CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

Geçit, Emre
e2521581@ceng.metu.edu.tr

Yancı, Baran
e2449015@ceng.metu.edu.tr

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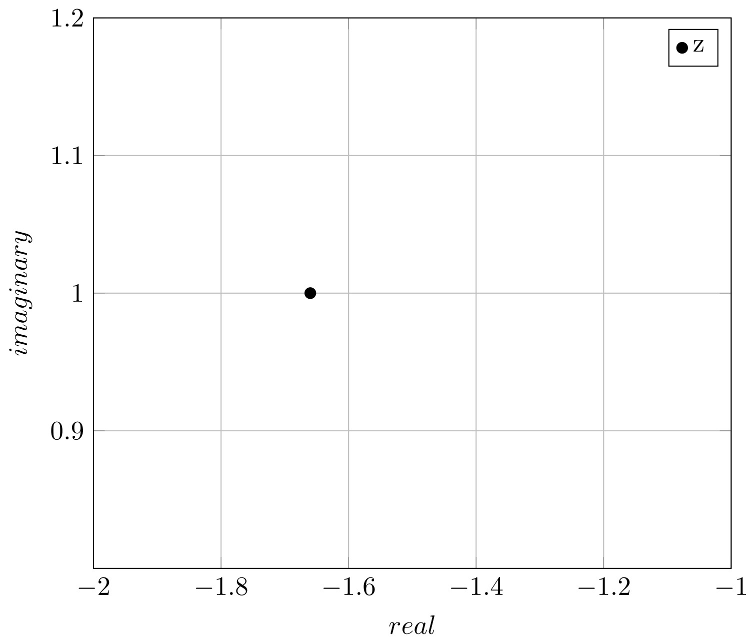
1. (a)

$$
\begin{aligned}
&z = x + yj \implies \bar{z} = x - yj \\
&2z + 5 = j - \bar{z} \\
&2(x + yj) + 5 = j - (x - yj) \\
&2x + 5 + 2yj = (1 + y)j - x \\
&y = 1, x = \frac{-5}{3} \\
&z = \frac{-5}{3} + j \\
&|z|^2 = \frac{25}{9} + 1 = \frac{34}{9}
\end{aligned}
$$



(b)

$$
\begin{aligned}
&z = re^{j\theta} \implies z^5 = r^5 e^{j5\theta} \\
&32j = 32e^{j\pi/2} \\
&32e^{j\pi/2} = r^5 e^{j5\theta} \implies r = 2, \theta = \pi/10 \\
&z = 2e^{j\pi/10}
\end{aligned}
$$

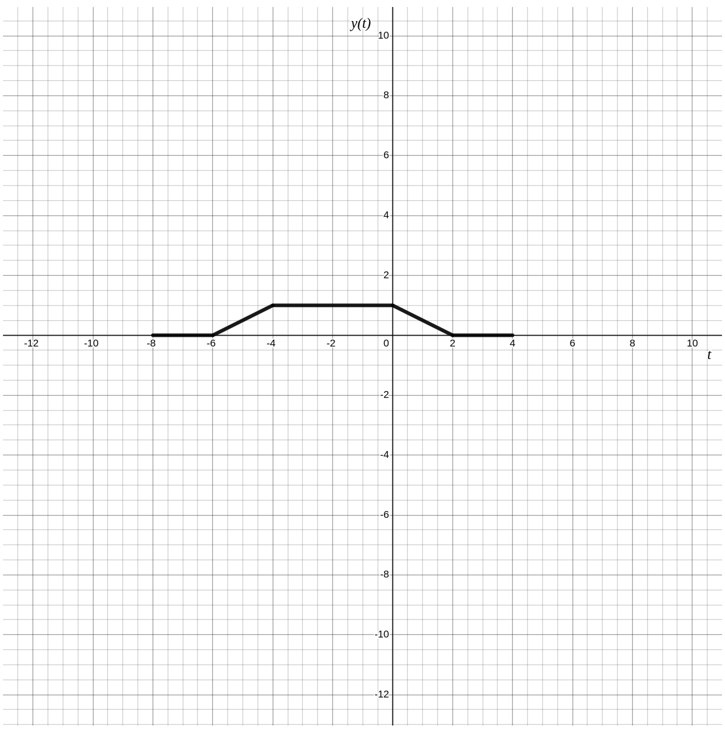
(c)

