Static and Dynamic Stiffness of the Milling Machine

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Introduction

- According to Zhang et al. [1]
  - 30% to 50% static stiffness of a machine tool depends on stiffness of joints,
  - The damping of a machine tool is derived from joints more than 90% and
  - More than 60% vibration problems that appear on a machine tool originate in joints

- There are two types of joints in the feed system:
  - Fixed joints (bolted joints) and
  - Rolling joints (ball screw assembly, rolling guide paired, and bearing jointed.)

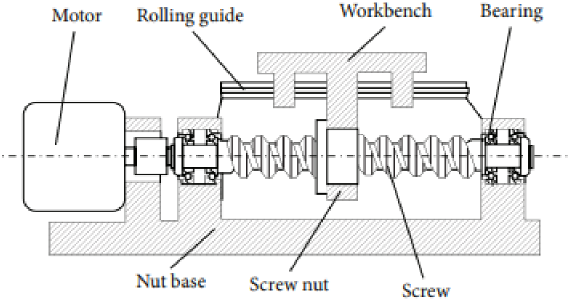

Figure: Feed system of a 3-axis CNC milling machine [2]

Static Stiffness

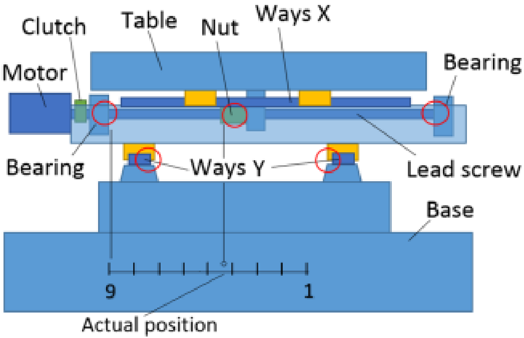

Figure: kinematic chain of the cross table with critical contact points marked with a circle. [3]

Dynamic Modeling of Feed System

- Wang et al. [2] proposed stiffness calculation methods

Modal orders	**1**	**2**	**3**	**4**
Measured results (Hz)	279.0	360.3	396.4	429.7
Theoretical results (Hz)	281.9	363.1	388.6	417.5
Relative error (%)	-1.03	-0.77	2.07	2.92

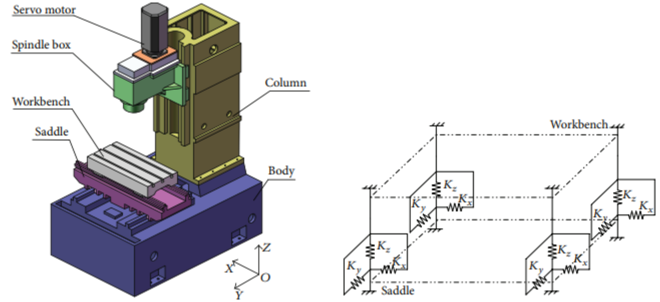

Figure: Structure (left) & dynamic modeling of the feed system (right) [2]

Stiffness of Rolling Guide Pair

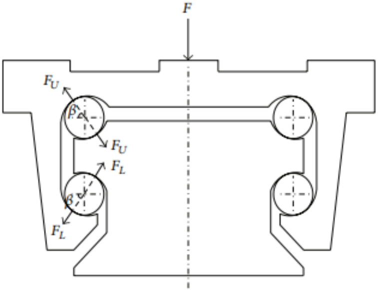

Figure: Force analysis of rolling guide joint [2]

- The pretightened force:
  - normal force (P_0),
  - initial deformation (δ_0) of each ball

Pretightened force with z number of balls:

$$F_0 = \sqrt{2}zP_0 sin\beta$$

Under external force (F):
Vertical (δ_n) & Tangential (δ_t) displacement:

$$\delta_n = (\delta_U - \delta_0)sin\beta = (\delta_0 - \delta_L)sin\beta$$

$$\delta_t = (\delta_U - \delta_0)cos\beta$$

Normal stiffness: $K_n = \frac{F}{\delta_n}$

Tangential stiffness: $K_t = \frac{zP_0 cos\beta}{2\delta_t}$

Equivalent Stiffness of Axial Feed Unit

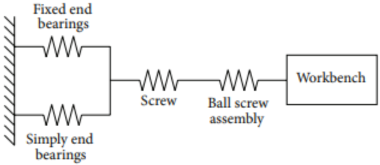

Figure: Dynamic model of ball screw feed unit [2]

