# *Statically determinate and indeterminate structures*

*Gebril El-Fallah*

**EG1101 – Mechanical Engineering – Mechanics of Materials**

gmae2@leicester.ac.uk

# Statically Determinate Structures – Example: Cantilever Beam

Balance of forces vertically, we have:

$$H_A = F$$

Taking Moments about Point A   +
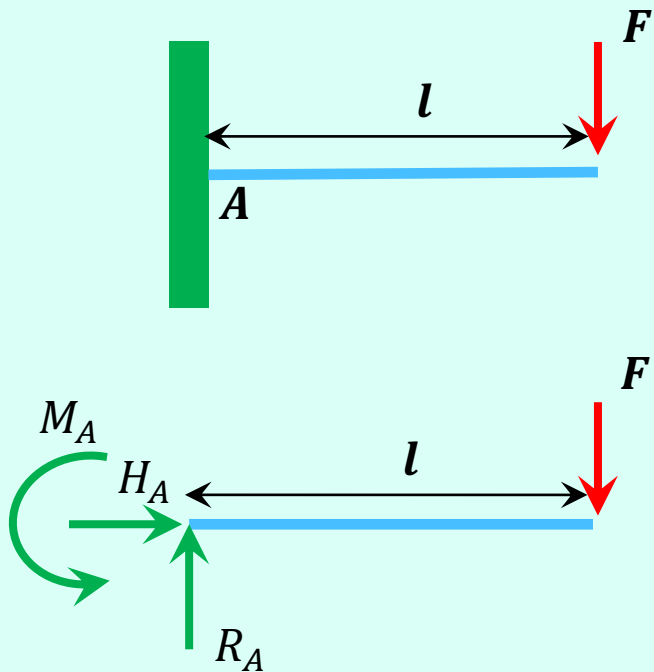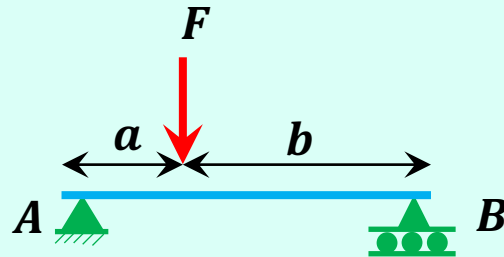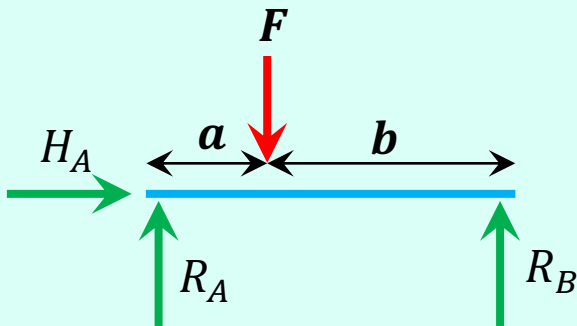
$$M_A - F.l = 0$$

Thus

$$M_A = F.l$$

$F$

$l$

$A$

$M_A$

$H_A$

$l$

$F$

$R_A$

**Free Body Diagram**

UNIVERSITY OF
LEICESTER

# Statically Determinate Structures – Example: Simply Supported Beam



**Free Body Diagram**



**Taking Moments about Point B**

$$R_A.(a+b) - F.b = 0$$

Thus

$$R_A = \frac{b}{(a+b)}F$$

And either by balance forces in the vertical direction or by taking Moments about Point A

$$R_B = \frac{a}{(a+b)}F$$

# Statically Determinate Structures

Reactions and Internal Forces can be determined solely from

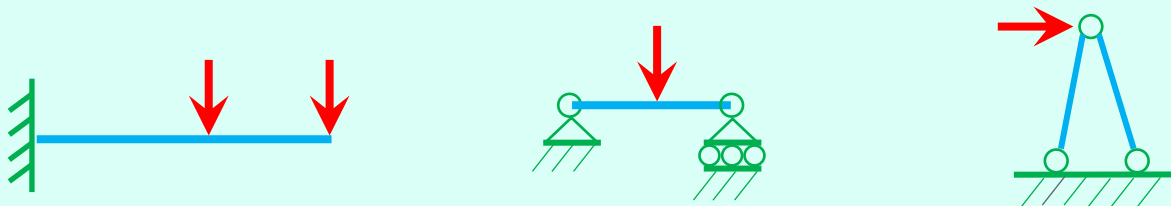**Free Body Diagrams** and **Equations of Equilibrium**

Number of Unknowns = Number of Equations of Equilibrium

Properties of the Material **NOT** required

UNIVERSITY OF
LEICESTER

# Statically Determinate Structures

- For this type of structures, you can find all the internal forces and reaction forces by using equilibrium conditions (balance of forces).



