HPSC - ASSIGNMENT 2

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Spring 2016, W831 Mon-Thu. Period 1-2

Due date: 2016/05/30

Problem

Measure the convergence rate of Laplace Problem. Laplace equation is given by:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0,$$

in a $x = [0, 2]$, $y = [0, 1]$ domains with boundary conditions:

$$p = 0 \ at \ x = 0,$$

$$p = y \ at \ x = 2,$$

$$\partial p / \partial y = 0 \ at \ y = 0, 1.$$

$$\text{error} = \sqrt{\sum_{i,j=1}^{nx,ny} \frac{(p_{exact} - p_{approx})^2}{p_{exact}^2}}$$

The exact solution is available from BEM `step02.py`:

$$p_{exact} = \frac{x}{4} - 4 \sum_{n=odd}^{\infty} \frac{1}{(nx)^2 \sinh 2n\pi} \sinh n\pi x \cos n\pi y$$

Compute:

1. Plot the log scale error against the number of iterations.

2. Change nx, ny.

3. Change order of the finite difference scheme.

The source code and jupyter notebook for this assignment can be found at:
`https://github.com/gear/HPSC/tree/master/hw`

Answer

Using the given code for FDM and exact solution in the lectures, I extract the boundary points from the solution of FDM and compare with the exact solution.

1. Plot the log scale error. Choose default values of $nx = ny = 32$. We have the following log-scale error result:

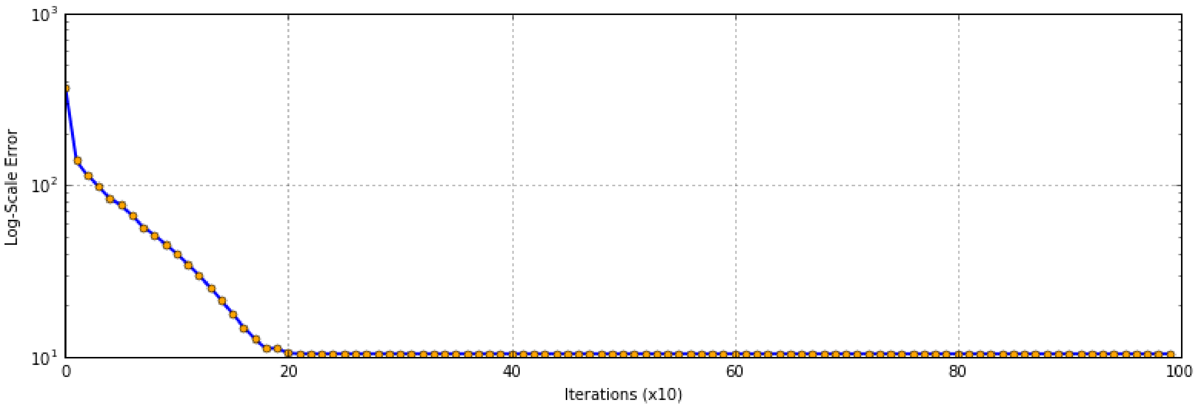

Figure 1: Error convergence. Torlerant $=10^{-6}$.

2. Change nx, ny. For simplicity, here we have log-scale error result of $nx = ny = n$ against error rate of 250 iterations.

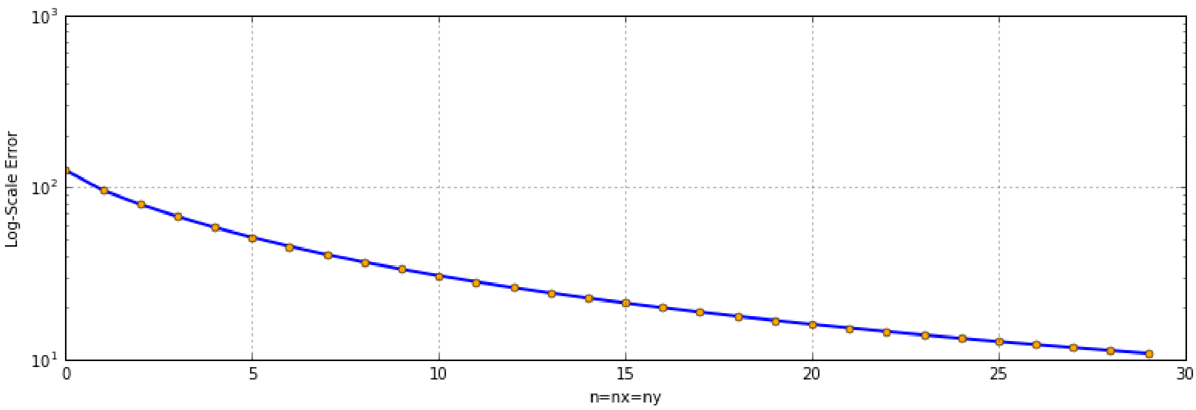

Figure 2: Error convergence for spatial resolution. iter=250.

3. Change to higher order finite diference scheme. The higher order Taylor expansion of the second order devivative terms in Laplace equation can be written in discretized form as:

$$\frac{\partial^2 p}{\partial x^2} = \frac{-p_{j,i+2} + 16p_{j,i+1} - 30p_{j,i} + 16p_{j,i-1} - p_{j,i-2}}{12(\Delta x)^2} + \mathcal{O}(\Delta x)^4$$

$$\frac{\partial^2 p}{\partial y^2} = \frac{-p_{j+2,i} + 16p_{j+1,i} - 30p_{j,i} + 16p_{j-1,i} - p_{j-2,i}}{12(\Delta y)^2} + \mathcal{O}(\Delta y)^4$$