139              179
## 6 Fri      162.         159              141              179
## 7 Sat      159.         156              137              178
```

Mortality tends to be a bit higher on weekdays than weekends, but it's not a dramatic difference.

We did the same check for temperature:

```
obs %>%
  group_by(dow) %>%
  summarize(mean(tmean),
            median(tmean),
            quantile(tmean, 0.25),
            quantile(tmean, 0.75))

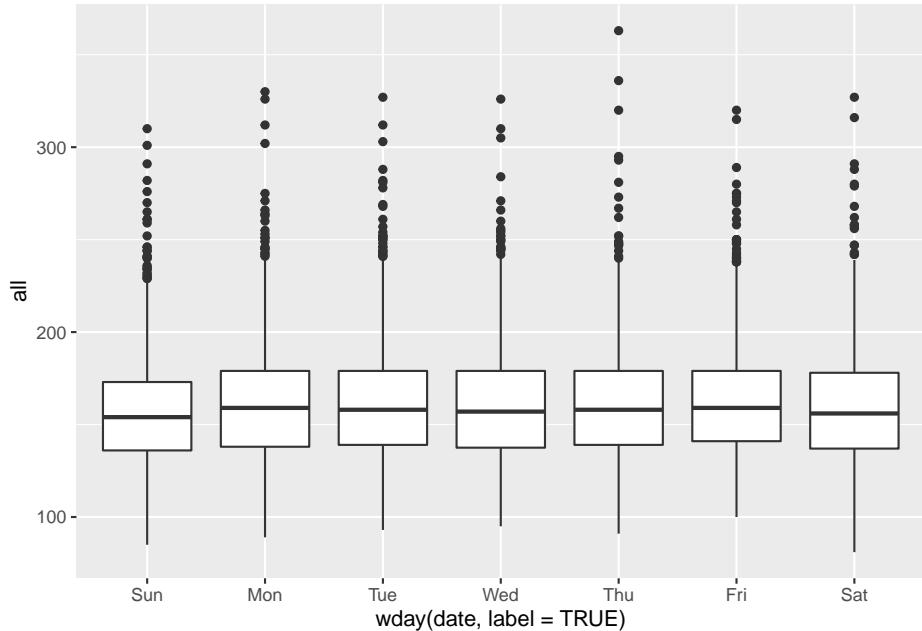
## # A tibble: 7 x 5
##   dow    `mean(tmean)` `median(tmean)` `quantile(tmean, 0.25)` `quantile(tmean, 0.75)`
## * <ord>     <dbl>        <dbl>             <dbl>             <dbl>
## 1 Sun      11.6         11.3             7.48             15.9
## 2 Mon      11.6         11.4             7.33             15.8
## 3 Tue      11.5         11.4             7.48             15.9
## 4 Wed      11.7         11.7             7.64             16.0
## 5 Thu      11.6         11.5             7.57             16.0
```

```
## 6 Fri          11.6          11.6        7.41      15.8
## 7 Sat          11.6          11.5        7.53      15.9
```

In this case, there does not seem to be much of a pattern by weekday.

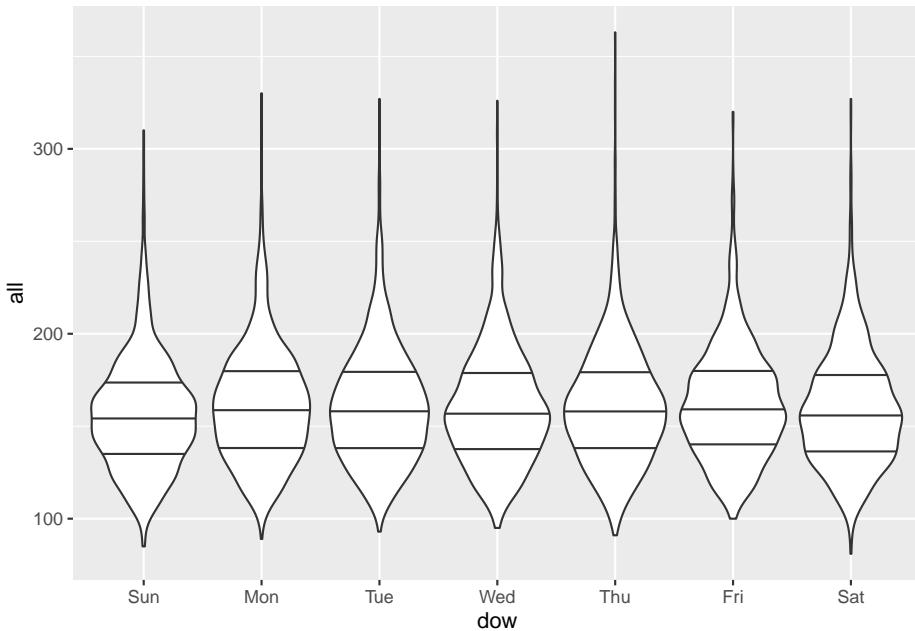
You can also visualize the association using boxplots:

```
ggplot(obs, aes(x = wday(date, label = TRUE), y = all)) +
  geom_boxplot()
```



You can also try violin plots—these show the full distribution better than boxplots, which only show quantiles.

```
ggplot(obs, aes(x = dow, y = all)) +
  geom_violin(draw_quantiles = c(0.25, 0.5, 0.75))
```



All these reinforce that there are some small differences in weekend versus weekday patterns for mortality. There isn't much pattern by weekday with temperature, so in this case weekday is unlikely to be a confounder (the same is not true with air pollution, which often varies based on commuting patterns and so can have stronger weekend/weekday differences). However, since it does help some in explaining variation in the health outcome, it might be worth including in our models anyway, to help reduce random noise.

Exploratory data analysis is an excellent tool for exploring your data before you begin fitting a statistical model, and you should get in the habit of using it regularly in your research. Dominici and Peng (2008a) provides another walk-through of exploring this type of data, including some more advanced tools for exploring autocorrelation and time patterns.

## 3.5 Statistical modeling for a time series study

Now that we've explored the data typical of a time series study in climate epidemiology, we'll look at how we can fit a statistical model to those data to gain insight into the relationship between the exposure and acute health effects. Very broadly, we'll be using a statistical model to answer the question: How does the relative risk of a health outcome change as the level of the exposure changes, after controlling for potential confounders?

In the rest of this chapter and the next chapter, we'll move step-by-step to build up to the statistical models that are now typically used in these studies. Along the way, we'll discuss key components and choices in this modeling process. The

statistical modeling is based heavily on regression modeling, and specifically generalized linear regression. To help you get the most of this section, you may find it helpful to review regression modeling and generalized linear models. Some resources for that include Dunn and Smyth (2018) and James et al. (2013).

One of the readings for this week, Vicedo-Cabrera et al. (2019), includes a section on fitting exposure-response functions to describe the association between daily mean temperature and mortality risk. This article includes example code in its supplemental material, with code for fitting the model to these time series data in the file named “01EstimationERAssociation.r”. Please download that file and take a look at the code.

The model in the code may at first seem complex, but it is made up of a number of fairly straightforward pieces (although some may initially seem complex):

- The model framework is a *generalized linear model (GLM)*
- This GLM is fit assuming an *error distribution* and a *link function* appropriate for count data
- The GLM is fit assuming an *error distribution* that is also appropriate for data that may be *overdispersed*
- The model includes control for day of the week by including a *categorical variable*
- The model includes control for long-term and seasonal trends by including a *spline* (in this case, a *natural cubic spline*) for the day in the study
- The model fits a flexible, non-linear association between temperature and mortality risk, also using a spline
- The model fits a flexible non-linear association between temperature on a series of preceding days and current day and mortality risk on the current day using a *distributed lag approach*
- The model jointly describes both of the two previous non-linear associations by fitting these two elements through one construct in the GLM, a *cross-basis term*

In this section and the next chapter, we will work through the elements, building up the code to get to the full model that is fit in Vicedo-Cabrera et al. (2019).

#### *Fitting a GLM to time series data*

The generalized linear model (GLM) framework unites a number of types of regression models you may have previously worked with. One basic regression model that can be fit within this framework is a linear regression model. However, the framework also allows you to also fit, among others, logistic regression models (useful when the outcome variable can only take one of two values, e.g., success / failure or alive / dead) and Poisson regression models (useful when the outcome variable is a count or rate). This generalized framework brings some unity to these different types of regression models. From a practical standpoint, it has allowed software developers to easily provide a common interface to fit these types of models. In R, the common function call to fit GLMs is `glm`.

Within the GLM framework, the elements that separate different regression models include the link function and the error distribution. The error distribution encodes the assumption you are enforcing about how the errors after fitting the model are distributed. If the outcome data are normally distributed (a.k.a., follow a Gaussian distribution), after accounting for variance explained in the outcome by any of the model covariates, then a linear regression model may be appropriate. For count data—like numbers of deaths a day—this is unlikely, unless the average daily mortality count is very high (count data tend to come closer to a normal distribution the further their average gets from 0). For binary data—like whether each person in a study population died on a given day or not—normally distributed errors are also unlikely. Instead, in these two cases, it is typically more appropriate to fit GLMs with Poisson and binomial “families”, respectively, where the family designation includes an appropriate specification for the variance when fitting the model based on these outcome types.

The other element that distinguishes different types of regression within the GLM framework is the link function. The link function applies a transformation on the combination of independent variables in the regression equation when fitting the model. With normally distributed data, an *identity link* is often appropriate—with this link, the combination of independent variables remain unchanged (i.e., keep their initial “identity”). With count data, a *log link* is often more appropriate, while with binomial data, a *logit link* is often used.

Finally, data will often not perfectly adhere to assumptions. For example, the Poisson family of GLMs assumes that variance follows a Poisson distribution (The probability mass function for Poisson distribution  $X \sim \text{Poisson}(\mu)$  is denoted by  $f(k; \mu) = \Pr[X = k] = \frac{\mu^k e^{-\mu}}{k!}$ , where  $k$  is the number of occurrences, and  $\mu$  is equal to the expected number of cases). With this distribution, the variance is equal to the mean ( $\mu = E(X) = \text{Var}(X)$ ). With real-life data, this assumption is often not valid, and in many cases the variance in real life count data is larger than the mean. This can be accounted for when fitting a GLM by setting an error distribution that does not require the variance to equal the mean—instead, both a mean value and something like a variance are estimated from the data, assuming an overdispersion parameter  $\phi$  so that  $\text{Var}(X) = \phi E(X)$ . In environmental epidemiology, time series are often fit to allow for this overdispersion. This is because if the data are overdispersed but the model does not account for this, the standard errors on the estimates of the model parameters may be artificially small. If the data are not overdispersed ( $\phi = 1$ ), the model will identify this when being fit to the data, so it is typically better to prefer to allow for overdispersion in the model (if the size of the data were small, you may want to be parsimonious and avoid unneeded complexity in the model, but this is typically not the case with time series data).

In the next section, you will work through the steps of developing a GLM to fit the example dataset `obs`. For now, you will only fit a linear association between mean daily temperature and mortality risk, eventually including control for day

of week. In later work, especially the next chapter, we will build up other components of the model, including control for the potential confounders of long-term and seasonal patterns, as well as advancing the model to fit non-linear associations, distributed by time, through splines, a distributed lag approach, and a cross-basis term.

*Applied: Fitting a GLM to time series data*

In R, the function call used to fit GLMs is `glm`. Most of you have likely covered GLMs, and ideally this function call, in previous courses. If you are unfamiliar with its basic use, you will want to refresh yourself on this topic—you can use some of the resources noted earlier in this section and in the chapter’s “Supplemental Readings” to do so.

1. Fit a GLM to estimate the association between mean daily temperature (as the independent variable) and daily mortality count (as the dependent variable), first fitting a linear regression. (Since the mortality data are counts, we will want to shift to a different type of regression within the GLM framework, but this step allows you to develop a simple `glm` call, and to remember where to include the data and the independent and dependent variables within this function call.)
2. Change your function call to fit a regression model in the Poisson family.
3. Change your function call to allow for overdispersion in the outcome data (daily mortality count). How does the estimated coefficient for temperature change between the model fit for #2 and this model? Check both the central estimate and its estimated standard error.
4. Change your function call to include control for day of week.

*Applied exercise: Example code*

1. **Fit a GLM to estimate the association between mean daily temperature (as the independent variable) and daily mortality count (as the dependent variable), first fitting a linear regression.**

This is the model you are fitting:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \epsilon$$

where  $Y_t$  is the mortality count on day  $t$ ,  $X_{1t}$  is the mean temperature for day  $t$  and  $\epsilon$  is the error term. Since this is a linear model we are assuming a Gaussian error distribution  $\epsilon \sim N(0, \sigma^2)$ , where  $\sigma^2$  is the variance not explained by the covariates (here just temperature).

To do this, you will use the `glm` call. If you would like to save model fit results to use later, you assign the output a name as an R object (`mod_linear_reg` in the example code). If your study data are in a dataframe, you can specify these data in the `glm` call with the `data` parameter. Once you do this, you can use column names directly in the model formula. In the model formula, the dependent variable is specified first (`all`, the column for daily mortality counts for all ages, in this example), followed by a tilde (~), followed by all independent

variables (only `tmean` in this example). If multiple independent variables are included, they are joined using `+`. We'll see an example when we start adding control for confounders later.

```
mod_linear_reg <- glm(all ~ tmean, data = obs)
```

Once you have fit a model and assigned it to an R object, you can explore it and use resulting values. First, the `print` method for a regression model gives some summary information. This method is automatically called if you enter the model object's name at the console:

```
mod_linear_reg
```

```
##  
## Call: glm(formula = all ~ tmean, data = obs)  
##  
## Coefficients:  
## (Intercept)      tmean  
##     187.647     -2.366  
##  
## Degrees of Freedom: 8278 Total (i.e. Null);  8277 Residual  
## Null Deviance:    8161000  
## Residual Deviance: 6766000  AIC: 79020
```

More information is printed if you run the `summary` method on the model object:

```
summary(mod_linear_reg)
```

```
##  
## Call:  
## glm(formula = all ~ tmean, data = obs)  
##  
## Deviance Residuals:  
##      Min       1Q   Median       3Q      Max  
## -77.301  -20.365   -1.605   17.502  169.280  
##  
## Coefficients:  
##                 Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 187.64658    0.73557 255.10  <2e-16 ***  
## tmean        -2.36555    0.05726  -41.31  <2e-16 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## (Dispersion parameter for gaussian family taken to be 817.4629)  
##  
## Null deviance: 8161196  on 8278  degrees of freedom  
## Residual deviance: 6766140  on 8277  degrees of freedom  
## AIC: 79019
```

```
##  
## Number of Fisher Scoring iterations: 2
```

Make sure you are familiar with the information provided from the model object, as well as how to interpret values like the coefficient estimates and their standard errors and p-values. These basic elements should have been covered in previous coursework (even if a different programming language was used to fit the model), and so we will not be covering them in great depth here, but instead focusing on some of the more advanced elements of how regression models are commonly fit to data from time series and case-crossover study designs in environmental epidemiology. For a refresher on the basics of fitting statistical models in R, you may want to check out Chapters 22 through 24 of Wickham and Grolemund (2016), a book that is available online, as well as Dunn and Smyth (2018) and James et al. (2013).

Finally, there are some newer tools for extracting information from model fit objects. The `broom` package extracts different elements from these objects and returns them in a “tidy” data format, which makes it much easier to use the output further in analysis with functions from the “tidyverse” suite of R packages. These tools are very popular and powerful, and so the `broom` tools can be very useful in working with output from regression modeling in R.

The `broom` package includes three main functions for extracting data from regression model objects. First, the `glance` function returns overall data about the model fit, including the AIC and BIC:

```
library(broom)  
glance(mod_linear_reg)
```

```
## # A tibble: 1 x 8  
##   null.deviance df.null  logLik     AIC     BIC deviance df.residual nobs  
##             <dbl>    <int>    <dbl>    <dbl>    <dbl>      <dbl>    <int> <int>  
## 1         8161196.     8278 -39507. 79019. 79041. 6766140.      8277  8279
```

The `tidy` function returns data at the level of the model coefficients, including the estimate for each model parameter, its standard error, test statistic, and p-value.

```
tidy(mod_linear_reg)
```

```
## # A tibble: 2 x 5  
##   term       estimate std.error statistic p.value  
##   <chr>        <dbl>     <dbl>     <dbl>     <dbl>  
## 1 (Intercept) 188.      0.736     255.      0  
## 2 tmean       -2.37     0.0573    -41.3     0
```

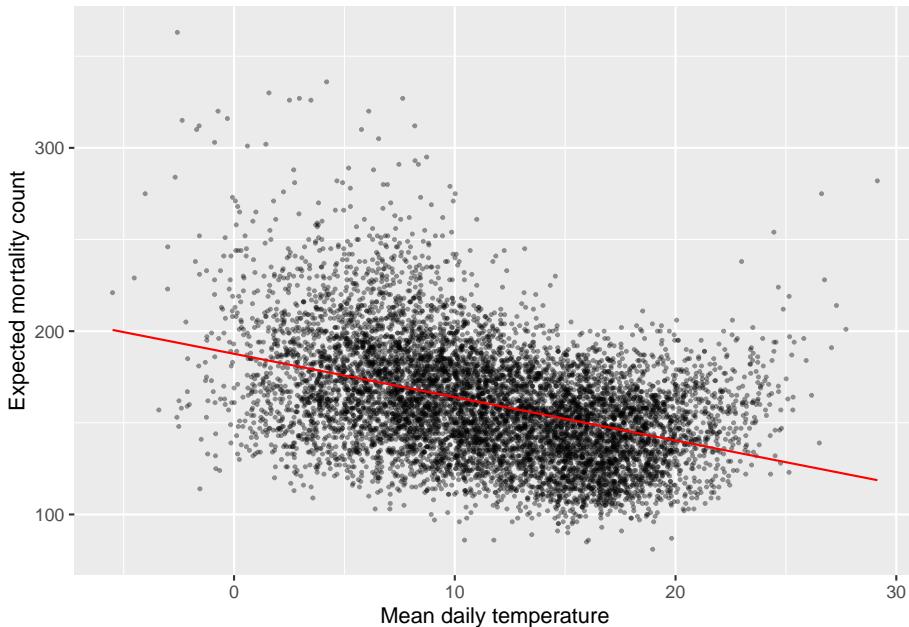
Finally, the `augment` function returns data at the level of the original observations, including the fitted value for each observation, the residual between the fitted and true value, and some measures of influence on the model fit.

```
augment(mod_linear_reg)

## # A tibble: 8,279 x 8
##       all tmean .fitted .resid .std.resid     .hat .sigma .cooksdi
##   <dbl> <dbl>   <dbl>  <dbl>     <dbl>   <dbl>   <dbl>
## 1    220  3.91    178.   41.6     1.46 0.000359  28.6  0.000380
## 2    257  5.55    175.   82.5     2.89 0.000268  28.6  0.00112
## 3    245  4.39    177.   67.7     2.37 0.000330  28.6  0.000928
## 4    226  5.43    175.   51.2     1.79 0.000274  28.6  0.000440
## 5    236  6.87    171.   64.6     2.26 0.000211  28.6  0.000539
## 6    235  9.23    166.   69.2     2.42 0.000144  28.6  0.000420
## 7    231  6.69    172.   59.2     2.07 0.000218  28.6  0.000467
## 8    235  7.96    169.   66.2     2.31 0.000174  28.6  0.000467
## 9    250  7.27    170.   79.5     2.78 0.000197  28.6  0.000761
## 10   214  9.51    165.   48.9     1.71 0.000139  28.6  0.000202
## # ... with 8,269 more rows
```

One way you can use `augment` is to graph the fitted values for each observation after fitting the model:

```
mod_linear_reg %>%
  augment() %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = .fitted), color = "red") +
  labs(x = "Mean daily temperature", y = "Expected mortality count")
```



For more on the `broom` package, including some excellent examples of how it can be used to streamline complex regression analyses, see Robinson (2014). There is also a nice example of how it can be used in one of the chapters of Wickham and Grolemund (2016), available online at <https://r4ds.had.co.nz/many-models.html>.

## 2. Change your function call to fit a regression model in the Poisson family.

A linear regression is often not appropriate when fitting a model where the outcome variable provides counts, as with the example data, since such data often don't follow a normal distribution. A Poisson regression is typically preferred.

For a count distribution were  $Y \sim \text{Poisson}()$  we typically fit a model such as

$g(Y) = \beta_0 + \beta_1 X_1$ , where  $g()$  represents the link function, in this case a log function so that  $\log(Y) = \beta_0 + \beta_1 X_1$ . We can also express this as  $Y = \exp(\beta_0 + \beta_1 X_1)$ .

In the `glm` call, you can specify this with the `family` parameter, for which "poisson" is one choice.

```
mod_pois_reg <- glm(all ~ tmean, data = obs, family = "poisson")
```

One thing to keep in mind with this change is that the model now uses a non-identity link between the combination of independent variable(s) and the dependent variable. You will need to keep this in mind when you interpret the estimates of the regression coefficients. While the coefficient estimate for `tmean` from the linear regression could be interpreted as the expected increase in mortality counts for a one-unit (i.e., one degree Celsius) increase in temperature, now the estimated coefficient should be interpreted as the expected increase in the natural log-transform of mortality count for a one-unit increase in temperature.

```
summary(mod_pois_reg)
```

```
## 
## Call:
## glm(formula = all ~ tmean, family = "poisson", data = obs)
## 
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max 
## -6.5945  -1.6365  -0.1167   1.3652  12.2221 
## 
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)    
## (Intercept) 5.2445409  0.0019704 2661.67  <2e-16 ***
## tmean      -0.0147728  0.0001583  -93.29  <2e-16 ***
## ---
```

```

## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
## Null deviance: 49297 on 8278 degrees of freedom
## Residual deviance: 40587 on 8277 degrees of freedom
## AIC: 97690
##
## Number of Fisher Scoring iterations: 4

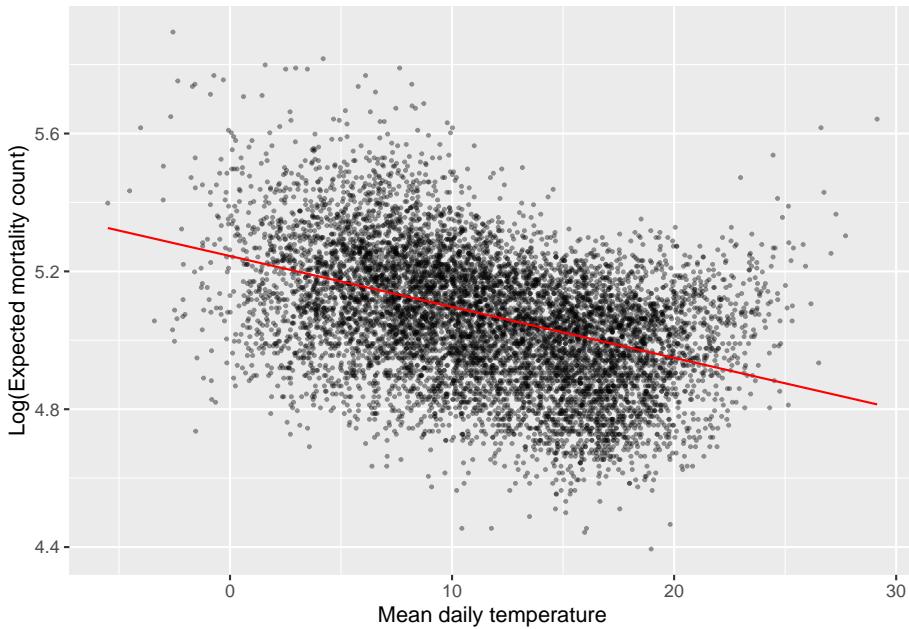
```

You can see this even more clearly if you take a look at the association between temperature for each observation and the expected mortality count fit by the model. First, if you look at the fitted values without transforming, they will still be in a state where mortality count is log-transformed. You can see by looking at the range of the y-scale that these values are for the log of expected mortality, rather than expected mortality (compare, for example, to the similar plot shown from the first model, which was linear), and that the fitted association for that *transformation*, not for untransformed mortality counts, is linear:

```

mod_pois_reg %>%
  augment() %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = log(all)), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = .fitted), color = "red") +
  labs(x = "Mean daily temperature", y = "Log(Expected mortality count)")

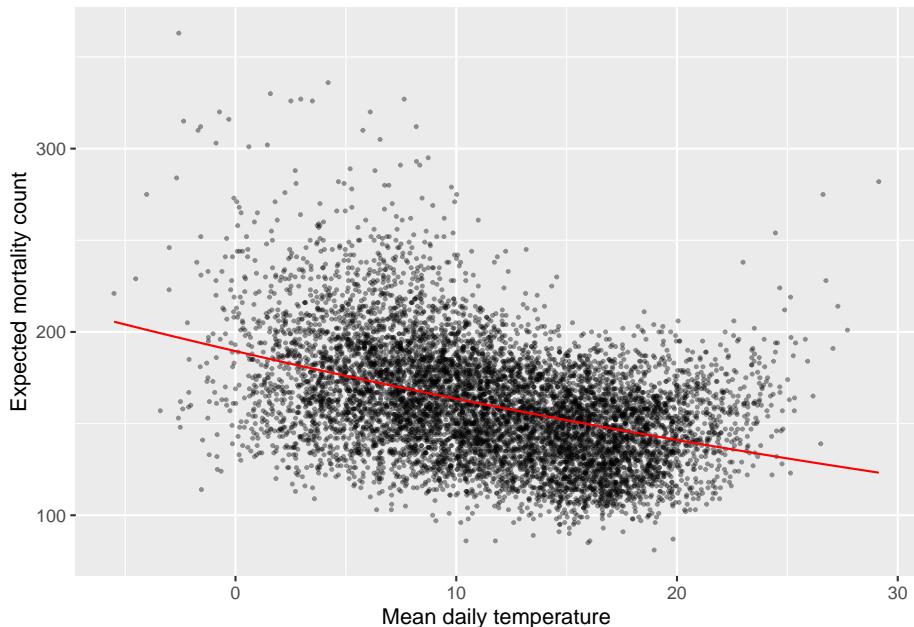
```



You can use exponentiation to transform the fitted values back to just be the

expected mortality count based on the model fit. Once you make this transformation, you can see how the link in the Poisson family specification enforced a curved relationship between mean daily temperature and the untransformed expected mortality count.

```
mod_pois_reg %>%
  augment() %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Mean daily temperature", y = "Expected mortality count")
```



For this model, we can interpret the coefficient for the temperature covariate as the expected log relative risk in the health outcome associated with a one-unit increase in temperature. We can exponentiate this value to get an estimate of the relative risk:

```
# Extract the temperature coefficient from the model
tmean_coef <- mod_pois_reg %>%
  tidy() %>%
  filter(term == "tmean") %>% # Extract the row with the tmean estimates
  pull(estimate) # Extract just the point estimate

# Estimate of the log relative risk for a one-unit increase in temperature
tmean_coef

## [1] -0.0147728
```

```
# Estimate of the relative risk for a one-unit increase in temperature
exp(tmean_coef)

## [1] 0.9853358
```

If you want to estimate the confidence interval for this estimate, you should calculate that *before* exponentiating.

3. Change your function call to allow for overdispersion in the outcome data (daily mortality count). How does the estimated coefficient for temperature change between the model fit for #2 and this model? Check both the central estimate and its estimated standard error.

In the R `glm` call, there is a family that is similar to Poisson (including using a log link), but that allows for overdispersion. You can specify it with the “quasipoisson” choice for the `family` parameter in the `glm` call:

```
mod_ovdisp_reg <- glm(all ~ tmean, data = obs, family = "quasipoisson")
```

When you use this family, there will be some new information in the summary for the model object. It will now include a dispersion parameter ( $\phi$ ). If this is close to 1, then the data were close to the assumed variance for a Poisson distribution (i.e., there was little evidence of overdispersion). In the example, the overdispersion is around 5, suggesting the data are overdispersed (this might come down some when we start including independent variables that explain some of the variation in the outcome variable, like long-term and seasonal trends).

```
summary(mod_ovdisp_reg)

##
## Call:
## glm(formula = all ~ tmean, family = "quasipoisson", data = obs)
##
## Deviance Residuals:
##      Min        1Q     Median        3Q       Max
## -6.5945   -1.6365   -0.1167    1.3652   12.2221
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 5.2445409  0.0044087 1189.6  <2e-16 ***
## tmean       -0.0147728  0.0003543   -41.7  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for quasipoisson family taken to be 5.006304)
##
## Null deviance: 49297  on 8278  degrees of freedom
```

```
## Residual deviance: 40587  on 8277  degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 4
```

If you compare the estimates of the temperature coefficient from the Poisson regression with those when you allow for overdispersion, you'll see something interesting:

```
tidy(mod_pois_reg) %>%
  filter(term == "tmean")

## # A tibble: 1 x 5
##   term estimate std.error statistic p.value
##   <chr>    <dbl>     <dbl>     <dbl>    <dbl>
## 1 tmean   -0.0148  0.000158    -93.3      0

tidy(mod_ovdisp_reg) %>%
  filter(term == "tmean")

## # A tibble: 1 x 5
##   term estimate std.error statistic p.value
##   <chr>    <dbl>     <dbl>     <dbl>    <dbl>
## 1 tmean   -0.0148  0.000354    -41.7      0
```

The central estimate (`estimate` column) is very similar. However, the estimated standard error is larger when the model allows for overdispersion. This indicates that the Poisson model was too simple, and that its inherent assumption that data were not overdispersed was problematic. If you naively used a Poisson regression in this case, then you would estimate a confidence interval on the temperature coefficient that would be too narrow. This could cause you to conclude that the estimate was statistically significant when you should not have (although in this case, the estimate is statistically significant under both models).

#### 4. Change your function call to include control for day of week.

Day of week is included in the data as a categorical variable, using a data type in R called a factor. You are now essentially fitting this model:

$$\log(Y) = \beta_0 + \beta_1 X_1 + \gamma' X_2,$$

where  $X_2$  is a categorical variable for day of the week and  $\gamma'$  represents a vector of parameters associated with each category.

It is pretty straightforward to include factors as independent variables in calls to `glm`: you just add the column name to the list of other independent variables with a `+`. In this case, we need to do one more step: earlier, we added order to `dow`, so it would “remember” the order of the week days (Monday before Tuesday, etc.). However, we need to strip off this order before we include the

factor in the `glm` call. One way to do this is with the `factor` call, specifying `ordered = FALSE`. Here is the full call to fit this model:

```
mod_ctrl_dow <- glm(all ~ tmean + factor(dow, ordered = FALSE),
                      data = obs, family = "quasipoisson")
```

When you look at the summary for the model object, you can see that the model has fit a separate model parameter for six of the seven weekdays. The one weekday that isn't fit (Sunday in this case) serves as a baseline—these estimates specify how the log of the expected mortality count is expected to differ on, for example, Monday versus Sunday (by about 0.03), if the temperature is the same for the two days.

