

# Time series decomposition and measurement of business cycles, trends and growth cycles

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## Abstract

A study of business cycles does not require trend estimation and elimination, but a study of growth cycles does. Major cyclical slowdowns and speedups deserve to be analyzed, but the needed time series decomposition presents difficult problems, mainly because trends and cycles influence each other. We compare cyclical movements in levels, deviations from trend, and smoothed growth rates for both the quarterly real GDP and the monthly U.S. Coincident Index—using the phase average trend (PAT). Then we compare alternative trend estimates, deterministic and stochastic, linear and nonlinear, and the corresponding series of deviations from these trends. We discuss how the resulting estimates differ for U.S. growth cycles in the post-World War II period. The results of PAT show great similarity to the results obtained with the Hodrick–Prescott, local linear trend, band-pass filtering methods.

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## 1. Background, motivation and plan of this study

Recent empirical research on business cycles devotes much attention to time series adjusted for long-term growth trends. This is in contrast to the early studies which defined business cycles as sequences of expansions and contractions in a large array of series representing the levels of total output, employment, and many other component and related processes. The mild or severe absolute declines in general economic activity (recessions or depressions) are of particular concern to economic agents and policy makers. The expansions and contractions in the level series could be analyzed without trend adjustments. This was the position taken by the “classical cycle” approach dominant particularly in the studies of the National Bureau of Economic Research (NBER).

For a number of reasons to be discussed, it is difficult to estimate and eliminate trends from economic aggregates and their components, and faulty trend estimates can cause significant errors. If trend adjustments are not required, the analysis can be comparatively simple and reliable. This is an additional reason why studies of cyclical indicators, at the NBER, the U.S. Department’s of Commerce Bureau of Economic Analysis (BEA), and most recently The Conference Board (TCB), concentrated on timing and other aspects of nonseasonal fluctuations in series that in many cases show pronounced long-term trends.

Intensive and detailed studies of a large number of diverse indicators laid the foundation for much of the existing knowledge of “what happens during business cycles” (Mitchell, 1951). Thus, discovery of comovements and timing characteristics led to the dating of business cycles and the distinction between groups of leading, coincident, and lagging indicators.

But the economy has at times suffered major slowdowns, that is declines not in levels but in growth rates that remain positive. It is certainly possible to conceive of a severe and long slowdown causing more hardship than a mild and short recession. In fact, long slowdowns in employment and demand growth have occurred repeatedly in recent times even while output and supply growth held up well, supported by the progress of technology and productivity. So phases of declining output are not the only periods of widespread discontent with economic conditions.

Indeed, sequences of serious acceleration and deceleration of macroeconomic activity have long attracted considerable public and professional concern. These “growth cycles” can be measured as fluctuations in the deviations of the principal indicators around their generally rising trends. So trend estimation, which is not needed for business cycle analysis, cannot be avoided in growth cycle analysis.<sup>3</sup>

Clearly, all recessions involve slowdowns, but not all slowdowns involve recessions; hence, growth cycles are more numerous than business cycles. Of course, the two sets of phenomena or processes are related, but they are distinct as defined and measured. Yet some of the recent papers proposing different detrending methods, which yield various growth cycle estimates, mistakenly seek verification in NBER chronologies or other measures relating to business cycles. What is frequently implied is that growth cycles and business cycles are not distinguishable, and slowdowns are treated like recessions.<sup>4</sup>

<sup>3</sup>As will be shown in Section 2.3, using fluctuations in smoothed growth rates is not a good substitute.

<sup>4</sup>For a recent debate on how “business cycle facts” are reflected in selected detrended aggregates, see Canova (1998a, 1998b, 1999) and Burnside (1998). For views similar to ours see Harding and Pagan (2002).

One can only welcome the revival of interest in methods of filtering and detrending economic and financial indicators that is associated with recent research in business cycles. However, these methods too often abstract from the main difficulty of time series decomposition, which is that trends and cycles interact and influence each other, as suggested by the literature and data. Also, there is much diversity and uncertainty about which indicators are to be used to define cyclical movements.

Despite the rather rich background of past and contemporary work, then, there is much in this area that needs to be clarified and improved. Our intention and hope here is to remove or at least reduce the sources of confusion in studies of economic fluctuations. This provides the basic motivation for the paper in which we focus on practical needs of research in the indicators of cyclical turning points and movements. How do the different approaches to trend estimation compare from this point of view?

The paper has two substantive parts. Part 2 shows how the study of business cycles, i.e., movements measured in time series that may include trends, is complemented by the study of growth cycles, which applies parallel criteria and methods to the same indicator series adjusted for longer-term trends. We describe the “phase average trend” (PAT) used for this purpose, then examine the possibility of using smoothed percentage changes to measure growth cycles. The data used, rules observed, and problems encountered in this work are discussed using the resulting chronologies for the United States in the post-World War II period.

Part 3 asks how useful are other approaches to time series decomposition for the analysis of growth cycles. We proceed from linear deterministic to linear stochastic trends, from unobserved component models to the Hodrick–Prescott, heuristic and band-pass filter methods of trend estimation. The last section of the paper collects and states briefly our main inferences and conclusions.

## 2. Cyclical movements in levels, deviations from trend, and growth rates

### 2.1. *Classical business cycles and trend adjustments*

In the traditional NBER approach, the business cycle included the intra-cycle trend. Secular forces were believed to influence the cycle and cyclical forces were believed to influence the trend in a way that made a clean separation of cycle and trend impossible.<sup>5</sup>

In the “classical” business cycles, expansions should tend to be longer and larger than contractions, but either phase must be persistent and pervasive enough to allow for the significant cumulative and interactive effects that are observed and analyzed in literature. The sequence of up and down phases that constitutes the business cycle is recurrent but not periodic. According to Burns and Mitchell (1946), **business cycles vary in duration “from more than one year to 10 or 12 years.”** This would still accommodate all the cycles identified so far, domestic and foreign, but the more typical range for the U.S. is not so

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<sup>5</sup>Burns and Mitchell (1946, Chapter 7) conclude that trend adjustments reduce the variations of cyclical behavior both across series and within series over time. They regard that as a disadvantage because these variations matter in the analysis of business cycles (pp. 307–309). As a result, a step function linking the average levels of a variable in successive business cycles was effectively the trend representation complementing the cyclical measures.

broad, since very few cycles are shorter than two or longer than eight years (however, the duration of business expansions may be increasing).<sup>6</sup>

We accept the NBER definition and the view that seasonally adjusted level data rather than detrended data should be used for the analysis of business cycles under the time-honored definition of recessions and recoveries. Yet, this is not the only interesting concept of generalized economic fluctuations. A market economy in an expanding, competitive world must grow to prosper, indeed even to survive. Hence it is important to ask how its sustainable potential trend changes over time, and how its actual course deviates from that trend.

In the traditional NBER approach, each monthly or quarterly time series was treated as a product (or, less likely, a sum) of three components: the seasonal (S), the irregular (I), and the trend-cycle (TC). After elimination of S, a combination of smoothing formulas served to reduce the effects on TC of the random irregular movements.<sup>7</sup>

Studies of business cycles can go a long way without requiring trend estimation and elimination. Yet, the TC decomposition is needed for several reasons and it is not intractable.

First, reasonably good trend estimates are required to study economic growth empirically and test related theories. This task cannot be accomplished without sufficiently long and reliable data and without confronting the question of how trends and cycles influence each other. Economic growth and its major sources are clearly important for the society's welfare.

Second, in the absence of business cycle recessions, sequences of slowdowns and speedups of substantial size, spread, and length attracted considerable public attention in many advanced and developing market-oriented economies.<sup>8</sup> In the first two post-World-War II decades—an era of reconstruction, democratization, foreign aid, and important monetary, fiscal and structural reforms—several countries in Europe and the Far East, notably West Germany and Japan, enjoyed very high rates of real growth. These expansions were interrupted by slowdowns rather than absolute declines in overall economic activity. The same applies to more recent expansions in some developing countries such as India.<sup>9</sup>

<sup>6</sup>According to the historical NBER chronology for the United States, which goes back to 1796 (annual before 1854, monthly thereafter), only four of the 45 peak-to-peak cycles exceeded eight years or 100 months, and only two were shorter than two years or 20 months. This count includes the last cycle with the especially long 10-year expansion. It deserves to be noticed that the three longest cycles occurred recently, in the 1960s, 1980s and 1990s. Business cycles in Europe have been longer than in the U.S., averaging about five instead of four years. (See Moore and Zarnowitz, 1986 and Glasner, ed. 1997, Appendix.)

