

CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

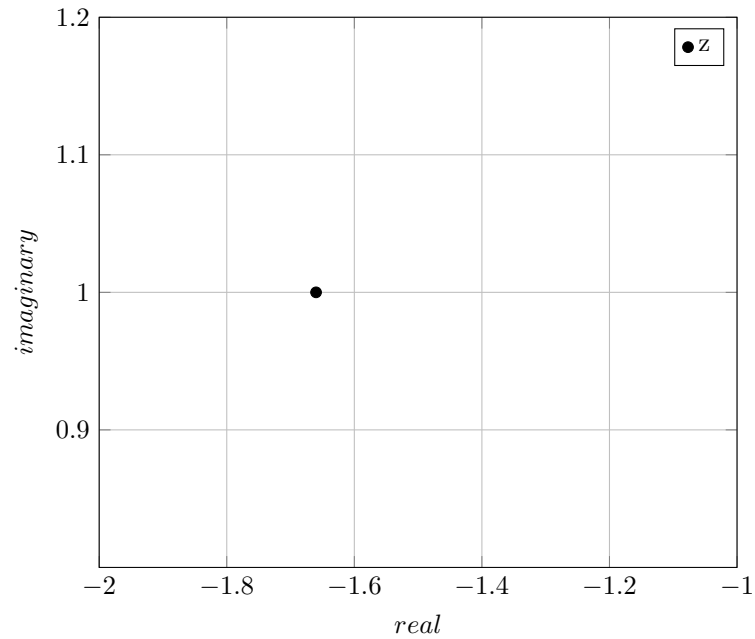
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1. (a)

$$\begin{aligned}z &= x + yj \implies \bar{z} = x - yj \\2z + 5 &= j - \bar{z} \\2(x + yj) + 5 &= j - (x - yj) \\2x + 5 + 2yj &= (1 + y)j - x \\y &= 1, x = \frac{-5}{3} \\z &= \frac{-5}{3} + j \\|z|^2 &= \frac{25}{9} + 1 = \frac{34}{9}\end{aligned}$$



(b)

$$\begin{aligned}z &= re^{j\theta} \implies z^5 = r^5 e^{j5\theta} \\32j &= 32e^{j\pi/2} \\32e^{j\pi/2} &= r^5 e^{j5\theta} \implies r = 2, \theta = \pi/10 \\z &= 2e^{j\pi/10}\end{aligned}$$

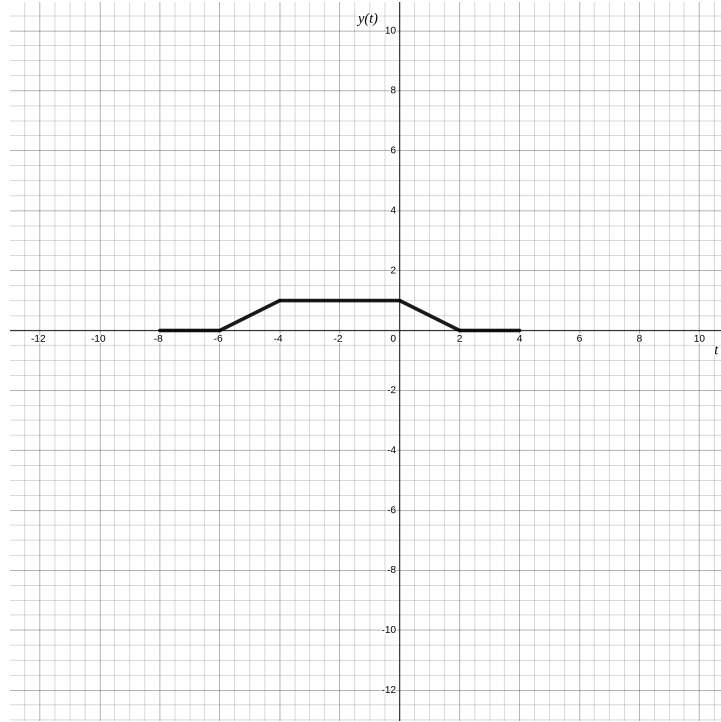
(c)

$$\begin{aligned}z &= \frac{(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{j-1} \\&= \frac{(j+1)(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{(j+1)(j-1)} \\&= \frac{(j+1)^2(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\&= \frac{(j^2 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\&= \frac{(-1 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\&= \frac{2j(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2} \\&= -j(\frac{1}{2} + \frac{\sqrt{3}}{2}) \\z &= r \cos \theta + r \sin \theta j \\j(-\frac{1}{2} - \frac{\sqrt{3}}{2}) &= r \cos \theta + r \sin \theta j \\r \cos \theta &= 0 \\r \sin \theta &= -\frac{1}{2} - \frac{\sqrt{3}}{2} \\\cos \theta &= 0 \\\sin \theta &= -1 \\r &= \frac{1}{2} + \frac{\sqrt{3}}{2} \\\theta &= -\pi/2\end{aligned}$$

