School of Engineering

# Pin Jointed Frames

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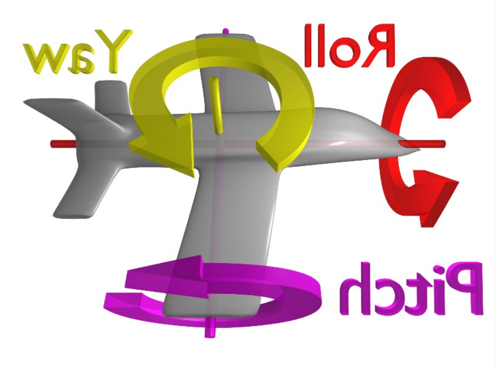
# Rigid Body




**<u>Rigid Body</u>** defined as:

- Solid Body whose Deformation is either Zero or Negligible
  i.e. Deformation so small that it can be ignored

- Distance between any 2 Points in Body effectively Constant
  *Regardless of any External Forces*

- **Rigid Body** considered as Continuous Distribution of Mass

# Statics

- Concerned with Analysis of Loads (Force and Torque, or ‘Moment’)
- Forces assumed to be in equilibrium (balance) within a body
- Body does NOT experience an Acceleration ($\underline{a} = \underline{0}$)
- Condition known as ‘**Static Equilibrium**’
- System is ‘**at rest**’ or ‘**moving at a constant velocity**’

e.g. Stationary Objects

Buildings, Bridges etc.

Objects in Stable Motion (constant velocity)

Aircraft in stable flight, Car cruising on motorway etc.

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# Static Equilibrium

Thus, for '**Static Equilibrium**' Conditions

No Linear Acceleration of the Body

$$\sum_i \underline{\boldsymbol{F}_i} = \underline{\mathbf{0}}$$

No Angular Acceleration of the Body

$$\sum_i \underline{\boldsymbol{M}_i} = \underline{\mathbf{0}}$$

# Moment of a Force

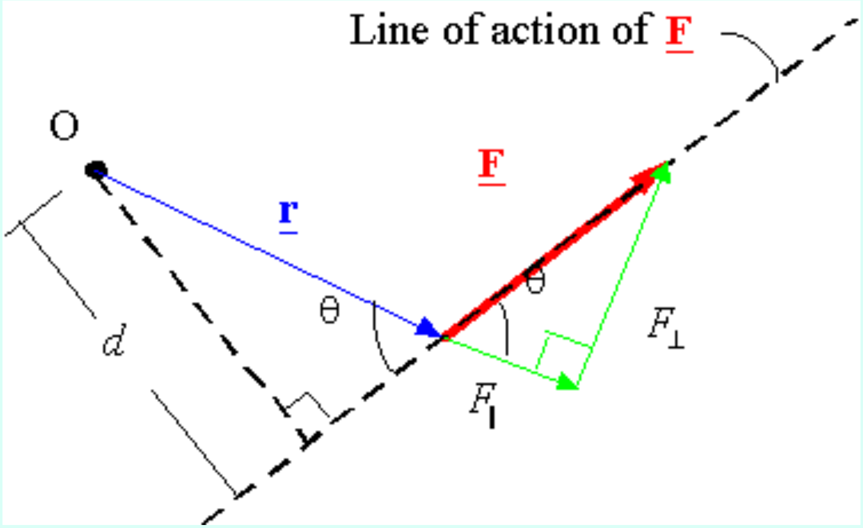
Force can also **ROTATE** a body about an AXIS or Point

Rotational Tendency known as: ***Moment*** (***M***) of the Force

(Moment can also be referred to as ***Torque***)

