Measuring The Equilibrium Real Exchange Rate for the Swiss Economy: An Unobserved Component Approach

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Abstract

In this paper, I jointly estimate the equilibrium real exchange rate, the potential output and the trend growth of potential output for Switzerland, using a Kalman filter approach, in a univariate and multivariate model, with or without microeconomic fundamental determination.

The equilibrium real exchange rate (ERER) shows significant variation over the past 30 years in Switzerland. In the multivariate models, my empirical tests show a strong positive and singificant relationship between the Swiss output gap and the real exchange rate gap as well as between inflation and the real exchange rate gap.

Surprisingly, my model does not show any significant evidence for a role for the real interest rate gap, in the case of Switzerland. It is the real exchange rate gap effect that dominates rather than the real interest rate gap. It is possible that the real interest rate gap has an effect for the output gap, albeit, a slower effect than the real exchange rate gap.

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1 Introduction

The object of monetary policy is to influence the performance of the economy, as reflected in such factors as inflation, economic output and employment. It works by affecting demand and supply across the economy, that is the population's willingness to spend on goods and services and firms' willingness to invest and produce.

Monetary policy has two basic goals: to promote maximum output and employment and to promote stable prices. In the long-run, the level of output and employment in the economy depends on factors other than monetary policy. These includes technology, people's preferences for saving, risk and work effort. So, promoting maximum output and employment is relative to levels that are consistent with these factors in the long-run.

In small open economies, the exchange rate is a key variable. Movements in exchange rates directly influence the domestic-currency price of goods that are traded in world markets. The exchange rate introduces a number of new channels in the monetary transmission mechanism: (i) the real exchange rate affects the relative price between domestic and foreign goods and thus contributes to aggregate demand channel; (ii) it affects consumer prices directly via the domestic currency price of imported goods and (iii) the real exchange rate affects the price of imported intermediate goods, and thus the pricing decision of firms.

Movements in the exchange rate are not well understood. A long-standing puzzle in international economics is the difficulty of explaining the exchange rate movement by some macroeconomic fundamentals, such as the money supply, output or interest rates. In particular, the parity conditions typically used in theoretical analysis – UIP and PPP – do not find much support in empirical studies. Meese and Rogoff (1983) first established the result that floating exchange rates between countries with roughly similar inflation rates are well approximated as random walks. Thus, the best predictor of next period change in exchange rate is this period nominal exchange rate change. As price levels are much less volatile than the exchange rate, and assumed sticky in the short-run, both at home and abroad, changes in the nominal exchange rate are reflected in changes in the real exchange rate.

As the economy goes through business cycles, output and employment could be above or below their long-run or potential levels. However, objects like potential levels or equilibrium levels of output, the natural rate of employment or unemployment, the equilibrium real exchange rate, are **unobservable**. What we measure is the **actual** levels of all these variables. Nonetheless, these equilibrium levels are key to the conduct of macroeconomic policy making. Monetary policy is based, among other things, on stabilizing the economy around its equilibrium values for output and inflation. If one ignores these equilibrium values or cannot estimate them properly, one has hard time converging, in the medium or long term, towards them.

A key variable in the conduct of modern monetary policy is the 'natural' or 'equilibrium' rate of interest that is consistent with a 'neutral' stance of policy – an equilibrium rate of interest that is consistent with output equalling potential and stable inflation. In an open economy approach, the simple expectation-augmented Phillips curve does not work due to the fact that inflation reacts to outside shocks. My conjecture for the Swiss economy, is that it is rather the exchange rate, more than the interest rate, that plays a prominent role in the transmission mechanism of monetary policy. In a small open economy, while Central Banks set a short-term rate of nominal interest or base money or GDP growth, in effect, by inducing exchange rate movements, monetary policy will affect the CPI inflation, even with a shorter lag than in the closed economy model. Thus, it is the exchange rate movement induced by the interest rate change that will, in the medium term, stabilize the economy, reaching potential output and having a stable inflation.

However, neither is the equilibrium real rate of exchange rate observable nor is the growth of potential output , another key variable for conducting monetary policy, observable . Moreover, these unobserved variables, which are often assumed constant in equilibrium, may instead be time-varying: for instance, PPP asserts that the equilibrium real exchange rate is constant. However, it may vary across business cycles, due to change in relative productivity, the Balassa-Samuelson effect, or change in terms of trade, the accumulated current account or the world real interest rate; in other words, if the underlying determinants of the equilibrium variable vary, then the equilibrium variable, over the business cycle, is itself time-varying. This applies to the equilibrium real exchange rate as well as to the growth of potential output, which may vary across business cycles. In this paper, we will assume a time-varying growth of potential output as well as time-varying equilibrium real exchange rate.

Four alternative methods will be used to estimate these long-run key variables, assuming that these equilibrium values are time-varying due to the time variation of the underlying fundamentals. The first method estimates the equilibrium real exchange rate (ERER) and potential output with a plain Hodrick-Prescott filter, using $\lambda_y = \lambda_e = 1600$ for the smoothness parameter in the HP filter series, for quarterly output data and quarterly real exchange rate smoothing. The larger the parameter λ is, the smoother is the solution series. The second method will decompose the observed real exchange rate into a random walk equilibrium process and a stationary cycle. The third method will use the long-run relationship between an observed RER and some economic fundamentals, assuming a random walk process for the equilibrium real exchange rate, as in the previous method. The fourth method, an extension of the former method, will conjecture and test which fundamentals drive the equilibrium real exchange rate process, following the economic theory.

The fundamentals for the equilibrium real exchange rate which I will test are as follows: (i) the productivity changes (the Balassa-Samuelson effect), using as proxy, the relative growth of potential output; (ii) the terms of trade; (iii) the accumulated Current Account as a ratio to GDP and (iv) the real interest rate. All variables are in relative terms, relative to the trade partner of Switzerland, which in my paper is Germany.

The first two estimation methods are purely econometric methods. The latter two methods are based on the Kalman filter signal extraction technique.

The Kalman filter method is a set of mathematical equations - (a) the measurement equations, relating the observables to the unobservable variables and (b) the transition equations defining the dynamics of the system of state variables – that provides an efficient computational (recursive) solution of the least-squares method. The Kalman filter state-space model was originally developed by control engineers (Kalman, 1960) to express dynamic systems that involved unobserved state variables. It is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. I am applying these two estimating methods to estimate jointly the equilibrium real exchange rate, potential output as well as the output gap.

The equilibrium real exchange rate will turn out to be highly significant for the output gap and the inflation rate in all the four models analyzed. This will prove to be significant and confirm my initial conjecture of the significant impact of the real exchange rate on the output gap. The model estimation in the case of Switzerland shows that the output gap depends on the change in the real exchange rate gap negatively but does not depend on the deviation from the natural rate of interest. Inflation also depends negatively on the change of the real exchange rate gap.

The rest of the paper is organised as follows. Section 2 presents the models; Section 3 presents the methodology; Section 4 presents the estimation issues; Section 5 the macro-fundamentals determination of the equilibrium real exchange rate; Section 6 presents the empirical results and Section 7 concludes.

2 The State Space Model

The model, a simple reduced-form VAR model, consists of an IS equation relating the output gap to lagged output gap, the real exchange rate gap and exogenous foreign demand variables and a Phillips curve equation relating inflation to lagged inflation, the output gap, the real exchange rate and exogenous terms affecting inflation. For inflation, we use CPI inflation. To close the state-space model, we specify the dynamics of the state variables.

2.1 The IS Equation

The IS equation consists of the output gap, lagged output gap and the change in real exchange rate gap:

$$\widetilde{y}_t = A_y(L)\widetilde{y}_{t-1} + A_e(L)\widetilde{e}_t + A_{dm}(L)dm\widetilde{y}_{t-1} + \varepsilon_{yt} \tag{1}$$

where $\widetilde{y}_t = 100 * (y_t - y_t^*)$ denotes the output gap, y_t is the logarithm of real GDP, y_t^* is the logarithm of the unobserved potential GDP, $\widetilde{e}_t = 100 * (e_t - e_t^*)$ is the real exchange rate gap between the actual and the unobserved equilibrium exchange rate and $dm\widetilde{y}$ is German real GDP gap, a proxy for foreign demand. ε_{1t} is a serially uncorrelated error term.

