Exchange Rates and Fundamentals

Author(s): Charles Engel and Kenneth D. West

Source: *Journal of Political Economy*, Vol. 113, No. 3 (June 2005), pp. 485-517
Published by: The University of Chicago Press
Stable URL: https://www.jstor.org/stable/10.1086/429137

Exchange Rates and Fundamentals

Charles Engel and Kenneth D. West

University of Wisconsin and National Bureau of Economic Research

We show analytically that in a rational expectations present-value model, an asset price manifests near–random walk behavior if fundamentals are I(1) and the factor for discounting future fundamentals is near one. We argue that this result helps explain the well-known puzzle that fundamental variables such as relative money supplies, outputs, inflation, and interest rates provide little help in predicting changes in floating exchange rates. As well, we show that the data do exhibit a related link suggested by standard models—that the exchange rate helps predict these fundamentals. The implication is that exchange rates and fundamentals are linked in a way that is broadly consistent with asset-pricing models of the exchange rate.

I. Introduction

A long-standing puzzle in international economics is the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. Our theories state that the exchange rate is determined by such fundamental variables, but floating exchange rates between countries with roughly similar inflation rates are in fact well approximated as random walks. Fundamental variables do not help predict future changes in exchange rates.

Meese and Rogoff (1983*a*, 1983*b*) first established this result. They evaluated the out-of-sample fit of several models of exchange rates, using

We thank Shiu-Sheng Chen, Akito Matsumoto, Benjamin T. West, and Yu Yuan for research assistance; the National Science Foundation for financial support; and two anonymous referees, the editor, and many seminar audiences for helpful comments. Portions of this paper were completed while West was a Houblon-Norman Fellow at the Bank of England and the Professorial Fellow in Monetary Economics at Victoria University and the Reserve Bank of New Zealand.

[*Journal of Political Economy*, 2005, vol. 113, no. 3]

data from the 1970s. They found that by standard measures of forecast accuracy, such as the mean-squared deviation between predicted and actual exchange rates, accuracy generally increased when one simply forecast the exchange rate to remain unchanged compared to when one used the predictions from the exchange rate models. While a large number of studies have subsequently claimed to find success for various versions of fundamentals-based models, sometimes at longer horizons and over different time periods, the success of these models has not proved to be robust. A recent comprehensive study by Cheung, Chinn, and Pascual (2002, 19) concludes that "the results do not point to any given model/specification combination as being very successful. On the other hand . . . , it may be that one model will do well for one exchange rate, and not for another."

In this paper, we take a new line of attack on the question of the link between exchange rates and fundamentals. We work with a conventional class of asset-pricing models in which the exchange rate is the expected present discounted value of a linear combination of observable fundamentals and unobservable shocks. Linear driving processes are posited for fundamentals and shocks.

We first present a theorem concerning the behavior of an asset price determined in a present-value model. We show analytically that in the class of present-value models we consider, asset prices will follow a process arbitrarily close to a random walk if (1) at least one forcing variable (observable fundamental or unobservable shock) has a unit autoregressive root and (2) the discount factor is near unity. So, in the limit, as the discount factor approaches unity, the change in the time t asset price will be uncorrelated with information known at time $t-1$. We explain below that our result is *not* an application of the simple efficient markets model of Samuelson (1965) and others. When that model is applied to exchange rates, it implies that cross-country interest rate differentials will predict exchange rate changes and thus that exchange rates will not follow a random walk.

Intuitively, as the discount factor approaches unity, the model puts relatively more weight on fundamentals far into the future in explaining the asset price. Transitory movements in the fundamentals become relatively less important than the permanent components. Imagine performing a Beveridge-Nelson decomposition on the linear combination of fundamentals that drive the asset price, expressing it as the sum of a random walk component and a transitory component. The class of theoretical models we are considering then expresses the asset price as the discounted sum of the current and expected future fundamentals. As the discount factor approaches one, the variance of the change of the discounted sum of the random walk component approaches infinity, whereas the variance of the change of the stationary component approaches a constant. So the variance of the change of the asset price is dominated by the change of the random walk component as the discount factor approaches one.

We view as unexceptionable the assumption that a forcing variable has a unit root, at least as a working hypothesis for our study. The assumption about the discount factor is, however, open to debate. We note that in reasonable calibrations of some exchange rate models, this discount factor in fact is quite near unity.

Of course our analytical result is a limiting one. Whether a discount factor of 0.9 or 0.99 or 0.999 is required to deliver a process statistically indistinguishable from a random walk depends on the sample size used to test for random walk behavior and the entire set of parameters of the model. Hence we present some correlations calculated analytically in a simple stylized model. We assume a simple univariate process for fundamentals, with parameters chosen to reflect quarterly data from the recent floating period. We find that discount factors above 0.9 suffice to yield near-zero correlations between the period t exchange rate and period $t-1$ information. We do not attempt to verify our theoretical conclusion that large discount factors account for random walk behavior in exchange rates using any particular fundamentals model from the literature. That is, we do not pick specific models that we claim satisfy the conditions of our theorem and then estimate them and verify that they produce random walks.

But if the present-value models of exchange rates imply random walk behavior, so that exchange rate changes are unpredictable, how then can we validate the models? We ask instead if these conventional models have implications for whether the exchange rate helps predict fundamentals. It is plausible to look in this direction. Surely much of the short-term fluctuation in exchange rates is driven by changes in expectations about the future. If the models are good approximations and expectations reflect information about future fundamentals, the exchange rate changes will likely be useful in forecasting these fundamentals. So these models suggest that exchange rates Granger-cause the fundamentals. Using quarterly bilateral dollar exchange rates, 1974–2001, for the dollar versus the currencies of the six other Group of Seven countries, we find some evidence of such causality, especially for nominal variables.

The statistical significance of the predictability is not uniform and suggests a link between exchange rates and fundamentals that perhaps is modest in comparison with the links between other sets of economic variables. But in our view, the statistical predictability is notable in light of the far weaker causality from fundamentals to exchange rates.

For countries and data series for which there is statistically significant evidence of Granger causality, we next gauge whether the Granger causality results are consistent with our models. We compare the correlation of exchange rate changes with two estimates of the change in the present discounted value of fundamentals. One estimate uses only the lagged value of fundamentals. The other uses both the exchange rate and own lags. We find that the correlation is substantially higher when the exchange rate is used in estimating the present discounted value.

To prevent confusion, we note that our finding that exchange rates predict fundamentals is distinct from our finding that large discount factors rationalize a random walk in exchange rates. It may be reasonable to link the two findings. When expectations of future fundamentals are very important in determining the exchange rate, it seems natural to pursue the question of whether exchange rates can forecast those fundamentals. But one can be persuaded that exchange rates Granger-cause fundamentals and still argue that the approximate random walk in exchange rates is not substantially attributable to a large discount factor. In the class of models we consider, all our empirical results are consistent with at least one other explanation, namely, that exchange rate movements are dominated by unobserved shocks that follow a random walk. The plausibility of this explanation is underscored by the fact that we generally fail to find cointegration between the exchange rate and observable fundamentals, a failure that is rationalized in our class of models by the presence of an I(1) (though not necessarily random walk) shock. As well, the random walk also can arise in models that fall outside the class we consider. It does so in models with small-sample biases, perhaps combined with nonlinearities/threshold effects (see Taylor, Peel, and Sarno 2001; Kilian and Taylor 2003; Rossi 2003). Exchange rates will still predict fundamentals in such models, though a nonlinear forecasting process may be required.

Our suggestion that the exchange rate will nearly follow a random walk when the discount factor is close to unity means that forecasting changes in exchange rates is difficult but perhaps still possible. Some recent studies have found success at forecasting changes in exchange rates at longer horizons or using nonlinear methods, and further research along these lines may prove fruitful. MacDonald and Taylor (1994), Chinn and Meese (1995), and Mark (1995) have all found some success in forecasting exchange rates at longer horizons imposing long-run restrictions from monetary models. Groen (2000) and Mark and Sul (2001) find greater success using panel methods. Kilian and Taylor (2003) suggest that models that incorporate nonlinear mean reversion can improve the forecasting accuracy of fundamentals models, though it will be difficult to detect the improvement in out-of-sample forecasting exercises.

The paper is organized as follows. Section II presents the theorem that the random walk in asset prices may result from a discount factor near one in a present-value model. Section III demonstrates how the theorem applies to some models of exchange rates. Section IV presents evidence that changes in exchange rates help predict fundamentals. Section V presents conclusions. The Appendix has some algebraic details. An additional appendix containing empirical results omitted from the paper to save space is available on request.

II. Random Walk in Asset Prices as the Discount Factor Goes to One

We consider models in which an asset price, s_t, can be expressed as a discounted sum of current and expected future "fundamentals." We examine asset-pricing models of the form

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(\boldsymbol{a}_1' \boldsymbol{x}_{t+j}) + b\sum_{j=0}^{\infty} b^j E_t(\boldsymbol{a}_2' \boldsymbol{x}_{t+j}), \quad 0 < b < 1, \tag{1}$$

where $\boldsymbol{x}_t$ is the $n \times 1$ vector of fundamentals, b is a discount factor, and $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ are $n \times 1$ vectors. For example, the model for stock prices considered by Campbell and Shiller (1987) and West (1988) has this form, where s_t is the level of the stock price, x_t the dividend (a scalar), $a_1 = 0$, and $a_2 = 1$. The log-linearized model of the stock price of Campbell and Shiller (1988) also has this form, where s_t is the log of the stock price, x_t is the log of the dividend, $a_1 = 1$, and $a_2 = 0$. The term structure model of Campbell and Shiller also is a present-value model, where s_t is the yield on a consol, x_t is the short-term rate, $a_1 = 1$, and $a_2 = 0$. In Section III, we review models in which s_t is the log of the exchange rate and $\boldsymbol{x}_t$ contains such variables as interest rates and logs of prices, money supplies, and income.

We spell out here the sense in which the asset price should follow a random walk for a discount factor b that is near one. Assume that at least one element of the vector $\boldsymbol{x}_t$ is an I(1) process, whose Wold innovation is the $n \times 1$ vector $\boldsymbol{\epsilon}_t$. Our result requires that either (1) $\boldsymbol{a}_1'\boldsymbol{x}_t \sim \mathrm{I}(1)$ and $\boldsymbol{a}_2 = \mathbf{0}$ or (2) $\boldsymbol{a}_2'\boldsymbol{x}_t \sim \mathrm{I}(1)$, with the order of integration of $\boldsymbol{a}_1'\boldsymbol{x}_t$ essentially unrestricted (I(0), I(1), or identically zero). In either case, for b near one, Δs_t will be well approximated by a linear combination of the elements of the unpredictable innovation $\boldsymbol{\epsilon}_t$. In a sense made precise in the Appendix, this approximation is arbitrarily good for b arbitrarily near one. This means, for example, that all autocorrelations of Δs_t will be very near zero for b very near one.

