

指数类 pdf 总结

$$p(x; \theta) = \exp[A(\theta)B(x) + C(x) + D(\theta)]$$

PDF 类型

$$\begin{aligned} \text{a 高斯} \quad p(x, \mu) &= \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{x^2 - 2\mu x + \mu^2}{2\sigma^2}\right) \\ &= \exp\left[\underbrace{\frac{\mu x}{\sigma^2}}_{A(\mu)B(x)} - \underbrace{\frac{x^2}{2\sigma^2}}_{C(x)} - \underbrace{\left(\frac{\mu^2}{2\sigma^2} + \ln \frac{1}{\sqrt{2\pi}}\right)}_{D(\mu)}\right] \end{aligned}$$

$$\begin{aligned} \text{b 伽马} \quad p(x; \delta^2) &= \frac{x}{\delta^2} e^{-\frac{1}{2}\frac{x^2}{\delta^2}} u(x) \\ &= \exp\left[\underbrace{-\frac{1}{2}\frac{x^2}{\delta^2}}_{A(\delta^2)B(x)} + \underbrace{\ln x u(x)}_{C(x)} - \underbrace{\ln \delta^2}_{D(\delta^2)}\right] \end{aligned}$$

$$\begin{aligned} \text{c 指数} \quad p(x; \lambda) &= \lambda e^{-\lambda x} \\ &= \exp\left[\underbrace{-\lambda x}_{A(\lambda)B(x)} + \underbrace{\ln u(x)}_{C(x)} + \underbrace{\ln \lambda}_{D(\lambda)}\right] \end{aligned}$$

充分统计量

$$\begin{aligned} p(x; \theta) &= \prod_{n=0}^{N-1} e^{A(\theta)B(x[n]) + C(x[n]) + D(\theta)} \\ &= \underbrace{e^{A(\theta) \sum_n B(x[n]) + ND(\theta)}}_{g(T(x), \theta)} \underbrace{e^{\sum_n C(x[n])}}_{h(x)} \end{aligned}$$

$$\text{其中 } T(x) = \sum_{n=0}^{N-1} B(x[n])$$

求解 MLE

$$P(x; \theta) = \exp[A(\theta) B(x) + C(x) + D(\theta)]$$

$$P(x; \theta) = \prod_{n=0}^{N-1} e^{A(\theta) B(x[n]) + C(x[n]) + D(\theta)}$$

令函数最小. 对其求导

$$\frac{dA(\theta)}{d\theta} \sum_{n=0}^{N-1} B(x[n]) + N \frac{dD(\theta)}{d\theta} = 0$$