CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

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1. (a)

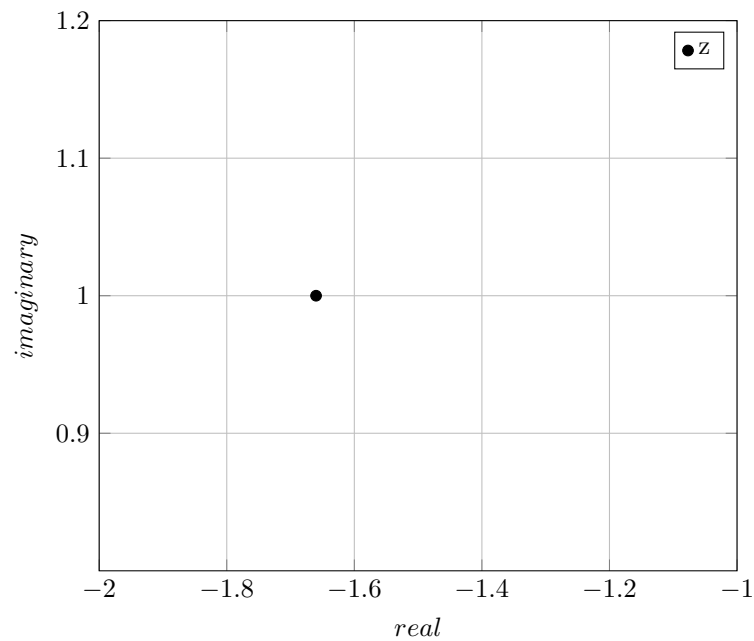$$z = x + yj \implies \bar{z} = x - yj$$
$$2z + 5 = j - \bar{z}$$
$$2(x + yj) + 5 = j - (x - yj)$$
$$2x + 5 + 2yj = (1 + y)j - x$$
$$y = 1, x = \frac{-5}{3}$$
$$z = \frac{-5}{3} + j$$
$$|z|^2 = \frac{25}{9} + 1 = \frac{34}{9}$$

(b)

$$z = re^{j\theta} \implies z^5 = r^5 e^{j5\theta}$$
$$32j = 32e^{j\pi/2}$$
$$32e^{j\pi/2} = r^5 e^{j5\theta} \implies r = 2, \theta = \pi/10$$
$$z = 2e^{j\pi/10}$$

(c)

$$z = \frac{(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{j-1}$$

$$= \frac{(j+1)(1+j)(\frac{1}{2} + \frac{\sqrt{3}}{2})j}{(j+1)(j-1)}$$

$$= \frac{(j+1)^2(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2}$$

$$= \frac{(j^2 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2}$$

$$= \frac{(-1 + 2j + 1)(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2}$$

$$= \frac{2j(\frac{1}{2} + \frac{\sqrt{3}}{2})}{-2}$$

$$= -j(\frac{1}{2} + \frac{\sqrt{3}}{2})$$

$$z = r cos\theta + r sin\theta j$$

$$j(-\frac{1}{2} - \frac{\sqrt{3}}{2}) = r cos\theta + r sin\theta j$$

$$r cos\theta = 0$$

$$r sin\theta = -\frac{1}{2} - \frac{\sqrt{3}}{2}$$
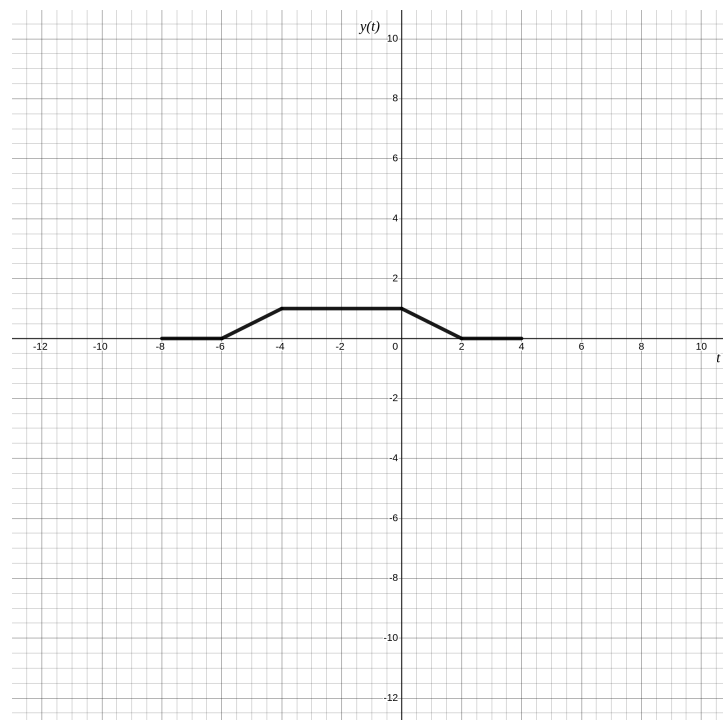
$$cos\theta = 0$$