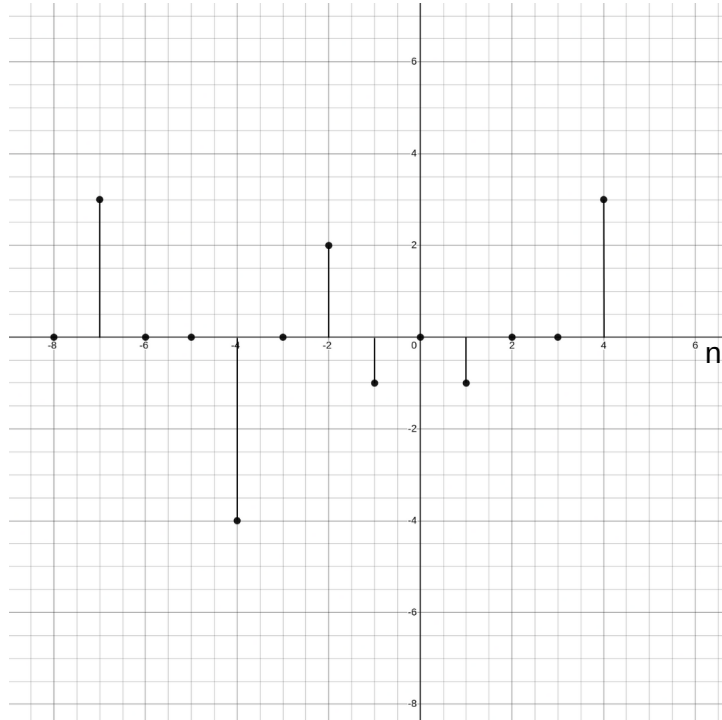
(d)

$$\begin{aligned}z &= je^{-j\pi/2} \\&= e^{j\pi/2}e^{-j\pi/2} \\&= e^0 = 1\end{aligned}$$

2. The graph of the function is given below.



3. (a) The graph of the function $x[-n] + x[2n - 1]$ is given below.



(b)

$$\begin{aligned}
 x[n] &= -\delta[n-1] + 2\delta[n-2] - 4\delta[n-4] + 3\delta[n-7] \\
 x[-n] &= -\delta[-n-1] + 2\delta[-n-2] - 4\delta[-n-4] + 3\delta[-n-7] \\
 &= -\delta[n+1] + 2\delta[n+2] - 4\delta[n+4] + 3\delta[n+7] \\
 x[2n-1] &= -\delta[2n-2] + 2\delta[2n-3] - 4\delta[2n-5] + 3\delta[2n-8] \\
 &= -\delta[n-1] + 3\delta[n-4] \\
 x[-n] + x[2n-1] &= -\delta[n+1] + 2\delta[n+2] - 4\delta[n+4] + 3\delta[n+7] - \delta[n-1] + 3\delta[n-4]
 \end{aligned}$$

4. (a)

$$\begin{aligned} \text{period}(5 \sin(3t - \frac{\pi}{4})) &= \text{period}(\sin(3t)) && \text{Scaling the amplitude does not affect the period, neither does shifting.} \\ &= \text{period}(\sin(t))/3 && \text{Time scaling inversely affects the period.} \\ &= \frac{2\pi}{3} && \text{Period of sin is } 2\pi. \end{aligned}$$

(b) $\cos[\frac{13\pi}{10}n]$ is periodic with period $\frac{2\pi \cdot 10}{13\pi} = \frac{20}{13}$
 $\sin[\frac{7\pi}{10}n]$ is periodic with period $\frac{2\pi \cdot 10}{7\pi} = \frac{20}{7}$

Least common multiple of $\frac{20}{13}$ and $\frac{20}{7}$ is 20, which is also an integer, so it satisfies the discrete-time periodicity condition.