Of course, there is continuity in the autocorrelations in the following sense: for b near one, the autocorrelations of Δs_t will be near zero if the previous paragraph's condition that certain variables are I(1) is replaced with the condition that those variables are I(0) but with an autoregres-

TABLE 1
Population Autocorrelations and Cross Correlations of Δs_t

	b (1)	φ_1 (2)	φ (3)	Correlation of Δs_t with: Δs_{t-1} (4)	Δs_{t-2} (5)	Δs_{t-3} (6)	Δx_{t-1} (7)	Δx_{t-2} (8)	Δx_{t-3} (9)
1.	.50	1.0	.3	.15	.05	.01	.16	.05	.01
2.			.5	.27	.14	.07	.28	.14	.07
3.			.8	.52	.42	.34	.56	.44	.36
4.	.90	1.0	.3	.03	.01	.00	.03	.01	.00
5.			.5	.05	.03	.01	.06	.03	.01
6.			.8	.09	.07	.06	.13	.11	.09
7.	.95	1.0	.3	.02	.01	.00	.02	.01	.00
8.			.5	.03	.01	.01	.03	.01	.01
9.			.8	.04	.04	.03	.07	.05	.04
10.	.90	.90	.5	.04	−.01	−.03	.02	−.03	−.05
11.	.90	.95	.5	.05	.01	−.01	.04	−.00	−.02
12.	.95	.95	.5	.02	−.00	−.01	.01	−.02	−.03
13.	.95	.99	.5	.02	.01	.00	.03	.01	−.00

Note.—The model is $s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t x_{t+j}$ or $s_t = b\sum_{j=0}^{\infty} b^j E_t x_{t+j}$. The scalar variable x_t follows an AR(2) process with autoregressive roots φ_1 and φ. When $\varphi_1 = 1.0$, $\Delta x_t \sim \text{AR}(1)$ with parameter φ. The correlations in cols. 4–9 were computed analytically. If $\varphi_1 = 1.0$, as in rows 1–9, then in the limit, as $b \to 1$, each of these correlations approaches zero.

sive root very near one. For a given autoregressive root less than one, the autocorrelations will not converge to zero as b approaches one. But they will be very small for b very near one.

Table 1 gives an indication of just how small "small" is. The table gives correlations of Δs_t with time $t-1$ information when x_t follows a scalar univariate AR(2). (One can think of $a_1 = 0$ and $a_2 = 1$ or $a_1 = 1$ and $a_2 = 0$. One can consider these two possibilities interchangeably since, for given $b < 1$, the autocorrelations of Δs_t are not affected by whether or not a factor of $1 - b$ multiplies the present value of fundamentals.) Rows 1–9 assume that $x_t \sim \text{I}(1)$—specifically, $\Delta x_t \sim \text{AR}(1)$ with parameter φ. We see that for $b = 0.5$ the autocorrelations in columns 4–6 and the cross correlations in columns 7–9 are appreciable. Specifically, suppose that one uses the conventional standard error of $1/\sqrt{T}$. Then when $\varphi = 0.5$, a sample size larger than 55 will likely suffice to reject the null that the first autocorrelation of Δs_t is zero (since row 2, col. 5, gives $\text{corr}(\Delta s_t, \Delta s_{t-1}) = 0.269$ and $0.269/[1/\sqrt{55}] \approx 2.0$). (In this argument, we abstract from sampling error in estimation of the autocorrelation.) But for $b = 0.9$, the autocorrelations are dramatically smaller. For $b = 0.9$ and $\varphi = 0.5$, a sample size larger than 1,600 will be required, since $0.051/(1/\sqrt{1{,}600}) \approx 2.0$. Finally, in connection with the previous paragraph's reference to autoregressive roots less than one, we see in rows 10–13 in the table that if the unit root in x_t is replaced by an autoregressive root of 0.9 or higher, the autocorrelations and cross correlations of Δs_t are not much changed.

To develop intuition on this result, consider the following example. Suppose that the asset price is determined by a simple equation:

$$s_t = (1-b)m_t + b\rho_t + bE_t(s_{t+1}).$$

The "no-bubbles" solution to this expectational difference equation is a present-value model like (1):

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t m_{t+j} + b\sum_{j=0}^{\infty} b^j E_t \rho_{t+j}.$$

Assume that the first differences of the fundamentals follow first-order autoregressions:

$$\Delta m_t = \phi \Delta m_{t-1} + \epsilon_{mt}; \quad \Delta \rho_t = \gamma \Delta \rho_{t-1} + \epsilon_{\rho t}.$$

Then we can write the solution as

$$\Delta s_t = \frac{\phi(1-b)}{1-b\phi}\Delta m_{t-1} + \frac{1}{1-b\phi}\epsilon_{mt} + \frac{b\gamma}{1-b\gamma}\Delta\rho_{t-1} + \frac{b}{(1-b)(1-b\gamma)}\epsilon_{\rho t}.$$

Consider first the special case of $\rho_t = 0$. Then as $b \to 1$, $\Delta s_t \approx [1/(1-\phi)]\epsilon_{mt}$. In this case, the variance of the change in the exchange rate is finite as $b \to 1$. If $\rho_t \neq 0$, then as $b \to 1$, $\Delta s_t \approx$ constant $\times$ $\epsilon_{\rho t}$. In this case, as b increases, the variance of the change in the exchange rate gets large, but the variance is dominated by the independently and identically distributed term $\epsilon_{\rho t}$.

In Section III, we demonstrate the applicability of this result to exchange rates.

III. Exchange Rate Models

Exchange rate models since the 1970s have emphasized that nominal exchange rates are asset prices and are influenced by expectations about the future. The "asset market approach to exchange rates" refers to models in which the exchange rate is driven by a present discounted sum of expected future fundamentals. Obstfeld and Rogoff (1996, 529) say that "one very important and quite robust insight is that *the nominal exchange rate must be viewed as an asset price.* Like other assets, the exchange rate depends on expectations of future variables" (italics in the original). Frenkel and Mussa's (1985) survey explains the asset market approach:

> These facts suggest that exchange rates should be viewed as prices of durable assets determined in organized markets (like stock and commodity exchanges) in which current prices reflect the market's expectations concerning present and future

> economic conditions relevant for determining the appropriate values of these durable assets, and in which price changes are largely unpredictable and reflect primarily new information that alters expectations concerning these present and future economic conditions. (726)

A variety of models relate the exchange rate to economic fundamentals and to the expected future exchange rate. We write this relationship as

$$s_t = (1 - b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + bE_t s_{t+1}. \tag{2}$$

Here, we define the exchange rate s_t as the log of the home currency price of foreign currency (dollars per unit of foreign currency if the United States is the home country). The terms f_{it} and z_{it} ($i = 1, 2$) are economic fundamentals that ultimately drive the exchange rate, such as money supplies, money demand shocks, productivity shocks, and so forth. We differentiate between fundamentals that are observable to the econometrician, f_{it}, and those that are not observable, z_{it}. One possibility is that the true fundamental is measured with error, so that f_{it} is the measured fundamental and the z_{it} include the measurement error; another is that the z_{it} are unobserved shocks.

Upon imposing the "no-bubbles" condition that $b^j E_t s_{t+j}$ goes to zero as $j \to \infty$, we have the present-value relationship

$$s_t = (1 - b)\sum_{j=0}^{\infty} b^j E_t(f_{1t+j} + z_{1t+j}) + b\sum_{j=0}^{\infty} b^j E_t(f_{2t+j} + z_{2t+j}). \tag{3}$$

This equation has the form of equation (1), where we have $\boldsymbol{a}'_1\boldsymbol{x}_{t+j} = f_{1t+j} + z_{1t+j}$ and $\boldsymbol{a}'_2\boldsymbol{x}_{t+j} = f_{2t+j} + z_{2t+j}$. We now outline some models that fit into this framework.

A. *Money Income Model*

Consider first the familiar monetary models of Frenkel (1976), Mussa (1976), and Bilson (1978) and their close cousins, the sticky-price monetary models of Dornbusch (1976) and Frankel (1979). Assume that in the home country there is a money market relationship given by

$$m_t = p_t + \gamma y_t - \alpha i_t + v_{mt}. \tag{4}$$

Here, m_t is the log of the home money supply, p_t is the log of the home price level, i_t is the level of the home interest rate, y_t is the log of output, and v_{mt} is a shock to money demand. Here and throughout we use the term "shock" in a somewhat unusual sense. Our "shocks" potentially include constant and trend terms, may be serially correlated, and may include omitted variables that in principle could be measured. Assume that a similar equation holds in the foreign country. The analogous foreign variables are m_t^*, p_t^*, i_t^*, y_t^*, and v_{mt}^*, and the parameters of the foreign money demand are identical to the home country's parameters.

The nominal exchange rate equals its purchasing power parity (PPP) value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \tag{5}$$

In financial markets, the interest parity relationship is

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t. \tag{6}$$

Here ρ_t is the deviation from rational expectations uncovered interest parity. It can be interpreted as a risk premium or an expectational error.

Putting these equations together and rearranging, we get

$$s_t = \frac{1}{1+\alpha}[m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_{mt} - v_{mt}^*) - \alpha\rho_t] + \frac{\alpha}{1+\alpha}E_t s_{t+1}. \tag{7}$$

This equation takes the form of equation (2) when the discount factor is given by $b = \alpha/(1+\alpha)$, the observable fundamentals are given by $f_{1t} = m_t - m_t^* - \gamma(y_t - y_t^*)$, and the unobservables are $z_{1t} = q_t - (v_{mt} - v_{mt}^*)$ and $z_{2t} = -\rho_t$. As in Mark (1995), our empirical work in Section IV sets $\gamma = 1$. We also investigate a version of this model setting $f_{1t} = m_t - m_t^*$ and moving $y_t - y_t^*$ to z_{1t}. We do so largely because we wish to conduct a relatively unstructured investigation into the link between exchange rates and various measures of fundamentals. But we could argue that we focus on $m_t - m_t^*$ because financial innovation has made standard income measures poor proxies for the level of transactions. Similarly, we investigate the relationship between s_t and $y_t - y_t^*$.

Equation (7) is implied by both the flexible-price and sticky-price versions of the monetary model. In the flexible-price monetarist models of Frenkel (1976), Mussa (1976), and Bilson (1978), output, y_t, and the real exchange rate, q_t, are exogenous. In the sticky-price models of Dornbusch (1976) and Frankel (1979), these two variables are endogenous. Because nominal prices adjust slowly, the real exchange rate is influenced by changes in the nominal exchange rate. Output is demand determined and may respond to changes in the real exchange rate, income, and real interest rates. Nonetheless, since equations (4) (and its foreign counterpart), (5), and (6) hold in the Dornbusch-Frankel model, one can derive relationship (7) in those models. Dornbusch and Frankel each consider special cases for the exogenous monetary processes (in Dornbusch's model, all shocks to the money supply are permanent; Frankel considers permanent shocks to the level and to the growth rate of money). As a result of their assumption that all shocks are permanent, they each can express the exchange rate purely in terms of current fundamentals, which may obscure the general implication that exchange rates depend on expected future fundamentals.

