Quiz 3 - Lecture 14 (Prof. Shinoda)

1. Prove that $p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$
2. Discuss the future prospect of deep learning and its related techniques.

Collaborators: None.

Exercise 3-1. Prove that $p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$

Solution: By definition, $\mathbf{z}$ is one-hot encoding representation, we have:

$$p(\mathbf{z}) = \prod_{k=1}^{K} \pi_k^{z_k}$$
$$p(\mathbf{x}|\mathbf{z}) = \prod_{k=1}^{K} \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)^{z_k}$$

By the product rule, we have the join probability of $\mathbf{x}$ and $\mathbf{z}$ as follow:

$$p(\mathbf{x}, \mathbf{z}) = p(\mathbf{x}|\mathbf{z})p(\mathbf{z})$$

Using the sum product to compute the marginal $p(\mathbf{x})$:

$$\begin{aligned} p(\mathbf{x}) &= \sum_{\mathbf{z}} p(\mathbf{x}|\mathbf{z})p(\mathbf{z}) \\ &= \sum_{\mathbf{z}} \prod_{k=1}^{K} \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)^{z_k} \pi_k^{z_k} \quad = \sum_{j=1}^{K} \prod_{k=1}^{K} (\pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k))^{\delta_{jk}}, \end{aligned}$$

where δ_{jk} is the Kronecker delta. Simply rewrite the product keeping not-1 values, we have the desired result:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

Exercise 3-2. Discuss the future prospect of deep learning and related techniques.

Solution: The area in deep learning research that I am interested in are active learning and multi-task learning. Active learning deals with problem of diversity in the training data.