Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin

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March 27, 2016

Abstract: The Supreme Court has held that partisan gerrymandering is recognizable by its asymmetry: for a given distribution of popular votes, if the parties switch places in popular vote, the numbers of seats will change in an unequal fashion. However, the asymmetry standard is only a broad statement of principle, and no analytical method for assessing asymmetry has yet been held to be manageable. Recently I have proposed three statistical tests to reliably assess asymmetry in state-level districting schemes: (a) unrepresentative distortion in the number of seats won based on expectations from nationwide district characteristics; (b) a discrepancy in winning vote margins between the two parties; and (c) the construction of reliable wins for the party in charge of redistricting, as measured by either the difference between mean and median vote share, or an unusually even distribution of votes across districts. The first test relies on computer simulation to estimate appropriate levels of representation for a given level of popular vote, and provides a way to measure the effects of a gerrymander. The second and third tests, which can identify evidence for intent, rely on well-established statistical principles and can be carried out using a hand calculator without examination of maps or redistricting procedures. These tests allow calculation of the exact probability that an outcome would have arisen by chance. Here I use these tests to show that both Maryland's Congressional districts (*Shapiro v. McManus*) and Wisconsin's state Assembly districts (*Whitford v. Nichol*) meet criteria for a partisan gerrymander. I propose that an intents-and-effects standard based on these tests is robust enough to mitigate the need to demonstrate predominant partisan intent. The three statistical standards offered here add to the judge's toolkit for rapidly and rigorously identifying the consequences of partisan redistricting.

I. INTRODUCTION

In this Article, I present three tests that address the problem of detecting extreme deviations from partisan symmetry in elected legislative bodies. I will first review court precedents that set partisan symmetry as a desirable outcome, and whose absence may help define a partisan gerrymander. I will describe mathematical approaches, grounded in longstanding statistical practice, to detect partisan asymmetry. I present two analyses: one that measures effects, defined as the number of seats that are gained by a gerrymander; and one that detects intents, as evidenced by a pattern of district-level partisan outcomes that is unlikely to have arisen by chance, and therefore imply deliberate actions by the legislature and/or the entity that drew the district lines. The number-of-seats measure specifically overcomes the central difficulty that representation is not necessarily proportional to public support. This consequence, which has long been known and results from the winner-take-all nature of individual elections[1], is intuitive but wrong. My calculation of effects replaces this concept. I will then use these analyses to construct three tests for use in evaluating cases of gerrymandering. These tests will be presented at greater length elsewhere[2].

I apply these tests to a variety of example cases, starting with the original Gerry-mander of 1812, and moving on to post-2010 Congressional districting plans in all 50 states. I then consider two current federal gerrymandering cases: the Maryland Congressional delegation (*Shapiro v. McManus*[3]) and Wisconsin state Assembly districts (*Whitford v. Nichol*[4]). The tests show that gerrymandering has created partisan distortions that are statistically highly significant. I will end by suggesting ways in which these tests can be used to construct a manageable standard for use by courts and legislatures.

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[1] See Edward R. Tufte, *The Relationship Between Votes and Seats in Two-Party Systems*, 67 AMERICAN POLITICAL SCIENCE REVIEW 540 (1973). For example, in a two-party system, it is theoretically possible for one political party to win 49% of the vote in every district, yet not win a single delegate. Although such an extreme case is highly improbable, strong deviations from proportionality are nevertheless an inherent risk of a winner-take-all district system. From a democratic standpoint, a central question is how to avoid the most extreme distortions. Actual nongerrymandered outcomes are considerably less distorted than the extreme hypothetical scenario described above.

[2] Samuel S.-H. Wang, *Three Tests for Practical Evaluation of Partisan Gerrymandering*, STANFORD LAW REVIEW, forthcoming (2016). Available on the SOCIAL SCIENCE RESEARCH NETWORK, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2671607 (last downloaded March 26, 2016).

[3] *Shapiro v. McManus,* 136 S. Ct. 450 (2015).

[4] *Whitford v. Nichol,* No. 3:15-cv-00421 (W.D. Wis.).

A. CONSTITUTIONAL INJURIES IN A PARTISAN GERRYMANDER

When districting plans are challenged for partisan gerrymandering, litigants assert that voters have lost the ability to elect representatives that fairly reflect their views. Also, redistricting efforts are said to confer specific advantage on one political party at the expense of another. In most partisan gerrymanders, the districting scheme results in the election of delegations that do not naturally reflect the overall preferences of the state's voters. The two fundamental strategies for achieving this outcome are "packing," in which a district is heavily loaded with supporters of the opposing party, so that their votes are wasted; and "cracking," in which a bloc of votes is split across districts to dilute their impact and prevent them from contributing to a majority in any one district.[5]

An important component of remedying a gerrymandering offense is identifying who is harmed, and how. The most obvious harm from partisan gerrymandering is representational. Partisan gerrymandering creates a situation in which the same overall statewide vote share would lead to a very different level of representation for the redistricting party and its opposing target. For example, in the Pennsylvania congressional election of 2012, Democrats won only 5 out of 18 congressional House seats, despite winning slightly more than half of the statewide vote. Democratic winners were packed into districts where they won an average of 76 percent of the vote, while Republican winners won an average of 59 percent.[6] In other words, partisan gerrymandering creates representational asymmetry between the two major political parties.

Partisan gerrymandering can also chill a voter's freedom to choose his/her favored political party. In gerrymandered districts, the noncompetitive nature of the general election leaves the primary election as the only avenue for voters to affect their representation. Such a situation creates a powerful incentive to compel voters to join the dominant political party, even if that party's issue positions do not encompass his or her political views. Since a partisan gerrymander creates noncompetitive districts for both major parties, voters on both sides may potentially feel the chill.

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[5] Justin Levitt, *A Citizen's Guide to Redistricting* , BRENNAN CENTER FOR JUSTICE, 57 (2010) http://www.brennancenter.org/sites/default/files/legacy/CGR%20Reprint%20Single%20Page.pdf (last visited Feb. 17, 2016), at12-13.

[6] Sam Wang, *Let Math Save Our Democracy*, N.Y. TIMES, Dec. 5, 2015, at SR6.

The justiciability (at least in principle) of partisan gerrymandering arises from a series of Supreme Court cases starting with *Davis v. Bandemer*[7], and continuing with *Vieth v. Jubelirer*[8] and *LULAC v. Perry*[9]. In 1986, the Supreme Court established justiciability in *Davis v. Bandemer*, in which Indiana Democrats asserted that they were systematically disadvantaged by their state's legislative map.[10] The Court did not rule in their favor, but they did lay out a cause for action based on a two-prong test: 1) intent—an established purpose to create a legislative districting map to disempower the voters for one party; and 2) effect—proof that an election based on the contested districting scheme led to a distorted outcome.[11]

Partisan gerrymandering's unconstitutionality rests on two rationales: the Fourteenth Amendment's Equal Protection Clause and "one person, one vote" principle, and the First Amendment-based protection of speech and association.[12]

An equal protection-based approach might suggest the possibility of taking a disparate-impact approach to partisan gerrymandering. The *Arlington Heights v. Metropolitan Housing*[13] housing discrimination case established a framework in which courts evaluate a number of factors to identify housing discrimination in the form of disparate impact and/or disparate treatment of groups of differing socioeconomic or racial characteristics. However, the Supreme Court has thus far not adopted standards resembling *Arlington Heights* criteria in the context of partisan gerrymandering. Indeed, the Court has developed an explicit distinction between racial and partisan gerrymandering, as seen in *Vieth v. Jubelirer*.

The *Vieth* case concerned whether Pennsylvania's Congressional districts constituted a partisan gerrymander. In that case, five justices voted to dismiss the claim. Justice Antonin

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[7] *Davis v. Bandemer*, 478 U.S. 109 (1986).

[8] *Vieth v. Jubelirer*, 541 U.S. 267 (2003).

[9] *League of United Latin American Citizens v. Perry*, 548 U.S. 399 (2004).

[10] *Bandemer*, 478 U.S. at 110.

[11] *Bandemer*, 478 U.S. at 128 (upholding the District Court's finding that the Bandemer plaintiffs were required to prove discriminatory intent and effect).

[12] *Bandemer*, 478 U.S. at 122-123; Vieth, 541 U. S., at 314 (J. Kennedy, concurring in judgment; "penalizing citizens because of their participation in the electoral process,. . . their association with a political party, or their expression of political views.", *citing* Elrod v. Burns, 427 U. S. 347 (1976) (plurality opinion)).

[13] *Village of Arlington Heights v. Metropolitan Housing Development Corp.*, 429 U. S. 252 (1977).

Scalia wrote a plurality opinion for four justices. He wrote that "to the extent that our racial gerrymandering cases represent a model of discernible and manageable standards, they provide no comfort here [in the partisan context]"[14]. Justice Kennedy wrote a separate concurrence, and declined to join Justice Stevens's opinion stating that Stevens "would apply the standard set forth in the *Shaw* [race] cases" in "evaluating a challenge to a specific district" on partisanship grounds.[15]

Instead of the *Shaw* standard, Justice Kennedy suggested a basis for determining partisan gerrymandering under the First Amendment. Unlike ethnicity or socioeconomic status, identification with a political party can be changed with little effort. In this respect, partisan identification can be regarded as an act of speech or free association, both of which are protected by the First Amendment. In *Vieth*, Justice Anthony Kennedy has noted that the First Amendment can be interpreted as a mandate for "not burdening or penalizing citizens because of their participation in the electoral process, their voting history, their association with a political party, or their expression of political views.[16] Under general First Amendment principles those burdens in other contexts are unconstitutional absent a compelling government interest."[17]

Although Justice Kennedy left this door open, he did not articulate an actual standard to evaluate partisanship under the First Amendment. The harms I have delineated above suggest two possibilities. First, packing voters into districts based on their partisan affiliation may constitute an infringement of public self-expression, or freedom of speech. Second, chilling of partisan choice may constitute an infringement of freedom of association. Together, these harms constitute a form of viewpoint discrimination. In this way, the purposeful creation of lopsided districts can be linked to First Amendment principles.

Since *Bandemer*, a central difficulty has been establishing a manageable standard, *i.e.* one that provides a reliable and usable determination of whether an offense has occurred. In *Bandemer*, the justices described the effects prong in general terms. Justice White advocated an analysis of an entire districting plan: "A statewide challenge, by contrast, would involve an

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[14] *Vieth*, 541 U.S. at 286.

[15] *Id.* at 321.

[16] *Vieth*, 541 U.S. at 314 (J. Kennedy, concurring); *Elrod v. Burns*, 427 U.S. 347, 362 (1976).

[17] *Elrod*, 427 U.S. at 362.

analysis of 'the voters' direct or indirect influence on the elections of the state legislature as a whole," while also acknowledging that this was "of necessity a difficult inquiry."[18] But eighteen years later in *Vieth*, the plurality opinion stated that no acceptable standard had been established in the intervening time, and therefore it was time to abandon the search.[19] The Court in *Vieth* was notably divided, culminating in five separate opinions.[20] In a separate concurrence, Justice Kennedy provided a fifth vote against invalidating the districts in Pennsylvania, but left the door open for future remedies in other cases if a clear standard could be established.[21] The dissenting four justices voted in favor of a finding of partisan gerrymandering and offered several possible standards, but none was backed by a majority of Justices.[22] This judicial stalemate was left unaltered by the *LULAC v. Perry*[23] case on mid-decade redistricting in Texas.

In this Article, I present three tests that address concerns expressed in the *Vieth* opinions of Justices Scalia and Kennedy. The tests are rooted in fundamental statistics and in a principle of symmetry endorsed by five justices (across multiple opinions) in *LULAC v. Perry*.[24] My method thus offers the advantages of mathematical rigor previously absent from the Court's partisan gerrymandering jurisprudence. As a result, these tests potentially provide a judicially manageable standard—based in the U.S. Constitution—for identifying partisan asymmetry, thereby filling a hole which has been left unfilled by the Court.

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[18] *Bandemer*, 478 U.S. at 143.

[19] *Vieth*, 541 U.S. at 279.

[20] *Vieth*, 541 U.S. 267, 271 (opinion. of J. Scalia, joined by C.J. Rehnquist, and O'Connor and Thomas, JJ.); *id.*, 306 (opinion of J. Kennedy, concurring in judgment); *id.*, 317 (opinion of J. Stevens, dissenting); *id.*, 343 (opinion of J. Souter, dissenting, joined by Ginsburg, J.); *id.*, 355 (opinion of J. Breyer, dissenting).

[21] *Vieth*, 541 U.S. at 306 ("I would not foreclose all possibility of judicial relief").

[22] *Id.*

[23] *League of United Latin American Citizens v. Perry*, 548 U.S. 399 (2004).

[24] *LULAC*, 548 U.S. at 468 (n.9) (opn. of Stevens, J. P., joined by Breyer, S) ("a helpful (though certainly not talismanic) tool"). *LULAC*, 548 U.S. at 473 (n. 11) (opn. of Stevens, J. P.; asymmetry as one of eight criteria he would use for determining effects-based violations). LULAC, 548 U.S. at 466 (opn. of Stevens, J.) ("Plan 1374C [the challenged plan] is inconsistent with the symmetry standard"). LULAC, 548 U.S. at 483 (opn. of Souter, J.)("do not rule out the utility of a criterion of symmetry"; "interest in exploring this notion is evident [on the Court]"). LULAC, 548 U.S. at 420 (opn. of Kennedy, J. joined by Justices Souter and Ginsburg)(indicating use as a standard based on election results, but not hypothetical future results).

B. SEARCHING FOR A MANAGEABLE STANDARD: THE CURRENT STATE OF PLAY

Repeatedly, Supreme Court opinions have expressed the desire to find a manageable standard for partisan gerrymandering. In *Vieth*, Justice Anthony M. Kennedy stated: "When presented with a claim of injury from partisan gerrymandering, courts confront two obstacles. First is the lack of comprehensive and neutral principles for drawing electoral boundaries. No substantive definition of fairness in districting seems to command general assent. Second is the absence of rules to limit and confine judicial intervention."[25] This concern has been longstanding. In *Bandemer*, Justice O'Connor expressed concern that the plurality's standard "will over time either prove unmanageable and arbitrary or else evolve towards some loose form of proportionality."[26] This statement was quoted in the *Vieth* plurality decision[27] by Justice Scalia, who also expressed pessimism that such standards could ever be established. However, the opinion in *LULAC* suggests partisan symmetry as a fresh start. Inspired by *LULAC*, I will build upon partisan symmetry to develop statistical ideas that are aimed at overcoming or bypassing past concerns.

As a guiding principle to defining fairness in districting, political scientists Bernard Grofman and Gary King have suggested[28] partisan symmetry: the idea that if the popular vote were reversed, the seat outcome should also reverse. A majority of the Supreme Court in *LULAC* was favorable to the symmetry concept. However, a consensus has not yet emerged on how to turn this idea into a specific standard.[29]

The current state of Constitutional law of partisan gerrymandering could be viewed with pessimism. In the words of the four-vote *Vieth* plurality, the application of the *Bandemer* standard "has almost invariably produced the same result (except for the incurring of attorney's fees) as would have obtained in the question were nonjusticiable: judicial intervention has been

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[25] *Vieth*, 541 U.S. at 306-7 (J. Kennedy, concurring).