We note here that some recent exchange rate models developed from the "new open economy macroeconomics" yield relationships very similar to the ones we describe in this section. For example, in Obstfeld and Rogoff (2003), the exchange rate is given by (their eq. [30])

$$s_t = \sum_{j=0}^{\infty} b^j E_t[(1-b)(m_{t+j} - m^*_{t+j}) - b\rho_{t+j}], \tag{8}$$

where we have translated their notation to be consistent with ours. Equation (8) is in fact the forward solution to a special case of equation (7) above. The discount factor, b, in Obstfeld and Rogoff's model is related to the semi-elasticity of money demand exactly as in equation (7). However, their money demand function is derived from a utility-maximizing framework in which real balances appear in the utility function, and their risk premium ρ_t is derived endogenously from first principles.

B. *Taylor Rule Model*

Here we draw on the burgeoning literature on Taylor rules. Let $\pi_t = p_t - p_{t-1}$ denote the inflation rate and y_t^g be the "output gap." We assume that the home country (the United States in our empirical work) follows a Taylor rule of the form

$$i_t = \beta_1 y_t^g + \beta_2 \pi_t + v_t. \tag{9}$$

In (9), $\beta_1 > 0$, $\beta_2 > 1$, and the shock v_t contains omitted terms.[1]

The foreign country follows a Taylor rule that explicitly includes exchange rates:

$$i_t^* = -\beta_0(s_t - \bar{s}_t^*) + \beta_1 y_t^{*g} + \beta_2 \pi_t^* + v_t^*. \tag{10}$$

In (10), $0 < \beta_0 < 1$, and $\bar{s}_t^*$ is a target for the exchange rate. We shall

[1] Much of the Taylor rule literature—wisely, in our view—puts expected inflation in the monetary policy rule. Among other benefits, this facilitates thinking of the monetary authority as setting an ex ante real rate. We use actual inflation for notational simplicity. If expected inflation is in the monetary rule, then inflation in the formulas below is replaced by expected inflation.

assume that monetary authorities target the PPP level of the exchange rate:

$$\bar{s}_t^* = p_t - p_t^*. \tag{11}$$

Since s_t is measured in dollars per unit of foreign currency, the rule indicates that, ceteris paribus, the foreign country raises interest rates when its currency depreciates relative to the target. Clarida, Gali, and Gertler (1998) estimate monetary policy reaction functions for Germany and Japan (using data from 1979–94) of a form similar to equation (10). They find that a 1 percent real depreciation of the mark relative to the dollar led the Bundesbank to increase interest rates (expressed in annualized terms) by five basis points, whereas the Bank of Japan increased rates by nine basis points in response to a real yen depreciation relative to the dollar.

As the next equation makes clear, our argument still follows if the United States were also to target exchange rates. We omit the exchange rate target in (9) on the interpretation that U.S. monetary policy has virtually ignored exchange rates except, perhaps, as an indicator.

Subtracting the foreign from the home money rule, we obtain

$$i_t - i_t^* = \beta_0(s_t - \bar{s}_t^*) + \beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^*. \tag{12}$$

Use interest parity (6) to substitute out for $i_t - i_t^*$ and (11) to substitute out for the exchange rate target:

$$\begin{aligned} s_t = {} & \frac{\beta_0}{1+\beta_0}(p_t - p_t^*) - \frac{1}{1+\beta_0}[\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) \\ & + v_t - v_t^* + \rho_t] + \frac{1}{1+\beta_0}E_t s_{t+1}. \end{aligned} \tag{13}$$

This equation has the general form of (2) of the expected discounted present-value models. The discount factor is equal to $1/(1+\beta_0)$. We have $f_{1t} = p_t - p_t^*$. In our empirical work (in Sec. IV), we shall treat the remaining variables as unobservable, so we have

$$z_{2t} = -[\beta_1(y_t^g - y_t^{*g}) + \beta_2(\pi_t - \pi_t^*) + v_t - v_t^* + \rho_t].$$

Equation (12) can be expressed another way, again using interest parity (6) and the equation for the target exchange rate (11):

$$\begin{aligned} s_t = {} & \beta_0(i_t - i_t^*) + \beta_0(p_t - p_t^*) - \beta_1(y_t^g - y_t^{*g}) - \beta_2(\pi_t - \pi_t^*) \\ & - v_t + v_t^* - (1-\beta_0)\rho_t + (1-\beta_0)E_t s_{t+1}. \end{aligned} \tag{14}$$

This equation is very much like (13), except that it incorporates the interest differential, $i_t - i_t^*$, as a "fundamental." The discount factor in this formulation is given by $1 - \beta_0$. The observed fundamental is given by $f_{1t} = i_t - i_t^* + p_t - p_t^*$. In our empirical work, we treat the remaining period t variables in equation (14) as unobserved.

C. *Discussion*

We begin by noting that the classic efficient markets model of Samuelson (1965) and others does *not* predict a random walk in exchange rates. The essence of this model is that there are no predictable profit opportunities for a risk-neutral investor to exploit. If the U.S. interest rate i_t is higher than the foreign interest rate i_t^* by x percent, then the U.S. dollar must be expected to fall by x percent over the period of the investment if there are to be no such opportunities. In terms of equation (6), then, the classic efficient markets model says that the risk premium ρ_t is zero and that a population regression of Δs_{t+1} on $i_t - i_t^*$ will yield a coefficient of one. (For equities, the parallel prediction is that on the day on which a stock goes ex-dividend, its price should fall by the amount of the dividend [e.g., Elton and Gruber 1970].)

Our explanation yields a random walk approximation even when, as in the previous paragraph, uncovered interest parity holds. The reader may wonder how the data can simultaneously satisfy the following conditions: (1) a regression of Δs_{t+1} on $i_t - i_t^*$ yields a nonzero coefficient, and (2) s_t is arbitrarily well approximated as a random walk (i.e., Δs_{t+1} is arbitrarily well approximated as white noise). The answer is that when b is arbitrarily close to one, the R^2 of the regression of Δs_{t+1} on $i_t - i_t^*$ will be arbitrarily close to zero and the correlation of Δs_{t+1} with $i_t - i_t^*$ will be arbitrarily small. It is in those senses that the random walk approximation will be arbitrarily good.

The key question is not the logic of our result but its empirical validity. The result does not require uncovered interest parity, which was maintained in the previous two paragraphs merely to clarify the relation of our result to the standard efficient markets result. Instead, two conditions are required. The first is that fundamentals variables be very persistent—I(1) or nearly so. This is arguably the case with our data on the observed fundamentals. We shall present evidence in Section IV that we cannot reject the null of a unit root in any of our data. Further, there is evidence in other research that the unobservable variables are very persistent. For the money income model (eq. [7]), this is suggested for v_{mt}, q_t, and ρ_t by the literature on money demand (e.g., Sriram 2000), PPP (e.g., Rogoff 1996), and interest parity (e.g., Engel 1996). (We recognize that theory suggests that a risk premium like ρ_t is I(0); our interpretation is that if ρ_t is I(0), it has a very large autoregressive root.) We are not concerned if ρ_t or other variables are highly persistent I(0) variables rather than I(1) variables, for we saw in rows 10–13 of table 1 that a near random walk can result for such processes.

A second condition for s_t to follow an approximate random walk is that b is sufficiently close to one. The evidence we present below in table 2 on the first-order autocorrelations for the exchange rate fundamentals suggests that the rows in table 1 most relevant to our data are those with $\varphi = 0.3$ or $\varphi = 0.5$. If so, table 1 suggests that if b is around 0.9 or above, the asset price appears to be nearly indistinguishable from a random walk.

In the money income models, b is related to the interest semi-elasticity of money demand: $b = \alpha/(1 + \alpha)$. Bilson (1978) estimates $\alpha \approx 60$ in the monetary model, whereas Frankel (1979) finds $\alpha \approx 29$. The estimates from Stock and Watson (1993, 802, table 2, panel I) give us $\alpha \approx 40$.[2] They imply a range for b of 0.97–0.98 for quarterly data.

To get a sense of the plausibility of this discount factor, compare it to the discount factor implied in a theoretical model in which optimal real balance holdings are derived from a money-in-the-utility-function framework. Obstfeld and Rogoff (2003) derive a money demand function that is very similar to equation (4) when utility is separable over consumption and real balances, and money enters the utility function as a power function:

$$\frac{1}{1-\epsilon}\left(\frac{M_t}{P_t}\right)^{1-\epsilon}.$$

They show that $\alpha \approx 1/\epsilon\bar{i}$, where $\bar{i}$ is the steady-state nominal interest rate in their model. They state that "assuming time is measured in years, then a value between 0.04 and 0.08 seems reasonable for $\bar{i}$. It is usually thought that ϵ is higher than one, though not necessarily by a large margin. Thus, based on a priori reasoning, it is not implausible to assume $1/\epsilon\bar{i} = 15$" (98). For our quarterly data, the value of α would be 60, which is right in line with the estimate from Bilson cited above.

In the Taylor rule model, the discount factor is large when the degree of intervention by the monetary authorities to target the exchange rate is small. The strength of intervention is given by the parameter β_0 from (12), and the discount factor is either $1/(1 + \beta_0)$ in the formulation of (13) or $1 - \beta_0$ in the representation in (14). In practice, it seems as

[2] Bilson (1978) uses quarterly interest rates that are annualized and multiplied by 100 in his empirical study. So his actual estimate of $\alpha = 0.15$ should be multiplied by 400 to construct a quarterly discount rate. MacDonald and Taylor (1993) estimate a discounted sum of fundamentals and test for equality with the actual exchange rate—following the methods of Campbell and Shiller (1987) for equity prices. MacDonald and Taylor rely on the estimates of Bilson to calibrate their discount factor but mistakenly use 0.15 instead of 60 as the estimate of α. Stock and Watson's data estimates also use annualized interest rates multiplied by 100, so we have multiplied their estimate by 400.

though foreign exchange intervention within the G7 has not been very active. For example, if the exchange rate were 10 percent above its PPP value, it is probably an upper bound to guess that a central bank would increase the short-term interest rate by one percentage point (expressed on an annualized basis). With quarterly data, this would imply a value of b of about 0.975, which is consistent with the discount factors we imputed in the monetary models. Clarida et al.'s (1998) estimates of the monetary policy reaction functions for Germany and Japan over the 1979–94 period find that a 10 percent real depreciation of the currency led the central banks to increase annualized interest rates by 50 and 90 basis points, respectively. This translates to quarterly discount factors of 0.988 and 0.978.

Our result does not require that the fundamentals evolve exogenously to the exchange rate. The result is not, however, consistent with a thought experiment that allows the stochastic process for the fundamentals to change as b gets near to one. But we can answer the following question: With given data for fundamentals and plausible values for b, will a present-value model yield an approximate random walk? For the values of b taken from the literature (which we have just discussed) and for serial correlation plausible for exchange rate fundamentals (reported in table 2 below), the figures in table 1 indicate near–random walk behavior.