<sup>7</sup>However, no attempt was made to segregate and eliminate the effects on longer movements of major episodic disturbances due to wars, strikes, industrial combinations and conflicts, technological innovations, abundant or poor harvests, etc. (Bry and Boschan, 1971, see especially Chapter 3). The risk that excessive or inappropriate smoothing may distort the measured fluctuations or even create spurious ones was recognized early in NBER studies (see Macaulay, 1931; Burns and Mitchell, 1946, Chapter 8).

<sup>8</sup>These “growth cycles” or “deviation cycles” were formally identified and dated for many countries. Measurement and analysis of cycles in the deviations from trend constitute a very worthwhile subject for the light they may throw on (a) the level and variability of growth, and (b) the sources of economic instability. The first, influential study of major fluctuations in detrended indicator series was by Mintz (1969) for West Germany. Her method has been applied and improved by several analysts at the NBER and adopted internationally, notably by the Organization for Economic Cooperation and Development (OECD).

<sup>9</sup>It is well to note that historical studies identified business cycles in the past for a number of the same countries, including Japan, India, and of course Germany. See Thorp (1926).

Third, the appraisal of cyclical indicators can be substantially improved by considering their trends and the fluctuations in the deviations from trends. Leading indicators are much more sensitive to all types of disturbances, whether associated with business cycles or with fluctuations at shorter frequencies; hence they are generally much more volatile than coincident indicators. They also have as a group fewer and weaker upward trends. Using deviations from trend or smoothed growth rates reduces these differences between the two sets of indicators.

Lastly, the leaders seldom miss turning points in coinciders but they show many “extra” turns of their own not connected with business cycles. But these apparent “false signals” in leading indicators are not random: most of them are associated with turning points in growth cycles. Numerous tests confirm that most economic slowdowns of cyclical proportions have been anticipated by leading indicators, in the U.S. and abroad (Klein and Moore, 1985).

Growth cycles have not replaced business cycles; but we conclude that business cycle research needs to be complemented by study of trends and growth cycles.

## 2.2. The phase average trend

Boschan and Ebanks (1978) describes a 10-step procedure to calculate the PAT. First, a 75-month moving average of the data is computed to approximate a secular trend. The deviations from this preliminary trend are calculated in order to determine cyclical turning points. These turning points determine the phases of fluctuations (periods of high growth and low growth). Then, mean values of the original data for each successive phase are computed and placed in the midpoint of each phase. From these phase averages a three-phase moving average (triplets) is calculated and also placed in the midpoint of the period spanned by three phases. Then, the triplets are connected by monthly interpolation. The level of the trend in each segment is adjusted by making sure the sum of the trend values equal the sum of the actual data in each segment. The transitions from segment to segment are smoothed by using a 12-month moving average which gives the final PAT.

The use of centered 75-month moving averages makes it necessary to extrapolate backward (forward) over the first (last) 37 months of the series covered. This is the main source of problems and errors here: note that the slope of the trend at the end of the period must be estimated while the relevant cyclical developments are still unknown. But this “end-period” difficulty is a general one, applying to all other methods of trend determination as well.

All procedures to estimate and eliminate trends are to some extent arbitrary, and PAT is certainly no exception. The method lacks the mathematical elegance or apparent simplicity of other approaches that can be summarized by formulae; however, the first impression of it as an excessively elaborate calculation is misleading. The multi-step, successive-approximations approach is dictated by the objective of deriving estimates that reflect in a reasonable way the interplay of longer (trend) and shorter (cyclical) movements. PAT passes smoothly through the principal segments of higher and lower average growth in the data, which makes it nonlinear and flexible.

Fig. 1 shows the *coincident index* (CI) for the United States monthly since 1948, with shaded areas representing business cycle recessions as dated by NBER.<sup>10</sup> This series,

<sup>10</sup>TCB publishes the U.S. CI for the period starting in January 1959. The CI used in this paper was extended back to January 1948 with the component data from the Business Cycle Indicators Database maintained by TCB.

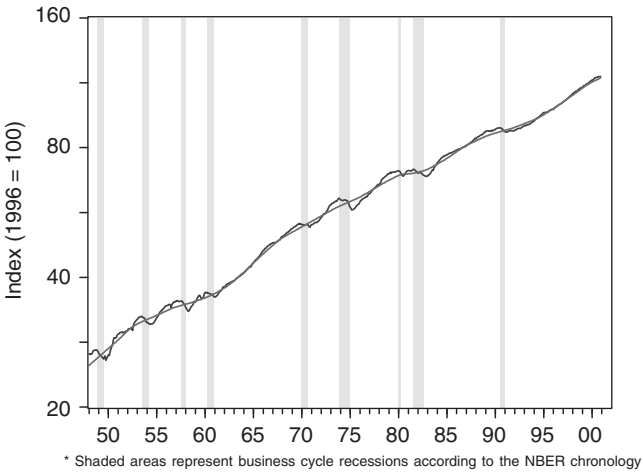


Fig. 1. U.S. coincident index and phase average trend monthly, 1948–2000. (Shaded areas represent business cycle recessions according to the NBER chronology.)

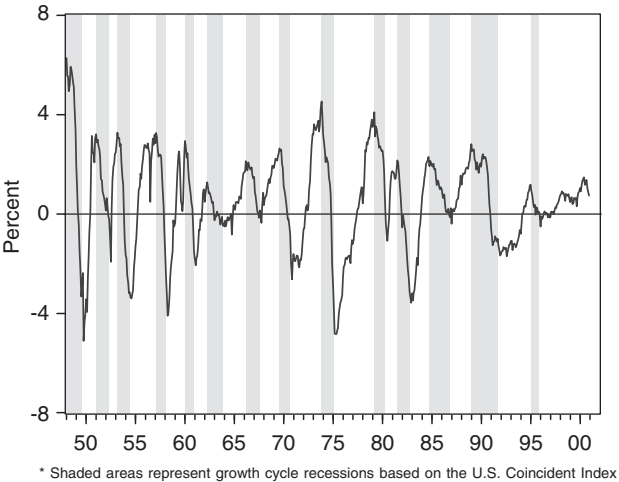


Fig. 2. U.S. coincident index deviations from phase average trend. (Shaded areas represent growth cycle recessions based on the U.S. coincident index.)

published since 1996 by TCB, was compiled earlier by the BEA in the U.S. Department of Commerce. It is evident that CI, which is based on the consensus of comprehensive measures of real personal income, real manufacturing and trade sales, industrial production, and nonfarm employment, had nine cyclical declines in this period, which represent well the nine recessions of 1948–1991. The PAT of the CI continued to rise in each recession with what appears to be only few and weak slope changes.

Fig. 2 shows that the percentage deviations from PAT of the U.S. CI do stand out very clearly, with sharp cyclical peaks (*P*) and troughs (*T*) alternating on the plus and minus sides of the zero line, respectively. The shaded columns represent the cyclical slowdowns

dated according to these  $P$ 's and  $T$ 's. The series is smooth, showing a month of cyclical dominance (MCD) ratio of one. (This is the ratio of cyclical to irregular components on the average, see Bry and Boschan, 1971.)

The dates of the 14 U.S. growth slowdowns completed between 1948 and 2000, as determined from the PAT deviations for CI according to the algorithm developed by Bry and Boschan (1971), are listed in Table 1, along with the durations of the corresponding above-trend and below-trend phases (columns 1–6). The table also includes a parallel listing of the turning dates and durations for the nine U.S. business cycles of the 1948–2001 period. It is clear that growth cycles are generally shorter, more frequent, less variable, and much more nearly symmetrical than business cycles. This is because long business cycle expansions are usually interrupted by significant slowdowns. In addition, most postwar recessions were preceded by marked retardations of growth.<sup>11</sup>

Fig. 3 shows that real GDP declined around the same dates as the CI, although not necessarily in two or more consecutive quarters; this popular rule of thumb would clearly have failed in some of the recent recessions. The deviations from PAT for real GDP shown in Fig. 4 follow closely the pattern of growth cycles derived from the data for the U.S. CI. This close correspondence is not surprising, since the cyclical movements in the CI are well correlated with those in real GDP.