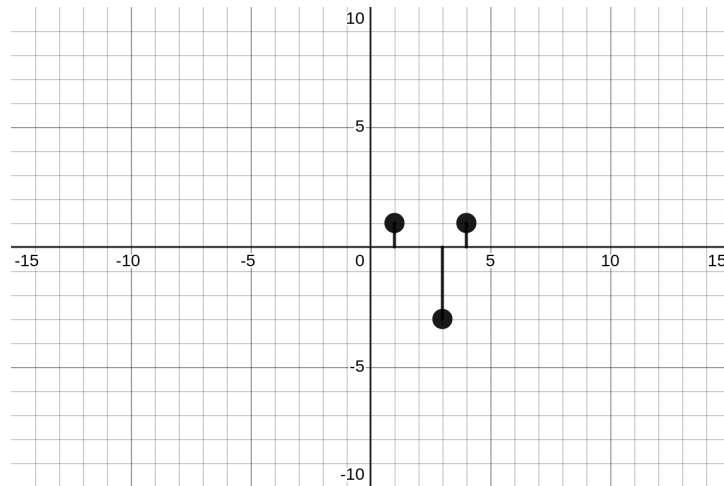
(c) In order $\frac{1}{2} \cos[7n - 5]$ to be periodic, its continuous-time counterpart must have a rational period. However, the signal $\frac{1}{2} \cos(7n - 5)$ has a fundamental period of $2\pi/7$.

There is no integer t_0 such that,

$$\frac{1}{2} \cos[7n - 5] = \frac{1}{2} \cos[7(n + t_0) - 5]$$

5. (a) $x(t) = u[t - 1] - 3u[t - 3] + u[t - 4]$

(b) $\frac{dx(t)}{dt} = \delta(t - 1) - 3\delta(t - 3) + \delta(t - 4)$ The graph of $\frac{dx(t)}{dt}$ is given below.



6. (a)

$$y(x) = tx(2x + 3)$$

Memory: The system has memory, as the output depends on the future values of the input. For example, for $t = 3$, the output value depends on the future value of the input, $y(3) = 3x(9)$.

Stability: The system is **not** stable, as there is no finite number b' such that $|y(t)| \leq b'$ for all t .

Causality: The system is **not** causal, as the output depends on the future values of the input.

Linearity: The system is linear, because the superposition principle holds.

$$x_1 \implies y_1(t) = tx_1(2t + 3)$$

$$x_2 \implies y_2(t) = tx_2(2t + 3)$$

$$x_3 = a_1x_1 + a_2x_2$$

assumption

$$y_3 = t(a_1x_1 + a_2x_2) = a_1y_1 + a_2y_2$$

Invertibility: The system is invertible for $t \neq 0$.

$$\begin{aligned}y(t) &= tx(2t + 3) \\ \frac{1}{t}y(t) &= x(2t + 3) \\ \frac{1}{t}y\left(\frac{t-3}{2}\right) &= x(t)\end{aligned}$$

Time invariance: The system is **not** time invariant.

$$\begin{aligned}y(t) &= tx(2t + 3) \\ y(t - t_0) &= (t - t_0)x(2(t - t_0) + 3) \neq tx(2(t - t_0) + 3)\end{aligned}$$

(b)

$$y[n] = \sum_{k=1}^{\infty} x[n - k]$$

Memory: The system has memory, as the output depends on the past values of the input. For example, for $n = 6$, the output value depends on the past values of the input, $y[6] = x[5] + x[4] + x[3] + \dots$.

Stability: The system is **not** stable, as there is no finite number b' such that $|y(t)| \leq b'$ for all t . For example, if we take $x(n)$ as the unit step function, the signal $\sum_{k=1}^{\infty} u[n - k]$ is unbounded for $n \Rightarrow \infty$.

Causality: The system is causal, as the output depends on the past values of the input.

Linearity: The system is linear, because the superposition principle holds.

$$\begin{aligned}x_1 &\Rightarrow y_1(n) = \sum_{k=1}^{\infty} x_1[n - k] \\ x_2 &\Rightarrow y_2(n) = \sum_{k=1}^{\infty} x_2[n - k] \\ a_1y_1 + a_2y_2 &= \sum_{k=1}^{\infty} (a_1x_1[n - k] + a_2x_2[n - k]) \\ \sum_{k=1}^{\infty} x_3[n - k] &= a_1 \sum_{k=1}^{\infty} x_1[n - k] + a_2 \sum_{k=1}^{\infty} x_2[n - k]\end{aligned}$$

Invertibility: The system is invertible, the fact that the system has the invertibility property can be seen by taking the difference of two consequent values of $y[n]$.