# Moment of a Force

Moment of a Force about a Point O




**Magnitude** of the Moment of Force (M) about Point O given by:

$$M_O = F.d$$

where

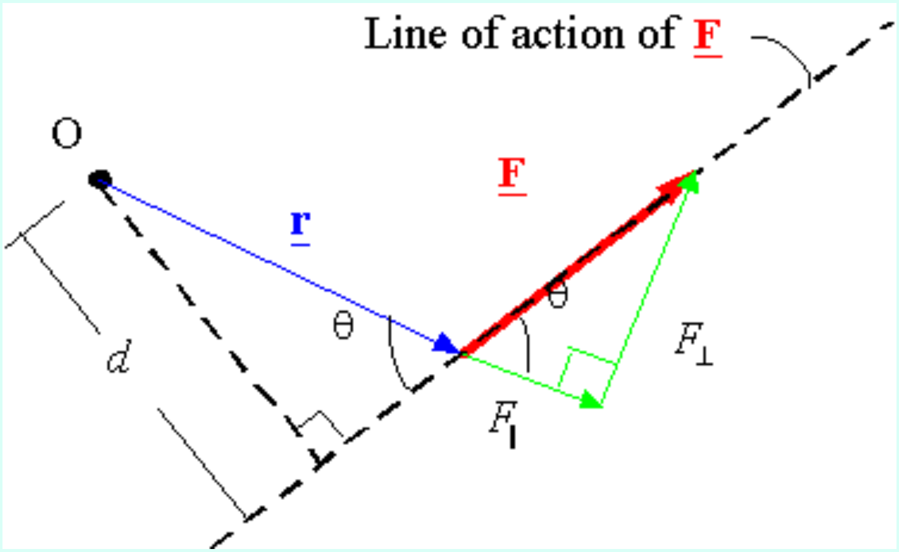
$F$ is the Magnitude of Applied Force

$d$ is **perpendicular** distance from the line of action of the Force

Note: Sign Convention for direction of Moments must be consistent in a given calculation

# Moment of a Force

Moment of a Force about a Point O




In Vector Format, Moment ($\underline{M}$) given by the **Vector Cross Product**:

$$\underline{M_O} = \underline{r} \times \underline{F}$$

where

$\underline{F}$ is the Force Vector

$\underline{r}$ is the radius vector from the Point O to the line of action

# Free Body Diagrams

- **Shows the Forces and Moments on a Body**
- Enables Calculation of the Resulting Reaction Forces
- Used to Determine the Loading of Individual Structural Components
- Also Calculates Internal Forces within a Structure
- Essentially a **VECTOR** diagram of all localized Forces
- Condition of **Static Equilibrium** assumed
    - **i.e. Sum of Forces and Moments must be zero**

# Free Body Diagrams

- Simplified Version of Structural Component
    - Often a Point, Line or Box
- Forces shown as Arrows pointing in direction they act on Body
- Moments shown as Curved Arrows in direction they act on Body
- Coordinate System
- Reactions to Applied Forces also Shown

# Free Body Diagrams

- Typically Provisional Free Body Diagram drawn before all Forces and Reactions are known so that unknowns can be evaluated
- Constraints replaced by Reaction Forces
- Note: If External Forces are small → Can Be Neglected
    - Buoyancy forces in Air
    - Atmospheric Pressure
- Free Body analysed by Summing all the Forces
    - Resolved into the coordinate system directions
    - Net Force in any direction is Zero for Static Equilibrium: $\sum \boldsymbol{F_x} = \mathbf{0} \quad \sum \boldsymbol{F_y} = \mathbf{0}$
    - Net Moment is Zero for Static Equilibrium: $\sum \boldsymbol{M} = \mathbf{0}$