Fig. 4 demonstrates that the series of percentage deviations from the trend declined strongly during the recessions of 1949, 1953, 1957, 1960, 1970, 1973–1975, and 1981–1982, less so during the milder and shorter recessions of 1980 and 1990–1991. In addition, smaller declines in the PAT detrended real GDP occurred in 1952, 1962–1963, 1966–1967, 1984–1987, and 1994–1996; all slowdowns interrupting long economic expansions.<sup>12</sup>

PAT was applied at the NBER to a large number of indicators with generally satisfactory results in terms of timing and conformity to aggregate growth cycles. For this reason, we will use PAT as a benchmark in the comparisons of alternate trend estimation methods described in part 3, and pay much attention to what the trends imply with respect to the dating of growth cycles. For growth cycle chronologies based on the NBER methodology and the PAT estimates for several countries, see Glasner, ed. (1997), Appendix Table 4, pp. 736–737.

### 2.3. *Cycles in smoothed growth rates vs. deviations from trend*

Students of growth cycles have long been fully aware that the selection of the trend curve is inevitably associated with considerable arbitrariness. This is why Friedman and Schwartz (1963) and Mintz (1969) looked for an alternative to detrending by attempting to identify cyclical fluctuations in series of growth rates directly.

<sup>11</sup>Note that the Bry and Boschan algorithm identifies a peak in March 1998 and a trough in September 1999. We did not use this peak–trough pair in our analysis since it is uncertain whether it qualifies as a growth cycle contraction; the deviations from PAT were not negative in the CI and were only briefly negative in real GDP.

<sup>12</sup>Working with related but earlier and inferior methodology, Mintz (1969) found that most detrended indicators for Germany in 1950–1967 displayed “clear-cut cyclical swings with unmistakable turning points” (see Mintz, 1969, p. 14). The set of centered 75-month moving averages could do at least rough justice to these diverse time series, something that according to Mintz “could not have been done with fitted trends,” and the methods succeeded in distinguishing well between leading, coincident, and lagging indicators (Mintz, 1969 pp. 12 and 28–43).



Table 1  
U.S. growth cycles and business cycles, 1948–2000 durations of cycles and their phases

Growth cycles			Duration in months of growth cycles and phases			Business cycles Peaks (P) and Troughs (T)			Duration of months of business cycles and phases		
P	T	P	P to T	T to P	P to P	P	T	P	P to T	T to P	P to P
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Jan-48	Oct-49	Jan-51	21	15	36	Nov-48	Oct-49	Jul-53	11	45	56
Jan-51	Jul-52	Mar-53	18	8	26						
Mar-53	Aug-54	Feb-57	17	30	47	Jul-53	May-54	Aug-57	10	39	49
Feb-59	Apr-58	Jan-60	14	21	35	Aug-57	Apr-58	Apr-60	8	24	32
Jan-60	Feb-61	Apr-62	13	14	27	Apr-60	Feb-61	Dec-69	10	106	116
Apr-62	Jan-64	Mar-66	21	26	47						
Mar-66	Oct-67	Aug-69	19	22	41						
Aug-69	Nov-70	Nov-73	15	36	51	Dec-69	Nov-70	Nov-73	11	36	47
Nov-73	Apr-75	Mar-79	17	47	64	Nov-73	Mar-75	Jan-80	16	58	74
Mar-79	Jul-80	Jul-81	16	12	28	Jan-80	Jul-80	Jul-81	6	12	18
Jul-81	Dec-82	Sep-84	17	21	38	Jul-81	Nov-82	Jul-90	16	92	108
Sep-84	Jan-87	Jan-89	28	24	52						
Jan-89	Dec-91	Jan-95	35	37	72	Jul-90	Mar-91	Mar-01	8	120	128
Jan-95	Jan-96	Jun-00	12	53	65						
Mean			18.8	26.1	44.9						
Median			17.0	23.0	44.0				10.7	59.1	69.8
Standard deviation			6.2	13.2	14.7				10.0	45.0	56.0
									3.4	38.0	39.1

Sources: NBER, Center for International Business Cycle Research (CIBCR); The Conference Board. For further detail, see Appendix to Glasner (1997) pp. 734–737.



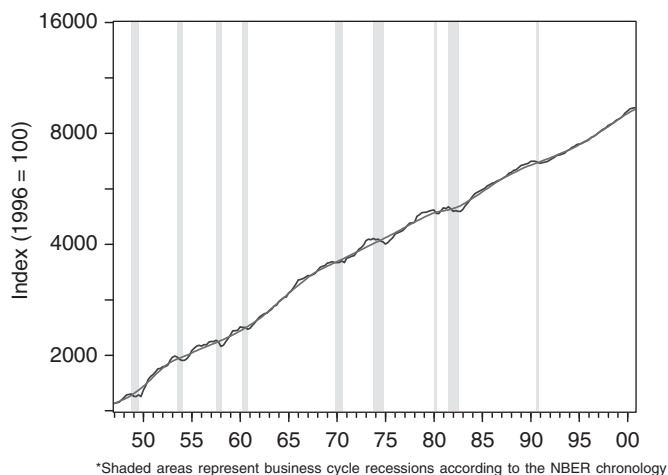


Fig. 3. U.S. real GDP and phase average trend quarterly, 1947–2000. (Shaded area represent business cycle recessions according to the NBER chronology.)

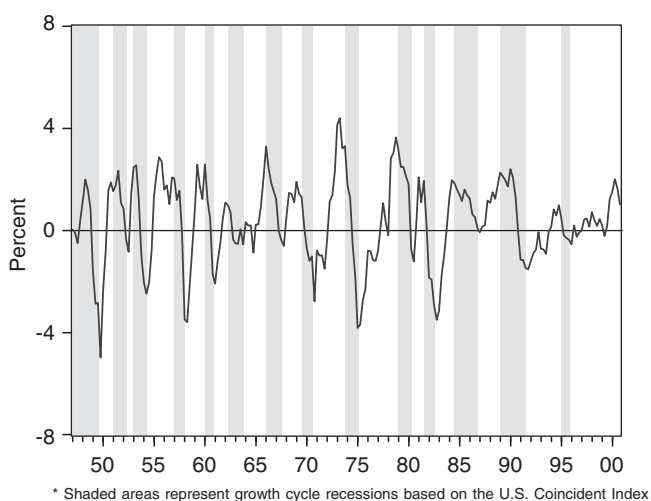


Fig. 4. U.S. real GDP deviations from phase average trend. (Shaded areas represent growth cycle recessions based on the U.S. coincident index.)

There are two major problems with this approach. First, series of growth rates computed over short unit periods tend to be very erratic. Their irregular components are often dominant and obscure their underlying cyclical movements. To bring out the latter, the series must be smoothed with fairly long and/or complex moving averages, which runs a certain risk of distorting the patterns, and perhaps especially the timing, of these movements.

The second perceived problem is the timing of the growth rates, which is very different from that of the corresponding level series. Early in an expansion growth is usually high,

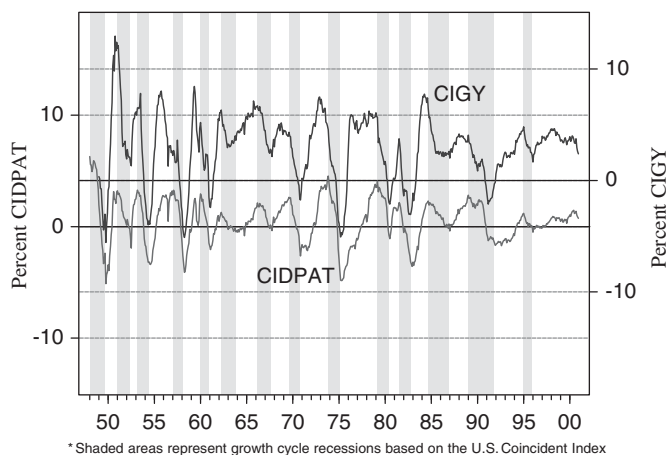


Fig. 5. Cycles in growth rates and deviations from trend U.S. coincident index, monthly, 1948–2000. (Shaded areas represent growth cycle recessions based on the U.S. coincident index.)

starting as it does from a low base after a period of decline. As the base itself rises during the expansion, growth rates fall lower and lower. Hence, cycles in growth rates can be expected to lead cycles in levels at peaks. Mintz (1969, p. 15) argued that these leads are so long (between one half and nearly one full business expansion phase) as to invalidate the use of peaks and troughs in growth rates for dating purposes. Instead she proposed to define the downturn in a growth cycle as the end of a period of relatively high growth and the upturn as the end of a period of relatively low growth. These are the “step-cycles” introduced earlier by Friedman and Schwartz in their 1963 work on the cyclical behavior of money.<sup>13</sup>

Despite much informative work on step cycles, the concept has not caught on, and for good reasons. Most series of rates of change, when smoothed, show cyclical fluctuations similar to those in detrended values; they do not move in steps. Thus, as noted by Moore (in Zarnowitz, 1972, pp. 178–179), “The step-fitting operation...seems to be imposing on the data something that is not obviously there.”

