# Econometría Financiera

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# I. Introduction

# Motivación I

Definition:

**Financial Econometrics** is concerned with the statistical analysis of financial data.

- The method of inference for the financial economist is model-based statistical inference-financial econometrics.
- While econometrics is also essential in other branches of economics, what distinguishes financial economics is the central role that **uncertainty** plays in both financial theory and its empirical implementation.

# Motivación II

Financial econometrics is concerned mainly about:

- asset pricing
- portfolio allocation
- risk management and diversification

studying issues like:

- market microstructure and liquidity,
- asset return volatility and correlation,
- and interest rate modeling

and also for understanding pivotal issues in:

- Money, credit and Banking
- Corporate finance
- Behavioral finance
- as well as regulatory purposes and more.

# Motivación III

Why is Financial econometrics important?

- **Financial economics** concentrates on decision making when two considerations are particularly important:
  1. some of the outcomes are risky
  2. both the decisions and the outcomes may occur at different times
- The past few decades have been characterized by an extraordinary growth in the use of quantitative methods in the analysis of various asset classes:
  - Equities (stocks),
  - Fixed income (gov't and corporate bonds),
  - Alternative investments: commodities (energy, agriculture, industrial metals, precious metals), hedge funds (pooled investment structure), real estate (mortages), insurance, etc.

  and different financial instruments:
  - Derivative and cash instruments
  - Debt and equity instruments
  - Foreign exchange instruments

# Motivación IV

So that, financial econometrics is related to the application of statistical and mathematical techniques to problems in finance.

Examples:

- Testing whether financial markets are efficient.
- Testing whether the CAPM or APT represent superior models for the determination of returns on risky assets.
- Measuring and forecasting the volatility of bond returns.
- Modelling long-term relationships between prices and exchange rates
- Determining the optimal hedge ratio for a spot position in oil.
- Forecasting the correlation between the returns to the stock indices of two countries.

# Risk, prices and returns I

- The price that can be observed can be interpreted as the market price in the sense that it is determined by demand and supply of the asset and can therefore deviate from the fundamental value, but in the long run will converge to the fundamental value of an asset (Smith, 1776).
  Definition:
  For assets, equity markets and bond markets, fundamental value is the present value of future cash flows.
- However, under the efficient markets hypothesis the price should always equal the fundamental value. So that, in many cases they are looked alike.
- Thus, there is a close relation between the fundamental value of an asset and their appropriate return.
- Much of finance is concerned with measuring and managing financial risk based on the analysis of returns.

# Risk, prices and returns II

- Risk must be correctly **measured** in order to select the quantity to be borne vs. to be hedged
- Several kinds of risk: market, liquidity (including funding), operational, business, credit

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- There are different kinds of risk we care for:
  - Market risk is defined as the risk to a financial portfolio from movements in market prices such as equity prices, foreign exchange rates, interest rates, and commodity prices
  - It is important to choose how much of this risk my be taken on (thus reaping profits and losses), and how much hedged away
  - Liquidity risk comes from a chance to have to trade in markets characterized by low trading volume and/or large bid-ask spreads
  - Under such conditions, the attempt to sell assets may push prices lower, and assets may have to be sold at prices below their fundamental values or within a time frame longer than expected

# Risk, prices and returns III

- Operational (op) risk is defined as the risk of loss due to physical catastrophe, technical failure, and human error in the operation of a firm, including fraud, failure of management, and process errors
  - Although it should be mitigated and ideally eliminated in any firm, this course has little to say about op risk because op risk is typically very difficult to hedge in asset markets
  - But cat bonds…
- Op risk is instead typically managed using self-insurance or third-party insurance
- Credit risk is defined as the risk that a counterparty may become less likely to fulfill its obligation in part or in full on the agreed upon date
- Banks spend much effort to carefully manage their credit risk exposure while nonfinancial firms try and remove it completely

# Risk, prices and returns IV

- Business risk is defined as the risk that changes in variables of a business plan will destroy that plan's viability
  - It includes quantifiable risks such as business cycle and demand equation risks, and non-quantifiable risks such as changes in technology
- These risks are integral part of the core business of firms

- Not always risks may be predicted or, even though these are predictable, they may be managed in asset markets
- When they are, then we care for them in this course

