CENG 384 - Signals and Systems for Computer Engineers
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Homework 3

Geçit, Emre
e2521581@ceng.metu.edu.tr

Yancı, Baran
exxxxxxx@ceng.metu.edu.tr

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1.

$$\begin{aligned}
\int_{-\infty}^{t} x(s)ds &= \int_{-\infty}^{t} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 s} ds \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0}\Big|_{-\infty}^{t}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0} - a_k \cdot \frac{e^{jkw_0(-\infty)}}{jkw_0}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0} - a_k \cdot \frac{0}{jkw_0}) \\
&= \sum_{k=-\infty}^{\infty} (a_k \cdot \frac{e^{jkw_0 t}}{jkw_0})
\end{aligned}$$

This equation is in the synthesis equation form where $a_k \frac{1}{jkw_0}$ is the Fourier series coefficients of the integrated signal.

Since w_0 is the frequency of the signal, $w_0 = \frac{2\pi}{T}$ where T is the period of the signal.

Substituting w_0 in the equation above, we prove the integration property of the Fourier series.

2. (a) $x(t)x(t) \leftrightarrow a_k * a_k$ (Multiplication Property)
   (b) $\mathcal{E}v\{x(t)\} \leftrightarrow b_k$ (Even Property)

$$b_k = \begin{cases} a_k & k \geq 0 \\ a_{-k} & k < 0 \end{cases}$$

   (c) $x(t+t_0) + x(t-t_0) \leftrightarrow a_k e^{jkw_0 t_0} + a_{-k} e^{-jkw_0 t_0}$ (Shifting and Linearity Properties)

3.

4. (a)
   (b)
   (c)
   (d)

5. (a)
   (b)
   (c)
   (d)

6. (a)
   (b)

7. (a)
   (b)
8.