In[51]:=
```
x = .
y = .
z = .
u = .
v = .
u[x_, y_, z_] := -x^2 + 2*x*y - 9*y^2 + 6*x*z + 18*y*z - 11*z^2 + 1;
u[x, y, z] // TraditionalForm
```

Out[57]//TraditionalForm=

$$-x^2+2\,x\,y+6\,x\,z-9\,y^2+18\,y\,z-11\,z^2+1$$

In[58]:=
```
A = {
  {-1, 1, 3},
  {1, -9, 9},
  {3, 9, -11}
};
B = {{0}, {0}, {0}};
a0 = 1;
```

In[61]:=
```
MatrixForm[A]
MatrixForm[B]
```

Out[61]//MatrixForm=

$$\begin{pmatrix} -1 & 1 & 3 \\ 1 & -9 & 9 \\ 3 & 9 & -11 \end{pmatrix}$$

Out[62]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

In[63]:=
```
(* Характеристический  многочлен *)
l = .
AE[l_] := A - IdentityMatrix[3] * l;
MatrixForm[AE[l]]
```

Out[65]//MatrixForm=

$$\begin{pmatrix} -1-l & 1 & 3 \\ 1 & -9-l & 9 \\ 3 & 9 & -11-l \end{pmatrix}$$

In[66]:=
```
charPoly = Det[AE[l]]
```

Out[66]= $128 - 28\,l - 21\,l^2 - l^3$

In[67]:=

```
(* Собственные значения *)
sols = Solve[charPoly == 0, l];
eigenValues = l /. sols;
(*eigenValues = N[eigenValues];*)
MatrixForm[N[eigenValues], TableDirections → Row]
```

Out[69]//MatrixForm=

```
(-19.1937  -3.63891  1.83265 )
```

In[70]:=

```
(* Собственные векторы *)
variables = {x, y, z};
```

In[71]:=

```
vector1 = Solve[AE[eigenValues[[1]]].variables == 0][[1]]
(* Возьмем y = 1 *)
vector1 = variables /. vector1 /. x → 1;
N[vector1]
```

Out[71]=

$$\left\{y \to -\frac{x\left(-2-12\sqrt{-19.2\ldots}-\left(\sqrt{-19.2\ldots}\right)^2\right)}{38+\sqrt{-19.2\ldots}},\ z \to \frac{3x\left(4+3\sqrt{-19.2\ldots}\right)}{38+\sqrt{-19.2\ldots}}\right\}$$

Out[73]=

```
{1., 7.44831, -8.54735}
```

In[74]:=

```
vector2 = Solve[AE[eigenValues[[2]]].variables == 0][[1]]
(* Возьмем x = 1 *)
vector2 = variables /. vector2 /. x → 1;
N[vector2]
```

Out[74]=

$$\left\{y \to -\frac{x\left(-2-12\sqrt{-3.64\ldots}-\left(\sqrt{-3.64\ldots}\right)^2\right)}{38+\sqrt{-3.64\ldots}},\ z \to \frac{3x\left(4+3\sqrt{-3.64\ldots}\right)}{38+\sqrt{-3.64\ldots}}\right\}$$

Out[76]=

```
{1., -0.827251, -0.603885}
```

In[77]:=

```
vector3 = Solve[AE[eigenValues[[3]]].variables == 0][[1]]
(* Возьмем x = 1 *)
vector3 = variables /. vector3 /. x → 1;
N[vector3]
```

Out[77]=

$$\left\{y \to -\frac{x\left(-2-12\sqrt{1.83\ldots}-\left(\sqrt{1.83\ldots}\right)^2\right)}{38+\sqrt{1.83\ldots}},\ z \to \frac{3x\left(4+3\sqrt{1.83\ldots}\right)}{38+\sqrt{1.83\ldots}}\right\}$$

Out[79]=

```
{1., 0.686633, 0.715339}
```

In[80]:=

```
eigenVectors = {vector1, vector2, vector3};
```

In[81]:=

```
(* Проверка собственных значений и векторов *)
{values , vectors} = Eigensystem[A];
eigenSystem  = {};
For[i = 1, i ≤ Length[values], i++,
  eigenSystem  = Append[eigenSystem , {values〚i〛, vectors〚i〛}]
];
(* Сортируем по собственным значениям *)
eigenSystem  = Sort[eigenSystem , (#1〚1〛 < #2〚1〛) &];
```

In[85]:=

```
values = Sort[values];
If[values == eigenValues ,
  For[i = 1, i ≤ Length[values], i++,
    (* Сравниваем нормированные векторы. Они могут отличаться знаком *)
    If[FullSimplify @ Normalize @ eigenSystem〚i, 2〛 ===
       FullSimplify @ Normalize @ eigenVectors〚i〛 ||
      FullSimplify @ Normalize @ eigenSystem〚i, 2〛 ===
       FullSimplify @ Normalize @ -eigenVectors〚i〛,
      Print["vectors ", i, " is equal"],
      Print["vectors ", i, " is not equal"]
    ]
  ]
]
```

```
vectors  1 is equal

vectors  2 is equal

vectors  3 is equal
```

In[37]:=

```
(* Матрица перехода из нормированых собственных векторов *)
S = Map[Normalize , eigenVectors] // Transpose ;
MatrixForm[N[S]]
```

Out[38]//MatrixForm=

$$\begin{pmatrix} 0.0878632 & 0.698597 & 0.7101 \\ 0.654433 & -0.577915 & 0.487578 \\ -0.750998 & -0.421873 & 0.507962 \end{pmatrix}$$

