常 用 积 分 公 式

（一） 含有$ax+b$的积分$(a\neq 0)$

1. $\int \frac{dx}{ax+b}=\frac{1}{a}\ln|ax+b|+C$

2. $\int (ax+b)^{\mu}\,dx=\frac{1}{a(\mu+1)}(ax+b)^{\mu+1}+C\ (\mu\neq -1)$

3. $\int \frac{x}{ax+b}dx=\frac{1}{a^2}(ax+b-b\ln|ax+b|)+C$

4. $\int \frac{x^2}{ax+b}dx=\frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2-2b(ax+b)+b^2\ln|ax+b|\right]+C$

5. $\int \frac{dx}{x(ax+b)}=-\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|+C$

6. $\int \frac{dx}{x^2(ax+b)}=-\frac{1}{bx}+\frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|+C$

7. $\int \frac{x}{(ax+b)^2}dx=\frac{1}{a^2}\left(\ln|ax+b|+\frac{b}{ax+b}\right)+C$

8. $\int \frac{x^2}{(ax+b)^2}dx=\frac{1}{a^3}\left(ax+b-2b\ln|ax+b|-\frac{b^2}{ax+b}\right)+C$

9. $\int \frac{dx}{x(ax+b)^2}=\frac{1}{b(ax+b)}-\frac{1}{b^2}\ln\left|\frac{ax+b}{x}\right|+C$

（二） 含有$\sqrt{ax+b}$的积分

10. $\int \sqrt{ax+b}\,dx=\frac{2}{3a}\sqrt{(ax+b)^3}+C$

11. $\int x\sqrt{ax+b}\,dx=\frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3}+C$

12. $\int x^2\sqrt{ax+b}\,dx=$

$\frac{2}{105a^3}(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3}+C$

13. $\int \frac{x}{\sqrt{ax+b}}dx=\frac{2}{3a^2}(ax-2b)\sqrt{ax+b}+C$

14. $\int \frac{x^2}{\sqrt{ax+b}}dx=\frac{2}{15a^3}(3a^2x^2-4abx+8b^2)\sqrt{ax+b}+C$

15. $\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}} \ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C, & (b > 0) \\ \frac{2}{\sqrt{-b}} \arctan\sqrt{\frac{ax+b}{-b}} + c, & (b < 0) \end{cases}$

16. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}$

17. $\int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$

18. $\int \frac{\sqrt{ax+b}}{x^2} dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}$

（三） 含有$x^2 \pm a^2$的积分

19. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2}\int \frac{dx}{(x^2+a^2)^{n-1}}$

21. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

（四） 含有$ax^2 + b(a > 0)$的积分

22. $\int \frac{dx}{ax^2+b} = \begin{cases} \frac{1}{\sqrt{ab}} \arctan\sqrt{\frac{a}{b}}x + C & (b > 0) \\ \frac{1}{2\sqrt{-ab}} \ln\left|\frac{\sqrt{a}x-\sqrt{-b}}{\sqrt{a}x+\sqrt{-b}}\right| + C & (b < 0) \end{cases}$

23. $\int \frac{x}{ax^2+b} dx = \frac{1}{2a}\ln|ax^2 + b| + C$

24. $\int \frac{x^2}{ax^2+b} dx = \frac{x}{a} - \frac{b}{a}\int \frac{dx}{ax^2+b}$

25. $\int \frac{dx}{x(ax^2+b)} = \frac{1}{2b}\ln\frac{x^2}{|ax^2+b|} + C$

26. $\int \frac{dx}{x^2(ax^2+b)} = -\frac{1}{bx} - \frac{a}{b}\int \frac{dx}{ax^2+b}$

27. $\int \frac{dx}{x^3(ax^2+b)} = \frac{a}{2b^2}\ln\frac{|ax^2+b|}{x^2} - \frac{1}{2bx^2} + C$