The axial stiffness of the deep groove ball bearing used at the simple end of screw is so small (neglected).

$$\frac{1}{k_x} = \frac{1}{k_s} + \frac{1}{k_a} + \frac{1}{k_b}$$

k_x equivalent axial stiffness
k_s is screw axial stiffness;
k_a is ball screw assembly axial stiffness;
k_b is bearings axial stiffness

Screw Axial Stiffness:

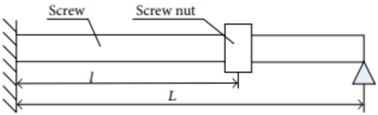

Figure: Schematic diagram of screw support mode [2]

$$k_s = \pi d^2 E/4l = \pi d^2 E/2L$$

d is minor diameter of the screw thread

Ball Screw Assembly Axial Stiffness

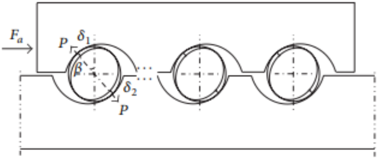

Figure: Force analysis of ball screw joint [2]

$$K_a = \frac{F_a}{\delta_a}$$

where:

δ_1 -deformation of the contact between ball and nut groove, and

δ_2 -deformation of the contact between ball and screw groove

$$\delta_a = (\delta_1 + \delta_2)/sin\beta cos\phi$$

$$\delta_i = \frac{K_i}{\pi a_i}\left(\frac{3P}{2}\left(\frac{1-u_1^2}{E_1} + \frac{1-u_2^2}{E_2}\right)\right)^{2/3}\left(\sum \rho_i\right)^{1/3}$$

$$\sum \rho_1 = \frac{2}{d_b} + \frac{2}{d_b} - \frac{1}{d_b f_1} - \frac{2cos\alpha cos\phi}{d + d_b cos\alpha}$$

$$\sum \rho_2 = \frac{2}{d_b} + \frac{2}{d_b} - \frac{1}{d_b f_2} - \frac{2cos\alpha cos\phi}{d + d_b cos\alpha}$$

f_i is a form factor, ratio of radius of curvatures

d_b s diameter of ball

Bearing Axial Stiffness

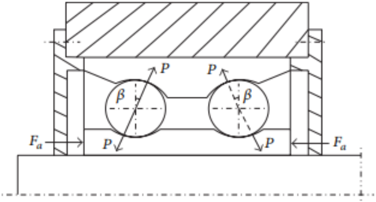

Figure: Force analysis of bearing joint [2]

- Bearing is comprised of inner ring, balls, outer ring
- Assumptions for Hertz contact
  - centrifugal force and gyroscopic moment induced by ball rotation are so small that can be ignored;
  - axial load is evenly distributed between all backing balls

$K_b = \frac{F_a}{\delta_a}$
where: $\delta_a = (\delta_1 + \delta_2)/sin\beta$

References I

[1] G. Zhang, Y. Huang, W. Shi, and W. Fu, "Predicting dynamic behaviours of a whole machine tool structure based on computer-aided engineering," *International Journal of Machine Tools and Manufacture*, vol. 43, no. 7, pp. 699–706, 2003.

[2] D. Wang, Y. Lu, T. Zhang, K. Wang, A. Rinoshika, *et al.*, "Effect of stiffness of rolling joints on the dynamic characteristic of ball screw feed systems in a milling machine," *Shock and Vibration*, vol. 2015, 2015.

[3] T. Stejskal, J. Svetlík, M. Dovica, P. Demeč, and J. Král', "Measurement of static stiffness after motion on a three-axis cnc milling table," *Applied Sciences*, vol. 8, no. 1, p. 15, 2017.

Thank You!