[26] *Bandemer*, 478 U.S. at 155 (C.J. Burger, concurring).

[27] *Vieth*, 541 U.S. at 281.

[28] Bernard Grofman & Gary King, *The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after* LULAC v. Perry, 6 ELECTION LAW JOURNAL 2 (2007).

[29] *Vieth*, 541 U.S. at 306-7 (J. Kennedy, concurring).

refused."[30] The *Vieth* plurality further stated that "no judicially discernible and manageable standards for adjudicating political gerrymandering claims have emerged. Lacking them, we must conclude that political gerrymandering claims are nonjusticiable and that *Bandemer* was wrongly decided."[31] However, the suggestion to overturn *Bandemer* is not controlling because of Justice Kennedy's concurring opinion.[32] Still, unless a manageable standard is found, partisan gerrymandering will be nonjusticiable in practice, leaving the *Bandemer* standard toothless.

A more optimistic view is to ask whether the partisan-symmetry idea in *LULAC* points to a way forward. An effective and manageable standard should be immune to the criticisms identified above. I suggest that such a standard should have the following minimum qualities: (1) It should be based on the general concept of partisan symmetry[33]. (2) It should not use circuitousness of geographic boundaries or districting procedures. (3) It should not use election results for offices other than the ones that are in dispute. Finally, any standard that can be clearly stated without case-specific or mathematics-intensive assumptions might even allow a court to instruct experts on how and where to apply more-detailed mathematical or other analysis.

C. MATHEMATICAL METHODS CAN IDENTIFY NATIONAL AND STATE-LEVEL IMBALANCES

In nationwide elections, majoritarian representativeness is the norm. In the U.S. House of Representatives, when a major party gets more than 50% of the vote, it almost always gets over 50% of the seats. In 35 elections, this basic principle has been violated only twice: in 1996 and in 2012.[34] However, national election results give only an aggregated view, and therefore may conceal many sins. Detecting problems in districting requires examination at a state-by-state level.

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[30] *Vieth*, 541 U.S. at 279.

[31] *Vieth*, 541 U.S. at 281.

[32] *League of United Latin American Citizens v. Perry*, 548 U.S. 399 (2004).

[33] Grofman & King, *supra* note 28.

[34] A failure rate of 2 out of 35, or 6%, may be considered acceptable, when one considers the following comparison: in the history of the United States, the popular vote winner has failed to win the Presidency in 4 out of 57 elections (see DAVE LEIP'S ATLAS OF PRESIDENTIAL ELECTIONS, http://uselectionatlas.org/ (last visited Aug. 20, 2015), a 7% rate. However, Presidential elections rely on fixed state boundaries. Retaining representative performance in legislative elections carries added risk due to changes in where and how district boundaries are drawn.

As previously discussed, anti-majoritarian outcomes do not by themselves constitute proof of deliberate distortion of electoral processes. But, they do present a preliminary clue that those who draw the districts can influence the relationship between voting and representative outcomes. For example, in the Congressional election of 2012, in five states the statewide popular vote favored the opposite party as the delegation that their votes elected: Arizona, Michigan, North Carolina, Pennsylvania and Wisconsin. Four of these five anti-majoritarian outcomes were enabled by their beneficiary, the Republican Party, which controlled the redistricting process.[35] Thus antimajoritarian outcomes often, but not always, reflect the partisan interests of those who draw the boundaries. As political parties become a greater predictor of legislative voting patterns, representing partisan loyalties accurately becomes increasingly important for getting policy outcomes to reflect popular sentiment.[36]

Even if some imagined ideal of districting could maximize the likelihood of a majoritarian outcome, lack of congruence with this standard could still arise by chance and small variations in opinion. In 2012, if a few thousand voters in Arizona had cast their ballots for a Republican instead of a Democrat in the 1st or 2nd District, the delegation would have been, like the state's popular vote, majority Republican.[37] Thus anti-majoritarian outcomes are not always accurate indicators of partisan maneuvering. Furthermore, a simple majoritarian standard is incomplete because it only addresses the issue of whether seats or votes fall above or below a 50% threshold. For example, if a party receives 51% of the vote, receiving 55% or 80% of the seats are both majoritarian outcomes, but might be viewed quite differently.

A statistical approach is needed to distinguish what degree of inequity is allowable. I will use natural variation and basic concepts of statistics to build three tests for state-level partisan gerrymandering. My approach allows the user to consider conceptual subtleties and at the same time obtain unambiguous judgments without need for elaborate computation using methods whose details have either not been widely adopted by political science researchers, and/or found

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[35] Griff Palmer & Michael Cooper, *How Maps Helped Republicans Keep and Edge in the House*, N.Y. TIMES, Dec. 14, 2012, at A10.

[36] Delia Baldassarri & Andrew Gelman, *Partisans without Constraint: Political Polarization and Trends in American Public Opinion*, 114 AMERICAN JOURNAL OF SOCIOLOGY 408 (2008) *See also* NOLAN MCCARTY, KEITH T. POOLE & HOWARD ROSENTHAL, POLARIZED AMERICA: THE DANCE OF IDEOLOGY AND UNEQUAL RICHES (2008).

[37] State of Arizona, canvass of election results, http://apps.azsos.gov/election/2012/General/Canvass2012GE.pdf, 4-6.

by courts not to be persuasive in the outcome. I hope that a more straightforward approach may meet with wide approval and serve as a universal tool to assess claims of partisan gerrymandering objectively. In this way, the approach described here may eventually serve as a core part of a court's analysis of partisan gerrymandering.

II. QUANTITATIVELY ANALYZING THE EFFECTS AND INTENTS OF PARTISAN GERRYMANDERING

The *Vieth* plurality opinion referred disparagingly to the concept of fairness as "flabby."[38] Quantitative approaches open the possibility of formulating a more muscular definition. This Article will provide methods to identify partisan unfairness at the whole-state level, resulting in proposed standards for partisan gerrymandering that do not require the drawing of hypothetical maps.

To establish statistical tests, it is first important to examine past patterns of gerrymandering. I will use several well-known examples to illustrate two analyses. Analysis #1 uses computer simulations to quantify the effects of gerrymandering. This analysis of effects can then be used as independent validation for Analysis #2, which identifies when win margins have been arranged to give a systematic pattern of reliable wins. Analysis #2, which reflects the intent of the redistricting party, can be done using a hand calculator and therefore can be done easily and rapidly.

This approach recalls Justice Kennedy's statement that "new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards."[39]

A. ANALYSIS OF EFFECTS: WHAT IS AN APPROPRIATE RANGE OF SEATS FOR A GIVEN SHARE OF VOTES?

1. DISTINGUISHING PARTISAN DISTORTION FROM VOTING RIGHTS ACT SECTION 2 CONSTRAINTS

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[38] *Vieth*, 541 U.S. at 281.

[39] *Id.* at 312-313 (J. Kennedy, concurring).

Although partisan gerrymandering is considered justiciable, another practice that uses similar districting methods is permitted and even mandated under Section 2 of the Voting Rights Act: the establishment of districts in which an ethnic minority constitutes a majority of the district's inhabitants.[40] These "majority-minority" districts are constructed to ensure that the interests of identified subgroups are represented. When such minorities are much less than 50% of a state's population, they can end up on the losing side of every election. To counteract this risk, majority-minority districts are constructed to cluster groups with shared interests[41].

This dual use of district-drawing methods opens the challenge of how to construct an analysis that identifies partisan gerrymandering as anomalous, but not single districts that are drawn to create ability-to-elect districts such as majority-minority districts. Such an analysis will require the evaluation of groups of districts at once. Existing doctrine may provide some guidance.

Among the standards for the proper establishment of majority-minority districts is the concept that majority-minority districts should comprise a fraction of all districts that does not exceed the proportion of the minority population.[42] In U.S. court precedent, the "no-more-than-proportional" concept contributes to "*Gingles* criteria" for evaluating districting schemes.[43] Where minority representation is concerned*, Gingles* criteria identify rough proportionality as a relevant factor in evaluating the fairness of a districting plan. Under that standard, the Court has held "that no violation of § 2 can be found here, where, in spite of continuing discrimination and racial bloc voting, minority voters form effective voting majorities in a number of districts roughly proportional to the minority voters' respective shares in the voting-age population. While such proportionality is not dispositive in a challenge to single-member districting, it is a relevant fact in the totality of circumstances to be analyzed when determining whether members of a minority group have 'less opportunity than other members of the electorate to participate in

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[40] Stephen Ansolabehere and Maxwell Palmer, A Two Hundred-Year Statistical History of the Gerrymander, Presentation at the Congress & History Conference, Vanderbilt University (May 22-23, 2015)..

[41] *How New York State's Approved Redistricting Lines Compare with Old District*s, REDISTRICTING AND YOU, http://www.urbanresearchmaps.org/nyredistricting/map.html (last visited Aug. 20, 2015).

[42] §2 of the Voting Rights Act of 1965, 79 Stat. 437, as amended, 42 U. S. C. § 1973.

[43] *Thornburg v. Gingles*, 478 U.S. 30 (1986); *Johnson v. De Grandy*, 512 U.S. 997 (1994).

the political process and to elect representatives of their choice.'"[44] For example, if a minority group with 20% of a state's eligible population is able to elect representatives in 20% of a state's districts, this argues against violation of *Gingles* criteria[45].

The idea underlying the *Gingles* criteria can be used to address the question of appropriate representation by political parties. I suggest that a redistricting plan is acceptable if it moves the seats-to-votes outcome toward partisan proportionality (eu-proportionality) as measured by prevailing national standards; and unacceptable if it moves the outcome away from proportionality (dys-proportionality) beyond the zone of chance. This standard can be understood at a glance using a plot (**Figure 1**) that I term a "representation
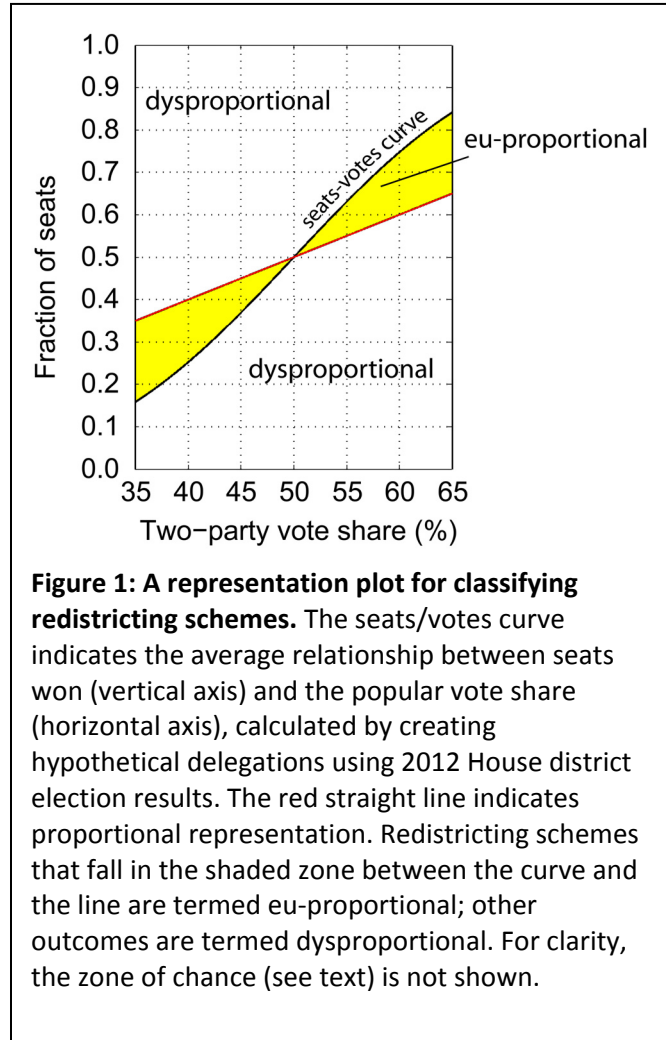

Figure 1: A representation plot for classifying redistricting schemes. The seats/votes curve indicates the average relationship between seats won (vertical axis) and the popular vote share (horizontal axis), calculated by creating hypothetical delegations using 2012 House district election results. The red straight line indicates proportional representation. Redistricting schemes that fall in the shaded zone between the curve and the line are termed eu-proportional; other outcomes are termed dysproportional. For clarity, the zone of chance (see text) is not shown.

plot," or alternatively a "bowtie plot," where euproportional outcomes are "inside the bowtie." Since dys-proportional outcomes are a major result of partisan gerrymandering, a standard should distinguish between eu-proportionality and dys-proportionality.[46]

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[44] *Id.* at 1000 (finding no violation of §2 of the Voting Rights Act of 1965, 79 Stat. 437, as amended, 42 U. S. C. § 1973).

[45] *Thornburg v. Gingles*, 478 U.S., 74-77 (describing near-proportional legislative representation of black voters as evidence of their ability to elect their preferred representatives).

[46] In this plot, the red line indicates proportionality and is a straight line drawn from zero vote share and zero seat fraction to 100% vote share and 100% seat fraction. The seats/votes curve is calculated by resampling to build "fantasy delegations" (see Section II.A.3) and is approximated by the mathematical function that is the area under a bell-shaped curve whose average is 50% vote share, and whose standard deviation is 14% vote share.

I note that the eu-proportionality concept specifically does *not* imply the establishment of proportional representation, a rule that is not to be found in the Constitution or in U.S. districting law, and which does not arise in a single-member district system. Single-member districts usually generate outcomes in which a party's share of seats tends to exceed its proportion of popular support.[47] Instead, the eu-proportionality concept relies on the idea that some deviations from an average seats-to-votes relationship are beneficial for representation, whereas other deviations are detrimental. Good districting seeks to establish "fair and effective representation for all citizens."[48] The concept that deviations toward proportionality are good encompasses a wide range of concepts that includes (a) establishing appropriate levels of representation for minority groups (*viz., Gingles* criteria); (b) allowing the possibility that lick a racial group, a political party with considerably less than 50% support might permissibly have enhanced representation relative to what would be predicted from national seats/votes relationships, but that reduced representation is impermissible; and (c) setting reasonable limits to how much enhancement from (b) is allowed. In this way, the Platonic ideal of proportionality does not set a specific goal, but instead defines a direction of acceptable deviation. It is simple to state, it is flexible, and it contains many permissible outcomes.

2. DEFINING THE ZONE OF CHANCE

In addition to defining desirable and undesirable directions, a standard for partisan gerrymandering requires a method for determining whether a change could have arisen as part of normal variation in districting as practiced across the United States. I will use the rules of probability to (a) describe that variation, (b) establish what the range of possible outcomes is, and (c) formulate a rule for identifying situations in which a state's new districting scheme has departed sufficiently from normal practice.

