IE407 - Homework 3

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1 Simplex Method

1.1 Formulation

Decision Variables

$$s = \text{Amount of } \textbf{Sweet Corn} \text{ used in the mix (grams)}$$
$$b = \text{Amount of } \textbf{Barbeque} \text{ used in the mix (grams)}$$

Constraints

$s + b = 100$	Weight of the package
$b - s \leq 35$	Difference between sweet corn and barbeque
$s - b \leq 35$	Difference between sweet corn and barbeque
$1.5 \cdot s + 3.5 \cdot b \geq 250$	Protein
$4 \cdot s + 6 \cdot b \leq 550$	Calories
$s, b \geq 0$	Non-negativity

Objective Function

$$\text{Minimize Total Cost} = 0.5 \cdot s + 0.7 \cdot b$$

1.2 Solution

1.2.1 Convert to Standard Form

$$\begin{array}{cccccccccccc}
z & -0.5 \cdot s & -0.7 \cdot b & & & & & & & = & 0 \\
 & s & +b & +a_1 & & & & & & = & 100 \\
 & -s & +b & & +s_1 & & & & & = & 35 \\
 & s & -b & & & +s_2 & & & & = & 35 \\
 & 1.5 \cdot s & +3.5 \cdot b & & & & -e_1 & +a_2 & & = & 250 \\
 & 4 \cdot s & +6 \cdot b & & & & & & +s_3 & = & 550
\end{array}$$

Where, $s, b, s_1, s_2, s_3, e_1, a_1, a_2$ are non-negative.

This is a system of equations with 6 equations and 9 variables.

The matrix representation of this system is as follows:

	z	s	b	s_1	s_2	s_3	e_1	a_1	a_2	RHS
z	1	-0.5	-0.7	0	0	0	0	0	0	0
a_1	0	1	1	0	0	0	0	1	0	100
s_2	0	-1	1	0	1	0	0	0	0	35
s_1	0	1	-1	1	0	0	0	0	0	35
a_2	0	1.5	3.5	0	0	0	-1	0	1	250
s_3	0	4	6	0	0	1	0	0	0	550

1.2.2 Finding a Basic Feasible Solution

Two-phase method will be used to find a basic feasible solution.

Objective: Minimize $w' = a_1 + a_2$

w'	s	b	s_1	s_2	s_3	e_1	a_1	a_2	RHS
1	0	0	0	0	0	0	-1	-1	0
0	1	1	0	0	0	0	1	0	100
0	-1	1	0	1	0	0	0	0	35
0	1	-1	1	0	0	0	0	0	35
0	1.5	3.5	0	0	0	-1	0	1	250
0	4	6	0	0	1	0	0	0	550

Add row 1 and row 4 from row 0 to get rid of a_1 and a_2.
Now, the matrix is ready for the first iteration of the simplex method.
Variable with the most positive coefficient in row 0 is b.

	w'	s	b	s_1	s_2	s_3	e_1	a_1	a_2	RHS	Ratio
w'	1	2.5	4.5	0	0	0	-1	0	0	350	
a_1	0	1	1	0	0	0	0	1	0	100	100
s_2	0	-1	**1**	0	1	0	0	0	0	35	35
s_1	0	1	-1	1	0	0	0	0	0	35	
a_2	0	1.5	3.5	0	0	0	-1	0	1	250	71.4
s_3	0	4	6	0	0	1	0	0	0	550	91.7

s_2 is the winner of the ratio test. s_2 will exit, and b will enter.

	w'	s	b	s_1	s_2	s_3	e_1	a_1	a_2	RHS	Ratio
w'	1	7	0	0	0	0	-1	0	0	192.5	
a_1	0	2	0	0	0	0	0	1	0	65	32.5
s_2	0	-1	1	0	1	0	0	0	0	35	
s_1	0	0	0	1	1	0	0	0	0	70	
a_2	0	**5**	0	0	0	0	-1	0	1	127.5	25.5
s_3	0	10	0	0	0	1	0	0	0	340	34

In the next iteration, a_2 will be replaced by s.

	w'	s	b	s_1	s_2	s_3	e_1	a_1	a_2	RHS	Ratio
w'	1	0	0	0	0	0	0.4	0	0	14	
a_1	0	0	0	0	0	0	0	1	0	14	
s_2	0	0	1	0	1	0	-0.2	0	0	60.5	
s_1	0	0	0	1	1	0	0	0	0	70	
s	0	1	0	0	0	0	-0.2	0	1	25.5	
s_3	0	0	0	0	0	1	-2	0	-10	85	

There is no ratio with a non-negative value. There was probably some mistake in the previous steps.

I do not have much time left to find my mistake until the deadline. From now on, I will include only the software outputs in my report.

2 Software Solution

Excel solution is shown below.

s:	50	>=	0
b:	50	>=	0
s+b:	100	=	100
b-s	0	<=	35
s-b	0	<=	35
1.5*s+3.5*	250	>=	250
4*s+6*b	500	<=	550
z:	60	(minimize)	

3 Changing the Constraints

Excel solution is shown below.

s:	37.5	>=	0
b:	62.5	>=	0
s+b:	100	=	100
b-s	25	<=	35
s-b	-25	<=	35
1.5*s+3.5*	275	>=	275
4*s+6*b	525	<=	550
z:	62.5	(minimize)	

4 Finding the Shadow Price

According to Excel output, shadow price for protein constraint is 10.

5 Examining other Shadow Prices

There is also the shadow price for the total weight constraint (0.35).

That means that there are no binding constraints other than the protein constraint and the weight constraint.