**Determinacy criteria for structures**:
- **Statically determinate structures**: the number of equilibrium equations equals the number of unknown forces (including reactions) – **Just stiff structure.**

- **Over-stiff structures**: more unknowns than equilibrium equations.

- **Mechanisms**: more equilibrium equations than unknown forces or reaction forces **(under-stiff structures)**
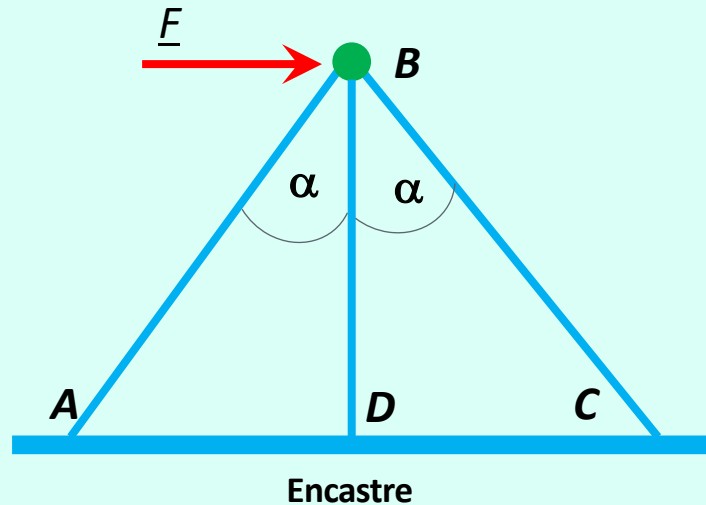
UNIVERSITY OF LEICESTER

# Statically Determinate Structures: Simple approach

- Structure would collapse if one of the supports or members is removed

UNIVERSITY OF
LEICESTER
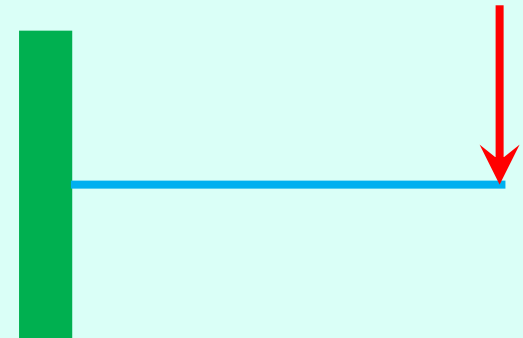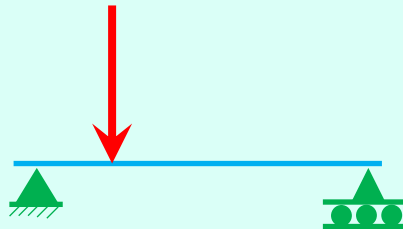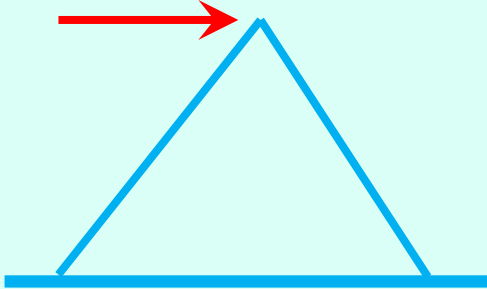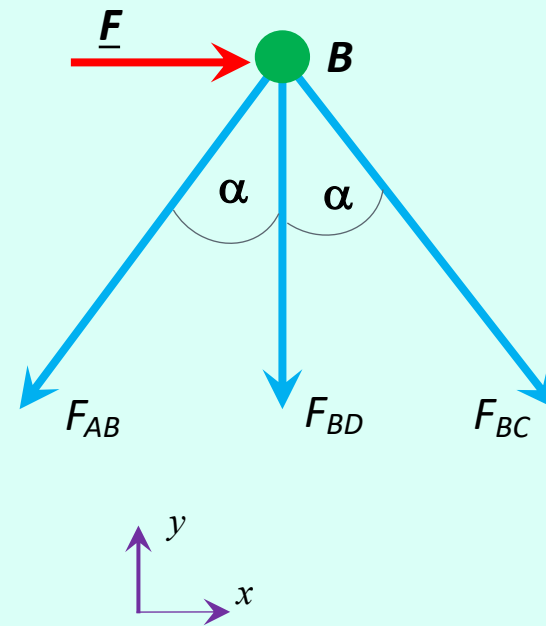
# Statically Indeterminate Structures – Example 1