```
summary(mod_ctrl_dow)
```

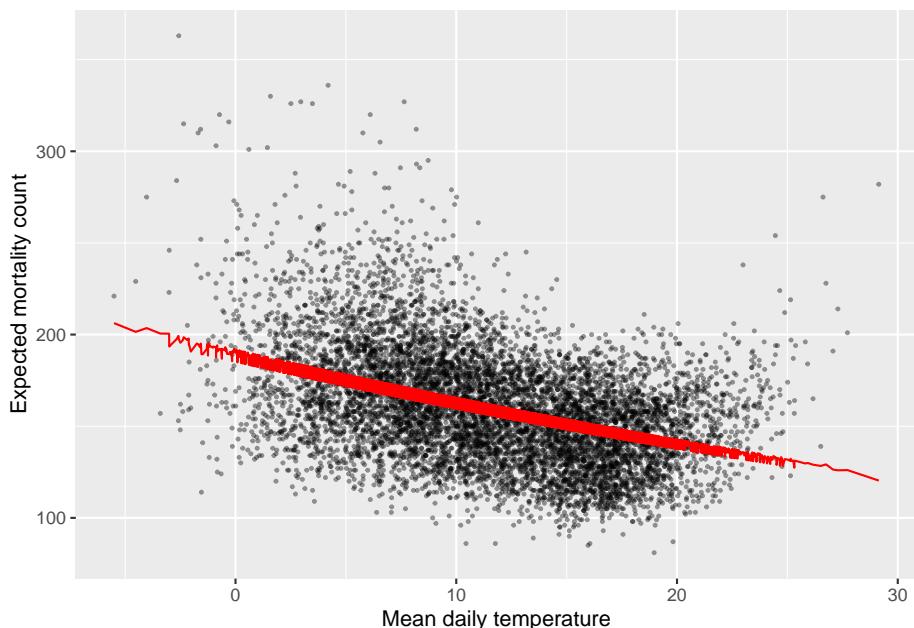
```
##
## Call:
## glm(formula = all ~ tmean + factor(dow, ordered = FALSE), family = "quasipoisson",
##      data = obs)
##
## Deviance Residuals:
##    Min      1Q  Median      3Q     Max
## -6.3211 -1.6476 -0.1313  1.3549 12.5286
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  5.2208502  0.0065277 799.804 < 2e-16 ***
## tmean                         -0.0147723  0.0003538 -41.750 < 2e-16 ***
## factor(dow, ordered = FALSE)Mon  0.0299282  0.0072910   4.105 4.08e-05 ***
## factor(dow, ordered = FALSE)Tue  0.0292575  0.0072920   4.012 6.07e-05 ***
## factor(dow, ordered = FALSE)Wed  0.0255224  0.0073020   3.495 0.000476 ***
## factor(dow, ordered = FALSE)Thu  0.0269580  0.0072985   3.694 0.000222 ***
## factor(dow, ordered = FALSE)Fri  0.0355431  0.0072834   4.880 1.08e-06 ***
## factor(dow, ordered = FALSE)Sat  0.0181489  0.0073158   2.481 0.013129 *
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for quasipoisson family taken to be 4.992004)
##
## Null deviance: 49297  on 8278  degrees of freedom
## Residual deviance: 40434  on 8271  degrees of freedom
## AIC: NA
##
## Number of Fisher Scoring iterations: 4
```

You can also see from this summary that the coefficients for the day of the week are all statistically significant. Even though we didn't see a big difference in

mortality counts by day of week in our exploratory analysis, this suggests that it does help explain some variance in mortality observations and will likely be worth including in the final model.

The model now includes day of week when fitting an expected mortality count for each observation. As a result, if you plot fitted values of expected mortality versus mean daily temperature, you'll see some "happiness" in the fitted line:

```
mod_ctrl_dow %>%
  augment() %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Mean daily temperature", y = "Expected mortality count")
```

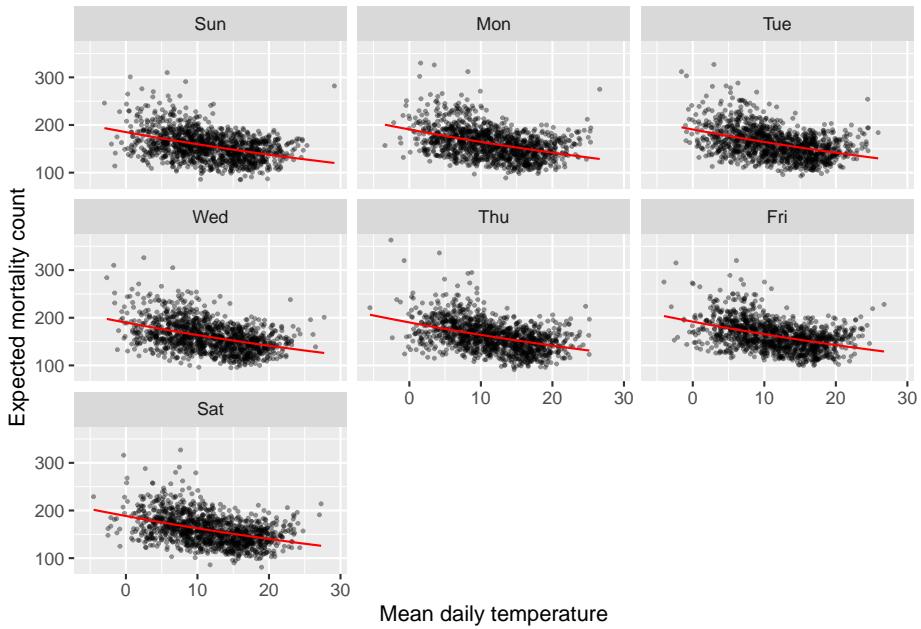


This is because each fitted value is also incorporating the expected influence of day of week on the mortality count, and that varies across the observations (i.e., you could have two days with the same temperature, but different expected mortality from the model, because they occur on different days).

If you plot the model fits separately for each day of the week, you'll see that the line is smooth across all observations from the same day of the week:

```
mod_ctrl_dow %>%
  augment() %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
```

```
geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Mean daily temperature", y = "Expected mortality count") +
  facet_wrap(~ obs$dow)
```



#### *Wrapping up*

At this point, the coefficient estimates suggests that risk of mortality tends to decrease as temperature increases. Do you think this is reasonable? What else might be important to build into the model based on your analysis up to this point?



# Chapter 4

## Generalized linear models

### 4.1 Readings

The readings for this chapter are:

- Bhaskaran et al. (2013) Provides an overview of time series regression in environmental epidemiology.
- Vicedo-Cabrera et al. (2019) Provides a tutorial of all the steps for a projecting of health impacts of temperature extremes under climate change. One of the steps is to fit the exposure-response association using present-day data (the section on “Estimation of Exposure-Response Associations” in the paper). In this chapter, we will go into details on that step, and that section of the paper is the only required reading for this chapter. Later in the class, we’ll look at other steps covered in this paper. Supplemental material for this paper is available to download by clicking <http://links.lww.com/EDE/B504>. You will need the data in this supplement for the exercises for class.
- Armstrong et al. (2014) This paper describes different data structures for case-crossover data, as well as how conditional Poisson regression can be used in some cases to fit a statistical model to these data. Supplemental material for this paper is available at <https://bmcmedresmethodol.biomedcentral.com/articles/10.1186/1471-2288-14-122#Sec13>.

The following are supplemental readings (i.e., not required, but may be of interest) associated with the material in this chapter:

- Armstrong (2006) Covers similar material as Bhaskaran et al. (2013), but with more focus on the statistical modeling framework (highly recommended!)
- Gasparrini and Armstrong (2010) Describes some of the advances made to time series study designs and statistical analysis, specifically in the context

of temperature

- Basu et al. (2005) Compares time series and case-crossover study designs in the context of exploring temperature and health. Includes a nice illustration of different referent periods, including time-stratified.
- Imai et al. (2015) Typically, the time series study design covered in this chapter is used to study non-communicable health outcomes. This paper discusses opportunities and limitations in applying a similar framework for infectious disease.
- Lu and Zeger (2007) Heavier on statistics. This paper shows how, under conditions often common for environmental epidemiology studies, case-crossover and time series methods are equivalent.
- Gasparrini (2014) Heavier on statistics. This provides the statistical framework for the distributed lag model for environmental epidemiology time series studies.
- Dunn and Smyth (2018) Introduction to statistical models, moving into regression models and generalized linear models. Chapter in a book that is available online through the CSU library.
- James et al. (2013) General overview of linear regression, with an R coding “lab” at the end to provide coding examples. Covers model fit, continuous, binary, and categorical covariates, and interaction terms. Chapter in a book that is available online through the CSU library.

## 4.2 Splines in GLMs

We saw from the last model, with a linear term for mean daily temperature, that the suggested effect on mortality is a decrease in daily mortality counts with increasing temperature. However, as you’ve probably guessed that’s likely not entirely accurate. A linear term for the effect of exposure restricts us to an effect that can be fitted with a straight line (either a null effect or a monotonically increasing or decreasing effect with increasing exposure).

This clearly is problematic in some cases. One example is when exploring the association between temperature and health risk. Based on human physiology, we would expect many health risks to be elevated at temperature extremes, whether those are extreme cold or extreme heat. A linear term would be inadequate to describe this kind of U-shaped association. Other effects might have a threshold—for example, heat stroke might have a very low risk at most temperatures, only increasing with temperature above a certain threshold.

We can capture non-linear patterns in effects, by using different functions of  $X$ . Examples are  $\sqrt{X}$ ,  $X^2$ , or more complex smoothing functions, such as polynomials or splines. Polynomials might at first make a lot of sense, especially since you’ve likely come across polynomial terms in mathematics classes since grade school. However, it turns out that they have some undesirable properties. A key one is that they can have extreme behavior, particularly when using a high-order polynomial, and particularly outside the range of data that are available

to fit the model.

An alternative that is generally preferred for environmental epidemiology studies is the regression spline. The word “spline” originally comes from drafting and engineering (ship building, in particular), where it described a flexible piece of wood or metal that you could use to draw a curved line—it created a curve that was flexible enough—but just flexible enough—to fit a space (see Wikipedia’s very interesting article on flat splines for more).

Splines follow a similar idea in mathematics, making them helpful tools when a line won’t fit your data well. In general, a spline fits together a few simpler functions to create something with a curve or non-linearity. Each simpler function operates within an interval of the data, and then they join together at “knots” along the range of the data. Regression splines are therefore simple parametric smoothing function, which fit separate polynomial in each interval of the range of the predictor; these can be linear, quadratic, and cubic.

The simplest example is a linear spline (also called a piecewise linear function). This type of spline creates non-linearity by having a breakpoint at the knot, allowing the slope of the line to be different in the intervals of data on either side of the knot. The following plot shows an example. Say you want to explore how mean temperature varies by the day in the year (Jan 1 = 1, Jan 2 = 2, and so on) in the London example dataset from the last chapter. Temperature tends to increase with day of year for a while, but then it changes around the start of August, after that decreasing with day of year. This pattern means that a line will give a bad fit for how temperature changes with day of year, since it smooths right through that change. On the other hand, you can get a very reasonable fit using a linear spline with a knot around August 1 (day 213 in the year). These two examples are shown in the following plot, with the linear function fit on the left and the linear spline on the right:



If you were to write out these regression models in mathematical notation, the linear one is very simple:

$$Y_t = \alpha + \beta X_t$$

where  $Y_t$  is the temperature on day  $t$ ,  $\alpha$  is the model intercept,  $X_t$  is the day of the year on day  $t$ , and  $\beta$  is the estimated coefficient for  $X_t$ .

The notation for the model with the linear spline is a bit more complex:

$$Y_t = \alpha + \beta_1 X_t + \beta_2 (X_t - k)_+$$

Here,  $Y_t$  is again the temperature on day  $t$ ,  $X_t$  is again the day of the year on day  $t$ , and  $\alpha$  is again the intercept. The term  $k$  is the “knot”—the value of  $X$  where we’re letting the slope change. In this example, we’re using  $k = 213$ . The term  $(X_t - k)_+$  has a special meaning—it takes the value 0 if  $X_t$  is in the interval to the left of the knot, while if  $X_t$  is in the interval to the right of the knot, it takes the value of  $X_t$  minus the knot value:

$$(X_t - k)_+ = \begin{cases} 0, & \text{if } X_t < k \\ X_t - k, & \text{if } X_t \geq k \end{cases}$$

In this model, the coefficient  $\beta_1$  estimates the slope of the line to the left of the knot, while  $\beta_2$  estimates how that slope will change to the right of the knot.

Fortunately, we usually won't have to get this complex in the model notation, especially when we use more complex splines (where the notation would get even more complex). Instead, we'll often write out the regression equation in a simpler way, just indicating that we're using a function of the covariate, rather than the covariate directly:

$$Y_t = \alpha + f(X_t | )$$

where we can note that  $f(X_t)$  is a function of day of the year ( $X_t$ ), fit in this case using a linear spline, and with a set of estimated coefficients for that function (see Armstrong (2006) for an example of using this model notation).

While a linear spline is the simplest conceptually (and mathematically), it often isn't very satisfying, because it fits a function with a sharp breakpoint, which often isn't realistic. For example, the linear spline fit above suggests that the relationship between day of the year and temperature changes abruptly and dramatically on August 1 of the year. In reality, we know that this change in the relationship between day of year and temperature is probably a lot smoother.

To fit smoother shapes, we can move to higher level splines. Cubic splines (“cubic” because they include terms of the covariate up to the third power) are very popular. An example of a cubic spline function is  $X + X^2 + X^3 + I((X > X_0) * (X - X_0)^3)$ . This particular function is a cubic spline with four degrees of freedom ( $df = 4$ ) and one knot ( $X_0$ ). A special type of cubic spline called a natural cubic spline is particularly popular. Unlike a polynomial function, a natural cubic spline “behaves” better outside the range of the data used to fit the model—they are constrained to continue on a linear trajectory once they pass beyond the range of the data.

Regression splines can be fit in a GLM via the package `splines`. Two commonly used examples of regression splines are b-splines and natural cubic splines. Vicedo-Cabrera et al. (2019) uses natural cubic splines, which can be fit with the `ns` (for “natural spline”) function from the `splines` package.

While splines are great for fitting non-linear relationships, they do create some challenges in interpreting the results. When you fit a linear relationship for a covariate, you will get a single estimate that helps describe the fitted relationship between that covariate and the outcome variable. However, when you use a non-linear function, you'll end up with a mix of coefficients associated with that function. Sometimes, you will use splines to control for a potential confounder (as we will in the exercises for this first part of the chapter). In this case, you don't need to worry about interpreting the estimated coefficients—you're just trying to control for the variable, rather than inferring anything about how it's associated with the outcome. In later parts of this chapter, we'll talk about how to interpret these coefficients if you're using a spline for the exposure that you're interested in, when we talk more broadly about basis functions.

*Applied: Including a spline in a GLM*

For this exercise, you will continue to build up the model that you began in the examples in the previous chapter. The example uses the data provided with one of this chapter's readings, Vicedo-Cabrera et al. (2019).

1. Start by fitting a somewhat simple model—how are daily mortality counts associated with (a) a linear and (b) a non-linear function of time? Is a linear term appropriate to describe this association? What types of patterns are captured by a non-linear function that are missed by a linear function?
2. In the last chapter, the final version of the model used a GLM with an overdispersed Poisson distribution, including control for day of week. Start from this model and add control for long-term and seasonal trends over the study period.
3. Refine your model to fit for a non-linear, rather than linear, function of temperature in the model. Does a non-linear term seem to be more appropriate than a linear term?

*Applied exercise: Example code*

1. Start by fitting a somewhat simple model—how are daily mortality counts associated with (a) a linear and (b) a non-linear function of time?

It is helpful to start by loading the R packages you are likely to need, as well as the example dataset. You may also need to re-load the example data and perform the steps taken to clean it in the last chapter:

```
# Load some packages that will likely be useful
library(tidyverse)
library(viridis)
library(lubridate)
library(broom)

# Load and clean the data
obs <- read_csv("data/lndn_obs.csv") %>%
  mutate(dow = wday(date, label = TRUE))
```

For this first question, the aim is to model the association between time and daily mortality counts within the example data. This approach is often used to explore and, if needed, adjust for temporal factors in the data.

There are a number of factors that can act over time to create patterns in both environmental exposures and health outcomes. For example, there may be changes in air pollution exposures over the years of a study because of changes in regulations or growth or decline of factories and automobile traffic in an area. Changes in health care and in population demographics can cause patterns in health outcomes over the study period. At a shorter, seasonal term, there are also factors that could influence both exposures and outcomes, including seasonal changes in climate, seasonal changes in emissions, and seasonal patterns

in health outcomes.

It can be difficult to pinpoint and measure these temporal factors, and so instead a common practice is to include model control based on the time in the study. This can be measured, for example, as the day since the start of the study period.

You can easily add a column for day in study for a dataset that includes date. R saves dates in a special format, which we're using the in `obs` dataset:

```
class(obs$date)
## [1] "Date"
```

However, this is just a fancy overlay on a value that's ultimately saved as a number. Like most Unix programs, the date is saved as the number of days since the Unix “epoch”, January 1, 1970. You can take advantage of this convention—if you use `as.numeric` around a date in R, it will give you a number that gets one unit higher for every new date. Here's the example for the first date in our example data:

```
obs$date[1]
## [1] "1990-01-01"
as.numeric(obs$date[1])
## [1] 7305
```

And here's the example for the next date:

```
obs$date[2]
## [1] "1990-01-02"
as.numeric(obs$date[2])
## [1] 7306
```

You can use this convention to add a column that gives days since the first study date. While you could also use the `1:n()` call to get a number for each row that goes from 1 to the number of rows, that approach would not catch any “skips” in dates in the data (e.g., missing dates if only warm-season data are included). The use of the dates is more robust:

```
obs <- obs %>%
  mutate(time = as.numeric(date) - first(as.numeric(date)))

obs %>%
  select(date, time)

## # A tibble: 8,279 x 2
##       date      time
```

```

## #> #> <date> <dbl>
## #> 1 1990-01-01 0
## #> 2 1990-01-02 1
## #> 3 1990-01-03 2
## #> 4 1990-01-04 3
## #> 5 1990-01-05 4
## #> 6 1990-01-06 5
## #> 7 1990-01-07 6
## #> 8 1990-01-08 7
## #> 9 1990-01-09 8
## #> 10 1990-01-10 9
## # ... with 8,269 more rows

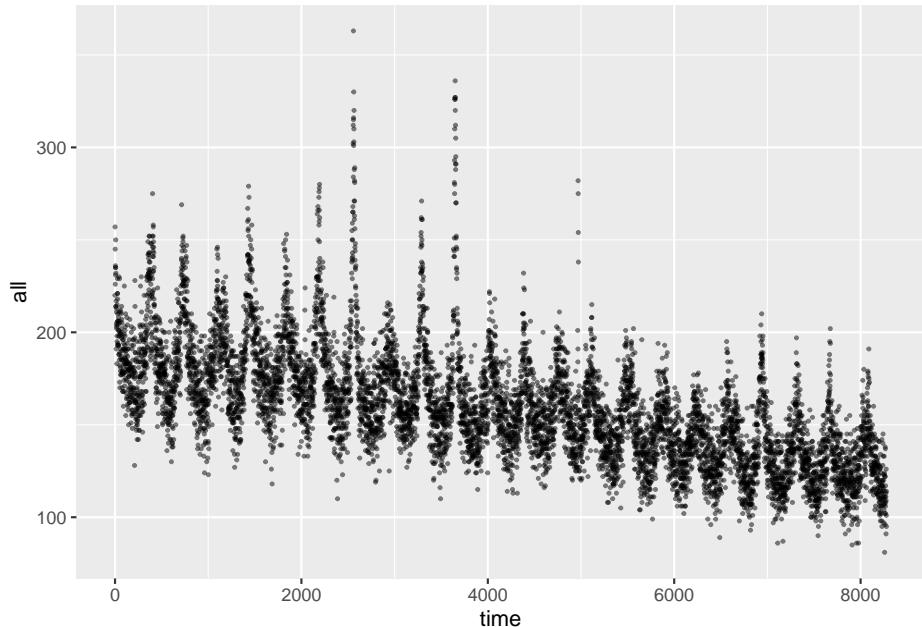
```

As a next step, it is always useful to use exploratory data analysis to look at the patterns that might exist for an association, before you start designing and fitting the regression model.

```

ggplot(obs,
       aes(x = time, y = all)) +
  geom_point(size = 0.5, alpha = 0.5)

```



There are clear patterns between time and daily mortality counts in these data. First, there is a clear long-term pattern, with mortality rates declining on average over time. Second, there are clear seasonal patterns, with higher mortality generally in the winter and lower rates in the summer.

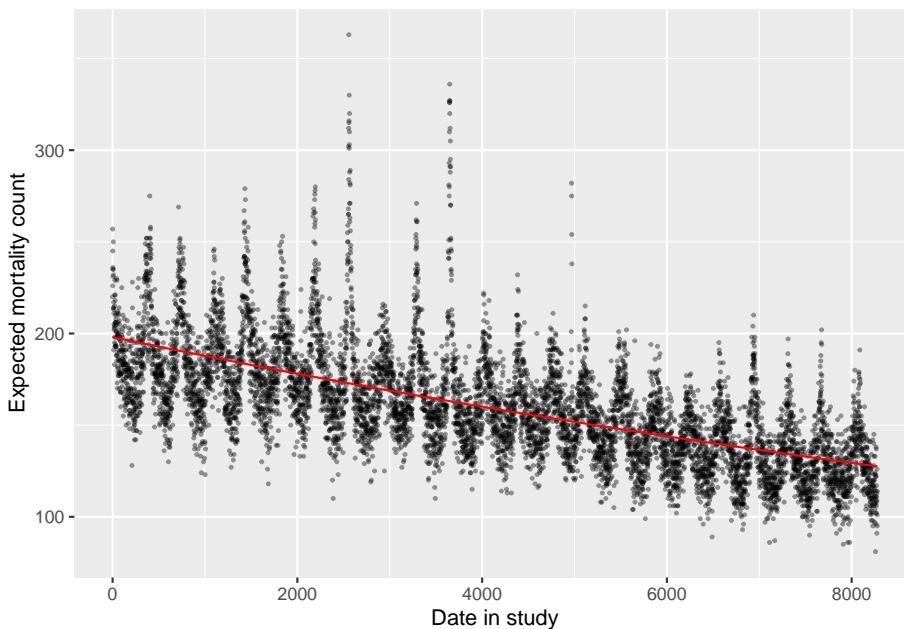
To model this, we can start with fitting a linear term. In the last chapter, we

determined that the mortality outcome data can be fit using a GLM with a Poisson family, allowing for overdispersion as it is common in real-life count data like these. To include time as a linear term, we can just include that column name to the right of the `~` in the model formula:

```
mod_time <- glm(all ~ time,
                  data = obs, family = "quasipoisson")
```

You can use the `augment` function from the `broom` package to pull out the fitted estimate for each of the original observations and plot that, along with the observed data, to get an idea of what this model has captured:

```
mod_time %>%
  augment() %>%
  ggplot(aes(x = time)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Date in study", y = "Expected mortality count")
```



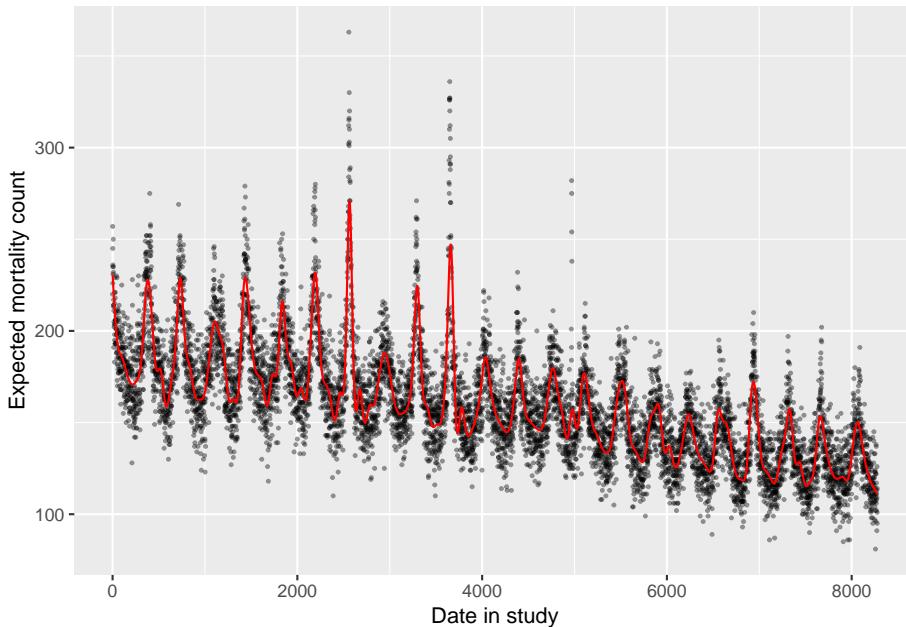
This linear trend captures the long-term trend in mortality rates fairly well in this case. This won't always be the case, as there may be some health outcomes—or some study populations—where the long-term pattern over the study period might be less linear than in this example. Further, the linear term is completely unsuccessful in capturing the shorter-term trends in mortality rate. These oscillate, and so would be impossible to capture over multiple years with a linear trend.

Instead, it's helpful to use a non-linear term for time in the model. We can use a natural cubic spline for this, using the `ns` function from the `splines` package. You will need to clarify how flexible the spline function should be, and this can be specified through the degrees of freedom for the spline. A spline with more degrees of freedom will be "wigglier" over a given data range compared to a spline with fewer degrees of freedom. Let's start by using 158 degrees of freedom, which translates to about 7 degrees of freedom per year:

```
library(splines)
mod_time_nonlin <- glm(all ~ ns(time, df = 158),
                        data = obs, family = "quasipoisson")
```

You can visualize the model results in a similar way to how we visualized the last model. However, there is one extra step. The `augment` function only carries through columns in the original data (`obs`) that were directly used in fitting the model. Now that we're using a transformation of the `time` column, by wrapping it in `ns`, the `time` column is no longer included in the `augment` output. However, we can easily add it back in using `mutate`, pulling it from the original `obs` dataset, and then proceed as before.

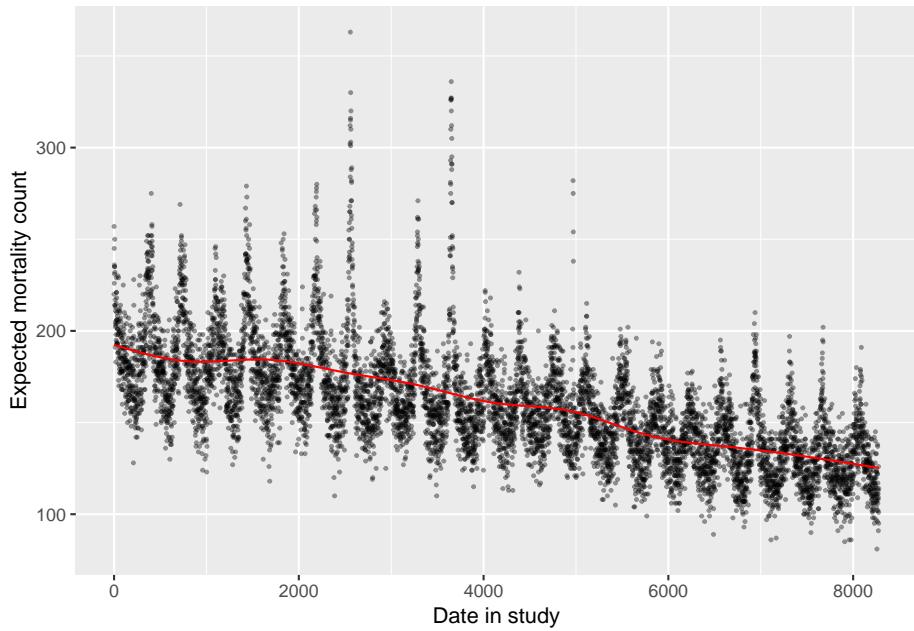
```
mod_time_nonlin %>%
  augment() %>%
  mutate(time = obs$time) %>%
  ggplot(aes(x = time)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Date in study", y = "Expected mortality count")
```



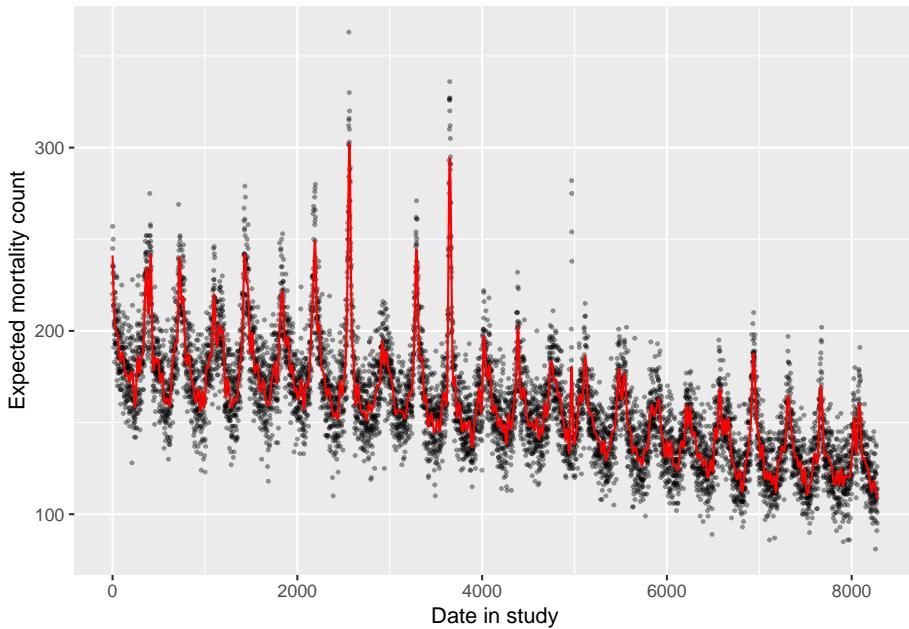
The non-linear term for time has allowed enough flexibility that the model now captures both long-term and seasonal trends in the data.

You might wonder how many degrees of freedom you should use for this time spline. In practice, researchers often use about 6–8 degrees of freedom per year of the study, in the case of year-round data. You can explore how changing the degrees of freedom changes the way the model fits to the observed data. As you use more degrees of freedom, the line will capture very short-term effects, and may start to interfere with the shorter-term associations between environmental exposures and health risk that you are trying to capture. Even in the example model we just fit, for example, it looks like the control for time may be capturing some patterns that were likely caused by heatwaves (the rare summer peaks, including one from the 1995 heatwave). Conversely, if too few degrees of freedom are used, the model will shift to look much more like the linear model, with inadequate control for seasonal patterns.