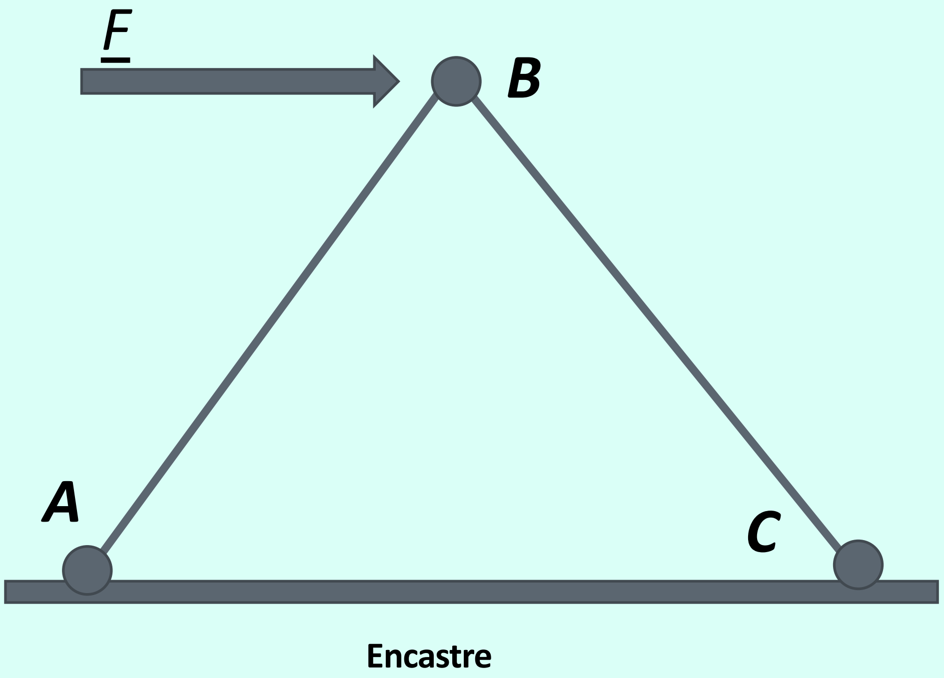
# Free Body Diagrams

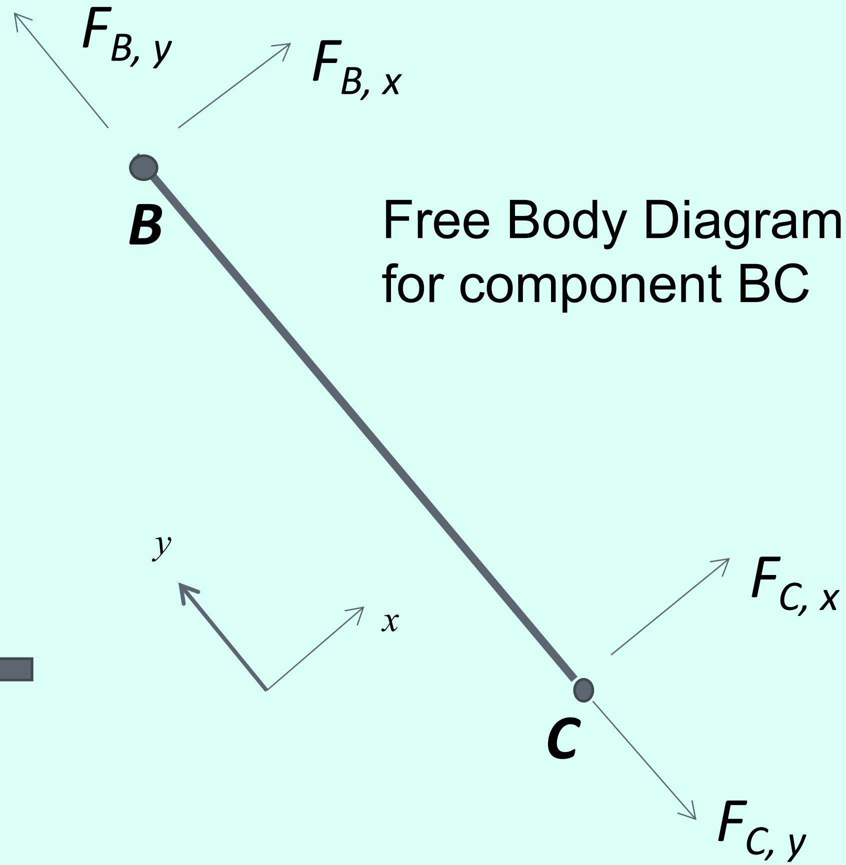
**A free body diagram consists of:**

- A coordinate system
- A simplified version of the isolated body
- Forces shown as straight arrows pointing in the direction they act on the body
- Moments shown as curved arrows pointing in the direction they act on the body
- Supports are replaced by reaction forces and moments

Free body diagrams can easily be constructed for simple problems

# Free Body Diagrams: Simple Example




Free Body Diagram for component BC

# Free Body Diagrams: Simple Example

**Balance of Forces**

Along Axis of Bar ***BC***

$$F_{B,y} - F_{C,y} = 0$$

Note: $F_{C,y}$ is a force in the negative $y$-direction

**Balance of Moments**

Taking Moment about Point B

Length of Bar BC is $l_{BC}$

$$0 + 0 + 0 + F_{C,x}.\, l_{BC} = 0$$

Which Implies $F_{C,x} = 0$

Similarly $F_{B,x} = 0$

if we take Moment about Point C.