We note that the presence of persistent deviations from uncovered interest parity, in the form of a risk premium or expectational error, could potentially play a large role in accounting for movements in exchange rates. Equation (3) draws a distinction between fundamentals that are multiplied by the discount factor, b (f_{2t} and z_{2t}), and fundamentals that are multiplied by $1 - b$ (f_{1t} and z_{1t}). As $b \rightarrow 1$, the former become increasingly dominant in determining exchange rate movements. In both the money income model and the Taylor rule model, the deviation from interest parity is like a z_{2t} variable—an unobservable fundamental multiplied by b in equation (3). This analysis alone cannot determine whether deviations from interest parity are very important. A more detailed model would determine the size of these deviations. (For example, in a particular model, it may be that the deviation from interest parity depends on the discount factor in such a way that as $b \rightarrow 1$, the deviation gets smaller.) We note one model in which a theoretical risk premium is derived—that of Obstfeld and Rogoff (2003). They refer to the effect of the risk premium on the level of the exchange rate—the discounted present value of the risk premium—as the "level risk premium." They explicitly note that in their model the discount factor b is large, and that in turn means that a volatile deviation from interest parity has a large impact on the variance of exchange rate changes (see eq. [8]).

IV. Empirical Findings

We have argued that when standard exchange rate models are plausibly calibrated, they have the property that the exchange rate should nearly follow a random walk. Evidence that the exchange rate change is not predictable is an implication of the models, not evidence against the models. But merely observing that exchange rates follow random walks is not a very complete validation of the models.

There are other possible explanations of the random walk behavior of exchange rates. The exchange rate may be dominated by unobservable shocks that are well approximated by random walks; that is, the z_{it} from equation (2) are well approximated by a random walk, and the variance of Δs_t is dominated by the changes in z_{it} rather than by changes in f_{it}. The standard set of fundamentals (money, income, prices, and interest rates) may not be important determinants of exchange rates, and instead there may be some other variable that models have not captured or that is unobserved that drives the exchange rate.

In this section, we consider an implication of asset-pricing models: that the asset price might help to predict the fundamentals. This basic insight led Campbell and Shiller (1987) to develop a test of present-value models of asset prices. We do not follow their method here because we acknowledge the possibility of unobserved fundamentals (the z_{it}), which make the exact method of Campbell and Shiller inapplicable. However, our approach to model validation is inspired by the Campbell-Shiller methodology.

A. *Data and Basic Statistics*

We use quarterly data, usually 1974:1–2001:3 (with exceptions noted below). With one observation lost to differencing, the sample size is $T = 110$.

We study bilateral U.S. exchange rates versus those of the other six members of the G7: Canada, France, Germany, Italy, Japan, and the United Kingdom. The International Financial Statistics (IFS) CD-ROM is the source for the end-of-quarter exchange rate s_t and consumer prices p_t. The Organization for Economic Cooperation and Development's (OECD's) Main Economic Indicators CD-ROM is the source for our data on the seasonally adjusted money supply, m_t (M4 in the United Kingdom and M1 in all other countries; 1978:1–1998:4 for France, 1974:1–1998:4 for Germany, and 1975:1–1998:4 for Italy). The OECD is also the source for real, seasonally adjusted gross domestic product, y_t, for all countries but Germany, which we obtain by combining IFS (1974:1–2001:1) and OECD (2001:2–2001:3) data, and Japan, which combines data from the OECD (1974:1–2002) with 2002:3 data from the Web site of the Japanese government's Economic and Social Research Institute. Datastream is the source for the interest rates, i_t, which are three-month Euro rates (1975:1–2001:3 for Canada and 1978:3–2001:3 for Italy and Japan). We convert all data but interest rates by taking logs and multiplying by 100. Throughout the rest of the paper, the symbols defined in this paragraph (s_t, m_t, y_t, and p_t) refer to the transformed data.

We focus on the bivariate relationship between s_t and the following five measures of fundamentals: m_t, p_t, i_t, y_t, and $m_t - y_t$. We briefly discuss results when we look at full systems of variables suggested by particular versions of the models sketched in Section III. As noted in that section, we focus on the simple bivariate relationships because we wish to conduct a relatively unstructured investigation.

Let f_t denote a measure of "fundamentals" in the United States relative to abroad (e.g., $f_t = m_t - m_t^*$). Using Dickey-Fuller tests with a time trend included, we were generally unable to reject the null of a unit root in any of the five measures of f_t (i.e., in m_t, p_t, i_t, y_t, and $m_t - y_t$). Hence our analysis presents statistics on Δf_t for all measures of fundamentals. Even though we fail to reject unit roots for interest differentials, we are uneasy using interest differentials only in differenced form. So we present statistics for both levels and differences of interest rates.

Some basic statistics are presented in table 2. Row 1 is consistent with much evidence that changes in exchange rates are serially uncorrelated and quite volatile. The standard deviation of the quarterly change is over five percentage points for all except the Canadian dollar exchange rate. First-order autocorrelations are small, under 0.15 in absolute value. Under the null of no serial correlation, the standard error on the estimator of the autocorrelation is approximately $1/\sqrt{T} \approx 0.1$, so none of the estimates are significant at even the 10 percent level.

Rows 2–7 present statistics on our measures of fundamentals. A positive value for the mean indicates that the variable has been growing faster in the United States than abroad. For example, the figure of -0.92 for the mean value of the United States–Italy inflation differential means that quarterly inflation was, on average, 0.92 percentage points lower in the United States than in Italy during the 1974–2001 period. Of particular note is that the vast majority of estimates of first-order autocorrelation coefficients suggest a rejection of the null of no serial correlation at the 10 percent level, and most do at the 5 percent level as well (again with an approximate standard error of 0.1). (An exception to this pattern occurs in output differentials in row 7. None of the autocorrelations are significant at the 5 percent level, and only one is [France, for which the estimate is 0.19] at the 10 percent level.) The magnitude of the autocorrelations—less than 0.5 for virtually all differenced series—suggests that for calibration of an exchange rate model,

TABLE 2
Basic Statistics

	Canada		France		Germany		Italy		Japan		United Kingdom	
	Mean	ρ_1	Mean	ρ_1	Mean	ρ_1	Mean	ρ_1	Mean	ρ_1	Mean	ρ_1
1. Δs	−.44	−.03	−.35	.10	.15	.07	−1.11	.14	.76	.13	−.44	.15
	(2.20)		(5.83)		(6.06)		(5.79)		(6.22)		(5.26)	
2. $\Delta(m-m^*)$	−.56	.19	.03	.25	−.55	.28	−1.19	.28	−.39	.46	−1.34	.54
	(2.59)		(2.41)		(2.38)		(2.24)		(2.18)		(1.94)	
3. $\Delta(p-p^*)$	−.04	.47	−.13	.62	.49	.42	−.92	.62	.50	.16	−.54	.27
	(.58)		(.68)		(.77)		(1.17)		(.86)		(1.29)	
4. $i-i^*$	−.92	.75	−1.89	.62	2.02	.84	−4.33	.66	3.64	.78	−2.40	.76
	(1.72)		(3.70)		(3.01)		(4.25)		(2.78)		(2.88)	
5. $\Delta(i-i^*)$	−.01	−.39	.06	−.37	−.01	−.34	.06	−.35	−.04	−.15	.06	−.13
	(1.21)		(3.23)		(1.70)		(3.51)		(1.83)		(2.00)	
6. $\Delta(m-m^*)-\Delta(y-y^*)$	−.60	.17	−.24	.17	−.72	.13	−1.42	.24	−.43	.35	−1.53	.41
	(2.65)		(2.59)		(2.92)		(2.35)		(2.54)		(2.19)	
7. $\Delta(y-y^*)$	.04	−.08	.21	.19	.17	.08	.20	.14	.04	.06	.19	−.04
	(.79)		(.88)		(1.47)		(1.01)		(1.21)		(1.06)	

Note.—The numbers in parentheses under the means are the standard deviations of the indicated variable; ρ_1 is the first-order autocorrelation coefficient of the indicated variable. Variable definitions: Δs is the percentage change in the dollar exchange rate (a higher value indicates depreciation). In other variables an asterisk indicates a non-U.S. value, and the absence of an asterisk indicates a U.S. value; Δm is the percentage change in M1 (M2 for the United Kingdom); Δy is the percentage change in real GDP; Δp is the percentage change in consumer prices; and i is the short-term rate on government debt. Money and output are seasonally adjusted. Data are quarterly, generally 1974:2–2001:3. Exceptions include an end date of 1998:4 for $m-m^*$ for France, Germany, and Italy; start dates for $m-m^*$ of 1978:1 for France, 1974:1 for Germany, and 1975:1 for Italy; and start dates for $i-i^*$ of 1975:1 for Canada and 1978:3 for Italy and Japan. See the text.

the relevant entries in table 1 are those with $\varphi = 0.3$ or $\varphi = 0.5$ but not $\varphi = 0.9$.

For each country we conducted five cointegration tests, between s_t and each of our measures of fundamentals, $m_t - m_t^*$, $p_t - p_t^*$, $i_t - i_t^*$, $y_t - y_t^*$, and $m_t - y_t - (m_t^* - y_t^*)$. We used Johansen's (1991) trace and maximum eigenvalue statistics, with critical values from Osterwald-Lenum (1992). Each bivariate vector autoregression (VAR) contained four lags. Of the 30 tests (six countries, five fundamentals), we rejected the null of no cointegration at the 5 percent level in five instances using the trace statistic: $m_t - m_t^*$, $p_t - p_t^*$, and $i_t - i_t^*$ for Italy and $p_t - p_t^*$ and $i_t - i_t^*$ for the United Kingdom. Of the 30 tests using the maximum eigenvalue statistic, the null was rejected only once, for the United Kingdom for $p_t - p_t^*$. We conclude that it will probably not do great violence to assume lack of cointegration, recognizing that a complementary analysis using cointegration would be useful.

We take the lack of cointegration to be evidence that unobserved variables such as real demand shocks, real money demand shocks, or possibly even interest parity deviations have a permanent component, or at least are very persistent. Alternatively, it may be that the data we use to measure the economic fundamentals of our model have some errors with permanent or very persistent components. For example, it may be that the appropriate measure of the money supply has permanently changed because of numerous financial innovations over our sample, so that the M1 money supply series varies from the "true" money supply by some I(1) errors.

B. *Granger Causality Tests*

Campbell and Shiller (1987) observe that when a variable s_t is the present value of a variable x_t, then either (1) s_t Granger-causes x_t relative to the bivariate information set consisting of lags of s_t and x_t or (2) s_t is an exact distributed lag of current and past values of x_t. That is, as long as s_t embodies *some* information in addition to that included in past values of x_t, s_t Granger-causes x_t.[3] As was emphasized in the previous section, however, exchange rate models must allow for unobservable fundamentals—the possibility that x_t is a linear combination of unobservable as well as observable variables, and thus x_t itself is unobservable. Failure to find Granger causality from s_t to observable variables no longer implies an obviously untenable restriction that the exchange rate is an exact distributed lag of observables. It is clear, though, that a finding of Granger causality is supportive of a view that exchange rates are

[3] In the Appendix, this additional information is formalized as additional random variables that are used by private agents in forecasting future fundamentals.