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[47] Proportional representation is achieved only in systems where it is enforced specifically and directly. For example, in Israel, members of the national legislative body, the Knesset, are assigned so that the number of a party's seats is proportional to the fraction of its popular vote. (article 4 of the Basic Law: The Knesset) Such a system embodies a legislature-centered form of the "one man, one vote" principle: each citizen's party preference is reflected proportionally at the national level.

[48] *Reynolds v. Sims*, 377 U. S. 533, 565 (1964).

Faulty bright-line standards such as a majoritarian standard can be repaired by identifying a "zone of chance,"[49] which I define as the range of outcomes that could have arisen, without deliberate planning, from variations in how districts are drawn.[50] I will calculate zones of chance for (a) the number of seats won in an election for any given statewide division of popular vote, and (b) the pattern of voting outcomes across districts.

The zone-of-chance approach recalls Justice Kennedy's statement that "new technologies may produce new methods of analysis that make more evident the precise nature of the burdens gerrymanders impose on the representational rights of voters and parties. That would facilitate court efforts to identify and remedy the burdens, with judicial intervention limited by the derived standards."[51] At the same time, I will also take advantage of longstanding statistical tests whose history assures their mathematical rigor. The use of statistical tests also allows judges to evaluate evidence more directly, with less need for assistance from external experts.

To understand the zone-of-chance concept. it is helpful to start by considering a case that is mathematically simple, and does not require computer simulation: equally matched parties.

As pointed out in the plurality opinion in *Vieth v. Jubelirer*, any districting scheme contains the possibility that a majority of votes will, by chance, lead to a minority of seats. To explore this concern, it is informative to calculate the exact probability that such a deviation could occur in the absence of intentional partisan districting. The calculation is simplest when the two-party popular-vote share (defined as the fraction of the top two parties' popular vote won by one party) is close to 50% for each party. In this circumstance, party A's seat-share for a random partitioning of N districts is on average N/2, and the probability of party A winning a particular district is 0.5. The actual number of districts won will vary, in the same way that a series of coin tosses are not guaranteed to yield equal numbers of heads and tails. The outcome

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[49] Sam Wang, *Let Math Save Our Democracy*, N.Y. TIMES, Dec. 5, 2015, at SR6. http://www.nytimes.com/2015/12/06/opinion/sunday/let-math-save-our-democracy.html (last visited Jan. 27, 2016).

[50] The zone of chance concept is a way to express the concept of significance testing in statistics. Statisticians calculate how far a measurement, such as the number of seats won by a party in a given election, is likely to stray from the expected average. In this article, I define the zone of chance as a region within which chance outcomes would fall 95% of the time, and outside the region 5% of the time. Statistics texts refer to this as a "p<0.05" or "α<0.05" standard. See Richard Lowry, Chapter 7. Tests of Statistical Significance: Three Overarching Concepts, VASSARSTATS, http://vassarstats.net/textbook/ch7pt1.html (last visited Dec. 27, 2015). Also see Wang, *supra* 49.

[51] *Vieth*, 541 U.S. at 312-313 (J. Kennedy, concurring).

will be within one standard deviation of the average about two-thirds of the time, and outcomes within this range would be fairly unsurprising[52]. And if the vote share is almost exactly 50%, then outcomes will give a majority to the other side close to half of the time.

To generalize the zone-of-chance calculation, I will use computer simulation. I will use existing districts in the year under examination as a source of information about how vote totals in districts may vary. The inputs to the calculation are: the Congressional vote totals for the state under examination; and national district-by-district Congressional results from the same year. This process escapes the burden of drawing boundaries, which requires the researcher to apply his/her standards about "good districting." This calculation will yield both a general seats/votes relationship and a statistical confidence interval (a.k.a. zone of chance) for the range of outcomes that could be expected in the absence of directed partisan intent. The zone of chance provides an answer to the question of whether a set of election outcomes has deviated sharply from national standards.

3. NATIONAL DISTRICTING PATTERNS CAN BE USED TO IDENTIFY A NATURAL SEATS/VOTES RELATIONSHIP

Computer simulations can be used to ask a simple question: if a given state's popular House vote were split into differently composed districts carved from the same statewide voting population, what would its Congressional delegation look like? The answer allows the definition of a range of seat outcomes that would arise naturally from districting standards that are extant at the time of the election in question.

It is possible to calculate each state's appropriate seat breakdown—in other words, how a Congressional delegation would be constituted if its districts were not contorted to protect a political party or an incumbent. This is done by randomly selecting combinations of districts

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[52] For example, if all N races are perfect toss-ups, then they behave like coin tosses, and according to the laws of probability the standard deviation of the outcome, a measure of variation often referred to as "sigma," or σ, is $0.5 * \sqrt{N}$. Thus if political parties A and B compete in a state that is composed of 16 Congressional districts, all of which are closely contested, then each party can expect to get 8 seats on average. Sigma for the specific case of all-close-races is $0.5 * \sqrt{16} = 2$ seats, suggesting that each party would typically get 6 to 10 seats. It must be noted that the foregoing formula for sigma is a substantial overestimate of real-life situations, because districting generates a mixture of more and less closely-contested districts, and only close contests contribute to uncertainty. To estimate the true value of sigma, which is typically smaller, a more sophisticated approach is required, as detailed in section III.B., Identifying a Natural Seats-from-Votes Relationship.

from around the United States that add up to the same statewide vote total for each party. Like a fantasy baseball team, a delegation put together this way is not constrained by the limits of geography. On a computer, it is possible to create millions of such unbiased delegations in short order. In this way, one can ask[53] what would happen if a state had districts whose distribution of voting populations was typical of the pattern found in rest of the nation. Because this approach uses existing districts, it uses as a baseline the asymmetries that are present nationwide.[54] Indeed, the average result of these simulations approximates a "natural" seats/votes relationship that can be defined with mathematical rigor and exactitude. In short, these simulations detect distortions in representativeness in one state, relative to the rest of the nation.

Using a standard ThinkPad X1 Carbon laptop computer equipped with the mathematical program MATLAB, simulation code[55] can perform one million simulations for a state in less than 20 seconds. **Figure 2** shows 1000 such "simulated delegations" for the state of Pennsylvania, along with the actual outcome in blue. The black curve defines a mathematically expected average seats/votes relationship.

It is apparent that most possible redistrictings would have resulted in a more equitable Congressional delegation. For outcomes with the same popular-vote split (50.7% D, 49.3% R), one million simulations gave a median result of 8 Democratic, 10 Republican seats (average, 8.5

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[53] This can be done by using all 435 House race outcomes. For a state X with N districts, calculate the total popular vote across all N districts. Now pick N races from around the country at random and add up their vote totals. If their vote total matches X's actual popular vote within 0.5%, score it as a comparable simulation. See Sam Wang, *The Great Gerrymander of 2012*, N.Y. TIMES, Feb. 2, 2013, at SR1.
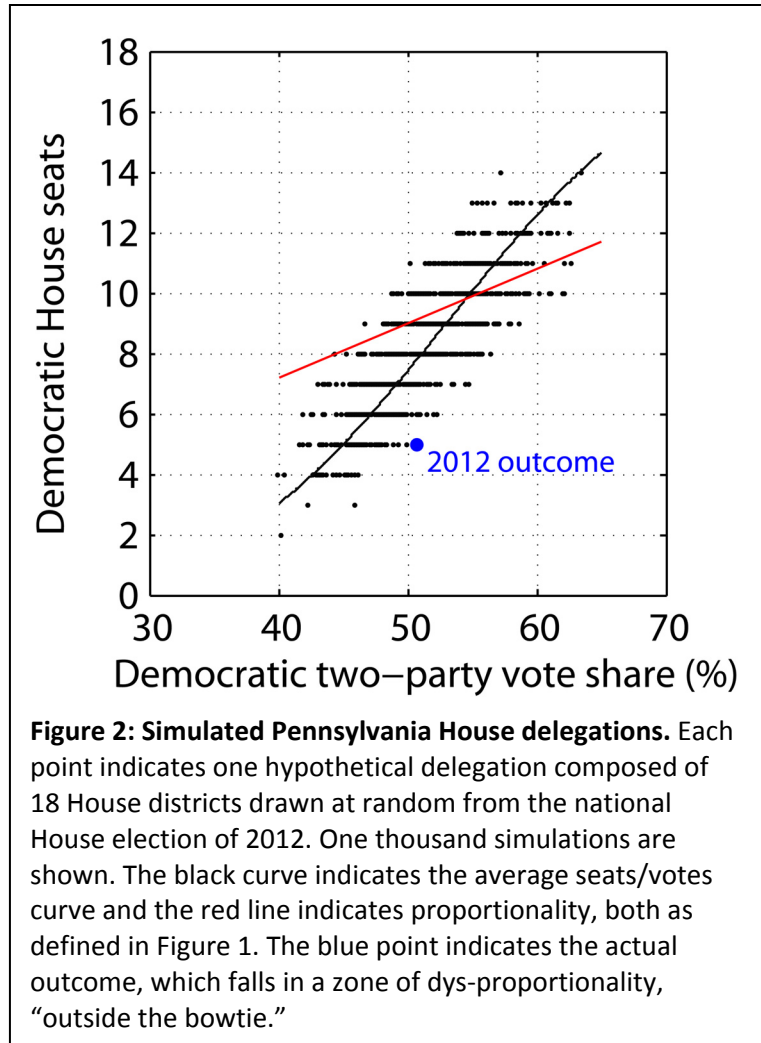
[54] It is possible to explore the properties of this simulation procedure by giving it a variety of hypothetical nationwide distributions of districts as starting data. These hypothetical scenarios reveal that the "fantasy delegation" procedure has important features that are required of a descriptor of partisan asymmetry. First, for a symmetric distribution of Congressional districts, *i.e.* a scenario in which Democrat-dominated districts are no more packed than Republican-dominated districts, fantasy delegations are typically majoritarian, awarding more representatives to the party that receives more votes. Second, the fantasy delegations have the same natural variation in partisan composition as the nationwide distribution, as measured by standard deviation. Third, when the nationwide distribution of districts has asymmetry, for instance containing a number of districts that are very packed with one party (as is the case in real life for Democrats), the fantasy delegations show a bias toward the other party, a phenomenon that is well analyzed (reviewed in Jowei Chen and Jonathan Rodden, *Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures*, 8 QUARTERLY JOURNAL OF POLITICAL SCIENCE 239, 248 (2013) [hereinafter Chen & Rodden, *Unintentional Gerrymandering*]).

[55] The MATLAB software is available at GITHUB, https://github.com/ (last visited Aug. 24, 2015).

D). The actual outcome was 5 Democratic, 13 Republican; however, only 0.2% of the million simulations led to such a lopsided (or a more lopsided) split favoring Republicans.

Pennsylvania is known to have been targeted by the Republican State Legislative Committee's project Redmap, a multiyear effort to facilitate and carry out aggressive redistricting after the 2010 Census.[56] A similar computational analysis of all 50 states can be done to test if additional Redmap states show statistical anomalies.

For all 50 states, **Figure 3** is calculated using the vote outcomes

Figure 2: Simulated Pennsylvania House delegations. Each point indicates one hypothetical delegation composed of 18 House districts drawn at random from the national House election of 2012. One thousand simulations are shown. The black curve indicates the average seats/votes curve and the red line indicates proportionality, both as defined in Figure 1. The blue point indicates the actual outcome, which falls in a zone of dys-proportionality, "outside the bowtie."

of non-extreme states (shaded in gray) to feed the simulations.[57] These results coincide strongly with targeted partisan redistricting efforts[58] and are highly unlikely to have arisen by chance. Red

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[56] Olga Pierce, Justin Elliott & Theodoric Meyer, *How Dark Money Helped Republicans Hold the House and Hurt Voters*, PROPUBLICA (Dec. 21, 2012, 3:36 PM), http://www.propublica.org/article/how-dark-money-helped-republicans-hold-the-house-and-hurt-voters; Gregory Giroux, *Republicans Win Congress as Democrats Get Most Votes*, BLOOMBERG BUSINESS (Mar. 18, 2013 8:00 PM), http://www.bloomberg.com/news/articles/2013-03-19/republicans-win-congress-as-democrats-get-most-votes; Tim Dickinson, *How Republicans Rig the Game*, ROLLING STONE, Nov. 21, 2013, at 36.

[57] Statewide vote totals may include some races that are uncontested. In these districts, it is not known how the voters would have decided if they had an alternative choice. In order to address this, it may also be necessary to assign those voters assuming a split other than 100%/0%. One established approach is to assume a 75%/25% split; see Andrew Gelman & Gary King, *A Unified Method of Evaluating Electoral Systems and Redistricting Plans*, 38 AMERICAN JOURNAL OF POLITICAL SCIENCE 541, 550 (1994). Generally, this alternative assumption does not affect the outcome of the tests in this Article.

shading indicates Republican Party control over redistricting, blue indicates Democratic Party control, and black indicates nonpartisan commission (AZ, Arizona) or a court-ordered map (TX, Texas). Out of 10 states with extreme outcomes, 8 favored the party that controlled the process, and none worked against the party in control.[59] Indeed, the extreme cases include all states with single-party control that have been mentioned on a redistricting watchlist published in 2011 by the *Washington Post*.[60]

Later in this Article, I will develop an analysis of intents (II.B.) that again uses the zone-of-chance concept. There, as here, the standard deviation, sigma, will be used as a yardstick of deviations from the average expected outcome. As before, the
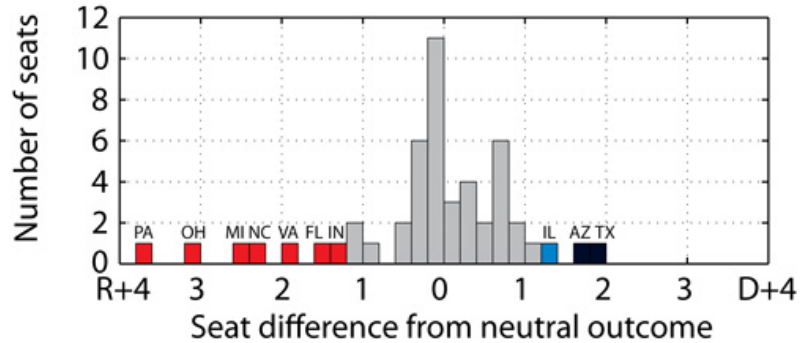

Figure 3: State-by-state differences between simulated and actual outcomes in the 2012 Congressional election. "R+" indicates that the actual outcome was more favorable to Republicans than random resampling from national races. "D+" indicates that the actual outcome was more favorable to Democrats. Color shading for discrepancies greater than 1.2 seats indicates who controlled redistricting: red for Republicans, blue for Democrats, and black for mixed control (AZ, nonpartisan commission, TX Republican Party and court-ordered changes).

general idea is that an average outcome only reflects one point in a range of outcomes, and the standard deviation (often referred to as sigma, or σ) is necessary to define a zone of chance. Generally speaking, for a bell-like curve, which these simulations approximately follow, a difference of 1.6 standard deviations or more occurs by chance in 5% of cases. 5% is a common

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[58] Pierce, Elliott & Theodoric Meyer, *supra* note 56; Giroux, *supra* note 56; Dickinson, *supra* note 56. *See also* Cynthia Canary & Kent Redfield, *Partisanship, Representation and Redistricting: An Illinois Case Study*, *Paper #38*, THE SIMON REVIEW OF THE PAUL SIMON PUBLIC POLICY INSTITUTE (2014), http://paulsimoninstitute.siu.edu/_common/documents/simon-review/Canary-Redfield%20Redistricting%20Paper%20Final%20Text.pdf.

