University of Texas at Austin, Cockrell School of Engineering
Data Mining – EE 380L

Problem Set # 2 (2b)

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Discussed Homework with Following Students:
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Q1] Part a]

FE (Fuel Efficiency) is a good candidate for dependent variable.

EngDispl, NumCyl, Transmission, NumGears, IntakeValvePerCyl, ExhaustValvesPerCyl are continuous variables.

AirAspirationMethod, TransLockup, TransCreeperGear, DriveDesc, CarlineClassDesc, VarValveTiming, VarValveLift are categorical variables.

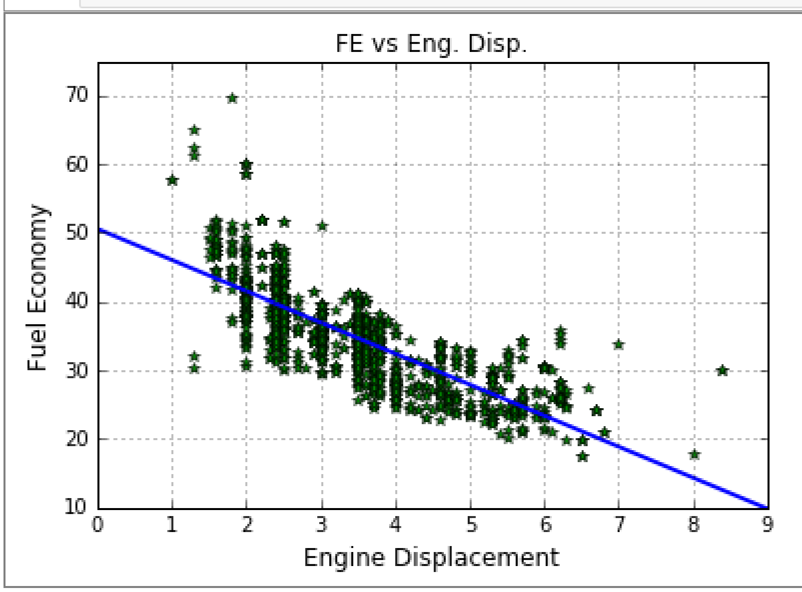
Part b]

In [5]:
```
#Part b
# http://docs.scipy.org/doc/numpy-1.10.0/reference/generated/numpy.linalg.lstsq.html
# B1 = m; B0 = c
# y = B1x + B0
# y = Ap, where A = [[x 1]] and p = [[B1], [B0]]
x = df["EngDispl"]
A = np.vstack([x, np.ones(len(x))]).T
#print A
y = np.array(df["FE"])
#m, c = np.linalg.lstsq(A, y)[0]
#B1, B0 = np.linalg.lstsq(A, y)[0]
model, resid = np.linalg.lstsq(A, y)[:2]
B1, B0 = model
#np.linalg.lstsq(A, y)
print("m(B1) is: {}.".format(round(B1,8)))
print("c(B0) is: {}.".format(round(B0,8)))
r2 = 1 - resid / (y.size * y.var())
print("R2 is: {}.".format(round(r2[0],8)))
```

```
m(B1) is: -4.52092928.
c(B0) is: 50.56322991.
R2 is: 0.61998904.
```

In [6]:
```
#Part b
# Add bestfit line to ScatterPlot using B1 and B0 values
# y = B1x + B0

part_b, = plt.plot(df["EngDispl"],df["FE"],'g*')
plt.grid(True)
plt.ylabel("Fuel Economy",fontsize=12)
plt.xlabel("Engine Displacement",fontsize=12)
plt.title("FE vs Eng. Disp.")
plt.xlim(0, 9)
plt.ylim(10, 75)
xVals = np.arange(0, 325)
yVals = (B1*xVals) + B0
plt.plot(xVals,yVals,'b-',linewidth=2.0)
plt.show()
```



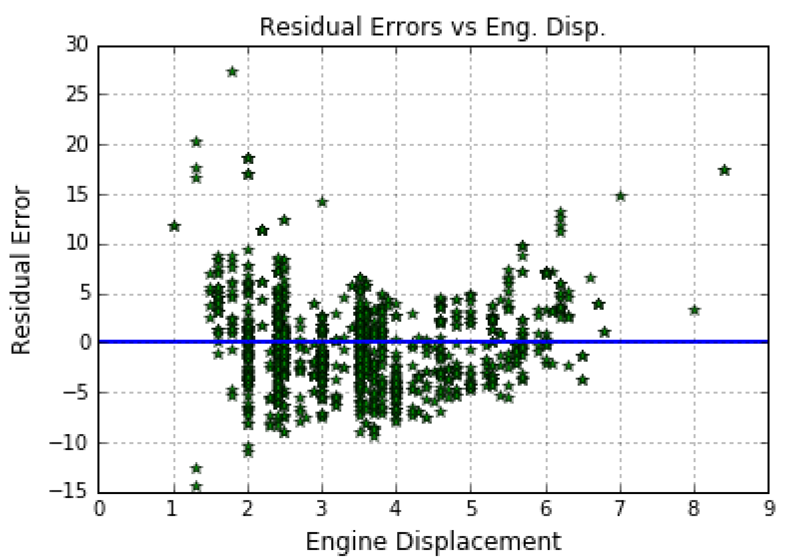
Part c]

