MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

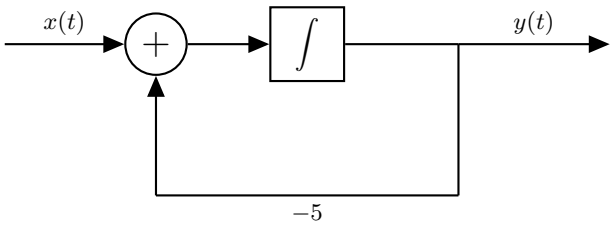
CENG 384

Spring 2023
Homework 2

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 16 April, 2023 (Sunday).
- **Late Submission:** Not allowed.

1. (15 pts) Consider an LTI system given by the following block diagram:



   (a) (5 pts) Find the differential equation which represents this system.
   (b) (10 pts) Find the output $y(t)$, when the input $x(t) = (e^{-t} + e^{-3t})u(t)$. Assume that the system is initially at rest.

2. (10 pts) Evaluate the following convolutions.
   (a) (5 pts) Given $x[n] = 2\delta[n] + \delta[n+1]$ and $h[n] = \delta[n-1] + 2\delta[n+1]$, compute and draw $y[n] = x[n] * h[n]$.
   (b) (5 pts) Given $x(t) = u(t-1) + u(t+1)$ and $h(t) = e^{-t}\sin(t)u(t)$, calculate $y(t) = \frac{dx(t)}{dt} * h(t)$.

3. (15 pts) Evaluate the following convolutions.
   (a) (7 pts) Given $h(t) = e^{-2t}u(t)$ and $x(t) = e^{-t}u(t)$, find $y(t) = x(t) * h(t)$.
   (b) (8 pts) Given $h(t) = e^{3t}u(t)$ and $x(t) = u(t) - u(t-1)$, find $y(t) = x(t) * h(t)$.

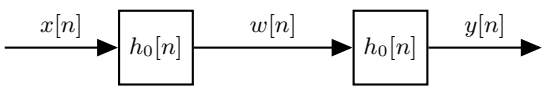
4. (10 pts) Solve the following homogeneous difference and differential equations with the specified initial conditions.
   (a) (5 pts) $y[n] - y[n-1] - y[n-2] = 0$, $y[0] = 1$ and $y[1] = 1$.
   (b) (5 pts) $y^{(3)}(t) - 6y''(t) + 13y'(t) - 10y(t) = 0$, $y''(0) = 3$, $y'(0) = \frac{3}{2}$ and $y(0) = 1$.

5. (15 pts) Consider the differential equation given below,

$$y''(t) + 5y'(t) + 6y(t) = x(t).$$

   (a) (6 pts) Find the particular solution for $x(t) = \cos(5t)$.
   (b) (3 pts) Find the homogeneous solution.
   (c) (6 pts) Find the general solution assuming that the system is initially at rest.

6. (20 pts) Consider the following discrete time LTI system which is initially at rest:



   where $w[n] - \frac{1}{2}w[n-1] = x[n]$.
   (a) (10 pts) Find $h_0[n]$.
   (b) (5 pts) Find the overall impulse response, $h[n]$, of this system.
   (c) (5 pts) Find the difference equation which represents the relationship between the input $x[n]$ and the output $y[n]$.

7. (15 pts) Programming.
Write a computer program to take discrete convolution of 2 signals. (You are not allowed to use any xx.convolve() function from any library.) Your function takes 4 inputs: the first signal x[n], the starting index of the first signal s_i^x, the second signal h[n] and the starting index of the second signal s_i^h (Starting indexes and signals are in the same format as the ones in HW1) and returns the output signal y[n] and the starting index of the output signal s_i^y.

   (a) (5 pts) Generate a shifted discrete impulse function $\delta[n-5]$ in the given signal form and plot the output function that is the result of your discrete convolution function when x[n]="the signal in hw2_signal.csv" and h[n]=$\delta[n-5]$. What is the effect of convolution with $\delta[n-5]$? Comment on that.

   (b) (10 pts) The N-Point moving average filter is defined as follows:

$$h[n] = \begin{cases} \frac{1}{N} & \text{if } 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

   Generate a N-point moving average filter m[n] in the given signal form and plot 4 output functions that is the result of your discrete convolution function when x[n]="the signal in hw2_signal.csv" and h[n]=m[n] by setting N=3,5,10,20. What is the effect of convolution with m[n]? What are the differences between different N values?

You should write your code in `Python 3`. You are not allowed to use any library other than **matplotlib.pyplot** and **numpy**.