As Fig. 5 shows, using simple year-over-year change in the CI (call it CIGY) produces a series that falls in each shaded period and rises in each unshaded period, where the shadings are dated according to the cyclical declines in the detrended value of the index

<sup>13</sup>With the average rate during a high step exceeding systematically the average rate during the preceding and succeeding low steps, they found the resulting cycles to correspond closely to the cycles in the trend adjusted series proper. Mintz, using a computerized version of the Friedman–Schwartz method, also concludes that deviation cycles and step-cycles show a high degree of agreement, particularly for composite indexes and at “strong turns.” (For individual indicators and at weak, i.e., not well articulated turns, the correspondence is at times attenuated.) For evidence, see Mintz (1969, 1972), tables and charts for West Germany and the United States, respectively. The turning points between the steps are determined by a program that maximizes the difference between the corresponding step variances. For the U.S., the turns in deviation cycles and in step cycles are approximately coincident in three-quarters of the cases. The worst matchings are related to the occurrence of flat bottoms and ceilings or of double turns.

(to be labeled CIDPAT). There is a one-to-one correspondence between the cyclical fluctuations in the two series. However, CIGY moves in larger and more varied swings than does CIDPAT. The latter is approximately symmetrical around the zero line, as would be expected of deviations from trend; it seldom exceeds the range of  $\pm 2\%$ . In contrast, CIGY consists largely of positive numbers, again as expected; they are concentrated in the range of 0–6%, with negative numbers limited to recessions and their immediate aftermath (see Fig. 5).<sup>14</sup>

The inflection point in the level of aggregate economic activity, which is the peak in the corresponding smoothed growth rate ( $P_{gr}$ ), is likely to occur well before the time when the composite activity index exceeds most its own long-term average value, which is the peak in its detrended value ( $P_{dt}$ ). The underlying eminently reasonable assumption is simply that growth starts decelerating in the vicinity of where economic activity rises above trend. Thus, the sequence on the upgrade is that  $P_{gr}$  leads  $P_{dt}$  which in turn leads the downturn in the level of economic activity itself, that is, the business cycle peak.

On the downgrade, a deceleration in growth may also start ahead of the time when activity falls farthest below the trend ( $T_{gr}$  leading  $T_{dt}$ ); however,  $T_{dt}$  may follow rather than lead the business cycle trough. These lags may well be rather short, but in sluggish recoveries, like those of 1991–1993 (and 2002–2003),  $T_{dt}$  could be much slower in arriving.

Growth rates are much more likely to lead detrended values at peaks than at troughs. Fig. 5 allows 10 comparisons at peaks, of which seven are leads of CIGY relative to CIDPAT and three are exact coincidences. The leads vary from 3 to 12 months, and average 6.2 months. Of the 10 comparisons at troughs, six are coincidences, two are short leads of 2–3 months, and two are long leads of 9 months each. The average is a lead of 2.3 months.

These observations underestimate the true leads of CIGY relative to CIDPAT because the year-over-year change is not centered whereas the detrended series is. Centering would add another 6 months to the leads but we purposely avoid it so as to retain the most recent data for CIGY which might be helpful with the “end-period” problem of PAT (or other trend) estimation.

### 3. Comparing different trend estimates

#### 3.1. Cyclical analysis with linear deterministic trends

For a long time, the dominant approach to modeling growth and fluctuations was to view them as a sum of a deterministic trend and stochastic deviations treated as the residual “cyclical” component. The deviations from this trend should then be themselves stationary; the economy’s output should exhibit reversion to the trend; and the effect of shocks to this trend-stationary (TS) model, though they may persist over time, should decline and eventually die out.

A linear deterministic trend is, of course, free of any cyclical or stochastic short-term movements. Such a clean separation of the long and short time-series components would

<sup>14</sup>In retrospect, CIGY can be easily made more symmetrical by taking deviations from a sequence of averages (3.427 for 1/60–11/70, 2.9595 for 12/70–4/75, 2.953 for 5/75–8/80, 2.104 for 9/80–4/91, and 2.677 for 5/91–12/2000). The adjustment seems fairly innocuous as it does not alter the timing of the cycles in CIGY, but it is based on hindsight and we decided not to use it.

be just fine if it were empirically acceptable. Unfortunately it is not. Indeed, it is very unlikely that any given type of linear deterministic trend would persist over long stretches of time, surviving major structural and technical changes, wars, business expansions and contractions, financial crises, rising and falling inflation, etc.

From the point of view of cyclical analysis, the cost of using a linear trend is that too much of the overall variation is attributed to business cycles. Moreover, the linear trends do a poor job of differentiating between good and bad economic times.

To prove this, consider the log-linear trend fitted to real GDP in Fig. 6 panel A and the deviations from it shown in panel B. The data run below the trend line in five periods: 1947–1951; 1954; 1956–1963; Q4 1981–Q1 1984; and Q4 1990–Q1 1999. These include some sluggish times such as all or some of the recessions in 1960–1961, 1975, 1981–1982, and 1990–1991, but also some times of good economic growth and prosperity in the early 1960s, mid-1980s, and especially the 1990s. The data run above the trend line in four periods: briefly in mid-1950s; Q2 1965–Q4 1975; 1976–Q3 1981; and Q2 1984–Q3 1990. These include the wartime boom of the late 1960s; almost all

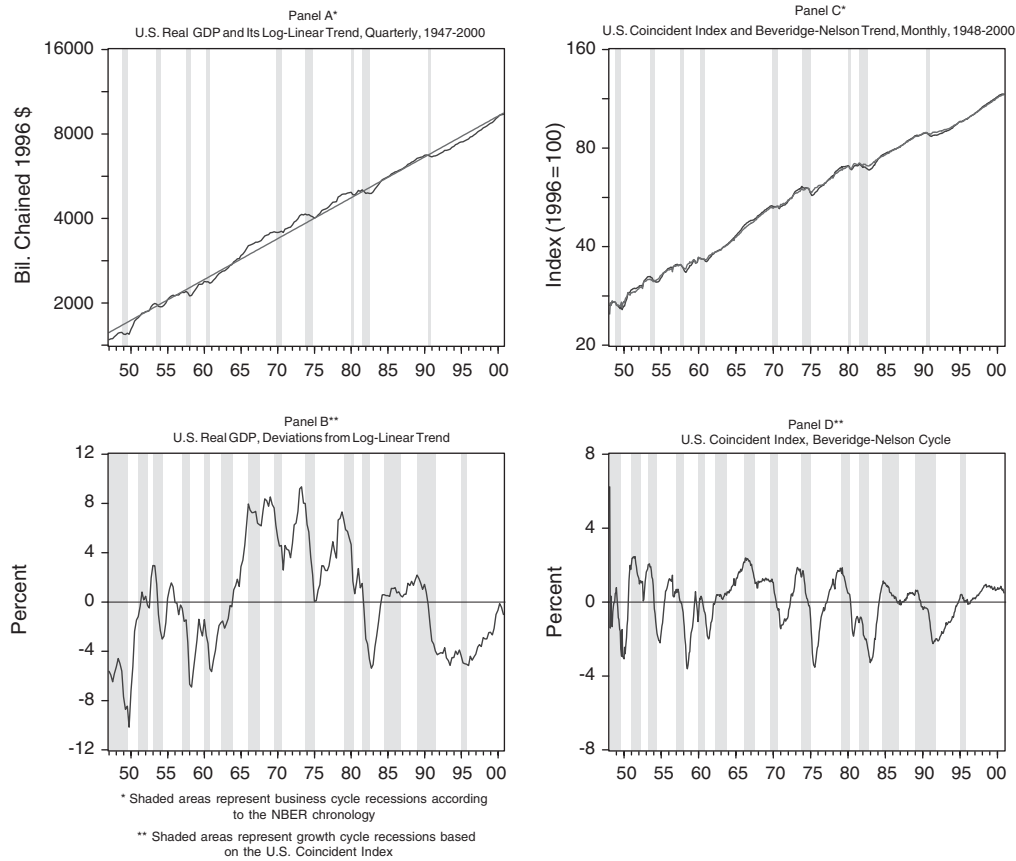


Fig. 6. Panel A: U.S. real GDP and its log-linear trend, quarterly, 1947–2000; Panel B: U.S. real GDP, deviations from log-linear trend; Panel C: U.S. coincident index and Beveridge–Nelson trend, monthly, 1948–2000; Panel D: U.S. coincident index, Beveridge–Nelson cycle.

of the turbulent, inflationary 1970s, which witnessed two recessions: a short recession and an incomplete recovery in 1980–1981; and the post-recovery part of the expansion of the 1980s.

