

CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

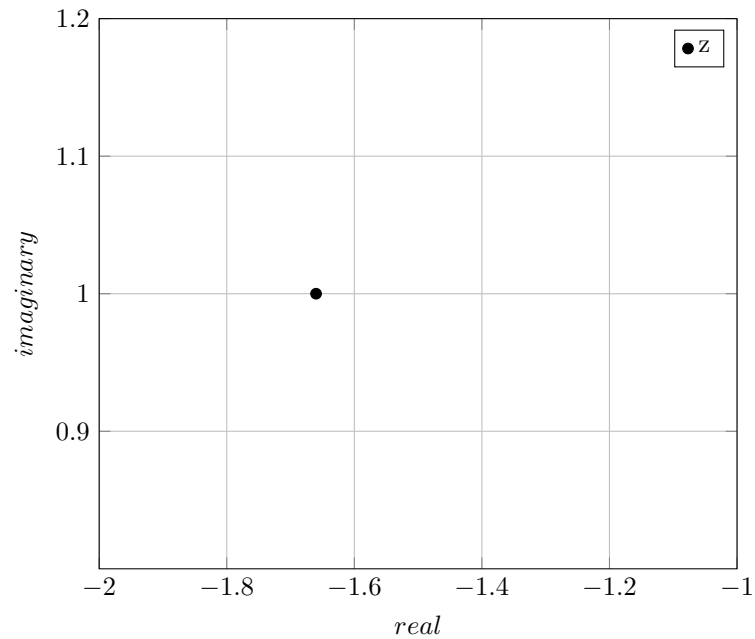
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1. (a)

$$\begin{aligned}z &= x + yj \implies \bar{z} = x - yj \\2z + 5 &= j - \bar{z} \\2(x + yj) + 5 &= j - (x - yj) \\2x + 5 + 2yj &= (1 + y)j - x \\y = 1, x &= \frac{-5}{3} \\z &= \frac{-5}{3} + j \\|z|^2 &= \frac{25}{9} + 1 = \frac{34}{9}\end{aligned}$$



(b)

$$\begin{aligned}z &= re^{j\theta} \implies z^5 = r^5 e^{j5\theta} \\32j &= 32e^{j\pi/2} \\32e^{j\pi/2} &= r^5 e^{j5\theta} \implies r = 2, \theta = \pi/10 \\z &= 2e^{j\pi/10}\end{aligned}$$

(c)

$$\begin{aligned}
z &= \frac{(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{j-1} \\
&= \frac{(j+1)(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{(j+1)(j-1)} \\
&= \frac{(j+1)^2(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(j^2 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(-1 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\
&= \frac{2j(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\
&= -j(\frac{1}{2} + \frac{\sqrt{3}}{2}) \\
z &= r\cos\theta + r\sin\theta j \\
j(-\frac{1}{2} - \frac{\sqrt{3}}{2}) &= r\cos\theta + r\sin\theta j \\
r\cos\theta &= 0 \\
r\sin\theta &= -\frac{1}{2} - \frac{\sqrt{3}}{2} \\
\cos\theta &= 0 \\
\sin\theta &= -1 \\
r &= \frac{1}{2} + \frac{\sqrt{3}}{2} \\
\theta &= -\pi/2
\end{aligned}$$

(d)

$$\begin{aligned}
z &= je^{-j\pi/2} \\
&= e^{j\pi/2}e^{-j\pi/2} \\
&= e^0 = 1
\end{aligned}$$

2. drawing

3. (a) drawing

(b)

$$\begin{aligned}
x[n] &= -\delta[n-1] + 2\delta[n-2] - 4\delta[n-4] + 3\delta[n-7] \\
x[-n] &= -\delta[-n-1] + 2\delta[-n-2] - 4\delta[-n-4] + 3\delta[-n-7] \\
x[2n-1] &= -\delta[2n-2] + 2\delta[2n-3] - 4\delta[2n-5] + 3\delta[2n-8] \\
x[-n] + x[2n-1] &= -\delta[-n-1] + 2\delta[-n-2] - 4\delta[-n-4] + 3\delta[-n-7] - \delta[2n-2] + 2\delta[2n-3] \\
&\quad - 4\delta[2n-5] + 3\delta[2n-8]
\end{aligned}$$

4. (a) $2\pi/3$

(b)

$$\begin{aligned}
x[n] &= x[n + t_0] \\
\cos\left[\frac{13\pi}{10}n\right] + \sin\left[\frac{7\pi}{10}n\right] &= \cos\left[\frac{13\pi}{10}(n + t_0)\right] + \sin\left[\frac{7\pi}{10}(n + t_0)\right] \\
\sin\left[\frac{\pi}{2} - \frac{13\pi}{10}n\right] + \sin\left[\frac{7\pi}{10}n\right] &= \sin\left[\frac{\pi}{2} - \frac{13\pi}{10}(n + t_0)\right] + \sin\left[\frac{7\pi}{10}(n + t_0)\right] \\
\sin\left[\frac{5\pi}{10} - \frac{13\pi}{10}n\right] + \sin\left[\frac{7\pi}{10}n\right] &= \sin\left[\frac{5\pi}{10} - \frac{13\pi}{10}(n + t_0)\right] + \sin\left[\frac{7\pi}{10}(n + t_0)\right] \\
\sin\left[\frac{\pi}{10}(13n - 5)\right] + \sin\left[\frac{7\pi}{10}n\right] &= \sin\left[\frac{\pi}{10}(13n + 13t_0 - 5)\right] + \sin\left[\frac{7\pi}{10}(n + t_0)\right] \\
2\sin\left(\frac{\frac{\pi}{10}(13n - 5) + \frac{7\pi}{10}n}{2}\right)\cos\left(\frac{\frac{\pi}{10}(13n - 5) - \frac{7\pi}{10}n}{2}\right) \\
&= 2\sin\left(\frac{\frac{\pi}{10}(13n + 13t_0 - 5) + \frac{7\pi}{10}(n + t_0)}{2}\right)\cos\left(\frac{\frac{\pi}{10}(13n + 13t_0 - 5) - \frac{7\pi}{10}(n + t_0)}{2}\right) \\
&\quad \sin\left(\frac{\pi}{20}(20n - 5)\right)\cos\left(\frac{\pi}{20}(6n - 5)\right) = \sin\left(\frac{\pi}{20}(20n + 20t_0 - 5)\right)\cos\left(\frac{\pi}{20}(6n + 6t_0 - 5)\right) \\
&\quad \sin\left(n\pi - \frac{\pi}{4}\right)\cos\left(\frac{3n\pi}{10} - \frac{\pi}{4}\right) = \sin\left(n\pi + t_0\pi - \frac{\pi}{4}\right)\cos\left(\frac{3n\pi + 3t_0\pi}{10} - \frac{\pi}{4}\right)
\end{aligned}$$

The smallest integer t_0 that satisfies the equation above is $t_0 = 20$.

(c) The signal is not periodic.

5. (a) $x(t) = u[t - 1] - 3u[t - 3] + u[t - 4]$
(b) $\frac{dx(t)}{dt} = \delta(t - 1) - 3\delta(t - 3) + \delta(t - 4)$ draw?
6. (a)
(b)
7. (a)
(b)