TABLE 3
Bivariate Granger Causality Tests, Different Measures of Δf_t, Full Sample: 1974:1–2001:3

	Canada	France	Germany	Italy	Japan	United Kingdom
	A. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H$_0$: Δs_t Fails to Cause Δf_t					
1. $\Delta(m - m^*)$		*		**	**	
2. $\Delta(p - p^*)$			***	***	***	
3. $i - i^*$		**			**	
4. $\Delta(i - i^*)$		**			***	
5. $\Delta(m - m^*) - \Delta(y - y^*)$		*		*		
6. $\Delta(y - y^*)$						
	B. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H$_0$: Δf_t Fails to Cause Δs_t					
1. $\Delta(m - m^*)$						
2. $\Delta(p - p^*)$	*					
3. $i - i^*$					**	
4. $\Delta(i - i^*)$						
5. $\Delta(m - m^*) - \Delta(y - y^*)$						
6. $\Delta(y - y^*)$						

Note.—See the notes to earlier tables for variable definitions. Statistics are computed from fourth-order bivariate VARs in $(\Delta s_t, \Delta f_t)'$. Because four observations were lost to initial conditions, the sample generally is 1975:2–2001:3, with exceptions as indicated in the note to table 2.

determined as a present value that depends in part on observable fundamentals.

Table 3 summarizes the results of our Granger causality tests on the full sample. We include a constant and four lags of each variable in all causality tests reported in this and all other tables. For all tests of no causality we use likelihood ratio statistics using the degrees of freedom correction suggested in Sims (1980).

We see in panel A that at the 5 percent level of significance, the null that Δs_t fails to Granger-cause $\Delta(m_t - m_t^*)$, $\Delta(p_t - p_t^*)$, $i_t - i_t^*$, $\Delta(i_t - i_t^*)$, $\Delta(y_t - y_t^*)$, and $\Delta[m_t - y_t - (m_t^* - y_t^*)]$ can be rejected in nine cases at the 5 percent level and three more cases at the 10 percent level. There are no rejections for Canada and the United Kingdom but rejections in 12 of the 24 tests for the other four countries. The strongest rejections pertain to prices, where the null is rejected in three cases at the 1 percent level.[4]

[4] The overall level of predictability, though not the pattern, is consistent with the point estimates in Stock and Watson (2003). Using inflation and output from the G7 countries (rather than for six countries relative to the United States) and a 1985–99 sample, Stock and Watson examine the ability of the exchange rate (and many other financial variables) to forecast out of sample. They find that the exchange rate lowers the mean squared prediction error for inflation in one country (Canada) and for GDP in four countries (Canada, Germany, Italy, and Japan). Thus the overall rate of success (five out of 14 data series) is comparable to ours, though the pattern (more success with real than nominal)

In a sense, this is not particularly strong evidence that exchange rates predict fundamentals.[5] After all, even if there were zero predictability, one would expect a handful of significant statistics just by chance. We accordingly are cautious in asserting that the posited link is well established. But one statistical (as opposed to economic) indication that the results are noteworthy comes from contrasting these results with ones for Granger causality tests running in the opposite direction. We see in panel B of table 3 that the null that the fundamentals fail to Granger-cause Δs_t can be rejected at the 5 percent level in only one test and at the 10 percent level in only one more test. So, however modest the evidence is that exchange rates help to predict fundamentals, the evidence is distinctly stronger than that on the ability of fundamentals to predict exchange rates.

There were some major economic and noneconomic developments during our sample that warrant investigation of subsamples. Several of the European countries' exchange rates and monetary policies became more tightly linked in the 1990s because of the evolution of the European Monetary Union. Germany's economy was transformed dramatically in 1990 because of reunification. We therefore look at causality results for two subsamples. Table 4 presents results for 1974:1–1990:2 and table 5 for the remaining part of the sample (1990:3–2001:3).

The results generally go in the same direction as for the whole sample. In panel A of table 4, we see that for the first part of the sample, we reject the null of no Granger causality from exchange rates to fundamentals at the 1 or 5 percent level in 10 cases and at the 10 percent level in two more cases. Panel B indicates that there are no cases in which we can reject the null of no Granger causality from fundamentals to exchange rates at the 5 percent level and only two cases at the 10 percent level.

Table 5 reports results for the second part of the sample. Panel A shows that we reject the null of no Granger causality from exchange rates to fundamentals in nine cases at the 1 or 5 percent level and five more cases at the 10 percent level. But for the test of no causality from fundamentals to exchange rates, panel B shows that we reject nine times at the 1 or 5 percent level and once at the 10 percent level. In the

is not. We have not investigated the extent to which results differ because different series are being fit or because of in- vs. out-of-sample.

[5] A referee has pointed out that for series other than interest rates, seasonal adjustment may lead to spurious findings of causality. We were not able to collect a complete set of not seasonally adjusted data. But we did repeat our Granger causality tests using money supply data that were not seasonally adjusted for the United States, France, and Japan from IFS. Our findings were not affected by the use of not seasonally adjusted money supply data: we reject no Granger causality at the 10 percent level for France and at the 5 percent level for Japan. (We were able to obtain only not seasonally adjusted M2 money supply for Italy. The p-value for the test of no causality was 20 percent.)

TABLE 4

Bivariate Granger Causality Tests, Different Measures of Δf_t, Early Part of Sample: 1974:1–1990:2

	Canada	France	Germany	Italy	Japan	United Kingdom
	A. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H_0: Δs_t Fails to Cause Δf_t					
1. $\Delta(m - m^*)$		**		*		
2. $\Delta(p - p^*)$			**	***	**	
3. $i - i^*$		***				*
4. $\Delta(i - i^*)$		***			**	**
5. $\Delta(m - m^*) - \Delta(y - y^*)$		**		**		
6. $\Delta(y - y^*)$					**	
	B. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H_0: Δf_t Fails to Cause Δs_t					
1. $\Delta(m - m^*)$						
2. $\Delta(p - p^*)$	*		*			
3. $i - i^*$						
4. $\Delta(i - i^*)$						
5. $\Delta(m - m^*) - \Delta(y - y^*)$						
6. $\Delta(y - y^*)$						

Note.—See the notes to the earlier tables for variable definitions.

TABLE 5

Bivariate Granger Causality Tests, Different Measures of Δf_t, Later Part of Sample: 1990:3–2001:3

	Canada	France	Germany	Italy	Japan	United Kingdom
	A. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H_0: Δs_t Fails to Cause Δf_t					
1. $\Delta(m - m^*)$		**			***	
2. $\Delta(p - p^*)$	*	***	*			
3. $i - i^*$			*		**	**
4. $\Delta(i - i^*)$			**		**	**
5. $\Delta(m - m^*) - \Delta(y - y^*)$		*				**
6. $\Delta(y - y^*)$					*	
	B. Rejections at 1% (***), 5% (**), and 10% (*) Levels of H_0: Δf_t Fails to Cause Δs_t					
1. $\Delta(m - m^*)$			**		**	
2. $\Delta(p - p^*)$		***		**		
3. $i - i^*$					***	
4. $\Delta(i - i^*)$		**			***	
5. $\Delta(m - m^*) - \Delta(y - y^*)$			**		**	
6. $\Delta(y - y^*)$		*				

Note.—See the notes to the earlier tables for variable definitions.

1990s, then, there appears to be more evidence of exchange rate predictability. This perhaps is not entirely surprising given the effort by the European countries to stabilize exchange rates. We note, however, that several of the rejections of the null pertain to the yen/dollar rate.

In addition to the causality tests we report from bivariate VARs, we also performed cointegration and causality tests based on some multivariate VARs. We chose several different combinations of variables to include in these VARs, based on the models outlined in Section III. There are five groupings: $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$, $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*))'$, $(\Delta s_t, \Delta(p_t - p_t^*), \Delta(y_t - y_t^*))'$, $(\Delta s_t, \Delta(m_t - m_t^*), \Delta(y_t - y_t^*), \Delta(p_t - p_t^*))'$, and $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), \Delta(i_t - i_t^*))'$. All variables were entered in differences because of results of tests for cointegration.[6] We performed causality tests for the null that Δs_t does not Granger-cause for each of the fundamentals or the fundamentals as a group, and conversely. For example, in the first grouping, that is, $(\Delta s_t, \Delta(y_t - y_t^*), \Delta(p_t - p_t^*), i_t - i_t^*)'$, there were four tests of Granger causality from Δs_t to each of the three fundamentals and to the block of fundamentals as a whole. There was also the corresponding set of four tests from fundamentals to Δs_t. Across the six countries, this yielded 24 tests of causality in each direction for this grouping. Across all five groupings, 108 test statistics were computed in each direction.

The results are very much like the results from the bivariate VARs. There is almost no evidence of causality from the fundamentals to the exchange rate. Of the 108 tests we performed, there are no cases in which we could reject at the 5 percent level the hypothesis of no causality from fundamentals to exchange rates and only four cases in which that hypothesis is rejected at the 10 percent level. In contrast, in 35 tests (out of 108 performed) we rejected the null of no causality from exchange rates to fundamentals at the 10 percent level, and these were significant at the 5 percent level in 16 cases. We present details for the Granger causality tests on the fundamentals as a group in table 6, relegating to the additional appendix details on the other tests. As table 6 demonstrates, there were no cases in which we rejected the joint null of no causality from the group of fundamentals to the exchange rate. Notable are the tests for whether the exchange rate does not Granger-cause any of the economic fundamentals. Table 6 reports that we reject the null of no causation in 16 of the 30 tests performed at the 10 percent level, and 12 of those were significant rejections at the 5 percent level. Nonetheless, there were many more cases in which the exchange rate

[6] According to Johansen's (1991) trace and maximum eigenvalue statistics, there were only three cases in which we were able to reject the null of no cointegration (one for Canada and two for Italy), so for uniformity we treated all variables as though they were not cointegrated.

TABLE 6
VAR Causality Tests, Full Sample: 1974:1–2001:3: Rejections at 1% (***), 5% (**), and 10% (*)

Variables in VAR	Canada	France	Germany	Italy	Japan	United Kingdom
1. $\Delta(y-y^*)$, $\Delta(p-p^*)$, $i-i^*$:						
Null hypothesis A		*	**	***	***	
Null hypothesis B						
2. $\Delta(y-y^*)$, $\Delta(p-p^*)$, $\Delta(i-i^*)$:						
Null hypothesis A		**	*	***	***	
Null hypothesis B						
3. $\Delta(m-m^*)$, $\Delta(y-y^*)$:						
Null hypothesis A		**				
Null hypothesis B						
4. $\Delta(m-m^*)$, $\Delta(y-y^*)$, $\Delta(p-p^*)$:						
Null hypothesis A		**	*	***	*	
Null hypothesis B						
5. $\Delta(y-y^*)$, $\Delta(p-p^*)$:						
Null hypothesis A			**	***		
Null hypothesis B						

Note.—Null hypothesis A: Δs_t fails to cause Δf_t jointly; null hypothesis B: Δf_t jointly fails to cause Δs_t. See the notes to the earlier tables for variable definitions.

could not help predict fundamentals. The exchange rate was found to be useful in forecasting real output in only two cases.

