CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

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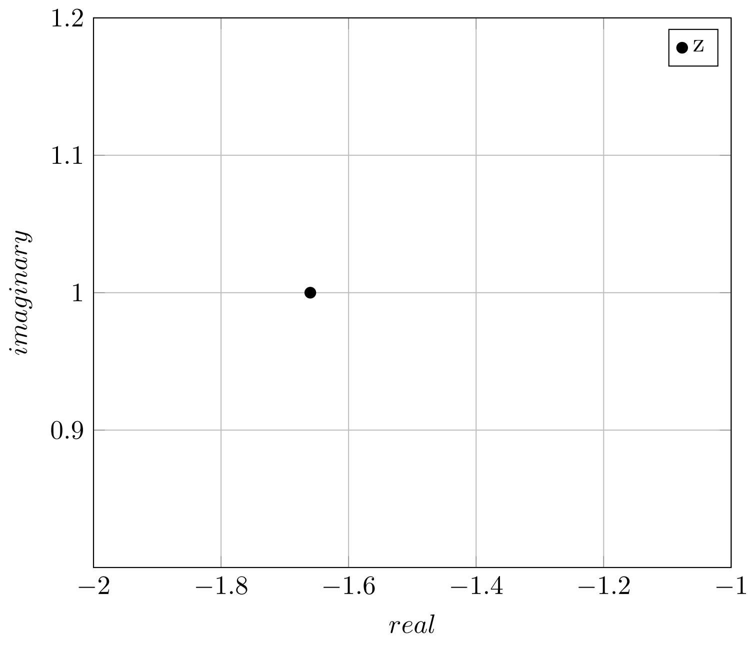
1. (a)

$$
\begin{aligned}
&z = x + yj \implies \bar{z} = x - yj \\
&2z + 5 = j - \bar{z} \\
&2(x + yj) + 5 = j - (x - yj) \\
&2x + 5 + 2yj = (1 + y)j - x \\
&y = 1, x = \frac{-5}{3} \\
&z = \frac{-5}{3} + j \\
&|z|^2 = \frac{25}{9} + 1 = \frac{34}{9}
\end{aligned}
$$



(b)

$$
\begin{aligned}
&z = re^{j\theta} \implies z^5 = r^5 e^{j5\theta} \\
&32j = 32e^{j\pi/2} \\
&32e^{j\pi/2} = r^5 e^{j5\theta} \implies r = 2, \theta = \pi/10 \\
&z = 2e^{j\pi/10}
\end{aligned}
$$

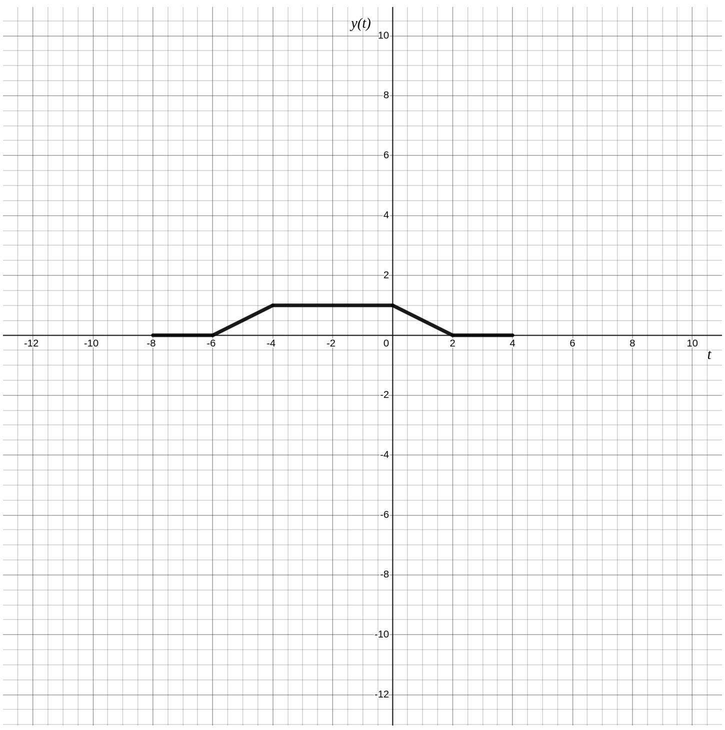
(c)