In [7]:
```
# Part C
# Plot the residual errors. Now sum the residual errors. What do you find?
# compute the predicted value for EngDispl
def predictedVals(coeff,intercept,engDisp):
    y_vals=list()
    for x in np.nditer(engDisp):
        y_vals.append((coeff * x )+ intercept)
    return y_vals
```

In [8]:
```
yPredict=predictedVals(B1,B0,df.as_matrix(["EngDispl"]))
```

In [9]:
```
# compute residual errors
residualErrors=[]
Y_FE = df.as_matrix(["FE"])
for i in np.arange(len(Y_FE)):
    residualErrors.append(Y_FE[i]-yPredict[i])
```

In [10]:
```
part_c, = plt.plot(df["EngDispl"],residualErrors,'g*')
plt.grid(True)
plt.ylabel("Residual Error",fontsize=12)
plt.xlabel("Engine Displacement",fontsize=12)
plt.title("Residual Errors vs Eng. Disp.")
plt.xlim(0, 9)
xVals1 = np.arange(0, 325)
yVals1 = (0*xVals)
plt.plot(xVals1,yVals1,'b-',linewidth=2.0)
plt.show()
```



In [11]:
```
#Sum of residual errors
print "Sum of Residual Errors: ",np.sum(residualErrors)
```

```
Sum of Residual Errors:  -2.2055246518e-11
```

Part d]

In [12]:
```
# Part D
# Now do the same where you fit a linear regression for FE on both EngDispl and NumCyl.
# Report the R2 value. Sum the residual errors. What do you find?
X1 = df.as_matrix(["EngDispl","NumCyl"])
FE_Y = df.as_matrix(["FE"])
model_lr = linear_model.LinearRegression()
model_lr.fit(X1,FE_Y)
print "Coeff, B0:", model_lr.coef_, model_lr.intercept_
print "r2 = ", model_lr.score(X1,FE_Y)
```

```
Coeff, B0: [[-3.74535214 -0.58802919]] [ 51.35414193]
r2 =  0.623958939799
```

In [13]:
```
def predictedVals1(coeff,intercept,X):
    y_vals1=list()
    for x in np.arange(len(X)):
        y_vals1.append(np.dot(coeff,X[x]) + intercept )
    return y_vals1
```

In [14]:
```
#yPredict=predictedVals(B1,B0,df.as_matrix(["EngDispl"]))
yPredict_D=predictedVals1(model_lr.coef_,model_lr.intercept_,X1)
```

In [15]:
```
# compute residual errors
residualErrors_D=[]
for i in np.arange(len(FE_Y)):
    residualErrors_D.append(FE_Y[i]-yPredict_D[i])
```

In [16]:
```
#Sum of residual errors
print "Sum of Residual Errors: ",np.sum(residualErrors_D)
```

```
Sum of Residual Errors:  0.0
```

Part e]

In [17]:
```
#PART E
#Now solve the OLS regression problem for FE against all the variables. Report the R2
#value. Sum the residual errors. What do you find?
```

In [18]:
```
X1 = df.as_matrix(["EngDispl","NumCyl","Transmission","NumGears","IntakeValvePerCyl",
                   "ExhaustValvesPerCyl","AirAspirationMethod","TransLockup",
                   "TransCreeperGear","DriveDesc","CarlineClassDesc","VarValveTiming",
                   "VarValveLift"])
FE_Y = df.as_matrix(["FE"])
model_lr = linear_model.LinearRegression()
model_lr.fit(X1,FE_Y)
print "Coeff, B0:", model_lr.coef_, model_lr.intercept_
print "r2 = ", model_lr.score(X1,FE_Y)
```

```
Coeff, B0: [[-3.73881937 -0.59854429  0.09853067 -0.26103154 -0.46677908 -1.41290715
  -0.32029424 -0.74947468 -0.75500417  0.83699599 -0.27189826  1.63506899
   0.8440168 ]] [ 54.26927826]
r2 =  0.707406484867
```