[59] In Arizona, small shifts in voting in two districts would have altered the overall outcome to near-neutrality (*supra* note 37). Texas is a complex case in which redistricting was constrained by multiple factors favoring both parties, including the establishment of multiple ability-to-elect districts (*infra* note 66).

[60] Aaron Blake & Chris Cillizza, *The Top 10 States to Watch in Redistricting*, WASH. POST POLITICS BLOG (Mar. 18, 2011) http://www.washingtonpost.com/blogs/the-fix/post/the-top-10-states-to-watch-in-redistricting/2011/03/18/ABju9Ar_blog.html.

threshold for determining statistical significance.[61] The standard deviation, sigma, is a handy and universal reference measure for detecting extreme outcomes, and it applies to all the analyses and tests in this Article. For convenience of notation in Tables, I define the quantity delta (δ) as the difference divided by sigma.

Table 1 shows states for which the partisan discrepancy was greater than 1 sigma in 2012. For comparison, discrepancies for the same states are shown for 2010 and 2014. Simulation-based values for sigma are given in the columns labeled "SD (sigma)".[62]

Five states showed deviations that were greater than one sigma and less than two sigma: Florida, Illinois, Indiana, Maryland, and Virginia. Six more states showed a deviation exceeding two sigma: Arizona, Michigan, North Carolina, Ohio, Pennsylvania, and Texas. Of these eleven states, Redmap's redistricting efforts are known to have targeted five:[63] Indiana and all four Republican-controlled states with two-sigma discrepancies, namely Michigan, North Carolina, Ohio, and Pennsylvania.[64] Of the remaining greater-than-two-sigma states, a fifth state, Texas, was redistricted by Republicans but showed a discrepancy favoring Democrats. A sixth state, Arizona, was redistricted by an independent commission and favored Democrats.

Of these six states, I will briefly describe how three cases of special interest: California, Texas, and Florida.

California. As a counterexample to the imbalanced states shown above, the example of California is worth mentioning. California was redistricted by an independent commission. In 2012, the California House popular vote was 62% Democratic resulting in 38 out of 53, or 72%,

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[61] A difference of Delta=1 or more in a dysproportional direction occurs in approximately 16% of cases. A difference of Delta = 2 or more occurs in approximately 2.3% of cases. A difference of Delta=3 or more occurs in approximately 0.13% of cases. These values are for Analysis #1, which uses a bell-shaped curve. Analyses #2 and #3 use the t-distribution, which gives slightly different values.

[62] These values are approximated reasonably well by the formula sigma = 0.52 * $\sqrt{s * (N\text{-}s) / N}$, where N is the number of a state's Congressional districts and s is the average number of seats won in that state by either major party in computer simulations. The principal difference from the "all tossups" example is the appearance of a factor of 0.52, which arises from the fact that some districts are competitive, and some are not; this factor fell within a narrow range of 0.50-0.53 between 2008 and 2014.

[63] *See* THE REDISTRICTING MAJORITY PROJECT, http://www.redistrictingmajorityproject.com/ (last visited Aug. 24, 2015).

[64] Pierce, Elliott & Theodoric Meyer, *supra* note 56; Giroux, *supra* note 56; Dickinson, *supra* note 56.

Democratic seats. However, the average simulated delegation was also 72% Democratic.[65] Thus election results in California exactly meet the expectations that arise from nationwide districting patterns.

Texas. Although the resampling simulations are a powerful and sensitive measure, the case of Texas demonstrates how examination of additional factors can be necessary. Before the 2012 election in Texas, a complex series of legal battles culminated in a court-ordered redistricting plan[66] and a Congressional election outcome in which over 60% of Texas voters voted for Republicans to elect 24 out of 36 seats. From a statistical standpoint, this was an underperformance for Republicans, who in a simulation would have won over 28 seats on average—a discrepancy of delta = 2.3 times sigma, which is outside the zone of chance, and therefore a statistically significant deviation. One major factor contributing to this discrepancy was the presence of Hispanic majorities in seven districts,[67] six of which elected Democratic Congressmen. These majority-minority districts, which have special status under the Voting Rights Act of 1965, reflect the growing Hispanic population in Texas, which as of the 2010 Census constituted 38% of Texans[68]. Democrats won approximately 40% of the statewide two-party popular vote and won 12 out of 36 seats (33% of seats); this outcome is eu-proportional compared with national standards.[69] The number of majority-minority districts (which usually favor Democrats) falls within the *Gingles* criteria. Thus the final outcome in Texas in 2012 favored the partisan minority for mandated race-based reasons, is in the opposite direction as dys-proportionality, and would not necessarily be grounds for further action.

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[65] A theoretical symmetric distribution of districts would, on average, give a delegation that is 79% Democratic. For a symmetrically distributed distribution of districts whose two-party vote share has standard deviation *SD*, the expected fraction of seats S for a given vote share V is normcdf((V-0.5)/*SD*), where normcdf is the integral of a bell-shaped normal curve with mean 0 and width parameter 1. For non-dysproportional states in 2012, SD=0.15, comparable to longstanding findings for seats/votes curves. GRAHAM GUDGIN & PETER J. TAYLOR, SEATS, VOTES, AND THE SPATIAL ORGANISATION OF ELECTIONS (1979), 20-31.

[66] *Redistricting in Texas*, BALLOTPEDIA, http://ballotpedia.org/wiki/index.php/Redistricting_in_Texas (last visited Aug. 24, 2015).

[67] *The Almanac of American Politics 2016*, Douglas Matthews, Grant Ujifusa, Michael Barone.

[68] Sharon R. Ennis, Merarys Rios-Vargas & Nora G. Albert, *The Hispanic Population: 2010*, 2010 CENSUS BRIEFS (2011), http://www.census.gov/prod/cen2010/briefs/c2010br-04.pdf

[69] NEW YORK TIMES ONLINE, http://elections.nytimes.com/2012/results/states/texas Last visited Feb. 18, 2016.

Florida. In this case, where the value of delta is between one and two, a similar but statistically stronger answer is given by a map-drawing approach. Chen and Rodden took a geographically intensive approach, drawing districts using automated rules of contiguity and community-preservation and implemented these rules thousands of times through detailed computer simulations.[70] They found that Florida's 2010 redistricting scheme was more favorable to Republicans than over 99% of their simulations, indicating that the Florida Legislature applied an approach that led to a more partisan outcome than Chen and Rodden's rules would support.[71] Florida's state Constitution mandates specific principles of districting and allows for judicial review by the state Supreme Court.[72] In December 2015, the Florida Supreme Court replaced the map to comply with the state Constitution.[73]

Nationwide, repairing the one-sigma and greater Republican-redistricted states (seven in all) would lead to an average swing of approximately 28 seats (an average of 27.7) toward Democrats; repairing the two Democratic-redistricted states, Illinois and Maryland, would lead to an average swing of approximately 6 seats (an average of 5.7) toward Republicans. Therefore, based on these measures, Republican gains in 2012 from aggressive redistricting (28 seats) were nearly five times the advantages gained by Democrats from the same process (6 seats). This sharp asymmetry coincides with a period during which state legislative processes have

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[70] Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 54; Jowei Chen and Jonathan Rodden, *Cutting Through the Thicket: Redistricting Simulations and the Detection of Partisan Gerrymanders*, 14 ELECTION LAW JOURNAL (forthcoming 2015) [hereinafter Chen & Rodden, *Cutting Through the Thicket*]; Jowei Chen and Jonathan Rodden, Report on Computer Simulations of Florida Congressional Districting Plans (February 15, 2013) (unpublished manuscript) (on file with author) [hereinafter Chen & Rodden, Report on Computer Simulations]; Jowei Chen and Jonathan Rodden, Supplemental Report on Partisan Bias in Florida's Congressional Redistricting Plan (October 21, 2013) (unpublished manuscript) (on file with author) [hereinafter Chen & Rodden, Supplemental Report].

[71] *id.*

[72] Fla. Const. art. III, §§ 20-21.

[73] League of Women Voters of Florida v. Detzner, Florida S. Ct. No. SC14-1905, 2015. See
http://redistricting.lls.edu/cases-FL.php#FL

Table 1: Discrepancies Between Simulated and Actual Delegations for the 2010-2014 House Elections. For 2010, 2012, and 2014, one million simulations were done for each state, resampling was done from nationwide House election returns for that year. The "SD (sigma)" column indicates the value of sigma calculated from the simulations. Color text indicates values of Delta (difference between simulation and actual) exceeding 1 times sigma favoring either party. Shading indicates differences exceeding 2 times sigma. Note the persistence of effects in 2014.

	2010						2012						2014				
	Total seats	Democratic vote share	Dem. Seats	Simulated average	SD (sigma)	Difference in SD	Total seats	Democratic vote share	Dem. Seats	Simulated average	SD (sigma)	Difference in SD	Democratic vote share	Dem. Seats	simulated average	SD (sigma)	Difference in SD
Arizona	8	46.74%	3	3.18	0.78	D by 0.2	9	45.60%	5	2.96	0.76	D by 2.7	43.17%	4	2.94	0.75	D by 1.4
Florida	25	42.53%	6	7.38	1.31	R by 1.1	27	50.00%	10	11.73	1.33	R by 1.3	49.02%	10	12.32	2.59	R by 0.9
Illinois	19	54.96%	8	11.15	1.23	R by 2.6	18	55.40%	12	10.04	1.11	D by 1.8	51.42%	10	9.14	1.12	D by 0.8
Indiana	9	42.04%	3	2.53	0.78	D by 0.6	9	45.80%	2	3.02	0.76	R by 1.3	38.90%	2	2.03	0.72	R by 0.04
Maryland	8	63.42%	6	6.13	0.78	R by 0.2	8	65.46%	7	6.11	0.72	D by 1.2	58.14%	7	5.13	0.76	D by 2.4
Michigan	15	47.87%	6	6.35	1.07	R by 0.3	14	52.70%	5	6.97	0.98	R by 2.0	50.88%	5	6.96	0.98	R by 2.0
North Carolina	13	42.47%	4	3.80	0.94	D by 0.2	13	50.90%	4	5.94	0.93	R by 2.1	45.31%	3	4.89	0.92	R by 2.1
Ohio	18	44.75%	5	6.26	1.14	R by 1.1	16	47.90%	4	6.48	1.03	R by 2.4	40.94%	4	4.40	0.98	R by 0.4
Pennsylvania	19	48.41%	7	8.27	1.21	R by 1.1	18	50.70%	5	8.14	1.10	R by 2.9	46.07%	5	7.06	1.09	R by 1.9
Texas	32	37.51%	7	5.69	1.30	D by 1.0	36	39.90%	12	8.68	1.47	D by 2.3	39.35%	11	8.65	1.43	D by 1.6
Virginia	11	44.94%	3	3.87	0.89	R by 1.0	11	49.00%	3	4.56	0.86	R by 1.8	44.84%	3	4.02	0.84	R by 1.2
Wisconsin	8	45.40%	3	2.91	0.77	D by 0.1	8	50.76%	3	3.64	0.73	R by 0.9	47.20%	3	3.36	0.73	R by 0.5

Three Tests for Gerrymandering: Application to Maryland and Wisconsin

come increasingly under single-party control[74]. Changes between decadal redistrictings favored Republicans, who controlled 13 state capitals in 2002, rising to 24 state capitals in 2012.[75] During that same interval, Democrats went from controlling 8 state capitals to controlling 13 state capitals.[76] Thus the potential for partisan control of districting has increased for both major parties, with a greater advantage for the Republican Party.

4. WHAT ACCOUNTED FOR THE ANTIMAJORITARIAN OUTCOME OF 2012?

With these analytical tools in hand, it is now possible to calculate the total effect of asymmetric partisan districting on the national House elections of 2012. The outcome was a 33-seat margin of control, with 234 Republican and 201 Democratic seats.[77] Applying party-neutral standards to the seven Republican-controlled states and two Democratic-controlled states would have given an average margin that was 22 seats smaller, or 212 Democrats and 223 Republicans. Because of the uncertainty contained in this analysis (the range of outcomes within two sigma of the average was 206 to 218 Democratic seats), it is just within the range of possibility that without partisan asymmetry, Democrats might have taken control of the chamber.

Republicans have a second advantage, one that arises from population clustering. Because this simulation-based analysis uses existing national districts, it includes effects of population clustering. It is possible to quantify the net impact of population clustering, which facilitates the drawing of districts that are heavily tilted toward Democrats.[78]

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[74] Carl E. Klarner, *State Partisan Balance Data, 1937 – 2011*, 2013, IQSS DATAVERSE NETWORK V1, http://hdl.handle.net/1902.1/20403 (last visited Aug. 21, 2015) (documenting one-party rule; updating Carl E. Klarner, *Measurement of the Partisan Balance of State Government*, 3 STATE POLITICS AND POLICY QUARTERLY 309 (2003)); U.S. CENSUS BUREAU, STATISTICAL ABSTRACT OF THE UNITED STATES 261 TABLE 418 (2012).

[75] *Id.*

[76] *Id.*

[77] Karen L. Haas, Statistics of the Presidential and Congressional Election of November 6, 2012. Clerk of the House of Representatives. http://clerk.house.gov/member_info/electionInfo/2012election.pdf. Last downloaded Feb. 18, 2016.

[78] Chen & Rodden, *Unintentional Gerrymandering*, *supra* note 54, at 260-264 (calculating the biases associated with simulated redistricting using compactness principles across the 50 states).

In the original simulations, states where I did not find dys-proportionality had a two-party vote share of 50.7% for Democrats and 180 out of 363 seats. I then calculated the expected share of seats if district-by-district vote shares were perfectly symmetrically distributed.[79] Such symmetry of population patterns predicts that a 50.7% vote share would lead to Democrats winning 51.8% of seats, or 188 seats, 8 seats more than the real-population-based simulation. Scaling this up to all 435 seats, this suggests that Republicans won 9 or 10 seats in non-dys-proportional states more than they would under symmetric population patterns; the swing, defined as margin between the parties, is be twice as large, 18 to 20 seats. This 18-to-20 seat swing effect in 41 states is smaller than the 22-seat effect of partisan dys-proportionality in just 9 states.[80] Therefore a considerable deviation from natural seats/votes relationships is driven not by political geography, but by political motivations and actors during the legislative process in a handful of states.

In summary, the effects of partisan redistricting exceeded the amount of asymmetry caused by natural patterns of population. Together, gerrymandering and population clustering are more than enough to account for the fact that in 2012, Democrats won the House popular vote but Republicans ended up in control of the chamber.