2.2 The Phillips Curve Equation

The Phillips curve equation links the CPI inflation with lags inflation, lagged output gap and the change in the real exchange rate gap, oil price inflation relative to CPI inflation and a serially uncorrelated error term:

$$\pi_t = B_y(L)\widetilde{y}_{t-1} + B_e(L)\widetilde{e}_t + B_\pi(L)\pi_{t-1} + B_x(L)x_t + \varepsilon_{\pi t} \tag{2}$$

where x_t captures other determinants of inflation and in our case, relative oil price inflation.

2.3 The Real Exchange Rate Equation

2.3.1 The Real Exchange Rate Equation Unobserved Component Approach

A third equation decomposes the real exchange rate as a random walk process, the trend, and a stationary gap component, the cycle:

$$e_t = e_t^* + \widetilde{e}_t \tag{3}$$

where e_t, is the real exchange rate, e_t^*, is the stochastic trend component and $\widetilde{e}_t$, is the stationary cyclical component, where $\widetilde{e}_t = e_t^* - e_t$. Moreover, e_t^* is defined to be a random walk with time-varying drift:

$$e_t^* = e_{t-1}^* + g\_e_{t-1} + \kappa_{et} \tag{4}$$

where g_e_t is the ERER growth.

and the real exchange rate gap, $\widetilde{e}_t$, is assumed to be a stationary AR(2) process:

$$\widetilde{e}_t = \phi_1\widetilde{e}_{t-1} + \phi_2\widetilde{e}_{t-2} + \varepsilon_{et} \tag{5}$$

2.3.2 A Model of The Equilibrium Real Exchange Rate with Fundamentals

In the context of an open-economy IS-Phillips curve model, it is clear that the real exchange rate affects both the output gap and the CPI inflation. But, what drives the real exchange rate. As stated earlier, the real exchange rate is driven by domestic as well as foreign factors. It is driven by domestic demand, fiscal and monetary policy but also by foreign demand.

My estimation of the equilibrium real exchange rate is defined in the medium to long-term equilibrium, when the economy is both in internal and external macroeconomic balance. By medium or long-term is meant when both output is at potential, inflation is stable on the internal side, and externally, when the currrent account is at a sustainable value, i.e, at a value that is sustainable as a ratio of debt to GDP and reflects desired net saving position levels for the country. Here I used "desired" instead of potential, in light of the Swiss case of running surpluses of several percents of GDP over several decades, reflecting for the Swiss economy, as a whole, higher desired savings preferences. Unlike deficits, current account surpluses, although not efficient economically, can be maintained for long periods wihtout real harm to the economy, except for over-appreciated real exchange rates, which, in the case of Switzerland, a highly open economy, would be detrimental in the medium to long-run, unless returns outside are expected to be higher than within, in the long-term.

I hypothesize that my model of the real exchange rate equation, in the case of Switzerland, is a function of (i) the relative trend growth of GDP, as a proxy for capturing the Balassa-Samuelson effect adn a wealth effect, (ii) accumulated current account ratio to GDP and (iii) gold stock price (iv) other fundamentals.

$$e_t^* = \alpha_0 + \alpha_1 \Pr od_t + \alpha_2 ACA_t + \alpha_3 Gold_t + Other_t \tag{6}$$

where $\mathrm{Prod}_t \equiv$ is the relative home to foreign growth trend of potential GDP, $ACA_t \equiv$ Accumulated Current Account, $Gold_t \equiv$ the US dollar stock price of gold; gold is highly and inversely correlated to the Swiss/DM real exchange rate and could function in a safe haven argument, $Other_t \equiv$ Other Fundamentals; for $Other_t$, we could interpret it, in the case of Switzerland, as an index of credibility, or institution. However, $Other_t$ will not be significant and will be eliminated from the final fundamental model.

This Fundamental equilibrium equation for the real exchange rate (6) is consistent with Williamson's "fundamental equilibrium exchange rate (FEER)" associated with the effective real exchange rate at which an economy is both at external and internal balance.

Although the current account variable is endogenously determined with the real exchange rate, the accumulated current account, ACA_t, is assumed to be predetermined.

Please note that all the variables entering this equation (20) determining e_t^* are **long-term or smoothed variables;** by long-term, I mean smoothed with an HP filter, except for g_t which will be estimated with the Kalman filter, along with potential output.

In my estimation, I test whether the growth trend of potential output, g_t, varies or remains constant. It is widely believed that growth trend, like other key equilibrium rates, vary over different periods. On the other hand, the foreign growth trend, h_t is considered to be exogenously determined. I will apply the Kalman filter to these unobserved low frequency variables, modeling the cyclical dynamics of output and inflation using a restricted VAR model that imposes little structure on short-run dynamics.

3 Data Source and Properties

3.1 Data Source

To empirically test my model, I analyse the Swiss economy, using quarterly data from 1972:1 to 2003:3 with the following series: (1) real GDP; (2) CPI price index; (3) the CHF/DM index of real exchange rate; a rising index value expresses an depreciation of the Swiss franc. The real exchange rate variable is derived from the CHF/DM nominal exchange rate doubly deflated by the ratio of German to Swiss GDP deflators (4) is the nominal government bond yield. This series was chosen over money market rates because of its longer time period (5) an oil price index deflated by the GDP deflator; (6) Germany real GDP as a proxy for foreign demand; (7) Germany potential Real GDP filtered by the Hodrick-Prescott filter; (8) the Swiss Terms of Trade, which is the ratio of an index of exports prices over import prices; (9) is the German Terms of Trade; (10) Swiss Current Account; (11) USD Gold stock price index.

Note that throughout my estimations, Germany, the main trading partner for Switzerland, is taken here as the rest of the world. According to a Swiss National Bank 2001 survey, Europe constituted over 70 percent of Switzerland's total export, of which the Euro currency area represented over 60 percent of trade shares, of which Germany accounts for more than 25 percent of Swiss exports; As Germany was the locomotive for Europe and the euro zone, I selected Germany as the main trade partner for Switzerland and used the DM/CHF real exchange rate instead of the REER, the real effective exchange rate. [1]

These variables are then transformed into: (1') log real GDP; (2') CPI inflation;(3') expected CPI inflation is modeled, using the naive approach, with lagged inflation as expected inflation; this approach is adopted as the hypothesis of a unit root is not rejected at the 1% level by an augmented Dickey-Fuller test, (see Table 1); (4') log DM/CHF Real exchange rate; (5') real interest rate is the nominal minus expected inflation, using the naive approach stated in (3'); (6') annualized world oil inflation deflated by GDP deflator; (7') log German real GDP; (8') log German potential HP-filtered real GDP; (9') the Swiss terms of trade; (10') the Swiss Accumulated Current Account Ratio to RGDP.[2]

(Place Fig.(1) here).

Figure (1) gives an idea of the historical evolution and relationship among the economic fundamentals considered in this paper with the DM/CHF index of real

[1] I introduce a level dummy for the German reunification, this dummy is negative but not significant:

The dummy, $w_t = 1$ for $t = 1990:3$; I also tried, $w_t = 1$ for $t = 1990:4$ or $1991:3$, but the dummy remains non-significant;

my del_e$_t^*$ equation becomes:

$$\Delta e_t^* = \alpha_0 + \alpha_1 (g_t - h_t) + \lambda w_t + \alpha_2 q_t + \alpha_3 ACA_t + \alpha_4 (r_{t-1} - dmr_{t-1})$$

[2] The accumulated current account ratio to GDP is used as a proxy for the net foreign asset position. The reason it is only a proxy is that it ignores valuation effects.