A method that shows the troublesome 1970s in a very positive light (strongly above-trend) and the prosperous 1960s and 1990s in a rather negative light (continuously below-trend) is quite difficult to accept. In each below-trend period, the economy was most of the time in a fast growing mode. Conversely, the above-trend periods include some of the recessions covered.<sup>15</sup>

### 3.2. *Modeling with linear stochastic trends*

In the 1980s, a sequence of influential papers reported favorably on the hypothesis of a single (positive, real) unit root in autoregressive representations of macroeconomic variables such as real income, output, and employment. In the popular alternative to the TS model, a difference-stationary (DS) process was inferred from the failure to reject the unit-root model. Autoregressive integrated moving-average (ARIMA) models were applied to reduce the time series to stationarity, replacing the assumption that the series are stationary around their trends. In the DS model, there is no time trend, only a constant and a stationary and invertible ARMA term. As argued by Nelson and Plosser (1982), if important economic aggregates have a DS representation then they have no tendency to return to linear trends. Rather, they have “permanent components” that show no such trend reversion as they reflect shocks which have long persistent effects. The trend is stochastic, perhaps simply exponential plus a random error.

However, there is no convincing evidence of the DS model outperforming the TS model and of the linear stochastic trends providing satisfactory estimates of the behavior of aggregate real variables over time (see part III of Diebold and Rudebusch, 1999). Series that look like DS can originate in TS processes with roots close to but distinct from one, and finite-sample tests cannot conclusively distinguish between DS and TS series in such cases. Not only that, but the tests display little power against TS models that are well estimated from the data and even favor such TS models when based on long spans of annual data (Rudebusch, 1992, 1993; Diebold and Senhadji, 1996).

Moreover, note that there is little to be done about the unpredictable shocks, their hypothetical long-term effects, and the stochastic variations in economic activity generally. Hence, countercyclical stabilization policies have very limited scope and promise to those who follow this approach. This is not necessarily a deficiency of the theory, but it may help explain the interest of many economists in exploring what may be causing business cycles beyond shocks (Fuhrer and Schuh, 1998).

We agree that the linear deterministic trends are inadequate, as already demonstrated. Trends are indeed variable because of interactions with shorter fluctuations as well as structural breaks. However, this points to the need to make the trends nonlinear or at least piecewise linear, but not necessarily stochastic. To be useful, trends should be smooth and predominantly in one direction (positive for growth and inflation). But an estimate of stochastic trend includes not only the long-term growth but also the major up-and-down movements. The latter are due to the random component of the trend, and the proponents

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<sup>15</sup>Loglinear trend fits better long series of U.S. real GNP per capita (1909–1970), as shown in Diebold and Senhadji (1996), but these are annual data not well suited for study of business cycles.

draw a strong conclusion, namely that “permanent innovations account for a substantial fraction of transitory economic fluctuations,” (King et al., 1991, p. 834).

Fig. 6 panel C shows the stochastic trend obtained by applying the Beveridge–Nelson decomposition to the monthly data on the U.S. CI (1948–2000).<sup>16</sup>

Fig. 6 panel D shows the cyclical component of the CI estimated by this procedure and compares it with the shaded areas based on the deviations of CI from its PAT. The detrended series in panel D of Fig. 6 is smooth ( $MCD = 1$ ) and roughly symmetric around zero. It shows declines during all of the growth slowdowns and recessions but some of these declines are poorly articulated, very small and mainly or entirely on the positive side of the zero line (1962–1963, 1995); some others are larger but also mainly or entirely on the positive or negative side only (1951–1952, 1960, 1981–1982). Thus, there is no expected transition from above to below trend and so several large fluctuations in this detrended series resemble business cycles more than growth cycles (1949–1954, 1960–1970, 1975–1982, and 1982–1991).

We found before that linear deterministic trends leave too much of the variation in the economic times series to business cycles; now we find that linear stochastic trends include much of the fluctuations and leave too little to business cycles.<sup>17</sup>

Table 2 compares the timing of growth cycles according to different methods of detrending the U.S. CI. The Beveridge–Nelson trend deviations lead at three peaks and at one trough in the PAT deviations by significant intervals but show lags, generally short, at most other turns (column 1). They miss only one PAT slowdown, in 1962–1964. On average, the timing of the two series is approximately coincident (with means and medians within the  $\pm 3$  months interval).

### 3.3. *The local linear trend model*

As noted by James H. Stock,<sup>18</sup> the theory of optimal linear filtering is very well developed and understood so one can assess the various filters well on a priori grounds for their suitability in the present context. Since it is desirable here to allow for persistent fluctuations in the mean growth rates, Stock (2000) suggested, a local linear trend model which allows the drifts to change needs to be considered.

This type of structural model is discussed by Harvey (1989, 2000) who assumes that a time series,  $y_t$ , is composed of a stochastic trend,  $\mu_t$ , and an irregular term,  $\varepsilon_t$ , as in

<sup>16</sup>We use the state-space approach proposed by Morley (2002) to estimate the trend in the U.S. CI on the assumption that the proper time series model is ARIMA(2,1,2). The cyclical component is defined as the difference between the index and its trend.

<sup>17</sup>This is consistent with the underlying theoretical preconception that attributes most of the cyclical instability to exogenous shocks, many of which are supposed to influence the levels of macroeconomic variables in the long run. With monetary shocks believed to have only temporary real effects, the attention here centers on real, especially productivity, shocks. But, as developed elsewhere (Zarnowitz, 1992, 2000; Glasner, 1997, pp. 557–560), there is much that is doubtful about this approach: the exclusion of any plausible endogenous sources of economic instability; the relative neglect of financial and expectational factors and of sectoral interactions; the uncertain measurement of productivity shocks. (Some of these points have been recognized by real business cycle (RBC) theorists; see King and Rebelo, 2000).

<sup>18</sup>We are grateful to James H. Stock for an invited comment on the first draft of this paper prepared for the meeting of The Conference Board Business Cycle Indicators Advisory Committee (March 2, 2000).

