Show & Tell

CIVE 503 – Final Project

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WIND

WAVE

TIDAL

COMMUNITIES

Contents

- Motivation
- Intent
- Methodology
- Interim Results
- 6 slides

Motivation

- Surrogate modelling is seeing uptake in a wide variety of domains (including model-based design optimization).

- The notions of “sampling efficiency” and “surrogate efficiency” appear in the literature, but there does not seem to be a standardized way of expressing these notions precisely.

Intent

- Propose and test a standardized surrogate model efficiency metric.

$$\eta_{\mathrm{SM}} \sim \frac{\text{surrogate utility}}{\text{surrogate cost}}$$

Methodology (Proposition)

- Consider standard benchmark problems of the form

$$y : \mathbb{R}^D \to \mathbb{R}$$
$$\vec{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_D \end{bmatrix}^\mathsf{T} \mapsto y(\vec{x})$$

  namely: Rastrigin, Rosenbrock, Griewank, and Styblinski–Tang.

- <u>Define</u> surrogate utility as inverse of surrogate error, where surrogate error is

$$\text{surrogate error} = \mu_{\text{d-APE}} + \text{IQR}_{\text{d-APE}}$$

- <u>Assume</u> surrogate cost is dominated by sampling cost (i.e., is proportional to number of samples N).

- <u>Proposition</u>:

$$\eta_{\text{SM}} = \exp\left[-\sqrt[D]{N}\left(\mu_{\text{d-APE}} + \text{IQR}_{\text{d-APE}}\right)\right]$$

Methodology (Testing)

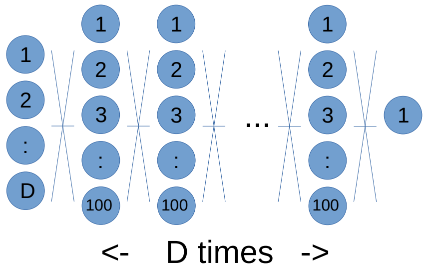
- Full factorial experiment considering combinations of
  - Benchmark problem [4]
  - Sampling scheme (simple random vs latin hypercube) [2]
  - Dimensionality 2 thru 6 [5]
  - Number of samples up to 10^5 [16]

- Monte Carlo trials for each combination considered. [50]

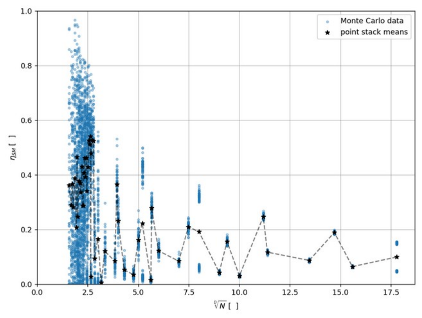
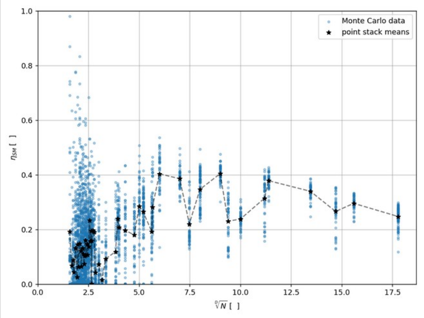
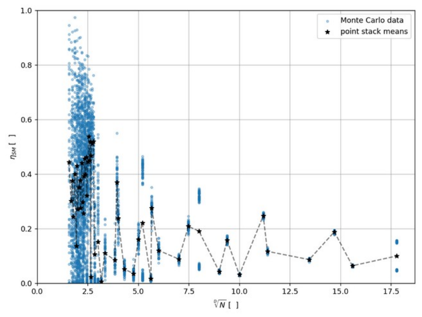
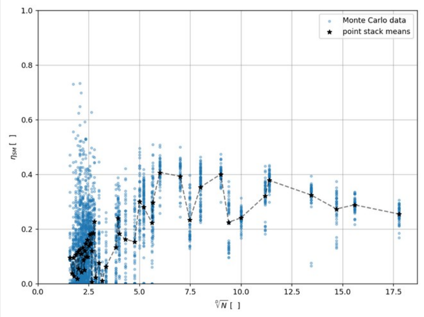
- Each Monte Carlo trial consists of
  - Sample objective ← some randomness here
  - Split sample data (train, validate, test) ← some randomness here
  - Train surrogate ← some randomness here
  - Test surrogate, compute efficiency

Methodology (Testing)

- Neural networks are commonly used as surrogate models in the literature, so the same is done here.
- Surrogate is a dense network with D hidden layers of 100 neurons (ReLU) each.
- Train/test split is 85/15, and then train is further split 85/15 into train/validation.

