Infinite Series of 1/n

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Date: 01/13/98 at 08:56:17
From: Michael Theemling
Subject: Sum of an infinite series of 1/n when n=1,2,3...

This is a question I encountered in calculus a long time ago while I
was still a young math student. I understand the answer is divergence,
or the sum is infinity, but do not conceptually understand why that is
so, especially since the terms eventually go to 0.

  1 +1/2+1/3+1/4+...

For n>1 and n<1 it is relatively easy to comprehend, but not this one.
I saw a proof before but never really got it.

I just want to add your site is outstanding. As a student (now
graduated) in mathematics, I found your information clear and your
knowledge in-depth.
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Date: 01/26/98 at 10:59:21
From: Doctor Joe
Subject: Re: Sum of an infinite series of 1/n when n=1,2,3...

Dear Michael,

The divergence of 1 + 1/2 + 1/3 + 1/4 + ... is famous (or rather
infamous!). The proof requires some geometrical interpretation and
a minimal knowledge of integral calculus.

Let f(x) = 1/x.

The next thing you need to do is to sketch this reciprocal function
over the interval (1,infinity).

Next, construct "boxes" that cover the region bounded by the curve
y=f(x), the y=1 line, the x-axis, and the y=n+1 line in the following
way:

On each integer interval a rectangle is constructed by taking the
height to be the maximum value over that interval. For example, on the
interval [1,2], the maximum value of the function y = 1/x is 1.
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```
       y

     1 |     ____
       |    |    |
       |    |    |
       |    |    |
   1/2 |    |    |____
       |    |    |    |
   1/3 |    |    |    |
       |    |    |    |
       |    |    |    |
       -----------------------  x
       0    1    2    3 ...
```

If we form the sum of the areas of the first n rectangles, then the value is:

    1 + 1/2 + 1/3 + 1/4 + ... + 1/n

Since the curve is concave downwards, the sum of areas of the n+1 rectangles is clearly greater than the area under the curve f(x) = 1/x on the interval [1,n+1].

So, we have the following inequality:

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         n+1
      Int | 1/x < 1 + 1/2 + 1/3 + 1/4 + ... + 1/n
           1
```

for each n, positive integer.

Evaluating the integral on the left, we have ln(n+1).

Suppose that the sum 1 + 1/2 + 1/3 + 1/4 + 1/5 + ... converges to, say, a real number S. Then,

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       lim      ln (n+1) < or equal to S.
    n->infinity
```

Note that the sequence {ln(n+1)} diverges to infinity. Thus, we have arrived at a contradiction, and therefore our supposition is false. Hence, 1 + 1/2 + 1/3 + ... diverges.

(Classical analysts call this series the harmonic series.)

-Doctor Joe, The Math Forum
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Date: 04/03/00 at 08:36:42
From: Dr. Schwa
Subject: harmonic series

To prove that the sum diverges, you can also use a comparison test to a series that obviously diverges.

Group the terms together:

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(1) + (1/2) + (1/3 + 1/4) + (1/5 + 1/6 + 1/7 + 1/8) + (1/9 + ... + 1/16)
+ ...
taking twice as many terms each time.

This is clearly greater than
(1) + (1/2) + (1/4 + 1/4) + (1/8 + 1/8+ 1/8 + 1/8) + (1/16 + ... + 1/16)
+ ...
since we replaced each term with something smaller or equal.

Then, each of the parenthesized groups is 1/2, so the original harmonic
series is greater than
1/2 + 1/2 + 1/2 + 1/2 + 1/2 + ...

Since there are infinitely many 1/2's, the series diverges.

-Dr. Schwa, The Math Forum
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