Algorithms Notes

Bliss Of Comprehension

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Contents

preface

This book is largely concerned with algorithms helpful for solving OJ problems.

Chapter 1

Stack

Stack is the suprising Data Structure that comes in handy in unexpected of times.

1.1 Balanced Parantheses

A bracket sequence is balanced if either of the below are true. As all of them are equivalent.

- Every element of prefix sum array is ≥ 0 and last element is 0.
- When open bracket is encountered push it to the stack and pop the top open bracket otherwise. If there is no open bracket to pop or the stack is not empty at the end, the sequence is not balanced.
- Base Case : Empty sequence is balanced.
  Constructor Case : If s,t (sequences) are balanced then s(t) is balanced.
  Now check if our sequence can be produced this way using recursion.

Overlapping two balanced sequences produces a balanced sequence. Prove it using the 1st definition.

C_i (catalan number) counts the number of sequences containing n pairs of parentheses which are correctly matched. Using this we can calculate in O(n) unlike other DP solutions.

Let s[0..n-1] be a balanced sequence

- s[0..i] is balanced iff s[i+1..n-1] is balanced.

When sequence is balanced, we can pairup open and closed brackets. This is done during the 2nd definition. When we are popping out the open bracket from stack(when closed br is encountered) we pair those two.

The pairing can also be done like this. For open bracket at i, find the minimum index j S.t sum[i..j]=0. (i,j) are paired

Let ith and jth index (i<j) are paired.

- s[i..k] $i < k < j$ is not balanced.
- If $i < l < j$ then the partner of l (named as r), $i < r < j$.
  Proof: We know $sum[i..l-1] > 0 => sum[l..j] < 0$ since $sum[l..l] = 0$ there exists $r < j$ S.t $sum[l..r] = 0$. We used the property that $sum[i..j]$ is continous over j.

Chapter 2

XOR

Competitive problem setters love XOR (never understood the physical significance of it).

Different techniques to solve problems on XOR

- Bit Trie.
- Solving for each individual bit and combining them at the end.
- Using Lexicographic property. If the most significant bit in binary form a number is 1 and the other number has 0 then first number is greater (no matter what the other bits are).
- In problems concerning XOR converting the array into prefix array can be useful.

Some tricks in solving XOR problems

- $4k \oplus (4k+1) \oplus (4k+2) \oplus (4k+3) = 0$ (used in finding XOR of numbers from 1..1e18)
- If $a \oplus b = 0$ and $b \neq c \implies c \oplus b \neq 0$. This trick is mostly used in number theory to prove theorems on nim.
- If the array is sorted, all the elements with the same first x bits will be contigous $\forall$x.

2.1 Game Theory

There are two type of operations involved in game theory

- MEX
- XOR

Sprague-Grundy Theorem :

If we know the MEX function of different games (G1, G2..) then the MEX function of combined games is XOR of all the functions.

Chapter 3

Counting

- Given a binary string, find number of 3-tuple (indices $i, j, k \ni i < j < k$) and exactly one of $s[i], s[j], s[k]$ is 1.