$$
\begin{aligned}
z &= \frac{(1+j)(\frac{1}{2}+\frac{\sqrt{3}}{2})j}{j-1} \\
&= \frac{(j+1)(1+j)(\frac{1}{2}+\frac{\sqrt{3}}{2})j}{(j+1)(j-1)} \\
&= \frac{(j+1)^2(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(j^2+2j+1)(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{(-1+2j+1)(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= \frac{2j(\frac{1}{2}+\frac{\sqrt{3}}{2})}{-2} \\
&= -j(\frac{1}{2}+\frac{\sqrt{3}}{2}) \\
z &= rcos\theta + rsin\theta j \\
j(-\frac{1}{2}-\frac{\sqrt{3}}{2}) &= rcos\theta + rsin\theta j \\
rcos\theta &= 0 \\
rsin\theta &= -\frac{1}{2}-\frac{\sqrt{3}}{2} \\
cos\theta &= 0 \\
sin\theta &= -1 \\
r &= \frac{1}{2}+\frac{\sqrt{3}}{2} \\
\theta &= -\pi/2
\end{aligned}
$$

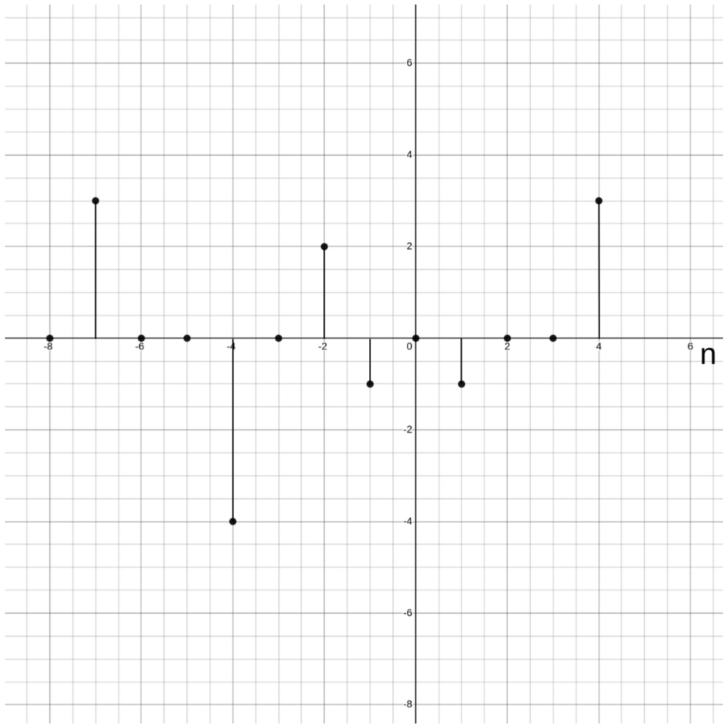
(d)

$$
\begin{aligned}
z &= je^{-j\pi/2} \\
&= e^{j\pi/2}e^{-j\pi/2} \\
&= e^0 = 1
\end{aligned}
$$

2. The graph of the function is given below.



3. (a) The graph of the function $x[-n] + x[2n-1]$ is given below.