B. ANALYSIS OF INTENTS: VOTER PACKING BY INTENTIONAL GERRYMANDERING AND SELF-ASSOCIATION

Analysis #1 of effects established a method for identifying states in which voter preferences lead to representation that is anomalous relative to national norms (see Section II.A.) Without gerrymandering, these anomalies could be rectified through the ballot box: if election outcomes shift sufficiently, legislators can be voted out, thus bringing outcomes more in line with the popular will. As an example of how gerrymandering vitiates this mechanism, the election of 2014 heralded a "wave year" in which Republicans won the national popular vote by

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[79] *See* GUDGIN & TAYLOR, *supra* note 65, at 20-31.

[80] This effect is consistent with previous work. See Nicholas Stephanopoulos & Eric McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 UNIV. OF CHICAGO L. REV. 831 (2015), 31-34 (calculating partisan-asymmetry trends over time from 1972 to 2012); John Sides, *Not Gerrymandering, but Districting: More Evidence on How Democrats Won the Popular Vote but Lost the Congress*, The Monkey Cage (Nov. 15, 2012), http://themonkeycage.org/blog/2012/11/15/not-gerrymandering-but-districting-more-evidence-on-how-democrats-won-the-popular-vote-but-lost-the-congress/.

5.9%, in sharp contrast to the Democratic popular vote win of 2012.[81] However, in the 12 states in Table 1, Republicans gained control of only five of 187 Democratic seats.[82] This small change indicates that representatives in these states were largely insulated from a large swing in opinion from 2012 to 2014. Considering the strength of partisan gerrymandering in 2012, the smallness of this change means that Republicans reaped most of their electoral gains two years earlier than their popular support would have merited.

Analysis #2 below presents a way to identify asymmetric reductions in the ability of legislative elections to respond to changes in voter opinion. It therefore can be used to measure a principal effect of partisan gerrymandering, reduction in the overall responsiveness of races across a state.

1. DISTINCTIVE PATTERNS OF WIN AND LOSS MARGINS ARISING FROM PARTISAN GERRYMANDERING AND VOTER SELF-ASSOCIATION

Partisan redistricting procedure creates a characteristic lopsided pattern of election results that can be used to identify when packing is likely to have occurred. State-level gerrymandering is more elaborate than single-district gerrymandering and relies on a two-part strategy. First, as before, map drawers will cram voters likely to favor their opponents into a few throwaway districts where the other side will win lopsided victories, a strategy known as "packing."[83] Second, they will draw the remaining, more numerous districts using boundaries to yield more-narrowly-won victories.[84] In this process, the critical requirement is asymmetry: the opposing party's voters must be more tightly packed than one's own voters.[85] The net result is an increased likelihood of unrepresentative outcomes.

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[81] New York Times online, http://elections.nytimes.com/2014/results/house. Accessed Feb. 18, 2016.

[82] *Id.*; Haas, *supra* note 77.

[83] Levitt, *supra* note 5.

[84] *Id.*

[85] Because members of both major parties get packed into districts in a partisan gerrymander, individual members of the opposing party may acquiesce or even be complicit in the process. *See*, *e.g.*, *League of United Latin American Citizens v. Perry*, 548 U.S. 399, 418 (2006) (noting "a number of line-drawing requests by Democratic state legislators were honored"). In other words, a single-district gerrymander can favor one party even as a partisan gerrymander favors the other party. For this reason, the use of intent as a standard for gerrymandering should distinguish between district-level and party-level motivations.

Here I will examine lopsided patterns in gerrymandered states, and compare them to nongerrymandered states. This provides a comparison of the effects of partisan gerrymandering with the effects of population variations and less-partisan districting. This analysis can be used to motivate Analysis #2, an index of gerrymandering that depends directly on the partisan redistricter's desired goal: the packing of opponents, as measured by election returns.

Gerrymandered districts show a distinctive pattern of lopsided votes (**Figure 4**). Figure 4a shows a histogram of two-party vote share for 2012 House districts that were asymmetric in favor of
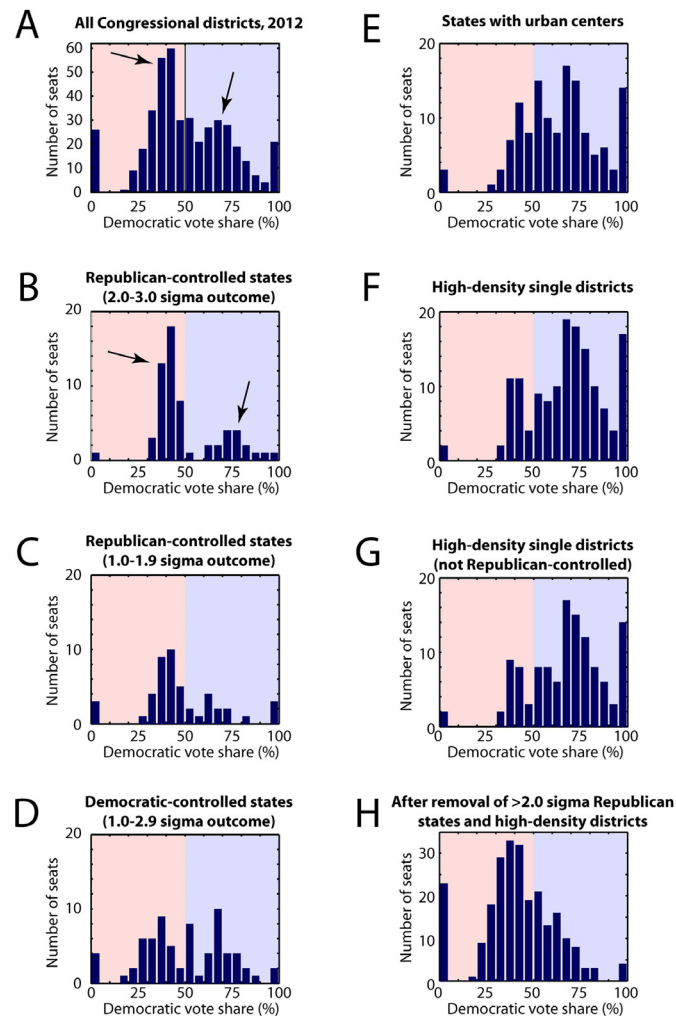

Figure 4: District-by-district histograms of 2012 House election results. **(a),** All districts, showing one peak with close Republican wins and one peak with lopsided Democratic wins. These peaks persist in **(b)**, districts from states showing an overall advantage to Republicans of 2 times sigma or greater, based on resampling simulations; and **(c)**, districts with a Republican advantage of 1 to 1.9 times sigma. **(d),** Peaks are shifted or absent in Democratic-advantaged states. **(e),** States with urbanized populations, defined as greater than 1 million people in cities larger than 250,000, in the 2010 Census. **(f),** Districts with more than 1000 persons per square mile. **(g),** Same as (f) but with the districts in (b) and (c) removed. (h), Same as (g) but with the districts in (f) removed.

Republicans. In this histogram, two peaks are apparent: a narrow peak centered near a 40% Democratic vote share and a broader peak centered near a 30% Republican vote share (indicated on the histograms by a 60% to 80% Democratic vote share). Both of these peaks are sufficiently prominent that they can also be seen in a histogram drawn using all states nationwide (**Figure**

4a). The peaks are considerably more prominent when the histogram includes only Republican-favoring states (**Figures 4b and 4c**) or Democratic-favoring states (**Figure 4d**).

However, voter packing can be asymmetric simply by virtue of the fact that voters arrange themselves in a manner that is not symmetric. Therefore any measure of gerrymandering-based packing must be done relative to a baseline of how voters "pack themselves."[86] Specifically, it has been suggested that structural factors such as concentration of Democrats in urban areas may have a greater effect than partisan redistricting.[87] I will now quantify the size of these two effects. Since both real packing by redistricters and virtual packing by structural factors are likely to have similar manifestations, they can be examined using the same statistical tools.

2. GERRYMANDERING EMULATES THE EFFECTS OF URBANIZATION

The establishment of competitive districts is made difficult by the fact that voters often choose to live near others of similar ethnic, religious, secular, and political affiliation.[88] Such self-selection is visible in urban regions that vote overwhelmingly for Democrats, and rural regions that vote overwhelmingly for Republicans. If natural population clustering favors increased Republican representation, then the distribution of vote share in urbanized districts should resemble that of Republican-gerrymandered states. Such a pattern is not apparent in high-population-density states (**Figure 5e**). However, urbanized districts (**Figure 5f**), defined as those with population density greater than 1000 persons/square mile, show both peaks, but with more emphasis on the high-Democratic-vote share peak. This pattern is visible even when putatively gerrymandered states (favoring both Democrats and Republicans) are omitted from the histogram (**Figure 5g**).

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[86] Sides, *supra* note 80.

[87] *Id.*

[88] BILL BISHOP, THE BIG SORT: WHY THE CLUSTERING OF LIKE-MINDED AMERICA IS TEARING US APART 5-15 (2009) (explaining the Big Sort phenomenon in politics). Wendy K. Tam Cho, James G. Gimpel, and Iris S. Hui, *Voter Migration and the Geographic Sorting of the American Electorate,* 103 ANNALS OF THE ASSOCIATION OF AMERICAN GEOGRAPHERS 856 (2013); Jesse Sussell, *New Support for the Big Sort Hypothesis: An Assessment of Partisan Geographic Sorting in California, 1992-2010,* 46 PS, POLITICAL SCIENCE & POLITICS 768 (2013) (providing new empirical support for the Big Sort hypothesis, and addressing a critique by Samuel J. Abrams & Morris P. Fiorina, *"The Big Sort" that Wasn't: A Skeptical Reexamination,* 45 PS, POLITICAL SCIENCE & POLITICS 203 (2012)).

Gerrymandering makes use of existing urbanization. In Republican-gerrymandered states, non-urbanized districts (**Figure 5b and 5c**) are dominated by Republican-packed districts, demonstrating that redistricters who seek a Republican advantage do so by creating numerous districts that avoid urban regions. Once Republican gerrymanders and urbanized areas are omitted, the remaining Congressional districts show a considerably lower tendency to have two peaks (**Figure 5h**). Democratic gerrymanders can achieve a converse advantage by carving out slices of urbanized areas and combining them artfully with more rural areas to create small but secure Democratic wins.

Although the representational effects of voter migration into urban communities are similar to the effects of partisan gerrymandering, the interpretations of the two phenomena are quite different. Voters who arrange themselves in this manner are voluntarily arranging themselves so that their representatives are at little risk of being turned out of office. In the case of partisan gerrymandering, voters are placed into political affiliation with one another—but without the consent of the citizens involved. Such a pattern contradicts the saying that "voters should choose their representatives, and not the other way around."[89] Gerrymandering thus penalizes voters based on their publicly available information, including partisan loyalty, all of which is present in Census data and commercial redistricting software.

3. A "LOPSIDED-MARGINS TEST" TO DETECT WHEN THE TARGETED PARTY WINS WITH UNUSUALLY LARGE MARGINS

In summary, the success of a gerrymandering scheme depends on the ability of the redistricting party to create safe margins of victory for both parties, with larger margins for their opponents. This pattern of outcomes can be quantified by sorting the districts into two groups, by winning party. Each party's winning vote shares can then be compared by what is said to be "the most widely used statistical test of all time"[90]: the t-test for comparing the averages of two groups of observations. In this way, the difference between each party's winning margins is used to carry out Analysis #2, which tests for intensive packing of one party's voters.

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[89] Mitchell N. Berman, *Managing Gerrymandering*, 83 TEXAS L. REV. 781, 781 (2005).

[90] Richard Lowry, Chapter 11. t-Test for the Significance of the Difference between the Means of Two Independent Samples, VASSARSTATS, http://vassarstats.net/textbook/ch11pt1.html (last visited Dec. 5, 2015).

4. THE MEAN-MEDIAN DIFFERENCE AS A MEASURE OF SKEWNESS

Now that I have identified states in which Republicans or Democrats gained an asymmetric advantage, I can examine these states to test the validity of a simpler statistic that does not require computer simulation: the difference between the mean (*i.e.* average) and the median vote share[91] for contested[92] districts. The mean-median difference is a simple measure of asymmetry[93] and allows for ready comparison with national standards. Notably, it does not require any inputs other than district-level election results for the state that is under examination.

As an example of the calculation, consider the 2012 Pennsylvania Congressional election. The Democratic two-party share of the total vote in all 18 districts was, in terms of percentages and sorted in ascending order:

34.4, 36, 37.1, 38.3, 40.3, 40.6, 41.5, 42.9, <u>*43.2, 43.4,*</u> *45.2, 45.2, 48.3,* 60.3, 69.1, 76.9, 84.9, 90.6.[94]

Races won by Republicans are indicated in *italics* and the two middle values are <u>underlined</u>. The median percentage is defined as the midpoint of the two middle values, 43.3%. The mean Democratic vote share is 51.0%. The difference between the median and the mean is 7.7%. This

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[91] The mean-median difference has also been suggested by Robin E. Best & Michael D. McDonald, *Unfair Partisan Gerrymanders in Politics and Law: A Diagnostic Applied to Six Cases.* 14 ELECTION LAW JOURNAL: RULES, POLITICS, AND POLICY 312 (2015). In the present paper I give mathematically rigorous confidence intervals on that statistic and describe the circumstances under which it is applicable.

[92] The presence of uncontested races reduces the value of the mean-minus-median statistic. In those cases, the partisan breakdown is not known with accuracy. Consider the example of a 20-district state, residents of an uncontested district would have voted at a rate of 80% for their party, instead of the nominal 100%. If their district were drawn differently, the appropriate mean for comparison would be based on the 80% figure, and shift the overall mean by 1%.

[93] The mean-median difference is a simple and old measure of "skewness," a statistical term for asymmetry. David P. Doane & Lori E. Seward, *Measuring Skewness: A Forgotten Statistic?*, 19 JOURNAL OF STATISTICS EDUCATION (2011), www.amstat.org/publications/jse/v19n2/doane.pdf; Karl Pearson, *Contributions to the Mathematical Theory of Evolution, II: Skew Variation in Homogeneous Material*, TRANSACTIONS OF THE ROYAL PHILOSOPHICAL SOCIETY, SERIES A, 186, 343-414 (1895). G. UDNY YULE AND MAURICE G. KENDALL, AN INTRODUCTION TO THE THEORY OF STATISTICS 162-3 (3d ed. 1950).

[94] Haas, *supra* note 77.

difference reflects the fact that counterintuitively, Republican vote shares were above average in considerably more than half of the districts: 72% (13 out of 18), to be exact.

The median serves as a measure of the overall behavior of the 18 district-level elections. The goal of a gerrymander is to maximize the number of districts won, which occurs when the median outcome is more unfavorable to the opposing party than that party's share of the vote. In other words, Pennsylvania's Democratic voters were empowered as if they comprised 43.3% of voters, even though they actually comprised 51.0%. The difference, 7.7%, is the number of voters who were effectively disenfranchised. Since approximately 5,400,000 Pennsylvanians cast votes in the 2012 Congressional election, redistricting achieved an effect equivalent to over 400,000 Democratic voters casting their ballots for Republicans. The probability is less than 1% that this difference arose by chance[95].