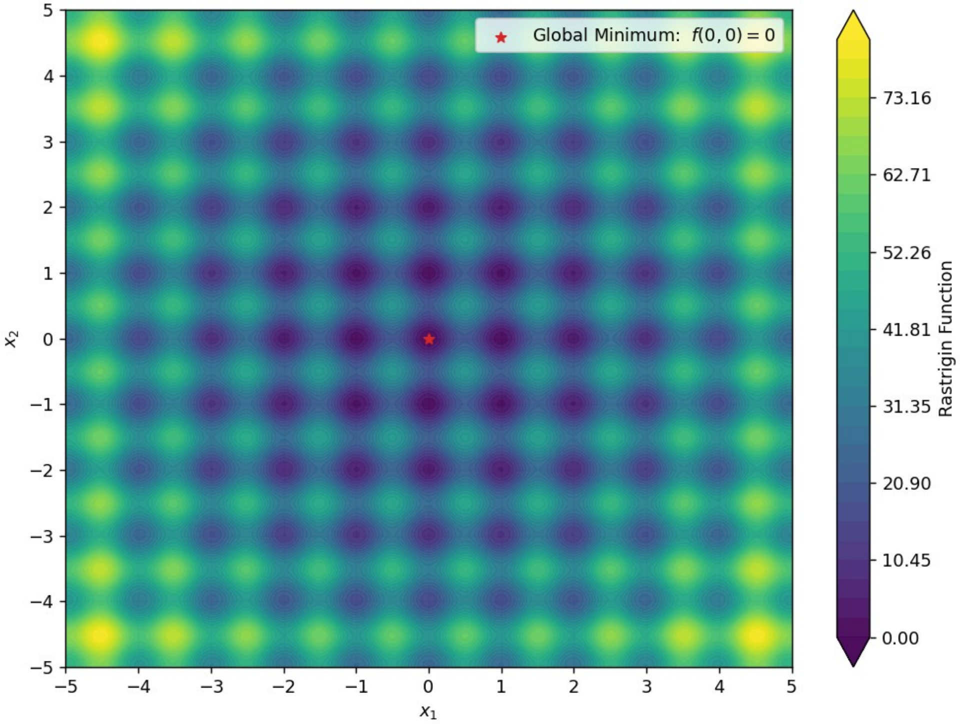


Interim Results

	Rastrigin ($R^4 \to R$)	Rosenbrock ($R^4 \to R$)
Simple Random		
Latin Hypercube		

Questions?

Rastrigin ($\mathbb{R}^2 \to \mathbb{R}$)

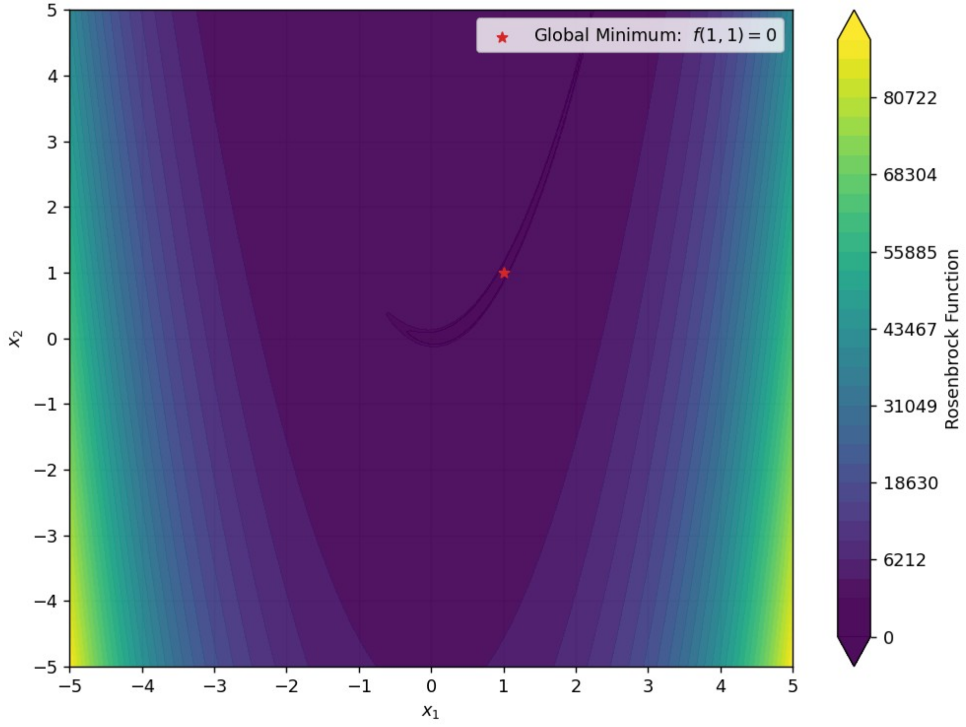


The Rastrigin function:

$$y(\vec{x}) = AD + \sum_{i=1}^{D} \left[x_i^2 - A\cos(2\pi x_i) \right]$$

where $A = 10$.