28. $\int \frac{dx}{(ax^2+b)^2} = \frac{x}{2b(ax^2+b)} + \frac{1}{2b}\int \frac{dx}{ax^2+b}$

（五） 含有$ax^2 + bx + c(a > 0)$的积分

29. $\int \frac{dx}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C, & (b^2 < 4ac) \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + C, & (b^2 > 4ac) \end{cases}$

30. $\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$

（六） 含有$\sqrt{x^2+a^2}$ $(a>0)$的积分

31. $\int \frac{dx}{\sqrt{x^2+a^2}} = \operatorname{arsh} \frac{x}{a} + C_1 = \ln\left(x+\sqrt{x^2+a^2}\right) + C$

32. $\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$

33. $\int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2} + C$

34. $\int \frac{x}{\sqrt{(x^2+a^2)^3}} dx = -\frac{x}{\sqrt{x^2+a^2}} + C$

35. $\int \frac{x^2}{\sqrt{x^2+a^2}} dx = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2} \ln(x+\sqrt{x^2+a^2}) + C$

36. $\int \frac{x^2}{\sqrt{(x^2+a^2)^3}} dx = -\frac{x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2}) + C$

37. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a} \ln \frac{\sqrt{x^2+a^2}-a}{|x|} + C$

38. $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x} + C$

39. $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \ln\left(x+\sqrt{x^2+a^2}\right) + C$

40. $\int \sqrt{(x^2+a^2)^3}\, dx = \frac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2} + \frac{3}{8}a^4 \ln\left(x+\sqrt{x^2+a^2}\right) + C$

41. $\int x\sqrt{x^2+a^2}\, dx = \frac{1}{3}\sqrt{(x^2+a^2)^3} + C$

42. $\int x^2\sqrt{x^2+a^2}\, dx = \frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \frac{a^4}{8} \ln\left(x+\sqrt{x^2+a^2}\right) + C$

43. $\int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} + a \ln \frac{\sqrt{x^2+a^2}-a}{|x|} + C$

44. $\int \frac{\sqrt{x^2+a^2}}{x^2} dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2}) + C$

（七） 含有$\sqrt{x^2-a^2}$ $(a>0)$的积分

45. $\int \frac{dx}{\sqrt{x^2-a^2}} = \frac{x}{|x|} \operatorname{arch} \frac{|x|}{a} + C_1 = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

46. $\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$

47. $\int \frac{x}{\sqrt{x^2-a^2}}dx = \sqrt{x^2-a^2} + C$

48. $\int \frac{x}{\sqrt{(x^2-a^2)^3}}dx = -\frac{x}{\sqrt{x^2-a^2}} + C$

49. $\int \frac{x^2}{\sqrt{x^2-a^2}}dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

50. $\int \frac{x^2}{\sqrt{(x^2-a^2)^3}}dx = -\frac{x}{\sqrt{x^2-a^2}} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

51. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\arccos\frac{a}{|x|} + C$

52. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$

53. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

54. $\int \sqrt{(x^2-a^2)^3}\,dx = \frac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \frac{3}{8}a^4\ln\left|x+\sqrt{x^2-a^2}\right| + C$

55. $\int x\sqrt{x^2-a^2}\,dx = \frac{1}{3}\sqrt{(x^2-a^2)^3} + C$

56. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

57. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\arccos\frac{a}{|x|} + C$

58. $\int \frac{\sqrt{x^2-a^2}}{x^2}dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

（八） 含有$\sqrt{a^2-x^2}(a>0)$的积分

59. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

60. $\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}} + C$

61. $\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} + C$

62. $\int \frac{x}{\sqrt{(a^2-x^2)^3}}dx = \frac{x}{\sqrt{a^2-x^2}} + C$

63. $\int \frac{x^2}{\sqrt{a^2-x^2}}dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

64. $\int \frac{x^2}{\sqrt{(a^2-x^2)^3}}dx = \frac{x}{\sqrt{a^2-x^2}} - \arcsin\frac{x}{a} + C$