**Conclusion: a solid bar (member) in a pin-jointed structure does not carry any forces <u>perpendicular</u> to the axis of the bar**

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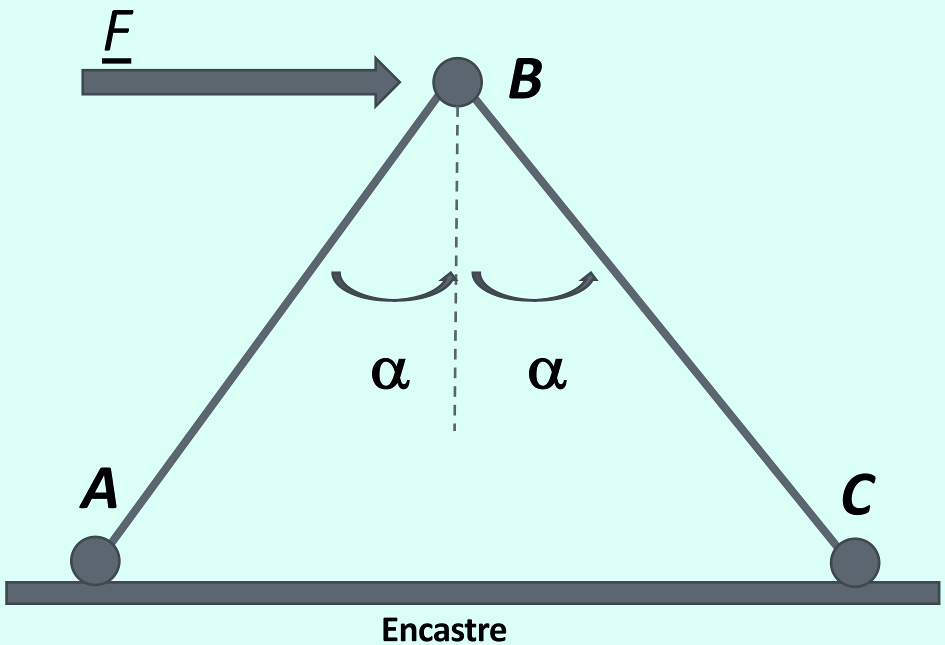
# Pin Jointed Structures

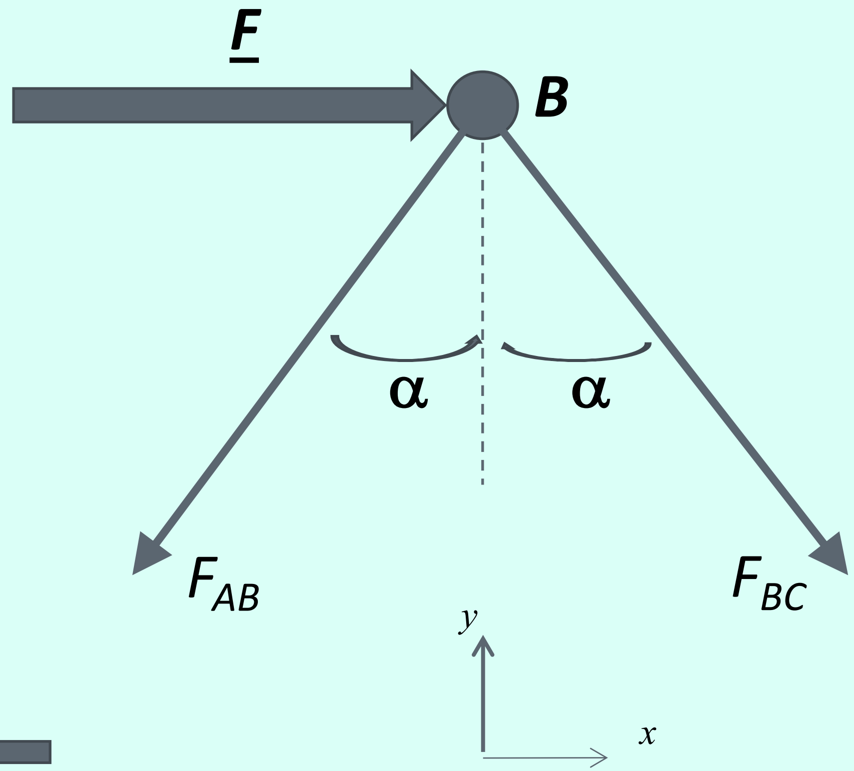
Free to Rotate at the Joints between Structural Members

- Solid Bar (member) in a Pin-Jointed Structure does not carry any Forces <u>perpendicular</u> to the axis of the bar