Table 2

Turning points in deviations from PAT and in deviations from six different trend estimates, U.S. Coincident Index (1948–2000)

PAT	Beveridge–Nelson	Local linear	Hodrick–Prescott ( $\lambda = 14,400$ )	Hodrick–Prescott ( $\lambda = 108,000$ )	Rotemberg	Band-pass
Leads (–) or Lags (+) relative to peaks in deviations from PAT						
	(1)	(2)	(3)	(4)	(5)	(6)
Jan-48	miss	6	7	7	6	6
Jan-51	6	0	0	0	miss <sup>c</sup>	0
Mar-53	3	0	0	0	0	0
Feb-57	–7	–2	0	0	–2	–2
Jan-60	4	0	0	0	–7	0
Apr-62	miss <sup>a</sup>	0	0	0	0	0
Mar-66	2	3	0	0	7	0
Aug-69	–7	0	2	0	0	0
Nov-73	–7	0	0	0	0	0
Mar-79	1	0	0	0	0	0
Jul-81	–3	–7	0	–7	miss <sup>d</sup>	0
Sep-84	1	6	0	0	miss <sup>d</sup>	0
Jan-89	1	0	0	0	0	0
Jan-95	3	0	0	0	miss <sup>d</sup>	0
Jun-00	–18	3	–2	–2	0	0
Mean	–1.62	0.60	0.47	–0.13	0.36	0.27
Median	1.00	0.00	0.00	0.00	0.00	0.00
Std. dev.	6.60	3.14	1.96	2.70	3.70	1.67
Leads (–) or Lags (+) relative to peaks in deviations from PAT						
Oct-49	3	0	0	0	0	0
Jul-52	1	0	0	0	miss <sup>c</sup>	0
Aug-54	3	0	0	0	0	0
Apr-58	3	0	0	0	1	0
Feb-61	3	0	0	0	0	0
Jan-64	miss <sup>a</sup>	–12	0 <sup>b</sup>	0	–12	0
Oct-67	1	0	0	0	0	0
Nov-70	2	0	0 <sup>b</sup>	0	9	9
Apr-75	4	1	–1	0	1	1
Jul-80	3	0	0	0	miss <sup>d</sup>	0
Dec-82	1	0	0	0	0	0
Jan-87	1	–7	0	0	miss <sup>d</sup>	0
Dec-91	–6	0	–9	0	miss <sup>d</sup>	0
Jan-96	1	0	0	0	miss <sup>d</sup>	0
Mean	1.54	–1.29	0.57	0.00	–0.11	0.71
Median	2.00	0.00	0.00	0.00	0.00	0.00
Std. dev.	2.50	3.63	4.29	0.00	5.33	2.40
All						
Mean	–0.04	–0.31	0.52	–0.07	0.15	0.48
Median	1.00	0.00	0.00	0.00	0.00	0.00
Std. dev.	5.15	3.45	3.24	1.91	4.38	2.03

Note: The turning points in the series are selected by the Bry–Boschan algorithm, except where noted.

<sup>a</sup>The early 1963 decline in the B–N series is too short and too small to qualify and has not been accepted by the algorithm (see Fig. 6 panel D).

<sup>b</sup>The algorithm selected dates lagging the PAT dates by 9 months in both these cases, but we overruled these choices by considering outliers (which are identical in the PAT and H–P series).

<sup>c</sup>The early 1951 decline in the Rotemberg series is too short and too small to qualify and has not been accepted by the algorithm (see Fig. 8 panel B).

<sup>d</sup>The Rotemberg series shows only a very short, though sharp, increase in 1980; it skips the 1984–1986 slowdown; and its decline in early 1995 is too short and too small to qualify (see panel B of Fig. 8).



Eq. (1).<sup>19</sup> Eqs. (2) and (3) describe how the slope of the stochastic trend,  $\beta_t$ , is itself a random walk process. The error terms  $\varepsilon_t$ ,  $\eta_t$ , and  $\zeta_t$  are normally and independently distributed with zero means and variances  $\sigma_\varepsilon^2$ ,  $\sigma_\eta^2$ , and  $\sigma_\zeta^2$ , respectively.

$$y_t = \mu_t + \varepsilon_t, \quad t = 1, \dots, T, \quad (1)$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \quad (2)$$

$$\beta_t = \beta_{t-1} - \zeta_t. \quad (3)$$

Depending on the assumptions about the variances of the error terms, different trends can be obtained in this framework.<sup>20</sup> The model can be cast in the state-space form and the Kalman filter can be used to get estimates of the unobserved trend and slope components (see Harvey, 1989, Chapter 4). Restricting our attention here to a specific set of assumptions without prior testing of their appropriateness, we estimate the model of equations (1)–(3) along these lines to see if it can give good results in terms of growth cycles.

We believe a good trend estimate should be influenced by the cyclical movements in the data but it should also be smooth. This implies that the trend should show only slow slope changes, with small innovations. Arguably, then, the variance of the residuals in the equation for the trend slope should be small, and so we assume  $\sigma_\zeta^2 = 10^{-9}$  while fixing  $\sigma_\eta^2$  at zero. The estimated value of  $\sigma_\varepsilon^2$  for the natural log of the U.S. CI for 1948–2000 is 0.0005. This is very close to the variance of the deviations from PAT, which is 0.00036, suggesting that the two trends may give similar results. Indeed, our estimated local linear trend is smooth (MCD = 1); its slope changes slowly and is relatively steeper during periods of high average growth rates (Fig. 7 panel A). Compared with the PAT in Fig. 1, this trend does not flatten as much during recessions.

The deviations from the local linear trend shown in panel B of Fig. 7 are quite similar to the deviations from PAT in Fig. 2, with just a few notable differences: they are somewhat smaller on the positive side in 1948, 1982, and 1985–1986; somewhat larger on the negative side in 1949, 1960–1965, 1971, 1975, 1983, and 1992–1996. The deviations from the Beveridge–Nelson trend (panel D of Fig. 6) are considerably smaller most of the time, except sporadically in the 1960s and 1990s.

The series of deviations from the local linear trend we have estimated shows timing coincident with that of PAT at most peaks and troughs (there are only eight nonzero entries here, and only four of them large; see Table 2, column 2). This correspondence is much better than that between the Beveridge–Nelson and PAT deviations.

<sup>19</sup>Note that no separate cycle component is distinguished here. A cyclical component is modeled explicitly in Harvey (1985), but the empirical applicability of this framework is severely limited, especially for the post-World War II period by the use of annual data only.

<sup>20</sup>If variances of both  $\eta_t$  and  $\zeta_t$  are zero, the model reduces to a deterministic linear trend. Alternatively, if only the variance of  $\zeta_t$  is zero, the trend is a random walk with drift as in the Beveridge–Nelson decomposition. When the variance of  $\eta_t$  is fixed at zero and the variance of  $\zeta_t$  is positive, the resulting local linear trend is an integrated random walk trend or a smooth trend (see Harvey, 2000, p. 3).

### 3.4. The Hodrick–Prescott trend

Let a time series  $y_t$  be viewed as the sum of a growth (trend) component  $g_t$  and a cyclical component  $c_t$ :  $y_t = g_t + c_t$  for  $t = 1, \dots, T$ . The growth component should be smooth, so that the procedure recommended by Hodrick and Prescott (1997) is to minimize

$$\sum_t c_t^2 + \lambda \sum_t [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2, \quad (4)$$

where the parameter  $\lambda$  is positive. The larger  $\lambda$ , the smoother is the result; if  $\lambda$ , which penalizes variability in  $g_t$ , is large enough,  $g_t$  approaches  $g_0 + \beta t$ . Hodrick and Prescott  $\lambda = 1,600$  favor for quarterly data, but show that the numbers change little if  $\lambda$  is reduced or increased by a factor of four.

The squared second difference of the trend component,  $g_t$ , which is multiplied by  $\lambda$ , is a very small term. Even very large changes in  $\lambda$ , therefore, influence the cyclical component  $c_t$  only modestly. For annual observations,  $\lambda$  values of 400 or 100 have been used, but Baxter and King (1999) argue that 10 gives better results. We are interested in applications to monthly data, and obtain reasonably good results with  $\lambda = 14,400$ . As shown in panels C and D of Fig. 7, the H–P trend shows both a considerable smoothness and some cyclical flexibility in this case, while the deviations from trend decline into negatives in each downward phase of growth cycles dated according to the PAT (see curves labeled  $\lambda = 14,400$  in panels C and D). The only exceptions to correspondence between the two cases occur in 1956 and 1959, when the H–P deviations series falls briefly below zero during apparent growth cycle upswings, and in 1989–1990, when it shows an overly strong extra rise during a slowdown. When the outliers are properly considered, the turning points in the H–P and the PAT growth cycles are either identical or very close in time, with but two significant deviations: the peak near the beginning of the sample period and the trough in 1991 (see Table 2, Column 3).

However, it is worth noting that the H–P series has smaller fluctuations and is less smooth than the PAT series (compare panel D of Fig. 7 with Fig. 2). In particular, the H–P cyclical component (with  $\lambda = 14,400$ ) seems to distort both the sluggish recovery in the early 1990s (by showing an early upswing) and the vigorous boom in the late 1990s (by showing only weak and erratic positive deviations from trend).