65. $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\ln\frac{a-\sqrt{a^2-x^2}}{|x|} + C$

66. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

67. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$

68. $\int \sqrt{(a^2-x^2)^3}\,dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3}{8}a^4\arcsin\frac{x}{a} + C$

69. $\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$

70. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\arcsin\frac{x}{a} + C$

71. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} + a\ln\frac{a-\sqrt{a^2-x^2}}{|x|} + C$

72. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\frac{\sqrt{a^2-x^2}}{x} - \arcsin\frac{x}{a} + C$

（九） 含有$\sqrt{\pm ax^2+bx+c}(a>0)$的积分

73. $\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C$

74. $\int \sqrt{ax^2+bx+c}\,dx =$

$$\frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C$$

75. $\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C$

76. $\int \frac{dx}{\sqrt{-ax^2+bx+c}} = -\frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$

77. $\int \sqrt{-ax^2+bx+c}\,dx =$

$$\frac{2ax-b}{4a}\sqrt{-ax^2+bx+c}+\frac{4ac+b^2}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C$$

78. $\int \frac{x}{\sqrt{-ax^2+bx+c}}dx =$

$$-\frac{1}{a}\sqrt{-ax^2+bx+c}+\frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}}+C$$

（十） 含有$\sqrt{\pm\frac{x-a}{x-b}}$或$\sqrt{(x-a)(b-x)}$的积分

79. $\int \sqrt{\frac{x-a}{x-b}}dx=(x-b)\sqrt{\frac{x-a}{x-b}}+(b-a)\ln\left(\sqrt{|x-a|}+\sqrt{|x-b|}\right)+C$

80. $\int \sqrt{\frac{x-a}{b-x}}dx=(x-b)\sqrt{\frac{x-a}{b-x}}+(b-a)\arcsin\sqrt{\frac{x-a}{b-x}}+C$

81. $\int \frac{dx}{\sqrt{(x-a)(b-x)}}=2\arcsin\sqrt{\frac{x-a}{b-x}}+C,\ (a<b)$

82. $\int \sqrt{(x-a)(b-x)}\,dx=\frac{2x-a-b}{4}\sqrt{(x-a)(b-x)}+\frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-x}}+C,\ (a<b)$

（十一） 含有三角函数的积分

83. $\int \sin x\,dx=-\cos x+C$

84. $\int \cos x\,dx=\sin x+C$

85. $\int \tan x\,dx=-\ln|\cos x|+C$

86. $\int \cot x\,dx=\ln|\sin x|+C$

87. $\int \sec x\,dx=\ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+C=\ln|\sec x+\tan x|+C$

88. $\int \csc x\,dx = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C$

89. $\int \sec^2 x\,dx = \tan x + C$

90. $\int \csc^2 x\,dx = -\cot x + C$

91. $\int \sec x \tan x\,dx = \sec x + C$

92. $\int \csc x \cot x\,dx = -\csc x + C$

93. $\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

94. $\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x + C$

95. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$

96. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

97. $\int \frac{dx}{\sin^n x} = -\frac{1}{n-1} * \frac{\cos x}{\sin^{n-1}x} + \frac{n-2}{n-1}\int \frac{dx}{\sin^{n-2}x}$

98. $\int \frac{dx}{\cos^n x} = \frac{1}{n-1} * \frac{\sin x}{\cos^{n-1}x} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2}x}$

99. $\int \cos^m x \sin^n x\,dx =$

$$\frac{1}{m+n}\cos^{m-1}x\sin^{n+1}x + \frac{m-1}{m+n}\int \cos^{m-2}x\sin^n x\,dx =$$

$$-\frac{1}{m+n}\cos^{m+1}x\sin^{n-1}x + \frac{n-1}{m+n}\int \cos^m x\sin^{n-2}x\,dx$$