- The lines between the different kinds of risk are often blurred; e.g., the securitization of credit risk via credit default swaps (CDS) is an example of a credit risk becoming a market risk (price of a CDS)

- When risk is quantifiable and manageable in asset markets, then we shall predict the distribution of risky asset returns

# Risk, prices and returns V

- How do we measure and predict risks? Studying **asset returns**
- Because returns have much better statistical properties than price levels, risk modeling focuses on describing the dynamics of returns

$$\boxed{r_{t+1} = (S_{t+1} - S_t)/S_t = S_{t+1}/S_t - 1}$$

(discretely compounded)

$$\boxed{R_{t+1} = \ln(S_{t+1}) - \ln(S_t)}$$

(continuously compounded)

Remarks:

- The return on an investment is its revenue as a fraction of the initial investment.
- It represents the net return for the holding period from $t$ to $t+1$.
- Risk means uncertainty in future returns from an investment, in particular, that the investment could earn less than the expected return and even result in a loss, that is, a negative return.

# Risk, prices and returns VI

Technical Remarks:

- At daily or weekly frequencies, the numerical differences between simple and compounded returns are minor

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- The two returns are typically fairly similar over short time intervals, such as daily:

  $R_{t+1} = \ln(S_{t+1}) - \ln(S_t) = \ln(S_{t+1}/S_t) = \ln(1 + r_{t+1}) \approx r_{t+1}$
- The approximation holds because $\ln(x) \approx x - 1$ when $x \cong 1$

# Risk, prices and returns VII

- Simple rates aggregate well **cross-sectionally** (in portfolios), while continuously compounded returns aggregate **over time**

## Cross-sectionally

- The simple rate of return definition has the advantage that **the rate of return on a portfolio is the portfolio of the rates of return**
  - If $V_{PF;t}$ is the value of the portfolio on day t so that $V_{PF,t} = \sum_{i=1}^{n} N_i S_{i,t}$
  - Then the portfolio rate of return is

$$r_{PF,t+1} \equiv \frac{V_{PF,t+1} - V_{PF,t}}{V_{PF,t}} = \frac{\sum_{i=1}^{n} N_i S_{i,t+1} - \sum_{i=1}^{n} N_i S_{i,t}}{\sum_{i=1}^{n} N_i S_{i,t}} = \sum_{i=1}^{n} w_i r_{i,t+1}$$

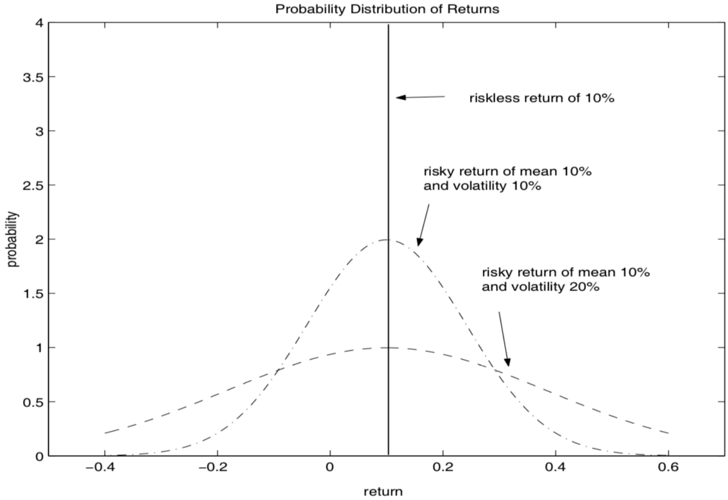
where $w_i = N_i S_{i,t} / V_{PF,t}$ is the portfolio weight in asset *i*