$$sin\theta = -1$$

$$r = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$\theta = -\pi/2$$
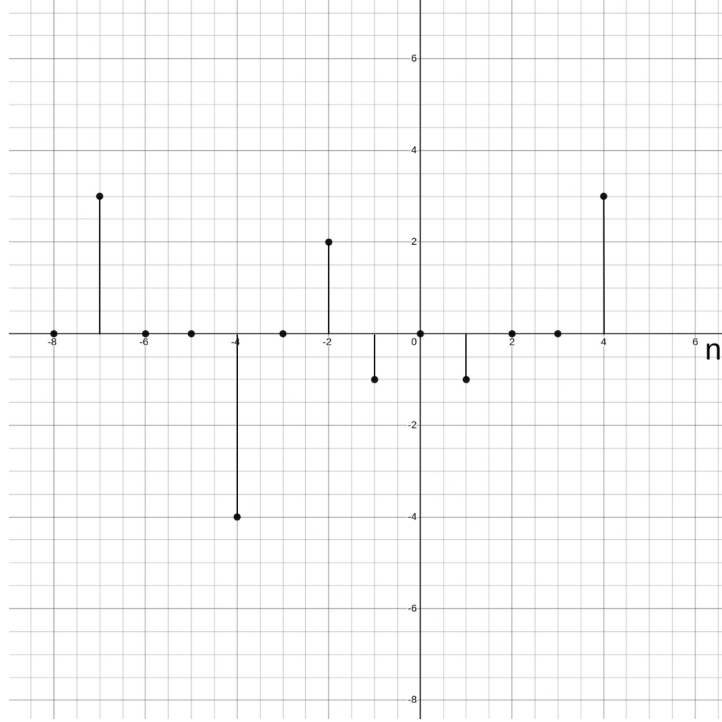
(d)

$$z = je^{-j\pi/2}$$

$$= e^{j\pi/2}e^{-j\pi/2}$$

$$= e^0 = 1$$

2. The graph of the function is given below.

2

3. (a) The graph of the function $x[-n] + x[2n - 1]$ is given below.

(b)

$$x[n] = -\delta[n - 1] + 2\delta[n - 2] + -4\delta[n - 4] + 3\delta[n - 7]$$
$$x[-n] = -\delta[-n - 1] + 2\delta[-n - 2] + -4\delta[-n - 4] + 3\delta[-n - 7]$$
$$x[2n - 1] = -\delta[2n - 2] + 2\delta[2n - 3] + -4\delta[2n - 5] + 3\delta[2n - 8]$$
$$x[-n] + x[2n - 1] = -\delta[-n - 1] + 2\delta[-n - 2] - 4\delta[-n - 4] + 3\delta[-n - 7] - \delta[2n - 2] + 2\delta[2n - 3]$$
$$- 4\delta[2n - 5] + 3\delta[2n - 8]$$

4. (a) $2\pi/3$

(b)

