HPSC - Final Assignment

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Problem

Solve the Laplace equation using multigrid.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0,$$

in a $x = [0, 2]$, $y = [0, 1]$ domains with boundary conditions:

$$p = 0 \ at \ x = 0,$$

$$p = y \ at \ x = 2,$$

$$\partial p/\partial y = 0 \ at \ y = 0, 1.$$

Modify the code in `demo.py` so that it runs with the sparse matrix from `step1.py`.

The source code and jupyter notebook for this assignment can be found at:
`https://github.com/gear/HPSC/tree/master/hw`
File name: `assign_final_worksheet.ipynb`

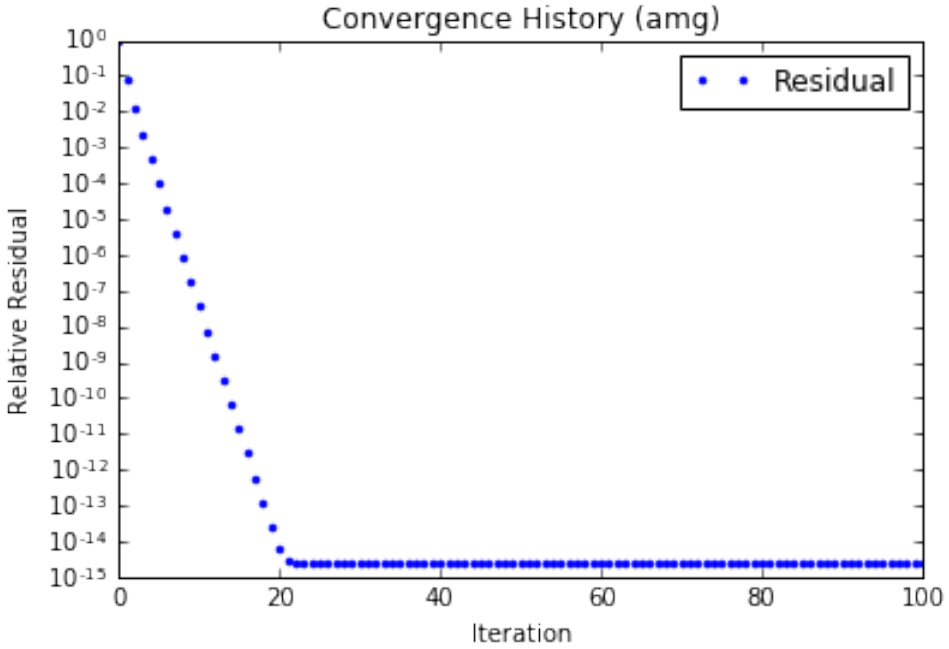
Answer

Using the given code in `step1.py`, I have the matrix A, vector b. This procedure is rewritten in `assign_final_worksheet.ipynb` as `laplace_eq`.

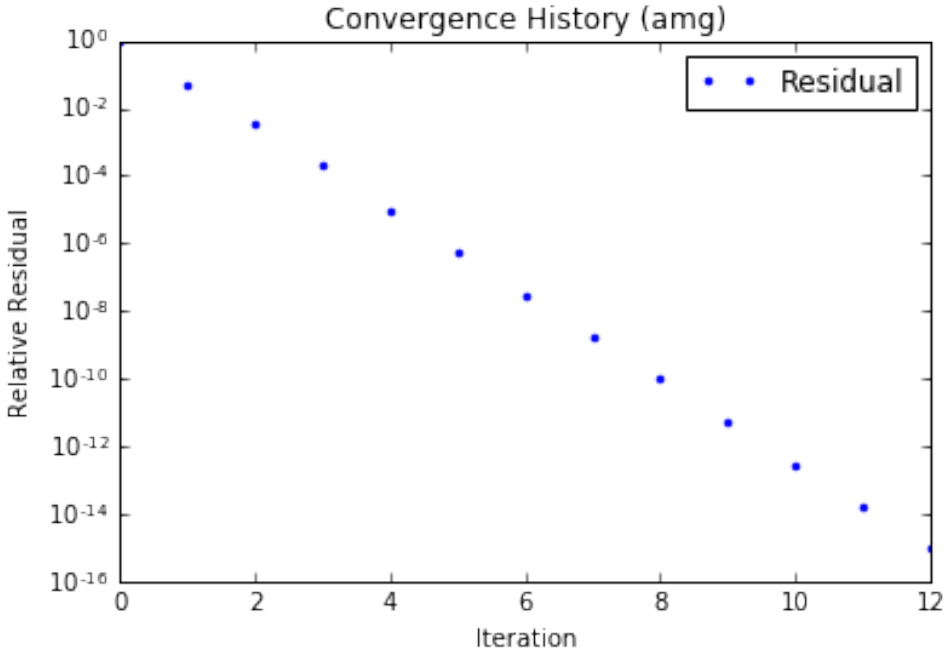
Listing 1: Solving Laplace equation using pyamg

```
    # Extracted from assign\_final\_worksheet.ipynb
    def amg_solver(A, b, cg=False, name='amg', tol=1e-10):
      mls = rootnode_solver(A)
      print mls
      # Solve Ax = b
      residuals = []
      if cg:
          x = mls.solve(b, tol=tol, accel='cg', residuals=
             residuals)
      else :
          x = mls.solve(b, tol=tol, accel=None, residuals=
             residuals)
      # Compute relative residuals
      residuals  = numpy.array(residuals)/residuals[0]
      # Plot convergence history
      import pylab
      pylab.figure()
      pylab.title('Convergence History (%s)' % name)
      pylab.xlabel('Iteration')
      pylab.ylabel('Relative Residual')
      pylab.semilogy(residuals, label='Residual', linestyle='
         None', marker='.')
      pylab.legend()
      pylab.show()
```

The result plots:



(a) Convergence of AMG without CG method.



(b) Convergence of AMG with CG method.

Figure 1: Convergence of AMG.