In [19]:
```
yPredict_E=predictedVals1(model_lr.coef_,model_lr.intercept_,X1)
```

In [20]:
```
# compute residual errors
residualErrors_E=[]
for i in np.arange(len(FE_Y)):
    residualErrors_E.append(FE_Y[i]-yPredict_E[i])
```

In [21]:
```
#Sum of residual errors
print "Sum of Residual Errors: ",np.sum(residualErrors_D)
```

```
Sum of Residual Errors:  0.0
```

Q2]

Q4]

PROBABILITY SECTION

4. For X, Y, and Z random variables, the *Covariance Matrix* of the random vector (X, Y, Z) is defined as:

$$\mathbb{E}\left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}\begin{pmatrix} X & Y & Z \end{pmatrix}\right] = \mathbb{E}\begin{bmatrix} X^2 & XY & XZ \\ XY & Y^2 & YZ \\ XZ & YZ & Z^2 \end{bmatrix}.$$

Suppose X, Y and Z represent the result of rolling three independent dice. What is the covariance matrix of (X, Y, Z)?

PMF of rolling a dice

$$f_X(y) = \begin{cases} \frac{1}{6} & if\ y \in \{1,2,3,4,5,6\} \\ 0 & otherwise \end{cases}$$

$$\mu_X = E(X) = \sum_{y=1}^{6} y \cdot f_Z(y) = (1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (3)\left(\frac{1}{6}\right) + (4)\left(\frac{1}{6}\right) + (5)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{6}\right)$$

$$\mu_X = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2} = 3.5$$

$$\sigma_X{}^2 = var(X) = \textstyle\sum_{y=1}^{6}(y-\mu_X)^2 \cdot f_Z(y)$$
$$\sigma_X{}^2 = \left(1-\frac{7}{2}\right)^2\left(\frac{1}{6}\right) + \left(2-\frac{7}{2}\right)^2\left(\frac{1}{6}\right) + \left(3-\frac{7}{2}\right)^2\left(\frac{1}{6}\right) + \left(4-\frac{7}{2}\right)^2\left(\frac{1}{6}\right) + \left(5-\frac{7}{2}\right)^2\left(\frac{1}{6}\right) + \left(6-\frac{7}{2}\right)^2\left(\frac{1}{6}\right)$$
$$\sigma_X{}^2 = \left(-\frac{5}{2}\right)^2\left(\frac{1}{6}\right) + \left(-\frac{3}{2}\right)^2\left(\frac{1}{6}\right) + \left(-\frac{1}{2}\right)^2\left(\frac{1}{6}\right) + \left(\frac{1}{2}\right)^2\left(\frac{1}{6}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{2}\right)^2\left(\frac{1}{6}\right)$$
$$\sigma_X{}^2 = \left(\frac{25}{24}\right) + \left(\frac{9}{24}\right) + \left(\frac{1}{24}\right) + \left(\frac{1}{24}\right) + \left(\frac{9}{24}\right) + \left(\frac{25}{24}\right) = \left(\frac{70}{24}\right) = \frac{35}{12} = 2\frac{11}{12}$$

Rules:
$E(XY) = E(X)E(Y)$
$E(X \pm a) = E(X) \pm a$

Covariance Matrix is Σ

$$\Sigma = \begin{pmatrix} E(X-3.5)^2 & E(X-3.5)(Y-3.5) & E(X-3.5)(Z-3.5) \\ E(X-3.5)(Y-3.5) & E(Y-3.5)^2 & E(Y-3.5)(Z-3.5) \\ E(X-3.5)(Z-3.5) & E(Y-3.5)(Z-3.5) & E(Z-3.5)^2 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 2\frac{11}{12} & E(X-3.5)E(Y-3.5) & E(X-3.5)E(Z-3.5) \\ E(X-3.5)E(Y-3.5) & 2\frac{11}{12} & E(Y-3.5)E(Z-3.5) \\ E(X-3.5)E(Z-3.5) & E(Y-3.5)E(Z-3.5) & 2\frac{11}{12} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 2\frac{11}{12} & 0 & 0 \\ 0 & 2\frac{11}{12} & 0 \\ 0 & 0 & 2\frac{11}{12} \end{pmatrix}$$

Q7]

> 7. Consider flipping a fair coin. Let Z denote the random variable that is the number of Heads that come up in a row. Thus, if the first flip comes up tails, $Z = 0$. If the flip sequence is
>
> $$HHTHHHHT....$$
>
> then $Z = 2$, and so on.
>
> - Write the probability mass function of Z.
> - Compute the mean and variance of Z.