5. STATE-BY-STATE COMPARISONS OF SKEWNESS WITH POPULATION CLUSTERING EFFECTS

To investigate the degree to which the mean-median difference arises as a function of population clustering patterns, I will make comparisons between a variety of states and years. For the 2012 congressional elections, the nationwide mean-median difference was 4.3% favoring Republicans across all 50 states and 1.9% favoring Republicans in non-dysproportional states. For Pennsylvania in 2012 the difference was 7.7%, greater than any of the other numbers, and comparable to the other four dysproportional states of Michigan (mean-median difference of 6.3%), North Carolina (7.3%), Ohio[96] (6.3%), and Virginia (6.3%). Overall, these mean-median differences are three to four times that seen in non-dysproportional states, indicating that the effects of partisan gerrymandering are three or four times larger than the effects of population clustering. Indeed, redistricting in a handful of states can generate a greater deviation from symmetry than population clustering in all 50 states combined.

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[95] The level of statistical significance is calculated using Test 3 in III.A. and Student's t-distribution. Lowry, *supra* note 90.

[96] In Ohio, one race, the 11th District, was uncontested and won by a Democrat, Marcia Fudge.

III. THREE QUANTITATIVE TESTS OF THE EFFECTS OF PARTISAN GERRYMANDERING

I will now use the two Analyses to propose three tests. The three tests have several advantages. First and foremost, they are simple to apply. None of the three tests requires the detailed drawing of maps. Because the tests can be stated with mathematical exactness, they can also provide a manageable standard for gerrymandering cases, giving predictable and sensible results—and unambiguous guidance to legislatures and judges. The tests are based on election outcomes, and therefore can be employed separately from, or in conjunction with, geographic and other criteria.

A. CONVERTING THE ANALYSES TO PRACTICAL TESTS

Now I use the analysis of effects, which is based on numerical simulation of seat outcomes, to construct Test 1, the excess seats test. I use the analysis of intents, which identifies narrow-but-reliable wins as a hallmark of gerrymandering, to construct two tests: Test 2, the lopsided outcomes test; and Test 3, a reliable-wins test.

Test 1 (the excess seats test): Calculate whether the outcome of an election after redistricting was dysproportional relative to a simulated seats/votes curve, and whether that outcome favors the redistricting party. For a state containing N districts, calculate the difference between the actual seats and the simulated expected number, and divide by the standard deviation to obtain the difference, Delta[97].

Test 2 (the lopsided outcomes test): Compare the difference between the share of Democratic votes in the districts that Democrats win, and the share of Republican votes in the districts that Republicans win. This test works because in a partisan gerrymander, the targeted party wins lopsided victories in a small number of districts, while the gerrymandering party's wins are engineered to be relatively narrow (see Section II.B.). To compare the winning vote shares for the two parties, use a grouped t-test, an extremely common statistical test.[98]

Test 3 (the reliable- wins test): Systematic rigging of total statewide outcomes occurs by the construction of districts that offer secure wins for the party in control of the map. These

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[97] This σ_1 can be calculated according to the formula for sigma in note 52.

[98] Lowry, *supra* note 90.

wins would be wide enough to guarantee victory, but not so wide as to waste votes that could be used to shore up other districts. How this intent is detected depends on whether the state's partisan vote is closely divided, or whether one party is dominant. In a closely divided state, reliable wins occur when the average and median vote differ from one another. In a state that is dominated by one party, reliable wins occur when that party's strength is spread highly evenly across districts.

In a closely divided state: Calculate the difference between a party's statewide average district vote share on the one hand, and the median vote share it receives on the other. In this situation a systematic gerrymander can be detected when a party's median vote share is substantially below its average vote share across districts.[99] For this test, calculate Delta by dividing the mean-median difference by σ_3, which is defined as 0.756 * (standard deviation of vote share across all N Congressional districts in a state)/$\sqrt{N}$.[100]

In a state where the redistricting party is dominant: Calculate the standard deviation of the redistricting party's vote share in the districts that it wins. Calculate the standard deviation of the party's vote share in the districts that it wins nationwide. Compare these two standard deviations using a well-established testing tool, the chi-square test for comparison of variances[101], to define zones of chance.

Test 1 evaluates whether a party gained a significant advantage in terms of seats, and calculates the size of the effect. Tests 2 and 3 determine whether the pattern of data could have arisen by chance; if not, this indicates an intent to gerrymander. A residual possibility exists of a false-positive result, *i.e.* identifying that a gerrymandering event occurred when in fact it did not. To reduce the possibility of such a false alarm, partisan gerrymandering could be assessed by

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[99] This is the mean-median test described in Wang, *supra* 2 and Wang, *supra* 17, and by Robin E. Best & Michael D. McDonald, *Unfair Partisan Gerrymanders in Politics and Law: A Diagnostic Applied to Six Cases.* 14 ELECTION LAW JOURNAL: RULES, POLITICS, AND POLICY 312 (2015)..

[100] Paul Cabilio & Joe Masaro, *A Simple Test of Symmetry About an Unknown Median*, 24 THE CANADIAN JOURNAL OF STATISTICS / LA REVUE CANADIENNE DE STATISTIQUE 349, 352 (1996); Tian Zheng & Joseph L. Gastwirth, *On Bootstrap Tests of Symmetry About an Unknown Median*, 8 JOURNAL OF DATA SCIENCE 397, 400-401 (2010).

[101] Pearson, Karl (1900). *On The Criterion That A Given System Of Deviations From The Probable In The Case Of A Correlated System Of Variables Is Such That It Can Be Reasonably Supposed To Have Arisen From Random Sampling.* 5 50 (302) PHILOSOPHICAL MAGAZINE SERIES: 157–175. Snedecor, George W. and Cochran, William G. (1989), *Statistical Methods*, Eighth Edition, Iowa State University Press.

evaluating both Test 2 and Test 3. If Delta is set to standard levels of statistical significance[102] in 2012, six states met both the Test 2 and Test 3 criteria: Michigan, North Carolina, Ohio, Pennsylvania, Virginia, and Wisconsin.

B. ADVANTAGES AND DISADVANTAGES OF THE THREE TESTS

The tests proposed here all have several advantages. First, the tests do not require the detailed drawing of maps. Second, because they are derived from election results only, the tests can be applied independently of evaluation of intent. Third, because their results are highly correlated, in situations where one test is unsuitable, another can be used instead. In this way the tests can be used separately or combined to reduce the risk of falsely identifying a gerrymander where none occurred, or conversely, failing to detect a gerrymander where one did occur. Finally, because the three tests do not use geography, they can easily be combined with other standards which may require circuitous geographic boundaries, such as state-mandated requirements[103], Section 2 of the Voting Rights Act, and other precedents that exist in federal law.

Before the judge (or other evaluator of a districting plan) chooses which test to apply, he/she should take the following advantages and disadvantages into account.

Test 1's most powerful use is to obtain an exact range for the appropriate number of seats for a given vote share. It addresses whether a redistricting scheme leads to an elected delegation that deviates from national districting norms, *i.e.* it measures effects. Test 1 can always be calculated from a set of election returns. Because it uses data from other states, it has the advantage of taking into account the overall nationwide demographic character of districts Therefore it has the virtue of measuring effects that go beyond those that arise naturally from population clustering.   However, because it requires computer simulation, it requires the use

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[102] A typical level of statistical significance is to set the threshold for Delta so that chance would give the observed result 5% of the time or less. When this occurs depends on the number of districts and follows the t-distribution, and typically occurs when Delta exceeds 1.75 for Analysis #2 and #3. *Supra* note 61.

[103] The three tests proposed here address the overall apportionment plan, but do not cover the case of individual self-dealing in single districts. Local laws may provide additional constraints. For example, the current Congressional districts in Florida do not violate the three tests presented here. Nonetheless, the Florida Supreme Court has found the map to violate the Florida Constitution redistricting provisions (article III, section 20(a) that reads "No apportionment plan or district shall be drawn with the intent to favor or disfavor a political party or an incumbent"). *Detzner*, 2015 WL 4130852. This stricter standard extends a mandate for competitive races to the level of single districts.

Table 2: Results of Three Tests for Partisan Asymmetry for the Congressional Elections of 2012. In all cases, the last column gives the difference between expectations and actual result, expressed in units of sigma, the standard deviation, to give a measure that is comparable across the three tests. Test #3 is done starting from raw percentage results, and also taking uncontested races and assuming that their voters are distributed 75%-25% for the winning party. The shaded boxes indicate statistically significant results. Test #2 could not be done for Maryland because the grouped t-test requires each group to include at least two wins.

| | Test #1 (simulation) | | | Test #2 (lopsided margins) | | | | Test #3 (skewed districts) | | | | | |
| | | | | | | | | directly from election returns | | | imputing uncontested races | | |
	Total seats	Simulated average	Δ (difference divided by sigma)	Democratic win %	Republican win %	sigma	Δ (difference divided by sigma)	Average minus median Dem vote (%)	sigma	Δ (Average-median divided by sigma)	Average minus median Dem vote (%)	sigma	Δ (Average-median divided by sigma)
Arizona	9	2.96	D by 2.7	63.1%	66.6%	9.5%	D by 0.4	-0.5%	3.8%	D by 0.1	-3.3%	3.8%	D by 0.9
Florida	27	11.73	R by 1.3	73.0%	67.4%	7.4%	R by 0.8	4.8%	3.8%	R by 1.2	4.8%	2.4%	R by 2.0
Illinois	18	10.04	D by 1.8	66.2%	62.1%	4.9%	R by 0.8	2.1%	3.1%	R by 0.7			
Indiana	9	3.02	R by 1.3	65.1%	59.5%	3.1%	R by 1.8	1.4%	2.1%	R by 0.7			
Maryland	8	6.11	D by 1.2	70.4%	66.5%	-	-	-2.8%	3.9%	D by 0.7			
Michigan	14	6.97	R by 2.0	74.4%	58.9%	4.9%	R by 3.2	6.9%	3.7%	R by 1.9			
North Carolina	13	5.94	R by 2.1	70.2%	57.5%	6.9%	R by 1.9	7.8%	3.2%	R by 2.5			
Ohio	16	6.48	R by 2.4	80.2%	62.2%	7.5%	R by 2.4	6.8%	4.3%	R by 1.6	6.8%	3.0%	R by 2.3
Pennsylvania	18	8.14	R by 2.9	76.3%	59.5%	5.5%	R by 3.1	7.6%	3.2%	R by 2.4			
Texas	36	8.68	D by 2.3	71.4%	72.1%	4.5%	D by 0.2	4.9%	3.1%	R by 1.6	7.0%	2.4%	R by 2.9
Virginia	11	4.56	R by 1.8	70.9%	58.8%	5.6%	R by 2.1	6.3%	3.4%	R by 1.9			
Wisconsin	8	3.64	R by 0.9	68.9%	59.6%	3.8%	R by 2.4	7.0%	4.2%	R by 1.7			

of a computer program (which can be obtained by contacting the Author).

Test 2 has the advantage of simplicity: it can be worked out using a spreadsheet program such as Microsoft Excel that can perform a two-sample t-test. If such a program is not available, it can be done using a hand calculator and a table of statistical values. It directly tests for noncompetitive races, a mainstay of gerrymandering. It identifies partisan asymmetry, though not bipartisan gerrymanders in which individual candidates of both parties benefit. Test 2 has the disadvantage that it can only be used if both parties win at least 2 seats each, since this is required to calculate standard deviations, a necessary step of the test.

Test 3 measures the reliability of wins for the redistricting party. Like Test 2, it is simple to calculate. Test 3 can always be done, since it is calculated using most or all of a state's district-level results. In the case of the mean-median difference, it does not rely on any data from other states, and is therefore self-contained. In the case of the chi-square test, national data must be used to provide a standard for comparison.

C. THREE EXAMPLES: THE ORIGINAL GERRY-MANDER, ARIZONA STATE LEGISLATIVE DISTRICTS, AND MARYLAND CONGRESSIONAL DISTRICTS
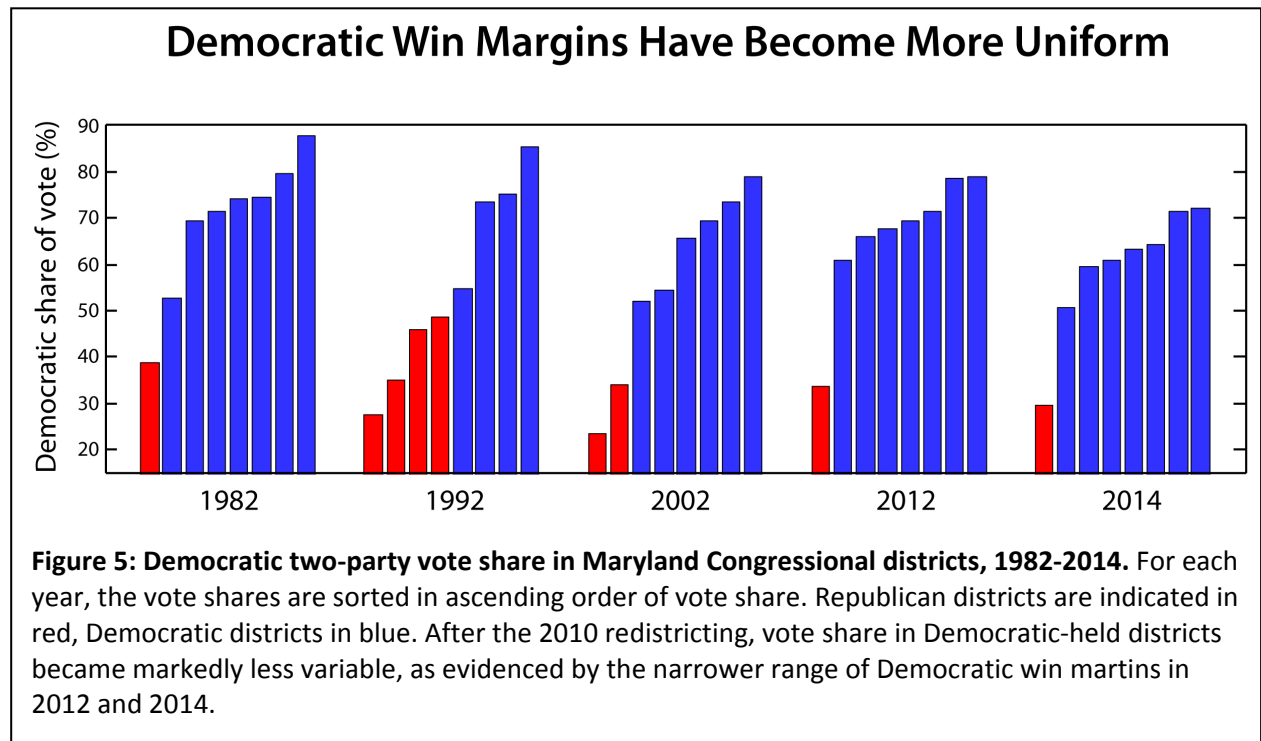
To examine the general applicability of these tests, let us consider three examples: (1) the original Gerry-mander of 1812, (2) post-2010 Maryland Congressional districts, which the Supreme Court recently remanded for consideration by a three-judge court[104], and (3) post-2010 Wisconsin state Assembly districts, which is currently under review in the Western District of Wisconsin.

