中 山 大 学 软 件 学 院 **2010** 级 软 件 工 程 专 业 (2012 春季学期)

# 《工程数学》期末试题试卷(A)

**(考 试 形 式： 闭 卷 考 试 时 间：2 小 时)**

警 示

**《中山大学授予学士学位工作细则》第六条**

**考 试 作 弊 不 授 予 学 士 学 位**

方向：________________ 姓名：________ 学号：__________

出卷：________________________ 审核：__________________

**注意：答案一定要写在答卷中，写在本试题卷中不给分。本试卷要和答卷一起交回。**

1. Find the value(s) of (10 Points)

(a) $(-8i)^{1/3}$ (b) $|e^{i\alpha}\dfrac{3-2i}{2+3i}|, \alpha \in R$

**Answers:**

$$(-8i)^{1/3}: c_k = 2\exp[i(-\frac{\pi}{6}+\frac{2k\pi}{3})], (k=0,1,2)$$

$$|e^{i\alpha}\frac{3-2i}{2+3i}| = |e^{i\alpha}||\frac{3-2i}{2+3i}| = 1 \times \frac{|3-2i|}{|2+3i|} = 1$$

2. Show that $f(z)=u(x,y)+iv(x,y)$ and that $f'(z)$ exists at a point $z_0=x_0+iy_0$. Then the first-order partial derivatives of u and v must exist at $(x_0,y_0)$, and they must satisfy the Cauchy-Riemann equations (10 Points)

$$u_x = v_y, u_y = -v_x$$

**Answers:**

Refer to pp.63-64 in the textbook.

3. Show that

(a) $Log(1+i)^2 = 2Log(1+i)$ (b) $Log(-1+i)^2 \neq 2Log(-1+i)$ (10 Points)

**Answers:**

(a) $$(1+i)^2 = 2i = 2e^{i\theta}, \theta = \pi/2 + 2k\pi, k = 0, \pm 1, \pm 2, \ldots$$

$$Log(1+i)^2 = \ln 2 + i\frac{\pi}{2}, k = 0, -\pi < \frac{\pi}{2} < +\pi$$

$$1+i = \sqrt{2}e^{i\theta}, \theta = \pi/4 + 2k\pi, k = 0, \pm 1, \pm 2, \ldots$$

$$2Log(1+i) = 2[\ln\sqrt{2} + i\frac{\pi}{4}], k = 0, -\pi < \frac{\pi}{4} < +\pi$$

Therefore, we have that

$$Log(1+i)^2 = 2Log(1+i) = \ln 2 + i\frac{\pi}{2}$$

(b) $(-1+i)^2 = -2i = 2e^{i\theta}, \theta = -\pi/2 + 2k\pi, k = 0, \pm1, \pm2, \ldots$

$$Log(-1+i)^2 = \ln 2 + i\frac{-\pi}{2}, k = 0, -\pi < \frac{-\pi}{2} < +\pi$$

$$-1+i = \sqrt{2}e^{i\theta}, \theta = 3\pi/4 + 2k\pi, k = 0, \pm1, \pm2, \ldots$$

$$2Log(-1+i) = 2[\ln\sqrt{2} + i\frac{3\pi}{4}] = \ln 2 + i\frac{3\pi}{2}, k = 0, -\pi < \frac{3\pi}{4} < +\pi$$

Therefore, we have that

$$Log(-1+i)^2 \neq 2Log(-1+i)$$

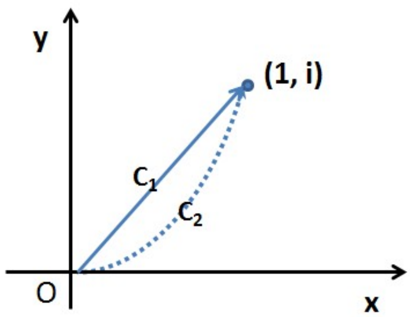
4. Suppose $f(z) = x^2 + iy$,

(a) Determine whether *f(z)* is analytic or not in the xy-plane

(b) Evaluate the integral $\int_C f(z)dz$, where C is

(b1) the line from 0 to (1,i)

(b2) the curve $y=x^2$ from 0 to (1,i) (15 points)




**Answers:**

(a) *f(z)* is not analytic, since

$$u(x,y) = x^2 \; \& \; v(x,y) = y$$

$$u_x = 2x \neq v_y = 1, (x \neq \frac{1}{2})$$

(b)

