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Infinite Series of $1/n$

Date: 01/13/98 at 08:56:17

From: Michael Theemling

Subject: Sum of an infinite series of $1/n$ when $n=1,2,3,\dots$

This is a question I encountered in calculus a long time ago while I was still a young math student. I understand the answer is divergence, or the sum is infinity, but do not conceptually understand why that is so, especially since the terms eventually go to 0.

$$1 + 1/2 + 1/3 + 1/4 + \dots$$

For $n > 1$ and $n < 1$ it is relatively easy to comprehend, but not this one. I saw a proof before but never really got it.

I just want to add your site is outstanding. As a student (now graduated) in mathematics, I found your information clear and your knowledge in-depth.

Date: 01/26/98 at 10:59:21

From: Doctor Joe

Subject: Re: Sum of an infinite series of $1/n$ when $n=1,2,3,\dots$

Dear Michael,

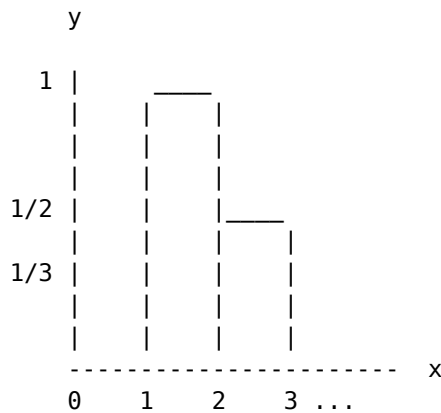
The divergence of $1 + 1/2 + 1/3 + 1/4 + \dots$ is famous (or rather infamous!). The proof requires some geometrical interpretation and a minimal knowledge of integral calculus.

Let $f(x) = 1/x$.

The next thing you need to do is to sketch this reciprocal function over the interval $(1, \text{infinity})$.

Next, construct "boxes" that cover the region bounded by the curve $y=f(x)$, the $y=1$ line, the x -axis, and the $y=n+1$ line in the following way:

On each integer interval a rectangle is constructed by taking the height to be the maximum value over that interval. For example, on the interval $[1,2]$, the maximum value of the function $y = 1/x$ is 1.



If we form the sum of the areas of the first n rectangles, then the value is:

$$1 + 1/2 + 1/3 + 1/4 + \dots + 1/n$$

Since the curve is concave downwards, the sum of areas of the $n+1$ rectangles is clearly greater than the area under the curve $f(x) = 1/x$ on the interval $[1, n+1]$.

So, we have the following inequality:

$$\int_1^{n+1} \frac{1}{x} < 1 + 1/2 + 1/3 + 1/4 + \dots + 1/n$$

for each n , positive integer.

Evaluating the integral on the left, we have $\ln(n+1)$.

Suppose that the sum $1 + 1/2 + 1/3 + 1/4 + 1/5 + \dots$ converges to, say, a real number S . Then,

$$\lim_{n \rightarrow \infty} \ln(n+1) < \text{or equal to } S.$$

Note that the sequence $\{\ln(n+1)\}$ diverges to infinity. Thus, we have arrived at a contradiction, and therefore our supposition is false. Hence, $1 + 1/2 + 1/3 + \dots$ diverges.

(Classical analysts call this series the harmonic series.)

-Doctor Joe, The Math Forum

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Date: 04/03/00 at 08:36:42

From: Dr. Schwa

Subject: harmonic series

To prove that the sum diverges, you can also use a comparison test to a series that obviously diverges.

Group the terms together:

$(1) + (1/2) + (1/3 + 1/4) + (1/5 + 1/6 + 1/7 + 1/8) + (1/9 + \dots + 1/16)$
+ ...
taking twice as many terms each time.

This is clearly greater than
 $(1) + (1/2) + (1/4 + 1/4) + (1/8 + 1/8 + 1/8 + 1/8) + (1/16 + \dots + 1/16)$
+ ...
since we replaced each term with something smaller or equal.

Then, each of the parenthesized groups is $1/2$, so the original harmonic series is greater than
 $1/2 + 1/2 + 1/2 + 1/2 + 1/2 + \dots$

Since there are infinitely many $1/2$'s, the series diverges.

-Dr. Schwa, The Math Forum

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