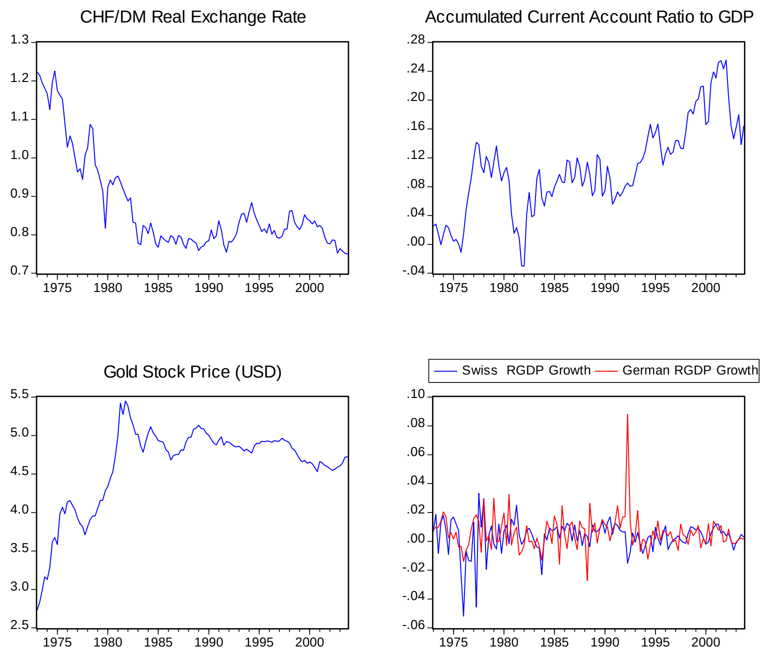


Figure 1: CHF/DM Real Exchange Rate and Its Determinants

exchange rate. All of these variables are in logarithmic transformation except for CPI inflation. From Figure 1, we see that the DM7CHF real exchange rate has an appreciating trend. We can notice also that is the case for the real GDP, both Swiss and German and the accumulated current account ratio to GDP. As for the relative productivity growth, HP-filtered estimation, it is mainly negative and goes through an increase trend during the 1970's then a decrease of similar magnitude followed by an increase to reach similar productivity growth in the end of the 1990's and early 2000. Terms of trade, another important determinant of the real exchange rate is decreasing in the mid-1970's and then has an increasing trend from the 1980's onward.

3.2 Unit Root Tests

Identification of the stationary cyclical components require that all variables in the system are stationary; in other words, taking the initial dependent variables in each equation, they need to be of the same order of integration.

Variable/ADF	Const., Trend			1st Diff		
	Lag	t-stat	p-stat	Lag	t-stat	p-stat
e1	0	-2.506186	0.324769	11	-7.33914***	1.93E-15
Rel. Prod.	0	-11.24575***	1.64E-15	2	-12.00327***	6.39E-58
ACA	3	-3.641553**	0.030335	5	-5.995882***	2.28E-09
Gold	3	-3.134585	0.103006	2	-3.70694***	2.77E-04
RGDP	0	-2.011347	0.589143	1	-5.634961***	2.85E-08
DM RGDP	0	-1.545762	0.808364	3	-2.837157***	4.83E-03

Figure 2: ADF Unit Root Tests

The unit root test for the real exchange rate and its determinants, is not rejected for the real exchange rate nor for gold, the Swiss and German real GDP but is rejected for the relative productivity and the accumulated current account at the 1 and 5 percent respectiviely. All the first-differenced variables are stationary. But, it is well known that unit root tests have a high false rejection rate under the null of a unit root when there is a nearly unit MA root. So, this result of Table (2) does not necessarily contradict the results of the median unbiased estimators of Stock and Watson. Table (2) presents the results for the unit root test, using the Augmented Dickey-Fuller test. The lag length is determined by AIC and SIC criteria. And Table (1) gives the results for the median unbiased estimators.

4 Estimation

4.1 Methodology

In Harvey's (1989) notation for the state-space model, the state-space model can be written in the following form:

$$Y_t = Z\alpha_t + Bx_t + u_t \quad (7)$$

$$\alpha_t = T\alpha_{t-1} + c + v_t \quad (8)$$

Here, Y_t and x_t are vectors of contemporaneous endogenous and exogenous and lagged endogenous variables respectively; α_t is a vector of unobserved states. The vectors of stochastic disturbances u_t and v_t are assumed to be Gaussian and mutually uncorrelated with mean zero and covariance matrices H and Q, respectively. The covariance matrix H of the measurement equation (7)is assumed to be diagonal.

The Measurement Equations The measurement equations that we estimate, in the last stage model, is the following:

$$\begin{aligned}
\widetilde{y}_t &= a_1\widetilde{y}_{t-1} + a_2\widetilde{y}_{t-2} + \alpha_3\widetilde{y}_{t-3} + a_4\widetilde{e}_{t-1} + a_5\widetilde{e}_{t-2} + \varepsilon_{yt} && (9)\\
\pi_t &= b_1\widetilde{y}_{t-3} + b_2\widetilde{y}_{t-4} + b_3\widetilde{e}_{t-1} + b_4\widetilde{e}_{t-2} + b_5\pi_{t-1} + && (10)\\
&\quad + b_6\pi_{t-2} + b_7\pi_{t-3} + b_8\pi_{t-4} + b_9 oil_t + b_{10} oil_{t-1} + \varepsilon_{\pi t}\\
\widetilde{e}_t &= \phi_1\widetilde{e}_{t-1} + \phi_2\widetilde{e}_{t-2} + \varepsilon_{et} && (11)
\end{aligned}$$

where a "~" variables are differences from potential. The lagged German output gap will be eliminated from the IS equation as it is not significant.

The Transition Equations To close the model, I need to determine the dynamics of the model state variables, the 'transition equations'. Modeling potential output as an I(2) process seems sensible if we assume that the trend growth follows a random walk process, something that I will test for. The transition equations for the state variables are as follows:

$$\begin{aligned}
y_t^* &= y_{t-1}^* + g_{t-1} + \eta_{y^*t} && (12)\\
g_t &= g_{t-1} + \eta_{gt} && (13)
\end{aligned}$$

where y_t^* is potential output, g_t is the the potential output's growth trend; η_t, ξ_t are assumed to be serially uncorrelated Gaussian innovations. To those above transition equations, I assume a random walk process for the equilibrium real exchange rate:

$$\begin{aligned}
e_t^* &= e_{t-1}^* + g_e_{t-1} + \kappa_{et} && (14)\\
g_e_t &= g_e_{t-1} + \kappa_{gt} && (15)
\end{aligned}$$

where e_t^* is the equilibrium real exchange rate (ERER),and g_e_t is the ERER growth, assumed to be time-varying.

The Model with Fundamentals In the model with fundamentals, I will replace equation the random walk with drift determination of the equilibrium real exchange rate equation (14) with its determination by the fundamentals equation, (20)

4.2 Estimation Issues

Three specific issues are at stake here in the MLE estimation: the issue pertaining to some variance parameters, the choice of initial values of the state variables and the computation of standard errors.

A problem in the estimation of unobserved component models is the **"pile-up" problem**, highlighted by Stock and Watson (1998). The MLE of the variance of the shocks to the time-varying growth of potential GDP and equilibrium real exchange rate, $\sigma_{\eta_{gt}}$, and $\sigma_{\kappa_{gt}}$, has the undesirable property that if the variance is too small, it has point mass at zero and the variance is mistaken for zero, even if its true value is greater than zero. When the signal-to-noise ratio is zero, $\frac{\sigma_{\eta_{gt}}}{\sigma_{\eta_{y^*t}}}$, the state variable is stationary, even if its true state is non-stationary. This discontinuity in the limiting distribution of the state is responsible for the pile-up effect. In our model, one would expect to find estimates of $\sigma_{\eta_{gt}}$, and $\sigma_{\kappa_{gt}}$ to be zero with high probability, even if their true value is strictly larger than zero. Stock and Watson (1998) propose an estimation procedure for obtaining median-unbiased estimates of the signal-to-noise ratio to deal with the "pile-up" effect.

The second problem is **choosing initial values** y_0^*, g_0 and e_0^* for the non-stationary state variables. Because these processes $\{y_t^*\}, \{g_t\}$ and $\{e_t^*\}$are non-stationary, setting the conditional expectation and covariance matrix of the initial state at the start of the Kalman filter equal to its unconditional expectation and covariance matrix is not feasible. The most commonly used approach in the presence of a non-stationary state is to integrate the initial value out of the likelihood by specifying an approximately diffuse prior, approximately because assuming an absolutely diffuse prior would need to set P_0, the variance-covariance matrix of the state to infinity, $P_0 = \infty$. In practice, we use $P_0 = \kappa I$, with $\kappa \to \infty$. However, as Harvey (1993) explains, starting the Kalman filter with a large κ is not entirely satisfactory as the presence of large numbers in the filter can lead to instability. Instead, I follow Harvey (1989, sec.3.3.4) who suggests to use a few initial observations to estimate the initial state by GLS and use the covariance matrix of the estimator as initial value for the conditional covariance matrix of the state. Harvey(1989) states that "*the use of a diffuse prior is equivalent to the construction of a proper prior from the first* m *sets of observations provided the model is observable*[3]*"*. For the measurement equations initial values of the parameters, they are computed by OLS regression, where potential GDP is proxied by a segmented trend fitted through log GDP; the growth of potential GDP is proxied by a constant. For the real exchange rate gap equation, a Hodrick-Prescott filter is used for the equilibrium real exchange rate. An OLS regression is then run to estimate the initial values for the parameters equation.