```
# A model with many less d.f. for the time spline
mod_time_nonlin_lowdf <- glm(all ~ ns(time, df = 10),
                                data = obs, family = "quasipoisson")
mod_time_nonlin_lowdf %>%
  augment() %>%
  mutate(time = obs$time) %>%
  ggplot(aes(x = time)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Date in study", y = "Expected mortality count")
```



```
# A model with many more d.f. for the time spline
# (Takes a little while to run)
mod_time_nonlin_highdf <- glm(all ~ ns(time, df = 400),
                                data = obs, family = "quasipoisson")
mod_time_nonlin_highdf %>%
  augment() %>%
  mutate(time = obs$time) %>%
  ggplot(aes(x = time)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_line(aes(y = exp(.fitted)), color = "red") +
  labs(x = "Date in study", y = "Expected mortality count")
```



In all cases, when you fit a non-linear function of an explanatory variable, it will make the model summary results look much more complicated, e.g.:

```
mod_time_nonlin_lowdf %>%
  tidy()

## # A tibble: 11 x 5
##   term            estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)    5.26      0.00948   555.    0.
## 2 ns(time, df = 10)1 -0.0260   0.0119    -2.18  2.93e- 2
## 3 ns(time, df = 10)2 -0.0860   0.0155    -5.56  2.85e- 8
## 4 ns(time, df = 10)3 -0.114     0.0139    -8.15  4.01e-16
## 5 ns(time, df = 10)4 -0.196     0.0151   -13.0   4.47e-38
## 6 ns(time, df = 10)5 -0.187     0.0148   -12.6   2.80e-36
## 7 ns(time, df = 10)6 -0.315     0.0154   -20.5   5.62e-91
## 8 ns(time, df = 10)7 -0.337     0.0154   -21.9   1.95e-103
## 9 ns(time, df = 10)8 -0.358     0.0135   -26.5   1.56e-148
## 10 ns(time, df = 10)9 -0.467     0.0244   -19.2   4.49e-80
## 11 ns(time, df = 10)10 -0.392     0.0126   -31.2   8.01e-202
```

You can see that there are multiple model coefficients for the variable fit using a spline function, one less than the number of degrees of freedom. These model coefficients are very hard to interpret on their own. When we are using the spline to *control* for a factor that might serve as a confounder of the association of interest, we typically won't need to try to interpret these model coefficients—

instead, we are interested in accounting for how this factor explains variability in the outcome, without needing to quantify the association as a key result. However, there are also cases where we want to use a spline to fit the association with the exposure that we are interested in. In this case, we will want to be able to interpret model coefficients from the spline. Later in this chapter, we will introduce the `dlnm` package, which includes functions to both fit and interpret natural cubic splines within GLMs for environmental epidemiology.

**2. Start from the last model created in the last chapter and add control for long-term and seasonal trends over the study period.**

The last model fit in the last chapter was the following, which fits for the association between a linear term of temperature and mortality risk, with control for day of week:

```
mod_ctrl_dow <- glm(all ~ tmean + factor(dow, ordered = FALSE),
                      data = obs, family = "quasipoisson")
```

To add control for long-term and seasonal trends, you can take the natural cubic spline function of temperature that you just fit and include it among the explanatory / independent variables from the model in the last chapter. If you want to control for only long-term trends, a linear term of the `time` column could work, as we discovered in the first part of this chapter's exercise. However, seasonal trends could certainly confound the association of interest. Mortality rates have a clear seasonal pattern, and temperature does as well, and these patterns create the potential for confounding when we look at how temperature and mortality risk are associated, beyond any seasonally-driven pathways.

```
mod_ctrl_dow_time <- glm(all ~ tmean + factor(dow, ordered = FALSE) +
                           ns(time, df = 158),
                           data = obs, family = "quasipoisson")
```

You can see the influence of this seasonal confounding if you look at the model results. When we look at the results from the model that did not control for long-term and seasonal trends, we get an estimate that mortality rates tend to be lower on days with higher temperature, with a negative term for `tmean`:

```
mod_ctrl_dow %>%
  tidy() %>%
  filter(term == "tmean")

## # A tibble: 1 x 5
##   term  estimate std.error statistic p.value
##   <chr>    <dbl>     <dbl>     <dbl>    <dbl>
## 1 tmean   -0.0148  0.000354    -41.7      0
```

Conversely, when we include control for long-term and seasonal trends, the estimated association between mortality rates and temperature is reversed, estimating increased mortality rates on days with higher temperature, *controlling*

for long-term and seasonal trends:

```
mod_ctrl_dow_time %>%
  tidy() %>%
  filter(term == "tmean")

## # A tibble: 1 x 5
##   term estimate std.error statistic p.value
##   <chr>    <dbl>     <dbl>     <dbl>    <dbl>
## 1 tmean    0.00370  0.000395    9.36  1.02e-20
```

### 3. Refine your model to fit for a non-linear, rather than linear, function of temperature in the model.

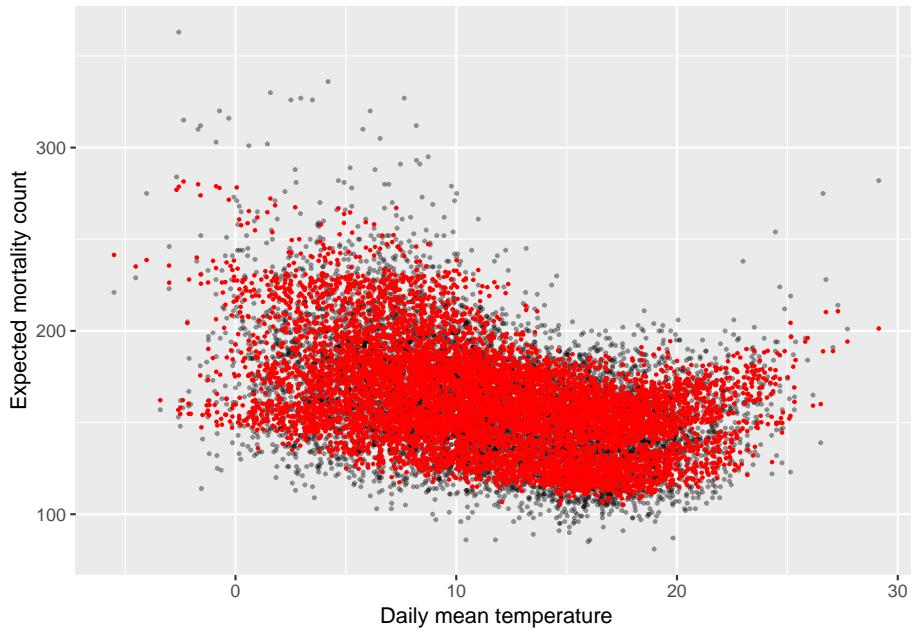
You can use a spline in the same way to fit a non-linear function for the exposure of interest in the model (temperature). We'll start there. However, as mentioned earlier, it's a bit tricky to interpret the coefficients from the fit model—you no longer generate a single coefficient for the exposure of interest, but instead several related to the spline. Therefore, once we show how to fit using `ns` directly, we'll show how you can do the same thing using specialized functions in the `dlnm` package. This package includes a lot of nice functions for not only fitting an association using a non-linear term, but also for interpreting the results after the model is fit.

First, here is code that can be used to fit the model using `ns` directly, similarly to the approach we used to control for temporal patterns with a flexible function:

```
mod_ctrl_nl_temp <- glm(all ~ ns(tmean, 4) + factor(dow, ordered = FALSE) +
                           ns(time, df = 158),
                           data = obs, family = "quasipoisson")
```

We can plot the predicted values from this fitted model (red points in the plot below) compared to the observed data (black dots) using our usual method of using `augment` to extract the predicted values:

```
mod_ctrl_nl_temp %>%
  augment() %>%
  mutate(tmean = obs$tmean) %>%
  ggplot(aes(x = tmean)) +
  geom_point(aes(y = all), alpha = 0.4, size = 0.5) +
  geom_point(aes(y = exp(.fitted)), color = "red", size = 0.4) +
  labs(x = "Daily mean temperature", y = "Expected mortality count")
```



However, these predictions are very variable at any given temperature. This reflects how other independent variables, like long-term and seasonal trends, explain variability in mortality. This makes sense, but it makes it a bit hard to investigate the role of temperature specifically. Let's look, then, at some other options for viewing the results that will help us focus on the association of the exposure we care about (temperature) with the outcome.

The next part has a lot of steps, so it might at first seem confusing. However, fortunately we won't always have to do all the steps ourselves—there is a nice R package that will help us. We'll look at the process first, though, and then the easier way to use a package to help with some of the steps. Also, this process gives us a first look at how the idea of basis functions work. We'll later expand on these to look at non-linear relationships with the exposure at different lag times, using cross-basis functions, so this forms a starting point for moving into the idea of a cross-basis.

First, we need to think about how temperature gets included in the regression model—specifically, as a function of *basis variables* rather than as a single variable. In other words, to include temperature as a nonlinear function, we're going to create a structure in the regression equation that includes the variable of temperature in several different terms, in different transformations in each term. A simple example of this is a second-degree polynomial. If we wanted to include a second-order polynomial function of temperature in the regression, then we'd use the following basis:

$$\beta_1 T + \beta_2 T^2$$

Notice that here we're using the same independent variable ( $T$ , which stands for daily temperature), but we're including both untransformed  $T$  and also  $T$  squared. This function of  $T$  might go into the regression equation as something like:

$$E(Y) = \alpha + \beta_1 T + \beta_2 T^2 + ns(time) + {}' \mathbf{D}$$

where  $\alpha$  is the intercept,  $ns(time)$  is a natural cubic spline that controls for time (long-term and seasonal trends) and  $\mathbf{D}$  is day of week, with  ${}'$  as the set of coefficients associated with day of week. (Here, I've included the outcome,  $Y$ , untransformed, as you would for a linear regression, but of course the same idea works with Poisson regression, when you'd instead have  $\log(E(Y))$  on the left of the equation.)

When you fit a regression model in a program like R, it sets up the observed data into something called a *model matrix*, where it has the values for each observation for each of the independent variables you want to include, plus a column of “1”s for the intercept, if you model structure includes one. The regression model will fit a coefficient for each of these columns in the model matrix. In a simple case, this model matrix will just repeat each of your independent variables. For example, the model matrix for the model `mod_overdisp_reg`, which we fit in the last chapter and which included only an intercept and a linear term for `tmean`, looks like this (the `model.matrix` function will give you the model matrix of any `glm` object that you get by running the `glm` function in R):

```
mod_ovdisp_reg %>%
  model.matrix() %>%
  head()

##   (Intercept)    tmean
## 1          1 3.913589
## 2          1 5.547919
## 3          1 4.385564
## 4          1 5.431046
## 5          1 6.867855
## 6          1 9.232628
```

We already start using the idea of basis variables when we include categorical variables, like day of week. Here is the model matrix for the `mod_ctrl_dow` model that we fit in the last chapter:

```
mod_ctrl_dow %>%
  model.matrix() %>%
  head()

##   (Intercept)    tmean factor(dow, ordered = FALSE)Mon
## 1          1 3.913589                      1
## 2          1 5.547919                      0
```

```

## 3      1 4.385564          0
## 4      1 5.431046          0
## 5      1 6.867855          0
## 6      1 9.232628          0
##   factor(dow, ordered = FALSE)Tue factor(dow, ordered = FALSE)Wed
## 1                  0          0
## 2                  1          0
## 3                  0          1
## 4                  0          0
## 5                  0          0
## 6                  0          0
##   factor(dow, ordered = FALSE)Thu factor(dow, ordered = FALSE)Fri
## 1                  0          0
## 2                  0          0
## 3                  0          0
## 4                  1          0
## 5                  0          1
## 6                  0          0
##   factor(dow, ordered = FALSE)Sat
## 1                  0
## 2                  0
## 3                  0
## 4                  0
## 5                  0
## 6                  1

```

You can see that the regression call broke the day-of-week variable up into a set of indicator variables, which equal either 1 or 0. There will be one less of these than the number of categories for the variable; in otherwords, if the categorical variable took two values (Weekend / Weekday), then this would be covered by a single column; since there are seven days in the week, the day-of-week variable breaks into six ( $7 - 1$ ) columns. The first level of the categories (Sunday in this case) serves as a baseline and doesn't get a value. The others levels (Monday, Tuesday, etc.) each get their own indicator variable—so, their own column in the model matrix—which equals “1” on that day (i.e., “1” for the Monday column if the date of the observation is a Monday) and “0” on all other days.

Now let's go back and look at the example of including temperature in the model as a nonlinear function, starting with the simple example of using a second-degree polynomial. What would the basis for that look like? We can find out by fitting the model and then looking at the model matrix (the `I()` function lets us specify a transformation of a column from the data when we set up the regression equation structure in `glm`):

```

mod_polynomial_temp <- glm(all ~ tmean + I(tmean ^ 2),
                           data = obs, family = "quasipoisson")
mod_polynomial_temp %>%

```

```
model.matrix() %>%
  head()

## # A tibble: 6 x 3
##   (Intercept)    tmean I(tmean^2)
##   <dbl>        <dbl>    <dbl>
## 1 3.913589    15.31618
## 2 5.547919    30.77941
## 3 4.385564    19.23317
## 4 5.431046    29.49627
## 5 6.867855    47.16743
## 6 9.232628    85.24142
```

You can see that this has created two columns based on the temperature variable, one with it untransformed and one with it squared. The regression will estimate coefficients for each of these basis variables of temperature, and then that allows you to fit a model with a function of temperature, rather than solely temperature. Here are the coefficients from this model:

```
mod_polynomial_temp %>%
  tidy()

## # A tibble: 3 x 5
##   term      estimate std.error statistic p.value
##   <chr>     <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept) 5.31     0.00675    787.  0.
## 2 tmean       -0.0298   0.00126   -23.6  2.27e-119
## 3 I(tmean^2)  0.000667 0.0000538   12.4  5.70e- 35
```

Therefore, it's fit the following model to describe the association between temperature and mortality:

$$\log(E(Y)) = 5.3092 - 0.0298T + 0.0007T^2$$

If you want to estimate the relative risk of mortality at 15 degrees Celsius versus 10 degrees Celsius with this model (which is confounded by long-term and seasonal trends, so has some issues we'll want to fix), you can take the following process. Take the two equations for the expected mortality when  $T$  is 15 and 10, respectively:

$$\log(E(Y|T = 15)) = 5.3092 - 0.0298(15) + 0.0007(15^2) \quad \log(E(Y|T = 10)) = 5.3092 - 0.0298(10) + 0.0007(10^2)$$

If you subtract the second from the first, you get:

$$\log(E(Y|T = 15)) - \log(E(Y|T = 10)) = (5.3092 - 5.3092) - 0.0298(15 - 10) + 0.0007(15^2 - 10^2)$$

which simplifies to (if the left part isn't clear, review the rules for how you can manipulate logarithms):

$$\log\left(\frac{E(Y|T=15)}{E(Y|T=10)}\right) = -0.0298(5) + 0.0007(125) = -0.0615$$

Exponentiate both sides, to get to the relative risk at 15 degrees versus 10 degrees (in other words, the ratio of expected mortality at 15 degrees to that at 10 degrees):

$$\frac{E(Y|T=15)}{E(Y|T=10)} = e^{-0.0615} = 0.94$$

You can see that the basis variables are great in helping us explore how an independent variable might be related to the outcome in a non-linear way, but there's a bit of a cost in terms of us needing to take some extra steps to interpret the results from that model. Also, note that there's not a single coefficient that we can extract from the model as a summary of the relationship. Instead, we needed to pick a reference temperature (10 degrees in this example) and compare to that to get an estimated relative risk. We'll see the same pattern as we move to using natural cubic splines to create the basis variables for temperature.

Now let's move to a spline. The function the spline runs to transform the temperature variable into the different basis variables is more complex than for the polynomial example, but the result is similar: you get several columns to fit in the model for a variable, compared to the single column you would include if you were only fitting a linear term. If you take a look at the output of running the `ns` function on temperature in the data, you can see that it creates several new columns (one for each degree of freedom), which will be the basis variables in the regression:

```
ns(obs$tmean, df = 4) %>%
  head()

##           1          2          3          4
## [1,] 0.1769619 -0.20432696 0.4777110 -0.2733841
## [2,] 0.2860253 -0.19686120 0.4602563 -0.2633951
## [3,] 0.2049283 -0.20410506 0.4771922 -0.2730872
## [4,] 0.2770456 -0.19804188 0.4630167 -0.2649748
## [5,] 0.4012496 -0.17595974 0.4113892 -0.2354295
## [6,] 0.6581858 -0.09844302 0.2476396 -0.1417190
```

The output from `ns` also has some metadata, included in the attributes of the object, that have some other information about the spline function, including where the knots were placed and where the boundary knots (which are at the outer ranges of the data) are placed. You can the `str` function to explore both the data and metadata stored in this object:

```
ns(obs$tmean, df = 4) %>%
  str()
```

```

##  'ns' num [1:8279, 1:4] 0.177 0.286 0.205 0.277 0.401 ...
## - attr(*, "dimnames")=List of 2
##   ..$ : NULL
##   ..$ : chr [1:4] "1" "2" "3" "4"
## - attr(*, "degree")= int 3
## - attr(*, "knots")= Named num [1:3] 7.47 11.47 15.93
##   ..- attr(*, "names")= chr [1:3] "25%" "50%" "75%"
## - attr(*, "Boundary.knots")= num [1:2] -5.5 29.1
## - attr(*, "intercept")= logi FALSE

```

This is saying, for example, that the internal knots for this spline were put at the 25th, 50th, and 75th quantiles of the temperature data, where were 7.47 degrees, 11.47 degrees, and 15.93 degrees. (The defaults for `ns` is to place knots evenly at percentiles, based on the number of degrees of freedom you specify. You can change this by placing the knots “by hand”, using the `knots` argument in the `ns` function.)

This spline object can be used not just for its basis values, but also to “predict” new basis values for a new set of temperature values. For example, you could figure out what the basis values from this spline would be for every degree of temperature between -6 and 29 using the following code:

```

temp_spline <- ns(obs$tmean, df = 4)
temp_spline_preds <- predict(temp_spline, newx = -6:29)
temp_spline_preds %>%
  head()

```

	1	2	3	4
## [1,]	0.000000e+00	0.01591874	-0.03721760	0.02129885
## [2,]	2.689545e-05	-0.01606406	0.03755734	-0.02149328
## [3,]	7.189726e-04	-0.04768237	0.11148013	-0.06379775
## [4,]	3.321933e-03	-0.07825364	0.18295494	-0.10470130
## [5,]	9.107577e-03	-0.10708098	0.25035250	-0.14327152
## [6,]	1.934771e-02	-0.13346753	0.31204355	-0.17857602

This is handy, because it will help us visualize the relationship we fit in a regression model—we can start with these basis values for each degree in our temperature range, and then use the regression coefficients for each basis variable to estimate relative risk at that temperature compared to a reference temperature, exactly as we did in the equations for the polynomial function of temperature earlier, comparing 15 degrees C to the reference of 10 degrees.

All we need now are the regression coefficients for each of these temperature basis variables. We can extract those from the model we fit earlier, where we included `ns(tmean, 4)` as one of the model terms. Here, I’m using `tidy` to get the model coefficients and then, because there are *lots* of them (from fitting a spline for time with lots of degrees of freedom), I’m using the `str_detect` function from the `stringr` package to pick out just those with “tmean” in the

term column:

```
mod_ctrl_nl_temp %>%
  tidy() %>%
  filter(str_detect(term, "tmean"))

## # A tibble: 4 x 5
##   term      estimate std.error statistic p.value
##   <chr>     <dbl>     <dbl>     <dbl>    <dbl>
## 1 ns(tmean, 4)1 -0.0700    0.0135    -5.18  2.30e- 7
## 2 ns(tmean, 4)2 -0.0660    0.0126    -5.24  1.61e- 7
## 3 ns(tmean, 4)3  0.0110    0.0313     0.350 7.26e- 1
## 4 ns(tmean, 4)4  0.347     0.0177    19.6   2.02e-83
```

You can add on `pull` to pull out just the `estimate` column as a vector, which will be helpful when we want to multiple these coefficients by the basis values for each degree of temperature across our temperature range:

```
temp_spline_estts <- mod_ctrl_nl_temp %>%
  tidy() %>%
  filter(str_detect(term, "tmean")) %>%
  pull(estimate)
temp_spline_estts

## [1] -0.06995454 -0.06596939  0.01095574  0.34659715
```

Now, let's put this together to see how relative risk of mortality changes as you move across the temperature range! First, we can set up a dataframe that has a column with each unit of temperature across our range—these are the temperatures where we want to estimate relative risk—and then the estimated relative risk at that temperature. By default, we'll be comparing to the lowest temperature in the original data, but we'll talk in a minute about how to adjust to a different reference temperature. You can use matrix multiplication (`%*%`) as a shorthand way to multiple each column of the spline basis variables from `temp_spline_preds` by its estimated coefficient from the regression model, saved in `temp_spline_estts`, and then add all of those values together (this is the same idea as what we did in the equations earlier, for the polynomial basis). Then, to get from log relative risk to relative risk, we'll exponentiate that with `exp`. You can see the first rows of the results below:

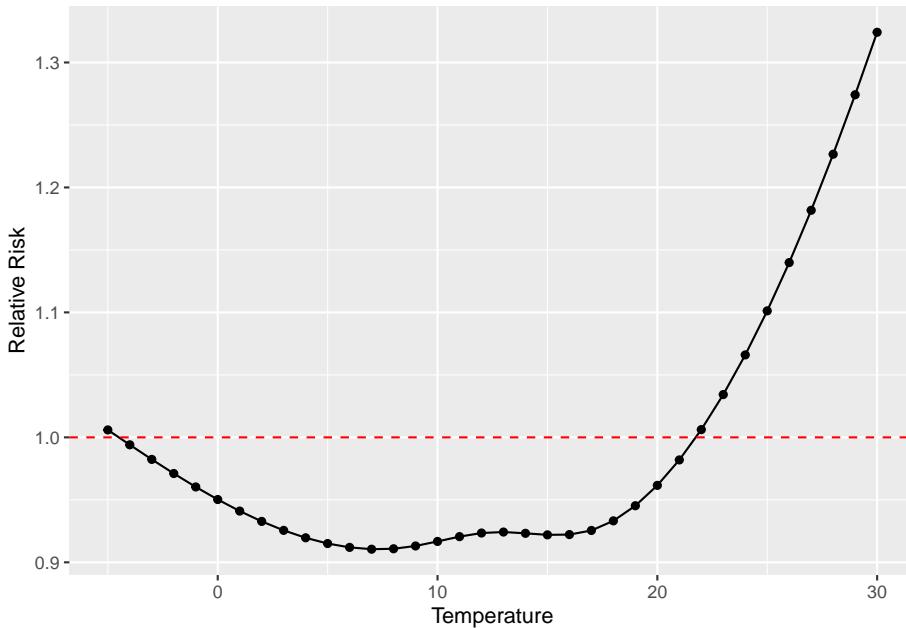
```
pred_temp_function <- tibble(
  temp = -5:30,
  temp_func = temp_spline_preds %*% temp_spline_estts,
  rr = exp(temp_func)
)

pred_temp_function %>%
  head()
```

```
## # A tibble: 6 x 3
##   temp temp_func[,1] rr[,1]
##   <int>      <dbl>  <dbl>
## 1    -5     0.00592  1.01
## 2    -4     -0.00598  0.994
## 3    -3     -0.0178   0.982
## 4    -2     -0.0294   0.971
## 5    -1     -0.0405   0.960
## 6     0     -0.0510   0.950
```

This is now very easy to plot, with a reference line added at a relative risk of 1.0:

```
ggplot(pred_temp_function, aes(x = temp, y = rr)) +
  geom_point() +
  geom_line() +
  labs(x = "Temperature",
       y = "Relative Risk") +
  geom_hline(yintercept = 1.0, color = "red", linetype = 2)
```



We might want to shift this, so we're comparing the temperature at which mortality risk is lowest (sometimes called the *minimum mortality temperature*). This tends to be at milder temperatures, in the middle of our range, rather than at the minimum temperature in the range. To start, let's see what temperature aligns with the lowest relative risk of mortality:

```

pred_temp_function %>%
  filter(rr == min(rr))

## # A tibble: 1 x 3
##   temp temp_func rr[,1]
##   <int>      <dbl>  <dbl>
## 1     7      -0.0938  0.911

```

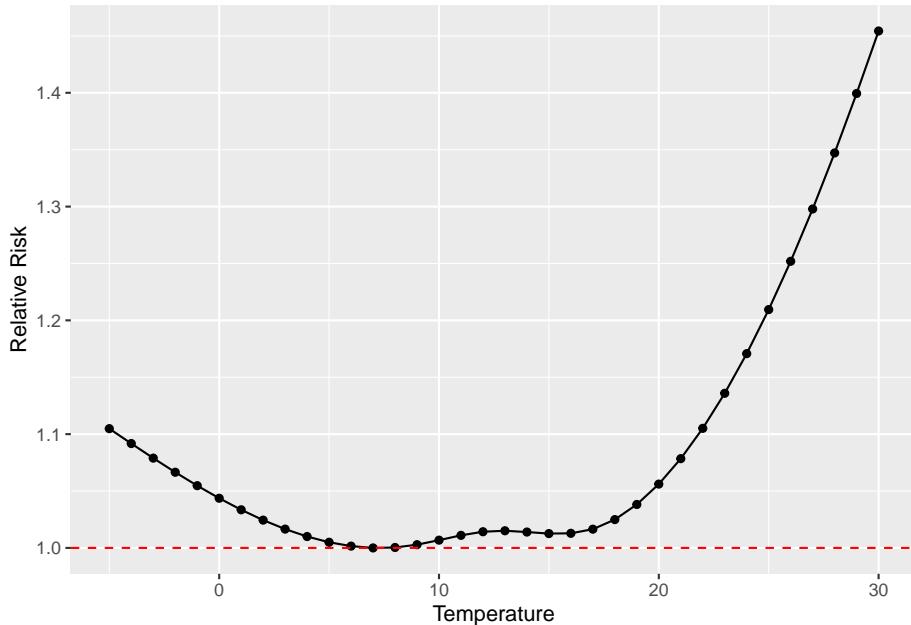
We can therefore realign so that the relative risk equals 1.0 when the temperature is 7 degrees C, and all other relative risks are relative to a reference temperature of 7 degrees C:

```

pred_temp_function <- pred_temp_function %>%
  mutate(temp_func_reset = temp_func - temp_func[temp == 7],
        rr_reset = exp(temp_func_reset))
)

ggplot(pred_temp_function, aes(x = temp, y = rr_reset)) +
  geom_point() +
  geom_line() +
  labs(x = "Temperature",
       y = "Relative Risk") +
  geom_hline(yintercept = 1.0, color = "red", linetype = 2)

```



This is a fairly cumbersome process, as you've seen with this example, and it would be a pain if we had to do it every time we wanted to visualize and

explore results from fitting regressions that include basis variables from splines. Fortunately, there's a nice R package called `dlnm` (for “distributed lag nonlinear models”) that will help take care of a lot of these “under the hood” steps for us. Even better, this package will let us move on to more complex crossbasis functions, where we fit non-linear functions in two dimensions, and help us visualize and interpret results from those models.

To start, make sure you have the `dlnm` package installed on your computer, and then load it in your R session. This package has a function called `crossbasis` that lets you build a crossbasis function to use in a regression model. We'll talk more about these types of functions later in this chapter; for right now, we'll be a bit simpler and just use it to create our spline function of temperature.

In the `crossbasis` function, you start by putting in the vector of the variable that you want to expand into basis variables. In our case, this is temperature, which we have saved as the `tmean` column in the `obs` dataset. You use the `argvar` to give some information about the basis you want to use for that variable. This will both include the type of basis function ("ns" for a natural cubic spline), and then also arguments you want to pass to that basis function, like degrees of freedom (`df`) or knot locations (`knots`) if you're fitting a spline. The other arguments allow you to specify a function of lagged time; we won't use that yet, so you can just include `lag = 0` and `arglag = list(fun = "integer")` to create a crossbasis where we only consider same-day effects in a simple way. If you look at the output, you'll see it's very similar to the basis created by the `ns` call earlier (in fact, the values in each column should be identical):

```
library(dlnm)
temp_basis <- crossbasis(obs$tmean, lag = 0,
                           argvar = list(fun = "ns", df = 4),
                           arglag = list(fun = "integer"))
temp_basis %>%
  head()

##           v1.11      v2.11      v3.11      v4.11
## [1,] 0.1769619 -0.20432696 0.4777110 -0.2733841
## [2,] 0.2860253 -0.19686120 0.4602563 -0.2633951
## [3,] 0.2049283 -0.20410506 0.4771922 -0.2730872
## [4,] 0.2770456 -0.19804188 0.4630167 -0.2649748
## [5,] 0.4012496 -0.17595974 0.4113892 -0.2354295
## [6,] 0.6581858 -0.09844302 0.2476396 -0.1417190
```

To estimate the regression coefficients, we'll put this whole crossbasis in as one of our terms in the `glm` regression equation:

```
dlnm_mod_1 <- glm(all ~ temp_basis + factor(dow, ordered = FALSE) +
                     ns(time, df = 158),
                     data = obs, family = "quasipoisson")
```

As when we fit the spline earlier, you can see that this gives us a set of coefficients,

one for each column in the matrix of crossbasis variables:

```
dlnm_mod_1 %>%
  tidy() %>%
  filter(str_detect(term, "temp_basis"))

## # A tibble: 4 x 5
##   term            estimate std.error statistic p.value
##   <chr>          <dbl>     <dbl>      <dbl>    <dbl>
## 1 temp_basisv1.11 -0.0700    0.0135     -5.18  2.30e- 7
## 2 temp_basisv2.11 -0.0660    0.0126     -5.24  1.61e- 7
## 3 temp_basisv3.11  0.0110    0.0313     0.350  7.26e- 1
## 4 temp_basisv4.11  0.347     0.0177     19.6   2.02e-83
```

However, there's an advantage with using `crossbasis`, even though it's looked pretty similar to using `ns` up to now. That's that there are some special functions that let us predict and visualize the model results without having to do all the work we did before.

For example, there's a function called `crosspred` that will give us a number of values, including the estimated relative risk compared to a reference value. To use this, we need to input the object name of our crossbasis object (`temp_basis`) and the name of our regression model object (`dlnm_mod_1`). We also want to tell it which temperature we want to use as our reference (`cen = 7` to compare everything to 7 degrees C) and what interval we want for predictions (`by = 1` will give us an estimate for every degree temperature along our range). The output is a list with a lot of elements, which you can get a view of with `str`:

```
crosspred(basis = temp_basis, model = dlnm_mod_1, cen = 7, by = 1) %>%
  str()

## List of 19
## $ predvar     : num [1:35] -5 -4 -3 -2 -1 0 1 2 3 4 ...
## $ cen         : num 7
## $ lag         : num [1:2] 0 0
## $ bylag       : num 1
## $ coefficients: Named num [1:4] -0.07 -0.066 0.011 0.347
##   ..- attr(*, "names")= chr [1:4] "temp_basisv1.11" "temp_basisv2.11" "temp_basisv3.11" "temp_basisv4.11"
## $ vcov        : num [1:4, 1:4] 1.83e-04 1.12e-04 3.85e-04 6.58e-05 1.12e-04 ...
##   ..- attr(*, "dimnames")=List of 2
##     ...$ : chr [1:4] "temp_basisv1.11" "temp_basisv2.11" "temp_basisv3.11" "temp_basisv4.11"
##     ...$ : chr [1:4] "temp_basisv1.11" "temp_basisv2.11" "temp_basisv3.11" "temp_basisv4.11"
## $ matfit      : num [1:35, 1] 0.0874 0.0756 0.064 0.0529 0.0424 ...
##   ..- attr(*, "dimnames")=List of 2
##     ...$ : chr [1:35] "-5" "-4" "-3" "-2" ...
##     ...$ : chr [1, 1] "lag0"
## $ matse      : num [1:35, 1] 0.01435 0.01254 0.01077 0.00907 0.00746 ...
##   ..- attr(*, "dimnames")=List of 2
```

```

## ...$ : chr [1:35] "-5" "-4" "-3" "-2" ...
## ...$ : chr [1, 1] "lag0"
## $ allfit      : Named num [1:35] 0.0874 0.0756 0.064 0.0529 0.0424 ...
## ..- attr(*, "names")= chr [1:35] "-5" "-4" "-3" "-2" ...
## $ allse       : Named num [1:35] 0.01435 0.01254 0.01077 0.00907 0.00746 ...
## ..- attr(*, "names")= chr [1:35] "-5" "-4" "-3" "-2" ...
## $ matRRfit    : num [1:35, 1] 1.09 1.08 1.07 1.05 1.04 ...
## ..- attr(*, "dimnames")=List of 2
## ...$ : chr [1:35] "-5" "-4" "-3" "-2" ...
## ...$ : chr [1, 1] "lag0"
## $ matRRlow   : num [1:35, 1] 1.06 1.05 1.04 1.04 1.03 ...
## ..- attr(*, "dimnames")=List of 2
## ...$ : chr [1:35] "-5" "-4" "-3" "-2" ...
## ...$ : chr [1, 1] "lag0"
## $ matRRhigh  : num [1:35, 1] 1.12 1.11 1.09 1.07 1.06 ...
## ..- attr(*, "dimnames")=List of 2
## ...$ : chr [1:35] "-5" "-4" "-3" "-2" ...
## ...$ : chr [1, 1] "lag0"
## $ allRRfit    : Named num [1:35] 1.09 1.08 1.07 1.05 1.04 ...
## ..- attr(*, "names")= chr [1:35] "-5" "-4" "-3" "-2" ...
## $ allRRlow   : Named num [1:35] 1.06 1.05 1.04 1.04 1.03 ...
## ..- attr(*, "names")= chr [1:35] "-5" "-4" "-3" "-2" ...
## $ allRRhigh  : Named num [1:35] 1.12 1.11 1.09 1.07 1.06 ...
## ..- attr(*, "names")= chr [1:35] "-5" "-4" "-3" "-2" ...
## $ ci.level    : num 0.95
## $ model.class : chr [1:2] "glm" "lm"
## $ model.link  : chr "log"
## - attr(*, "class")= chr "crosspred"

```

The one we'll look at right now is `allRRfit`. You can extract it with (here I'm showing how you can do it with piping and the `pluck` function to pull out an element of a list, but you could do other approaches, too):

```

est_rr <- dlnm_mod_1 %>%
  crosspred(basis = temp_basis, model = ., cen = 7, by = 1) %>%
  pluck("allRRfit")
est_rr

##      -5      -4      -3      -2      -1       0       1       2
## 1.0913393 1.0785207 1.0661255 1.0543222 1.0432720 1.0331298 1.0240458 1.0161668
##      3       4       5       6       7       8       9      10
## 1.0096382 1.0046057 1.0012180 0.9996288 1.0000000 1.0024680 1.0064490 1.0106777
##     11      12      13      14      15      16      17      18
## 1.0138437 1.0147024 1.0135774 1.0122284 1.0124622 1.0160921 1.0245314 1.0378080
##     19      20      21      22      23      24      25      26
## 1.0557076 1.0780540 1.1046963 1.1354974 1.1703226 1.2090286 1.2514526 1.2974015
##     27      28      29

```

```
## 1.3466411 1.3988856 1.4537866
```

To make it easier to work with, let's put this in a dataframe. Note that the temperatures are included as the names in the vector, so we can extract those with `names`:

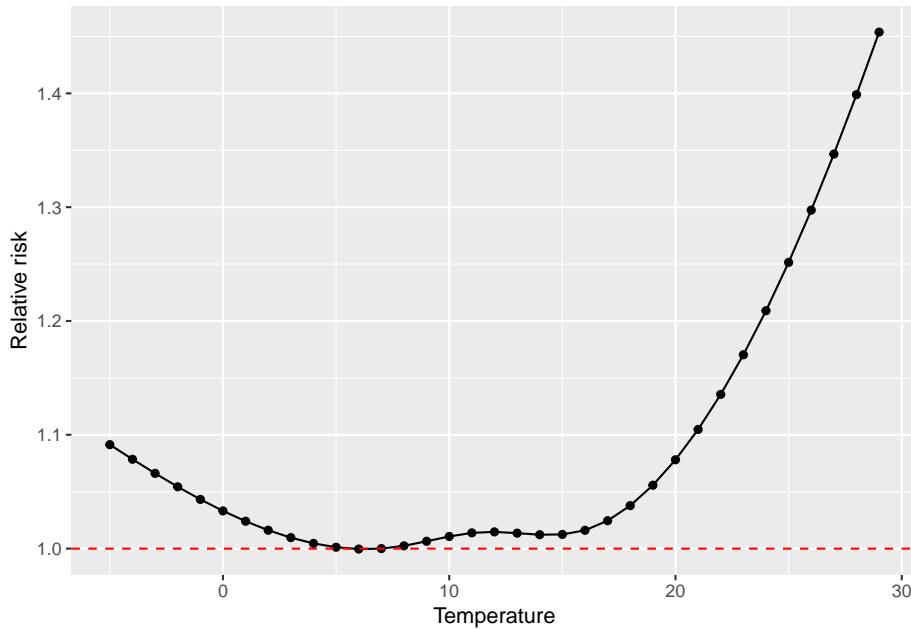
```
dlnm_temp_function <- tibble(tmean = as.numeric(names(est_rr)),
                                rr = est_rr)

dlnm_temp_function %>%
  head()
```

```
## # A tibble: 6 x 2
##   tmean     rr
##   <dbl> <dbl>
## 1 -5    1.09
## 2 -4    1.08
## 3 -3    1.07
## 4 -2    1.05
## 5 -1    1.04
## 6  0    1.03
```

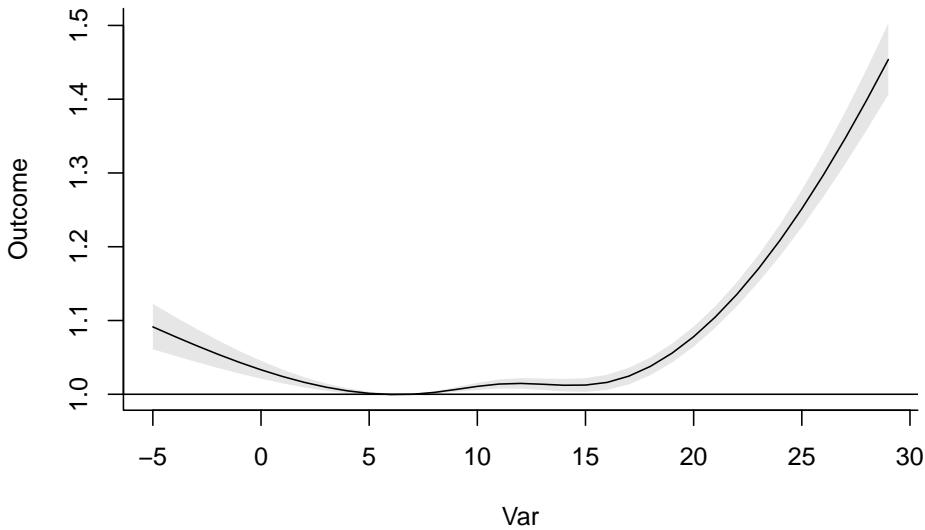
This has gotten us (much more quickly!) to estimates of the relative risk of mortality at each temperature compared to a reference of 7 degrees C. We can plot this the same way we did earlier, and you'll notice that this plot is identical to the one we created based on the regression with `ns` earlier:

```
ggplot(dlnm_temp_function, aes(x = tmean, y = rr)) +
  geom_point() +
  geom_line() +
  labs(x = "Temperature", y = "Relative risk") +
  geom_hline(yintercept = 1.0, color = "red", linetype = 2)
```



The `dlnm` package also includes some functions specifically for plotting. One downside is that they're in base R, rather than based on `ggplot2`, which makes them a bit harder to customize. However, they're a great way to get a first look at your results:

```
dlnm_mod_1 %>%
  crosspred(basis = temp_basis, model = ., cen = 7, by = 1) %>%
  plot()
```



Don't worry if all of this about basis functions is a lot! If this is the first time you've seen this, it will likely take a few passes to get comfortable with it—and while the idea of basis variables is fairly straightforward, it's certainly not straightforward to figure out how to interpret the results of the model that you fit. It is worth the effort, though, as this is a very powerful tool when working with regression modeling in environmental epidemiology.

### 4.3 Distributed lags and cross-basis functions in GLMs

Next, let's explore how we can add into our model something that lets us look at delayed effects and the potential for mortality displacement. So far, we have only considered how temperature affects mortality risk on the day of exposure—in other words, if it is very hot today, does risk of mortality go up substantially today? For many exposures, however, risk can persist after the day of exposure.

In some cases, this may be caused by a delay in detection of the outcome in the data we have. For example, extreme heat today could precipitate a respiratory or cardiac event that the person may not seek treatment for until tomorrow, and so in the data the outcome would be recorded at a one-day delay from the exposure.

In other cases, the path from the exposure to the outcome may take several days. For example, cold temperatures are often found to be associated with respiratory outcomes a week or more after exposures (something you can look for in the example data!), and one potential pathway is that cold temperatures might promote the spread of respiratory infectious disease, like influenza and colds, both through making people stay inside in more crowded, less ventilated conditions and also through effects on humidity and other conditions that might influence the behavior of respiratory droplets, which are key players in spreading some respiratory diseases. Since there is an incubation time up to several days for these infectious diseases, we might expect a lagged association between temperature and respiratory outcomes under this scenario.

There is another lagged pattern that can be important to check for, as well—one that indicates mortality displacement. For many ambient environmental exposures, there are clear indications that associations with health outcomes are strongest among older adults or other populations with a disproportionately high percent of people who are frail before exposure. One question that comes up, then, is if, when an exposure is associated with excess deaths, these are deaths that are displaced in time by only a few days or weeks, rather than deaths that represent a larger time of life lost. This question—of whether deaths observed to be associated with an environmental exposure were displaced by only a small amount of time—can be explored by investigating whether an observed increase in mortality across a community during an exposure is then offset by a lower-than-expected rate of mortality in the following days and weeks.

All of these questions can be explored through a type of model called a *distributed lag model*. Conceptually, these models investigate how an exposure today is associated with risk of a health outcome not only today, but also in the following days. Because we are concerned about potential confounding by seasonal trends—and have incorporated control for these trends in our models—we often restrict these models to only look up to about a month following exposure. However, this can help in identifying delayed effects of an exposure days to weeks following the initial exposure, and can also help in determining the role of mortality displacement in accounting for some or all of the excess deaths that are associated with an exposure. While some environmental exposures of interest have mostly immediate effects (e.g., hot temperature), others, including many air pollutants, tend to have effects that are stretched over a longer period of one or more days from exposure. As a result, distributed lag models are used widely in environmental epidemiology studies of time series data, to ensure that important associations aren't missed by limiting the model to focus on risk that are detectable on the same day as the exposure.

To fit a distributed lag model, you can start with a fairly straightforward extension of the regression models we've been fitting, especially if you are investigating an exposure that can be plausibly fit with a linear term (rather than a more complex non-linear basis function) in the regression model. Say you are starting, for example, from the following regression model:

$$\log(E(Y)) = \alpha + \beta X + {}' \mathbf{W} + ns(time)$$

In this model, you are using a Poisson regression to investigate how risk of the health outcome ( $Y$ ) changes for every unit increase in the exposure ( $X$ ), assuming a linear relationship between the exposure and the log of the health outcome. The model can include control for confounders like day of week ( $\mathbf{W}$ ) and long-term and seasonal time trends ( $ns(time)$ ). This model should look familiar from some of the models we've fit earlier to the example temperature data.

To expand this model to explore associations of temperature at different lags, the simplest approach is to add a term for each lag—in other words, instead of only having a term for temperature on the same day as the outcome measurement, we'll also include a term for temperature the previous day, and the day before that, and so on. For example, here's what the model would look like if we included terms up to lag 2:

$$\log(E(Y)) = \alpha + \beta_0 X_0 + \beta_1 X_1 + \beta_2 X_2 + {}' \mathbf{W} + ns(time)$$

where  $X_0$  is temperature on the same day as the measured outcome  $Y$  (with an associated coefficient  $\beta_0$ ),  $X_1$  is the temperature the day before (with an associated coefficient  $\beta_1$ ), and  $X_2$  is the temperature two days before (with an associated coefficient  $\beta_2$ ). The interpretation of each of these coefficients is

similar— $\exp(\beta_2)$ , for example, gives our estimate of the change in the relative risk of the outcome  $Y$  for every one-unit increase in the exposure two days prior.

It can become cumbersome to write out the separate terms for each lag day, so you'll often see equations for distributed lag models written using a shorter approach with the  $\sum$  symbol. The  $\sum$  symbol expresses several terms that are added together and so is well-suited for shortening the expression of a distributed lag model. For example, the previous model equation could be rewritten as:

$$\log(E(Y)) = \alpha + \sum_{l=0}^2 \beta_l X_l + {}' \mathbf{W} + ns(time)$$

This version of a distributed lag model is nice because it is fairly simple—we're taking the idea of fitting a linear term for an exposure, and then expanding it to look include the exposure on earlier days by adding the same type of term for exposure measured at each lag. However, it does have some downsides. The main downside is that exposure tends to be very strongly correlated from one day to the next. Any time you put different terms in a regression model that are correlated, you create the risk of *collinearity*, and associated problems with fitting the model. The term *collinearity* refers to the idea that you have two or more columns in your model matrix that give identical (or, if you loosen the definition a bit, very similar) information. To fit coefficients for each term, the modeling algorithm is trying to identify weights to place on each column in the model matrix that result in a model that gives the best likelihood of the data you see. If two or more columns have (almost) identical information, then the algorithm struggles to determine how to define weight across these two columns. The implications are that you can end up with a lot of instability in model coefficients for the columns that are collinear, as well as very inflated estimates of the standard error. Long story short, it can cause problems in your regression model if you include independent variables that are very strongly correlated with each other, and so we usually try to avoid doing that.

There are alternative distributed lag models that can help avoid this instability that can come from fitting a distributed lag by using a separate term for each lag. All of them share a common feature—they somehow fit a function of the lags instead of each separate lag, so that we end up adding fewer terms to the regression model (and, in turn, are somewhat constraining that pattern that the distributed lag effects follow). The most extreme is to use a term that reduces all the lag terms to a single term, which you can do by averaging the exposure across all lags. In other words, you could take the same-day temperature, yesterday's temperature, and the temperature the day before that, and average those three temperatures to create the term you put in the model. The model equation in this case would look like:

$$\log(E(Y)) = \alpha + \beta_{\overline{0-2}} X_{\overline{0-2}} + {}' \mathbf{W} + ns(time)$$

where  $\overline{0-2}$  represents an average over lags 0 to 2, and so  $X_{\overline{0-2}}$  is the average of the exposure on lags 0, 1, and 2 compared to the day that  $Y$  is measured.

This approach avoids any issues with collinearity across exposure terms from different lags, because you're only including a single term for exposure in the model. However, there is a downside—this model will provide you with a single estimate for the lagged effects, without allowing you to explore patterns across lags (for example, is the association mostly on the day of exposure, or are there some delayed effects on following days?).

Another approach is more complex, but it allows you to explore patterns across lags. Instead of reducing the lagged exposure to a single term in the model, you can reduce it to a few terms that represent the basis for a smooth function. Often, a polynomial or natural cubic spline function will be used for this. The details of setting up the model become more complex, but fortunately there are software packages (like the `dlnm` package in R) that help in managing this set-up, as we'll explore in the exercise.

Packages like `dlnm` can help in extending the complexity of the distributed lag model in other ways, too. So far, we have explored using a distributed lag model where we are happy to model the exposure using a linear term. As we've seen in earlier exercises, this isn't ideal for temperature. Instead, temperature has a non-linear association with mortality risk, with the lowest risk at mild temperatures and then increased risk at both lower (colder) and higher (hotter) temperatures.

We can fit a GLM that incorporates the exposure using a non-linear shape while, at the same time, including a non-linear function to describe how the association evolves across lagged time following the exposure. The approach is to extend the idea of using a basis function—instead of using a basis function in a single dimension, we'll use cross of basis functions in two dimensions. One dimension is the level of exposure (e.g., cold to hot temperatures), and the other is the timing between the exposure and the outcome (e.g., same-day to lagged by many days). In terms of the mechanics of building a model matrix to use to fit the regression model, everything follows directly from the idea we explored early of using a function like a spline to create basis variables in that model matrix for a term with a non-linear association with the outcome variable. However, the mechanics do get quite cumbersome as so many terms are included, and so again we will use the `dlnm` package to handle these mechanics as we start to incorporate these cross-basis functions within a GLM.

#### *Applied: Including a spline in a GLM*

For this exercise, you will continue to build up the model that you began in the examples in the previous chapter. The example uses the data provided with one of this chapter's readings, Vicedo-Cabrera et al. (2019).

1. Start with the simple model describing how are daily mortality counts are associated with a linear function of temperature. Include control for

day of week and long-term / seasonal trends. Add to this model a term that describes the lagged effects of temperature up to a week following exposure, using a separate model term for each lag. What patterns do you detect in the lagged effects of temperature?

2. Start with the model from the first question, but change it to use a smooth function of lag to detect lagged effects, rather than fitting a separate term for each lag. Extend the model to look at lagged effects up to a month following exposure. What patterns do you see in these lagged effects?
3. Refine your model to fit for a non-linear, rather than linear, function of temperature, while also incorporating a function to describe lagged effects up to a month following exposure. Based on this model, what does the association between temperature and mortality look like at lag 0 days? How about at lag 21 days? What does the pattern of lagged effects look like when comparing the relative risk of mortality at a hot temperature (e.g., 28 degrees C) to a milder temperature (7 degrees C)? What about when comparing mortality at a cold temperature (e.g., -4 degrees C) to a milder temperature (7 degrees C)? What is the cumulative relative risk at a hot temperature (e.g., 28 degrees C) to a milder temperature (7 degrees C), once you incorporated all lagged effects up to a month after exposure?

*Applied exercise: Example code*

1. Start with the simple model describing how daily mortality counts are associated with a linear function of temperature. Include control for day of week and long-term / seasonal trends. Add to this model a term that describes the lagged effects of temperature up to a week following exposure, using a separate model term for each lag. What patterns do you detect in the lagged effects of temperature?

We'll start by looking at distributed lag associations by fitting a model with separate terms for each of the lags we want to fit. Here, we want to fit up to a week, so we'll want eight terms in total: ones for the same day ( $X_0$ ), the previous day ( $X_1$ ), two days before ( $X_2$ ), three days before ( $X_3$ ), four days before ( $X_4$ ), five days before ( $X_5$ ), six days before ( $X_6$ ), and seven days before ( $X_7$ ).

Right now, our data have temperature lined up with the date of the recorded deaths:

```
obs %>%
  select(date, all, tmean) %>%
  head()

## # A tibble: 6 x 3
##   date      all tmean
##   <date>    <dbl> <dbl>
## 1 1990-01-01  220  3.91
## 2 1990-01-02  257  5.55
```

```

## 3 1990-01-03 245 4.39
## 4 1990-01-04 226 5.43
## 5 1990-01-05 236 6.87
## 6 1990-01-06 235 9.23

```

To add lagged temperature to the model, then, we'll need to add some columns to our data, so that the row for an observation includes not only the temperature on that day, but also on each of the seven previous days. The `lag` function from the `tidyverse` suite of packages can be used to create these columns:

```
obs <- obs %>%
  mutate(tmean_1 = lag(tmean, n = 1),
        tmean_2 = lag(tmean, n = 2),
        tmean_3 = lag(tmean, n = 3),
        tmean_4 = lag(tmean, n = 4),
        tmean_5 = lag(tmean, n = 5),
        tmean_6 = lag(tmean, n = 6),
        tmean_7 = lag(tmean, n = 7))
```

You can see that each of these have taken `tmean` and then offset it by moving it down by one or more rows (down one row for lag 1, two for lag 2, etc.). As a result, we now have columns that give us today's temperature, yesterday's, and so on, all lined up with the row with the observation for daily mortality:

```
obs %>%  
  select(date, all, tmean, tmean_1:tmean_7) %>%  
  head()  
  
## # A tibble: 6 x 10  
##   date       all     tmean tmean_1 tmean_2 tmean_3 tmean_4 tmean_5 tmean_6 tmean_7  
##   <date>    <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>   <dbl>  
## 1 1990-01-01   220    3.91     NA      NA      NA      NA      NA      NA      NA  
## 2 1990-01-02   257    5.55     3.91    3.91     NA      NA      NA      NA      NA  
## 3 1990-01-03   245    4.39     5.55    3.91     NA      NA      NA      NA      NA  
## 4 1990-01-04   226    5.43     4.39    5.55    3.91     NA      NA      NA      NA  
## 5 1990-01-05   236    6.87     5.43    4.39    5.55    3.91     NA      NA      NA  
## 6 1990-01-06   235    9.23     6.87    5.43    4.39    5.55    3.91     NA      NA
```

You'll notice that this process has created some missing data right at the beginning of the dataset—we don't know what the temperature was the day before Jan. 1, 1990, for example, since the study data starts on Jan. 1, 1990. With most time series, we'll have plenty of days in the study period, so we'll usually just allow these early dates to drop when the model is fit.

Now we'll add all these terms to our regression model:

```
ns(time, df = 158),
data = obs, family = "quasipoisson")
```

If you look at the output from the model, you can see that a regression coefficient has been fit for each of these distributed lag terms:

```
dist_lag_mod_1 %>%
  tidy() %>%
  filter(str_detect(term, "tmean"))

## # A tibble: 8 x 5
##   term      estimate std.error statistic p.value
##   <chr>     <dbl>    <dbl>     <dbl>    <dbl>
## 1 tmean     0.00749  0.000592   12.6   2.67e-36
## 2 tmean_1   -0.00255  0.000788  -3.23   1.25e- 3
## 3 tmean_2   -0.00152  0.000797  -1.91   5.68e- 2
## 4 tmean_3   -0.00243  0.000798  -3.05   2.31e- 3
## 5 tmean_4   -0.00106  0.000798  -1.32   1.86e- 1
## 6 tmean_5   -0.000515 0.000797  -0.645  5.19e- 1
## 7 tmean_6   -0.00107  0.000789  -1.36   1.73e- 1
## 8 tmean_7   -0.00209  0.000593  -3.52   4.30e- 4
```

We can pull out all those estimates, calculate the 95% confidence intervals (do this before you take the exponential to get to the relative risk estimate!), and then exponentiate all the terms to get estimates of relative risk of mortality per unit increase in temperature.

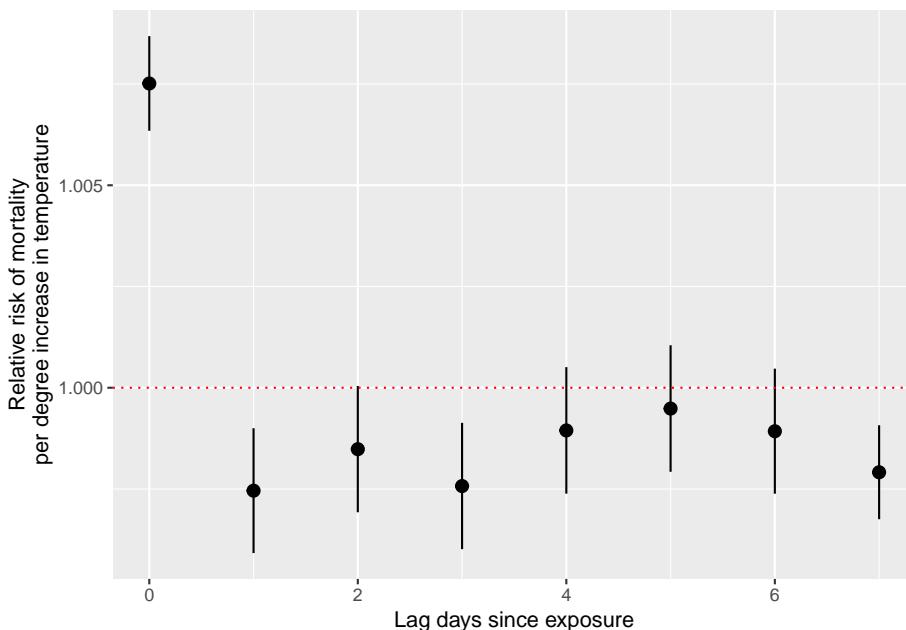
```
dist_lag_terms <- dist_lag_mod_1 %>%
  tidy() %>%
  filter(str_detect(term, "tmean")) %>%
  mutate(lag = 0:7,
        low_ci = estimate - 1.96 * std.error,
        high_ci = estimate + 1.96 * std.error,
        rr = exp(estimate),
        low_rr = exp(low_ci),
        high_rr = exp(high_ci)) %>%
  select(term, lag, rr, low_rr, high_rr)
dist_lag_terms

## # A tibble: 8 x 5
##   term      lag      rr  low_rr high_rr
##   <chr>   <int> <dbl>  <dbl>   <dbl>
## 1 tmean      0  1.01   1.01    1.01
## 2 tmean_1    1  0.997  0.996   0.999
## 3 tmean_2    2  0.998  0.997   1.00
## 4 tmean_3    3  0.998  0.996   0.999
## 5 tmean_4    4  0.999  0.997   1.00
```

```
## 6 tmean_5      5 0.999  0.998  1.00
## 7 tmean_6      6 0.999  0.997  1.00
## 8 tmean_7      7 0.998  0.997  0.999
```

You can plot these estimates to explore how the association changes by lag (notice how the geom `geom_pointrange` is very helpful here!):

```
dist_lag_terms %>%
  ggplot(aes(x = lag, y = rr)) +
  geom_point() +
  geom_pointrange(aes(ymin = low_rr, ymax = high_rr)) +
  labs(x = "Lag days since exposure",
       y = "Relative risk of mortality\nper degree increase in temperature") +
  geom_hline(yintercept = 1, color = "red", linetype = 3)
```



You can also use the `dlnm` package to fit this model, using the `crossbasis` function to set up the distributed lag basis to include in the regression, and then the `crosspred` function to extract results after fitting the regression model and to plot results.

First, you'll again use `crossbasis` to create the basis, as you did earlier when using this package to fit a non-linear function of temperature. You will use the `lag` argument to say the maximum lag you want to include—in this case, seven days (`lag = 7`). You can use the `argvar` and `arglag` functions to specify the basis function for the exposure variable and the distributed lag component. For right now, we'll use a very basic linear function for the exposure (temperature), using `fun = "lin"` within the `argvar` list of arguments. To fit a separate co-

efficient for each lag, we can specify `fun = "integer"` in the `arglag` list of arguments (later we'll explore some other functions for the lag component).

