Math 320 / HOMEWORK 1 *Due: Friday 1/25/19*

Name: ______________________ **ID:** ______________

NOTE: *Include all necessary steps to receive full or partial credits. Write clearly on the given papers only.* **No scratch papers are accepted;** *use the back of the paper if necessary.*

(1) (8 points) Prove that $\sqrt{3}$ is irrational.

(2) (a) (5 points) Write a formal definition for the *greatest lower bound* $\inf A$ for a set $A \subseteq \mathbf{R}$, in the style of $\sup A$.

(b) (7 points) Use the Archimedean Property to prove that $\inf\{\frac{1}{n} : n \in \mathbf{N}\} = 0$.

(3) (10 points) Let $A \subseteq \mathbf{R}$ be nonempty and bounded below, and define $B = \{b \in \mathbf{R} : b \text{ is a lower bound for } A\}$. Show that $\sup B = \inf A$.

(4) (10 points) Let S be the set consisting of *all sequences* of digits 0 and 1. Show that S is **not** countable.

(5) Let $\mathbf{Q}$ be the set of rational numbers and $\mathbf{I}$ be the set of irrational numbers.

(a) (3 points) Show that if $a, b \in \mathbf{Q}$ then $ab \in \mathbf{Q}$ and $a + b \in \mathbf{Q}$.

(b) (4 points) Show that if $a \in \mathbf{Q}$ and $t \in \mathbf{I}$, then $a + t \in \mathbf{I}$, and $at \in \mathbf{I}$ as long as $a \neq 0$.

(c) (3 points) Given any two real numbers $a < b$, show that there exists a number $t \in \mathbf{I}$ such that $a < t < b$.