$$x[n] = x[n + t_0]$$
$$cos[\frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] = cos[\frac{13\pi}{10}(n + t_0)] + sin[\frac{7\pi}{10}(n + t_0)]$$
$$sin[\frac{\pi}{2} - \frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] = sin[\frac{\pi}{2} - \frac{13\pi}{10}(n + t_0)] + sin[\frac{7\pi}{10}(n + t_0)]$$
$$sin[\frac{5\pi}{10} - \frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] = sin[\frac{5\pi}{10} - \frac{13\pi}{10}(n + t_0)] + sin[\frac{7\pi}{10}(n + t_0)]$$
$$sin[\frac{\pi}{10}(13n - 5)] + sin[\frac{7\pi}{10}n] = sin[\frac{\pi}{10}(13n + 13t_0 - 5)] + sin[\frac{7\pi}{10}(n + t_0)]$$
$$2sin(\frac{\frac{\pi}{10}(13n - 5) + \frac{7\pi}{10}n}{2})cos(\frac{\frac{\pi}{10}(13n - 5) - \frac{7\pi}{10}n}{2})$$
$$= 2sin(\frac{\frac{\pi}{10}(13n + 13t_0 - 5) + \frac{7\pi}{10}(n + t_0)}{2})cos(\frac{\frac{\pi}{10}(13n + 13t_0 - 5) - \frac{7\pi}{10}(n + t_0)}{2})$$
$$sin(\frac{\pi}{20}(20n - 5))cos(\frac{\pi}{20}(6n - 5)) = sin(\frac{\pi}{20}(20n + 20t_0 - 5))cos(\frac{\pi}{20}(6n + 6t_0 - 5))$$
$$sin(n\pi - \frac{\pi}{4})cos(\frac{3n\pi}{10} - \frac{\pi}{4}) = sin(n\pi + t_0\pi - \frac{\pi}{4})cos(\frac{3n\pi + 3t_0\pi}{10} - \frac{\pi}{4})$$

The smallest integer t_0 that satisfies the equation above is $t_0 = 20$.

(c) The signal is not periodic.

5. (a) $x(t) = u[t - 1] - 3u[t - 3] + u[t - 4]$

(b) $\frac{dx(t)}{dt} = \delta(t - 1) - 3\delta(t - 3) + \delta(t - 4)$ The graph of $\frac{dx(t)}{dt}$ is given below.

6. (a)

   (b)