# Pin Jointed Structures: Simple Example




Taking Joint B as a Free Body Diagram



# Pin Jointed Structures: Simple Example

**At Point B**

Balance of Forces in $x$-direction

$$F + F_{BC} \sin \alpha - F_{AB} \sin \alpha = 0$$

Balance of Forces in $y$-direction

$$-F_{BC} \cos \alpha - F_{AB} \cos \alpha = 0$$

which gives: $F_{AB} = -F_{BC}$

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# Pin Jointed Structures: Simple Example

Then, By Substitution

$$F + F_{BC}\sin\alpha + F_{BC}\sin\alpha = 0$$

giving

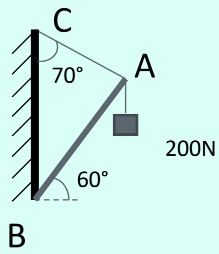
$$F_{BC} = -\frac{F}{2.\sin\alpha}$$

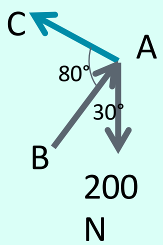
thus

$$F_{AB} = -F_{BC} = \frac{F}{2.\sin\alpha}$$

# Example 1

- A rigid rod is hinged to a vertical support and held at 60° to the horizontal by means of a cable when a weight of 200N is suspended as shown in the figure.



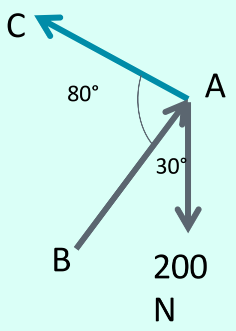




Associated Free Body Diagram

# Example 1

Balance of forces in x-direction:

$$-F_{AC} \cdot \cos 20° + F_{AB} \cdot \cos 60° = 0 \quad (1)$$



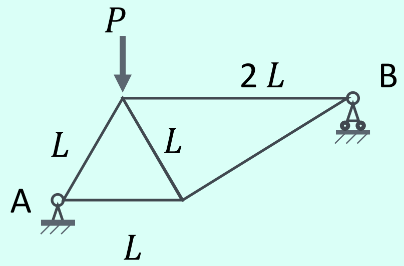

$$F_{AB} = 1.88F_{AC} \quad (2)$$

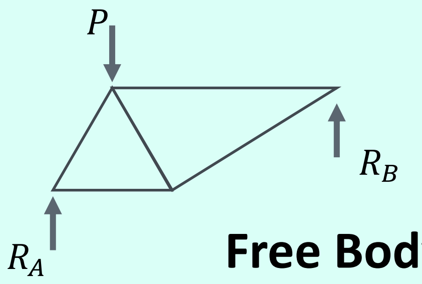
Balance of forces in y-direction:

$$F_{AB} \sin 60° + F_{AC} \sin 20° - 200 = 0 \quad (3)$$

By either substitute (2) into (1), we have: $F_{AC} = 102\ N\ and\ F_{AB} = 192\ N$

# Example 2




**Free Body Diagram**

Balance of forces vertically, we have:

$$R_A + R_B = P \qquad (1)$$

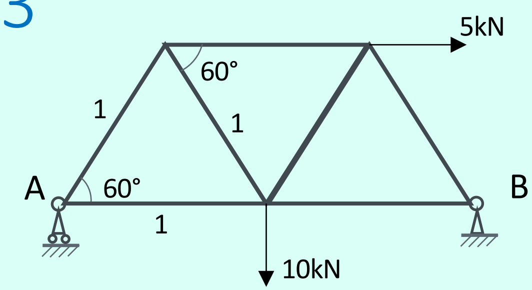
Taking moment about B, +

$$R_A \cdot \left(\frac{1}{2}L + 2L\right) - P \cdot 2L = 0 \qquad (2)$$

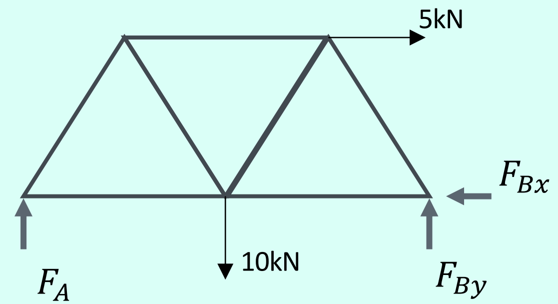
By either substitute (2) into (1), or taking moment about A, we obtain:

$$R_B = \frac{1}{5} \cdot P$$

# Example 3




**Free Body Diagram**




Balance of forces horizontally, we have: $F_{Bx} = 5kN$

Balance of forces vertically, we have:

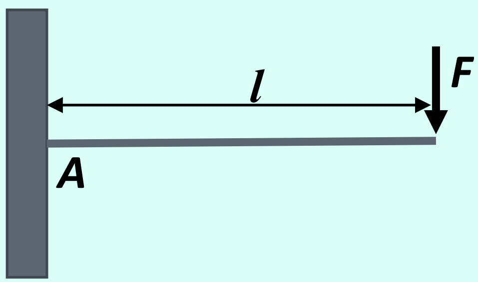
$$F_A + F_{By} = 10kN \qquad (1)$$

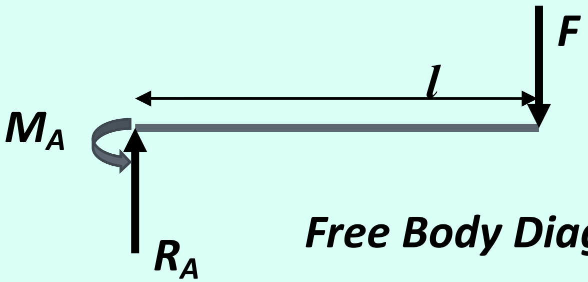
Taking moment about A,

$$F_{By} \cdot 2 - 10\times1 - \frac{\sqrt{3}}{2}\times5 = 0 \qquad (2)$$

By either substitute (2) into (1), we obtain: $F_A = 2.83\ kN\ and\ F_{By} = 7.17\ kN$

# Statically Determinate Structures – Example: Cantilever Beam




***Free Body Diagram***

Taking Moments about Point A

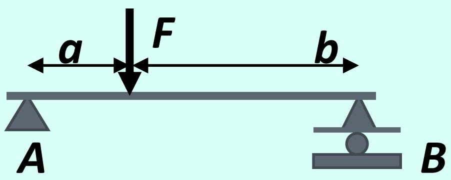
(Moment in Anti-Clockwise Direction)

$$M_A - F.l = 0$$

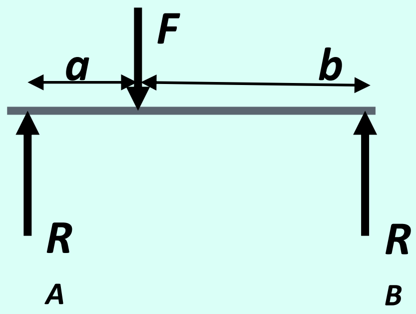
Thus

$$M_A = F.l$$

# Statically Determinate Structures – Example: Simply Supported Beam




***Free Body Diagram***




**Taking Moments about Point B**

$$R_A.(a+b) - F.b = 0$$

Thus

$$R_A = \frac{b}{(a+b)}F$$

And either by substitution or by taking Moments about Point A

$$R_B = \frac{a}{(a+b)}F$$

# Statically Determinate Structures

Reactions and Internal Forces can be determined solely from

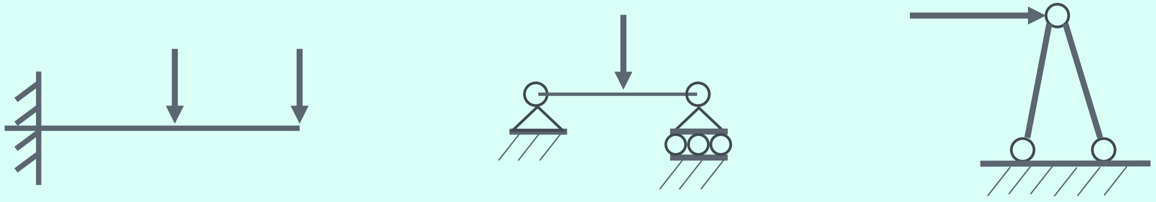
**Free Body Diagrams** and **Equations of Equilibrium**

Number of Unknowns = Number of Equations of Equilibrium

Properties of the Material **NOT** required

# Statically Determinate Structures

- For this type of structures, you can find all the internal forces and reaction forces by using equilibrium conditions (balance of forces).