(b1) The points of $C_1$ by means of the equation

$$z(t) = t + it, (0 \le t \le 1)$$

$$\int_{C_1} f(z)dz = \int_0^1 f(z(t))z'(t)dt = \int_0^1 (t^2 + it)(1+i)dt = -\frac{1}{6} + \frac{5}{6}i$$

(b2) The points of $C_2$ by means of the equation

$$z(t) = t + it^2, (0 \le t \le 1)$$

$$\int_{C_2} f(z)dz = \int_0^1 f(z(t))z'(t)dt = \int_0^1 (t^2 + it^2)(1+2it)dt = \frac{1}{6}(-1+5i)$$

5. Expand the function

$$f(z) = \frac{1+2z^2}{z^3+z^5}$$

into a series involving powers of z, and find the residue (10 points)

**Answers:**

$$f(z) = \frac{1+2z^2}{z^3+z^5} = \frac{1}{z^3}(\frac{1+2z^2}{1+z^2}) = \frac{1}{z^3}(2 - \frac{1}{1+z^2})$$

$$\frac{1}{1+z^2} = 1 - z^2 + z^4 - z^6 + z^8 - \ldots (|z| < 1)$$

$$f(z) = \frac{1}{z^3}(2 - 1 + z^2 - z^4 + z^6 - z^8 + \ldots) = \frac{1}{z^3} + \frac{1}{z} - z + z^3 - z^5 + \ldots$$

The residue is 1.

Refer to pp.195 Ex. 5 in the textbook

6. Let C be the counterclockwise circle with center at 0 and radius r. Evaluate the following integrals

$$\oint_C \frac{e^z}{z^2 - 2z - 8} dz$$

where r=1, 3 and 5 (10 points)

**Answers:**

$$f(z) = \frac{e^z}{z^2 - 2z - 8} = \frac{e^z}{(z+2)(z-4)}$$

When r=1, there is no singular points in the circle. Thus the integral is zero.

When r=3, there is a singular point (z=-2) in the circle. Thus the integral is

$$\oint_C \frac{e^z}{z^2 - 2z - 8} dz = 2\pi i (\operatorname*{Res}_{z=-2} f(z))$$

$$f(z) = \frac{\phi(z)}{z+2}, \phi(z) = \frac{e^z}{z-4}$$ $\phi(z)$ is analytic at z=-2 and $\phi(-2) \ne 0$

$$\oint_C \frac{e^z}{z^2-2z-8}dz = 2\pi i(\operatorname*{Res}_{z=-2} f(z)) = 2\pi i\phi(-2) = \frac{-\pi e^{-2}}{3}i$$

When r=5, there are two singular points (z=-2, z=4) in the circle. Similarly we obtain that

$$\oint_C \frac{e^z}{z^2-2z-8}dz = 2\pi i(\operatorname*{Res}_{z=-2} f(z) + \operatorname*{Res}_{z=4} f(z)) = \pi i(-\frac{e^{-2}}{3}+\frac{e^4}{3})$$

7. Evaluate the following improper integrals

(a) $\int_0^{+\infty} \frac{x^2}{x^6+1}dx$

(b) $\int_0^{+\infty} \frac{x\sin x}{x^2+a^2}dx, (a>0)$ (15 points)

**Answers:**

(a) The integral is π/6 (refer to pp. 265-267 in the textbook)

(b) The integral is $\pi/2e^{-a}$.

8. Find the special case of linear fractional transformation

$$w = \frac{az+b}{cz+d}, (ad-bc \neq 0)$$

that maps the points

(a) $z_1=1$, $z_2=0$, $z_3=i$ onto the point $w_1=(3+i)/5$, $w_2=-i$, $w_3=0$

(b) $z_1=1$, $z_2=0$, $z_3=i$ onto the point $w_1=\infty$, $w_2=2$, $w_3=i$ (10 points)

**Answers:**

**(a)**

$$w = \frac{iz+1}{2z+i}$$

(b)

$$w = \frac{(1-i)z-2i}{iz-i}$$

Refer to the implicit form in pp. 322 in the textbook

9. Suppose that $f = u + iv$ is an analytic function, find $v$ given $u$: (10 points)

$$u(x,y) = x^2 - y^2$$

**Answers:**

$v(x,y) = 2xy + C$, C is a complex constant.

Refer to pp. 81 (way #1), and pp. 365 (way #2) in the textbook.