CENG 384 - Signals and Systems for Computer Engineers
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Homework 4

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1. (a)

$$\begin{aligned}
H(j\omega) &= \frac{j\omega - 1}{j\omega + 1} \\
\frac{Y(j\omega)}{X(j\omega)} &= \frac{j\omega - 1}{j\omega + 1} \\
Y(j\omega)(j\omega + 1) &= X(j\omega)(j\omega - 1) \\
y'(t) + y(t) &= x'(t) - x(t)
\end{aligned}$$

(b)

$$\begin{aligned}
H(j\omega) &= \frac{j\omega - 1}{j\omega + 1} \\
h(t) &= \mathcal{F}^{-1}\{H(j\omega)\} \\
&= \mathcal{F}^{-1}\{\frac{j\omega - 1}{j\omega + 1}\} \\
&= \mathcal{F}^{-1}\{\frac{j\omega + 1 - 2}{j\omega + 1}\} \\
&= \mathcal{F}^{-1}\{\frac{j\omega + 1}{j\omega + 1}\} - \mathcal{F}^{-1}\{\frac{2}{j\omega + 1}\} \\
&= \mathcal{F}^{-1}\{1\} - 2\mathcal{F}^{-1}\{\frac{1}{j\omega + 1}\} \\
&= \delta(t) - 2e^{-t}u(t)
\end{aligned}$$

(c)

$$
\begin{aligned}
y'(t) + y(t) &= x'(t) - x(t) \\
y'(t) + y(t) &= -2e^{-2t}u(t) - e^{-2t}u(t) \\
y'(t) + y(t) &= -3e^{-2t}u(t) \\
y_p(t) &= Ae^{-2t} \\
y_p'(t) &= -2Ae^{-2t} \\
-2Ae^{-2t} + Ae^{-2t} &= -3e^{-2t}u(t) \\
A &= 3 \\
y_p(t) &= 3e^{-2t} \\
y_h(t) &= c_1 e^{-t}u(t) \\
y(t) &= y_p(t) + y_h(t) \\
&= 3e^{-2t} + c_1 e^{-t}u(t) \\
y(0) &= 0 \\
0 &= 3e^{-2(0)} + c_1 e^{-0}u(0) \\
0 &= 3 + c_1 \\
c_1 &= -3 \\
y(t) &= 3e^{-2t} - 3e^{-t}u(t)
\end{aligned}
$$

(d)

2. (a)

$$
\begin{aligned}
y[n+1] - \frac{1}{2}y[n] &= x[n+1] \\
e^{j\omega}Y(e^{j\omega}) - \frac{1}{2}Y(e^{j\omega}) &= e^{j\omega}X(e^{j\omega}) \\
H(e^{j\omega}) &= \frac{Y(e^{j\omega})}{X(e^{j\omega})} \\
H(e^{j\omega}) &= \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H(e^{j\omega}) &= \frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}} \\
h[n] &= \mathcal{F}^{-1}\{H(e^{j\omega})\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega} - \frac{1}{2} + \frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{\frac{e^{j\omega} - \frac{1}{2}}{e^{j\omega} - \frac{1}{2}} + \frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{1 + \frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \mathcal{F}^{-1}\{1\} + \mathcal{F}^{-1}\{\frac{\frac{1}{2}}{e^{j\omega} - \frac{1}{2}}\} \\
&= \delta[n] + \frac{1}{2}\mathcal{F}^{-1}\{\frac{1}{e^{j\omega} - \frac{1}{2}}\} \\
&= \delta[n] + \frac{1}{2}e^{\frac{1}{2}n}u[n]
\end{aligned}
$$

