指数类 pdf 总形

$$p(x;\theta) = \exp[A(\theta)B(x) + C(x) + D(\theta)]$$

PDF类型

a 高斯 $p(x;\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2-2\mu x+\mu^2}{2\sigma^2}\right)$

$$= \exp\left[\underbrace{\frac{\mu x}{\sigma^2}}_{A(\mu)B(x)} - \underbrace{\frac{x^2}{2\sigma^2}}_{C(x)} - \underbrace{\left(\frac{\mu^2}{2\sigma^2} + \ln\frac{1}{\sqrt{2\pi}}\right)}_{D(\mu)}\right]$$

b 瑞利 $p(x;\sigma^2) = \frac{x}{\sigma^2} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} u(x)$

$$= \exp\left[\underbrace{-\frac{1}{2}\frac{x^2}{\sigma^2}}_{A(\sigma^2)B(x)} + \underbrace{\ln x\, u(x)}_{C(x)} - \underbrace{\ln\sigma^2}_{D(\sigma^2)}\right]$$

c 指数 $p(x;\lambda) = \lambda e^{-\lambda x}$

$$= \exp\left[\underbrace{-\lambda x}_{A(\lambda)B(x)} + \underbrace{\ln u(x)}_{C(x)} + \underbrace{\ln\lambda}_{D(\lambda)}\right]$$

充分统计量

$$p(x;\theta) = \prod_{n=0}^{N-1} e^{A(\theta)B(x[n]) + C(x[n]) + D(\theta)}$$

$$= \underbrace{e^{A(\theta)\sum_n B(x[n]) + ND(\theta)}}_{g(T(x),\theta)} \underbrace{e^{\sum_n C(x[n])}}_{h(x)}$$

其中 $T(x) = \sum_{n=0}^{N-1} B(x[n])$

求解 MLE

$$p(x;\theta) = \exp\left[A(\theta)B(x) + C(x) + D(\theta)\right]$$

$$p(x;\theta) = \prod_{n=0}^{N-1} e^{A(\theta)B(x[n]) + C(x[n]) + D(\theta)}$$

令的数取对，对其求导

$$\frac{dA(\theta)}{d\theta}\sum_{n=0}^{N-1} B(x[n]) + N\frac{dD(\theta)}{d\theta} = 0$$