IE407 - Homework 2

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a)

x_{ij}: The amount of product produced at Plant i and shipped to Warehouse j (in tons).

y_{ij}: The amount of product shipped from Warehouse i to Customer j (in tons).

w_{ij}: The cost of shipping product from Plant i to Warehouse j (in dollars per ton).

z_{ij}: The cost of shipping product from Warehouse i to Customer j (in dollars per ton).

Minimize Cost: $c = \sum\sum w_{ij}x_{ij} + \sum\sum z_{ij}y_{ij} + 40p_1 + 50p_2 + 45p_3 + 50p_4 + 45p_5 + 30w_1 + 40w_2 + 30w_3$

Subject to,

Plant Constraints: $\sum x_{1j} \leq 300$, $\sum x_{2j} \leq 200$, $\sum x_{3j} \leq 300$, $\sum x_{ij} \leq 200$, $\sum y_{1j} \leq 400$

Customer Constraints: $\sum y_{i1} \geq 200$, $\sum y_{i2} \geq 300$, $\sum y_{i3} \geq 250$, $\sum y_{i4} \geq 350$

Transportation Constraints: $\sum_{i=1}^{5} x_{ij} = \sum_{k=1}^{4} y_{jk} \quad \forall j$

To minimize the cost of meeting customer demand, decisions should be made as follows:

The amount of product produced at Plant i and shipped to Warehouse j should be as in the following table:

Table 1: Amount produced at Plant i and shipped to Warehouse j (tons)

		To			
		Warehouse 1	Warehouse 2	Warehouse 3	Total
From	Plant 1	0	0	0	0
	Plant 2	0	200	0	200
	Plant 3	0	50	250	300
	Plant 4	200	0	0	200
	Plant 5	350	50	0	400
	Total	550	300	250	

The amount of product shipped from Warehouse i to Customer j should be as in the following table:

Table 2: Amount shipped from Warehouse i to Customer j (tons)

		To				
		Customer 1	Customer 2	Customer 3	Customer 4	Total
From	Warehouse 1	200	0	0	350	550
	Warehouse 2	0	300	0	0	300
	Warehouse 3	0	0	250	0	250
	Total	200	300	250	350	

When these decisions are made, minimized total cost is $C = 53800$.

b)

New constraints and variables should be added to the model to solve this problem.

p_i: Binary variable that is equal to 1 if Plant i is used and 0 otherwise.

w_i: Binary variable that is equal to 1 if Warehouse i is used and 0 otherwise.

Plant Constraints: $\sum x_{1j} \leq 300p_1$, $\sum x_{2j} \leq 200p_2$, $\sum x_{3j} \leq 300p_3$, $\sum x_{ij} \leq 200p_4$, $\sum y_{1j} \leq 400p_5$

Warehouse Constraints: $\sum x_{ij} \leq 1100w_1$, $\sum x_{ij} \leq 1100w_2$, $\sum x_{ij} \leq 1100w_3$

To minimize the cost of meeting customer demand, decisions should be made as follows:

All warehouses and plants should be used except for plant 1. Mathematically, $p_i = 1 \quad \forall i, i \neq 1$ and $w_i = 1 \quad \forall i$.

The amount of product produced at Plant i and shipped to Warehouse j should be as in the following table:

Table 3: Amount produced at Plant i and shipped to Warehouse j (tons)

		To			
		Warehouse 1	Warehouse 2	Warehouse 3	Total
From	Plant 1	8.52651E-14	0	0	8.52651E-14
	Plant 2	0	200	0	200
	Plant 3	0	50	250	300
	Plant 4	200	0	0	200
	Plant 5	350	50	0	400
	Total	550	300	250	

The amount of product shipped from Warehouse i to Customer j should be as in the following table:

Table 4: Amount shipped from Warehouse i to Customer j (tons)

		To				
		Customer 1	Customer 2	Customer 3	Customer 4	Total
From	Warehouse 1	200	0	0	350	550
	Warehouse 2	0	300	0	0	300
	Warehouse 3	0	0	250	0	250
	Total	200	300	250	350	

When these decisions are made, minimized total cost is $C = 54090$.