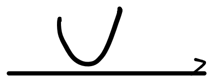
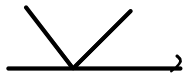
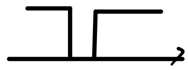
第十一章笔记

误差	$\varepsilon=\theta-\hat{\theta}$		
代价函数	$C(\varepsilon)$		
贝叶斯风险	$R=E(C(\varepsilon))$		
类型	二次		$p(\theta\|x)$的均值
	绝对		$p(\theta\|x)$的中值
	成功-失败		$p(\theta\|x)$的众数

绝对值误差代价函数

$$g(\hat{\theta})=\int|\theta-\hat{\theta}|\,p(\theta|x)\,d\theta$$

$$=\int_{-\infty}^{\hat{\theta}}(\hat{\theta}-\theta)\,p(\theta|x)\,d\theta+\int_{\hat{\theta}}^{+\infty}(\theta-\hat{\theta})\,p(\theta|x)\,d\theta$$

又有 $\frac{\partial}{\partial u}\int_{\phi_1(u)}^{\phi_2(u)}h(u,v)\,dv$

$$=\int_{\phi_1(u)}^{\phi_2(u)}\frac{\partial h(u,v)}{\partial u}dv+\frac{d\phi_2(u)}{du}h(u,\phi_2(u))-\frac{d\phi_1(u)}{du}h(u,\phi_1(u))$$

所以 $\frac{dg(\hat{\theta})}{d\hat{\theta}}=\int_{-\infty}^{\hat{\theta}}p(\theta|x)\,d\theta-\int_{\hat{\theta}}^{+\infty}p(\theta|x)\,d\theta$

令其为0则 $\int_{-\infty}^{\hat{\theta}}p(\theta|x)\,d\theta=\int_{\hat{\theta}}^{+\infty}p(\theta|x)\,d\theta$

即$\hat{\theta}$为后验pdf的中值 $\Leftrightarrow$ $Pr(\theta\le\hat{\theta}|x)=\frac{1}{2}$的点

成功-失败型函数

$$g(\hat{\theta}) = \int_{-\infty}^{\hat{\theta}-\delta} 1 \cdot p(\theta|x)\,d\theta + \int_{\hat{\theta}+\delta}^{\infty} 1 \cdot p(\theta|x)\,d\theta$$

$$= 1 - \int_{\hat{\theta}-\delta}^{\hat{\theta}+\delta} p(\theta|x)\,d\theta$$

对于任意的 δ 选择 $\hat{\theta}$ 为后验 pdf 的众数

矢量参数 多参量

$$p(\theta|x) = \frac{p(x|\theta)p(\theta)}{\int p(x|\theta)p(\theta)\,d\theta}$$

$$\hat{\theta}_1 = E(\theta_1 x) = \int \theta_1\, p(\theta_1|x)\,d\theta_1$$

$$\hat{\theta}_i = E(\theta_i x) = \int \theta_i\, p(\theta_i|x)\,d\theta_i$$

$$E[(\theta_i - \hat{\theta}_i)^2] = \int (\theta_i - \hat{\theta}_i)\, p(x, \theta_i)\,dx\,d\theta_i$$

而 $p(\theta_i|x) = \int \cdots \int p(\theta|x)\,d\theta_1 \cdots d\theta_p$

$$\int \theta_i\, p(\theta_i|x)\,d\theta_i = \int \theta_i \left[\int \cdots \int p(\theta|x)\,d\theta_1 \cdots d\theta_p\right] d\theta_1$$

$$= \int \theta_1\, p(\theta|x)\,d\theta$$

所以 $$\hat{\theta} = \begin{bmatrix} \int \theta_1 p(\theta|x)\,d\theta \\ \int \theta_2 p(\theta|x)\,d\theta \\ \vdots \\ \int \theta_n p(\theta|x)\,d\theta \end{bmatrix} = \int \theta\, p(\theta|x)\,d\theta$$

最大后验估计量MAP

$$\hat{\theta} = \arg\max_{\theta} p(\theta|x)$$

$$= \arg\max_{\theta} p(x|\theta)p(\theta)$$

$$= \arg\max_{\theta} [\ln p(x|\theta) + \ln p(\theta)]$$