In summary, while the evidence is far from overwhelming, there does appear to be a link from exchange rates to fundamentals, going in the directions that exchange rates help forecast fundamentals.

C. *Correlation between* Δs *and the Present Value of Fundamentals*

The previous subsection established a statistically significant link between exchange rates and certain fundamentals. We now examine such links to ask whether the signs of the regression coefficients are in some sense right. The statistic we propose is broadly similar to one developed in Campbell and Shiller (1987). The modification of the Campbell-Shiller statistic is necessary for two reasons. First, in contrast to Campbell and Shiller, our variables are not well approximated as cointegrated. Second, we allow for unobservable forcing variables, again in contrast to Campbell and Shiller.

Write the present-value relationship for exchange rates as

$$s_t = \sum_{j=0}^{\infty} b^j E_t f_{t+j} + \sum_{j=0}^{\infty} b^j E_t z_{t+j} \equiv F_t + U_t. \tag{15}$$

Now

$$\sum_{j=0}^{\infty} b^j E_t f_{t+j} = \frac{1}{1-b}\left(f_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j}\right).$$

Thus

$$s_t - \frac{1}{1-b} f_{t-1} = \frac{1}{1-b}\sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j} + U_t. \qquad (16)$$

Our unit root tests indicate that Δf_t and hence $\sum_{j=0}^{\infty} b^j E_t \Delta f_{t+j}$ are I(0) and that s_t and f_t are not cointegrated. For (3) to be consistent with a lack of cointegration between s_t and f_t, we must have $U_t \sim$ I(1). A stationary version of (15) is then

$$\Delta s_t = \Delta F_t + \Delta U_t. \qquad (17)$$

Let F_{it} be the present value of future f's computed relative to an information set indexed by the i subscript. The two information sets we use are univariate and bivariate:

$$F_{1t} \equiv E\left(\sum_{j=0}^{\infty} b^j f_{t+j} | f_t, f_{t-1}, \ldots\right) \qquad (18)$$

and

$$F_{2t} \equiv E\left(\sum_{j=0}^{\infty} b^j f_{t+j} | s_t, f_t, s_{t-1}, f_{t-1}, \ldots\right). \qquad (19)$$

We hope to get a feel for whether either of these information sets yields economically meaningful present values by estimating corr(ΔF_{it}, Δs_t), the correlation between ΔF_{it} and Δs_t. The findings of Granger causality from exchange rates to observable fundamentals support the view that exchange rates are determined as a present value that depends in part on these observables. A more demanding verification of the relationship between exchange rates and observed fundamentals implied by the model is that corr(ΔF_{it}, Δs_t) be high.[7]

We estimate corr(ΔF_{it}, Δs_t) using estimates of ΔF_{it} constructed from univariate autoregressions (F_{1t}) or bivariate VARs (F_{2t}). If the estimated correlation is substantially stronger using the bivariate estimate, we take that as evidence that the coefficients of Δs_t in the VAR equation for Δf_t are economically reasonable and important. We limit our analysis to the variables in which there is a statistically significant relationship between Δf_t and Δs_t, as indicated by the Granger causality tests in table 3.

Note that a low value of the correlation is not necessarily an indica-

[7] Engel and West (2004) propose a method for calculating the variance of ΔF_t (from eq. [17]) relative to the variance of Δs_t.

tion that s_t is little affected by the present value of f_t. A low correlation will result from a small covariance between ΔF_{it} and Δs_t. But since $\mathrm{Cov}(\Delta F_{it}, \Delta s_t) = \mathrm{Cov}(\Delta F_{it}, \Delta F_t) + \mathrm{Cov}(\Delta F_{it}, \Delta U_t)$, this covariance might be small because a sharply negative covariance between ΔF_{it} and ΔU_t offsets a positive covariance between ΔF_{it} and ΔF_t. Conversely, of course, a high correlation might reflect a tight relationship between ΔF_{it} and ΔU_t with little connection between ΔF_{it} and ΔF_t.[8]

We do, however, take as reasonable the notion that if the correlation is higher for the bivariate than for the univariate information set, the coefficients on lags of Δs_t in the Δf_t equation are economically meaningful.

We construct $\hat{F}_{1t}$ from estimates of univariate autoregressions and $\hat{F}_{2t}$ from bivariate VARs, imposing a value of the discount factor b. The lag length is four in both the univariate and bivariate estimates. We then estimate the correlations $\mathrm{corr}(\Delta F_{it}, \Delta s_t)$ using these estimated $\hat{F}_{it}$. We report results only for data that show Granger causality from Δs_t to Δf_t at the 10 percent level or higher in the whole sample (panel A of table 3). We construct confidence intervals using the percentile method and a nonparametric bootstrap. We sample with replacement from the bivariate VAR residuals, with actual data used as initial conditions. We use 5,000 replications. For $\hat{F}_{1t}$ and $\hat{F}_{2t}$, we construct 90 percent confidence intervals using the .05 and .95 quantiles. For $\hat{F}_{2t} - \hat{F}_{1t}$, we use the .10 and 1.0 quantiles. We do not attempt to control for the data-dependent fact that we study only samples in which the previous subsection found Granger causality.

We tried three values of the discount factor, $b = 0.5$, $b = 0.9$, and $b = 0.98$. Results were strongest for $b = 0.98$. So to be conservative we report results only for $b = 0.5$ and $b = 0.9$. See panels A and B, respectively, of table 7. For the univariate information set (F_{1t}), the three discount factors give very similar results. Of the 10 estimated correlations, only two are positive for each value of b. (All the relations should be positive for the four variables reported in table 7—$\Delta(m_t - m_t^*)$, $\Delta(p_t - p_t^*)$, $\Delta(i_t - i_t^*)$, and $\Delta[m_t - y_t - (m_t^* - y_t^*)]$—according to the models of Sec. III, if the contribution of ΔU_t is sufficiently small.) So if one relies on univariate estimates of the present value, one would find little support for the notion that changes in exchange rates reflect changes in the present value of fundamentals.

The bivariate estimates lend rather more support for this notion, especially for $b = 0.9$. The estimated correlation between ΔF_{2t} and Δs_t

[8] Since s_t is an element of the bivariate information set, projection of both sides of (2) onto this information set yields $s_t = F_{2t} + E(U_t | s_t, f_t, s_{t-1}, f_{t-1}, \ldots)$. It may help readers familiar with Campbell and Shiller (1987) to note that because our models include unobserved forcing variables (i.e., because U_t is present), we may not have $s_t = F_{2t} = F_t$. These equalities hold only if $E(U_t | s_t, f_t, s_{t-1}, f_{t-1}, \ldots) = 0$.

TABLE 7

Correlation between Δs_t and ΔF_t

Variables and Information Set	France	Germany	Italy	Japan
	A. Discount Factor $b = 0.5$			
1. $\Delta(m - m^*)$:				
F_{1t}	$-.02$		$-.13$	.24
	$(-.24, .16)$		$(-.32, .06)$	(.08, .37)
F_{2t}	.10		$-.05$	.23
	$(-.14, .31)$		$(-.26, .17)$	(.05, .38)
$F_{2t} - F_{1t}$	.13		.08	$-.01$
	(.05, .36)		(.02, .29)	$(-.08, .16)$
2. $\Delta(p - p^*)$:				
F_{1t}		$-.03$	.19	$-.21$
		$(-.20, .14)$	$(-.03, .34)$	$(-.35, -.06)$
F_{2t}		.10	.27	$-.13$
		$(-.09, .28)$	(.04, .44)	$(-.29, .06)$
$F_{2t} - F_{1t}$		.13	.08	.09
		(.07, .28)	(.02, .24)	(.03, .25)
3. $\Delta(i - i^*)$:				
F_{1t}	$-.21$			$-.05$
	$(-.38, -.03)$			$(-.25, .13)$
F_{2t}	$-.07$			.13
	$(-.27, .13)$			$(-.09, .34)$
$F_{2t} - F_{1t}$	.14			.18
	(.07, .34)			(.11, .42)
4. $\Delta(m - m^*) - \Delta(y - y^*)$:				
F_{1t}	$-.01$		$-.10$	
	$(-.23, .17)$		$(-.28, .07)$	
F_{2t}	.10		$-.05$	
	$(-.15, .31)$		$(-.25, .16)$	
$F_{2t} - F_{1t}$	.11		.05	
	(.03, .33)		$(-.01, .31)$	
	B. Discount Factor $b = 0.9$			
1. $\Delta(m - m^*)$:				
F_{1t}	$-.05$		$-.13$	.19
	$(-.24, .12)$		$(-.31, .05)$	(.04, .33)
F_{2t}	.25		$-.03$	$-.05$
	$(-.15, .55)$		$(-.33, .34)$	$(-.32, .24)$
$F_{2t} - F_{1t}$	.30		.10	$-.24$
	(.05, .89)		$(-.07, .69)$	$(-.41, .30)$
2. $\Delta(p - p^*)$:				
F_{1t}		$-.01$	.17	$-.17$
		$(-.18, .16)$	$(-.03, .34)$	$(-.31, -.02)$
F_{2t}		.49	.51	.31
		(.19, .68)	(.18, .71)	(.00, .53)
$F_{2t} - F_{1t}$		.50	.34	.47
		(.32, .81)	(.16, .71)	(.28, .84)
3. $\Delta(i - i^*)$:				
F_{1t}	$-.21$			$-.06$
	$(-.39, -.03)$			$(-.27, .12)$
F_{2t}	.15			.54
	$(-.19, .45)$			(.19, .75)
$F_{2t} - F_{1t}$	.37			.60
	(.15, .86)			(.41, .95)

TABLE 7
(*Continued*)

Variables and Information Set	France	Germany	Italy	Japan
4. $\Delta(m - m^*) - \Delta(y - y^*)$:				
F_{1t}	−.04		−.10	
	(−.23, .14)		(−.28, .06)	
F_{2t}	.23		−.04	
	(−.17, .53)		(−.34, .31)	
$F_{2t} - F_{1t}$	.27		.06	
	(.02, .78)		(−.11, .67)	

NOTE.—F_{1t} and F_{2t} are the expected discounted values of fundamentals, computed using lagged fundamentals alone (F_{1t}) or lagged fundamentals and lagged exchange rates (F_{2t}). The point estimates are the correlation between the change in the estimates of the expected present discounted values and the change in the actual exchange rate. They may be interpreted as correlations between fitted and actual values. The numbers in parentheses are 90 percent confidence intervals, computed from a nonparametric bootstrap.

is positive in six of the 10 cases for $b = 0.5$ (though significantly different from zero at the 90 percent level in only one case [Japan, $\Delta(m - m^*)$]); it is positive in seven of the 10 cases for $b = 0.9$ and significant in four of these (all three inflation series and $\Delta(i - i^*)$ in Japan). The sharpest result is that the correlation is higher for ΔF_{2t} than for ΔF_{1t}: the difference between the two is positive and significant in eight cases for $b = 0.5$ and positive in nine cases and significant in seven for $b = 0.9$.[9] The median correlations can be summarized as follows:

Information Set	$b = 0.5$	$b = 0.9$
F_{1t}	−.04	−.05
F_{2t}	.10	.24

It is clear that using lags of Δs_t to estimate the present value of fundamentals results in an estimate that is more closely tied to Δs_t itself than when the present value of fundamentals is based on univariate estimates. But even when we limit ourselves to data in which there is Granger causality from Δs_t to Δf_t, the largest single correlation in the table is 0.51 (Germany, for $\Delta(p_t - p_t^*)$, when $b = 0.9$). A correlation less than one may be due to omitted forcing variables, U_t. In addition, we base our present values on the expected present discounted value of fundamental variables one at a time, instead of trying to find the appropriate linear combination (except when we use $m - y$ as a fundamental). So we should not be surprised that the correlations are still substantially below one.