Example 1: The original "Gerry-mander," the Massachusetts state Senate election of 1812. Test 1 is evaluated by starting from the fact that there were 18 races.[105] The average expectation of a nearly evenly divided popular vote is 9 races for each party. The upper theoretical value to sigma is $0.5*\sqrt{18} = 2.1$ races; computational simulation reveals a true value of sigma of 1.4 races. The Federalists won only five races[106], and therefore Test 1 is met to a standard of $(9-5)/1.4 = 2.9$ sigma, statistically significant.

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[104] *Shapiro v. McManus,* 136 S. Ct. 450 (2015).

[105] In that election, multimember districts of unequal population were allowed. For the calculation of Test 1, each district election is used as one data value.

[106] *Lampi Collection of American Electoral Returns, 1787–1825. American Antiquarian Society, 2016.*

Democratic Win Margins Have Become More Uniform

Figure 5: Democratic two-party vote share in Maryland Congressional districts, 1982-2014. For each year, the vote shares are sorted in ascending order of vote share. Republican districts are indicated in red, Democratic districts in blue. After the 2010 redistricting, vote share in Democratic-held districts became markedly less variable, as evidenced by the narrower range of Democratic win martins in 2012 and 2014.

For Test 2, the Federalists won five races (which accounted for 11 districts); in these races, their two-party vote share averaged 55.6%, with a standard deviation of 4.6%. The Democratic-Republicans won 13 races (which accounted for 29 districts), with an average vote share of 70.7% and a standard deviation of 5.3%. The resulting Delta (also called t-score) is 5.5, and therefore Test 2 is met to a standard of 5.5 sigma. This is an unusually high level of significance, and is reached by chance 0.0025% of the time.

Test 3 cannot be used because districts are not equal in size. In 1812 the number of votes per legislator ranged from Dukes/Nantucket (1,078 votes cast in total for 1 legislator) to Franklin (4,469 votes for 1 legislator).[107]

Example 2: Maryland Congressional districts. Maryland has eight Congressional districts. Steven Shapiro and other plaintiffs filed suit in district court that the post-2010 districting plan violated their rights to political association and equal representation under the First and Fourteenth Amendments.[108] This complaint was dismissed, an outcome that was

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[107] Id.

[108] *Shapiro*, brief of petitioners, at 12 and 35-39. http://www.scotusblog.com/wp-content/uploads/2015/08/14-990-ts.pdf

affirmed by the U.S. Court of Appeals for the Fourth Circuit.[109] However, in December 2015 the Supreme Court reversed the decision, remanding the case to a three-judge court for further consideration[110].

In Maryland, Democrats typically win around 60% of the vote at a statewide level – the same as the margin needed for a safe victory. Artful arrangement is accomplished – and can be detected – in the form of many districts of near-identical partisan composition (**Figure 5**).

Test 1 identifies Maryland as a gerrymander. In the pre-redistricting election of 2010, Democrats won 63.2% of the statewide vote and six seats[111], compared with a simulated average of 6.1 seats – not statistically significant (**Table 1**). After redistricting, in 2012 Democrats won 65.5% of the statewide vote and won seven seats[112], compared with a simulated average of 6.1 seats. The value of Delta was 1.2 favoring Democrats, not quite statistically significant. In 2014, Democrats' vote share declined to 58.1% but they retained all seven of their seats.[113] In this case, the simulated average was 5.1 seats, and the value of Delta was 2.4, statistically significant. These results indicate that redistricting gained Democrats a 1-seat advantage in a strong Democratic year, 2012; and that this advantage was retained in the national wave election of 2014 that swept dozens of Republicans into office in states outside Maryland.

Test 2 cannot be applied because with only one Republican Congressman, the standard deviation of the Republican winning vote share cannot be calculated.

Test 3 should be done for the case of partisan dominance, a situation that calls for the chi-square test to test whether Democratic votes are spread unusually uniformly across Congressional districts. **Figure 6** shows the classical measure of variability, the standard deviation.[114] The standard deviation of Maryland Democrats' winning vote share in seven districts was 6.6% in 2012 and 7.3% in 2014. I compared the variability of Maryland Democratic
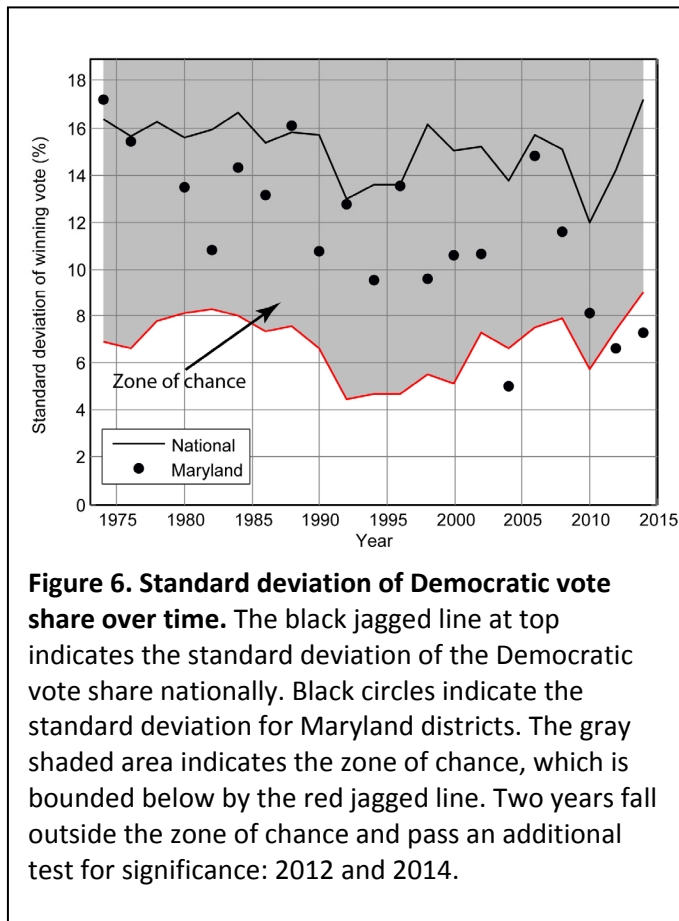
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[109] 4th Cir. No. 14-1417.

[110] *Shapiro*, s*upra* note 3.

[111] Karen L. Haas, Clerk of the House, Statistics of the Congressional Election of November 2, 2010, at 22. http://clerk.house.gov/member_info/electionInfo/2010election.pdf

[112] Haas, *supra* note 77, at 27-28. http://clerk.house.gov/member_info/electionInfo/2012election.pdf

[113] New York Times online, http://elections.nytimes.com/2014/maryland-elections. Downloaded Feb. 18, 2016.

[114] The standard deviation is the square root of the variance.

Figure 6. Standard deviation of Democratic vote share over time. The black jagged line at top indicates the standard deviation of the Democratic vote share nationally. Black circles indicate the standard deviation for Maryland districts. The gray shaded area indicates the zone of chance, which is bounded below by the red jagged line. Two years fall outside the zone of chance and pass an additional test for significance: 2012 and 2014.

districts with the variability of Democratic districts nationwide. The values for Maryland fall outside the zone of chance.

Maryland's standard deviations would have arisen by chance in only 2.8% of cases in 2012, and 1.7% of cases in 2014.[115] A third year, 2004, also showed an unusually low standard deviation[116]. These findings show that the Democrats' partisan advantage was achieved by spreading their partisan support in a highly even manner across their winning districts.

Example 3: Wisconsin state Assembly districts. After the 2010 election, the Republican Party controlled the Wisconsin state Senate, Assembly, and governorship, bringing post-Census redistricting into its control. The resulting state Assembly map was challenged by a group of Wisconsin Democratic voters who have alleged partisan gerrymandering under the First and Fourteenth Amendments.[117]

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[115] For a lower one-tailed test at significance level p<0.05, the lower bound of the zone of chance is equal to sqrt(2.167/(N-1))*(national s.d.). http://www.itl.nist.gov/div898/handbook/eda/section3/eda358.htm http://sites.stat.psu.edu/~mga/401/tables/Chi-square-table.pdf It should also be noted that the chi-square test assumes normally distributed vote shares. An additional test, the Ansari-Bradley test, does not make this assumption, and still identifies 2012 and 2014 (but not 2004) as being statistically significant departures from national Democratic districts. A.R.Ansari and R.A. Bradley, *Rank-Sum Tests for Dispersions*, 31 ANN. MATH. STATIST. 1174-1189 (1960).

[116] Without partisan intent, the Maryland standard deviation would still be expected to fall outside the zone of chance in five percent of cases – one in twenty. Maryland's 2004 Congressional delegation was within the zone of chance by Test 1, indicating that the result of Test 3 is a chance result, i.e. a "false positive."
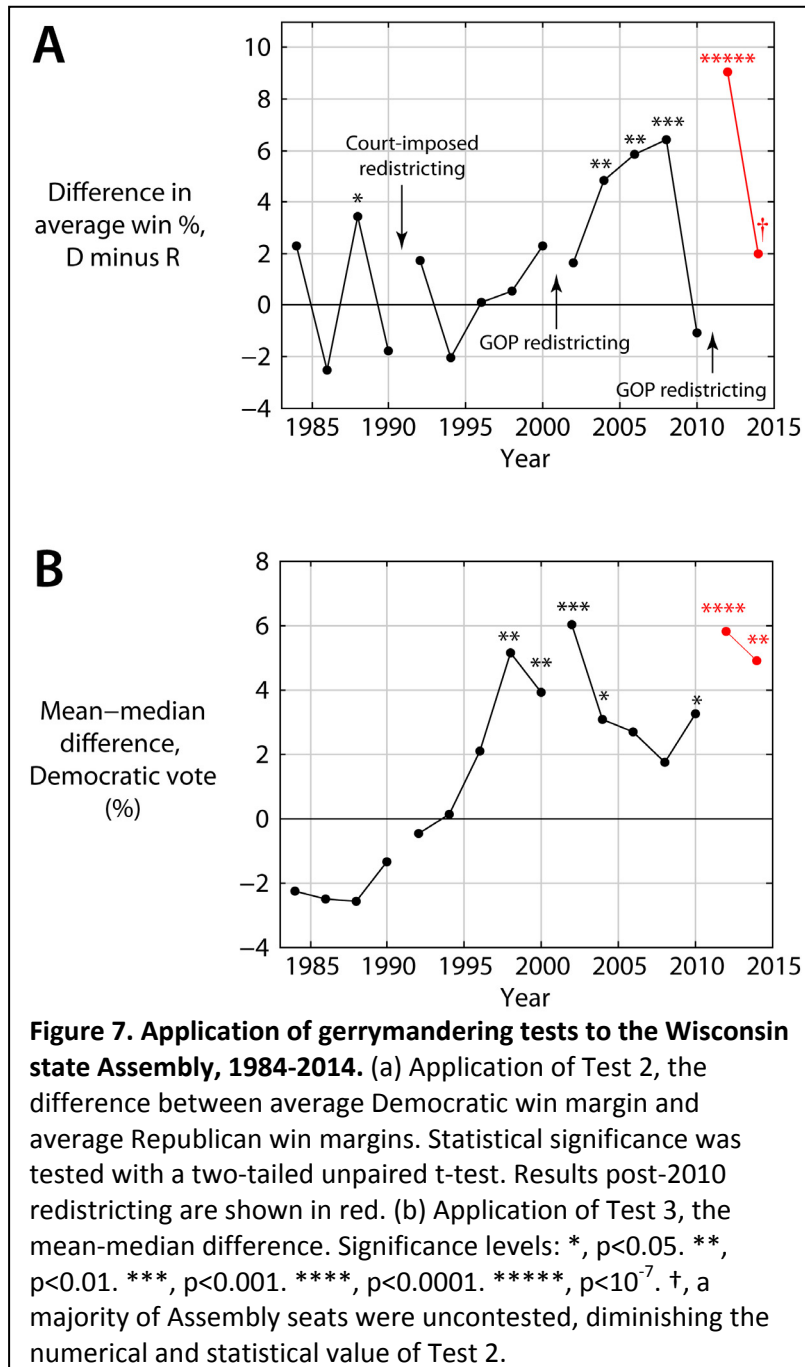
[117] *Whitford*, supra note 4.

The Wisconsin Assembly has 99 seats. To evaluate its partisan asymmetry in historical context, I applied Test 2 (the lopsided-outcomes test) and Test 3 (the reliable-wins test). Test 1 (quantifying the number of excess seats) was not done because it optimally requires a population of districts from the same year for purposes of simulation.

I analyzed state elections from 1984 to 2014. During this period, the average two-party vote across districts was between 45% and 55% for both parties. This condition of near-parity provides the greatest potential advantage to the party that can impose a partisan gerrymander. Over the entire 30-year period, the difference in winning vote share between the two parties (Test 2) was at its greatest in the 2012 election (**Figure 7**). Democrats won 39 seats with an average vote share of 68.8% (standard deviation 8.3%), while Republicans won 60 seats with an average vote share of 59.7% (standard deviation 6.5%). The difference, 9.1%, is statistically significant: this outcome would have arisen from a partisan-symmetric process by chance with a probability of less than 1 in 10 million (*i.e.* a two-sample t-test shows that $p < 10^{-7}$).[118] Of particular note is the fact that this partisan advantage appeared immediately after redistricting. Such a sudden jump would not be expected from population-clustering effects, which should change more gradually over time.

From 1984 to 2010, the overall results of Test 2 were consistent with partisan control. In 1990, Democrats and Republicans jointly controlled redistricting, leading to an impasse and a court-ordered redistricting. In the following five elections from 1992 to 2000, the difference in average winning vote share was not statistically significant, and never exceeded 2% in either direction. Then, in 2000, redistricting came under single-party Republican control, and in the following five election cycles from 2002 to 2010, the median value of the lopsided-outcomes test was a 5.0% advantage in favor of Republicans, reaching statistical significance three times.

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[118] In such a calculation, provision must be made for how to score uncontested races. The calculation in the main text was done counting uncontested races as 75%-25% victories. This assumption is established in previous literature (Gelman and King, *supra* note 57) as a means of evaluating likely imputed amounts of support in a situation where one party is dominant. In the case of the 2012 election, 23 Democratic seats and 4 Republican seats were uncontested. If these 27 races were counted as 100%-0% splits, the average vote share would be 83.5% for Democrats and 61.4% for Republicans, with even greater statistical significance ($p<10^{-9}$). Generally, imputed support is a conservative assumption that tends to reduce differences between the two parties.