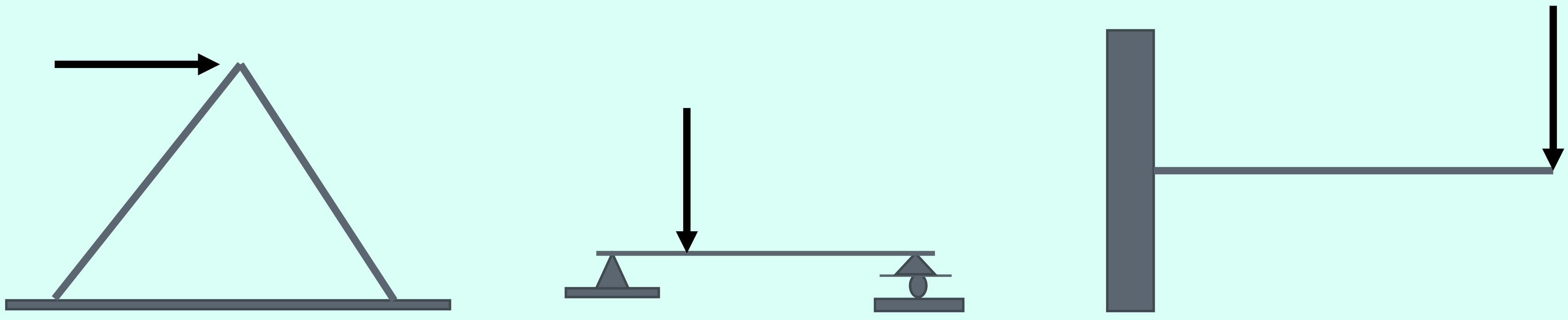


**Determinacy criteria for structures**:

- **Statically determinate structures**: the number of equilibrium equations is equal to the number of unknown forces (including reactions)
- **Mechanisms**: more equilibrium equations than unknowns (**under-stiff structures**)
- **Over-stiff structures**: more unknowns than equilibrium equations

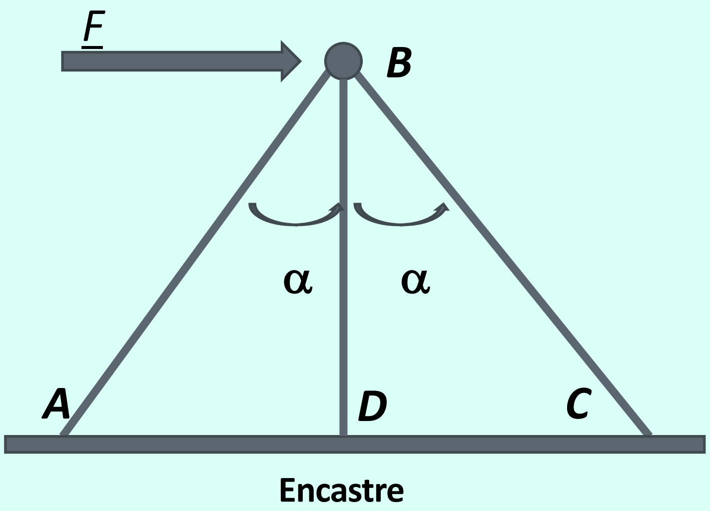
# Statically Determinate Structures: Simple approach