The long literature on random walks in exchange rates causes us to

[9] Here it is advisable to recall that we examine only series that display Granger causality. So the statistical significance of the difference is unsurprising. On the other hand, the sign of the difference (positive) was not foretold by our Granger causality tests.

interpret the correlations in table 7 as new evidence that exchange rates are tied to fundamentals. We recognize, however, that these estimates leave a vast part of the movements in exchange rates not tied to fundamentals. The results may suggest a direction for future research into the link between exchange rates and fundamentals—looking for improvements in the definition of fundamentals used to construct F_{2t}.

V. Conclusions

We view the results of this paper as providing some counterbalance to the bleak view of the usefulness (especially in the short run) of rational expectations present-value models of exchange rates that has become predominant since Meese and Rogoff (1983*a*, 1983*b*). We find that exchange rates may incorporate information about future fundamentals, a finding consistent with the present-value models. We also show theoretically that under some empirically plausible circumstances the inability to forecast exchange rates is a natural implication of the models. The models do suggest that innovations in the exchange rate ought to be highly correlated with news about future fundamentals—a link that seems to garner support from the recent study of Andersen et al. (2003), who find strong evidence of exchange rate reaction to news (and in a direction consistent with standard models) in intraday data.

On the other hand, our findings certainly do not provide strong direct support for these models, and indeed there are several caveats that deserve mention. First, while our Granger causality results are consistent with the implications of the present-value models—that exchange rates should be useful in forecasting future economic variables such as money, income, prices, and interest rates—there are other possible explanations for these findings. It may be, for example, that exchange rates Granger-cause the domestic consumer price level simply because exchange rates are passed on to prices of imported consumer goods with a lag. Exchange rates might Granger-cause money supplies because monetary policy makers react to the exchange rate in setting the money supply. In other words, the present-value models are not the only models that imply Granger causality from exchange rates to other economic variables. Table 7, which concerns the correlation of exchange rate changes with the change in the expected discounted fundamentals, provides some evidence that the Granger causality results are generated by the present-value models, but it is far from conclusive.

Second, the empirical results are not uniformly strong. As well, it remains to be seen how well they hold upon, for example, use of panel data or out-of-sample techniques such as in Groen (2000), Mark and Sul (2001), or Stock and Watson (2003).

Third, while we read the exchange rate literature as agreeing with us that there is a role for "unobserved" fundamentals—money demand shocks, real exchange rate shocks, and risk premiums—we recognize that others might view such a role as evidence of a failure of the model. We do not find much evidence that the exchange rate is explained only by the "observable" fundamentals. Our bivariate cointegration tests generally fail to find cointegration between exchange rates and fundamentals. Moreover, we know from Mark (2001) that actual exchange rates are likely to have a much lower variance than a discounted sum of observable fundamentals. Our view is that it is perhaps unrealistic to believe that only fundamentals that are observable by the econometrician should affect exchange rates, but it is nonetheless important to note that observables do not obviously dominate exchange rate changes.

But perhaps our findings shift the terms of the exchange rate debate. We have shown analytically that if discount factors are large (and fundamentals are I(1)), then it may not be surprising that present-value models cannot outforecast the random walk model of exchange rates. We have found some support for the link between fundamentals and the exchange rate in the other direction: exchange rates can help forecast the fundamentals. We tentatively conclude that exchange rates and fundamentals are linked in a way that is broadly consistent with asset-pricing models of the exchange rate.

Finally, our analytical results may also help explain the near–random walk behavior of other asset prices. It is well known that as a theoretical matter, asset prices follow random walks only under very special circumstances. A priority for future research is investigating the power of our results to explain the time-series behavior of a variety of asset prices.

Appendix

In this appendix, we prove the statement in the text concerning random walk behavior in s_t as the discount factor $b \to 1$.

We suppose that there is an $n \times 1$ vector of fundamentals $\boldsymbol{x}_t$. This vector includes all variables, observable as well as unobservable (to the economist), that private agents use to forecast f_{1t}, f_{2t}, z_{1t}, and z_{2t}. For example, we may have $n = 9$, $\boldsymbol{x}_t = (m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, \rho_t, u_t)'$, and $f_t = m_t - m_t^* - (y_t - y_t^*)$, with u_t a variable that helps predict one or more of m_t, m_t^*, y_t, y_t^*, v_{mt}, v_{mt}^*, q_t, and ρ_t. We assume that u_t is a scalar only as an example; there may be a set of variables like u_t. We assume that $\Delta \boldsymbol{x}_t$ follows a stationary finite-order autoregressive moving average process (possibly with one or more unit moving average roots; we allow $\boldsymbol{x}_t$ to include stationary variables as well as cointegrated I(1) variables). Let $\boldsymbol{\epsilon}_t$ denote the $n \times 1$ innovation in $\Delta \boldsymbol{x}_t$ and L the lag operator, $L\boldsymbol{x}_t = \boldsymbol{x}_{t-1}$. For notational simplicity we assume tentatively that $\Delta \boldsymbol{x}_t$ has zero mean. Write the Wold representation of $\Delta \boldsymbol{x}_t$ as

$$\Delta \boldsymbol{x}_t = \boldsymbol{\theta}(L)\boldsymbol{\epsilon}_t = \sum_{j=0}^{\infty} \boldsymbol{\theta}_j \boldsymbol{\epsilon}_{t-j}, \quad \boldsymbol{\theta}_0 \equiv \mathbf{I}. \tag{A1}$$

We define $E_t \Delta \boldsymbol{x}_{t+j}$ as $E(\Delta \boldsymbol{x}_{t+j} | \boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_{t-1}, \ldots)$ and assume that mathematical expectations and linear projections coincide.

Define the $n \times 1$ vectors $\boldsymbol{w}_{1t}$ and $\boldsymbol{w}_{2t}$ as

$$\boldsymbol{w}_{1t} = (1-b) \sum_{j=0}^{\infty} b^j E_t \boldsymbol{x}_{t+j},$$

$$\boldsymbol{w}_{2t} = b \sum_{j=0}^{\infty} b^j E_t \boldsymbol{x}_{t+j},$$

$$\boldsymbol{w}_t = (\boldsymbol{w}'_{1t}, \boldsymbol{w}'_{2t})'. \quad \text{(A2)}$$

Then s_t is a linear combination of the elements of $\boldsymbol{w}_{1t}$ and $\boldsymbol{w}_{2t}$, say

$$s_t = \boldsymbol{a}'_1 \boldsymbol{w}_{1t} + \boldsymbol{a}'_2 \boldsymbol{w}_{2t} \quad \text{(A3)}$$

for suitable $n \times 1$ $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. We assume that either (a) $\boldsymbol{a}'_1 \boldsymbol{w}_{1t} \sim \mathrm{I}(1)$ and $\boldsymbol{a}_2 \equiv \mathbf{0}$ or (b) if $\boldsymbol{a}_2 \neq \mathbf{0}$, $\boldsymbol{a}'_2 \boldsymbol{w}_{2t} \sim \mathrm{I}(1)$, with $\boldsymbol{a}'_1 \boldsymbol{w}_{1t}$ essentially unrestricted (stationary, I(1), or identically zero).

We show the following below.

1. Suppose that $\boldsymbol{a}_2 \equiv \mathbf{0}$ (i.e., $\rho_t = 0$ in the monetary model). Then

$$\mathrm{plim}_{b \to 1} [\Delta s_t - \boldsymbol{a}'_1 \boldsymbol{\theta}(1) \boldsymbol{\epsilon}_t] = 0. \quad \text{(A4)}$$

Here, $\boldsymbol{\theta}(1)$ is an $n \times n$ matrix of constants, $\boldsymbol{\theta}(1) = \sum_{j=0}^{\infty} \boldsymbol{\theta}_j$, for $\boldsymbol{\theta}_j$ defined in (A1). We note that if $\boldsymbol{a}'_1 \boldsymbol{x}_t$ were stationary (contrary to what we assume when $\boldsymbol{a}_2 = \mathbf{0}$), then $\boldsymbol{a}'_1 \boldsymbol{\theta}(1) = \mathbf{0}$, and (A3) states that as b approaches one, s_t approaches a constant. But if $\boldsymbol{a}'_1 \boldsymbol{x}_t$ is I(1), as is arguably the case in our data, we have the claimed result: for b very near one, Δs_t will behave very much like the unpredictable sequence $\boldsymbol{a}'_1 \boldsymbol{\theta}(1) \boldsymbol{\epsilon}_t$.

2. Suppose that $\boldsymbol{a}_1 \equiv \mathbf{0}$ and $\boldsymbol{a}_2 \neq \mathbf{0}$. Then

$$\mathrm{plim}_{b \to 1} \{[(1-b)\Delta s_t] - b \boldsymbol{a}'_2 \boldsymbol{\theta}(1) \boldsymbol{\epsilon}_t\} = 0. \quad \text{(A5)}$$

By assumption, $\boldsymbol{a}'_2 \boldsymbol{x}_t \sim \mathrm{I}(1)$, so $\boldsymbol{a}'_2 \boldsymbol{\theta}(1) \neq \mathbf{0}$. Then for b near one, $(1-b)\Delta s_t$ will behave very much like the unpredictable sequence $\boldsymbol{a}'_2 \boldsymbol{\theta}(1) \boldsymbol{\epsilon}_t$. This means in particular that the correlation of $(1-b)\Delta s_t$ with any information known at time $t-1$ will be very near zero. Since the correlation of Δs_t with such information is identical to that of $(1-b)\Delta s_t$, Δs_t will also be almost uncorrelated with such information.

