CENG 384 - Signals and Systems for Computer Engineers
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Homework 3

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1.

$$\begin{aligned}
\int_{-\infty}^{t} x(s)ds &= \int_{-\infty}^{t} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 s} ds \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0}\Big|_{-\infty}^{t}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0} - a_k \cdot \frac{e^{jkw_0(-\infty)}}{jkw_0}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0} - a_k \cdot \frac{0}{jkw_0}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0})
\end{aligned}$$

This equation is in the synthesis equation form where $a_k \frac{1}{jkw_0}$ is the Fourier series coefficients of the integrated signal.

Since w_0 is the frequency of the signal, $w_0 = \frac{2\pi}{T}$ where T is the period of the signal.

Substituting w_0 in the equation above, we prove the integration property of the Fourier series.

2. (a) $x(t)x(t) \leftrightarrow a_k * a_k$ (Multiplication Property)

   (b) $\mathcal{E}v\{x(t)\} \leftrightarrow b_k$ (Even Property)

$$b_k = \begin{cases} a_k & k \geq 0 \\ a_{-k} & k < 0 \end{cases}$$

   (c) $x(t+t_0) + x(t-t_0) \leftrightarrow a_k e^{jkw_0 t_0} + a_{-k} e^{-jkw_0 t_0}$ (Shifting and Linearity Properties)

3.

$$x(t) = \begin{cases} 1 & x \in (0,1) \\ 0 & x \in (1,2) \\ -1 & x \in (2,3) \\ 0 & x \in (3,4) \\ \text{Periodic} & x \notin (0,4) \end{cases}$$

$$
\begin{aligned}
a_k &= \frac{1}{T}\int_0^T x(t)e^{-jkw_0t}dt \\
&= \frac{1}{4}(\int_0^1 e^{-jkw_0t}dt + \int_1^2 0dt + \int_2^3 -e^{-jkw_0t}dt + \int_3^4 0dt) \\
&= \frac{1}{4}(\frac{e^{-jkw_0t}}{-jkw_0}\Big|_0^1 + \frac{e^{-jkw_0t}}{-jkw_0}\Big|_2^3) \\
&= \frac{1}{4}(\frac{e^{-jkw_0}}{-jkw_0} - \frac{1}{-jkw_0} + \frac{e^{-3jkw_0}}{-jkw_0} - \frac{e^{-2jkw_0}}{-jkw_0}) \\
&= \frac{1}{-4jkw_0}(e^{-jkw_0} - 1 + e^{-3jkw_0} - e^{-2jkw_0})
\end{aligned}
$$

Substitute $w_0 = \frac{2\pi}{T} = \frac{2\pi}{4}$

$$
\begin{aligned}
a_k &= \frac{1}{-4jk\frac{2\pi}{4}}(e^{-jk\frac{2\pi}{4}} - 1 + e^{-3jk\frac{2\pi}{4}} - e^{-2jk\frac{2\pi}{4}}) \\
&= \frac{1}{-2jk\pi}(e^{-jk\frac{\pi}{2}} - 1 + e^{-3jk\frac{\pi}{2}} - e^{-jk\pi}) \\
&= \frac{1}{-2jk\pi}(cos(-k\frac{\pi}{2}) + jsin(-k\frac{\pi}{2}) - 1 + cos(-3k\frac{\pi}{2}) + jsin(-3k\frac{\pi}{2}) - cos(-k\pi) - jsin(-k\pi)) \\
&= \frac{1}{-2jk\pi}(2cos(k\frac{\pi}{2}) - 1 - cos(k\pi)) \\
&= \frac{1}{2jk\pi}(1 - 2cos(k\frac{\pi}{2}) + cos(k\pi))
\end{aligned}
$$

4. (a)
   (b)
   (c)
   (d)
5. (a)
   (b)
   (c)
   (d)
6. (a)
   (b)
7. (a)
   (b)
8.