```
library(dlnm)
dl_basis <- crossbasis(obs$tmean, lag = 7,
                        argvar = list(fun = "lin"),
                        arglag = list(fun = "integer"))
```

If you take a peak at this crossbasis, you'll notice that it's doing something similar to what we did when we added columns for lagged temperatures. The first column gives temperature on the same day, the second on the previous day (and so is offset one down from the first column), and so on. The crossbasis function deals with the missing early dates by setting *all* column values to be missing on those days; in practice, this difference in set up won't create any difference in how the model is fit, as the `glm` function will by default exclude any observations with *any* columns of missing data.

```
dl_basis %>%
  head(n = 12)

##          v1.11     v1.12     v1.13     v1.14     v1.15     v1.16     v1.17     v1.18
## [1,]       NA       NA       NA       NA       NA       NA       NA       NA
## [2,]       NA       NA       NA       NA       NA       NA       NA       NA
## [3,]       NA       NA       NA       NA       NA       NA       NA       NA
## [4,]       NA       NA       NA       NA       NA       NA       NA       NA
## [5,]       NA       NA       NA       NA       NA       NA       NA       NA
## [6,]       NA       NA       NA       NA       NA       NA       NA       NA
## [7,]       NA       NA       NA       NA       NA       NA       NA       NA
## [8,] 7.958644 6.686216 9.232628 6.867855 5.431046 4.385564 5.547919 3.913589
## [9,] 7.268950 7.958644 6.686216 9.232628 6.867855 5.431046 4.385564 5.547919
## [10,] 9.510644 7.268950 7.958644 6.686216 9.232628 6.867855 5.431046 4.385564
## [11,] 10.424808 9.510644 7.268950 7.958644 6.686216 9.232628 6.867855 5.431046
## [12,] 7.368595 10.424808 9.510644 7.268950 7.958644 6.686216 9.232628 6.867855
```

We can use this matrix of crossbasis variables now in our regression call:

```
dist_lag_mod_2 <- glm(all ~ dl_basis +
                       factor(dow, ordered = FALSE) +
                       ns(time, df = 158),
                       data = obs, family = "quasipoisson")
```

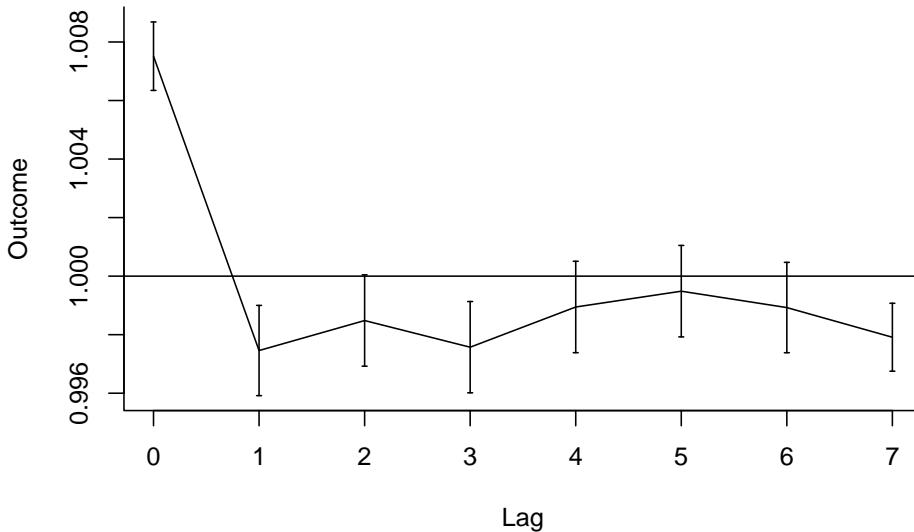
Again, you can pull the regression coefficients directly out of the regression model object (and you can see how they're identical to those from our first distributed lag model):

```
dist_lag_mod_2 %>%
  tidy() %>%
  filter(str_detect(term, "dl_basis"))
```

```
## # A tibble: 8 x 5
##   term      estimate std.error statistic p.value
##   <chr>     <dbl>    <dbl>     <dbl>    <dbl>
## 1 dl_basisv1.11  0.00749  0.000592   12.6   2.67e-36
## 2 dl_basisv1.12 -0.00255  0.000788  -3.23   1.25e- 3
## 3 dl_basisv1.13 -0.00152  0.000797  -1.91   5.68e- 2
## 4 dl_basisv1.14 -0.00243  0.000798  -3.05   2.31e- 3
## 5 dl_basisv1.15 -0.00106  0.000798  -1.32   1.86e- 1
## 6 dl_basisv1.16 -0.000515 0.000797  -0.645  5.19e- 1
## 7 dl_basisv1.17 -0.00107  0.000789  -1.36   1.73e- 1
## 8 dl_basisv1.18 -0.00209  0.000593  -3.52   4.30e- 4
```

You can proceed from here to plot estimates by lag using ggplot, but you can also use some of the special functions for plotting in dlnm. We can plot the “slice” of comparing the relative risk at 1 degree C (var = 1) (this will give us the one-unit increase, because by default the crossbasis set-up with 0 degrees as the baseline, so this is one degree above that value).

```
crosspred(dl_basis, dist_lag_mod_2) %>%
  plot(ptype = "slices", var = 1, ci = "bars")
```



If you look at the correlations between temperature, however, you’ll see they are very strongly correlated, which means that we could have some issues with collinearity if we use this simpler distributed lag model, with a separate coefficient for each lag:

```
obs %>%
  select(tmean, tmean_1:tmean_7) %>%
  cor(use = "complete.obs")
```

```

##          tmean   tmean_1   tmean_2   tmean_3   tmean_4   tmean_5   tmean_6
## tmean  1.0000000 0.9431416 0.8845767 0.8462589 0.8215974 0.8031790 0.7891306
## tmean_1 0.9431416 1.0000000 0.9431447 0.8846236 0.8463419 0.8216464 0.8032469
## tmean_2 0.8845767 0.9431447 1.0000000 0.9431403 0.8846198 0.8463094 0.8216348
## tmean_3 0.8462589 0.8846236 0.9431403 1.0000000 0.9431414 0.8846132 0.8463072
## tmean_4 0.8215974 0.8463419 0.8846198 0.9431414 1.0000000 0.9431482 0.8846132
## tmean_5 0.8031790 0.8216464 0.8463094 0.8846132 0.9431482 1.0000000 0.9431506
## tmean_6 0.7891306 0.8032469 0.8216348 0.8463072 0.8846132 0.9431506 1.0000000
## tmean_7 0.7781337 0.7892093 0.8032205 0.8216271 0.8463046 0.8846283 0.9431482
##          tmean_7
## tmean  0.7781337
## tmean_1 0.7892093
## tmean_2 0.8032205
## tmean_3 0.8216271
## tmean_4 0.8463046
## tmean_5 0.8846283
## tmean_6 0.9431482
## tmean_7 1.0000000

```

Next, we'll explore a model that constrains the lagged effects to follow a smooth function, to help avoid this potential instability in the modeling.

2. Start with the model from the first question, but change it to use a smooth function of lag to detect lagged effects, rather than fitting a separate term for each lag. Extend the model to look at lagged effects up to a month following exposure. What patterns do you see in these lagged effects?

```

dl_basis_2 <- crossbasis(obs$tmean, lag = 7,
                         argvar = list(fun = "lin"),
                         arglag = list(fun = "ns", df = 4))

dl_basis_2 %>%
  head(n = 12)

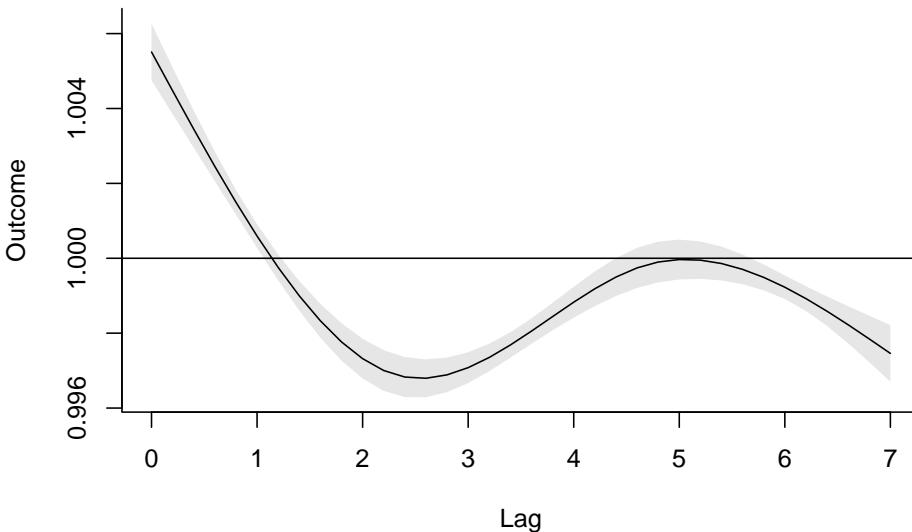
##          v1.11    v1.12    v1.13    v1.14
## [1,]      NA      NA      NA      NA
## [2,]      NA      NA      NA      NA
## [3,]      NA      NA      NA      NA
## [4,]      NA      NA      NA      NA
## [5,]      NA      NA      NA      NA
## [6,]      NA      NA      NA      NA
## [7,]      NA      NA      NA      NA
## [8,] 10.53142 5.938107 17.26767 -2.9117904
## [9,] 11.46186 7.311815 17.99659 -2.0873124
## [10,] 10.91751 8.769682 19.69413 -3.2804334
## [11,] 10.94122 8.867758 22.33138 -2.7169313
## [12,] 13.00391 8.984807 22.20899 -0.3726342

```

```
dist_lag_mod_3 <- glm(all ~ dl_basis_2 +
                      factor(dow, ordered = FALSE) +
                      ns(time, df = 158),
                      data = obs, family = "quasipoisson")
```

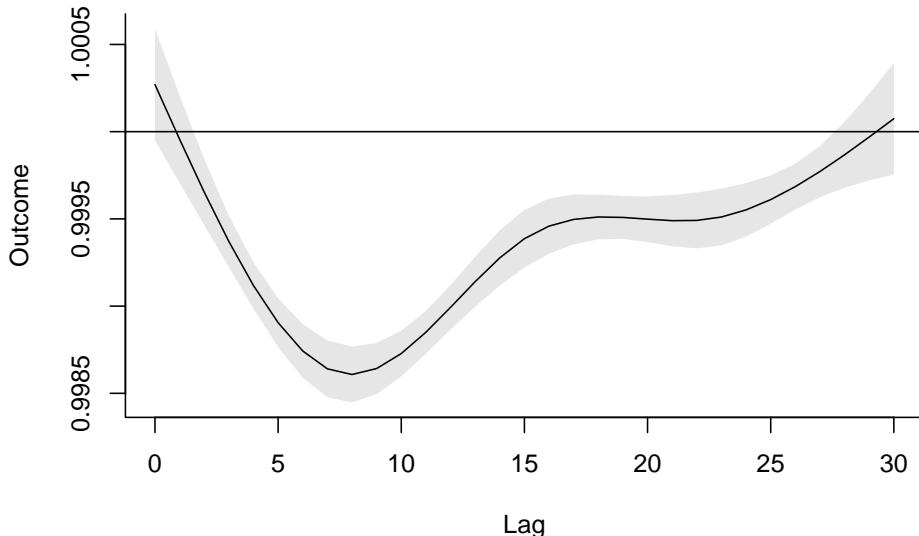
Again, you can plot the results to see the pattern of associations by lag. The plot is just visualizing the results of predicting the model to certain values. By default, it will predict at each lag; this makes the plot a little “jumpy” if you’re just looking at lags for a week or so, so to make it smoother, you might want to ask `crosspred` to predict to a finer resolution along the lag dimension (here, we’re using `bylag = 0.2`).

```
crosspred(dl_basis_2, dist_lag_mod_3, at = 1, bylag = 0.2) %>%
  plot(ptype = "slices", var = 1)
```



Extend out to about month after exposure:

```
dl_basis_3 <- crossbasis(obs$tmean, lag = 30,
                           argvar = list(fun = "lin"),
                           arglag = list(fun = "ns", df = 5))
dist_lag_mod_4 <- glm(all ~ dl_basis_3 +
                      factor(dow, ordered = FALSE) +
                      ns(time, df = 158),
                      data = obs, family = "quasipoisson")
crosspred(dl_basis_3, dist_lag_mod_4, at = 1) %>%
  plot(ptype = "slices", var = 1)
```



3. Refine your model to fit for a non-linear, rather than linear, function of temperature, while also incorporating a function to describe lagged effects up to a month following exposure. Based on this model, what does the association between temperature and mortality look like at lag 0 days? How about at lag 21 days? What does the pattern of lagged effects look like when comparing the relative risk of mortality at a hot temperature (e.g., 28 degrees C) to a milder temperature (7 degrees C)? What about when comparing mortality at a cold temperature (e.g., -4 degrees C) to a milder temperature (7 degrees C)? What is the cumulative relative risk at a hot temperature (e.g., 28 degrees C) to a milder temperature (7 degrees C), once you incorporated all lagged effects up to a month after exposure?

Extend to include non-linear function of temperature:

```
dl_basis_4 <- crossbasis(obs$tmean, lag = 30,
                           argvar = list(fun = "ns", df = 4),
                           arglag = list(fun = "ns", df = 5))
```

```
dl_basis_4 %>%
  as.data.frame() %>%
  slice(28:38)
```

	v1.11	v1.12	v1.13	v1.14	v1.15	v2.11	v2.12
## 1	NA	NA	NA	NA	NA	NA	NA
## 2	NA	NA	NA	NA	NA	NA	NA
## 3	NA	NA	NA	NA	NA	NA	NA
## 4	2.761039	3.997877	2.270266	3.233986	-0.57618666	-0.6476740	-0.9204305
## 5	2.668897	4.023218	2.331192	3.379841	-0.44045281	-0.6808901	-0.9105162

```

## 6 2.597078 4.038829 2.390537 3.454790 -0.33703666 -0.7105258 -0.9052107
## 7 2.564832 4.043947 2.422375 3.492696 -0.08551391 -0.7326065 -0.9045334
## 8 2.491398 4.033905 2.404209 3.593013 0.08504674 -0.7635506 -0.9097586
## 9 2.342457 4.003091 2.320497 3.862538 0.03369753 -0.8334343 -0.9226668
## 10 2.282906 3.949254 2.322582 3.954232 -0.17572292 -0.8553998 -0.9441819
## 11 2.259979 3.877105 2.258828 4.202319 -0.19706918 -0.8828778 -0.9729276
##          v2.13      v2.14      v2.15     v3.11     v3.12     v3.13     v3.14
## 1          NA        NA        NA        NA        NA        NA        NA
## 2          NA        NA        NA        NA        NA        NA        NA
## 3          NA        NA        NA        NA        NA        NA        NA
## 4 -0.5810436 -1.1792423 -0.103868127 1.675142 2.374818 1.442488 2.821684
## 5 -0.5532776 -1.1558093 -0.079483003 1.733576 2.358711 1.396614 2.765277
## 6 -0.5285540 -1.1403549 -0.049230096 1.781872 2.349637 1.356949 2.729928
## 7 -0.5085435 -1.1357676 0.005314667 1.811710 2.347824 1.326218 2.722338
## 8 -0.5025200 -1.1122547 0.050393725 1.862702 2.355930 1.325598 2.672637
## 9 -0.5327496 -0.9920020 0.015000395 1.973101 2.377416 1.380339 2.479005
## 10 -0.5286118 -0.9552027 -0.046539254 2.016020 2.413900 1.377029 2.414106
## 11 -0.5615277 -0.8240561 -0.082721353 2.043549 2.462664 1.424478 2.227474
##          v3.15      v4.11      v4.12     v4.13     v4.14     v4.15
## 1          NA        NA        NA        NA        NA        NA
## 2          NA        NA        NA        NA        NA        NA
## 3          NA        NA        NA        NA        NA        NA
## 4  0.21289849 -0.9586487 -1.359059 -0.8255059 -1.614791 -0.121837367
## 5  0.16334484 -0.9920895 -1.349841 -0.7992530 -1.582510 -0.093478849
## 6  0.10071654 -1.0197280 -1.344648 -0.7765534 -1.562281 -0.057637978
## 7 -0.01746731 -1.0368037 -1.343611 -0.7589669 -1.557937  0.009996179
## 8 -0.11182911 -1.0659854 -1.348250 -0.7586120 -1.529495  0.063997470
## 9 -0.06963965 -1.1291648 -1.360546 -0.7899392 -1.418683  0.039853323
## 10 0.06149683 -1.1537262 -1.381425 -0.7880451 -1.381542 -0.035193354
## 11 0.08860827 -1.1694805 -1.409331 -0.8151992 -1.274737 -0.050708665

dist_lag_mod_5 <- glm(all ~ dl_basis_4 +
  factor(dow, ordered = FALSE) +
  ns(time, df = 158),
  data = obs, family = "quasipoisson")

cp_dl <- crosspred(dl_basis_4, dist_lag_mod_5, cen = 7)

cp_dl %>%
  pluck("matRRfit")

##          lag0      lag1      lag2      lag3      lag4      lag5      lag6
## -5.5 1.0562560 1.0529660 1.0497248 1.0465704 1.0435402 1.0406715 1.0380011
## -5   1.0528499 1.0498015 1.0467970 1.0438712 1.0410584 1.0383925 1.0359071
## -4.5 1.0494653 1.0466560 1.0438860 1.0411867 1.0385892 1.0361243 1.0338226
## -4   1.0461126 1.0435390 1.0410002 1.0385244 1.0361395 1.0338731 1.0317527

```

```

## -3.5 1.0428019 1.0404597 1.0381479 1.0358916 1.0337156 1.0316445 1.0297027
## -3   1.0395434 1.0374272 1.0353373 1.0332958 1.0313243 1.0294444 1.0276776
## -2.5 1.0363469 1.0344505 1.0325767 1.0307443 1.0289720 1.0272786 1.0256825
## -2   1.0332223 1.0315387 1.0298741 1.0282442 1.0266652 1.0251527 1.0237225
## -1.5 1.0301792 1.0287006 1.0272374 1.0258028 1.0244101 1.0230723 1.0218025
## -1   1.0272274 1.0259449 1.0246746 1.0234272 1.0222132 1.0210431 1.0199274
## -0.5 1.0243763 1.0232803 1.0221936 1.0211243 1.0200807 1.0190707 1.0181023
## 0    1.0216353 1.0207153 1.0198021 1.0189013 1.0180188 1.0171605 1.0163320
## 0.5   1.0190138 1.0182586 1.0175078 1.0167650 1.0160338 1.0153181 1.0146214
## 1    1.0165210 1.0159187 1.0153185 1.0147223 1.0141318 1.0135489 1.0129753
## 1.5   1.0141662 1.0137038 1.0132418 1.0127801 1.0123189 1.0118584 1.0113986
## 2    1.0119587 1.0116226 1.0112852 1.0109452 1.0106013 1.0102520 1.0098960
## 2.5   1.0099074 1.0096832 1.0094564 1.0092245 1.0089849 1.0087351 1.0084724
## 3    1.0080217 1.0078941 1.0077629 1.0076248 1.0074760 1.0073131 1.0071325
## 3.5   1.0063106 1.0062636 1.0062124 1.0061528 1.0060805 1.0059913 1.0058810
## 4    1.0047833 1.0048001 1.0048123 1.0048153 1.0048045 1.0047752 1.0047228
## 4.5   1.0034490 1.0035120 1.0035703 1.0036192 1.0036541 1.0036701 1.0036627
## 5    1.0023170 1.0024077 1.0024940 1.0025714 1.0026353 1.0026815 1.0027054
## 5.5   1.0013966 1.0014958 1.0015910 1.0016785 1.0017544 1.0018147 1.0018556
## 6    1.0006973 1.0007847 1.0008692 1.0009477 1.0010174 1.0010752 1.0011182
## 6.5   1.0002285 1.0002832 1.0003362 1.0003859 1.0004305 1.0004685 1.0004981
## 7    1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
## 7.5   1.0000216 0.9999440 0.9998686 0.9997972 0.9997321 0.9996753 0.9996289
## 8    1.0002841 1.0001072 0.9999349 0.9997716 0.9996220 0.9994905 0.9993816
## 8.5   1.0007143 1.0004243 1.0001416 0.9998735 0.9996272 0.9994098 0.9992287
## 9    1.0012257 1.0008183 1.0004211 1.0000441 0.9996974 0.9993910 0.9991348
## 9.5   1.0017315 1.0012119 1.0007054 1.0002247 0.9997825 0.9993917 0.9990649
## 10   1.0021451 1.0015280 1.0009267 1.0003563 0.9998323 0.9993697 0.9989839
## 10.5  1.0023792 1.0016891 1.0010170 1.0003803 0.9997964 0.9992828 0.9988567
## 11   1.0023470 1.0016179 1.0009084 1.0002376 0.9996247 0.9990886 0.9986482
## 11.5  1.0019617 1.0012371 1.0005331 0.9998697 0.9992670 0.9987450 0.9983234
## 12   1.0011781 1.0005056 0.9998538 0.9992431 0.9986937 0.9982257 0.9978594
## 12.5  1.0000856 0.9994987 0.9989323 0.9984064 0.9979410 0.9975562 0.9972719
## 13   0.9988008 0.9983154 0.9978501 0.9974246 0.9970588 0.9967723 0.9965850
## 13.5  0.9974399 0.9970542 0.9966886 0.9963627 0.9960965 0.9959098 0.9958223
## 14   0.9961184 0.9958133 0.9955288 0.9952855 0.9951038 0.9950043 0.9950075
## 14.5  0.9949512 0.9946901 0.9944514 0.9942572 0.9941299 0.9940915 0.9941645
## 15   0.9940528 0.9937820 0.9935367 0.9933422 0.9932238 0.9932068 0.9933167
## 15.5  0.9935374 0.9931859 0.9928648 0.9926043 0.9924344 0.9923855 0.9924877
## 16   0.9935193 0.9929988 0.9925157 0.9921073 0.9918106 0.9916628 0.9917009
## 16.5  0.9940799 0.9932894 0.9925462 0.9918967 0.9913872 0.9910640 0.9909734
## 17   0.9952121 0.9940521 0.9929517 0.9919691 0.9911620 0.9905882 0.9903055
## 17.5  0.9968953 0.9952691 0.9937179 0.9923134 0.9911272 0.9902306 0.9896947
## 18   0.9991095 0.9969234 0.9948306 0.9929187 0.9912749 0.9899860 0.9891385
## 18.5  1.0018357 0.9989985 0.9962761 0.9937742 0.9915974 0.9898496 0.9886346
## 19   1.0050550 1.0014781 0.9980411 0.9948694 0.9920868 0.9898164 0.9881806

```

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## 19.5 1.0087493 1.0043464 1.0001122 0.9961937 0.9927358 0.9898817 0.9877739
## 20 1.0129007 1.0075877 1.0024764 0.9977369 0.9935366 0.9900405 0.9874124
## 20.5 1.0174917 1.0111866 1.0051206 0.9994887 0.9944819 0.9902880 0.9870935
## 21 1.0225048 1.0151279 1.0080321 1.0014389 0.9955641 0.9906196 0.9868149
## 21.5 1.0279228 1.0193962 1.0111980 1.0035773 0.9967758 0.9910303 0.9865743
## 22 1.0337282 1.0239763 1.0146055 1.0058937 0.9981096 0.9915154 0.9863693
## 22.5 1.0399038 1.0288530 1.0182418 1.0083780 0.9995581 0.9920703 0.9861976
## 23 1.0464319 1.0340108 1.0220939 1.0110199 1.0011139 0.9926900 0.9860567
## 23.5 1.0532949 1.0394343 1.0261489 1.0138094 1.0027694 0.9933700 0.9859445
## 24 1.0604747 1.0451076 1.0303936 1.0167359 1.0045174 0.9941054 0.9858584
## 24.5 1.0679528 1.0510149 1.0348150 1.0197893 1.0063502 0.9948915 0.9857963
## 25 1.0757105 1.0571400 1.0393994 1.0229590 1.0082604 0.9957235 0.9857558
## 25.5 1.0837285 1.0634661 1.0441334 1.0262345 1.0102404 0.9965967 0.9857345
## 26 1.0919869 1.0699765 1.0490031 1.0296051 1.0122825 0.9975063 0.9857302
## 26.5 1.1004653 1.0766538 1.0539944 1.0330601 1.0143793 0.9984474 0.9857404
## 27 1.1091426 1.0834803 1.0590932 1.0365884 1.0165228 0.9994153 0.9857630
## 27.5 1.1179972 1.0904378 1.0642847 1.0401791 1.0187054 1.0004052 0.9857955
## 28 1.1270065 1.0975077 1.0695541 1.0438210 1.0209192 1.0014121 0.9858356
## 28.5 1.1361475 1.1046709 1.0748864 1.0475026 1.0231563 1.0024313 0.9858810
## 29 1.1453959 1.1119076 1.0802659 1.0512124 1.0254087 1.0034579 0.9859295
##          lag7      lag8      lag9      lag10     lag11     lag12     lag13
## -5.5 1.0355654 1.0333990 1.0314958 1.0298094 1.0282922 1.0268971 1.0255771
## -5 1.0336357 1.0316099 1.0298243 1.0282365 1.0268031 1.0254809 1.0242272
## -4.5 1.0317142 1.0298281 1.0281593 1.0266695 1.0253194 1.0240699 1.0228821
## -4 1.0298056 1.0280577 1.0265045 1.0251118 1.0238444 1.0226670 1.0215448
## -3.5 1.0279144 1.0263027 1.0248637 1.0235670 1.0223813 1.0212755 1.0202184
## -3 1.0260451 1.0245672 1.0232405 1.0220385 1.0209335 1.0198983 1.0189056
## -2.5 1.0242022 1.0228552 1.0216387 1.0205295 1.0195041 1.0185386 1.0176096
## -2 1.0223902 1.0211708 1.0200618 1.0190436 1.0180962 1.0171994 1.0163332
## -1.5 1.0206135 1.0195179 1.0185134 1.0175841 1.0167131 1.0158837 1.0150794
## -1 1.0188766 1.0179004 1.0169973 1.0161543 1.0153579 1.0145946 1.0138511
## -0.5 1.0171837 1.0163223 1.0155169 1.0147576 1.0140337 1.0133350 1.0126512
## 0 1.0155392 1.0147875 1.0140759 1.0133972 1.0127437 1.0121080 1.0114825
## 0.5 1.0139475 1.0132999 1.0126778 1.0120765 1.0114911 1.0109165 1.0103480
## 1 1.0124129 1.0118633 1.0113261 1.0107987 1.0102788 1.0097635 1.0092504
## 1.5 1.0109396 1.0104816 1.0100243 1.0095672 1.0091099 1.0086519 1.0081927
## 2 1.0095320 1.0091587 1.0087760 1.0083852 1.0079877 1.0075848 1.0071778
## 2.5 1.0081943 1.0078983 1.0075846 1.0072559 1.0069151 1.0065649 1.0062083
## 3 1.0069307 1.0067043 1.0064537 1.0061826 1.0058952 1.0055954 1.0052873
## 3.5 1.0057454 1.0055805 1.0053867 1.0051685 1.0049310 1.0046790 1.0044174
## 4 1.0046428 1.0045308 1.0043871 1.0042169 1.0040256 1.0038188 1.0036016
## 4.5 1.0036271 1.0035588 1.0034583 1.0033310 1.0031822 1.0030176 1.0028427
## 5 1.0027024 1.0026685 1.0026040 1.0025139 1.0024036 1.0022784 1.0021434
## 5.5 1.0018731 1.0018637 1.0018275 1.0017691 1.0016931 1.0016041 1.0015067
## 6 1.0011434 1.0011481 1.0011323 1.0010997 1.0010536 1.0009976 1.0009353
## 6.5 1.0005176 1.0005256 1.0005220 1.0005089 1.0004882 1.0004620 1.0004321