# Risk, prices and returns VIII

Over time

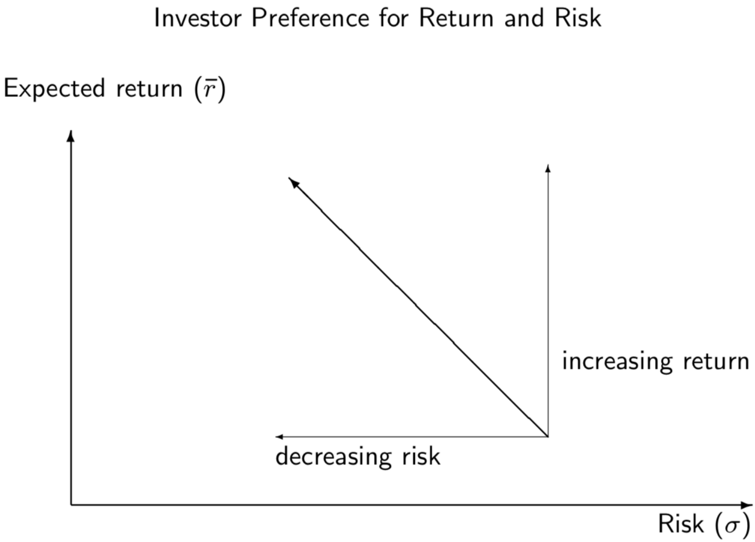
- This relationship does not hold for log returns because the log of a sum is not the sum of the logs
- However, most assets have a lower bound of zero on the price. Log returns are more convenient for preserving this lower bound in risk models because an arbitrarily large negative log return tomorrow will still imply a positive price at the end of tomorrow:
$$S_{t+1} = \exp(R_{t+1})\, S_t$$
    - If we instead we use the rate of return definition, then tomorrow's closing price is $S_{t+1} = (1 + r_{t+1})\, S_t$ so that the price might go negative in the model unless the assumed distribution of tomorrow's return, $r_{t+1}$, is bounded below by -1
- An advantage of the log return definition is that we can calculate the compounded return at the K-day horizon simply as the sum of the daily returns:
$$R_{t+1:t+K} = \ln(S_{t+K}) - \ln(S_t) = \sum_{k=1}^{K} [\ln(S_{t+k}) - \ln(S_{t+k-1})] = \sum_{k=1}^{K} R_{t+k}$$

# Return distribution




Which asset would you choose?

# Return distribution




Investors care about expected return and risk !

# Stylized facts on asset returns I

- At daily or weekly frequencies, asset returns display **weak serial correlations** (in absolute value)
- Returns **are not normal and display asymmetries and fat tails**
- At high frequencies, the standard deviation of asset returns completely dominates the mean which is often not significant
- Squared and absolute returns have **strong serial correlations** and there is a leverage effect
- Correlations between asset returns are time-varying

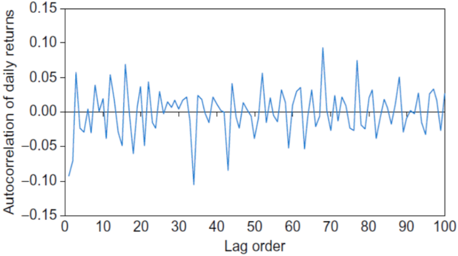
Note:

- The leverage effect stems from the fact that losses have a greater influence on future volatilities than do gains.
- Asymmetry means that the distribution of losses has a heavier tail than the distribution of gains

# Stylized facts on asset returns II

Example: Standard and Poors 500 Index

- Asset returns display a few **stylized facts** that tend to generally apply and that are well-known
  - Refer to daily returns on the S&P 500 from January 1, 2001, through December 31, 2010
  - But these properties are much more general, see below



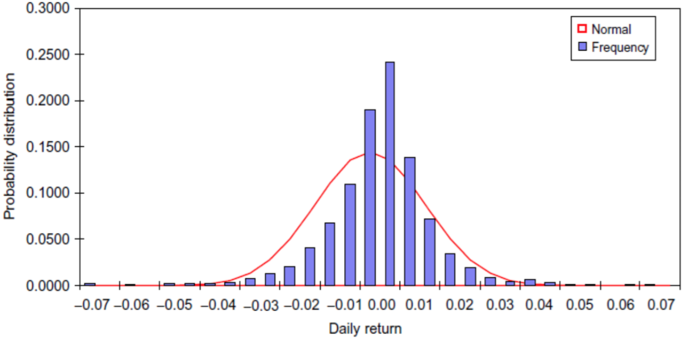

1. Daily returns show weak autocorrelation:

$$Corr\left(R_{t+1}, R_{t+1-\tau}\right) \approx 0, \quad \text{for } \tau = 1, 2, 3, \ldots, 100$$