$$\begin{aligned}y[n + 1] - y[n] &= \sum_{k=1}^{\infty} x[n + 1 - k] - \sum_{k=1}^{\infty} x[n - k] \\ y[n + 1] - y[n] &= x[n]\end{aligned}$$

Time invariance: The system is time invariant, because a delay in the input signal $x[n]$ by x , causes a corresponding delay for the output signal $y[n]$ by x as well.

$$\begin{aligned}y[n] &= \sum_{k=1}^{\infty} x[n - k] \\ y[n - n_0] &= \sum_{k=1}^{\infty} x[(n - n_0) - k]\end{aligned}$$

7. (a)

```
def decompose(signal_name):
    """Read the CSV file with the signal name, decompose the signal into even and odd components,
    and save the results as PNG files."""

    with open(signal_name + ".csv", "r", encoding="ascii") as file:
        data = [float(item) for item in file.read().split(",")]
        start = int(data[0])
        signal = data[1:]
        end = start + len(signal) - 1

    pyplot.title("Original Signal")
    pyplot.plot(range(start, end + 1), signal)
    pyplot.savefig(IMAGES_PATH + signal_name + "_original.png")
    pyplot.clf()

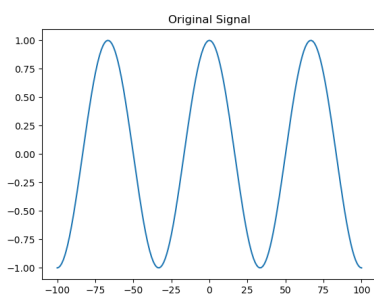
    if abs(start) > end:
        signal = signal + [0] * (abs(start) - end)
        end = -start
    else:
        signal = [0] * (end - abs(start)) + signal
        start = -end

    even = [(x + y) / 2 for x, y in zip(signal, signal[::-1])]
    odd = [(x - y) / 2 for x, y in zip(signal, signal[::-1])]

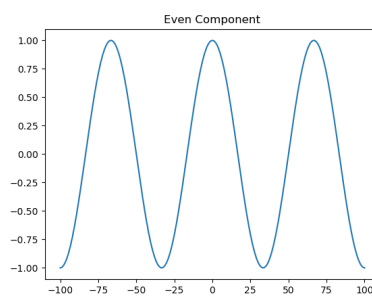
    pyplot.title("Even Component")
    pyplot.plot(range(start, end + 1), even)
    pyplot.savefig(IMAGES_PATH + signal_name + "_even.png")
    pyplot.clf()

    pyplot.title("Odd Component")
    pyplot.plot(range(start, end + 1), odd)
    pyplot.savefig(IMAGES_PATH + signal_name + "_odd.png")
    pyplot.clf()
```