State-space models estimations allow for quantifying the **uncertainty** around the estimates. There are two sources of uncertainty described by Hamilton (1994, sec. 13.7). There is an uncertainty due to the fact that the true state is unobserved and thus, has to be inferred. These estimates of the prediction error variance for the state at time t are conditioned either on data observed up to time t or on the whole sample. Either of these two methods assume that the parameters are known. A second source of uncertainty arises from the maximum-likelihood estimation of the parameters themselves. Given es-

[3] The state vector α_t, is said to be 'observable' if it can be determined excatly, given the observations $y_t, ..., y_{t+m-1}$. A necessary for the system to be observable is $Rank[Z', T'Z', ...(T')^{m-1}Z'] = m$

timates of the model parameters and their covariance matrix, Hamilton's MC procedure is used to obtain standard errors for the state taking account of both uncertainties.

Using Monte Carlo simulations to obtain standard errors for the state causes problems as it is conditioned on initial states to be estimated. When obtaining estimates of the initial state by GLS, the simulations involve draws of parameters that yield a badly conditioned covariance matrix of the GLS estimator. To avoid this problem, Laubach and Williams (2003) suggest, during the simulations, the initial state is drawn randomly from a normal distribution with the mean and variance given by the smoothed estimates obtained by the estimation.

4.3 Estimation of the Univariate Model

I apply Kim and Nelson's (1999) unobserved component model originally applied to US real GDP to decompose the quarterly Swiss RER, e_t, into a stochastic trend and a cyclical component. The univariate model is as follows:

$$e_t = e_t^* + \widetilde{e}_t \tag{16}$$

$$e_t^* = e_{t-1}^* + g\_e_{t-1} + \kappa_{et} \tag{17}$$

$$g\_e_t = g\_e_{t-1} + \kappa_{gt} \tag{18}$$

$$\widetilde{e}_t = \phi_1 \widetilde{e}_{t-1} + \phi_2 \widetilde{e}_{t-2} + \varepsilon_{et} \tag{19}$$

where e_t is the real exchange rate, e_t^* is the stochastic equilibrium real exchange rate and $\widetilde{e}_t$ is the stationary real exchange rate gap. κ_{et}, κ_{gt} and ε_{et} are independent white noise processes.

4.4 Estimation of the Multivariate Model with ERER modeled as a Random Walk With Stochastic Drift

The model that I now estimate is equation (9) to equation (15). The estimation of the state variables is done in three stages. I follow Laubach and Williams (2003) method and apply the Kalman approach to my system. In this first estimation, I apply the Kalman filter to estimate potential output, omitting the real exchange rate gap term from the IS equation and assuming that the trend growth rate, g, is constant. I compute the Exponential Wald statistic (EW) of Andrews and Ploberger (1994) for a structural break with unknown break date on this preliminary estimate of potential output. I then use Stock and Watson's result to convert the exponential Wald statistic into an estimate of $\widehat{\mu}_g = 0.0940$ for the signal-to-noise ratio of the growth potential output. Applying Stock and Watson's method to the preliminary estimate of potential GDP, instead of directly to the GDP data, has the advantage that it is consistent with the assumption of long-term or permanent effects on the level of the series under consideration. When applying the Kalman filter, I always test for no serial correlation in the error term.

I, then impose this estimate of $\widehat{\mu}_g$ to my second-stage estimation of the model, where the real exchange rate gap is included, assuming now a constant growth trend for the equilibrium real exchange rate. Repeating the Stock and Watson's Median Unbiased estimation (MUE) for the real exchange rate, I find a value for $\widehat{\mu}_e = 0.057$ for the signal-to-noise ratio of the growth potential real exchange rate. In the third-stage estimation of the model, I estimate the full model, imposing these two values of $\widehat{\mu}_g$, $\widehat{\mu}_e$ for the signal-to noise ratios of the growth of potential GDP and equilibrium real exchange rate.

4.5 The State-Space Model With Fundamentals

In this section, I specify the macroeconomic fundamentals which determine the the equilibrium real exchange rate (ERER).

$$e_t^* = \alpha_0 + \alpha_1 (g_t - h_t) + \alpha_2 ACA_t + \alpha_3 Gold_t + Other_t \tag{20}$$

where g_t is the home growth trend of potential GDP, h_t is the foreign (here German) growth trend of GDP, ACA_t is the home accumulated current account ratio to GDP; the variable $Other_t$ will be eliminated from the final analysis as non-significant. All the variables entering this equation (20) determining e_t^* are long-term or equilibrium variables.

4.5.1 Effects of Fundamentals on ERER

Relative Productivity Growth Rate Here productivity changes is proxied by the long-term growth of potential GDP. The classic Balassa-Samuelson effect says that an increase in the relative productivity change for tradables relative to non-tradables will have a positive effect on the ERER, ie an appreciation, given that I'm considering the CHF_DM real exchange rate. GDP is a crude measure of the development of productivity in the tradable sector of the economy. But, what really matters, in the long run, is the trend growth of output. Moreover, an increase in relative productivity should produce an appreciation of the real exchange rate through an asset approach. When productivity increases, firms produce more at a given cost and become more attractive as an asset, which appreciates the domestic currrency. On the other hand, an increased productivity can bring about a real depreciation through a goods market argument: an increase in relative efficiency raises output which raises the supply of home goods which decreases their relative price, relative to foreign but increases the domestic income of firm owners and firm workers. Part of this increase in income is spent on foreign goods. However, a fall in the price, both of traded and non-trqaded goods eliminates the excess supply but the price change does bring an increase in the real exchange rate, i.e, a long-run real depreciation. Thus, it is difficult to guess the sign of the relative productivity growth rate, depending on whether the income or wealth effect dominates. I will show that in my model, the relative productivity effect will be positive but not significant.

Accumulated Current Account ratio to GDP Several theoretical approaches predict that an increase in the net foreign asset position of the country should appreciate the equilibrium real exchange rate. Countries with large external liabilities need to run large trade surpluses in order to service them, and achieving those surpluses would require a more depreciated level of the real exchange rate.

US Gold Stock Price The Swiss currency usually works as a safe haven variable in times of crisis of the US currency. Gold follows the same pattern. There is indeed a strongly and inverse correlation betweent the CHF/DM real exchange rate and the stock gold price. I use the gold stock price as a proxy for a safe haven argument. One would then predict that a rise in the long-term price of gold would imply an appreciation of the Swiss currency, in real terms.

4.5.2 The State-Space Model With Fundamentals

After substituting equation (20) for e_t^*, the equilibrium real exchange rate in the full model, I reapply LW method together with the SW's MUE to re-estimate potential output and the growth trend. Substituting e_t^*, (20) into our system, we get the following equations:

$$\begin{aligned}
\widetilde{y}_t &= a_1\widetilde{y}_{t-1} + a_2\widetilde{y}_{t-2} + a_3\widetilde{y}_{t-3} + a_4 e_{t-1} + a_5 e_{t-2} + \\
&\quad -a_4\left[\alpha_1\left(g_{t-1}-h_{t-1}\right) + +\alpha_2 ACA_{t-1} + \alpha_3 Gold_{t-1}\right] + \qquad (21)\\
&\quad -a_5\left[\alpha_1\left(g_{t-2}-h_{t-2}\right) + \alpha_2 ACA_{t-2} + \alpha_3 Gold_{t-2}\right] + \varepsilon_{yt}\\
\pi_t &= b_1\widetilde{y}_{t-3} + b_2\widetilde{y}_{t-4} + b_3 e_{t-1} + b_4 e_{t-2} + \\
&\quad -b_3\left[\alpha_1\left(g_{t-1}-h_{t-1}\right) + +\alpha_2 ACA_{t-1} + \alpha_3 Gold_{t-1}\right] + \qquad (22)\\
&\quad -b_4\left[\alpha_1\left(g_{t-2}-h_{t-2}\right) + \alpha_2 ACA_{t-2} + \alpha_3 Gold_{t-2}\right] + \\
&\quad +b_5\pi_{t-1} + b_6\pi_{t-2} + b_7\pi_{t-3} + b_8\pi_{t-4} + b_9 oil_t + b_{10} oil_{t-1} + \varepsilon_{\pi t}\\
\widetilde{e}_t &= \phi_1\widetilde{e}_{t-1} + \phi_2\widetilde{e}_{t-2} + \varepsilon_{et} \qquad (23)
\end{aligned}$$

Estimated confidence intervals and corresponding standard errors for the estimates of the states, uses Hamilton (1986, 13.7) Monte Carlo procedure, which accounts for both filter and parameter uncertainty.