Figure 7. Application of gerrymandering tests to the Wisconsin state Assembly, 1984-2014. (a) Application of Test 2, the difference between average Democratic win margin and average Republican win margins. Statistical significance was tested with a two-tailed unpaired t-test. Results post-2010 redistricting are shown in red. (b) Application of Test 3, the mean-median difference. Significance levels: *, $p<0.05$. **, $p<0.01$. ***, $p<0.001$. ****, $p<0.0001$. *****, $p<10^{-7}$. †, a majority of Assembly seats were uncontested, diminishing the numerical and statistical value of Test 2.

In 2014, a majority of Assembly seats were uncontested: 29 out of 63 Republican seats, and 23 out of 36 Democratic seats. In this situation, the average winning vote share is dominated by imputed values. For example, if all races were uncontested, the difference in average winning vote share would be defined as zero. Therefore an abundance of uncontested races tends to underestimates of the degree of partisan asymmetry. In this case, the difference in average winning vote share was 2.0% favoring Republicans. This case demonstrates that when many races are uncontested, an additional measure of partisan asymmetry is needed.

As a second test for gerrymandering, I used Test 3, the mean-median difference. The mean-median difference is applicable since the parties are closely matched in statewide strength. After redistricting, the average Democratic vote share in 2012 was 51.5% and the median vote share was 45.7%. The difference, 5.8% favoring Republicans, was statistically highly significant at $p<10^{-5}$, meaning that under symmetric conditions, the mean-median difference would reach 5.8% by chance less than once in one hundred thousand cases. In 2014, Democrats' average vote share declined to 46.0% and their median vote share was 41.1%. The

difference, 4.9% favoring Republicans, was again statistically significant (p<0.01). Both 2012 and 2014 had a higher mean-median difference than the pre-redistricting election of 2010, in which the mean-median difference was 3.2% favoring Republicans. These findings are consistent with the idea, based on past analysis, that the effects of a gerrymander tend to decrease over time – but can increase again when a new gerrymandering scheme is put into place.

IV. DISCUSSION

In this Article I have presented three tests for rapid identification of partisan gerrymanders. These tests can be used to evaluate intents and effects, the two prongs articulated in *Davis v. Bandemer*. The two intents tests can be done with computing resources already available on a judge's or clerk's desk, and the effects test requires some additional software.[119] All three tests rely on well-established statistical principles. The tests measure different aspects of partisan asymmetry, and therefore fall within the scope of principles that have been expressed by the Supreme Court. I suggest that these tests may constitute a manageable standard for courts to evaluate the impact of a state's districting scheme on its residents' Equal Protection and First Amendment rights.

The broader implications of this Article are threefold. First, I have used statistical science to express the idea that a pattern of election results might have arisen by chance, and therefore not warrant judicial intervention. By establishing "zones of chance" in which the partisan impacts of a districting plan are ambiguous, the three tests presented here can help a judge evaluate whether an identifiable injury has occurred in the first place. Second, my statistical analysis shows that in 2012, the effects of partisan asymmetry were so large as to exceed the effects of population clustering across the whole nation. This demonstrates the importance of measuring the degree of distortion from the natural relationship between votes and representation. Third, an intents-and-effects standard based on the tests is unambiguous, and may mitigate the need to demonstrate predominant partisan intent. For these reasons, these statistical tests comprise a valuable and timely addition to the judge's toolkit for rapid and rigorous identification of partisan gerrymanders.

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[119] A version of this software is available on GitHub at https://github.com/.

A. ZONES OF CHANCE AND THE FIRST AMENDMENT

My statistical analysis of the effects of gerrymandering may be of particular relevance to First Amendment analysis, which "allows a pragmatic or functional assessment that accords some latitude to the States."[120] By allowing for a normal amount of statistical variation, the three tests proposed in this Article build in zones of chance where any of a range of outcomes would lead to an acceptable amount of asymmetry.

Any statistical approach contains some possibility of accidentally identifying gerrymandering where it does not exist (in statistical terminology, "false positives"), or missing cases where it did occur (false negatives). Tests may also sometimes not be usable, for instance Test 2 when one party only wins one seat. For these reasons, I have provided two separate tests of intents. These tests are oriented toward the outcomes of elections rather than the specifics of map boundaries or district procedures. The tests hew closely to the electoral goals of redistricters and do not rely on geographically-oriented approaches which require normative assumptions of what constitutes good districting procedure.

B. THE TRANSPARENCY OF WELL-KNOWN STATISTICAL STANDARDS

If statistical tests for gerrymandering are sufficiently complex, the use of expert witnesses becomes necessary. However, complex arguments are subject to challenge on technical grounds[121], creating the secondary question of the credibility not just of the statistical method, but of the experts themselves. Although the use of expert testimony and statistical reasoning is commonplace in courts[122], for Constitutional questions where statutory guidance is lacking a judge may wish to conduct his or her own evaluation in a more direct manner.

Whitford v. Nichol provides an example of the complications that may arise. In *Whitford*, the districting plan was evaluated using a recently-developed measure of asymmetry, the efficiency gap.[123] Expert witness Prof. Simon Jackman established the statistical properties of the

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[120] *See e.g., Eu v. San Francisco County Democratic Central Comm.*, 489 U. S. 214. (1989).

[121] *Daubert v. Merrell Dow Pharmaceuticals*, 509 U.S. 579 (1993)

[122] *Tex. Dep't of Housing and Comm. Aff. v. Inclusive Communities Project,* 136 S. Ct. 2507 (2015).

[123] E. McGhee. *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGISLATIVE STUDIES QUARTERLY 55–85 (2014); Stephanopoulos and McGhee, *supra* note 80.

efficiency gap in a presentation that included 36 figures.[124] This report was challenged by the state's expert witness, who focused on the question of how much asymmetry came from population clustering; that expert was, in turn, counter-challenged.[125]

While such challenges are an inevitable part of complex litigation, the use of longstanding and simple statistical tests may reduce the need for expert witnesses and detailed presentations. In particular, the Tests 2 and 3 proposed here use well-known statistical tests with established procedures for significance testing, can be explained succinctly[126], and can be worked out by hand. These qualities confer transparency to my proposed analysis.

In addition, I have used this Article's tests to separate the contributions of gerrymandering and population clustering (see section II.B.). Since gerrymandering relies on the ability to sequester voting populations, the geographic patterns that give Republicans a naturally-occurring advantage can also be used to construct further artificial advantages. Conceptually, this addresses the concern about natural clustering expressed in the *Whitford* testimony.

C. WHAT IS THE ROLE OF INTENT?

The intent prong in *Bandemer* initially required that the intent be predominantly partisan.[127] This presented a higher bar to proving injury than simply showing that partisanship was one of multiple factors. It is a far higher bar than the evaluation of disparate impact alone. Such a stringent standard may have been appropriate in the absence of legislative guidance or a large body of court precedent. In the *Bandemer/Vieth* framework, the lack of simple and reliable tests made it necessary to assess the link between redistricters' actions and the injury. Indeed,

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[124] S. Jackman, *Assessing the Current Wisconsin State Legislative Districting Plan.* Exhibit 3 in Whitford v. Nichol. http://www.fairelectionsproject.org/s/Exhibit-3.pdf (last downloaded March 27, 2016).

[125] S. Trende, State Expert's Declaration, *Whitford v. Nichol.* http://www.fairelectionsproject.org/s/Declaration-of-Sean-Trende-Dkt-55.pdf (last downloaded March 27, 2016); S. Jackman, Rebuttal Report, *Whitford v. Nichol.* http://www.fairelectionsproject.org/s/Jackman-Rebuttal-Report-Dec-21-2015.pdf (last downloaded March 27, 2016).

[126] S. Wang, amicus brief, *Harris v. Arizona Independent Redistricting Commission*, 993 F.Supp.2d 1042 (D. Ariz. 2014). https://web.math.princeton.edu/~sswang/14-232_bsac_SamWang.pdf (last downloaded March 27, 2016); also see Wang, *supra* note 6.

[127] *Bandemer*, 478 U.S. 109, 128.

current approaches to proving gerrymanders focus on intent, are diverse in approach, and sometimes do not agree with one another.[128]

An example of ambiguous intent is found in *LULAC v. Perry*.[129] The Republican majority was able to involve individual Democratic legislators in the districting process.[130] However, in matters of redistricting, a party as a whole has motivations that can be at odds with those of some of their own party's individual legislators[131]. Therefore intent is most fairly evaluated at the state level or at the individual level, but not both at the same time. In addition, the majority in *Crawford v. Marion County Election Board* held that partisan intent is insufficient as a reason to strike down voting restrictions[132].

The identification of intent begins with a fact-specific inquiry into the state of mind of the legislature and/or the entity that drew the district lines. Statistical testing such as my proposed Tests 2 and 3 allows the identification of patterns of districting that are highly unlikely to have arisen by chance, thereby providing concrete evidence that a legislature or other district-drawing body acted specifically to produce partisan outcomes. This rigorous standard should aid tremendously in the identification of intent.

Furthermore, I suggest that districting can impose a burden on a group's representational rights whether or not the effects (as measured by Test 1) are intentional. Even where intentions are nonpartisan, bipartisan, or unknown, the effect of a districting plan with partisan asymmetry is to produce legislative blocs whose size is unrepresentative of the popular will. The construction of a reliable measure of effect provides clear guidance when an injury has taken place, and a template for how the injury can be repaired. Just as a road worker may act to right an upended orange traffic cone even if he/she does not know how the cone came to be tipped over, a court may act when effects are sufficiently strong, as in disparate impact cases in racial

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[128] Micah Altman, Brian Amos, Michael P. McDonald, & Daniel A. Smith, *Revealing preferences: why gerrymanders are hard to prove, and what to do about it*, SOCIAL SCIENCE RESEARCH NETWORK, March 22, 2015, 11-36 (enumerating the strengths and weaknesses of different approaches to evaluating partisan gerrymanders). http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2583528. Last accessed August 27, 2015.

[129] *LULAC*, 548 U.S. 399, 417-418 (describing cooperation of individual Democratic legislators).

[130] *Id.*

[131] See discussion of mixed partisan motivations, *supra* note 85.

[132] *Crawford v. Marion County. Elec. Bd.*, 553 U.S. 181, 203-204 (2008).

discrimination cases[133]. Although partisan gerrymandering cases are governed by different doctrine (Constitutional) from racial discrimination cases (statutory interpretation), both types of case concern the issue of intent.

C. EVALUATING THE PARTISAN IMPACT OF DISTRICT MAPS BEFORE IMPLEMENTATION

Although in this Article I used election results to calculate the three tests, the tests could alternatively use other inputs. For example, to rule out the possibility that the tests may be influenced by variations in the quality of specific candidates, it would be possible to use district-level presidential vote shares as inputs.[134]

In current federal precedent, the need for redrawing a set of districts often relies on forensic evidence; that is, on elections that have already occurred.[135] However, by that time an injury to voters has already occurred. To preempt such an injury from occurring, the three tests could be calculated using information that is available before an election. Under the First Amendment rationale of not penalizing groups for their partisan preference, party registration might be used as an input to calculate the three tests. Political scientists, redistricters, and commercial redistricting software also use other variables to predict overall partisan preference; these predictions could also serve as inputs to the tests. Doing so would allow a hypothetical districting scheme to be assessed before it has passed into law.

The standards presented here can quantify the benefits of reform efforts directed at reducing the likelihood of partisan gerrymandering. One such route is the establishment of nonpartisan districting commissions that remove districting from the direct control of legislators. In California, a voter referendum in 2008 established the formation of the California Citizens

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[133] In one recent example, in a racial discrimination case the Supreme Court ruled that demonstration of disparate impact was sufficient to prove discrimination, and that a demonstration of intent was not necessary. *Tex. Dep't of Housing and Comm. Aff., supra* note 121. This case held that in light of results-oriented statutory language in the Fair Housing Act, determination of disparate impacts was sufficient to warrant a remedy, even without discriminatory intent. I argue that if gerrymandering has a sufficiently large effect on a party's supporters, such an injury should still be remedied even when redistricters are not motivated purely by partisan intent.

[134] LULAC, 548 U.S. at 420 (Justice Kennedy, joined by Justices Souter and Ginsburg) states in regard to a partisan gerrymandering claim that "such a challenge could be litigated if and when the feared inequity arose." Redistricting software is capable of using quantities such as the presidential vote share to estimate the partisan tendency of a hypothetical district. Redistricters use such measures to judge the likely outcome of a district, and could use them as inputs to the three tests in this Article to evaluate a districting plan before it is implemented.

[135] LULAC, 548 U.S. at 2638 (n.9) (opn. of Stevens, J. P., joined by Breyer, S).

Redistricting Commission.[136] The commission is composed of 14 members who are drawn from members of the general public, including five Democrats, five Republicans, and four members who decline to state a partisan loyalty.[137] The commission's mandate is to draw districts that respect principles of contiguity, compactness, and representation of a community's interests.[138] The resulting Congressional districts have become more competitive: margins of victory have become smaller, and incumbents have lost their re-election races at higher rates than before the formation of the commission.[139] Like the Arizona commission, the work of the California commission has led to closer races and more euproportional overall outcomes.

These tests could also be used in approaches that leave districting under the control of state legislators, but place constraints on how and what they produce. Such an approach has been taken in Florida, ballot initiatives known as Amendments 5 and 6 were passed in 2010, becoming Article III, §§ 20 and 21 of the Florida Constitution.[140] Together with Article III, § 16[141], the Florida Constitution stipulates that district lines "must be contiguous, compact, and use existing political geographical boundaries where available."[142] Districts also may not be drawn to "favor or disfavor a political party or incumbent."[143] The resulting plans are subject to review by the the Florida Supreme Court for review, leading either to approval or return to the legislature for a further attempt to meet districting criteria.[144] The tests described in this Article could be useful in identifying statewide partisan favor. Individual districts would still need to be evaluated separately, for example to comply with Voting Rights Act restrictions and other principles set

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[136] CALIFORNIA CITIZENS REDISTRICTING COMMISSION, http://wedrawthelines.ca.gov/regulation_archive.html (last visited Aug. 24, 2015).

[137] Calif. Const. art. XXII, § 2(c)(2).

[138] Calif. Const. art. XXII, § 2(d).

[139] *Id.*

[140] Justin Levitt, *Florida,* ALL ABOUT REDISTRICTING, http://redistricting.lls.edu/states-FL.php (last visited Aug. 24, 2015).

[141] Fla. Const. art. III, § 16.

[142] Fla. Const. art. III, §§ 20-21.

[143] *Id*.

[144] Fla. Const. art. III, § 3(b).

down in federal or state law. These tests, which address the properties of combinations of districts, can complement these other constraints without conflict.

CONCLUSION

Partisan gerrymandering distorts relationships between voting and representation that would otherwise arise naturally, generates seats that are unresponsive to shifts in public opinion, and chills the freedom of voters to associate with a political party of their choosing. The health of democratic processes would be considerably improved by reducing the ability of legislative processes to impose partisan distortions on redistricting maps. The three tests for asymmetry presented here may contribute to a manageable standard for identifying partisan gerrymanders, with the eventual goal of reducing or eliminating them.