  - Returns are almost impossible to predict from their own past

# Stylized facts on asset returns III

2. The unconditional distribution of daily returns does not follow the normal distribution

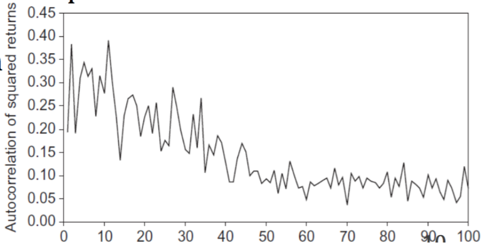


# Stylized facts on asset returns IV

- The histogram is more peaked around zero than a normal distribution
- Daily returns tend to have more small positive and fewer small negative returns than the normal distribution (fat tails)
- The stock market exhibits occasional, very large drops but not equally large upmoves
- Consequently, the distribution is asymmetric or negatively skewed

# Stylized facts on asset returns V

3. Std. dev. completely dominates the mean at short horizons
   - S&P 500: daily mean of 0.0056% and daily std. dev. of 1.3771%
4. Variance, measured, for example, by squared returns, displays positive correlation with its own past
5. Equity and equity indices display negative correlation between variance and returns, the leverage effect
6. Correlation between assets appears to be time varying



# Basic model for asset returns I

- Based on the previous list of stylized facts, our model of asset returns will take the generic form:

$$\boxed{R_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1}, \quad \text{with } z_{t+1} \sim \text{i.i.d. } D(0,1)}$$

  - $z_{t+1}$ is an innovation term, which we assume is identically and independently distributed (i.i.d.) according to the distribution D(0, 1), which has a mean equal to zero and variance equal to one
  - The **conditional mean** of the return, $E_t[R_{t+1}]$, is thus $\mu_{t+1}$, and the **conditional variance**, $E_t\{[R_{t+1} - \mu_{t+1},]^2\}$; is $\sigma^2_{t+1}$
  - Often assume $\mu_{t+1} = 0$ as for daily data this is a reasonable assumption
  - Notice that D(0, 1) does not have to be a normal distribution
  - Our task will consist of building and estimating models for both the conditional variance and the conditional mean
    - E.g., $\mu_{t+1} = \phi_0 + \phi_1 R_t$ and $\sigma^2_{t+1} = \lambda\sigma^2_t + (1 - \lambda)R^2_t$
  - However, robust conditional mean relationships are not easy to find, and assuming a zero mean return may be a prudent choice

# Basic model for asset returns II

- One important notion in this course distinguishes between unconditional vs. conditional moments and/or densitiies

- Unconditional moments and densities represent the long-run, average properties of times series of interest
- Conditional moments and densities capture how our perceptions of RV dynamics changes over time as news arrive

- An unconditional moment or density represents the long-run, average, "stable" properties of one or more random variables
  - Example 1: $E[R_{t+1}] = 11\%$ means that on average, over all data, one expects that an asset gives a return of 11%

# Basic model for asset returns III

- Example 2: $E[R_{t+1}] = 11\%$ is not inconsistent with $E_t[R_{t+1}] = -6\%$ if news are bad today, e.g., after a bank has defaulted on its obligations
- Example 3: One good reason for the conditional mean to move over time is that $E_t[R_{t+1}] = \alpha + \beta X_t + \epsilon_{t+1}$, which is a predictive regression
- Example 4: This applies also to variances, i..e, there is a difference between $Var[R_{t+1}] \equiv \sigma^2$ and $Var_t[R_{t+1}] \equiv \sigma^2_{t+1}$
- Example 5: Therefore the **unconditional density** of a time series represents long-run average frequencies in one observed sample
- Example 6: The **conditional density** describes the expected frequencies (probabilities) of the data based on currently available info

# Basic model for asset returns IV

- When a series (or a vector of series) is identically and independently (i.i..d. or IID) distributed over time, then the conditional objects collapse into being unconditional ones

- When they are different, conditional distribution is relevant for issues involving **predictability and asset pricing**.
- **Asset pricing** tries to understand the prices of claims with uncertain payments.