CENG 384

Spring 2023

Homework 3

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Submission:** You need to submit a pdf file named 'hw3.pdf' to the odtuclass page of the course. You need to use the given template 'hw3.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 10 May, 2023 (Wednesday).
- **Late Submission:** Not allowed.

1. (10 pts) Prove the integration property of the Fourier series, showing all steps clearly. Integration property is stated as follows in Table 3.1 of Oppenheim's text book:

   If the Fourier series coefficients of the periodic continuous function $x(t)$ with period T is a_k and $a_0 = 0$ then the Fourier series coefficients of $\int_{-\infty}^{t} x(s)ds$ are $\left(\frac{1}{jk(\frac{2\pi}{T})}\right) a_k$.

2. (12 pts) Given that Fourier series coefficients of periodic continuous function $x(t)$ with fundamental period T are a_k, determine Fourier series coefficients of each of the following signals in terms of a_k.

   (a) (4 pts) $x(t)x(t)$

   (b) (4 pts) $\mathcal{E}v\{x(t)\}$

   (c) (4 pts) $x(t+t_0) + x(t-t_0)$

3. (10 pts) Find the Fourier series representation for the signal given below. Show your work clearly and in order.

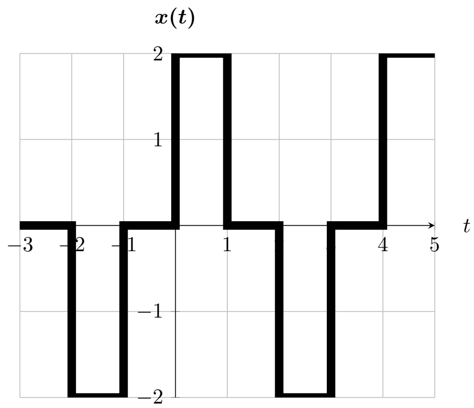


Figure 1: t vs. $x(t)$.

4. (16 pts) Consider the following system:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

   where the input is denoted with $x(t)$, the impulse response is denoted with $h(t)$ and the output is denoted with $y(t)$. The relation between $x(t)$ and $y(t)$ is known to be $\frac{dy(t)}{dt} + y(t) = x(t)$.

   (a) (4 pts) $x(t) = 1 + \sin(\omega_0 t) + 2\cos(\omega_0 t) + \cos(2\omega_0 t + \frac{\pi}{4})$ with $\omega_0 = 2\pi$. Find Fourier series coefficients a_k of $x(t)$ and sketch magnitude and phase of a_k, i.e., sketch $|a_k|$ and $\angle a_k$.

   (b) (4 pts) Write transfer function of the system. What is the eigenvalue of the system?

   (c) (4 pts) Find Fourier series coefficients b_k of $y(t)$ and sketch magnitude and phase of b_k, i.e., sketch $|b_k|$ and $\angle b_k$.

   (d) (4 pts) Write the expression for $y(t)$.

5. (16 pts) Consider the following discrete time signals:

$$x[n] = \sin \tfrac{\pi}{2} n$$

$$y[n] = 1 + \cos \tfrac{\pi}{2} n$$

(a) (4 pts) Find the Fourier series coefficients of $x[n]$.

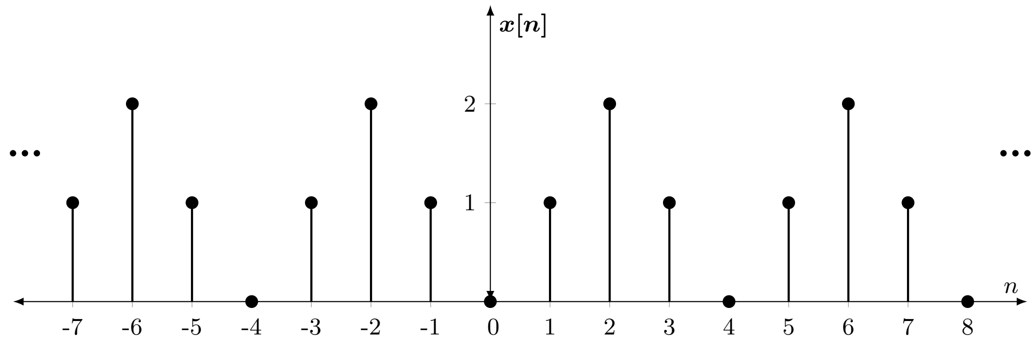
(b) (4 pts) Find the Fourier series coefficients of $y[n]$.

