中 山 大 学 软 件 学 院 **2009** 级 软 件 工 程 专 业 (2010 学年春季学期)

《SE-208 工程数学》 期末考试试卷（B）

(考 试 形 式 ： 闭 卷 考 试 时 间 ： 2 小 时)

警 示

《中山大学授予学士学位工作细则》第六条

考 试 作 弊 不 授 予 学 士 学 位

方向：__________________ 姓名：__________ 学号：____________

注意：答案一定要写在答卷中，写在本试题卷中不给分。本试卷要和答卷一起交回。

1. (5 pts.)(Short answer question. No proofs required) State Cauchy's residue theorem.
2. (5 pts.)Sketch the image of the half plane $y>1$ under the transformation

   $w=(1+i)z$.
3. (7 pts.)Find the linear fractional transformation that maps the points

   $z_1=1$, $z_2=i$, $z_3=-2$ onto the points $w_1=0, w_2=1, w_3=\infty$.
4. (10 pts.)Find a harmonic conjugate of the harmonic function $u(x,y)=x^3-3xy^2$.

   Write the resulting analytic function in terms of the complex variable z.
5. (7 pts.) If f(z) is an entire function and f(x + 2π) = f(x) for all real x, does f(z + 2π) = f(z) for all complex z ? Proof or counterexample.
6. (10 pts.)Show that if C is the boundary of a triangle with vertices at the points 0, 3i and −4, with counter-clock-wise orientation, then: $\left|\int_C (e^z-\bar{z})dz\right|\le 60$. Can you do better (smaller) than 60?
7. (10 pts.)Find the Taylor series expansion of $f(z)=\dfrac{z}{3-2z}$ at $z_0=0$ and its circle of convergence.
8. (21 pts.) Evaluate the following integrations along the indicated contours (in the positive sense).

   (a) $\int_C f(z)dz$, where $f(z)=\begin{cases}1, & y<0\\ 4y, & y\ge 0\end{cases}$, and C is the arc from −1−i to 1+i along the curve $y=x^3$;

   (b) $\displaystyle\int_{C:|z|=3}\frac{e^z}{(z-1)(z+2)}dz$;

   (c) $\displaystyle\int_{C:|z+2|=3}\frac{z^3+2z}{(z-i)^3}dz$.

9. (15 pts.)Let $f(z) = z^2 e^{1/z}$ (a) Find the Laurent expansion of $f(z)$ at z = 0. (b) What is the residue of $f(z)$ at z = 0? (c) Where does the Laurent series converge? (d) What type of isolated singularity does $f(z)$ have at 0? (e) Evaluate the integration $\int_C z^2 e^{1/z} dz$, where C: |z|=3 in the positive sense.

10. (10pts.)Prove that if $f(z) = \begin{cases} \dfrac{1-\cos z}{z^2}, & z \neq 0 \\ 1/2 & z = 0 \end{cases}$, then $f(z)$ is an entire function.