100. $\int \sin ax \cos bx\,dx = -\frac{1}{2(a+b)}\cos(a+b)x - \frac{1}{2(a-b)}\cos(a-b)x + C$

101. $\int \mathrm{Sin}\, ax \sin bx\,dx = -\frac{1}{2(a+b)}\sin(a+b)x + \frac{1}{2(a-b)}\sin(a-b)x + C$

102. $\int \cos ax \cos bx\,dx = \frac{1}{2(a+b)}\sin(a+b)x + \frac{1}{2(a-b)}\sin(a-b)x + C$

103. $\int \frac{dx}{a+b\sin x} = \begin{cases} \frac{2}{\sqrt{a^2-b^2}}\arctan\frac{a\tan\frac{x}{2}+b}{\sqrt{a^2-b^2}} + C, & (a^2 > b^2) \\ \frac{1}{\sqrt{b^2-a^2}}\ln\left|\frac{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\frac{x}{2}+b-\sqrt{b^2-a^2}}\right| + C, & (a^2 < b^2) \end{cases}$

104. $\int \frac{dx}{a+b\cos x} = \begin{cases} \frac{2}{a+b}\sqrt{\frac{a+b}{a-b}}\arctan\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) + C, & (a^2 > b^2) \\ \frac{1}{a+b}\sqrt{\frac{a+b}{b-a}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{a+b}{b-a}}}{\tan\frac{x}{2}-\sqrt{\frac{a+b}{b-a}}}\right| + C, & (a^2 < b^2) \end{cases}$

105. $\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{1}{ab}\arctan\left(\frac{b}{a}\tan x\right) + C$

106. $\int \frac{dx}{a^2\cos^2 x-b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\tan x+a}{b\tan x-a}\right| + C$

107. $\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{1}{a}x\cos ax + C$

108. $\int x^2\sin ax\,dx = -\frac{1}{a}x^2\cos ax + \frac{2}{a^2}x\sin ax + \frac{2}{a^3}\cos ax + C$

109. $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{1}{a}x\sin ax + C$

110. $\int x^2\cos ax\,dx = \frac{1}{a}x^2\sin ax + \frac{2}{a^2}x\cos ax - \frac{2}{a^3}\sin ax + C$

（十二）　含有反三角函数的积分（其中$a > 0$）

111. $\int \arcsin\frac{x}{a}dx = x\arcsin\frac{x}{a} + \sqrt{a^2-x^2} + C$

112. $\int x\arcsin\frac{x}{a}dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arcsin\frac{x}{a} + \frac{x}{4}\sqrt{a^2-x^2} + C$

113. $\int x^2\arcsin\frac{x}{a}dx = \frac{x^3}{3}\arcsin\frac{x}{a} + \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$

114. $\int \arccos\frac{x}{a}dx = x\arccos\frac{x}{a} - \sqrt{a^2-x^2} + C$

115. $\int x\arccos\frac{x}{a}dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\arccos\frac{x}{a} - \frac{x}{4}\sqrt{a^2-x^2} + C$

116. $\int x^2\arccos\frac{x}{a}dx = \frac{x^3}{3}\arccos\frac{x}{a} - \frac{1}{9}(x^2+2a^2)\sqrt{a^2-x^2} + C$

117. $\int \arctan\frac{x}{a}dx = x\arctan\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2) + C$

118. $\int x\arctan\frac{x}{a}dx=\frac{1}{2}(a^2+x^2)\arctan\frac{x}{a}-\frac{a}{2}x+C$

119. $\int x^2\arctan\frac{x}{a}dx=\frac{x^3}{3}\arctan\frac{x}{a}-\frac{a}{6}x^2+\frac{a^3}{6}\ln(a^2+x^2)+C$