(c) (4 pts) Using the multiplication property of the discrete time Fourier series, find the Fourier series coefficients of $x[n] \times y[n]$.

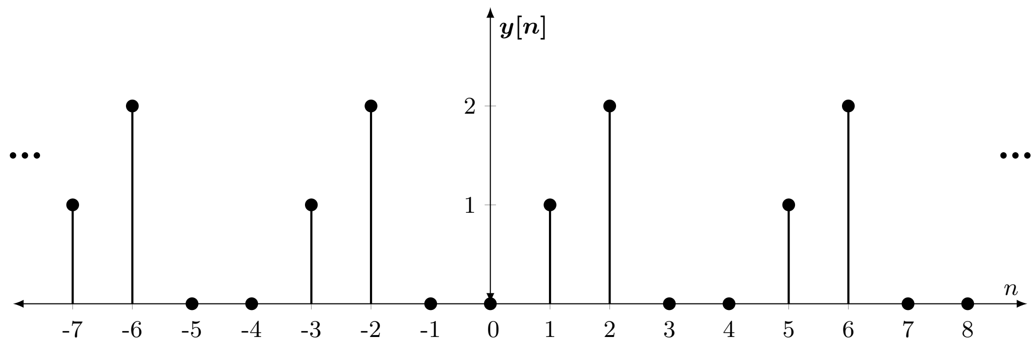
(d) (4 pts) Using Fourier analysis equation (direct evaluation), find the Fourier series coefficients of $x[n] \times y[n]$. Compare the result with the result of part c.

6. (10 pts)

(a) (5 pts) Find and plot the spectral coefficients of the Fourier series representation for the following discrete time signal, $x[n]$:



(b) Consider the following discrete time signal, $y[n]$:



i. (2 pts) Define $y[n]$ in terms of $x[n]$.

ii. (3 pts) Find and plot the spectral coefficients of Fourier series for $y[n]$.

7. (10 pts) Consider a LTI system with the following transfer function,

$$H(j\omega) = \begin{cases} 1 & \text{for } |\omega| \leq 80 \\ 0 & \text{otherwise.} \end{cases}$$

When we feed $x(t)$, a CT periodic signal with Fourier spectral coefficients a_k, as input to this system, the output is $y(t)$. The fundamental period of $x(t)$ is π/K, where K is a constant.

(a) (5 pts) What can we conclude about a_k if we know that $y(t) = x(t)$.

(b) (5 pts) What about when $y(t) \neq x(t)$?

8. (16 pts) Programming.
In this programming task, we try to approximate two different periodic functions by using their Fourier Series representations.

(a) Firstly, write a function that computes the first n+1 Fourier Series coefficients of a given signal. Your function takes the given signal, period of the signal and number of coefficients as input. You will need to compute DC component and the coefficients of n harmonic.(For safety you can compute one DC coefficient, n coefficients for cosine components and n coefficients for sine components.)

(b) Write a function to generate the approximate function by using Fourier Series coefficients.

(c) Generate following square wave function by dividing [-0.5, 0.5] range into 1000 points.

$$s[n] = \begin{cases} -1 & \text{if } -0.5 < n < 0 \\ 1 & \text{if } 0 < n < 0.5 \end{cases}$$

You can assume that this function is periodic and above definition belongs to one cycle of the signal. Compute n Fourier Series coefficients of the given function by using the function you implemented in the first part. Then, generate the approximate function by using the function you implemented in the second part. Plot both original function and approximated function on the same plot by setting n=[1, 5, 10, 50, 100].(You can use plt.plot() function for better visualization.)

(d) Generate following sawtooth function by dividing [-0.5, 0.5] range into 1000 points. (You can use **scipy.signal.sawtooth()** function or you can implement it by hand.)

$$s[n] = \begin{cases} 1 + 2n & \text{if } -0.5 < n < 0 \\ -1 + 2n & \text{if } 0 < n < 0.5 \end{cases}$$

Apply the procedure in the third part to the new signal. What is the effect of increasing n?

You should write your code in `Python` and no library is allowed other than `matplotlib.pyplot`, `numpy` and `scipy.signal.sawtooth()`.