```

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## 7   1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
## 7.5 0.9995949 0.9995753 0.9995699 0.9995763 0.9995921 0.9996147 0.9996418
## 8   0.9993000 0.9992497 0.9992305 0.9992371 0.9992641 0.9993060 0.9993574
## 8.5 0.9990909 0.9990034 0.9989655 0.9989686 0.9990042 0.9990633 0.9991374
## 9   0.9989388 0.9988126 0.9987550 0.9987542 0.9987980 0.9988741 0.9989703
## 9.5 0.9988148 0.9986535 0.9985795 0.9985774 0.9986315 0.9987258 0.9988446
## 10  0.9986899 0.9985024 0.9984192 0.9984217 0.9984903 0.9986058 0.9987487
## 10.5 0.9985355 0.9983357 0.9982547 0.9982704 0.9983603 0.9985015 0.9986714
## 11  0.9983225 0.9981297 0.9980662 0.9981073 0.9982272 0.9984003 0.9986009
## 11.5 0.9980223 0.9978606 0.9978341 0.9979156 0.9980769 0.9982898 0.9985261
## 12  0.9976148 0.9975111 0.9975434 0.9976823 0.9978978 0.9981596 0.9984374
## 12.5 0.9971082 0.9970839 0.9971930 0.9974045 0.9976864 0.9980066 0.9983330
## 13  0.9965165 0.9965856 0.9967849 0.9970816 0.9974411 0.9978295 0.9982122
## 13.5 0.9958538 0.9960230 0.9963213 0.9967128 0.9971602 0.9976265 0.9980743
## 14  0.9951339 0.9954029 0.9958040 0.9962975 0.9968420 0.9973963 0.9979188
## 14.5 0.9943710 0.9947320 0.9952351 0.9958351 0.9964848 0.9971372 0.9977450
## 15  0.9935788 0.9940170 0.9946167 0.9953249 0.9960869 0.9968478 0.9975523
## 15.5 0.9927713 0.9932647 0.9939507 0.9947664 0.9956467 0.9965265 0.9973401
## 16  0.9919622 0.9924817 0.9932391 0.9941588 0.9951626 0.9961719 0.9971079
## 16.5 0.9911620 0.9916734 0.9924840 0.9935023 0.9946336 0.9957831 0.9968552
## 17  0.9903717 0.9908414 0.9916871 0.9927984 0.9940613 0.9953611 0.9965828
## 17.5 0.9895909 0.9899868 0.9908502 0.9920493 0.9934474 0.9949075 0.9962915
## 18  0.9888192 0.9891104 0.9899751 0.9912569 0.9927938 0.9944234 0.9959821
## 18.5 0.9880564 0.9882134 0.9890636 0.9904231 0.9921022 0.9939103 0.9956553
## 19  0.9873018 0.9872968 0.9881173 0.9895500 0.9913745 0.9933695 0.9953120
## 19.5 0.9865553 0.9863616 0.9871382 0.9886396 0.9906124 0.9928024 0.9949530
## 20  0.9858164 0.9854088 0.9861279 0.9876937 0.9898178 0.9922103 0.9945791
## 20.5 0.9850847 0.9844394 0.9850882 0.9867145 0.9889924 0.9915946 0.9941910
## 21  0.9843599 0.9834545 0.9840209 0.9857039 0.9881380 0.9909566 0.9937896
## 21.5 0.9836415 0.9824549 0.9829278 0.9846639 0.9872565 0.9902977 0.9933756
## 22  0.9829292 0.9814418 0.9818105 0.9835963 0.9863496 0.9896193 0.9929499
## 22.5 0.9822227 0.9804161 0.9806708 0.9825032 0.9854191 0.9889226 0.9925133
## 23  0.9815214 0.9793789 0.9795104 0.9813866 0.9844667 0.9882091 0.9920664
## 23.5 0.9808252 0.9783310 0.9783312 0.9802484 0.9834943 0.9874801 0.9916103
## 24  0.9801335 0.9772735 0.9771347 0.9790904 0.9825036 0.9867369 0.9911455
## 24.5 0.9794461 0.9762075 0.9759228 0.9779147 0.9814964 0.9859809 0.9906730
## 25  0.9787625 0.9751337 0.9746970 0.9767232 0.9804744 0.9852133 0.9901934
## 25.5 0.9780823 0.9740533 0.9734592 0.9755178 0.9794393 0.9844356 0.9897077
## 26  0.9774053 0.9729672 0.9722110 0.9743004 0.9783930 0.9836490 0.9892165
## 26.5 0.9767310 0.9718764 0.9709541 0.9730728 0.9773372 0.9828548 0.9887207
## 27  0.9760592 0.9707818 0.9696902 0.9718371 0.9762735 0.9820545 0.9882211
## 27.5 0.9753893 0.9696844 0.9684209 0.9705950 0.9752038 0.9812492 0.9877184
## 28  0.9747210 0.9685852 0.9671479 0.9693485 0.9741296 0.9804403 0.9872134
## 28.5 0.9740541 0.9674850 0.9658728 0.9680993 0.9730528 0.9796292 0.9867068
## 29  0.9733880 0.9663849 0.9645974 0.9668494 0.9719750 0.9788170 0.9861996
##           lag14    lag15    lag16    lag17    lag18    lag19    lag20

```

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## -5.5 1.0242855 1.0229759 1.0216148 1.0202197 1.0188205 1.0174469 1.0161287
## -5 1.0229992 1.0217545 1.0204626 1.0191401 1.0178151 1.0165156 1.0152694
## -4.5 1.0217174 1.0205374 1.0193144 1.0180642 1.0168131 1.0155873 1.0144129
## -4 1.0204431 1.0193274 1.0181729 1.0169945 1.0158168 1.0146642 1.0135611
## -3.5 1.0191792 1.0181272 1.0170406 1.0159335 1.0148286 1.0137486 1.0127160
## -3 1.0179284 1.0169396 1.0159203 1.0148837 1.0138507 1.0128424 1.0118797
## -2.5 1.0166937 1.0157673 1.0148145 1.0138475 1.0128856 1.0119480 1.0110541
## -2 1.0154778 1.0146132 1.0137259 1.0128275 1.0119354 1.0110675 1.0102412
## -1.5 1.0142837 1.0134799 1.0126571 1.0118260 1.0110026 1.0102029 1.0094429
## -1 1.0131141 1.0123701 1.0116107 1.0108457 1.0100895 1.0093565 1.0086612
## -0.5 1.0119718 1.0112866 1.0105893 1.0098888 1.0091982 1.0085303 1.0078981
## 0 1.0108597 1.0102321 1.0095953 1.0089579 1.0083311 1.0077265 1.0071556
## 0.5 1.0097805 1.0092091 1.0086315 1.0080553 1.0074905 1.0069472 1.0064355
## 1 1.0087369 1.0082205 1.0077004 1.0071835 1.0066786 1.0061945 1.0057398
## 1.5 1.0077318 1.0072688 1.0068044 1.0063449 1.0058978 1.0054705 1.0050705
## 2 1.0067680 1.0063568 1.0059463 1.0055420 1.0051502 1.0047773 1.0044295
## 2.5 1.0058481 1.0054870 1.0051284 1.0047770 1.0044381 1.0041169 1.0038187
## 3 1.0049749 1.0046622 1.0043534 1.0040525 1.0037639 1.0034916 1.0032400
## 3.5 1.0041511 1.0038850 1.0036238 1.0033708 1.0031296 1.0029034 1.0026954
## 4 1.0033796 1.0031581 1.0029420 1.0027344 1.0025377 1.0023543 1.0021869
## 4.5 1.0026630 1.0024841 1.0023108 1.0021456 1.0019902 1.0018465 1.0017163
## 5 1.0020041 1.0018655 1.0017325 1.0016068 1.0014896 1.0013821 1.0012856
## 5.5 1.0014055 1.0013052 1.0012097 1.0011204 1.0010380 1.0009632 1.0008966
## 6 1.0008702 1.0008058 1.0007450 1.0006889 1.0006377 1.0005917 1.0005514
## 6.5 1.0004008 1.0003698 1.0003409 1.0003146 1.0002909 1.0002700 1.0002519
## 7 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
## 7.5 0.9996707 0.9996990 0.9997248 0.9997475 0.9997672 0.9997839 0.9997976
## 8 0.9994129 0.9994670 0.9995153 0.9995570 0.9995923 0.9996215 0.9996446
## 8.5 0.9992177 0.9992954 0.9993634 0.9994206 0.9994677 0.9995052 0.9995337
## 9 0.9990743 0.9991738 0.9992589 0.9993284 0.9993837 0.9994259 0.9994562
## 9.5 0.9989719 0.9990920 0.9991919 0.9992708 0.9993309 0.9993743 0.9994032
## 10 0.9988998 0.9990395 0.9991524 0.9992380 0.9992998 0.9993414 0.9993661
## 10.5 0.9988472 0.9990062 0.9991304 0.9992202 0.9992810 0.9993179 0.9993362
## 11 0.9988033 0.9989817 0.9991159 0.9992078 0.9992650 0.9992947 0.9993046
## 11.5 0.9987574 0.9989557 0.9990990 0.9991911 0.9992422 0.9992627 0.9992627
## 12 0.9987011 0.9989204 0.9990721 0.9991627 0.9992057 0.9992148 0.9992035
## 12.5 0.9986334 0.9988757 0.9990359 0.9991234 0.9991558 0.9991509 0.9991264
## 13 0.9985550 0.9988234 0.9989926 0.9990754 0.9990945 0.9990724 0.9990317
## 13.5 0.9984664 0.9987652 0.9989444 0.9990212 0.9990239 0.9989806 0.9989198
## 14 0.9983681 0.9987027 0.9988936 0.9989630 0.9989458 0.9988769 0.9987911
## 14.5 0.9982609 0.9986376 0.9988425 0.9989032 0.9988623 0.9987624 0.9986462
## 15 0.9981454 0.9985716 0.9987933 0.9988441 0.9987754 0.9986387 0.9984854
## 15.5 0.9980220 0.9985063 0.9987482 0.9987880 0.9986871 0.9985069 0.9983091
## 16 0.9978915 0.9984434 0.9987095 0.9987372 0.9985993 0.9983685 0.9981178
## 16.5 0.9977543 0.9983841 0.9986788 0.9986935 0.9985135 0.9982244 0.9979119
## 17 0.9976107 0.9983285 0.9986561 0.9986567 0.9984298 0.9980751 0.9976920

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## 17.5 0.9974609 0.9982765 0.9986411 0.9986267 0.9983483 0.9979206 0.9974586
## 18 0.9973053 0.9982278 0.9986333 0.9986032 0.9982687 0.9977613 0.9972124
## 18.5 0.9971441 0.9981824 0.9986326 0.9985858 0.9981910 0.9975973 0.9969538
## 19 0.9969776 0.9981401 0.9986386 0.9985744 0.9981151 0.9974289 0.9966835
## 19.5 0.9968059 0.9981007 0.9986510 0.9985686 0.9980410 0.9972563 0.9964020
## 20 0.9966294 0.9980642 0.9986694 0.9985681 0.9979685 0.9970796 0.9961099
## 20.5 0.9964483 0.9980303 0.9986937 0.9985727 0.9978976 0.9968991 0.9958077
## 21 0.9962629 0.9979989 0.9987233 0.9985821 0.9978281 0.9967150 0.9954960
## 21.5 0.9960734 0.9979699 0.9987581 0.9985960 0.9977600 0.9965276 0.9951754
## 22 0.9958800 0.9979431 0.9987977 0.9986142 0.9976933 0.9963369 0.9948464
## 22.5 0.9956831 0.9979184 0.9988418 0.9986363 0.9976277 0.9961433 0.9945096
## 23 0.9954829 0.9978956 0.9988901 0.9986621 0.9975633 0.9959470 0.9941656
## 23.5 0.9952796 0.9978746 0.9989423 0.9986913 0.9974999 0.9957481 0.9938149
## 24 0.9950735 0.9978552 0.9989981 0.9987237 0.9974375 0.9955469 0.9934580
## 24.5 0.9948649 0.9978374 0.9990570 0.9987589 0.9973759 0.9953436 0.9930956
## 25 0.9946539 0.9978209 0.9991189 0.9987967 0.9973152 0.9951383 0.9927282
## 25.5 0.9944409 0.9978056 0.9991835 0.9988367 0.9972552 0.9949314 0.9923563
## 26 0.9942261 0.9977913 0.9992503 0.9988788 0.9971957 0.9947230 0.9919806
## 26.5 0.9940098 0.9977780 0.9993191 0.9989227 0.9971369 0.9945133 0.9916015
## 27 0.9937922 0.9977654 0.9993896 0.9989680 0.9970784 0.9943026 0.9912196
## 27.5 0.9935736 0.9977535 0.9994614 0.9990145 0.9970204 0.9940910 0.9908356
## 28 0.9933542 0.9977420 0.9995343 0.9990620 0.9969626 0.9938787 0.9904498
## 28.5 0.9931342 0.9977309 0.9996079 0.9991100 0.9969050 0.9936661 0.9900630
## 29 0.9929140 0.9977199 0.9996819 0.9991585 0.9968475 0.9934532 0.9896756
##          lag21     lag22     lag23     lag24     lag25     lag26     lag27
## -5.5 1.0148952 1.0137760 1.0127991 1.0119686 1.0112642 1.0106648 1.0101492
## -5 1.0141043 1.0130479 1.0121267 1.0113444 1.0106818 1.0101188 1.0096354
## -4.5 1.0133158 1.0123220 1.0114563 1.0107220 1.0101010 1.0095742 1.0091229
## -4 1.0125316 1.0115999 1.0107893 1.0101027 1.0095230 1.0090322 1.0086126
## -3.5 1.0117535 1.0108834 1.0101273 1.0094879 1.0089490 1.0084939 1.0081057
## -3 1.0109833 1.0101740 1.0094718 1.0088789 1.0083804 1.0079603 1.0076031
## -2.5 1.0102229 1.0094735 1.0088242 1.0082772 1.0078182 1.0074327 1.0071058
## -2 1.0094740 1.0087834 1.0081861 1.0076840 1.0072639 1.0069121 1.0066149
## -1.5 1.0087384 1.0081053 1.0075589 1.0071007 1.0067185 1.0063996 1.0061313
## -1 1.0080180 1.0074410 1.0069442 1.0065287 1.0061833 1.0058963 1.0056560
## -0.5 1.0073145 1.0067921 1.0063434 1.0059693 1.0056596 1.0054034 1.0051901
## 0 1.0066296 1.0061601 1.0057579 1.0054238 1.0051485 1.0049220 1.0047346
## 0.5 1.0059653 1.0055467 1.0051894 1.0048937 1.0046512 1.0044531 1.0042904
## 1 1.0053233 1.0049536 1.0046391 1.0043802 1.0041691 1.0039978 1.0038585
## 1.5 1.0047053 1.0043823 1.0041087 1.0038846 1.0037032 1.0035573 1.0034399
## 2 1.0041131 1.0038344 1.0035995 1.0034083 1.0032548 1.0031327 1.0030357
## 2.5 1.0035485 1.0033116 1.0031131 1.0029527 1.0028252 1.0027251 1.0026469
## 3 1.0030132 1.0028155 1.0026509 1.0025190 1.0024155 1.0023355 1.0022743
## 3.5 1.0025091 1.0023477 1.0022143 1.0021087 1.0020269 1.0019650 1.0019190
## 4 1.0020379 1.0019097 1.0018049 1.0017229 1.0016606 1.0016148 1.0015821
## 4.5 1.0016013 1.0015033 1.0014241 1.0013631 1.0013179 1.0012859 1.0012644

```

```

## 5   1.0012011 1.0011300 1.0010733 1.0010306 1.0010000 1.0009795 1.0009670
## 5.5 1.0008391 1.0007914 1.0007540 1.0007267 1.0007080 1.0006966 1.0006909
## 6   1.0005171 1.0004891 1.0004677 1.0004527 1.0004432 1.0004383 1.0004370
## 6.5 1.0002368 1.0002248 1.0002159 1.0002101 1.0002068 1.0002057 1.0002064
## 7   1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
## 7.5 0.9998085 0.9998164 0.9998215 0.9998239 0.9998240 0.9998222 0.9998189
## 8   0.9996620 0.9996738 0.9996803 0.9996818 0.9996791 0.9996729 0.9996639
## 8.5 0.9995539 0.9995662 0.9995712 0.9995698 0.9995627 0.9995511 0.9995357
## 9   0.9994758 0.9994860 0.9994880 0.9994829 0.9994716 0.9994553 0.9994349
## 9.5 0.9994198 0.9994262 0.9994245 0.9994163 0.9994026 0.9993842 0.9993622
## 10  0.9993776 0.9993792 0.9993744 0.9993652 0.9993523 0.9993364 0.9993180
## 10.5 0.9993410 0.9993377 0.9993314 0.9993247 0.9993177 0.9993105 0.9993031
## 11  0.9993020 0.9992945 0.9992894 0.9992899 0.9992954 0.9993051 0.9993181
## 11.5 0.9992524 0.9992422 0.9992420 0.9992560 0.9992822 0.9993188 0.9993635
## 12  0.9991856 0.9991746 0.9991839 0.9992183 0.9992747 0.9993493 0.9994385
## 12.5 0.9991000 0.9990893 0.9991116 0.9991729 0.9992680 0.9993914 0.9995374
## 13  0.9989948 0.9989844 0.9990223 0.9991158 0.9992575 0.9994394 0.9996535
## 13.5 0.9988695 0.9988582 0.9989131 0.9990431 0.9992382 0.9994875 0.9997802
## 14  0.9987236 0.9987090 0.9987814 0.9989511 0.9992053 0.9995298 0.9999107
## 14.5 0.9985562 0.9985351 0.9986241 0.9988359 0.9991539 0.9995606 1.0000385
## 15  0.9983669 0.9983347 0.9984386 0.9986935 0.9990792 0.9995742 1.0001567
## 15.5 0.9981550 0.9981060 0.9982219 0.9985201 0.9989764 0.9995646 1.0002587
## 16  0.9979199 0.9978475 0.9979712 0.9983119 0.9988406 0.9995262 1.0003378
## 16.5 0.9976613 0.9975580 0.9976849 0.9980663 0.9986685 0.9994549 1.0003892
## 17  0.9973800 0.9972385 0.9973637 0.9977843 0.9984608 0.9993512 1.0004133
## 17.5 0.9970770 0.9968901 0.9970092 0.9974672 0.9982191 0.9992164 1.0004109
## 18  0.9967531 0.9965142 0.9966228 0.9971167 0.9979446 0.9990518 1.0003834
## 18.5 0.9964092 0.9961119 0.9962058 0.9967340 0.9976389 0.9988587 1.0003316
## 19  0.9960464 0.9956845 0.9957597 0.9963209 0.9973034 0.9986384 1.0002568
## 19.5 0.9956655 0.9952331 0.9952859 0.9958786 0.9969396 0.9983922 1.0001599
## 20  0.9952673 0.9947590 0.9947859 0.9954088 0.9965488 0.9981214 1.0000422
## 20.5 0.9948529 0.9942634 0.9942609 0.9949128 0.9961324 0.9978272 0.9999046
## 21  0.9944232 0.9937476 0.9937125 0.9943922 0.9956920 0.9975111 0.9997484
## 21.5 0.9939791 0.9932127 0.9931421 0.9938484 0.9952290 0.9971742 0.9995745
## 22  0.9935214 0.9926600 0.9925510 0.9932829 0.9947448 0.9968179 0.9993840
## 22.5 0.9930512 0.9920906 0.9919406 0.9926972 0.9942408 0.9964435 0.9991782
## 23  0.9925692 0.9915059 0.9913125 0.9920928 0.9937184 0.9960523 0.9989580
## 23.5 0.9920765 0.9909070 0.9906679 0.9914711 0.9931791 0.9956456 0.9987245
## 24  0.9915740 0.9902950 0.9900083 0.9908336 0.9926244 0.9952246 0.9984788
## 24.5 0.9910625 0.9896713 0.9893351 0.9901817 0.9920555 0.9947907 0.9982221
## 25  0.9905430 0.9890370 0.9886497 0.9895170 0.9914741 0.9943452 0.9979554
## 25.5 0.9900163 0.9883934 0.9879534 0.9888409 0.9908814 0.9938893 0.9976799
## 26  0.9894834 0.9877415 0.9872477 0.9881548 0.9902789 0.9934243 0.9973965
## 26.5 0.9889452 0.9870827 0.9865339 0.9874602 0.9896680 0.9929516 0.9971064
## 27  0.9884026 0.9864181 0.9858134 0.9867585 0.9890501 0.9924723 0.9968106
## 27.5 0.9878564 0.9857489 0.9850876 0.9860512 0.9884266 0.9919878 0.9965104

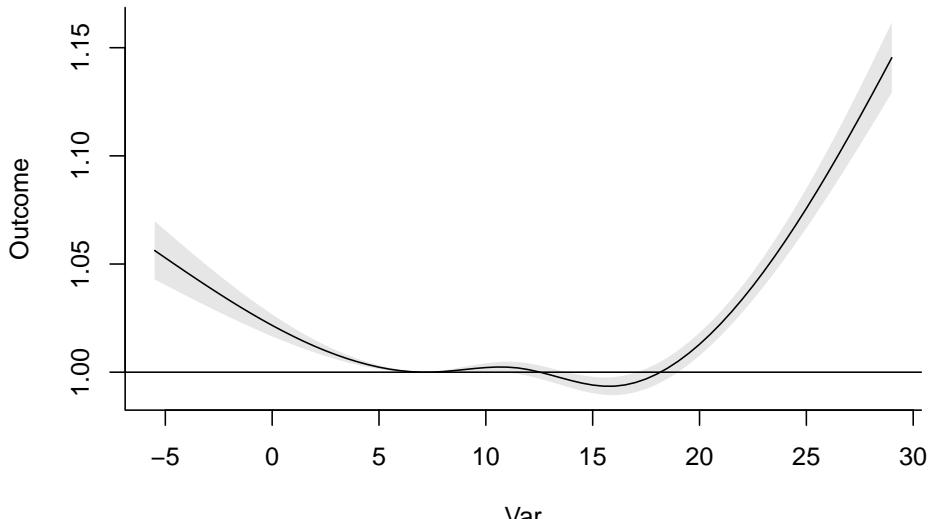
```

```

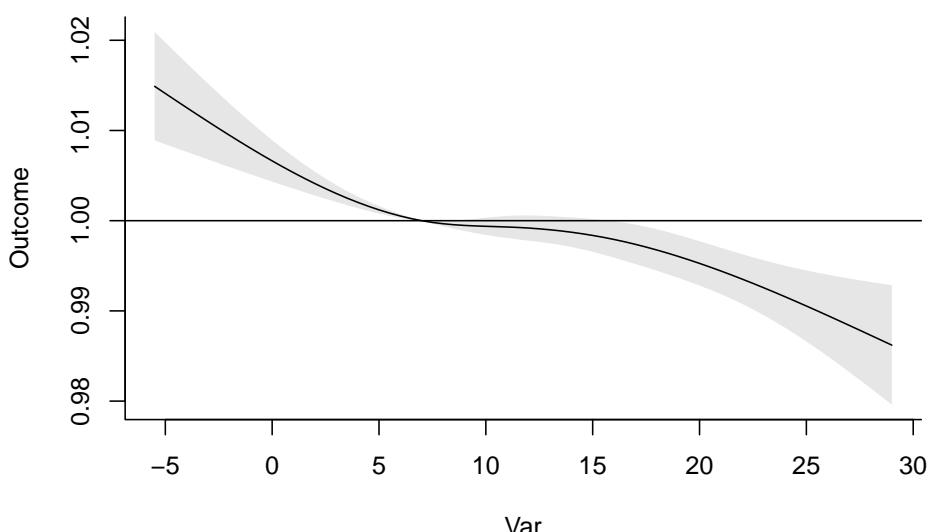
## 28    0.9873076 0.9850763 0.9843579 0.9853398 0.9877990 0.9914994 0.9962066
## 28.5   0.9867571 0.9844015 0.9836256 0.9846256 0.9871686 0.9910084 0.9959005
## 29     0.9862058 0.9837256 0.9828921 0.9839100 0.9865368 0.9905159 0.9955931
##          lag28      lag29      lag30
## -5.5   1.0096966 1.0092858 1.0088961
## -5     1.0092117 1.0088278 1.0084638
## -4.5   1.0087280 1.0083708 1.0080325
## -4     1.0082464 1.0079156 1.0076027
## -3.5   1.0077677 1.0074631 1.0071754
## -3     1.0072929 1.0070141 1.0067510
## -2.5   1.0068229 1.0065694 1.0063305
## -2     1.0063586 1.0061297 1.0059144
## -1.5   1.0059010 1.0056959 1.0055035
## -1     1.0054508 1.0052688 1.0050986
## -0.5   1.0050090 1.0048492 1.0047002
## 0      1.0045765 1.0044380 1.0043092
## 0.5    1.0041542 1.0040358 1.0039263
## 1      1.0037431 1.0036436 1.0035520
## 1.5    1.0033439 1.0032620 1.0031873
## 2      1.0029576 1.0028920 1.0028326
## 2.5    1.0025851 1.0025342 1.0024889
## 3      1.0022272 1.0021896 1.0021567
## 3.5    1.0018850 1.0018589 1.0018368
## 4      1.0015592 1.0015429 1.0015299
## 4.5    1.0012507 1.0012423 1.0012366
## 5      1.0009605 1.0009581 1.0009577
## 5.5    1.0006895 1.0006910 1.0006939
## 6      1.0004384 1.0004417 1.0004459
## 6.5    1.0002083 1.0002111 1.0002143
## 7      1.0000000 1.0000000 1.0000000
## 7.5    0.9998144 0.9998092 0.9998035
## 8      0.9996527 0.9996402 0.9996270
## 8.5    0.9995175 0.9994975 0.9994766
## 9      0.9994115 0.9993860 0.9993595
## 9.5    0.9993373 0.9993107 0.9992831
## 10     0.9992978 0.9992764 0.9992543
## 10.5   0.9992957 0.9992881 0.9992805
## 11     0.9993336 0.9993508 0.9993688
## 11.5   0.9994144 0.9994694 0.9995265
## 12     0.9995387 0.9996462 0.9997574
## 12.5   0.9997004 0.9998746 1.0000546
## 13     0.9998918 1.0001463 1.0004088
## 13.5   1.0001056 1.0004527 1.0008109
## 14     1.0003340 1.0007857 1.0012516
## 14.5   1.0005697 1.0011367 1.0017218
## 15     1.0008050 1.0014974 1.0022122

```

```
## 15.5 1.0010324 1.0018594 1.0027136
## 16 1.0012442 1.0022143 1.0032167
## 16.5 1.0014350 1.0025557 1.0037146
## 17 1.0016048 1.0028834 1.0042065
## 17.5 1.0017542 1.0031977 1.0046926
## 18 1.0018843 1.0034994 1.0051731
## 18.5 1.0019957 1.0037888 1.0056484
## 19 1.0020893 1.0040666 1.0061186
## 19.5 1.0021660 1.0043333 1.0065840
## 20 1.0022266 1.0045894 1.0070448
## 20.5 1.0022719 1.0048355 1.0075013
## 21 1.0023027 1.0050721 1.0079538
## 21.5 1.0023199 1.0052998 1.0084023
## 22 1.0023243 1.0055191 1.0088473
## 22.5 1.0023168 1.0057305 1.0092889
## 23 1.0022982 1.0059346 1.0097274
## 23.5 1.0022693 1.0061321 1.0101631
## 24 1.0022309 1.0063233 1.0105961
## 24.5 1.0021840 1.0065088 1.0110267
## 25 1.0021293 1.0066893 1.0114552
## 25.5 1.0020677 1.0068652 1.0118819
## 26 1.0020000 1.0070371 1.0123069
## 26.5 1.0019270 1.0072056 1.0127305
## 27 1.0018496 1.0073711 1.0131529
## 27.5 1.0017687 1.0075344 1.0135745
## 28 1.0016850 1.0076958 1.0139954
## 28.5 1.0015994 1.0078560 1.0144159
## 29 1.0015128 1.0080155 1.0148363
plot(cp_dl, ptype = "slices", lag = 0)
```