Median Unbiased Estimator μ				
Variable	**Med.Wald Stat**	**Point Est**	**90% Conf.Int**	
		$\widehat{\mu}$	**Low**	**High**
Growth GDP (g)	0.7	0.0256	0.000	0.1
Growth ERER (g_e)	5.63	0.1026	0.000	0.163

Table 1: Estimation of the MUE

5 Empirical Results

5.1 Estimation Results

5.1.1 First-Stage Estimation of the Median Unbiased Estimator (MUE) for The Growth Trend of Output and the ERER

The first state estimation of potential output shows that the trend growth rate, g_t, exhibits only a relatively modest amount of variation over time. Table (1) reports the value of the Median Wald statistic (MW) testing for an intercept shift in the first difference of potential GDP. Using Stock and Watson's median unbiased estimator, this significant evidence of a change in the growth rate of potential GDP translates into an estimate of μ_g of 0.0256, shown in the second column of the table. When multiplied by the standard deviation of potential GDP, $4 * \left(\mu_g * \sigma_\eta\right) = 4 * (0.0256 * 0.9) = 0.092\,16$, at an annual rate, the implied standard deviation of the trend growth change is estimated to be about 10 percentage point. The 90% confidence intervals for the estimate of μ_g is computed by Monte Carlo simulations, and are given as the low and high in Table (1). Note that the lower end includes 0.00, the case in which the growth trend rate is constant and potential output is integrated of order 1. The upper end estimate for μ_g is approximately 0.1.

This value for the signal-to-noise ratio for μ_g, is imposed in the second-stage estimation of the signal-to-noise ratio estimate for ERER, μ_e. This value for the MUE is 5.63, is evidence of a change in the growth rate of the equilibrium real exchange rate, shown in the last row of the table, Table 2. When multiplied by the standard deviation of the equilibrium real exchange rate, $4 * (\mu_e * \sigma_\kappa) = 4 * (0.1026 * 1.126) = 0.462\,11$, at an annual rate, the implied standard deviation of the trend growth change is estimated to be about 46 percentage point. The 90% confidence intervals for the estimate of μ_e is again computed by Monte Carlo simulations, and are given as the low and high in Table 2. Note that the lower end includes 0.00, the case in which the growth trend rate is constant and

ERER is integrated of order 1. The upper end estimate for μ_e is approximately 0.16.

These two values for $\widehat{\mu}_g, \widehat{\mu}_e$ are then imposed for the third and final stage estimation of the model.

5.1.2 Parameter Estimation

There is empirical evidence of time variation in the US trend growth rate ([8]). This would imply that potential output would be integrated of order 2, I(2), not simply I(1). Although this hypothesis is generally rejected by the unit root tests, there is ample evidence for the US real GDP ([11]). Stock and Watson (1998) ([11]) explain that when the variance of the change in growth rate is small, the I(1) component, the model implies that the change in real GDP, the I(2) component, has a nearly unit MA root. And tests for a unit AR root have a high false rejection rate under the null of a unit AR root when there is a nearly unit MA root ([11]). I will apply the same technique to test for time variation in the Swiss trend growth rate and trend real exchange rate.

Some specification concerns the selection of lags for the VAR system. The theoretical discussion suggests what variables to include in the empirical model but it does not give any indication with respect to the dynamics on how many lags should be included. The lag lengths in the output gap and inflation equations have been determined in a way that is flexible enough to capture the lagged effects and to fulfill other statistical criteria, such as no serial correlation in the error term in the measurement equations and parsimony principles, using a general-to-specific specification approach based on AIC and BIC criteria. The model is estimated by maximum likelihood estimation, using the Kalman filter. Specifically, for the IS equation, I include three lags of the output gap, no inflation lag, and no lag for the German output gap, as the latter is not significant up to a fourth lag. For the Phillips equation, I include the 3rd and 4th lags of the output gap, four lags of inflation, the first difference of the relative oil price inflation. For the real exchange rate gap equation, I assume an AR(2) process, although we could assume an AR(1) process, as the second coefficient is not significant, in either case.

Discussion I have estimated four models:

- the Hodrick-Prescott filter Model (HP Model)
- the Univariate Model (Uni Model)

Parameter	**Uni Model**	**Multi. RW**	**Multi. FD. Mod 1**
λ_g		0.0256	0.0256
λ_e		0.1026	
IS Equation			
Σa_y		0.9147** (0.34)	0.9348** (0.37)
$a_{e,t-1}$		0.0774* (0.05)	0.0646* (0.0381)
$a_{e,t-2}$		-0.05 (0.0515)	-0.0634*** (0.037)
Phillips Curve Equ.			
$b_{y,t-3}$		1.523*** (0.4367)	1.6234*** (0.4376)
$b_{y,t-4}$		-1.445***(0.4245)	-1.5858*** (0.434)
$b_{e,t-1}$		0.1847** (0.1026)	0.1671*** (0.0836)
$b_{e,t-2}$		-0.208*** (0.1056)	-0.1731***(0.0764)
Σb_π		0.897*** (0.146)	0.906*** (0.155)
b_{oil}		0.0375*** (0.009)	0.0361*** (0.0109)
RER Gap Equation			
ϕ_1	072*** (0.08)	0.81*** (0.134)	0.8164*** (0.075)
ϕ_2	-0.04 (0.043)	-0.131 (0.1357)	-0.1067 (0.1067)
S.E.of equations			
σ_y		0.5547*** (0.1642)	0.4375*** (0.171)
σ_π		1.781*** (0.1568)	1.812*** (0.176)
σ_e	0.0257*** (0.00165)	2.3684*** (0.24)	2.6158*** (0.15)
σ_{y^*}		0.8464*** (0.116)	0.9*** (0.2)
σ_g		0.0216	0.023
σ_{e^*}	0.00009 (0.001)	1.126***(0.26)	
σ_{g_e}	0.001*** (0.0003)	0.006	
Log Liklhd	272.11	763.97	757.84
S.E.of States (Ave)			
y*		2.0714	2.81
g		0.447	1.151
e*		2.262	
g_e*		1.135	

Table 2: Parameter Estimation

Parameter	Uni Model	Multi. RW	Multi. FD. Mod 1
Estar Equation			
α_0			1.8554***(0.0904)
$\alpha_{\Pr od.}$			0.0204 (0.0373)
α_{ACA_t}			-0.0178*** (0.006)
α_{Gold_t}			-0.201*** (0.0205)

Table 3: Estar Equation

- the Multivariate Random Walk Process (Multi RW Model)
- the Multivariate Model with Fundamentals

In all the four models, the Box-Pierce Q statistic test for no serial correlation is satisfied for all equations at least up to the 16th lag.

IS Equation: The sum of the coefficients on the output gap $\Sigma a_y = 0.91$ (for the RW model), (resp. $\Sigma a_y = 0.93$ for the model with fundamentals) is significantly positive and strictly less, albeit close to 1. This shows strong persistence in the output gap but the effect, in the long-run dies off. The model is stable in the long-run. There remains the long-term effect of the first difference in the real exchange rate gap, the effect of which, $\alpha_{e,t-1} = 0.077$ for the RW model, (resp.$\alpha_{e,t-1} = 0.064$ for the model with fundamentals) is positive and significant. An increase in the real exchange rate gap, meaning an increased depreciation of the real exchange rate, has a positive effect on the output gap. But this latter effect of the real exchange rate gap is mostly reversed in the second period with a negative and significant coefficient of almost the same size .

