# A CFD CODE BUILDING

Chenhui Ge

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# 1 Introduction

Build some CFD code and test my skills. Code will be written in Fortran for I am more familiar with it. Might do another version in Java due to its better support for object-oriented. Will stat with some time and space schemes and solve some simple cases. Will build a incompressible solver for N-S in the end.

# 2 Time Scheme

## 2.1 Euler Scheme

Explicit Euler:

$$\frac{T_{n+1} - T_n}{\Delta t} = g(T_n) \tag{1}$$

Implicit Euler:

$$\frac{T_{n+1} - T_n}{\Delta t} = g(T_n, T_{n+1}) \tag{2}$$

### 2.1.1 Stability and Accuracy

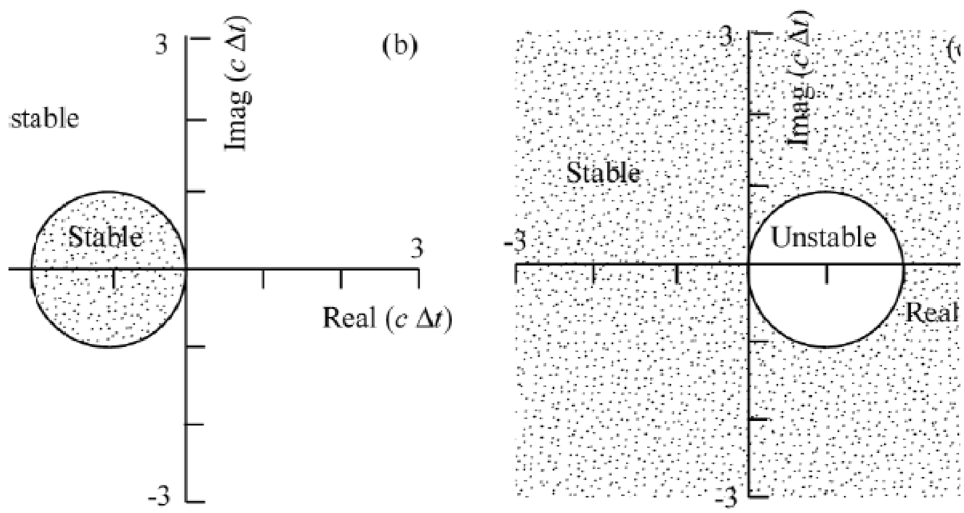


**Figure 3.1** Stability diagrams for the different schemes. (a) The actual stability region for the equation (3.2.19); (b) Region of stability and instability for the explicit Euler scheme; (c) Region of stability and instability for the implicit Euler scheme.indexEuler scheme!implicit!stability

## 2.2 AB Family

**Table 3.1** Multistep Adams–Bashforth schemes and their local truncation error.

| $n$ | *Name* | *Scheme* | *Local Truncation Error* |
|---|---|---|---|
| 0 | One-step AB scheme | $\xi_{k+1} = \xi_k + \Delta t\, g_k$ | $\frac{1}{2}(\Delta t)\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2}$ |
| 1 | Two-step AB scheme | $\xi_{k+1} = \xi_k + \Delta t\left[\frac{3}{2}g_k - \frac{1}{2}g_{k-1}\right]$ | $\frac{5}{12}(\Delta t)^2\frac{\mathrm{d}^3\xi}{\mathrm{d}t^3}$ |
| 2 | Three-step AB scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{12}\left[23g_k - 16g_{k-1} + 5g_{k-2}\right]$ | $\frac{3}{8}(\Delta t)^3\frac{\mathrm{d}^4\xi}{\mathrm{d}t^4}$ |
| 3 | Four-step AB scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{24}\left[55g_k - 59g_{k-1} + 37g_{k-2} - 9g_{k-3}\right]$ | $\frac{251}{720}(\Delta t)^4\frac{\mathrm{d}^5\xi}{\mathrm{d}t^5}$ |

