CENG 384 - Signals and Systems for Computer Engineers
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Homework 2

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1. (a)

$$y(t) = x(t) - 5\dot{y}(t)$$

(b)

$$\begin{aligned}
y(t) &= (e^{-t} + e^{-3t})u(t) - 5\dot{y}(t) \\
y(t) + 5\dot{y}(t) &= (e^{-t} + e^{-3t})u(t) \\
y(t) &= y_p(t) + y_h(t) \\
y_p(t) &= Ke^{-t}u(t) + Le^{-3t}u(t) \\
Ke^{-t}u(t) + Le^{-3t}u(t) + 5(-Ke^{-t}u(t) - 3Le^{-3t}u(t)) &= (e^{-t} + e^{-3t})u(t) \\
Ke^{-t}u(t) + Le^{-3t}u(t) - 5Ke^{-t}u(t) - 15Le^{-3t}u(t) &= (e^{-t} + e^{-3t})u(t) \\
e^{-t}u(t)(K - 5K) + e^{-3t}u(t)(L - 15L) &= (e^{-t} + e^{-3t})u(t) \\
K - 5K &= 1 \\
K &= -1/4 \\
L - 15L &= 1 \\
L &= -1/14 \\
y_p(t) &= \frac{-1}{4}e^{-t}u(t) + \frac{-1}{14}e^{-3t}u(t) \\
y_h(t) &= c_1 e^{\alpha t} \\
c_1 e^{\alpha t} + 5\alpha c_1 e^{\alpha t} &= 0 \\
c_1 + 5\alpha c_1 &= 0 \\
\alpha &= \frac{-1}{5} \\
y_h(t) &= c_1 e^{\frac{-1}{5}t} \\
y(t) &= y_p(t) + y_h(t) \\
&= \frac{-1}{4}e^{-t}u(t) + \frac{-1}{14}e^{-3t}u(t) + c_1 e^{\frac{-1}{5}t} \\
y(0) &= 0 \\
0 &= \frac{-1}{4} + \frac{-1}{14} + c_1 \\
c_1 &= \frac{9}{28} \\
y(t) &= \frac{-1}{4}e^{-t}u(t) + \frac{-1}{14}e^{-3t}u(t) + \frac{9}{28}e^{\frac{-1}{5}t}
\end{aligned}$$

2. (a)

$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=0}^{n} x[k]h[n-k] \\
&= \sum_{k=0}^{n} \left(2\delta[k] + \delta[k+1]\right)\left(\delta[n-(1+k)] + 2\delta[n+1-k]\right) \\
&= 2\sum_{k=0}^{n} \delta[k]\delta[n-(1+k)] + 4\sum_{k=0}^{n} \delta[k]\delta[n+1-k] + \sum_{k=0}^{n} \delta[k+1]\delta[n-(1+k)] + 2\sum_{k=0}^{n} \delta[k+1]\delta[n+1-k] \\
&= 2\delta\left[\frac{n-1}{2}\right] + 4\delta\left[\frac{n+1}{2}\right] + \delta\left[\frac{n-2}{2}\right] + 2\delta\left[\frac{n}{2}\right]
\end{aligned}
$$

(b)

$$
\begin{aligned}
y(t) &= \frac{dx(t)}{dt} * h(t) \\
&= \frac{d}{dt}\left(u(t-1) + u(t+1)\right) * e^{-t}\sin(t)u(t) \\
&= \left(\delta(t-1) - \delta(t+1)\right) * e^{-t}\sin(t)u(t) \\
&= \int_{-\infty}^{\infty} \left(\delta(\tau-1) - \delta(\tau+1)\right) e^{-t-\tau}\sin(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} \delta(\tau-1)e^{-t-\tau}\sin(t-\tau)d\tau - \int_{-\infty}^{\infty} \delta(\tau+1)e^{-t-\tau}\sin(t-\tau)d\tau \\
&= e^{-t-1}\sin(t-1)u(t) + e^{-t+1}\sin(t+1)u(t)
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{-(t-\tau)}e^{-2\tau}d\tau \\
&= \int_{0}^{t} e^{-(t-\tau)}e^{-2\tau}d\tau \\
&= e^{-t}\int_{0}^{t} e^{\tau}d\tau \\
&= e^{-t}\left(1 - e^{-t}\right)u(t)
\end{aligned}
$$

(b)

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau \\
&= \int_{-\infty}^{\infty} \left(u(t-\tau) - u(t-(\tau+1))\right)e^{3\tau}d\tau \\
&= \int_{-\infty}^{\infty} u(t-\tau)e^{3\tau}d\tau - \int_{-\infty}^{\infty} u(t-(\tau+1))e^{3\tau}d\tau \\
&= \int_{-\infty}^{t} e^{3\tau}d\tau - \int_{-\infty}^{t-1} e^{3\tau}d\tau \\
&= \frac{e^{3t}}{3} - \frac{e^{3t-3}}{3}
\end{aligned}
$$

4. (a)

$$
\begin{aligned}
y[n] - y[n-1] - y[n-2] &= 0 \\
y[n] &= y[n-1] + y[n-2] \\
y[2] &= y[1] - y[0] = 2 \\
y[3] &= y[2] - y[1] = 3 \\
y[4] &= y[3] - y[2] = 5 \\
y[5] &= y[4] - y[3] = 8 \\
&\ldots
\end{aligned}
$$

It is the Fibonacci sequence.

$$y[n] = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

(b)

$$
\begin{aligned}
y^{(3)}(t) - 6y''(t) + 13y'(t) - 10y(t) &= 0 \\
K^3 - 6K^2 + 13K - 10 &= 0 \\
K(K-5)(K-2) + (K+5)(K-2) &= 0 \\
(K-2)(K^2 - 4K + 5) &= 0
\end{aligned}
$$

$$K = 2, 2-i, 2+i$$

$$
\begin{aligned}
y_h(t) &= c_1 e^{2t} + c_2 e^{(2+i)t} + c_3 e^{(2-i)t} \\
y_h(t) &= c_1 e^{2t} + c_2\left(e^{2t}\cos(t) + ie^{2t}\sin(t)\right) + c_3\left(e^{2t}\cos(t) - ie^{2t}\sin(t)\right) \\
y_h(t) &= c_1 e^{2t} + c_2 e^{2t}\cos(t) + c_3 e^{2t}\cos(t) + c_2 ie^{2t}\sin(t) - c_3 ie^{2t}\sin(t) \\
y_h(t) &= c_1 e^{2t} + (c_2 + c_3)e^{2t}\cos(t) + i(c_2 - c_3)e^{2t}\sin(t) \\
y_h(t) &= C_1 e^{2t} + C_2 e^{2t}\cos(t) + C_3 e^{2t}\sin(t)
\end{aligned}
$$

$$
\begin{aligned}
y''(0) &= 3 \\
&= 4C_1 + 3C_2 + 4C_3 \\
y'(0) &= 1.5 \\
&= 2C_1 + 2C_2 + C_3 \\
y(0) &= 1 \\
&= C_1 + C_2
\end{aligned}
$$

$$
\begin{aligned}
C_1 &= 2 \\
C_2 &= -1 \\
C_3 &= -0.5
\end{aligned}
$$

$$y_h(t) = 2e^{2t} - e^{2t}\cos(t) - \frac{1}{2}e^{2t}\sin(t)$$

5. (a)
   (b)
   (c)
6. (a)
   (b)
   (c)
7. (a)
   (b)