The Phillips Curve Equation $b_{y,t-3} = 1.52$ (resp.$b_{y,t-3} = 1.62$) is the coefficient on the third and fourth lags of the output gap, in the Phillips curve equation. An increase in the output gap has a positive effect on inflation; but this effect is reversed in the next period by a reversal sign of almost equal size on the next period output gap. So the effect of the output gap is positive but not longer than four quarters. The coefficient on the real exchange rate gap, $b_{e,t-1} = 0.18$ (resp. $b_{e,t-1} = 0.17$) is significant and positive. Like in the IS equation, an increase in the real exchange rate gap, meaning an increase in the depreciation of the real exchange rate, will increase the rate of inflation. But this latter effect of the real exchange rate is reversed the next period. so the overall direct effect of the real exchange rate on inflation lasts over two periods. But the overall effect of lagged inflation, $\Sigma b_\pi = 0.9$ (resp. $\Sigma b_\pi = 0.9$) is significant and positive, and close to 1. An increase in past inflation will raise current inflation; this effect is long-lasting but eventually dies off in the long-run. Finally, $b_{oil,t} = 0.0375$,(resp.$b_{oil,t} = 0.036$) is the coefficient on the contemporary oil inflation, is positive, as expected. It has an impact on current inflation but one 30th of the combined effect of past lagged inflation. The overall effect of the real exchange rate gap is larger, in absolute values, for the Phillips curve equation than the IS equation, as there is a comnbined direct and indirect effect of the real exchange rate on the CPI inflation.

The Equilibrium Real Exchange Rate, e_t^* The equilibrium real exchange rate (ERER) is initially modeled as a random walk process with a stochastic drift. The cyclical component of the real exchange rate is significantly modeled as an AR(2) process, with an AR(1) coefficient which is significant, positive but less than 1 and an AR(2) coefficient which is negative but not significant.

Turning now to the determination of the ERER with fundamentals, (3) , the sign of the coefficient on the relative productivity growth, $\alpha_{\Pr od.} = 0.02$, on the relative productivity growth is positive but not significant. The relative growth of potential output is not a good proxy for relative productivity of traded goods versus non-traded goods, a proxy for the Balassa-Samuelson effect. Moreover, one would not expect a strong Balassa-Samuelson effect for a well-established industrial economy. The sign on the accumulated current account coefficient, $\alpha_{ACA_t} = -0.018$, is significantly different from zero and negative. The impact of the current account on the equilibrium exchange rate could go both ways. A country with current account surplus must experience a currency appreciation to induce domestic residents to acquire foreign securities, in equal amount to the current account surplus; that is an asset market approach. On the other hand, with rational expectations, supposing there is an anticipated future disturbance, say in the money supply. An initial jump in the exchange rate would be followed by further gradual depreciation. Under these circumstances, one would observe the opposite: a current account surplus would be accompanied by a currency depreciation; that is a wealth effect approach. The coefficient on the gold price, $\alpha_{Gold_t} = -0.2$ is negative and significant, as the safe have argument would predict. Economic theory would predict that all three coefficients, except for the constant, would have be negative. The constant term, $\alpha_0 = 1.85$ is significant and positive. One would expect a constant coefficient to be close to the mean, should the other fundamentals revert to their mean of zero, which is not necessasrily the case in our exemple.

I introduced a dummy for the year 1990-1991, corresponding to the German reunification period. But the coefficient was not significant. This was cancelled in subsequent analysis. It could be that the non-significance of the relative productivity term does not create a structural break for the equilibrium real exchange rate.

5.1.3 The Output Gap and the Growth Trend

The Kalman filter estimates of potential output and the output gap, using the multivariate model with a random walk (RW) determination for ERER or the one using fundamentals (FD) for the ERER determination, are close to the results produced applying the Hodrick-Prescott filter to those variables. Figure

3 to Figure 5 show these results obtained, using KF and HP. Estimates of the trend growth, using KF and HP are given in Fig. 5. Comparing the KF-filtered potential output with the HP's, we see that HP underestimates KF during booms and overestimates under recessions. One-step-ahead prediction and using an all sample prediction remain very close all along except during the first 10 periods of the sample.

(Place Fig. (3) here).

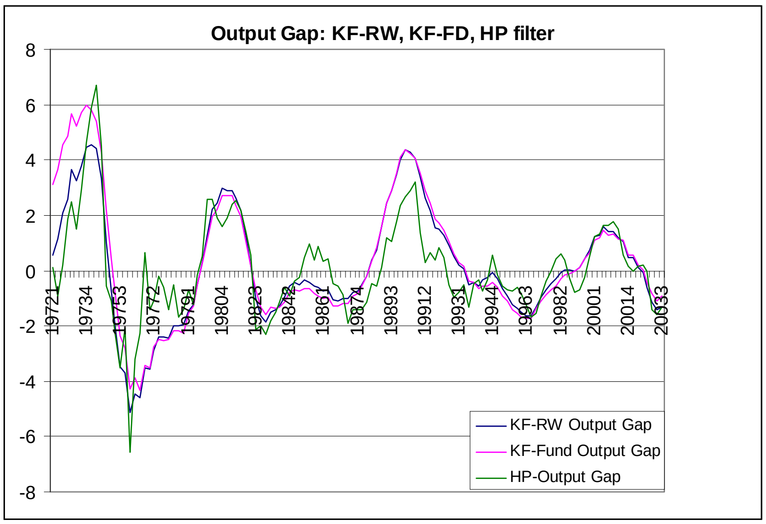


Figure 3: The Output Gap: KF-RW, KF-FD, HP filter

The average standard error for the growth trend is around 0.95, nearly one percentage point. This standard error can be decomposed into two parts: the uncertainty coming from the filter and that surrounding the parameters. The 'filter' uncertainty comes from the smoothing iteration and represents uncertainty about the predicted state variable that would be present even if the parameter vector, θ_0, was known with certainty. Then, we have also the 'parameter' uncertainty as the parameters themselves have to be estimated and thus, the estimated parameter vector, $\widehat{\theta}$, differs from the true θ_0. For more details on statistical inference with the Kalman filter, I would refer you to Hamilton (1994).

Estimates for the growth rate, using an Hodrick-Prescott filter or a Kalman filter differ somewhat, for the growth rate of potential output. The two Kalman smoothing estimations, using a RW process or using fundamentals, the results are very similar, except for the initial period. On the other hand, the Hodrick-Prescott filter estimation, is much more volatile with higher peaks (over 2.7 percent in the mid-1980s) and lower troughs (negative growth in mid-1970s).

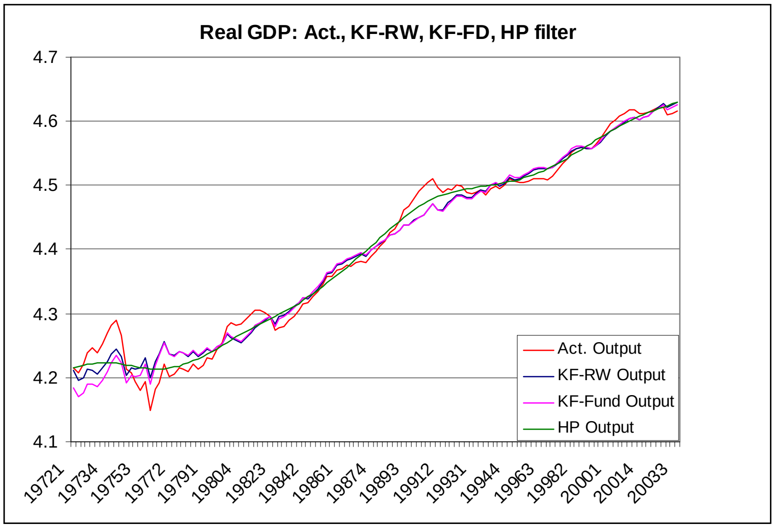


Figure 4: Real GDP: Actual,, KF-RW, KF-FD, HP filter

In either case, we can infer, using a KF estimation, that growth trend of potential output did vary between the 1970's and 2000 with an increase in the growth rate from around 1 to 1.2 percent at the beginning of the sample period, increasing in the 1980's with a peak of 1.5 percent in the mid-1980s (resp. 2.7 percent, using a HP filter) and lowered to around 1.4 percent at the end of the period, (resp. 1 percent, using a HP filter). Is growth trend a good proxy for productivity in traded versus non-traded goods ? Does it reflect a change in total factor productivity, a change in capital stock or in labour supply. If one assumes constant capital stock and labour supply during this period, then the growth trend would indeed reflect change in total factor productivity. However, one cannot assume constant these equilibrium factors over such a long period. Unfortunately, my measure of potential GDP growth cannot entangle one from another nor can it measure growth in traded goods versus non-traded goods.