**Taking Joint B as a Free Body Diagram**

**Third Member Added**

# Statically Indeterminate Structures – Example 1

- Balance of Forces in Vertical Direction

$$F_{BD} + F_{BA} \cos \alpha + F_{BC} \cos \alpha = 0$$

- Balance of Forces in Horizontal Direction

$$F + F_{BC} \sin \alpha - F_{BA} \sin \alpha = 0$$

Giving **2 Equations in 3 Unknowns**

Another Equation required for solution

# Statically Indeterminate Structures

- Reactions and Internal Forces can **<u>NOT</u>** be determined solely from Free Body Diagrams and Equations of Equilibrium

- **More than one unknown in the system of Equations**

- **Additional Equation(s) are required for solution**

    Relating to Displacements of the Structure

    Called **Equation(s) of Compatibility**

- Properties of the Material are required

# Statically Indeterminate Structures: Simple approach

- Structure would still stand if one (or more) of the supports or members is removed
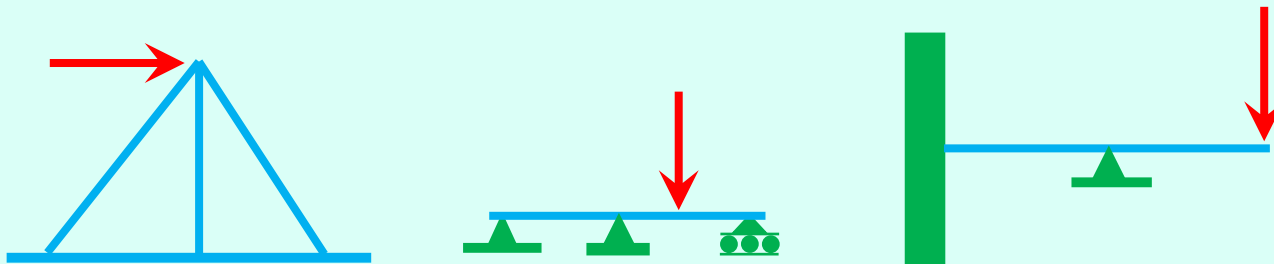
# Statically Indeterminate Structures
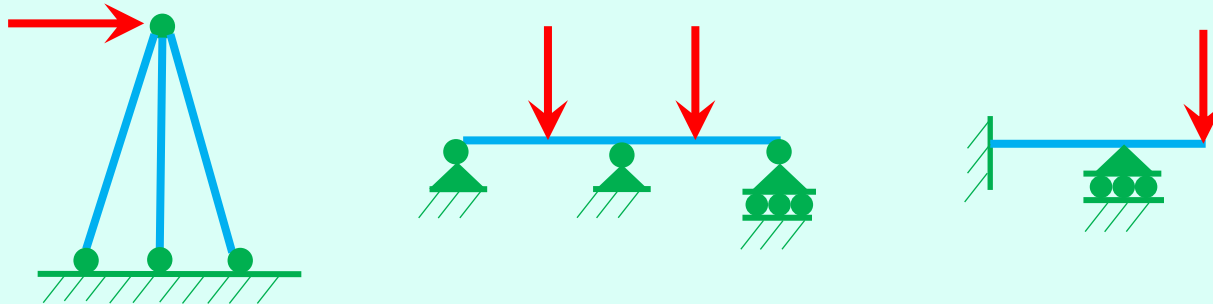


For this type of structures, it is not possible to find the internal forces or support forces by using equilibrium condition alone. Condition about displacement of the structure has to be added in order to find the forces.

The structures have members or support that are not absolutely necessary. The structure would stand if some of them are removed.

# Statically Indeterminate Structures

Statically determinate STRESS systems
- The stresses can be calculated purely from equilibrium conditions
- Example: tie, strut

Statically indeterminate STRESS systems

In general, solutions require:
- Equilibrium of forces (internal and external forces)
- Compatibility of displacements (displacement –strain)
- Constitutive law (stress-strain)

UNIVERSITY OF LEICESTER