(a) Original Signal



(b) Even Component



(c) Odd Component

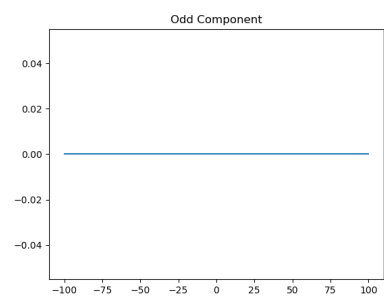


Figure 1: Sinusoidal Signal Decomposition

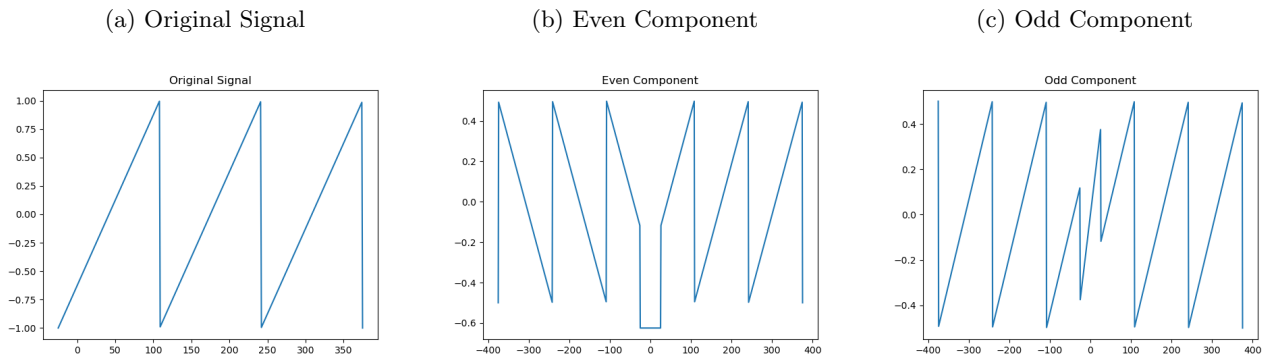


Figure 2: Shifted Sawtooth Signal Decomposition

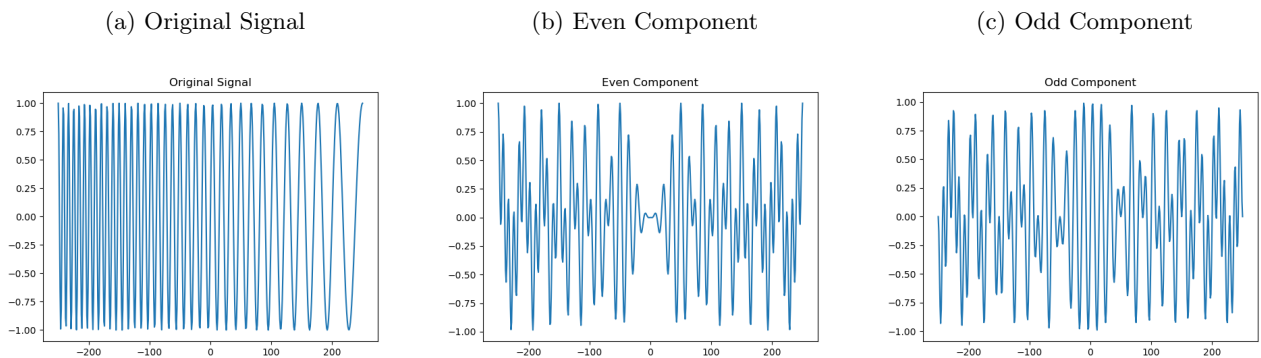


Figure 3: Chirp Signal Decomposition

(b)

```
def shift_n_scale(signal_name):
    """
    Read the CSV file with the signal name, shift and scale the signal,
    and save the results as PNG files.

    This functions reads a signal  $x[n]$ , and produces  $x[a*n + b]$  for  $a$  and  $b$ .
    """

    with open(signal_name + ".csv", "r", encoding="ascii") as file:
        data = [float(item) for item in file.read().split(",")]
    start = int(data[0])
    a = int(data[1])
    b = int(data[2])
    signal = data[3:]
    end = start + len(signal) - 1

    new_start = (start - b) // a
    new_end = (end - b) // a
    pyplot.xlim(new_start, new_end)

    pyplot.plot(range(start, end + 1), signal, linewidth=1)

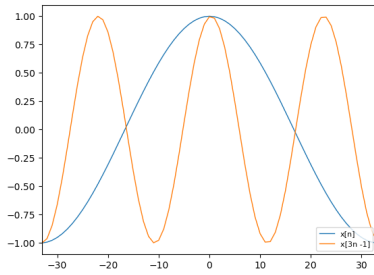
    if new_start > new_end:
        domain = range(new_start, new_end, -1)
    else:
        domain = range(new_start, new_end + 1)
    pyplot.plot(
        domain,
        [signal[a*i+b-start] for i in domain],
        linewidth=1,
```

```

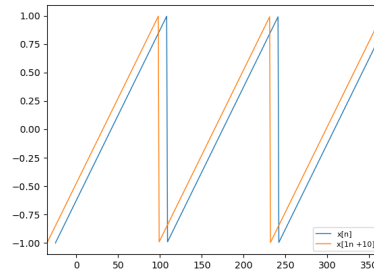
)
pyplot.legend(
    ["x[n]", "x[" + str(a) + "n " + ("+" if b >= 0 else "") + str(b) + "]" ],
    loc="lower right",
    fontsize=8,
)
pyplot.savefig(IMAGES_PATH + signal_name)
pyplot.clf()

```

(a) Sinusoidal Signal



(b) Shifted Sawtooth Signal



(c) Chirp Signal

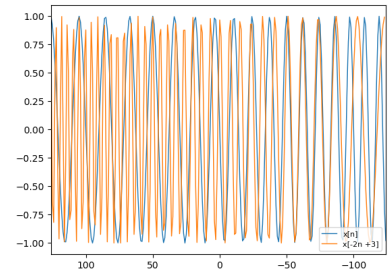


Figure 4: Shift and Scale