$$
\begin{aligned}
z &= \frac{(1+j)(\frac{1}{2}+\frac{\sqrt{3}}{2})j}{j-1} \\
&= \frac{(j+1)(1+j)(\frac{1}{2}+\frac{\sqrt{3}}{2})j}{(j+1)(j-1)} \\
&= \frac{(j+1)^2(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(j^2+2j+1)(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(-1+2j+1)(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{2j(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= -j(\frac{1}{2}+\frac{\sqrt{3}}{2}) \\
z &= r\cos\theta + r\sin\theta j \\
j(-\frac{1}{2}-\frac{\sqrt{3}}{2}) &= r\cos\theta + r\sin\theta j \\
r\cos\theta &= 0 \\
r\sin\theta &= -\frac{1}{2}-\frac{\sqrt{3}}{2} \\
\cos\theta &= 0 \\
\sin\theta &= -1 \\
r &= \frac{1}{2}+\frac{\sqrt{3}}{2} \\
\theta &= -\pi/2
\end{aligned}
$$

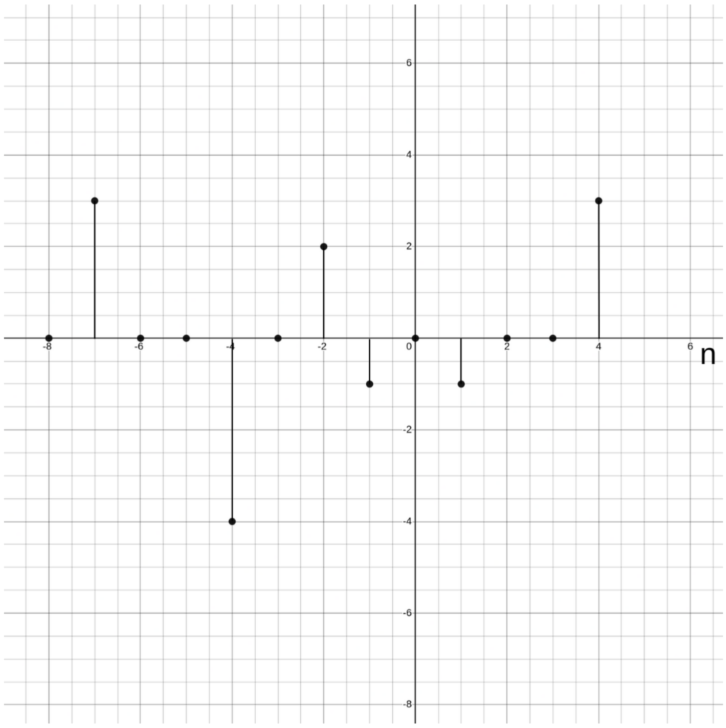
(d)

$$
\begin{aligned}
z &= je^{-j\pi/2} \\
&= e^{j\pi/2}e^{-j\pi/2} \\
&= e^0 = 1
\end{aligned}
$$

2. The graph of the function is given below.



3. (a) The graph of the function $x[-n]+x[2n-1]$ is given below.



(b)

$$
\begin{aligned}
&x[n]=-\delta[n-1]+2\delta[n-2]+-4\delta[n-4]+3\delta[n-7]\\
&x[-n]=-\delta[-n-1]+2\delta[-n-2]+-4\delta[-n-4]+3\delta[-n-7]\\
&x[2n-1]=-\delta[2n-2]+2\delta[2n-3]+-4\delta[2n-5]+3\delta[2n-8]\\
&x[-n]+x[2n-1]=-\delta[-n-1]+2\delta[-n-2]-4\delta[-n-4]+3\delta[-n-7]-\delta[2n-2]+2\delta[2n-3]\\
&-4\delta[2n-5]+3\delta[2n-8]
\end{aligned}
$$

4. (a)