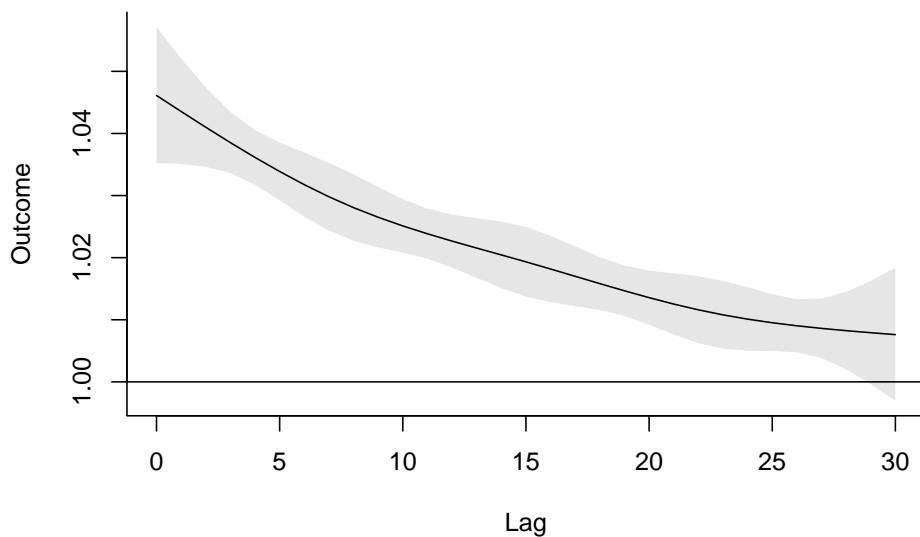
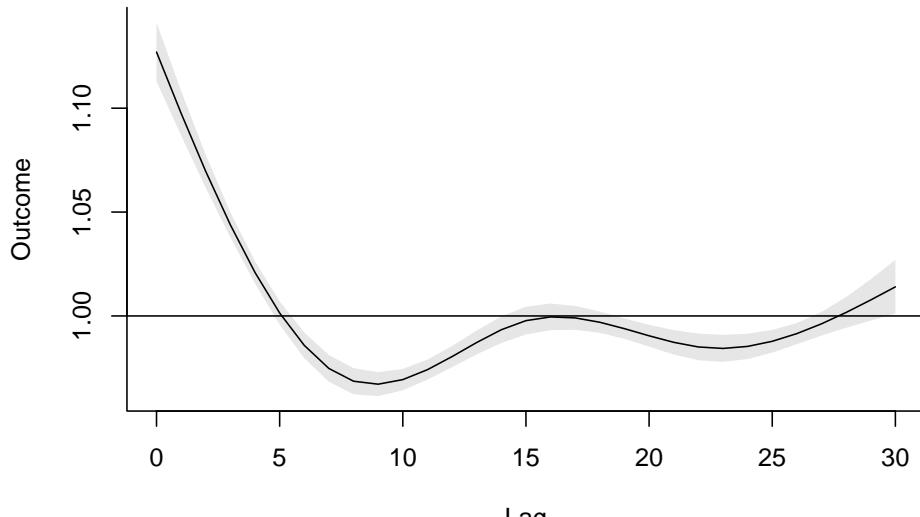


```
plot(cp_dl, ptype = "slices", lag = 21)
```



```
plot(cp_dl, ptype = "slices", var = 28)
```

4.3. DISTRIBUTED LAGS AND CROSS-BASIS FUNCTIONS IN GLMS101





# Chapter 5

## Natural experiments

The readings for this chapter are:

- Bernal et al. (2017) (on interrupted time series), with a correction to an equation in the paper at <https://academic.oup.com/ije/article/49/4/1414/5900884>. Example data and R code for the paper are available to download through a Supplemental Appendix.
- Barone-Adesi et al. (2011), the scientific paper highlighted as an example in the tutorial in the previous reading
- Bor et al. (2014) (on interrupted time series)
- Casey et al. (2018) (on difference-in-differences)
- Mendola (2018), an Invited Commentary on the previous reading

### 5.1 Interrupted time series

[Interrupted time series assessing effects of policy/intervention in specific point in time]

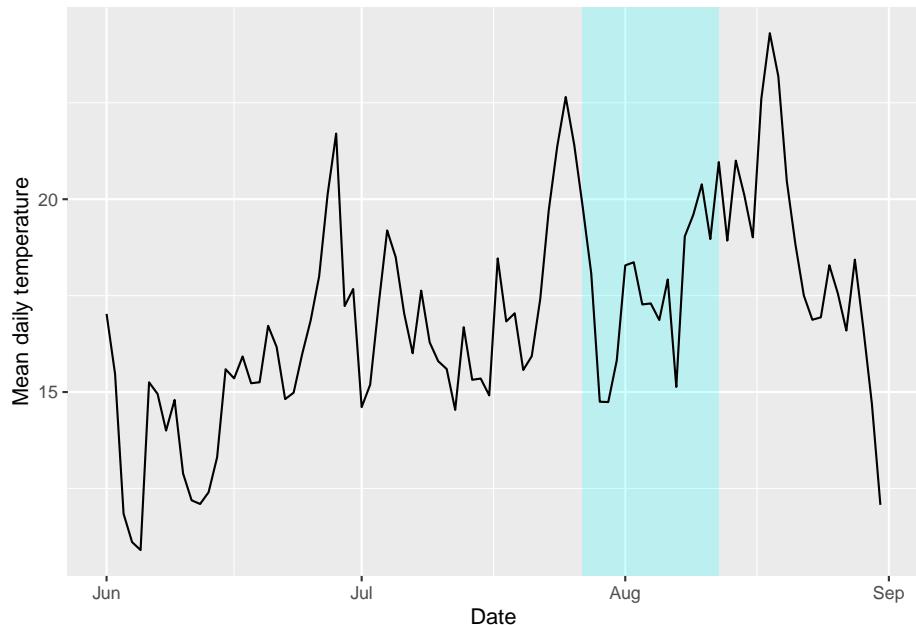
```
# Load some packages that will likely be useful
library(tidyverse)
library(viridis)
library(lubridate)
library(broom)

# Load and clean the data
obs <- read_csv("data/lndn_obs.csv") %>%
  mutate(dow = wday(date, label = TRUE)) %>%
  mutate(time = as.numeric(date) - first(as.numeric(date)))

london_summer_2012 <- obs %>%
  filter(ymd("2012-06-01") <= date & date <= ymd("2012-09-30"))
```

```
london_olympic_dates <- tibble(date = ymd(c("2012-07-27", "2012-08-12")))

ggplot() +
  geom_polygon(aes(x = ymd(c("2012-07-27", "2012-08-12",
                             "2012-08-12", "2012-07-27")),
                     y = c(Inf, Inf, -Inf, -Inf)), fill = "cyan", alpha = 0.2) +
  geom_line(data = london_summer_2012, aes(x = date, y = tmean)) +
  labs(x = "Date", y = "Mean daily temperature")
```



Example data from Bernal et al. (2017):

```
sicily <- read_csv("data/sicily.csv") %>%
  mutate(date = paste(year, month, "15"), # Use middle of the month for plotting
        date = ymd(date))
```

Identify dates of the smoking band:

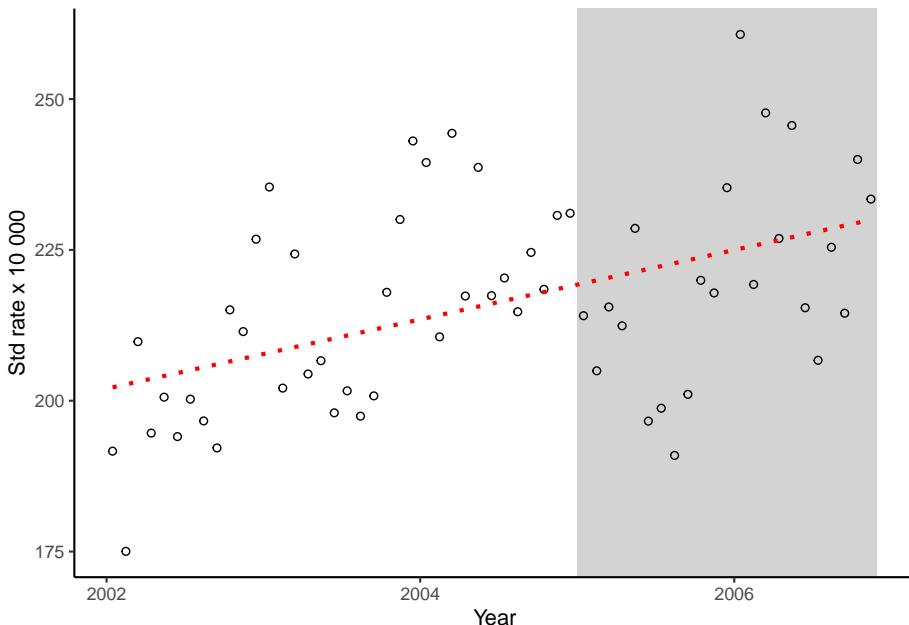
```
sicily %>%
  group_by(smokban) %>%
  slice(c(1, n()))

## # A tibble: 4 x 8
## # Groups:   smokban [2]
##   year month aces time smokban      pop stdpop date
##   <dbl> <dbl> <dbl> <dbl>    <dbl>   <dbl>   <dbl> <date>
## 1  2002     1    728     1       0 364277. 379875. 2002-01-15
```

```
## 2 2004    12   886    36      0 364700. 383428. 2004-12-15
## 3 2005     1   831    37      1 364421. 388153. 2005-01-15
## 4 2006    11   912    59      1 363833. 390712. 2006-11-15
```

Recreate Figure 1 from Bernal et al. (2017):

```
ggplot() +
  geom_polygon(aes(x = ymd(c("2005-01-01", "2006-11-30",
                           "2006-11-30", "2005-01-01"))),
               y = c(Inf, Inf, -Inf, -Inf)), fill = "lightgray") +
  geom_point(data = sicily,
             aes(x = date, y = 10000 * 10 * aces / stdpop), shape = 21) +
  geom_smooth(data = sicily,
              aes(x = date, y = 10000 * 10 * aces / stdpop), # Need the extra 10 to line up with
              method = "lm", se = FALSE, color = "red", linetype = 3) +
  labs(x = "Year", y = "Std rate x 10 000") +
  theme_classic()
```



## 5.2 Difference-in-differences

[Difference-in differences application for intervention introduced in one point in time]



## **Chapter 6**

# **Risk assessment**

[Predict expected heat-related mortality under a climate change scenario]



# Chapter 7

## Longitudinal cohort study designs

The readings for this chapter are

- Andersson et al. (2019)
- Wong et al. (1989)

The following are a series of instructional papers on survival analysis, that are meant as general background on how to fit survival analysis models.

- Clark et al. (2003)
- Bradburn et al. (2003a)
- Bradburn et al. (2003b)

### 7.1 Longitudinal cohort data

Example datasets are available online, but also made available to you on the course website. For the Framingham Heart Study the example data are available as the file “frmgham2.csv”. It is saved in a csv format, and so they can be read into R using the `read_csv` function from the `readr` package (part of the tidyverse). You can use the following code to read in these data, assuming you have saved them in a “data” subdirectory of your current working directory:

```
library(tidyverse) # Loads all the tidyverse packages, including readr
fhs <- read_csv("data/frmgham2.csv")
fhs

## # A tibble: 11,627 x 39
##   RANDID    SEX TOTCHOL     AGE SYSBP DIABP CURSMOKE CIGPDAY     BMI DIABETES BPMEDS
```

```

##   <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl>  <dbl>
## 1 2448     1    195   39  106   70      0     0  27.0     0     0
## 2 2448     1    209   52  121   66      0     0  NA       0     0
## 3 6238     2    250   46  121   81      0     0  28.7     0     0
## 4 6238     2    260   52  105  69.5     0     0  29.4     0     0
## 5 6238     2    237   58  108   66      0     0  28.5     0     0
## 6 9428     1    245   48  128.   80      1    20  25.3     0     0
## 7 9428     1    283   54  141   89      1    30  25.3     0     0
## 8 10552    2    225   61  150   95      1    30  28.6     0     0
## 9 10552    2    232   67  183   109     1    20  30.2     0     0
## 10 11252   2    285   46  130   84     1    23  23.1     0     0
## # ... with 11,617 more rows, and 28 more variables: HEARTRTE <dbl>,
## # GLUCOSE <dbl>, educ <dbl>, PREVCHD <dbl>, PREVAP <dbl>, PREVMI <dbl>,
## # PREVSTRK <dbl>, PREVHYP <dbl>, TIME <dbl>, PERIOD <dbl>, HDLC <dbl>,
## # LDLC <dbl>, DEATH <dbl>, ANGINA <dbl>, HOSPMI <dbl>, MI_FCHD <dbl>,
## # ANYCHD <dbl>, STROKE <dbl>, CVD <dbl>, HYPERTEN <dbl>, TIMEAP <dbl>,
## # TIMEMI <dbl>, TIMEMIFC <dbl>, TIMECHD <dbl>, TIMESTRK <dbl>, TIMECVDF <dbl>,
## # TIMEDTH <dbl>, TIMEHYP <dbl>

```

- One important difference compared to a time-series dataset is the **RANDID** variable. This is the unique identifier for unit for which we have repeated observations for over time. In this case the **RANDID** variable represents a unique identifier for each study participant, with multiple observations (rows) per participant over time.
- The **TIME** variable indicates the number of days that have elapsed since beginning of follow-up of each observation. (**TIME=0** for the first observation of each participant).
- Number of observations varies between participants (typical)
- The time spacing between observations is not constant. This is because the repeated observations in the Framingham Heart Study are the result of follow-up exams happening 3 to 5 years apart. Many longitudinal cohorts will instead have observations over a fixed time interval (monthly, annual, biannual etc), resulting in a more balanced dataset.
- Observations are given for various risk factors, covariates and cardiovascular outcomes. Some will be invariant for each participant over time (**SEX**, **educ**), while others will vary with each exam.

From a data management perspective, we might want to change all the column names to be in lowercase, rather than uppercase. This will save our pinkies some work as we code with the data! You can make that change with the following code, using the **str\_to\_lower** function from the **stringr** package (part of the **tidyverse**):

```

fhs <- fhs %>%
  rename_all(.funs = str_to_lower)
fhs

```

```

## # A tibble: 11,627 x 39
##   randid   sex totchol    age sysbp diabp cursmoke cigpday    bmi diabetes bpmeds
##   <dbl> <dbl>
## 1 2448     1    195    39   106    70        0      0  27.0      0     0
## 2 2448     1    209    52   121    66        0      0  NA       0     0
## 3 6238     2    250    46   121    81        0      0  28.7      0     0
## 4 6238     2    260    52   105   69.5       0      0  29.4      0     0
## 5 6238     2    237    58   108    66        0      0  28.5      0     0
## 6 9428     1    245    48   128.   80        1      20  25.3      0     0
## 7 9428     1    283    54   141    89        1      30  25.3      0     0
## 8 10552    2    225    61   150    95        1      30  28.6      0     0
## 9 10552    2    232    67   183   109       1      20  30.2      0     0
## 10 11252   2    285    46   130    84       1      23  23.1      0     0
## # ... with 11,617 more rows, and 28 more variables: heartrte <dbl>,
## # glucose <dbl>, educ <dbl>, prevchd <dbl>, prevap <dbl>, prevmi <dbl>,
## # prevstrk <dbl>, prevhyp <dbl>, time <dbl>, period <dbl>, hdlc <dbl>,
## # ldlc <dbl>, death <dbl>, angina <dbl>, hospmi <dbl>, mi_fchd <dbl>,
## # anychd <dbl>, stroke <dbl>, cvd <dbl>, hyperten <dbl>, timeap <dbl>,
## # timemi <dbl>, timemifc <dbl>, timechd <dbl>, timestrk <dbl>, timecvd <dbl>,
## # timedth <dbl>, timehyp <dbl>

```

*Applied exercise:* Exploring longitudinal cohort data Read the example cohort data in R and explore it to answer the following questions:

1. What is the number of participants and number of observations in the `fhs` dataset?
2. Is there any missingness in the data?
3. How many participants die? What is the distribution of age at time of death?
4. What is the distribution of age at time of incident MI? Are there differences between males and females? Are there differences in smoking between males and females?
5. What is the distribution of BMI among MI cases and non-cases? How about between smokers and non-smokers

Based on this exploratory exercise in this section, talk about the potential for confounding when these data are analyzed to estimate the association between smoking and risk of incident MI.

*Applied exercise:* Example code

1. **What is the number of participants and the number of observations in the `fhs` dataset? (i.e what is the sample size and number of person-time observations)**

In the `fhs` dataset, the number of participants will be equal to the number of unique ID's (The `RANDID` variable which takes a unique value for each participant). We can extract this using the `unique` function nested within the `length` function

```
length(unique(fhs$randid))
```

```
## [1] 4434
```

If you'd like to use `tidyverse` tools to answer this question, you can do that, as well. The pipe operator (`%>%`) works on any type of object—it will take your current output and include it as the first parameter value for the function call you pipe into. If you want to perform operations on a column of a dataframe, you can use `pull` to extract it from the dataframe as a vector, and then pipe that into vector operations:

```
fhs %>%
  pull(randid) %>%
  unique() %>%
  length()
```

```
## [1] 4434
```

It's entirely a personal choice whether you use the `$` operator and “nesting” of function calls, versus `pull` and piping to do a series of function calls. You can see you get the same result, so it just comes down to the style that you will find easiest to understand when you look at your code later.

The number of person-time observations will actually be equal to the length of the dataset. The `dim` function gives us the length (number of rows) and width (number of columns) for a dataframe or any matrix like object in R.

```
dim(fhs)
```

```
## [1] 11627    39
```

We see that there is approximately an average of 2 to 3 observations per participants.

When you know there are repeated measurements, it can be helpful to explore how much variation there is in the number of observations per study subject. You could do that in this dataset with the following code:

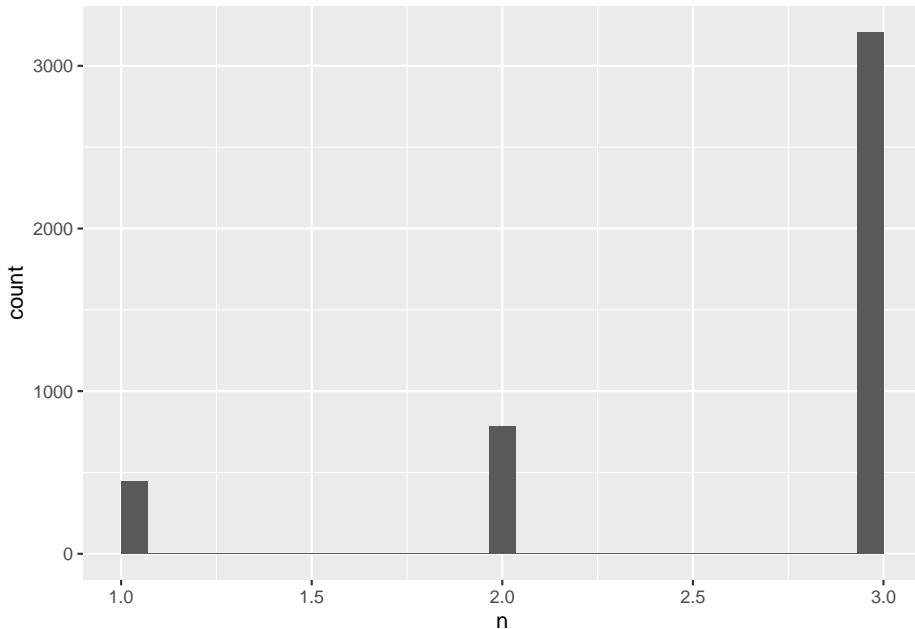
```
fhs %>%
  # Group by the study subject identifier and then count the rows for each
  group_by(randid) %>%
  count() %>%
  # Reorder the dataset so the subjects with the most observations come first
  arrange(desc(n)) %>%
  head()

## # A tibble: 6 x 2
## # Groups:   randid [6]
##   randid     n
##   <dbl> <int>
```

```
## 1   6238    3
## 2   11252    3
## 3   11263    3
## 4   12806    3
## 5   14367    3
## 6   16365    3
```

You can visualize this, as well. A histogram is one good choice:

```
fhs %>%
  # Group by the study subject identifier and then count the rows for each
  group_by(randid) %>%
  count() %>%
  ggplot(aes(x = n)) +
  geom_histogram()
```



All study subjects have between one and three measurements. Most of the study subjects (over 3,000) have three measurements recorded in the dataset.

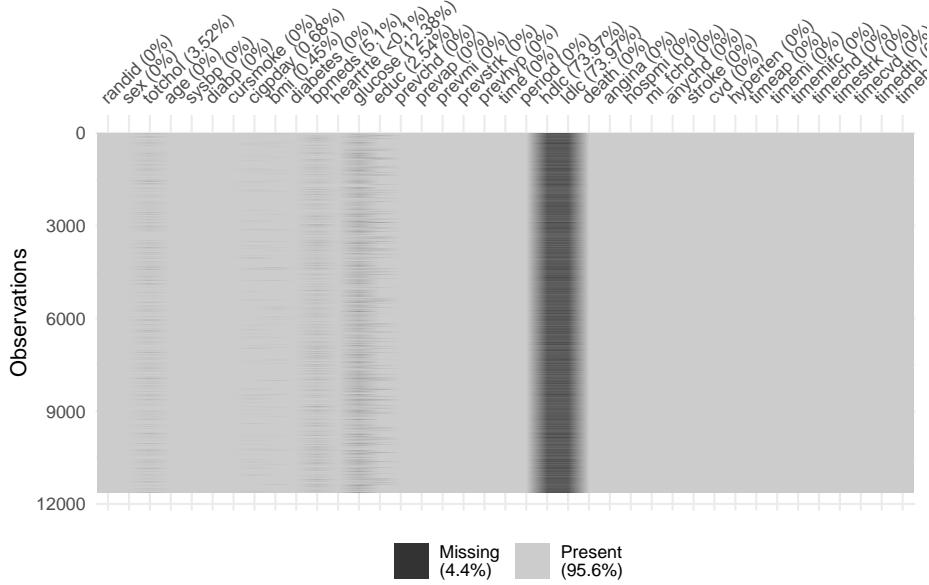
## 2. Is there any missingness in the data?

We can check for missingness in a number of ways. There are a couple of great packages, `visdat` and `naniar`, that include functions for investigating missingness in a dataset. If you don't have these installed, you can install them using `install.packages("naniar")` and `install.packages("visdat")`. The `naniar` package has a vignette with examples that is a nice starting point for working with both packages.

The `vis_miss` function shows missingness in a dataset in a way that lets you

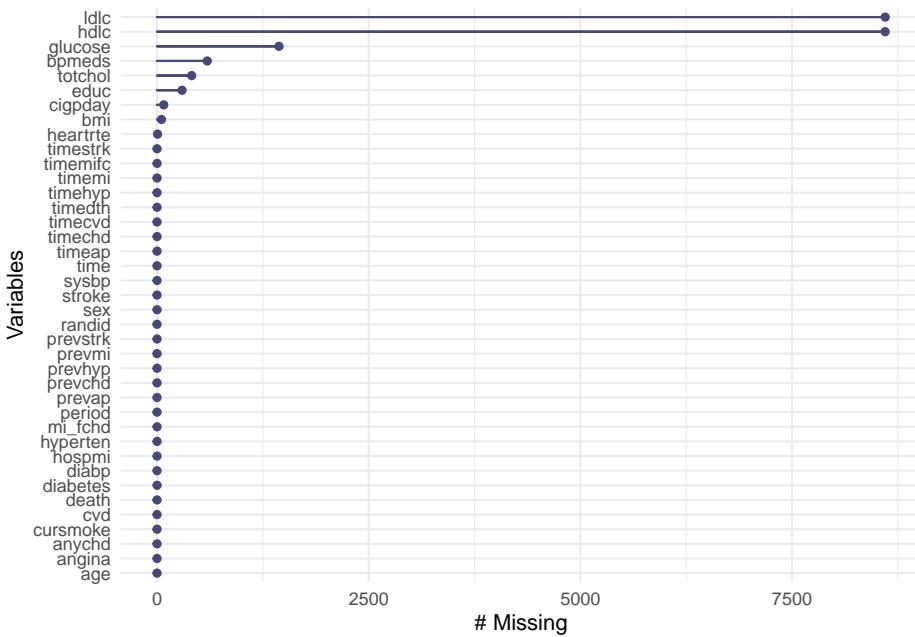
get a top-level snapshot:

```
library(visdat)
vis_miss(fhs)
```



Another was to visualize this is with `gg_miss_var`:

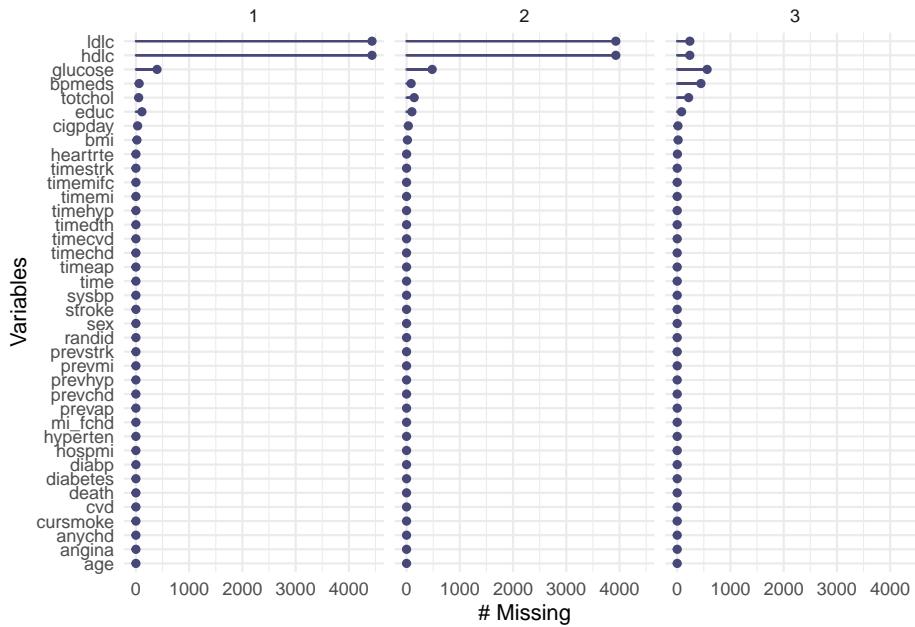
```
library(naniar)
gg_miss_var(fhs)
```



Many of the variables are available for all observations, with no missingness, including records of the subject's ID, measures of death, stroke, CVD, and other events, age, sex, and BMI. Some of the measured values from visits are missing occasionally, like the total cholesterol, and glucose. Other measures asked of the participants (number of cigarettes per day, education) are occasionally missing. Two of the variables—hd1c and ld1c—are missing more often than they are available.

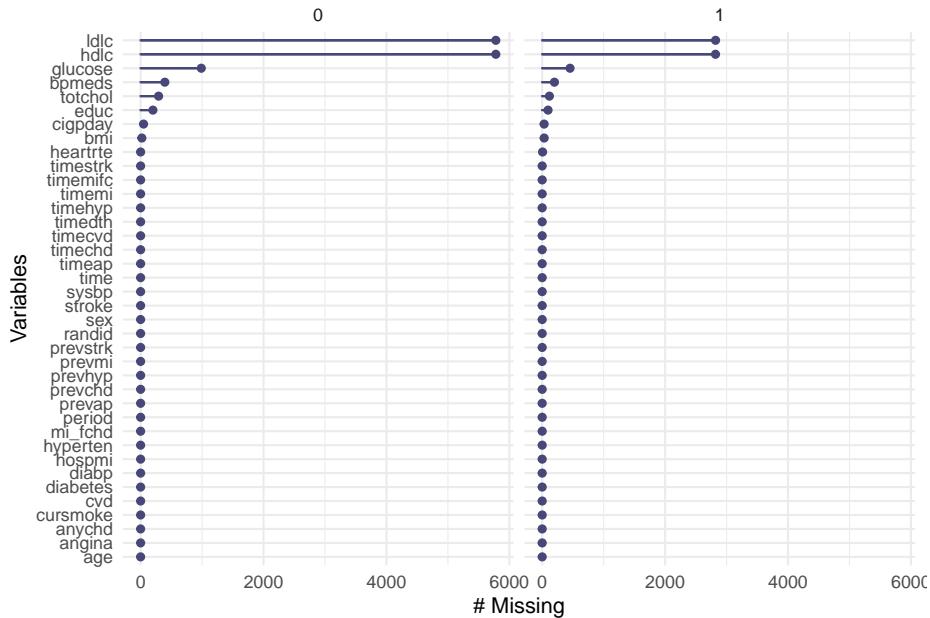
You can also do faceting with the `gg_miss_var` function. For example, you could see if missingness varies by the period of the observation:

```
gg_miss_var(fhs, facet = period)
```



You may also want to check if missingness varies with whether an observation was associated with death of the study subject:

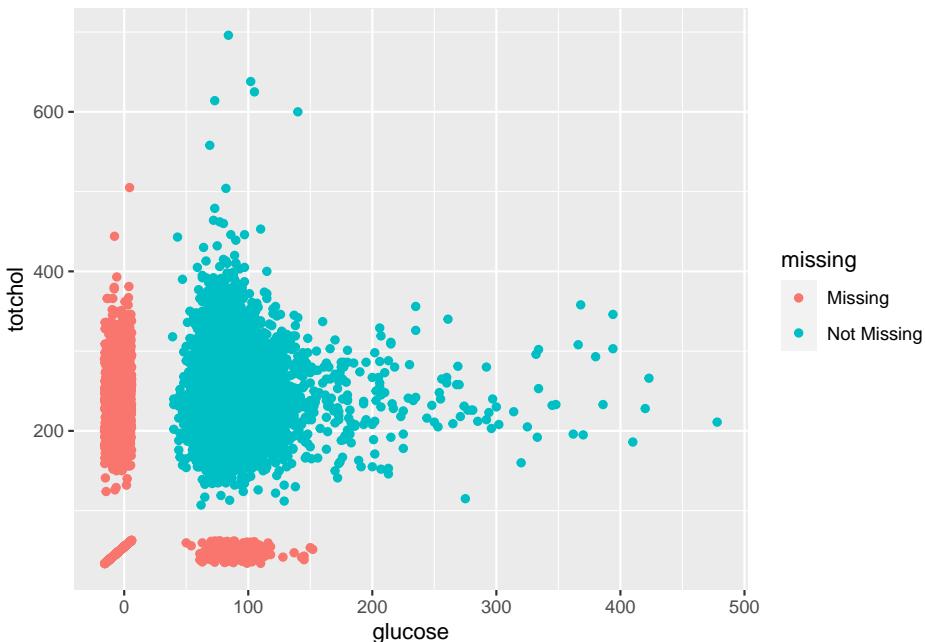
```
gg_miss_var(fhs, facet = death)
```



There are also functions in these packages that allow you to look at how miss-

ingness is related across variables. For example, both `glucose` and `totchol` are continuous variables, and both are occasionally missing. You can use the `geom` function `geom_miss_point` from the `naniar` package with a `ggplot` object to explore patterns of missingness among these two variables:

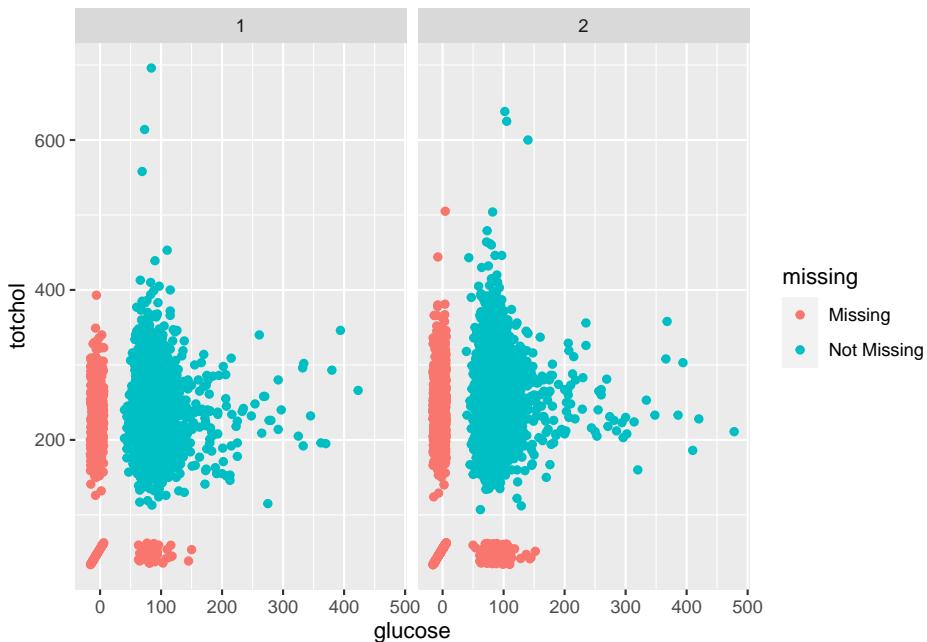
```
fhs %>%
  ggplot(aes(x = glucose, y = totchol)) +
  geom_miss_point()
```



The lower left corner shows the observations where both values are missing—it looks like there aren't too many. For observations with one missing but not the other (the points in red along the x- and y-axes), it looks like the distribution across the non-missing variable is pretty similar to that for observations with both measurements available. In other words, `totchol` has a similar distribution among observations where `glucose` is available as observations where `glucose` is missing.