As an experiment, we performed a search to locate a value of  $\lambda$  that would minimize  $d = \sum_t (p_t - h_t)^2 / T$ , where  $p_t$  and  $h_t$  represent the series of PAT and the H–P trend, respectively. With  $\lambda = 14,400$ ,  $d = 0.45$ ; when  $\lambda$  is increased by increments of 100,  $d$  reaches a minimum of 0.16 for  $\lambda = 108,000$ . We add the corresponding trend and detrended values to panels C and D of Fig. 7, (see curves labeled  $\lambda = 108,000$ ). The 7.5-fold increase in  $\lambda$  produces moderate improvements in the new H–P estimate of the “cyclical component.” The H–P deviations series for  $\lambda = 108,000$  has somewhat larger and on the whole, more symmetrical fluctuations than the series for  $\lambda = 14,400$ .<sup>21</sup> The estimate with larger  $\lambda$  is quite similar to the PAT, even in many small details.

<sup>21</sup>However, note that in the years 1962–1967 the series for  $\lambda = 108,000$  is less symmetrical around zero than the series for  $\lambda = 14,400$  (see panel D of Fig. 7).

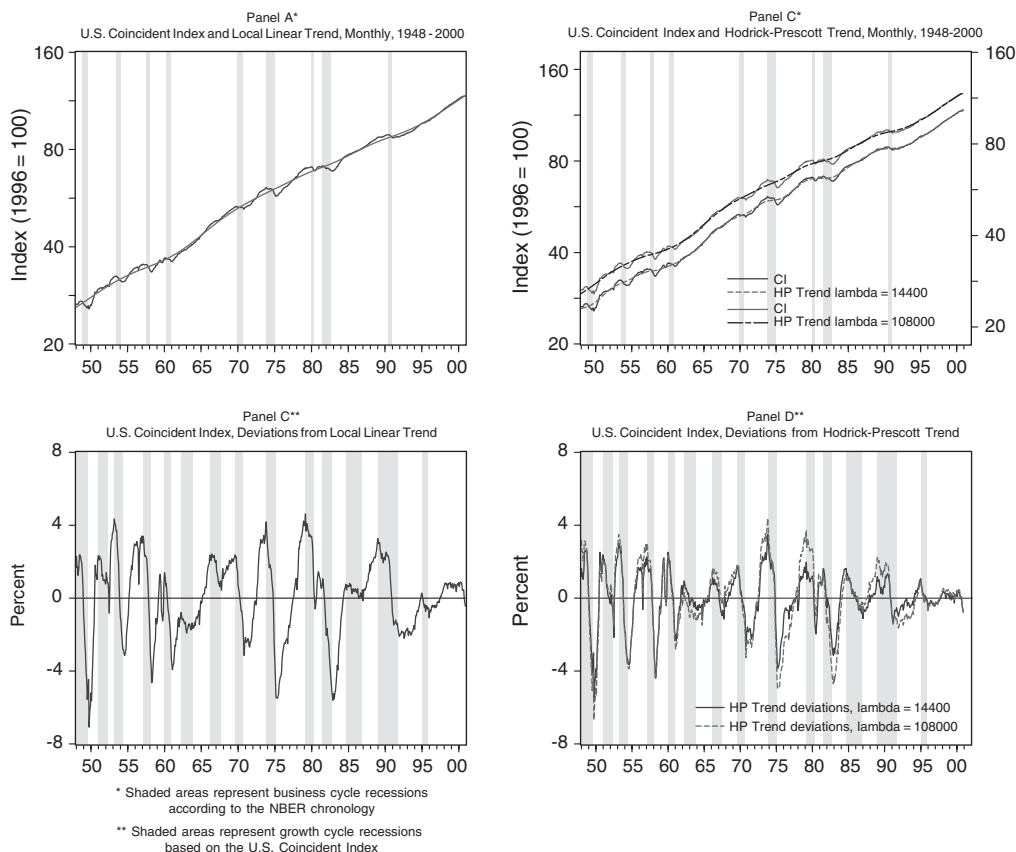


Fig. 7. Panel A: U.S. coincident index and local linear trend, monthly, 1948–2000; Panel B: U.S. coincident index, deviations from local linear trend; Panel C: U.S. coincident index and Hodrick–Prescott trend, monthly, 1948–2000; Panel D: U.S. coincident index, deviations from Hodrick–Prescott trend.

### 3.5. The Rotemberg trend

Rotemberg (1999) estimates the trend by minimizing

$$\sum_{t=1+k}^T c_t^* c_{t-k} + \lambda \sum_{t=2}^{T-1} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \quad (5)$$

$\lambda$  is chosen as the lowest parameter value such that the following constraint holds:

$$\sum_{t=k+v}^{T-k-v} c_t * [(g_{t+v} - g_t) - (g_t - g_{t-v})] = 0. \quad (6)$$

The parameter  $k$  ensures that the estimated trend minimizes the covariance of two values of the cyclical component,  $c_t$  and  $c_{t+k}$ . The parameter  $v$  ensures that the trend and cycle components,  $g_t$  and  $c_t$ , are orthogonal over the horizon of  $v$  periods.

Rotemberg recommends that  $k$  be set to equal 16 quarters on the admittedly somewhat arbitrary ground that historically NBER business cycle troughs for the U.S. have been four years apart on average. (The dispersion around this average is very large.) With large  $k$ , the minimization of (5) results in a trend that is quite smooth and not very sensitive to either the cyclical movements of the series or the choice of  $v$ . With low  $k$  – *a fortiori*, with zero  $k$ , which is the case in the H–P trend—the effects are opposite. Rotemberg chooses  $v$  to equal five quarters.

Fig. 8 panel A shows what happens when this procedure ( $k = 48$  months and  $v = 15$  months) is applied to the U.S. CI. The trend here is exceedingly smooth, as the estimate of  $\lambda$  is equal to 4,335,498(!) and it is rising persistently, with few slope changes.<sup>22</sup> The deviations from the Rotemberg trend shown in panel B of Fig. 8 track the growth cycle turns shown by the shaded areas according to PAT much better during recessions than during the other slowdowns. They are not very symmetric around zero, their amplitudes vary widely, and the trend is probably not high enough at the end of the sample after 1998 (a shortcoming noted by Rotemberg). The growth cycles are reduced strongly (those in 1951–1952, 1963–1964, 1966–1967, 1984–1986, and 1995 are almost eliminated) so that the general appearance of the fluctuations in this figure tracks business cycles rather than growth cycles. Table 2 column 4 shows several close correspondences but also large discrepancies between the timing of this trend and that of PAT.

### 3.6. Band-pass filters

Baxter–King (1999) propose a band-pass filter that isolates the components of the time series with fluctuations between six and 32 quarters and removes the components of higher and lower frequencies as “noncyclical”.<sup>23</sup>

The resulting series are therefore relatively smooth, often with well-articulated turning points. But the latter may well be misdated if the short-period variations of the indicators around their peaks and troughs are not taken into account properly. Hence, the elimination of high frequency changes from data may affect the results adversely. Furthermore, it is doubtful that one can precisely identify the exclusively “cyclical” frequencies and assume that the resulting band-pass filter remains valid and constant over time. In some very long phases (e.g., the contraction of the 1930s, the expansions of the 1960s and the 1990s), the relevant frequency mix may be rather different from that applying to some very short phases (e.g., the back-to-back recessions separated by the incomplete recovery in the early 1980s). Thus, much flexibility is needed in reconciling reasonable trend estimates with both unusually long and unusually short cyclical

<sup>22</sup>Still, Rotemberg’s trend decelerates slightly in late 1960s, early 1980s and early 1990s, accelerates slightly in between, notably in the long expansions of the 1960s and the 1990s. So there is some mutual dependence of trend and cycles, but it is minimized.

<sup>23</sup>They refer to Burns and Mitchell’s definition of business cycles in support of this choice but not quite convincingly. First, NBER research always required that business cycles involve absolute declines (contractions) in economic activity, whereas Baxter and King (1999) like most others attempting statistical TC decompositions make no distinction between business cycles and growth cycles. Hence, to begin with, it must again be noted that they all obtain estimates of growth cycles, not business cycles. Second, Burns and Mitchell define business cycles as ranging up to 10 or 12 (not eight) years, and Table 1 Column 12, suggests that cycles exceeding 8 years in duration may no longer be so rare. In contrast, growth cycles lasted two to six years in the recent era (Table 1, column 6). Finally, NBER researchers would always determine cyclical timing and other properties of time series in seasonally adjusted but otherwise unsmoothed time series, using moving averages cautiously for guidance only.

