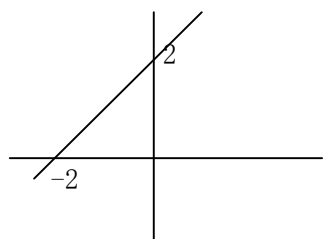


《SE-208 工程数学》 期末考试试卷 (B) 答案

1. 课本 P. 235 定理。

2.



3.
$$w = \frac{(z-1)(i+2)}{(z+2)(i-1)}$$

4.
$$v(x, y) = 3x^2y - y^3, \quad f(z) = z^3$$

5. True. Let $g(z) := f(z + 2\pi) - f(z)$. then g is entire and vanishes on the real axis. Consequently $g(z) = 0$ everywhere.

6. $|e^z - \bar{z}| \leq |e^z| + |z| \leq 1 + 4$, so $\left| \int_C (e^z - \bar{z}) dz \right| \leq 5 \times (3 + 4 + 5) = 60$ In fact,
 $|e^z - \bar{z}| < 4.5$, so we can do better.

7.
$$\sum_{n=0}^{\infty} \frac{2^n z^{n+1}}{3^{n+1}}, \quad |z| < \frac{3}{2}$$

8. (a) $2+3i$

(b)
$$\int_{C:|z|=3} \frac{e^z}{(z-1)(z+2)} dz = \frac{2}{3}(e - e^{-2})\pi i$$

(c)
$$\int_{C:|z+2|=3} \frac{z^3 + 2z}{(z-i)^3} dz = 2\pi i \cdot 3i = -6\pi$$

9. (a)
$$z^2 e^{1/z} = z^2 + z + \frac{1}{2} + \frac{1}{6z} + \cdots + \frac{1}{n! z^{n-2}} + \cdots$$

(b) $\text{Res}(f, 0) = 1/6$

(c) $z \neq 0$

(d) essential singular point

(e)
$$\frac{\pi i}{3}$$

10.
$$f(z) = \frac{1}{2} - \frac{z^2}{4!} + \frac{z^4}{6!} - \cdots, \quad |z| < \infty.$$