You can also do things like facet by sex to explore patterns at a finer level:

```
fhs %>%
  ggplot(aes(x = glucose, y = totchol)) +
  geom_miss_point() +
  facet_wrap(~ sex)
```



3. How many participants die? What is the distribution of age at time of death?

The **death** variable in the **fhs** data is an indicator for mortality if a participant died at any point during follow-up. It is time-invariant taking the value 1 if a participant died at any point or 0 if they were alive at their end of follow-up, so we have to be careful on how to extract the actual number of deaths.

If you arrange by the random ID and look at **period** and **death** for each subject, you can see that the **death** variable is the same for all periods for each subject:

```
fhs %>%
  arrange(randid) %>%
  select(randid, period, death)
```

```
## # A tibble: 11,627 x 3
##   randid period death
##   <dbl>   <dbl> <dbl>
## 1 2448     1     0
## 2 2448     3     0
## 3 6238     1     0
## 4 6238     2     0
## 5 6238     3     0
## 6 9428     1     0
## 7 9428     2     0
## 8 10552    1     1
## 9 10552    2     1
```

```
## 10 11252      1      0
## # ... with 11,617 more rows
```

We need to think some about this convention of recording the data when we count the deaths.

It is often useful to extract the first (and sometimes last) observation, in order to assess certain covariate statistics on the individual level. We can create a dataset including only the first (or last) observation per participant from the `lhs` data using `tidyverse` tools. The `group_by` function groups data by unique values of designated variables (here `randid`) and the `slice` function selects rows as designated.

```
fhs_first <- fhs %>%
  group_by(randid) %>%
  slice(1L)%>%
  ungroup()
```

Alternatively you can use the `slice_head` function, which allows us to slice a designated number of rows beginning from the first observation. Because we are piping this in the `group_by` function, we will be slicing rows beginning from the first observation for each `randid`

```
fhs_first <- fhs %>%
  group_by(randid) %>%
  slice_head(n=1)%>%
  ungroup()
```

We can similarly select the last observation for each participant

```
fhs_last <- fhs %>%
  group_by(randid) %>%
  slice(n())%>%
  ungroup()
```

or using the `slice_tail` function

```
fhs_last <- fhs %>%
  group_by(randid) %>%
  slice_tail(n=1)%>%
  ungroup()
```

In this dataset we can extract statistics on baseline covariates on the individual level, but also assess the number of participants with specific values, including `death=1`. For example, we can use the `sum` function in base R, which generates the sum of all values for a given vector. In this case since each death has the value of 1 the `sum` function will give as the number of deaths in the sample.

```
sum(fhs_first$death)
```

```
## [1] 1550
```

Conversely using `tidyverse` tools we can extract the number of observations with `death=1` using the `count` function

```
fhs_first %>%
  count(death)
```

```
## # A tibble: 2 x 2
##   death     n
## * <dbl> <int>
## 1     0    2884
## 2     1    1550
```

Note that survival or time-to-event outcomes in longitudinal cohort data will often be time-varying. For example, a variable for mortality will take the value of zero until the person-time observation that represents the time interval that the outcome actually happens in. For outcomes such as mortality this will typically be the last observation. We will construct a variable like this in `fhs` below.

In order to estimate the distribution of age at death among those participants who died during follow-up we need to create a new age at death variable. The `age` variable in `fhs` represents the participants age at each visit. Typically a death would happen between visits so the last recorded value for `age` would be less than the age at death. We will use the `timedth` variable to help us determine the actual age at death. The value of `timedth` is the number of days from beginning of follow-up until death for those with `death=1`, while it is a fixed value of `timedth=8766` (the maximum duration of follow-up) for those with `death=0`.

We can create a new age at death variable for those with `death=1` using the `age` at baseline and `timedth` values

```
fhs_first<-fhs_first %>%
  mutate(agedth=age+timedth/365.25)
```

We can then get summary statistics on this new variable

```
fhs_first %>%
  summarize(min_agedth = min(agedth),
  mean_agedth = mean(agedth),
  max_agedth = max(agedth))
```

```
## # A tibble: 1 x 3
##   min_agedth mean_agedth max_agedth
##       <dbl>        <dbl>        <dbl>
## 1      38.4        70.5        93
```

We can also check on these values by groups of interest such as sex

```
fhs_first %>%
  group_by(sex) %>%
  summarize(min_agedth = min(agedth),
            mean_agedth = mean(agedth),
            max_agedth = max(agedth))

## # A tibble: 2 x 4
##   sex min_agedth mean_agedth max_agedth
## * <dbl>      <dbl>        <dbl>
## 1     1       41.6        69.5      91.1
## 2     2       38.4        71.3      93
```

4. *What is the distribution of age at time of incident MI? Are there differences between males and females? Are there differences in smoking between males and females?*

Similar to the question about death (all-cause mortality) we can look at disease incidence, for example myocardial infarction (MI). The `fhs` dataset has the `hospmi` variable as an indicator for any participant who had a hospitalization due to MI and `timemi` gives the number of days from beginning of follow up to the hospitalization due to MI. We can create an age at incident MI hospitalization in a similar fashion as the example for age at death.

```
fhs_first<-fhs_first %>%
  mutate(agemi=age+timemi/365.25)
```

We can then get summary statistics on this new `agemi` variable

```
fhs_first %>%
  summarize(min_agemi = min(agemi),
            mean_agemi = mean(agemi),
            max_agemi = max(agemi))

## # A tibble: 1 x 3
##   min_agemi mean_agemi max_agemi
##       <dbl>      <dbl>      <dbl>
## 1       37       69.7       93
```

And by sex

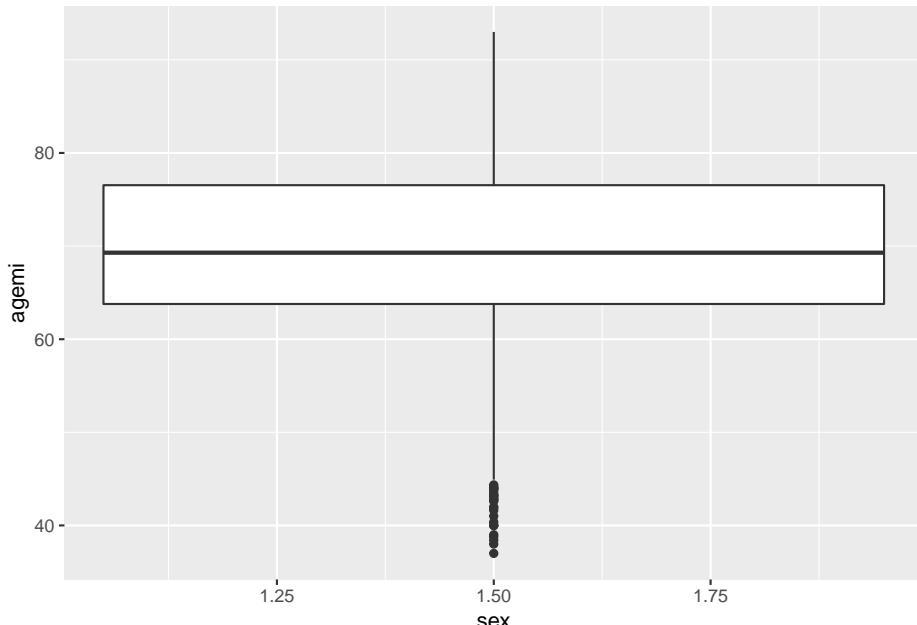
```
fhs_first %>%
  group_by(sex) %>%
  summarize(min_agemi = min(agemi),
            mean_agemi = mean(agemi),
            max_agemi = max(agemi))

## # A tibble: 2 x 4
##   sex min_agemi mean_agemi max_agemi
## * <dbl>      <dbl>        <dbl>
## 1     1       41.6        69.5      91.1
## 2     2       38.4        71.3      93
```

```
## 1      1      37      68.2     91.1
## 2      2     38.4      70.9      93
```

We can see that the mean age at incident MI hospitalization among males and females is similar, but with males being somewhat younger on average at the time of incident MI. We can take a closer look at the distribution using boxplots:

```
fhs_first %>%
  # define the axes for the boxplot
  ggplot(aes(x = sex, y=agemi)) +
  geom_boxplot()
```



We see that R didn't return two separate boxplots by sex, but rather one centered between the two values of `sex=1` and `sex=2` which are the values for males and females respectively. This is an indicator that the `sex` variable is of class `numeric` and is treated as a continuous values rather than categorical. We can verify that this is in fact the case:

```
class(fhs_first$sex)
```

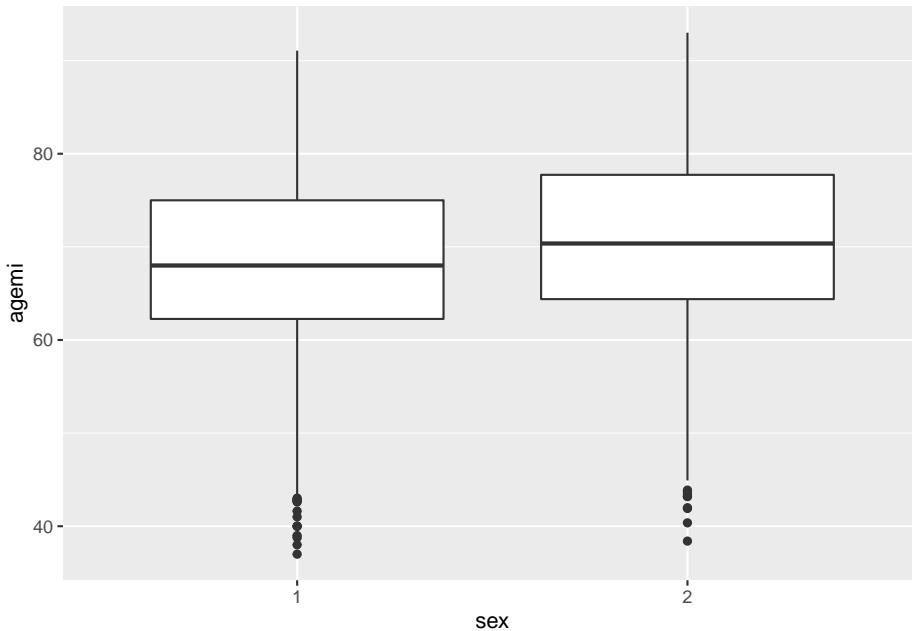
```
## [1] "numeric"
```

We can transform the variable to one of class `factor` in order for it to be treated as a categorical variable

```
fhs_first<-fhs_first %>%
  mutate(sex=as.factor(sex))
```

If we repeat the function for the boxplot now we get separate boxplots by sex

```
fhs_first %>%
  # define the axes for the boxplot
  ggplot(aes(x = sex, y=agemi)) +
  geom_boxplot()
```



We can once again see from the the boxplots that females tend to be a little older at incidence of MI.

5. *What is the distribution of BMI among MI cases and non-cases? How about between smokers and non-smokers*

Similar to the exercise above we can compare BMI distributions by MI case status.

## 7.2 Coding a survival analysis

[R package **survival**]

In the context of survival analysis what is modelled is time to an event (also referred to as survival time or failure time). This is a bit different than the models in the linear or `glm` family that model an outcome that may follow a gaussian (linear regression), binomial (logistic model) or Poisson distribution. Another difference is that the outcome (time to event) will not be determined in some participants, as they will not have experienced the event of interest during their follow-up. These participants are considered ‘censored’. Censoring can occur in three ways:

- the participant does not experience the event of interest before the study end
- the participant is lost to follow-up before experiencing the event of interest
- the participant experiences a difference event that makes the event of interest impossible (for example if the event of interest is acute MI a participant that dies from a different cause is considered censored)

These are all types of right censoring and in simple survival analysis they are considered to be uninformative (typically not related to exposure). If the censoring is related to the exposure and the outcome then adjustment for censoring has to happen.

Let's assume that we are interested in all cause mortality as the event of interest let's denote  $T$  is time to death and  $T \geq 0$ . We define the survival function as  $S(t) = Pr[T > t] = 1 - F(t)$ , where the survival function  $S(t)$  is the probability that a participant survives past time  $t$  ( $Pr[T > t] = 1$ ).  $F(t)$  is the Probability Density Function, (sometimes also denoted as the Cumulative Incidence Function,  $R(t)$ ) or the probability that that an individual will have a survival time less than or equal to  $t$  ( $Pr(T \leq t)$ )

Time to event  $t$  is bounded by  $[0, \infty)$  and  $S(t)$  is non-increasing as  $t$  becomes greater. At  $t = 0$ ,  $S(t) = 1$  and conversely as  $t$  approaches  $\infty$ ,  $S(t) = 0$ . A property of the survival and probability density function is  $S(t) = 1 - F(t)$ : the survival function and the probability density function (or cumulative incidence function ( $R(t)$ )) sum to 1.

Another useful function is the hazard Function,  $h(t)$ , which is the instantaneous potential of experiencing an event at time  $t$ , conditional on having survived to that time ( $h(t) = \frac{Pr[t < T \leq t + \Delta t | T > t]}{\Delta t} = \frac{f(t)}{S(t)}$ ). The cumulative Hazard Function,  $H(t)$  is defined as the integral of the hazard function from time 0 to time  $t$ , which equals the area under the curve  $h(t)$  between time 0 and time  $t$  ( $H(t) = \int_0^t h(u)du$ ). If we know any of  $S(t)$ ,  $H(t)$  or  $h(t)$ , we can derive the rest based on the following relationships:

$$h(t) = \frac{\partial \log(S(t))}{\partial t}$$

$$H(t) = -\log(S(t)) \text{ and conversely } S(t) = \exp(-H(t))$$

The **survival** package in R allows us to fit these types of models, including a very popular model in survival analysis, the Cox proportional hazards model that was also applied in Wong et al. (1989). A very simple way to estimate survival is the non-parametric Kaplan-Meier estimator.

In R we would estimate Survival  $S(t)$  with all-cause mortality representing failure as follows:

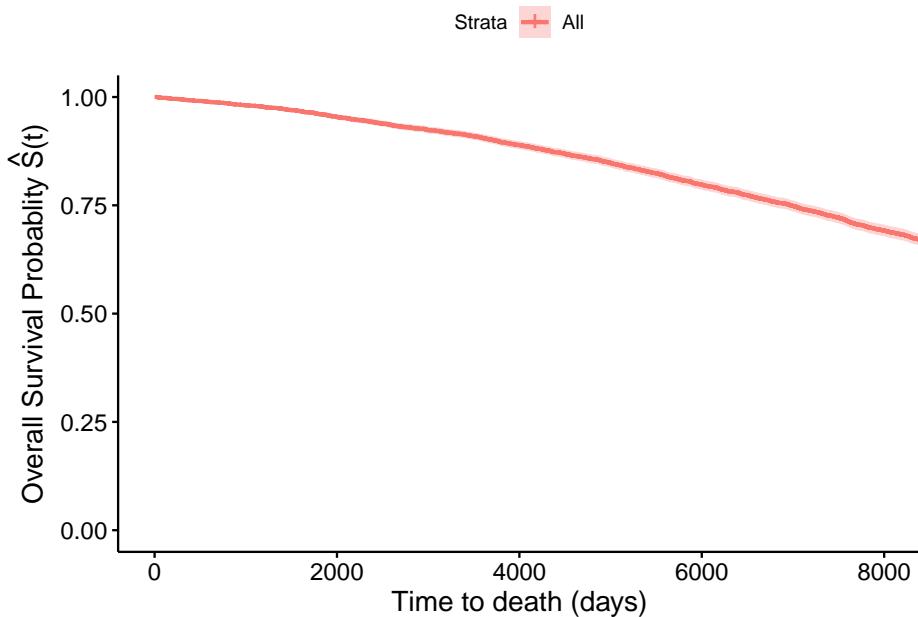
```
library(survival)
S1=Surv(fhs_first$timedth,fhs_first$death)
head(S1)

## [1] 8766+ 8766+ 8766+ 2956  8766+ 8766+
```

The numbers assigned to each individual represent their censoring times, with each number with a plus sign indicating that the participant was censored at that time without developing the outcome (haven't failed/died), while those without the plus sign are the times at which participants developed the outcome (failure/death).

```
library(survminer) ##for plotting survival plots with ggplot2

## Loading required package: ggpublisher
fit1<-survfit(Surv(timedth, death) ~ 1, data = fhs_first)
fit1 %>%
ggsurvplot(xlab="Time to death (days)", ylab=expression(paste('Overall Survival Probability ', hat(S(t))))
```

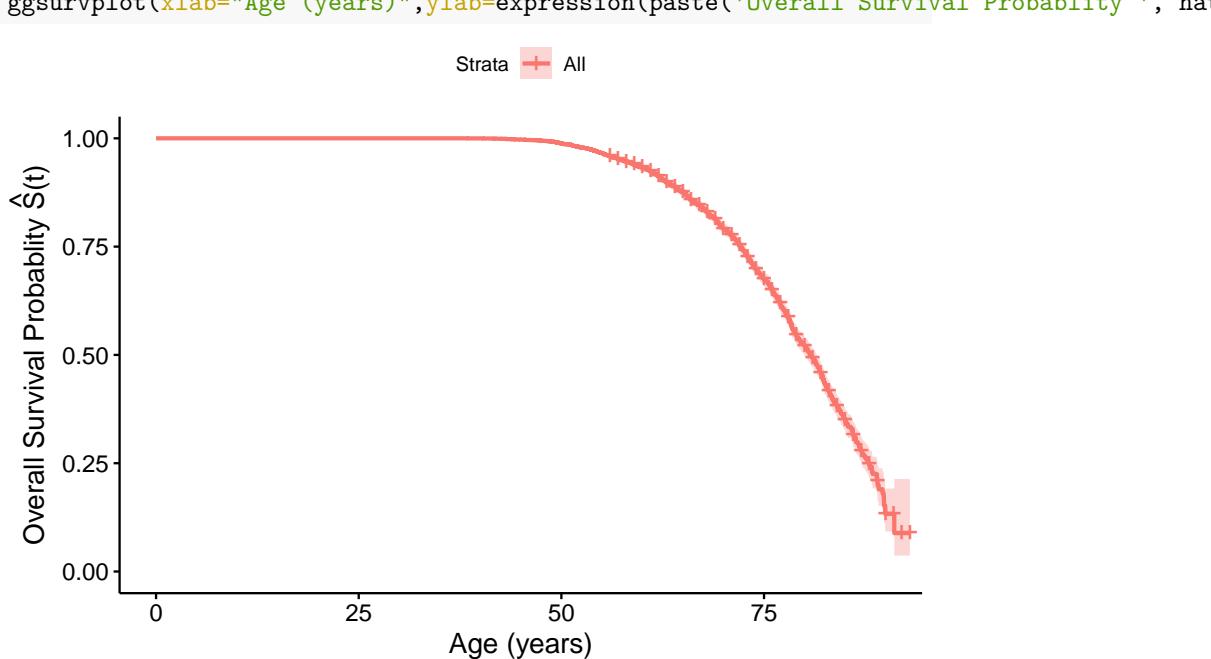


We can see that as follow-up time increases survival decreases rather monotonically over time, or in other words the number of people who have died increases. Survival  $\hat{S}(t)$  drops to about 0.65 at the end of follow-up, or in other words about 35% of participants have died, which is what is expected as we already know that 1550 of 4434 participants have died.

We can repeat this estimation with a different time-scale of interest. Other than follow-up times we may also be interested in Survival and failure (mortality) with respect to age. We repeat the same code only changing the first argument

in the `Surv` function, substituting time of death with respect to follow-up time with age at death.

```
fit2<-survfit(Surv(agedth, death) ~ 1, data = fhs_first)
fit2 %>%
  ggsurvplot(xlab="Age (years)", ylab=expression(paste('Overall Survival Probability ', ha
```



We see that the shape of this survival curve is different, with virtually no one dying until they reach their 40s, and then a sharper drop in survival as age increases.

The Cox proportional hazards model in a simple form has this form

$$\log(\lambda(t|X)) = \log(\lambda_0(t)) + \beta_1 \times X$$

where  $\lambda(t)$  represent the hazard at time  $t$ ,  $\lambda_0(t)$  is the baseline hazard at time  $t$ , and  $\beta_1$  is the log hazard for those with  $X = 1$  compared to  $X = 0$ . The baseline hazard  $\lambda_0(t)$  is similar to the intercept term in a linear model or `glm` and is the value of the hazard when all covariates equal 0. However, unlike the intercept term in a linear model or `glm`,  $\lambda_0(t)$  is not estimated by the model. The above model can also be written as

$$\lambda(t|X) = \lambda_0(t) \times e^{\beta_1 \times X}$$

$e^{\beta_1}$  is the hazard ratio comparing those with  $X = 1$  and  $X = 0$

Using the `fhs` data we will fit a simple Cox proportional hazard for the effect of smoking on the hazard for MI.

*Note: Variables of interest to continue with: for mixed models, `sysbp`, `diabp`,*

*totchol* compared to *cigpday*, *bmi* smoking or not for long. analysis, *timemi* and *timestrk* and *hyperten*, exposure: *cigpday*, *sysbp*, *diabp*, *bmi*

## 7.3 Handling complexity

### 7.3.1 Multi-level exposure

### 7.3.2 Recurrent outcome

### 7.3.3 Time-varying coefficients

### 7.3.4 Using survey results

[e.g., NHANES]



## **Chapter 8**

# **Some approaches for confounding**

### **8.1 Inverse probability weighting**

### **8.2 Propensity scores**

[Modeling for weights/propensity scores, involves machine learning]



# **Chapter 9**

## **Mixed models**

[Using a mixed modeling framework to help analyze repeated measures]



## Chapter 10

# Instrumental variables



## Chapter 11

# Causal inference



# Bibliography

- Andersson, C., Johnson, A. D., Benjamin, E. J., Levy, D., and Vasan, R. S. (2019). 70-year legacy of the framingham heart study. *Nature Reviews Cardiology*, page 1.
- Armstrong, B. (2006). Models for the relationship between ambient temperature and daily mortality. *Epidemiology*, pages 624–631.
- Armstrong, B., Hajat, S., Kovats, S., Lloyd, S., Scovronick, N., and Wilkinson, P. (2012). Commentary: Climate changehow can epidemiology best inform policy? *Epidemiology*, 23(6):780–784.
- Armstrong, B. G., Gasparrini, A., and Tobias, A. (2014). Conditional Poisson models: a flexible alternative to conditional logistic case cross-over analysis. *BMC Medical Research Methodology*, 14(1):122.
- Barone-Adesi, F., Gasparrini, A., Vizzini, L., Merletti, F., and Richiardi, L. (2011). Effects of italian smoking regulation on rates of hospital admission for acute coronary events: a country-wide study. *PloS one*, 6(3):e17419.
- Basu, R., Dominici, F., and Samet, J. M. (2005). Temperature and mortality among the elderly in the united states: a comparison of epidemiologic methods. *Epidemiology*, pages 58–66.
- Bernal, J. L., Cummins, S., and Gasparrini, A. (2017). Interrupted time series regression for the evaluation of public health interventions: a tutorial. *International Journal of Epidemiology*, 46(1):348–355.
- Bhaskaran, K., Gasparrini, A., Hajat, S., Smeeth, L., and Armstrong, B. (2013). Time series regression studies in environmental epidemiology. *International Journal of Epidemiology*, 42(4):1187–1195.
- Bor, J., Moscoe, E., Mutevedzi, P., Newell, M.-L., and Bärnighausen, T. (2014). Regression discontinuity designs in epidemiology: causal inference without randomized trials. *Epidemiology*, 25(5):729.
- Bradburn, M. J., Clark, T. G., Love, S. B., and Altman, D. G. (2003a). Survival analysis part ii: multivariate data analysis—an introduction to concepts and methods. *British journal of cancer*, 89(3):431–436.

- Bradburn, M. J., Clark, T. G., Love, S. B., and Altman, D. G. (2003b). Survival analysis part iii: multivariate data analysis—choosing a model and assessing its adequacy and fit. *British journal of cancer*, 89(4):605–611.
- Casey, J. A., Karasek, D., Ogburn, E. L., Goin, D. E., Dang, K., Braveman, P. A., and Morello-Frosch, R. (2018). Retirements of coal and oil power plants in California: association with reduced preterm birth among populations nearby. *American Journal of Epidemiology*, 187(8):1586–1594.
- Chang, W. (2018). *R graphics cookbook: practical recipes for visualizing data*. O'Reilly Media.
- Clark, T. G., Bradburn, M. J., Love, S. B., and Altman, D. G. (2003). Survival analysis part i: basic concepts and first analyses. *British journal of cancer*, 89(2):232–238.
- Dominici, F. and Peng, R. D. (2008a). *Exploratory Data Analysis*, chapter 5, pages 41–67. Springer.
- Dominici, F. and Peng, R. D. (2008b). *Statistical Issues in Estimating the Health Effects of Spatial–Temporal Environmental Exposures*, chapter 4, pages 31–40. Springer.
- Dominici, F. and Peng, R. D. (2008c). *Studies of Air Pollution and Health*, chapter 1, pages 1–6. Springer.
- Dunn, P. K. and Smyth, G. K. (2018). *Statistical Models*, chapter 1, pages 1–30. Springer.
- Gasparrini, A. (2014). Modeling exposure–lag–response associations with distributed lag non-linear models. *Statistics in Medicine*, 33(5):881–899.
- Gasparrini, A. and Armstrong, B. (2010). Time series analysis on the health effects of temperature: advancements and limitations. *Environmental research*, 110(6):633–638.
- Healy, K. (2018). *Data visualization: a practical introduction*. Princeton University Press.
- Imai, C., Armstrong, B., Chalabi, Z., Mangtani, P., and Hashizume, M. (2015). Time series regression model for infectious disease and weather. *Environmental research*, 142:319–327.
- James, G., Witten, D., Hastie, T., and Tibshirani, R. (2013). *Linear Regression*, chapter 3, pages 59–126. Springer.
- Lu, Y. and Zeger, S. L. (2007). On the equivalence of case-crossover and time series methods in environmental epidemiology. *Biostatistics*, 8(2):337–344.
- Mendola, P. (2018). Invited commentary: The power of preterm birth to motivate a cleaner environment. *American journal of epidemiology*, 187(8):1595–1597.

- Robinson, D. (2014). broom: An r package for converting statistical analysis objects into tidy data frames. *arXiv preprint arXiv:1412.3565*.
- Vicedo-Cabrera, A. M., Sera, F., and Gasparini, A. (2019). Hands-on tutorial on a modeling framework for projections of climate change impacts on health. *Epidemiology*, 30(3):321–329.
- Wickham, H. and Grolemund, G. (2016). *R for data science: import, tidy, transform, visualize, and model data.* ” O'Reilly Media, Inc.”.
- Wong, N. D., Cupples, A., Ostfeld, A. M., Levy, D., and Kannel, W. B. (1989). Risk factors for long-term coronary prognosis after initial myocardial infarction: the framingham study. *American Journal of Epidemiology*, 130(3):469–480.