(b)

$$
\begin{aligned}
&x[n] = -\delta[n-1] + 2\delta[n-2] + -4\delta[n-4] + 3\delta[n-7] \\
&x[-n] = -\delta[-n-1] + 2\delta[-n-2] + -4\delta[-n-4] + 3\delta[-n-7] \\
&x[2n-1] = -\delta[2n-2] + 2\delta[2n-3] + -4\delta[2n-5] + 3\delta[2n-8] \\
&x[-n] + x[2n-1] = -\delta[-n-1] + 2\delta[-n-2] - 4\delta[-n-4] + 3\delta[-n-7] - \delta[2n-2] + 2\delta[2n-3] \\
&\ - 4\delta[2n-5] + 3\delta[2n-8]
\end{aligned}
$$

4. (a) $2\pi/3$

(b)

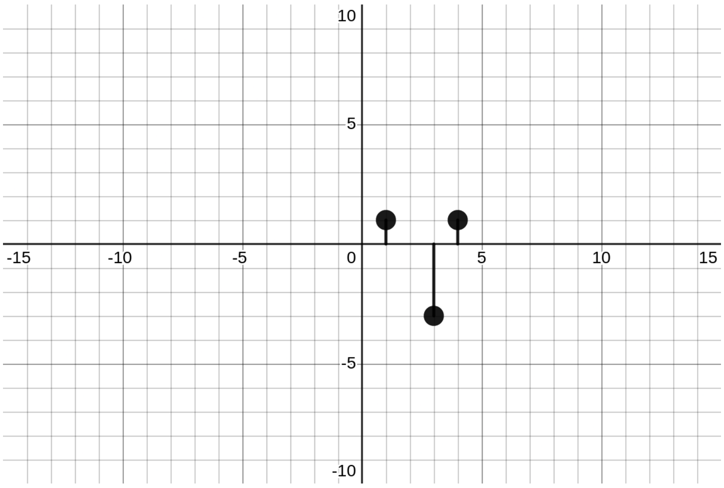
$$
\begin{aligned}
x[n] &= x[n+t_0] \\
cos[\frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] &= cos[\frac{13\pi}{10}(n+t_0)] + sin[\frac{7\pi}{10}(n+t_0)] \\
sin[\frac{\pi}{2} - \frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] &= sin[\frac{\pi}{2} - \frac{13\pi}{10}(n+t_0)] + sin[\frac{7\pi}{10}(n+t_0)] \\
sin[\frac{5\pi}{10} - \frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n] &= sin[\frac{5\pi}{10} - \frac{13\pi}{10}(n+t_0)] + sin[\frac{7\pi}{10}(n+t_0)] \\
sin[\frac{\pi}{10}(13n-5)] + sin[\frac{7\pi}{10}n] &= sin[\frac{\pi}{10}(13n+13t_0-5)] + sin[\frac{7\pi}{10}(n+t_0)] \\
2sin(\frac{\frac{\pi}{10}(13n-5) + \frac{7\pi}{10}n}{2})cos(\frac{\frac{\pi}{10}(13n-5) - \frac{7\pi}{10}n}{2}) & \\
= 2sin(\frac{\frac{\pi}{10}(13n+13t_0-5) + \frac{7\pi}{10}(n+t_0)}{2})&cos(\frac{\frac{\pi}{10}(13n+13t_0-5) - \frac{7\pi}{10}(n+t_0)}{2}) \\
sin(\frac{\pi}{20}(20n-5))cos(\frac{\pi}{20}(6n-5)) &= sin(\frac{\pi}{20}(20n+20t_0-5))cos(\frac{\pi}{20}(6n+6t_0-5)) \\
sin(n\pi - \frac{\pi}{4})cos(\frac{3n\pi}{10} - \frac{\pi}{4}) &= sin(n\pi + t_0\pi - \frac{\pi}{4})cos(\frac{3n\pi + 3t_0\pi}{10} - \frac{\pi}{4})
\end{aligned}
$$

The smallest integer t_0 that satisfies the equation above is $t_0 = 20$.

(c) The signal is not periodic.

5. (a) $x(t) = u[t-1] - 3u[t-3] + u[t-4]$

(b) $\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-3) + \delta(t-4)$ The graph of $\frac{dx(t)}{dt}$ is given below.



6. (a)
   (b)

7. (a)
   (b)