Rosenbrock ($R^2 \rightarrow R$)

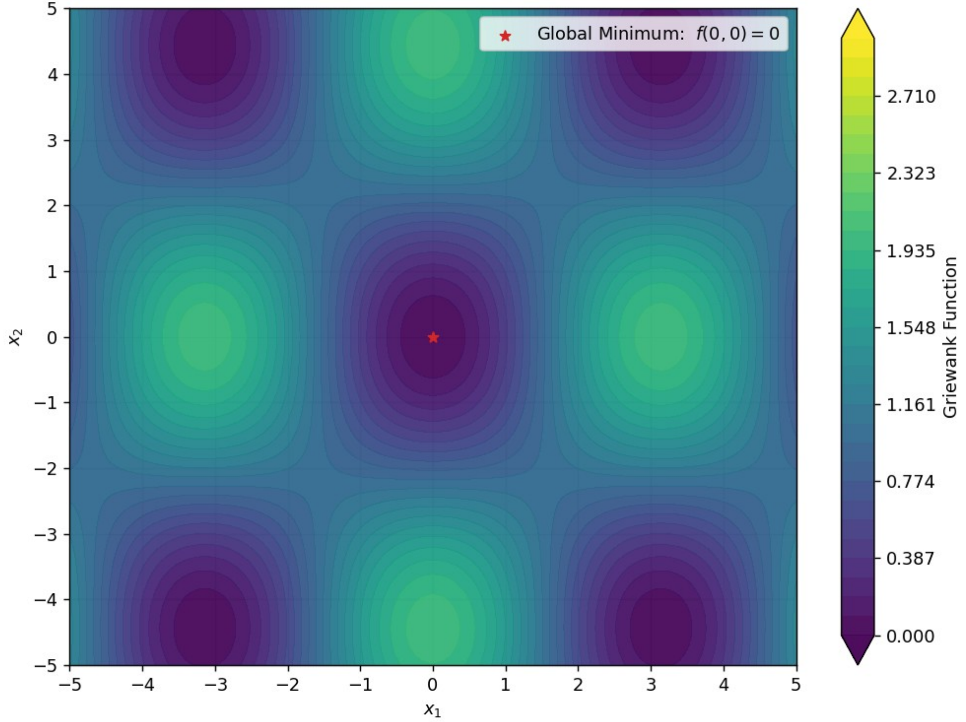


The Rosenbrock function:

$$y(\vec{x}) = \sum_{i=1}^{D-1} \left[A(x_{i+1} - x_i^2)^2 + (1 - x_i)^2\right]$$

where $A = 100$.

Griewank ($\mathbb{R}^2 \rightarrow \mathbb{R}$)

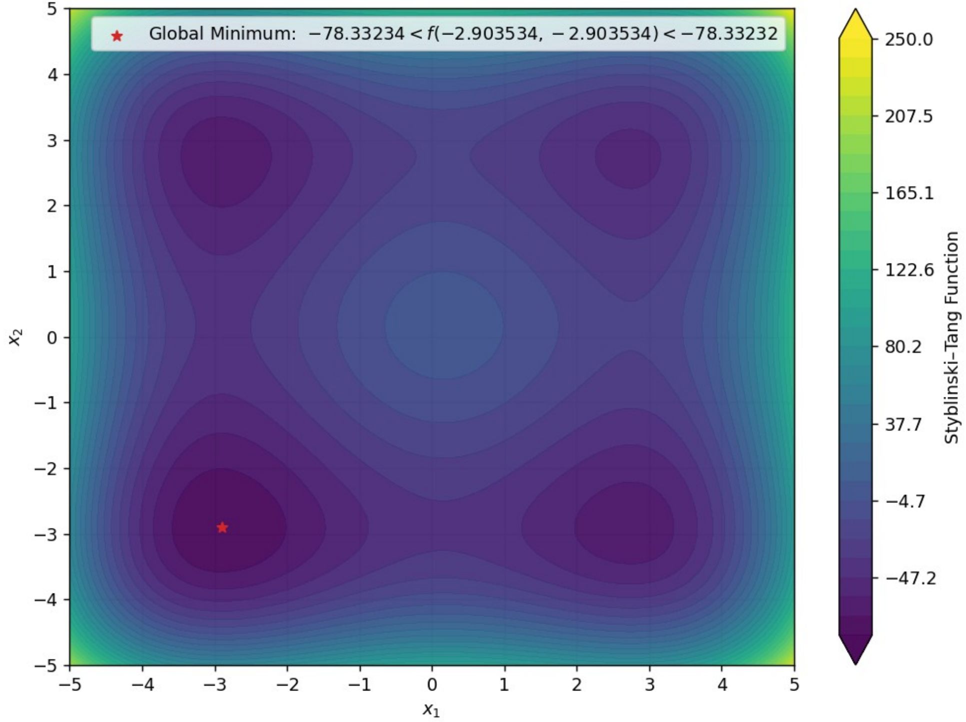


The Griewank function:

$$y(\vec{x}) = 1 + \frac{1}{A}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right)$$

where $A = 4000$.

Styblinski–Tang ($R^2 \rightarrow R$)



The Styblinski–Tang function:

$$y(\vec{x}) = \frac{1}{2}\sum_{i=1}^{D}\left[x_i^4 - Ax_i^2 + Bx_i\right]$$

where $A = 16$ and $B = 5$.