$$
\begin{aligned}
x(t)&=x(t+t_0)\\
5\sin(3t-\frac{\pi}{4})&=5\sin(3(t+t_0)-\frac{\pi}{4})\\
\sin(3t-\frac{\pi}{4})&=\sin(3t+3t_0-\frac{\pi}{4})\\
\sin(3t-\frac{\pi}{4})&=\sin(3t)\cos(3t_0-\frac{\pi}{4})+\cos(3t)\sin(3t_0-\frac{\pi}{4})\\
\sin(3t)\cos(-\frac{\pi}{4})+\cos(3t)\sin(-\frac{\pi}{4})&=\sin(3t)\cos(3t_0-\frac{\pi}{4})+\cos(3t)\sin(3t_0-\frac{\pi}{4})\\
\sin(3t)\frac{\sqrt{2}}{2}-\cos(3t)\frac{\sqrt{2}}{2}&=\sin(3t)\cos(3t_0-\frac{\pi}{4})+\cos(3t)\sin(3t_0-\frac{\pi}{4})\\
\sin(3t)\frac{\sqrt{2}}{2}-\sin(3t)\cos(3t_0-\frac{\pi}{4})&=\cos(3t)\frac{\sqrt{2}}{2}+\cos(3t)\sin(3t_0-\frac{\pi}{4})\\
\sin(3t)\left(\frac{\sqrt{2}}{2}-\cos(3t_0-\frac{\pi}{4})\right)&=\cos(3t)\left(\frac{\sqrt{2}}{2}+\sin(3t_0-\frac{\pi}{4})\right)\\
\frac{\sqrt{2}}{2}-\cos(3t_0-\frac{\pi}{4})&=0\\
\frac{\sqrt{2}}{2}+\sin(3t_0-\frac{\pi}{4})&=0\\
t_0&=\frac{2\pi}{3}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x[n] &= x[n+t_0] \\
\cos[\frac{13\pi}{10}n] + \sin[\frac{7\pi}{10}n] &= \cos[\frac{13\pi}{10}(n+t_0)] + \sin[\frac{7\pi}{10}(n+t_0)] \\
\sin[\frac{\pi}{2} - \frac{13\pi}{10}n] + \sin[\frac{7\pi}{10}n] &= \sin[\frac{\pi}{2} - \frac{13\pi}{10}(n+t_0)] + \sin[\frac{7\pi}{10}(n+t_0)] \\
\sin[\frac{5\pi}{10} - \frac{13\pi}{10}n] + \sin[\frac{7\pi}{10}n] &= \sin[\frac{5\pi}{10} - \frac{13\pi}{10}(n+t_0)] + \sin[\frac{7\pi}{10}(n+t_0)] \\
\sin[\frac{\pi}{10}(13n-5)] + \sin[\frac{7\pi}{10}n] &= \sin[\frac{\pi}{10}(13n+13t_0-5)] + \sin[\frac{7\pi}{10}(n+t_0)]
\end{aligned}
$$

$$
2\sin(\frac{\frac{\pi}{10}(13n-5)+\frac{7\pi}{10}n}{2})\cos(\frac{\frac{\pi}{10}(13n-5)-\frac{7\pi}{10}n}{2})
$$

$$
= 2\sin(\frac{\frac{\pi}{10}(13n+13t_0-5)+\frac{7\pi}{10}(n+t_0)}{2})\cos(\frac{\frac{\pi}{10}(13n+13t_0-5)-\frac{7\pi}{10}(n+t_0)}{2})
$$

$$
\begin{aligned}
\sin(\frac{\pi}{20}(20n-5))\cos(\frac{\pi}{20}(6n-5)) &= \sin(\frac{\pi}{20}(20n+20t_0-5))\cos(\frac{\pi}{20}(6n+6t_0-5)) \\
\sin(n\pi - \frac{\pi}{4})\cos(\frac{3n\pi}{10} - \frac{\pi}{4}) &= \sin(n\pi + t_0\pi - \frac{\pi}{4})\cos(\frac{3n\pi + 3t_0\pi}{10} - \frac{\pi}{4})
\end{aligned}
$$

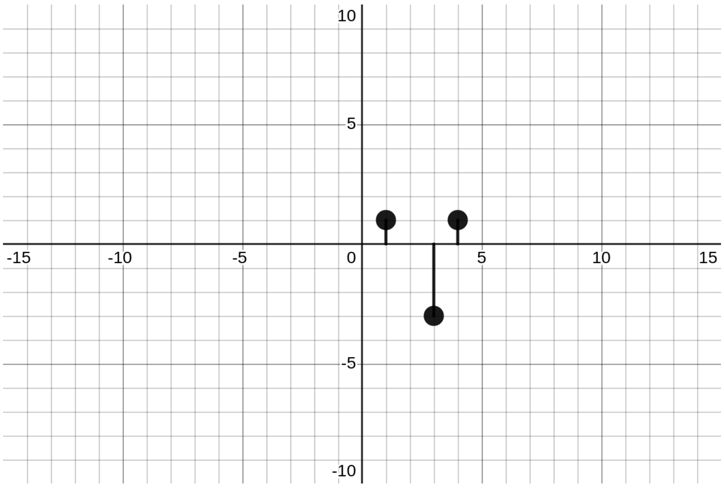
The smallest integer t_0 that satisfies the equation above is $t_0 = 20$.

(c) The signal is not periodic as there is no integer that satisfies the equation below.

$$
\frac{1}{2}\cos(7n-5) = \frac{1}{2}\cos(7(n+t_0)-5)
$$

5. (a) $x(t) = u[t-1] - 3u[t-3] + u[t-4]$

   (b) $\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-3) + \delta(t-4)$ The graph of $\frac{dx(t)}{dt}$ is given below.



6. (a)

   (b)

7. (a)

   (b)