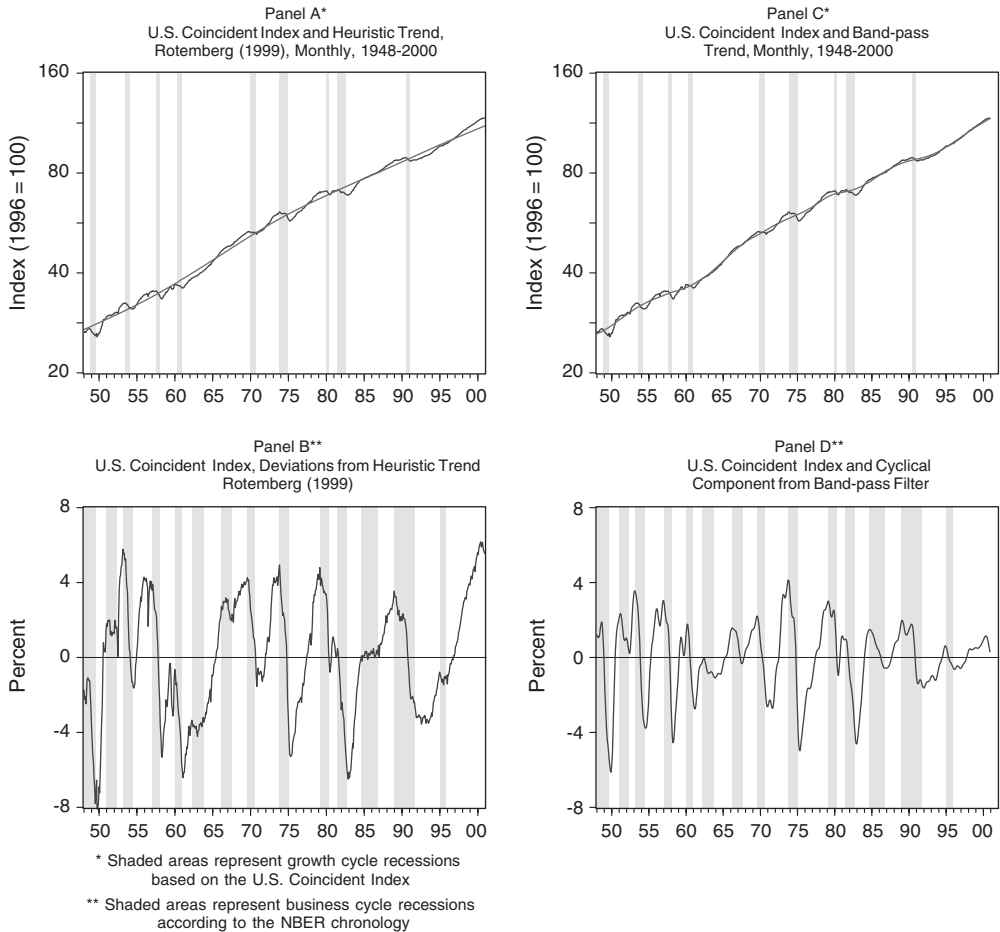


Fig. 8. Panel A: U.S. coincident index and heuristic trend, Rotemberg (1999), monthly, 1948–2000; Panel B: U.S. coincident index, deviations from heuristic trend Rotemberg (1999); Panel C: U.S. coincident index and band-pass trend, monthly, 1948–2000; Panel D: U.S. coincident index and cyclical component from band-pass filter.

movements. On that, the PAT method is probably at a significant advantage vis-à-vis the H-P and band-pass filtering methods.

We experimented with a wide range of band-pass filters so as to accommodate the wide range of growth cycles of actual experience. Generally, the narrower the band, the more numerous and the smaller are the fluctuations in the deviations from trend. The evidence suggests that narrow bands are inferior to wide bands in that they produce too many estimated growth cycles. For example, using the band-pass filter with a frequency range of nine to 50 months one gets 18 growth cycles; using a band of 12–50 months, one gets 19 cycles.

Fig. 8 panel C shows the trend and panel D shows the deviations from trend based on a band-pass filter of wide range (9–96 months). The trend is what is left after the cyclical component and the irregular component (less than 9 months) are subtracted from the

seasonally adjusted U.S. CI. Again, the shaded areas on the figure indicate the growth cycle slowdowns according to the PAT. A comparison of Figs. 2 and 8 suggests that the band-pass filter for the selected frequency range produced almost a smoothed version of the PAT deviations series. The only blemishes we can find here is that the declines in 1951–1952 and 1962–1963 are underrepresented (the former is limited to positives, the latter to negatives; see panel D of Fig. 8).

Table 2 column 5 compares the peak and trough dates in the cyclical component derived by using the band-pass filter with those in the deviations from PAT. The timing of the two series is virtually identical; evidently, smoothing has not distorted it in this case.

#### 4. Conclusion

A trend denotes a long-run tendency in economic time series, for example, an upward inclination reflecting real growth or cost (price) inflation. There is so much variability in intermediate and short run components of many series—cyclical, seasonal, irregular—that it is useful to define trend by contrast as changing rather slowly, without sudden disturbances that are difficult to explain. The smoothest are the linear trends, yet linear trends are hard to find, for several good reasons. Economies grow through cycles of expansions and contractions that vary widely in amplitude and duration. Long expansions (contractions) can tilt growth trends upward (downward). Great technological innovations speed up growth temporarily; disruptions of supply or demand in times of war, civil conflict or industrial dispute deter growth temporarily. Monetary and exchange regimes differ, periods of inflation and deflation have historically alternated. Thus, trends may be smooth but they vary and change.

We have looked at different methods of time series decomposition from the viewpoint of how best to separate trends and growth cycles, i.e. fluctuations in data adjusted for both long-term and seasonal movements. Smoothness in trends is desirable, linearity over short periods may be, linearity over long periods is not. The most difficult problem for time-series decomposition in the present context arises because trends and business cycles interact. Just as a long phase of depressed or stagnating business conditions can lower the long trend so a long period of maintained high growth can reduce cyclical instability by replacing recessions with slowdowns. So a nonlinear or at least a local or piecewise linear trend, which allow for the TC interaction, is apt to do a better job than a linear trend.

The PAT program developed at the NBER (Boschan and Ebanks, 1978) exemplifies such a trend, with the explicit purpose of growth cycle measurement. PAT addresses the task directly by using preliminary growth cycle turning points and it serves the purpose on hand well. This is revealed by a close examination of cyclical movements since 1948 in two principal measures of U.S. aggregate economic activity—the monthly CI and the quarterly real GDP. Growth cycles appear clearly in both the deviations from trends and the smoothed growth rates for the series. They complement well the business cycles as measured in the corresponding data on levels of CI and real GDP.

We have compared the patterns, timing, amplitudes, and smoothness of several estimates of U.S. growth cycles in the post World War II period obtained by using different methods of detrending the monthly CI 1948–2000. Log-linear trends are clearly inferior for our purposes. The stochastic Beveridge and Nelson (1981) trend is much better but it includes too much of cyclical fluctuations and the deviations from it often lack the proper patterns and timing. The local linear trend of Harvey (1989, 2000) leaves deviation

cycles that are substantially similar to those produced by PAT, and the two sets are close in timing. The Hodrick–Prescott (1997) approach is flexible, and with very high  $\lambda$  it produces growth cycles quite similar to PAT but it falls short on smoothness. Rotemberg's (1999) heuristic trend is very smooth, but the deviations from it vary widely in amplitude, miss some growth cycles, and are not as symmetric as the deviations from PAT and some other trends. The band-pass filter works quite well for wide frequency ranges (much worse for narrow ranges) and produces very smooth growth cycles very similar to those of PAT.

Some recent literature on various growth cycle estimates using different detrending methods mistakenly seeks verification in NBER chronologies or other measures relating to business cycles. It is implicitly assumed that growth cycles and business cycles are not distinguishable, and slowdowns are treated like recessions. In this paper we have argued that business cycles and growth cycles are two distinct but related sets of phenomena or processes. The decomposition of time series data into trend and cycle is very important in making this distinction and great care should be taken to estimate trends properly when performing cyclical analysis. In general, it is reassuring that the results of the wide range of detrending methods covered in this paper exhibit great similarity for U.S. data and conform to the chronology of growth cycles identified in earlier NBER studies.

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