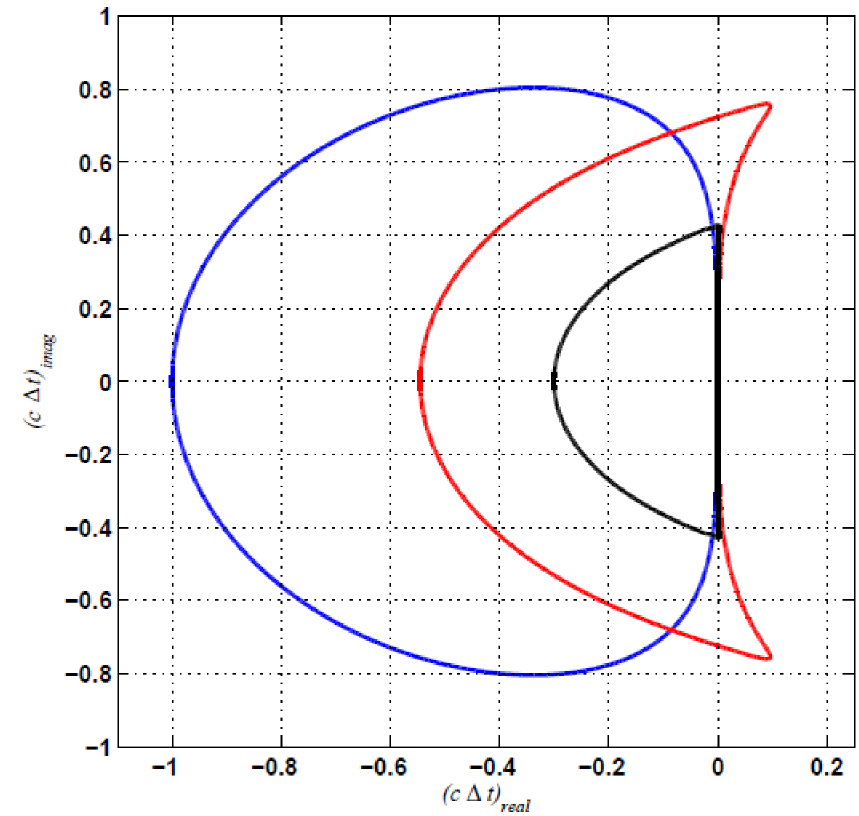


**Figure 3.5** Stability diagrams for the Adams–Bashforth (AB) family of schemes. Blue line: AB 2-step Scheme; Red line: AB 3-step Scheme; Black line: AB 4-step Scheme. Note that AB 1-step scheme is the same as the explicit Euler scheme and the stability region decreases with increasing order of the scheme. Also note that AB 2-step stability region does not include any portion of the imaginary axis and therefore will be unstable for a pure advection problem.

Table 3.2 Multistep Adams–Moulton schemes and their local truncation error.

| $n$ | *Name* | *Scheme* | *Local Truncation Error* |
|---|---|---|---|
| 0 | Crank–Nicholson scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{2}\left[g_{k+1} + g_k\right]$ | $-\frac{1}{12}(\Delta t)^2 \frac{\mathrm{d}^3\xi}{\mathrm{d}t^3}$ |
| 1 | Two-step AM scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{12}\left[5g_{k+1} + 8g_k - g_{k-1}\right]$ | $-\frac{1}{24}(\Delta t)^3 \frac{\mathrm{d}^4\xi}{\mathrm{d}t^4}$ |
| 2 | Three-step AM scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{24}\left[9g_{k+1} + 19g_k - 5g_{k-1} + g_{k-2}\right]$ | $-\frac{19}{720}(\Delta t)^4 \frac{\mathrm{d}^5\xi}{\mathrm{d}t^5}$ |
| 3 | Four-step AM scheme | $\xi_{k+1} = \xi_k + \frac{\Delta t}{720}\left[251g_{k+1} + 646g_k - 264g_{k-1} + 106g_{k-2} - 19g_{k-3}\right]$ | $-\frac{3}{160}(\Delta t)^5 \frac{\mathrm{d}^6\xi}{\mathrm{d}t^6}$ |

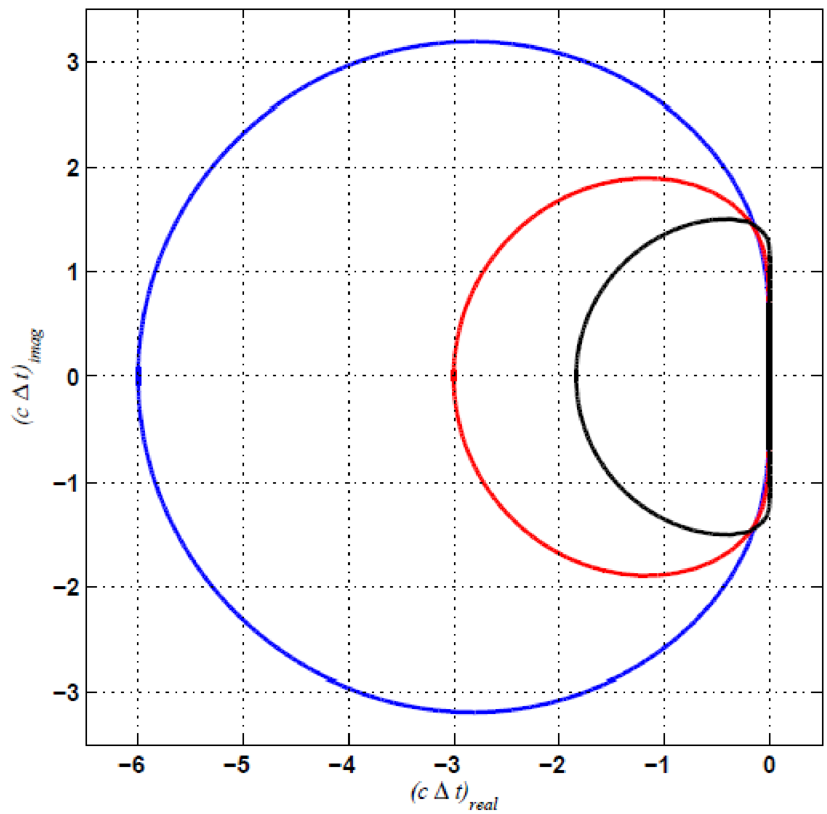


**Figure 3.6** Stability diagrams for the Adams–Moulton (AM) family of schemes. Blue line: AM 2-step Scheme; Red line: AB 3-step Scheme; Black line: AB 4-step Scheme. Note that AB 1-step scheme is commonly known as the Crank–Nicholson scheme and it is stable over the entire left half of the complex plane and the stability region decreases with increasing order of the AM scheme. Also note that AM 2-step and 3-step schemes' stability regions do not include any portion of the imaginary axis and therefore are unstable for a pure advection problem.