(Place Fig. (5)

In the next graph Fig.(5.1.3) , we compare the actual growth of GDP with the HodricK-Prescott filtered estimations and the Kalman filter smoothing.

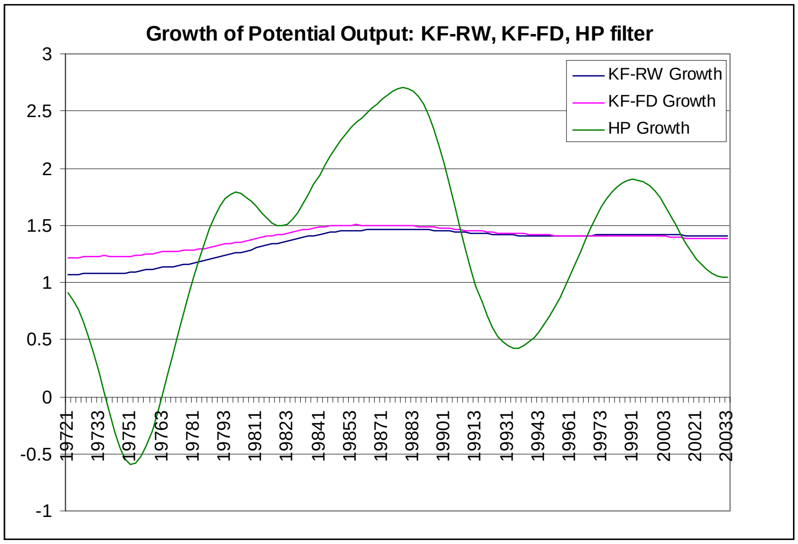
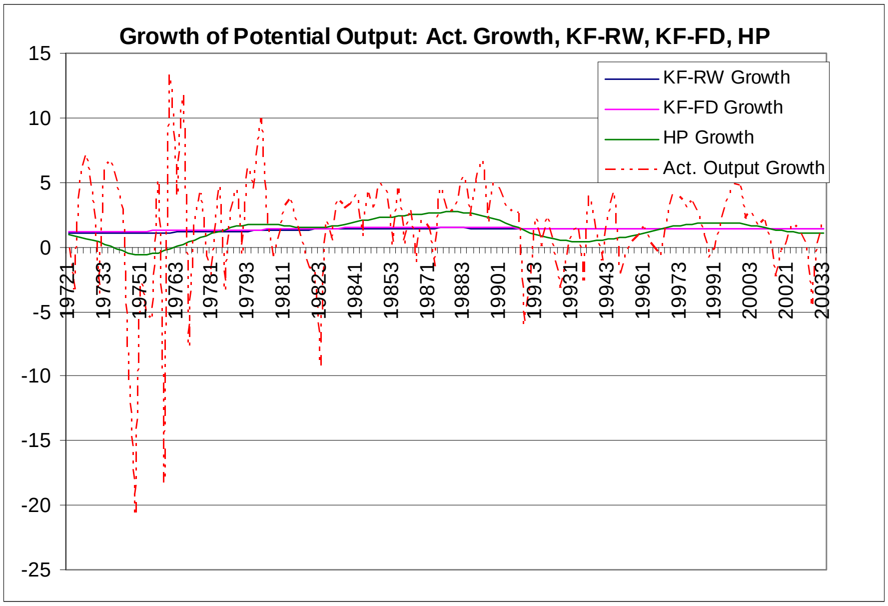


Figure 5: Growth of POtential Output: KF-RW, KF-FD, HP filter



Growth of Pot. RGDP: Actual, KF-RW, KF-FD, HP filter

5.1.4 The Real Exchange Rate, e*

Contrary to the PPP relationship, absolute or relative, the equilibrium real exchange rate, e^*, is not constant but time-varying. Fig. (6) shows that the CHF/DM equilibrium real exchange rate is constantly appreciating, just as the actual real exchange rate. Looking at the growth rate of the long-term real exchange rate, estimated through the four different models, we come to the same remarks:

- the equilibrium real exchange rate is steadily appreciating, albeit at a slower pace;
- the real exchange rate shows some mean-reverting process over a period of thirty years without fully converging, the long-term mean remains negative;

(Place Fig. (6) here).

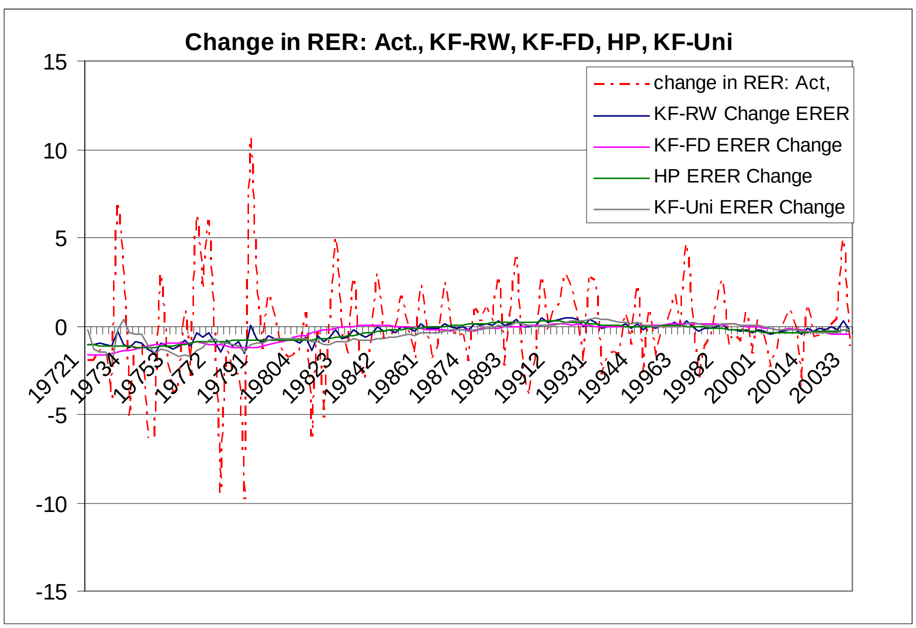


Figure 6: Change in Real Exchange Rate: Act., KF-RW, KF-FD, HP filter

Comparing Figure (1) with Figure (7), the equilibrium real exchange rate does not seem to be proportional to the relative GDP growth. The real exchange rate and the accumulated current account (ACA) seems to have proportional negative co-movements in the long run as well as for the gold stock price.

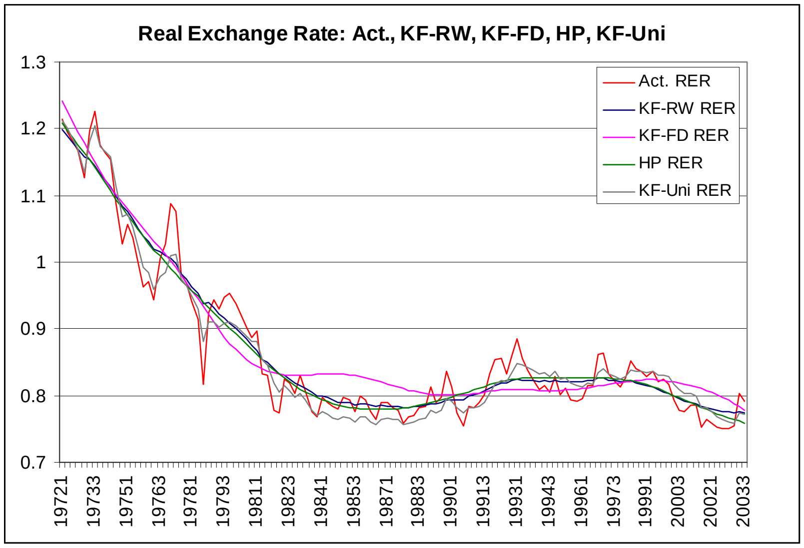


Figure 7: CHF/DM Real Exchange Rate: Act., KF-RW, KF-FD, HP filter

(Place Fig. (7) here).

(Place Fig. (8) here).

The equilibrium real exchange rate gap is shown in Fig.(8). By and large, although measurements are different from model to model, the gaps seem to move in the same directions, with more volatility for the model with fundamentals.

Considering a misalignment as a persistence departure from equilibrium, I cannot conclude to any persistent misalignment in the real exchange rate compared to equilirbium, looking at the four model estimations. The fundamental model shows some misalignments, over-appreciation during the 1980s, a phenomenon that is not supported by the other three models.

