IE407 - Homework 2

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a)

Decision variables:

- x_{ij}: The amount of product produced at Plant i and shipped to Warehouse j (in tons).
- y_{ij}: The amount of product shipped from Warehouse i to Customer j (in tons).
- w_{ij}: The cost of shipping product from Plant i to Warehouse j (in dollars per ton).
- z_{ij}: The cost of shipping product from Warehouse i to Customer j (in dollars per ton).

The cost function to be minimized:

$$C = \sum \sum w_{ij} x_{ij} + \sum \sum z_{ij} y_{ij} \tag{1}$$

Constraints:

- Plant Constraints: $\sum x_{1j} \leq 300$, $\sum x_{2j} \leq 200$, $\sum x_{3j} \leq 300$, $\sum x_{ij} \leq 200$, $\sum y_{1j} \leq 400$
- Customer Constraints: $\sum y_{i1} \geq 200$, $\sum y_{i2} \geq 300$, $\sum y_{i3} \geq 250$, $\sum y_{i4} \geq 350$
- Transportation Constraints: $\sum_{i=1}^{5} x_{ij} = \sum_{k=1}^{4} y_{jk} \quad \forall j$

To minimize the cost of meeting customer demand, decisions should be made as follows:

- The amount of product produced at Plant i and shipped to Warehouse j should be as in Table 1.
- The amount of product shipped from Warehouse i to Customer j should be as in Table 2.

When these decisions are made, minimized total cost is $C = 53800$.

b)

New constraints and variables should be added to the model to solve this problem.

Decision variables:

- p_i: Binary variable that is equal to 1 if Plant i is used and 0 otherwise.
- w_i: Binary variable that is equal to 1 if Warehouse i is used and 0 otherwise.

Table 1: Amount produced at Plant i and shipped to Warehouse j (tons)

		To			
		Warehouse 1	Warehouse 2	Warehouse 3	Total
From	Plant 1	0	0	0	0
	Plant 2	0	200	0	200
	Plant 3	0	50	250	300
	Plant 4	200	0	0	200
	Plant 5	350	50	0	400
	Total	550	300	250	

Table 2: Amount shipped from Warehouse i to Customer j (tons)

		To				
		Customer 1	Customer 2	Customer 3	Customer 4	Total
From	Warehouse 1	200	0	0	350	550
	Warehouse 2	0	300	0	0	300
	Warehouse 3	0	0	250	0	250
	Total	200	300	250	350	

The cost function to be minimized:

$$C = \sum\sum w_{ij}x_{ij} + \sum\sum z_{ij}y_{ij} + 40p_1 + 50p_2 + 45p_3 + 50p_4 + 45p_5 + 30w_1 + 40w_2 + 30w_3 \tag{2}$$

Constraints:

- Plant Constraints: $\sum x_{1j} \leq 300p_1$, $\sum x_{2j} \leq 200p_2$, $\sum x_{3j} \leq 300p_3$, $\sum x_{ij} \leq 200p_4$, $\sum y_{1j} \leq 400p_5$
- Warehouse Constraints: $\sum x_{ij} \leq 1100w_1$, $\sum x_{ij} \leq 1100w_2$, $\sum x_{ij} \leq 1100w_3$

To minimize the cost of meeting customer demand, decisions should be made as follows:

- All warehouses and plants should be used except for plant 1. Mathematically, $p_i = 1 \quad \forall i, i \neq 1$ and $w_i = 1 \quad \forall i$.
- The amount of product produced at Plant i and shipped to Warehouse j should be as in Table 3.
- The amount of product shipped from Warehouse i to Customer j should be as in Table 4.

When these decisions are made, minimized total cost is $C = 54090$.

Table 3: Amount produced at Plant i and shipped to Warehouse j (tons)

		To			
		Warehouse 1	Warehouse 2	Warehouse 3	Total
From	Plant 1	8.52651E-14	0	0	8.52651E-14
	Plant 2	0	200	0	200
	Plant 3	0	50	250	300
	Plant 4	200	0	0	200
	Plant 5	350	50	0	400
	Total	550	300	250	

Table 4: Amount shipped from Warehouse i to Customer j (tons)

		To				
		Customer 1	Customer 2	Customer 3	Customer 4	Total
From	Warehouse 1	200	0	0	350	550
	Warehouse 2	0	300	0	0	300
	Warehouse 3	0	0	250	0	250
	Total	200	300	250	350	

c)

In addition to existing variables and constraints, the following should be added to the model:

Decision variables:

- s_i: explained in Equation 3. A binary variable.
- t_i: equal to the amount of prouct sold to Customer i more than 100 tons. A continuous variable.

$$s_i = \begin{cases} 1 & \text{if Warehouse 1 sells to Customer } i \text{ more than 100 tons} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Constraints:

- $t_i \geq 100 s_i$
- $t_i \leq 350 s_i$
- $100 * (1 - s_i) + t_i = y_{1i}$

The new cost function is as follows:

$$C = \sum\sum w_{ij}x_{ij} + \sum\sum z_{ij}y_{ij} + 40p_1 + 50p_2 + 45p_3 + 50p_4 + 45p_5 + 30w_1$$
$$+ 40w_2 + 30w_3 + 1000s_1 - 10t_1 + 4000s_2 - 40t_2 + 6000s_3 - 60t_3 + 2000s_4 - 20t_4$$

The minimized total cost is $C = 52455$.