（十三）　含有指数函数的积分

120. $\int a^x\,dx=\frac{1}{\ln a}a^x+C$

121. $\int e^{ax}\,dx=\frac{1}{a}e^{ax}+C$

122. $\int xe^{ax}\,dx=\frac{1}{a^2}(ax-1)e^{ax}+C$

123. $\int x^n e^{ax}\,dx=\frac{1}{a}x^n e^{ax}-\frac{n}{a}\int x^{n-1}e^{ax}\,dx$

124. $\int xa^x\,dx=\frac{x}{\ln a}a^x-\frac{1}{(\ln a)^2}a^x+C$

125. $\int x^n a^x\,dx=\frac{1}{\ln a}x^n a^x-\frac{n}{\ln a}\int x^{n-1}a^x\,dx$

126. $\int e^{ax}\sin bx\,dx=\frac{1}{a^2+b^2}e^{ax}(a\sin bx-b\cos bx)+C$

127. $\int e^{ax}\cos bx\,dx=\frac{1}{a^2+b^2}e^{ax}(b\sin bx+a\cos bx)+C$

128. $\int e^{ax}\sin^n bx\,dx=$

$$\frac{1}{a^2+b^2n^2}e^{ax}\sin^{n-1}bx(a\sin bx-nb\cos bx)+\frac{n(n-1)b^2}{a^2+b^2n^2}\int e^{ax}\sin^{n-2}bx\,dx$$

129. $\int e^{ax}\cos^n bx\,dx=\frac{1}{a^2+b^2n^2}e^{ax}\cos^{n-1}bx(a\cos bx+n\sin bx)+\frac{n(n-1)b^2}{a^2+b^2n^2}\int e^{ax}\cos^{n-2}bx\,dx$

（十四）　含有对数函数的积分

130. $\int\ln x\,dx=x\ln x-x+C$

131. $\int\frac{dx}{x\ln x}=\ln|\ln x|+C$

132. $\int x^n\ln x\,dx=\frac{1}{n+1}x^{n+1}\left(\ln x-\frac{1}{n+1}\right)+C$

133. $\int(\ln x)^n\,dx=x(\ln x)^n-n\int(\ln x)^{n-1}\,dx$

134. $\int x^m (\ln x)^n \, dx = \frac{1}{m+1} x^{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, dx$

（十五）　含有双曲函数的积分

135. $\int \operatorname{sh} x \, dx = \operatorname{ch} x + C$

136. $\int \operatorname{ch} x \, dx = \operatorname{sh} x + C$

137. $\int \operatorname{th} x \, dx = \ln \operatorname{ch} x + C$

138. $\int \operatorname{sh}^2 x \, dx = -\frac{x}{2} + \frac{1}{4} \operatorname{sh} 2x + C$

139. $\int \operatorname{ch}^2 x \, dx = \frac{x}{2} + \frac{1}{4} \operatorname{sh} 2x + C$

（十六）　定积分

140. $\int_{-\pi}^{\pi} \cos nx \, dx = \int_{-\pi}^{\pi} \sin nx \, dx = 0$

141. $\int_{-\pi}^{\pi} \cos mx \sin nx \, dx = 0$

142. $\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

143. $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

144. $\int_0^{\pi} \sin mx \sin nx \, dx = \int_0^{\pi} \cos mx \cos nx \, dx = \begin{cases} 0, & m \neq n \\ \frac{\pi}{2}, & m = n \end{cases}$

145. (1) $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx = \int_0^{\frac{\pi}{2}} \cos^n x \, dx$

(2) $I_n = \frac{n-1}{n} I_{n-2}$

(3) $I_n = \frac{n-1}{n} * \frac{n-3}{n-2} * \ldots\ldots * \frac{4}{5} * \frac{2}{3}$，$(n$ 为大于 1 的正奇数$)$，$I_1 = 1$

(4) $I_n = \frac{n-1}{n} * \frac{n-3}{n-2} * \ldots\ldots * \frac{3}{4} * \frac{1}{2} * \frac{\pi}{2}$，$(n$ 为正偶数$)$，$I_0 = \frac{\pi}{2}$