(c)

$$
\begin{aligned}
y[n+1]-\frac{1}{2}y[n] &= x[n+1] \\
y[n+1]-\frac{1}{2}y[n] &= \left(\frac{3}{4}\right)^{n+1}u[n+1] \\
y_p[n] &= A\left(\frac{3}{4}\right)^{n} \\
y_p[n+1] &= A\left(\frac{3}{4}\right)^{n+1} \\
A\left(\frac{3}{4}\right)^{n+1}-\frac{1}{2}A\left(\frac{3}{4}\right)^{n} &= \left(\frac{3}{4}\right)^{n+1}u[n+1] \\
\frac{3}{4}A\left(\frac{3}{4}\right)^{n}-\frac{1}{2}A\left(\frac{3}{4}\right)^{n} &= \frac{3}{4}\left(\frac{3}{4}\right)^{n}u[n+1] \\
\frac{3}{4}A-\frac{1}{2}A &= \frac{3}{4}u[n+1] \\
A &= 3 \\
y_p[n] &= 3\left(\frac{3}{4}\right)^{n} \\
y_h[n] &= c_1\left(\frac{1}{2}\right)^{n}u[n] \\
y[n] &= y_p[n]+y_h[n] \\
&= 3\left(\frac{3}{4}\right)^{n}+c_1\left(\frac{1}{2}\right)^{n}u[n] \\
y[0] &= 0 \\
0 &= 3\left(\frac{3}{4}\right)^{0}+c_1\left(\frac{1}{2}\right)^{0}u[0] \\
0 &= 3+c_1 \\
c_1 &= -3 \\
y[n] &= 3\left(\frac{3}{4}\right)^{n}-3\left(\frac{1}{2}\right)^{n}u[n]
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
H(j\omega) &= H_1(j\omega)H_2(j\omega) \\
&= \frac{1}{j\omega+1}\frac{1}{j\omega+2} \\
&= \frac{Y(j\omega)}{X(j\omega)} \\
Y(j\omega)(j\omega+1)(j\omega+2) &= X(j\omega) \\
Y(j\omega)(j^2\omega^2+3j\omega+2) &= X(j\omega) \\
y''(t)+3y'(t)+2y(t) &= x(t)
\end{aligned}
$$

(b)

$$
\begin{aligned}
H(j\omega) &= H_1(j\omega)H_2(j\omega) \\
H_1(j\omega) &= \frac{1}{j\omega+1} \\
H_2(j\omega) &= \frac{1}{j\omega+2} \\
H(j\omega) &= \frac{1}{j\omega+1}\frac{1}{j\omega+2} \\
h_1(t) &= \mathcal{F}^{-1}\{H(j\omega)\} \\
&= \mathcal{F}^{-1}\{\frac{1}{j\omega+1}\frac{1}{j\omega+2}\} \\
&= \mathcal{F}^{-1}\{\frac{1}{j\omega+1}\} * \mathcal{F}^{-1}\{\frac{1}{j\omega+2}\} \\
&= e^{-t}u(t) * e^{-2t}u(t) \\
&= \int_{-\infty}^{\infty} e^{-(t-\tau)}u(t-\tau)e^{-2\tau}u(\tau)d\tau \\
&= \int_0^t e^{-(t-\tau)}e^{-2\tau}d\tau \\
&= \int_0^t e^{-t+\tau}e^{-2\tau}d\tau \\
&= \int_0^t e^{-t}e^{\tau}e^{-2\tau}d\tau \\
&= e^{-t}\int_0^t e^{-\tau}d\tau \\
&= e^{-t}\left[-e^{-\tau}\right]_0^t \\
&= e^{-t}\left[-e^{-t}+1\right] \\
&= e^{-t}-e^{-2t}
\end{aligned}
$$

(c)

$$
\begin{aligned}
X(j\omega) &= j\omega \\
Y(j\omega) &= H(j\omega)X(j\omega) \\
&= \frac{1}{j\omega+1}\frac{1}{j\omega+2}j\omega \\
y(t) &= \mathcal{F}^{-1}\{Y(j\omega)\} \\
&= \mathcal{F}^{-1}\{\frac{j\omega}{(j\omega+1)(j\omega+2)}\} \\
&= \mathcal{F}^{-1}\{\frac{-1}{j\omega+1}+\frac{2}{j\omega+2}\} \\
&= \mathcal{F}^{-1}\{\frac{-1}{j\omega+1}\} + \mathcal{F}^{-1}\{\frac{2}{j\omega+2}\} \\
&= -e^{-t}u(t)+2e^{-2t}u(t)
\end{aligned}
$$

4. (a)
   (b)
   (c)
5.