In[39]:=

```
(* Диагональная матрица *)
MatrixForm[N[A]]
A1 = Transpose[S] . A .S;
MatrixForm[N[A1]]
```

Out[39]//MatrixForm=

$$\begin{pmatrix} -1. & 1. & 3. \\ 1. & -9. & 9. \\ 3. & 9. & -11. \end{pmatrix}$$

Out[41]//MatrixForm=

$$\begin{pmatrix} -19.1937 & -8.88178 \times 10^{-16} & 8.88178 \times 10^{-16} \\ -2.22045 \times 10^{-16} & -3.63891 & 0. \\ -7.77156 \times 10^{-16} & -2.22045 \times 10^{-16} & 1.83265 \end{pmatrix}$$

In[87]:=

```
(* Приведенное уравнение *)
v = 0;
a1 = a0;
```

In[89]:=

```
(*Приводим к полному квадрату*)
For[i = 1, i ≤ Length[variables], i++,
 If[A1[[i, i]] ≠ 0, v += A1[[i, i]] * (variables[[i]]^ 2); ]]
v = (v + a1) // N;
(*Создаем в функцию *)
v = Function[{x, y, z}, Evaluate[v]];
```

In[47]:=

```
v[x, y, z] // TraditionalForm
```

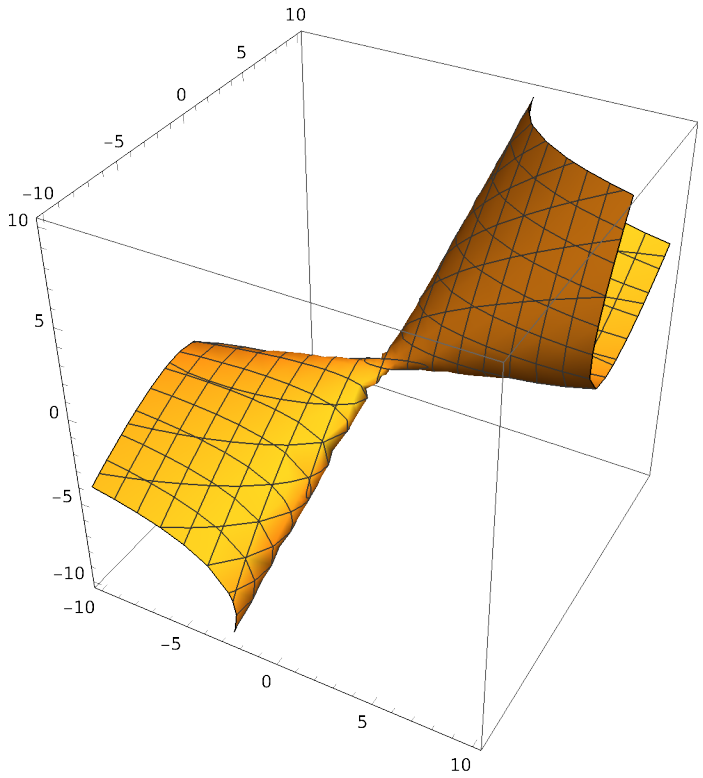
Out[47]//TraditionalForm=

$$-19.1937\ x^2 - 3.63891\ y^2 + 1.83265\ z^2 + 1.$$

In[92]:=

```
ContourPlot3D[u[x, y, z] == 0, {x, -10, 10}, {y, -10, 10}, {z, -10, 10}]
```

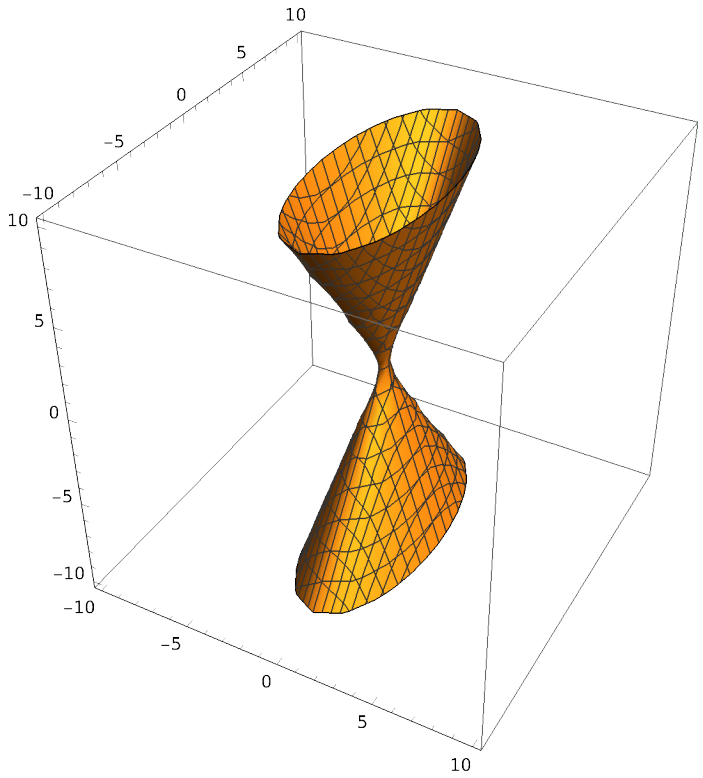
Out[92]=



In[93]:=

```
ContourPlot3D[v[x, y, z] == 0, {x, -10, 10}, {y, -10, 10}, {z, -10, 10}]
```

Out[93]=



In[94]:=