Let us combine (A4) and (A5). Then for b near one, Δs_t will be approximately uncorrelated with information known at $t-1$, since for b near one

$$\Delta s_t \approx \left[\boldsymbol{a}_1 + \left(\frac{b \boldsymbol{a}_2}{1-b}\right)\right]' \boldsymbol{\theta}(1) \boldsymbol{\epsilon}_t. \quad \text{(A6)}$$

Two comments. First, for any given $b < 1$, the correlation of Δs_t with period $t-1$ information will be very similar for (1) $\boldsymbol{x}_t$ processes that are stationary, but barely so, in the sense of having autoregressive unit roots near one; and (2) $\boldsymbol{x}_t$ processes that are I(1). This is illustrated in the calculations in table 2.

Second, suppose that $\Delta \boldsymbol{x}_t$ has nonzero mean $\boldsymbol{\mu}$ ($n \times 1$). Then (A6) becomes

$$\Delta s_t \approx \left[\boldsymbol{a}_1 + \left(\frac{b \boldsymbol{a}_2}{1-b}\right)\right]' [\boldsymbol{\mu} + \boldsymbol{\theta}(1)] \boldsymbol{\epsilon}_t. \quad \text{(A7)}$$

Thus the exchange rate approximately follows a random walk with drift $\{\boldsymbol{a}_1 + [b\boldsymbol{a}_2/(1-b)]\}'\boldsymbol{\mu}$ if $\{\boldsymbol{a}_1 + [b\boldsymbol{a}_2/(1-b)]\}'\boldsymbol{\mu} \neq 0$.

Proof of (A4)

With elementary rearrangement, we have

$$\boldsymbol{w}_{1t} = \boldsymbol{x}_{t-1} + \sum_{j=0}^{\infty} b^j E_t \Delta \boldsymbol{x}_{t+j}. \tag{A8}$$

Project (A8) on period $t-1$ information and subtract from (A8). Since $\boldsymbol{w}_{1t} - E_{t-1}\boldsymbol{w}_{1t} = \Delta\boldsymbol{w}_{1t} - E_{t-1}\Delta\boldsymbol{w}_{1t}$ and $\boldsymbol{x}_{t-1} - E_{t-1}\boldsymbol{x}_{t-1} = \boldsymbol{0}$, we get

$$\Delta\boldsymbol{w}_{1t} - E_{t-1}\Delta\boldsymbol{w}_{1t} = \sum_{j=0}^{\infty} b^j(E_t\Delta\boldsymbol{x}_{t+j} - E_{t-1}\Delta\boldsymbol{x}_{t+j}) = \boldsymbol{\theta}(b)\boldsymbol{\epsilon}_t, \tag{A9}$$

the last equality following from Hansen and Sargent (1981). Next, difference (A8). Upon rearranging the right-hand side, we get $\Delta\boldsymbol{w}_{1t} = \sum_{j=0}^{\infty} b^j(E_t\Delta\boldsymbol{x}_{t+j} - bE_{t-1}\Delta\boldsymbol{x}_{t+j})$. Project on period $t-1$ information and rearrange to get

$$E_{t-1}\Delta\boldsymbol{w}_{1t} = (1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta\boldsymbol{x}_{t+j}. \tag{A10}$$

From (A3) (with $\boldsymbol{a}_2 = \boldsymbol{0}$, by assumption), (A8), and (A9),

$$\Delta s_t = \boldsymbol{a}_1'\boldsymbol{\theta}(b)\boldsymbol{\epsilon}_j + \boldsymbol{a}_1'(1-b)\sum_{j=0}^{\infty} b^j E_{t-1}\Delta\boldsymbol{x}_{t+j}. \tag{A11}$$

Since $\boldsymbol{a}_1'\Delta\boldsymbol{x}_t$ is stationary, $\boldsymbol{a}_1'\sum_{j=0}^{\infty} b^j E_{t-1}\Delta\boldsymbol{x}_{t+j}$ converges in probability to a stationary variable as $b \rightarrow 1$. Since $\lim_{b\rightarrow 1}(b-1) = 0$, $(1-b)\boldsymbol{a}_1'\sum_{j=0}^{\infty} b^j E_{t-1}\Delta\boldsymbol{x}_{t+j}$ converges in probability to zero as $b \rightarrow 1$. Hence $\Delta s_t - \boldsymbol{a}_1'\boldsymbol{\theta}(b)\boldsymbol{\epsilon}_t$ converges in probability to zero, from which (A4) follows.

Result (A5) results simply by noting that when $\boldsymbol{a}_1 \equiv \boldsymbol{0}$, $(1-b)s_t = \boldsymbol{a}_2'(1-b)b\sum_{j=0}^{\infty} b^j E_t\boldsymbol{x}_{t+j}$, and the argument for (A4) may be applied to $(1-b)s_t$.

References

Andersen, Torban G., Tim Bollerslev, Francis X. Diebold, and Clara Vega. 2003. "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange." *A.E.R.* 93 (March): 38–62.

Bilson, John F. O. 1978. "The Monetary Approach to the Exchange Rate: Some Empirical Evidence." *IMF Staff Papers* 25 (March): 48–75.

Campbell, John Y., and Robert J. Shiller. 1987. "Cointegration and Tests of Present Value Models." *J.P.E.* 95 (October): 1062–88.

———. 1988. "Stock Prices, Earnings, and Expected Dividends." *J. Finance* 43 (July): 661–76.

Cheung, Yin-Wong, Menzie D. Chinn, and Antonio Garcia Pascual. 2002. "Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive?" Working Paper no. 9393 (December), NBER, Cambridge, MA.

Chinn, Menzie D., and Richard A. Meese. 1995. "Banking on Currency Forecasts: How Predictable Is Change in Money?" *J. Internat. Econ.* 38 (February): 161–78.

Clarida, Richard, Jordi Gali, and Mark Gertler. 1998. "Monetary Policy Rules in

Practice: Some International Evidence." *European Econ. Rev.* 42 (June): 1033–67.

Dornbusch, Rudiger. 1976. "Expectations and Exchange Rate Dynamics." *J.P.E.* 84 (December): 1161–76.

Elton, Edwin J., and Martin J. Gruber. 1970. "Marginal Stockholder Tax Rates and the Clientele Effect." *Rev. Econ. and Statis.* 52 (February): 68–74.

Engel, Charles. 1996. "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence." *J. Empirical Finance* 3 (June): 123–91.

Engel, Charles, and Kenneth D. West. 2004. "Accounting for Exchange-Rate Variability in Present-Value Models When the Discount Factor Is Near 1." *A.E.R. Papers and Proc.* 94 (May): 119–25.

Frankel, Jeffrey A. 1979. "On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials." *A.E.R.* 69 (September): 610–22.

Frenkel, Jacob A. 1976. "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence." *Scandinavian J. Econ.* 78 (2): 200–224.

Frenkel, Jacob A., and Michael L. Mussa. 1985. "Asset Markets, Exchange Rates, and the Balance of Payments." In *Handbook of International Economics*, vol. 2, edited by Ronald W. Jones and Peter B. Kenen. Amsterdam: North-Holland.

Groen, Jan J. J. 2000. "The Monetary Exchange Rate Model as a Long-Run Phenomenon." *J. Internat. Econ.* 52 (December): 299–319.

Hansen, Lars Peter, and Thomas J. Sargent. 1981. "A Note on Wiener-Kolmogorov Prediction Formulas for Rational Expectations Models." *Econ. Letters* 8 (3): 255–60.

Johansen, Soren. 1991. "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models." *Econometrica* 59 (November): 1551–80.

Kilian, Lutz, and Mark P. Taylor. 2003. "Why Is It So Difficult to Beat the Random Walk Forecast of Exchange Rates?" *J. Internat. Econ.* 60 (May): 85–107.

MacDonald, Ronald, and Mark P. Taylor. 1993. "The Monetary Approach to the Exchange Rate: Rational Expectations, Long-Run Equilibrium, and Forecasting." *IMF Staff Papers* 40 (March): 89–107.

———. 1994. "The Monetary Model of the Exchange Rate: Long-Run Relationships, Short-Run Dynamics and How to Beat a Random Walk." *J. Internat. Money and Finance* 13 (June): 276–90.

Mark, Nelson C. 1995. "Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability." *A.E.R.* 85 (March): 201–18.

———. 2001. *International Macroeconomics and Finance: Theory and Econometric Methods.* Oxford: Blackwell. [Errata: http://www.nd.edu/~nmark/book/errata/Errata.pdf.]

Mark, Nelson C., and Donggyu Sul. 2001. "Nominal Exchange Rates and Monetary Fundamentals: Evidence from a Small Post–Bretton Woods Sample." *J. Internat. Econ.* 53 (February): 29–52.

Meese, Richard A., and Kenneth S. Rogoff. 1983*a*. "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *J. Internat. Econ.* 14 (February): 3–24.

———. 1983*b*. "The Out of Sample Failure of Empirical Exchange Models." In *Exchange Rates and International Macroeconomics*, edited by Jacob A. Frenkel. Chicago: Univ. Chicago Press (for NBER).

Mussa, Michael L. 1976. "The Exchange Rate, the Balance of Payments and Monetary and Fiscal Policy under a Regime of Controlled Floating." *Scandinavian J. Econ.* 78 (2): 229–48.

Obstfeld, Maurice, and Kenneth S. Rogoff. 1996. *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.

———. 2003. "Risk and Exchange Rates." In *Economic Policy in the International Economy: Essays in Honor of Assaf Razin*, edited by Elhanan Helpman and Efraim Sadka. Cambridge: Cambridge Univ. Press.

Osterwald-Lenum, Michael. 1992. "A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics." *Oxford Bull. Econ. and Statis.* 54 (August): 461–72.

Rogoff, Kenneth S. 1996. "The Purchasing Power Parity Puzzle." *J. Econ. Literature* 34 (June): 647–68.

Rossi, Barbara. 2003. "Testing Long-Horizon Predictive Ability with High Persistence, and the Meese-Rogoff Puzzle." Manuscript, Dept. Econ., Duke Univ.

Samuelson, Paul A. 1965. "Proof That Properly Anticipated Prices Fluctuate Randomly." *Indus. Management Rev.* 6 (2): 41–49.

Sims, Christopher. 1980. "Macroeconomics and Reality." *Econometrica* 48 (January): 1–48.

Sriram, Subramanian S. 2000. "A Survey of Recent Empirical Money Demand Studies." *IMF Staff Papers* 47 (3): 334–65.

Stock, James H., and Mark W. Watson. 1993. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica* 61 (July): 783–820.

———. 2003. "Forecasting Output and Inflation: The Role of Asset Prices." *J. Econ. Literature* 41 (September): 788–829.

Taylor, Mark P., David A. Peel, and Lucio Sarno. 2001. "Nonlinear Mean-Reversion in Real Exchange Rates: Toward a Solution to the Purchasing Power Parity Puzzles." *Internat. Econ. Rev.* 42 (November): 1015–42.

West, Kenneth D. 1988. "Dividend Innovations and Stock Price Volatility." *Econometrica* 56 (January): 37–61.