CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 4

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1. (a)

$$\begin{aligned} H(j\omega) &= \frac{j\omega - 1}{j\omega + 1} \\ \frac{Y(j\omega)}{X(j\omega)} &= \frac{j\omega - 1}{j\omega + 1} \\ Y(j\omega)(j\omega + 1) &= X(j\omega)(j\omega - 1) \\ y'(t) + y(t) &= x'(t) - x(t) \end{aligned}$$

(b)

$$\begin{aligned} H(j\omega) &= \frac{j\omega - 1}{j\omega + 1} \\ h(t) &= \mathcal{F}^{-1}\{H(j\omega)\} \\ &= \mathcal{F}^{-1}\{\frac{j\omega - 1}{j\omega + 1}\} \\ &= \mathcal{F}^{-1}\{\frac{j\omega + 1 - 2}{j\omega + 1}\} \\ &= \mathcal{F}^{-1}\{\frac{j\omega + 1}{j\omega + 1}\} - \mathcal{F}^{-1}\{\frac{2}{j\omega + 1}\} \\ &= \mathcal{F}^{-1}\{1\} - 2\mathcal{F}^{-1}\{\frac{1}{j\omega + 1}\} \\ &= \delta(t) - 2e^{-t}u(t) \end{aligned}$$

(c)

$$
\begin{aligned}
y'(t) + y(t) &= x'(t) - x(t) \\
y'(t) + y(t) &= -2e^{-2t}u(t) - e^{-2t}u(t) \\
y'(t) + y(t) &= -3e^{-2t}u(t) \\
y_p(t) &= Ae^{-2t} \\
y_p'(t) &= -2Ae^{-2t} \\
-2Ae^{-2t} + Ae^{-2t} &= -3e^{-2t}u(t) \\
A &= 3 \\
y_p(t) &= 3e^{-2t} \\
y_h(t) &= c_1e^{-t}u(t) \\
y(t) &= y_p(t) + y_h(t) \\
&= 3e^{-2t} + c_1e^{-t}u(t) \\
y(0) &= 0 \\
0 &= 3e^{-2(0)} + c_1e^{-0}u(0) \\
0 &= 3 + c_1 \\
c_1 &= -3 \\
y(t) &= 3e^{-2t} - 3e^{-t}u(t)
\end{aligned}
$$

(d)

2. (a)

$$
\begin{aligned}
y[n+1] - \frac{1}{2}y[n] &= x[n+1] \\
e^{j\omega}Y(e^{j\omega}) - \frac{1}{2}Y(e^{j\omega}) &= e^{j\omega}X(e^{j\omega}) \\
H(e^{j\omega}) &= \frac{Y(e^{j\omega})}{X(e^{j\omega})} \\
H(e^{j\omega}) &= \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H(e^{j\omega}) &= \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}} \\
h[n] &= \mathcal{F}^{-1}\{H(e^{j\omega})\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega} - \frac{1}{2} + \frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega} - \frac{1}{2}}{e^{j\omega} - \frac{1}{2}} + \frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{1 + \frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{1\} + \mathcal{F}^{-1}\{\frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \delta[n] + \frac{1}{2}\mathcal{F}^{-1}\{\frac{1}{e^{j\omega} - \frac{1}{2}}\} \\
&= \delta[n] + \frac{1}{2}e^{\frac{1}{2}n}u[n]
\end{aligned}
$$

(c)

$$
\begin{aligned}
y[n+1] - \frac{1}{2}y[n] &= x[n+1] \\
y[n+1] - \frac{1}{2}y[n] &= \left(\frac{3}{4}\right)^{n+1} u[n+1] \\
y_p[n] &= A\left(\frac{3}{4}\right)^{n} \\
y_p[n+1] &= A\left(\frac{3}{4}\right)^{n+1} \\
A\left(\frac{3}{4}\right)^{n+1} - \frac{1}{2}A\left(\frac{3}{4}\right)^{n} &= \left(\frac{3}{4}\right)^{n+1} u[n+1] \\
\frac{3}{4}A\left(\frac{3}{4}\right)^{n} - \frac{1}{2}A\left(\frac{3}{4}\right)^{n} &= \frac{3}{4}\left(\frac{3}{4}\right)^{n} u[n+1] \\
\frac{3}{4}A - \frac{1}{2}A &= \frac{3}{4}u[n+1] \\
A &= 3 \\
y_p[n] &= 3\left(\frac{3}{4}\right)^{n} \\
y_h[n] &= c_1\left(\frac{1}{2}\right)^{n} u[n] \\
y[n] &= y_p[n] + y_h[n] \\
&= 3\left(\frac{3}{4}\right)^{n} + c_1\left(\frac{1}{2}\right)^{n} u[n] \\
y[0] &= 0 \\
0 &= 3\left(\frac{3}{4}\right)^{0} + c_1\left(\frac{1}{2}\right)^{0} u[0] \\
0 &= 3 + c_1 \\
c_1 &= -3 \\
y[n] &= 3\left(\frac{3}{4}\right)^{n} - 3\left(\frac{1}{2}\right)^{n} u[n]
\end{aligned}
$$

3. (a)
   (b)
   (c)
4. (a)
   (b)
   (c)
5.