7. (a)
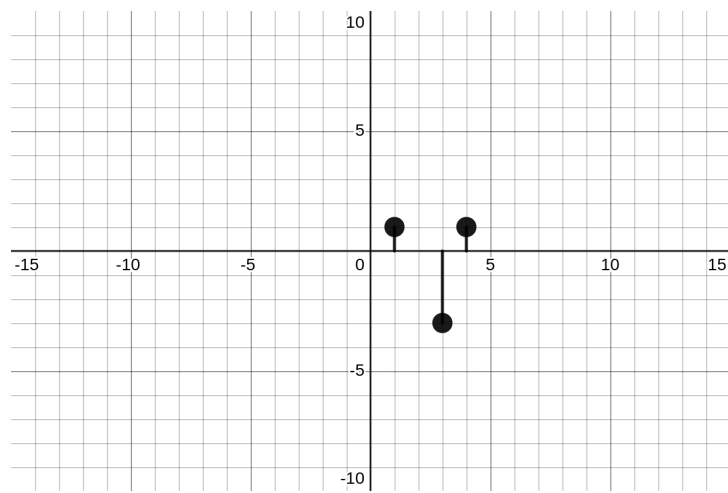
```python
def decompose(signal_name):
    """Open the CSV file with the signal name, decompose it into even and odd components,
    and save the results as PNG files."""

    with open(signal_name + ".csv", "r", encoding="ascii") as file:
        data = [float(item) for item in file.read().split(",")]
    start = int(data[0])
    print(start)
    signal = data[1:]
    end = start + len(signal) - 1

    pyplot.title("Original Signal")
    pyplot.plot(range(start, end + 1), signal)
    pyplot.savefig(IMAGES_PATH + signal_name + "_original.png")
    pyplot.clf()

    if abs(start) > end:
        signal = signal + [0] * (abs(start) - end)
        end = -start
    else:
        signal = [0] * (end - abs(start)) + signal
        start = -end

    even = [(x + y) / 2 for x, y in zip(signal, signal[::-1])]
    odd = [(x - y) / 2 for x, y in zip(signal, signal[::-1])]

    pyplot.title("Even Component")
    pyplot.plot(range(start, end + 1), even)
    pyplot.savefig(IMAGES_PATH + signal_name + "_even.png")
    pyplot.clf()

    pyplot.title("Odd Component")
    pyplot.plot(range(start, end + 1), odd)
    pyplot.savefig(IMAGES_PATH + signal_name + "_odd.png")
    pyplot.clf()
```
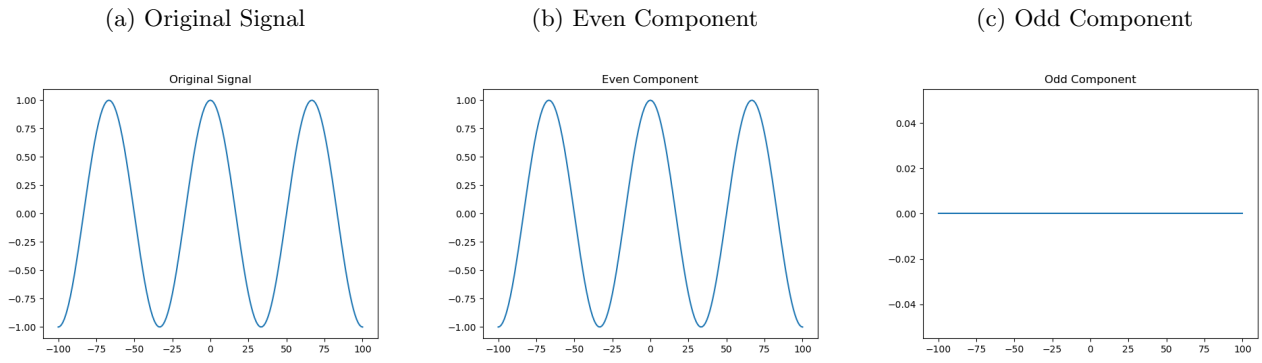
   (b)
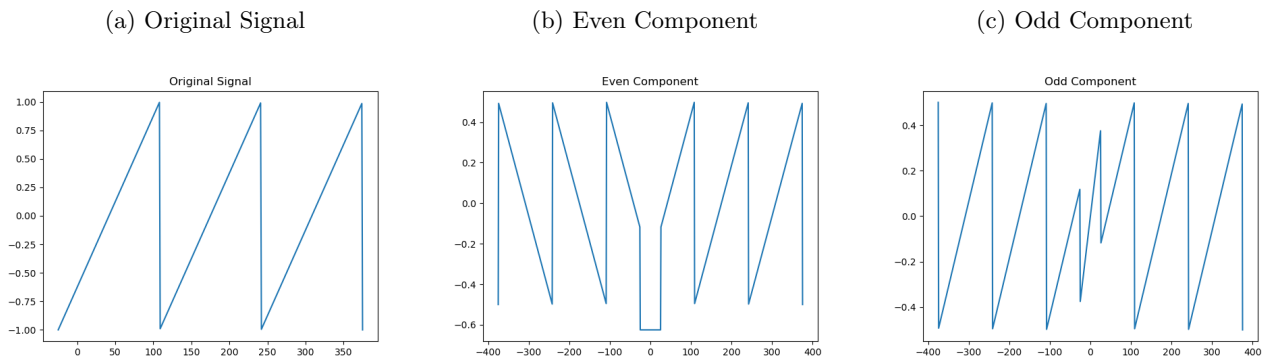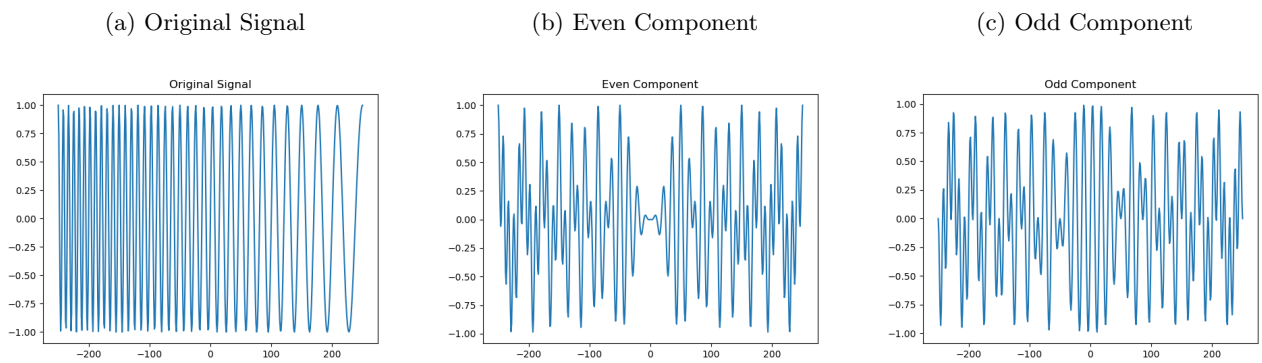
Figure 1: Sinusoidal Signal Decomposition

Figure 2: Shifted Sawtooth Signal Decomposition

Figure 3: Chirp Signal Decomposition