6 Conclusion

In this paper, I have jointly estimated the equilibrium real exchange rate, the potential output and the trend growth of potential output, using a Kalman filter approach, in a univariate and multivariate model, which I compare to a Hodrick-Prescott estimation approach. I compare four estimations of the equilibrium real exchange rate: a univariate unobserved component model with a multivariate determination of the equilibrium real exchange rate as a random walk with

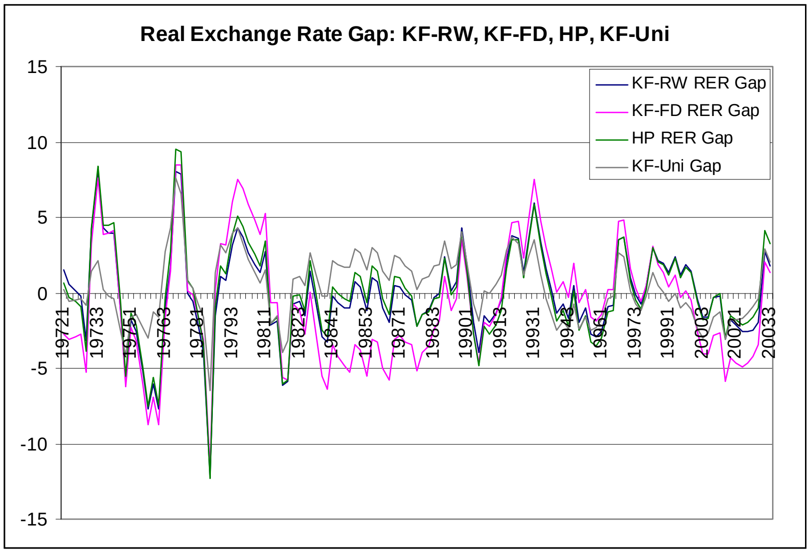


Figure 8: CHF/DM Real Exchange Rate Gap: KF-RW, KF-FD, HP filter

drift and determining the ERER with some macroeconomic fundamentals as well as a Hodrick-Prescott filter estimation.

I find that the equilibrium real exchange rate shows significant variation over the past 30 years in Switzerland. In the multivariate models, my empirical tests show a strong positive and significant relationship between the Swiss output gap and the real exchange rate gap as well as between inflation and the real exchange rate gap.

I cannot conclude to any persistent misalignment in the real exchange rate compared to equilibrium, looking at the four model estimations. The model with the fundmantals shows some misalignments, over-appreciation during the 1980s, a phenomenon that is not supported by the other three models.

Moreover, variation in the trend growth is important. However, it does not reflect directly into a relationship with the equilibrium real exchange rate. The coefficient on the relative trend growth differential between Switzerland and Germany is not significant. My estimations of the trend growth shows an increase growth trend in the mid-1980's in Switzerland, reaching nearly 1.5 percent, annualized, which then decreases to 1.4 percent in the late 1990's and early 2000, above the initial estimates for the mid-1970's.

Finally, my estimations, surprisingly, do not show any significant evidence for a role for the real interest rate gap, in the case of Switzerland. It is the real exchange rate gap that matters more than the real interest rate gap. It is possible that the real interest rate gap has an effect for the output gap, albeit, a slower effect than the real exchange rate gap, possibly longer than four quarters, maybe over eight or twelve quarters, which my Kalman filter approach estimation could not test.

7 Appendix

7.1 The State-Space Representation

Using Harvey (1989) notation, a rich class of dynamic models for y_t, an observed variable at time t, can be described in terms of a possibly unobserved $(m \times 1)$ vector α_t, the state vector, via a measurement equation

$$Y_t = Z_t \alpha_t + d_t + \varepsilon_t \tag{24}$$

where Z_t is an $(N \times m)$ matrix, d_t, an $(N \times 1)$ vector and ε_t, an $(N \times 1)$ vector of serially uncorrelated disturbance with mean zero and covariance matrix H_t.

The elements of the state vector α_t are unobservable. However, they are known to be generated by a first-order Markov process:

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t \tag{25}$$

where T_t is an $(m \times m)$ matrix, c_t, an $(m \times 1)$ vector, R_t an $(m \times g)$ matrix and η_t, an $(g \times 1)$ vector of serially uncorrelated disturbance with mean zero and covariance matrix Q_t.

Equation (24) is the "measurement" equation and (25) , the "transition equation".

The specification of the state-space system is completed by two further assumptions:

- the initial state vector α_0 with mean a_0 and covariance matrix P_0;
- the disturbances ε_t, η_t are uncorrelated with each other in all time periods and uncorrelated with the initial state.

7.2 The Kalman Filter

The Kalman filter is a recursive procedure for calculating the optimal estimator of the state vector, given all the information which is currently available. On the other hand, smoothing is a backward recursion, which enables optimal estimators of the state vector to be calculated at all possible points in time, using the full sample.

The Kalman filter is obtained in two steps:

- Step 1: the Prediction equations
- Step 2: the Updating equations

7.2.1 The Prediction Equations

Let a_t denote the optimal estimator of the state vector α_t, based on all observations, up to y_t and P_t, be the $(m \times m)$ covariance matrix of the associated estimated error

$$P_t = E\left(\alpha_t - a_t\right)\left(\alpha_t - a_t\right)' \tag{26}$$

This is referred to as the Mean Squarred Error (MSE) matrix of a_t.

Suppose now that a_{t-1}, P_{t-1} have been estimated. then, the optimal estimator of α_t is given by the prediction equation:

$$
\begin{aligned}
a_{t|t-1} &= T_t a_{t-1} + c_t &\quad (27)\\
P_{t|t-1} &= T_t P_{t-1} T_t' + R_t Q_t R_t' &\quad (28)
\end{aligned}
$$

with the corresponding estimator of Y_t:

$$Y_{t|t-1} = Z_t a_{t|t-1} + d_t \quad (29)$$

The MSE of the prediction error or innovation vector:

$$\nu_t = Y_t - Y_{t|t-1} = Z_t \left(\alpha_t - a_{t|t-1}\right) + \varepsilon_t \quad (30)$$

is

$$F_t = Z_t P_{t|t-1} Z_t^{'} + H_t \quad (31)$$

Once the new observation becomes available, the estimator of the state can be updated:

7.2.2 Updating Equations

$$a_t = a_{t|t} = a_{t|t-1} + P_{t|t-1} Z_t^{'} F_t^{-1} \left(Y_t - Z_t a_{t|t-1} - d_t\right) \quad (32)$$

and

$$P_t = P_{t|t} = P_{t|t-1} - P_{t|t-1} Z_t^{'} F_t^{-1} Z_t P_{t|t-1} \quad (33)$$

The prediction error vector, ν_t , plays a key role in the updating. The more the uncertainty associated with the predicted state $a_{t|t-1}$, the bigger the weight is given to new information in the prediction error ν_t . This is quite intuitive, since an increase in the uncertainty in $a_{t|t-1}$ may be interpreted as a deterioration of the information content of $a_{t|t-1}$, relative to that of ν_t . Given initial conditions, a_0, P_0, the Kalman filter delivers the optimal estimator of the state, as each new observation becomes available. When all T observations have been processed, the estimator a_T contains all information needed to make predictions of future observations.

7.3 Smoothing

The aim of filtering is to estimate α_t conditional on the information available at time t. The aim of smoothing is to take account of **all** the information up to time T, the end of sample period, $T \geq t$. The smoother $a_{t|T}$ is based on more information than the filtered estimator, thus, should have a smaller MSE, $P_{t|T}$ than the filtered one.

There are many different smoothing algorithms:

- Fixed point smoothing, $a_{\tau|t}$, at a given point τ.

- Fixed lag smoothing, $a_{t-j|t}$, at a given point $j = 1...m$.
- Fixed interval smoothing computes the full set of smoothed estimators.

The fixed-interval smoothing algorithm consists of a set of recursions which start with the final quantities, a_T and P_T, given by the Kalman filter and works backwards. The equations are:

$$a_{t|T} = a_t + P_t^* \left(a_{t+1|T} - T_{t+1}a_t - c_{t+1}\right) \tag{34}$$

$$P_{t|T} = P_t + P_t^* \left(P_{t+1|T} - P_{t+1|t}\right) P_t^{*\prime} \tag{35}$$

where

$$P_t^* = P_t T_{t+1}' P_{t+1|t}^{-1} \tag{36}$$

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