For a Fair Coin.
P(H) = $p = \frac{1}{2}$
P(T) = $(1-p) = \frac{1}{2}$

Random Variable

Compound Event	Elementary Event	Probability
(Z=0)	(T..) =	$\frac{1}{2}$
(Z=1)	(HT..)	$\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right) = \frac{1}{4}$
(Z=2)	(HHT..)	$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{8}$
(Z=3)	(HHHT..)	$\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{1}{16}$
(Z=4)	(HHHHT..)	$\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) = \frac{1}{32}$

PMF:

y	$f_Z(y)$	
0	$\frac{1}{2}$	$p^0(1-p)$
1	$\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right) = \frac{1}{4}$	$p^1(1-p)$
2	$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{8}$	$p^2(1-p)$
3	$\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{1}{16}$	$p^3(1-p)$
4	$\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) = \frac{1}{32}$	$p^4(1-p)$

So **PMF** for Z is $f_Z(y) = p^y(1-p)$ or for fair coin where $p = (1-p)$

$$f_Z(y) = p^{y+1} = \left(\tfrac{1}{2}\right)^{y+1} \qquad \text{where } y = 0,1,2,\dots,n$$

Compute Mean & Variance:

$$\mu = E(Z) = \sum_{y=0}^{n} y \cdot f_Z(y) = \sum_{y=0}^{n} y \cdot \left(\frac{1}{2}\right)^{y+1} = \sum_{y=1}^{n} y \cdot \left(\frac{1}{2}\right)^{y+1} = \frac{1}{4}\sum_{y=1}^{n} y \cdot \left(\frac{1}{2}\right)^{y-1}$$

From http://www.trans4mind.com/personal_development/mathematics/series/airthmeticGeometricSeries.htm

$$\boxed{1 + 2r + 3r^2 \dots n\,r^{(n-1)} = \frac{\left(1-(n+1)\,r^n + n\,r^{(n+1)}\right)}{(1-r)^2}}$$

$r = \frac{1}{2}$

$$\mu = E(Z) = \frac{1}{4}\sum_{y=1}^{n} y \cdot \left(\frac{1}{2}\right)^{y-1} = \frac{1}{4}\frac{\left(1-(n+1)\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}\right)}{\left(\frac{1}{2}\right)^2}$$

$$\mu = \frac{1}{4}\frac{\left(1-(n+1)\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}\right)}{\frac{1}{4}} = \left(1 + n\left(\frac{1}{2}\right)^{n+1} - (n+1)\left(\frac{1}{2}\right)^n\right) =$$

$$\mu = \left(1 + n\left(\frac{1}{2}\right)^{n+1} - (n+1)\left(\frac{1}{2}\right)^n\right) = \left(1 + \left(\frac{1}{2}\right)n\left(\frac{1}{2}\right)^n - (n+1)\left(\frac{1}{2}\right)^n\right)$$

$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$, then $\mu = 1$

Variance:
Use:

Properties of Variance

- $\mathrm{var}(X) = E\{(X - E[X])^2\} = E[X^2] - (E[X])^2$

Take advantage of:
$E[Z^2] = E[Z(Z-1)] + E[Z]$

Compute:
$E[Z(Z-1)] = \sum_{y=1}^{n} y(y-1)\cdot\left(\frac{1}{2}\right)^{y+1}$

$\sigma_Z{}^2 = E[(Z-\mu)^2] = E[Z^2] - (E[Z])^2 = E[Z^2] - \mu^2$

$$\sum_{y=0}^{n}(y-\mu)^2 f_Z(y) = \sum_{y=0}^{n}(y-1)^2\left(\frac{1}{2}\right)^{y+1} = \sum_{y=0}^{n}(y-1)^2\left(\frac{1}{2}\right)^{y+1}$$

$$\sigma_Z{}^2 = (0-1)^2\left(\frac{1}{2}\right)^{0+1} + \sum_{y=1}^{n}(y-1)^2\left(\frac{1}{2}\right)^{y+1} = \frac{1}{2} + \sum_{y=1}^{n}(y-1)^2\left(\frac{1}{2}\right)^{y+1}$$

Use from http://math2.org/math/expansion/power.htm:

$\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1) \quad \rightarrow \quad \sum_{j=1}^{n}(j-1)^2 = (1-1)^2 + \sum_{j=2}^{n}(j-1)^2 = \sum_{k=1}^{n} k^2$

Use from http://math2.org/math/expansion/geom.htm:

$$\sum_{k=1}^{n} k^2$$

Revisit with:
http://arnoldkling.com/apstats/geometric.html