- Structure would collapse if one of the supports or members is removed

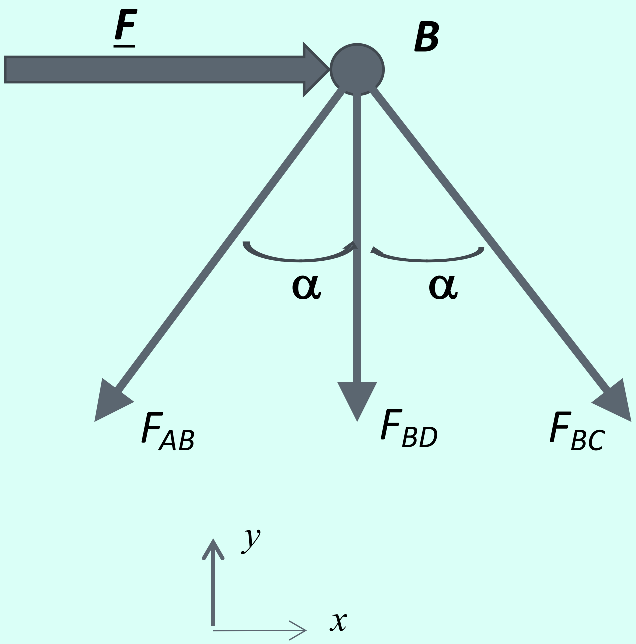


# Statically Indeterminate Structures – Example 1

## Third Member Added




## Taking Joint B as a Free Body Diagram



# Statically Indeterminate Structures – Example 1

- Balance of Forces in Vertical Direction

$$F_{BD} + F_{BA}\cos\alpha + F_{BC}\cos\alpha = 0$$

- Balance of Forces in Horizontal Direction

$$F + F_{BC}\sin\alpha - F_{BA}\sin\alpha = 0$$

Giving **2 Equations in 3 Unknowns**

Another Equation required for solution

# Statically Indeterminate Structures

- Reactions and Internal Forces can **NOT** be determined solely from Free Body Diagrams and Equations of Equilibrium
- **More than one unknown in the system of Equations**
- **Additional Equation(s) are required for solution**

  Relating to Displacements of the Structure

  Called **Equation(s) of Compatibility**
- Properties of the Material are required

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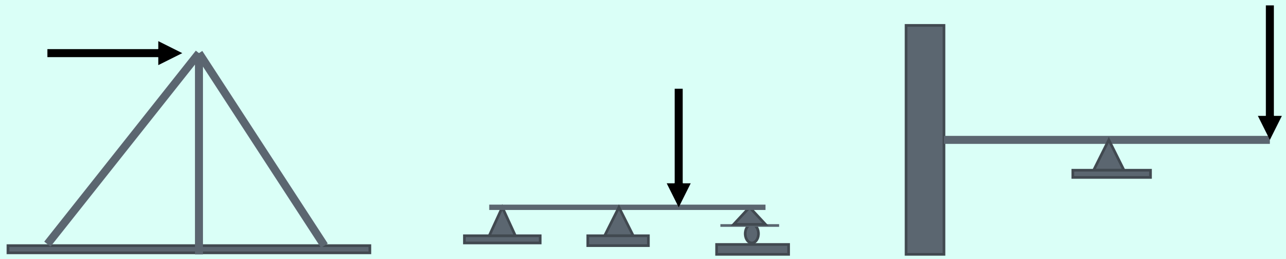
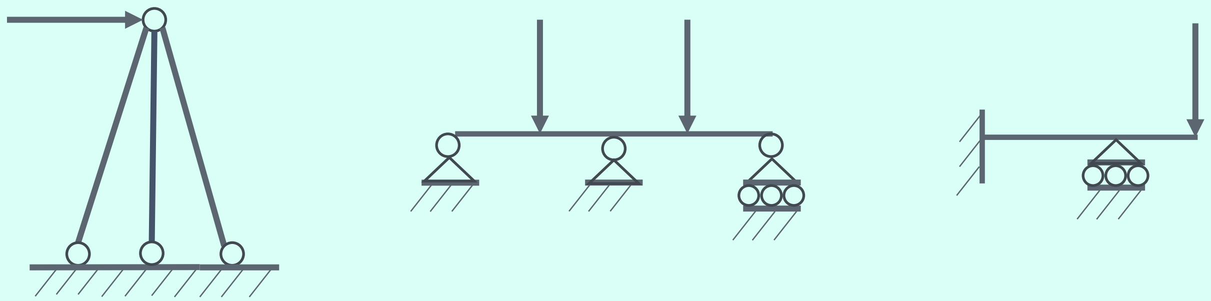
# Statically Indeterminate Structures: Simple approach

- Structure would still stand if one (or more) of the supports or members is removed



# Statically Indeterminate Structures



For this type of structures, it is not possible to find the internal forces or support forces by using equilibrium condition alone. Condition about displacement of the structure has to be added in order to find the forces.

The structures have members or support that are not absolutely necessary. The structure would stand if some of them are removed.

# Statically Indeterminate Structures

Statically determinate STRESS systems

- The stresses can be calculated purely from equilibrium conditions
- Example: tie, strut

Statically indeterminate STRESS systems

In general, solutions require:

- Equilibrium of forces (internal and external forces)
- Compatibility of displacements (displacement –strain)
- Constitutive law (stress-strain)

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