# Econometría Financiera

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# 3. Measuring financial Risk

# Motivation I

Value-at-Risk (VaR) and Expected Shortfall (ES) play an essential role in economics, finance, and insurance. They are lying at the core of the regulatory framework settled by the Solvency II regulation, the Swiss Solvency Test, and the Basel II and III accords, all intended to strengthen the financial stability of the banking system.

Beyond this, VaR and ES constitute a basic tool for investors, hedge fund managers, traders, and all decision-makers involved in risk management, portfolio allocation, trading desk limits, investment strategies, and the development of trading algorithms.

## A. Introduction:

Why measure and manage risk?

- The risk-return tradeoff is at the core of any business and financial decision
- Modern companies and financial institutions are large enterprises with many (thousands?) of individuals making decisions in an uncertain environment
- How can we measure, aggregate, and manage (= control) the overall risk exposure of an organization?
- What type of risks are we trying to measure and control?

# Introduction II

## Types of financial risk

- **Market risk**: losses due to movements in the level and/or volatility of asset prices
  - *directional*: exposure to movements of the market
  - *non-directional*: exposure to changes of volatility
- **Credit risk**: losses arising from defaults on bonds and loans; changes to credit ratings or market perception of default (sub-prime, mortgage-backed securities etc)
- **Liquidity risk**:
  - *Asset liquidity risk*: arises when transactions cannot be executed at prevailing market prices due to size of the position or thinness of the market
  - *Funding liquidity risk*: inability to meet margin calls on leveraged portfolios (LTCM)
- **Operational Risk**: due to human or technical mistakes, employees' fraud, inadequacy of risk management procedures,inadequacy of pricing models (model risk)

# Introduction III

## Derivatives and Risk Management

- Many of the financial disasters are due to the use of derivatives and leverage
- Derivatives are used to hedge but also speculate (directional bets) while leverage is used to boost profits from limited capital
- The problem is not so much the use of derivatives and/or leverage, rather the *uncontrolled* risk-taking allowed by these positions
- Good risk management practice means *measuring and managing* these risk (not *eliminating* it!)

# Introduction IV

## Risk management policies and regulation

- The increasingly frequent financial disasters has called the attention of the profession and regulators on the need for a more stringent risk management practice
- Professional initiatives:
  - Derivatives Policy Group
    - management controls
    - enhanced reporting
    - evaluation of risk versus capital
    - measure market risk via **Value at Risk** (**VaR**) at 99% level and 2-week horizon
  - RiskMetrics from J.P. Morgan
    - established a framework for **VaR** calculation
- Regulators:
  - Basel Accord on Banking Supervision (1988 and later amendments)

# Introduction V

## Basel Accord I and II and III and ...

- The Accords set rules on bank **capital adequacy** as a way to monitor/control the amount of risk-taking
- Banks are allowed to use their internal models to determine the capital charge
- They should demonstrate to have an appropriate risk management system, conduct regular stress testing, having independent risk-control unit and external audits
- Each bank is subject to the **Market Risk Charge (MRC)** which is calculated as:

$$\text{MRC} = \text{k} * \max\left[\text{yesterday's } \mathtt{VaR},\ \text{60-days average } \mathtt{VaR}\right]$$

where **VaR** is defined as **maximum loss that we expect at a certain confidence level and target horizon**

# Introduction VI

- The calculation of MRC is based on the following parameters:
  - **Horizon**: 10 trading days (2 weeks); allowed to scale up the 1-day **VaR** using $\sqrt{10}$
  - **Confidence level**: 99%
  - **Estimation**: at least 1 year of data and update of estimates at least once a quarter
  - **Correlation**: can be accounted among and across broad categories (bonds, equities, commodities, etc.)
  - **Multiplicative Factor k**: at least 3
  - **Plus Factor**: **k** can be increased by regulators to values higher than 3 if backtesting indicates inadequacy of the internal risk model

# VaR I

B. Value at Risk:

What is Value at Risk (VaR)?

# VaR II

- Denote the profit/loss of a financial institution in day $t+1$ by $R_{t+1} = 100 * ln(W_{t+1}/W_t)$, where $W_{t+1}$ is the portfolio value in day $t+1$
- We define **Value-at-Risk** (`VaR`) as the *maximum loss that it is expect at a certain confidence level and target horizon*; using a 99% confidence level and 1-day horizon `VaR` for day $t+1$, $VaR_{t+1}$, is defined as

$$P(R_{t+1} > VaR_{t+1}^{0.99}) = 0.99$$

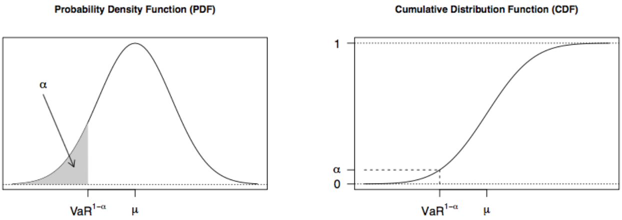
- `VaR` can also be expressed as the minimum loss that we expect at a certain confidence level and target horizon, that is,

$$P(R_{t+1} \leq VaR_{t+1}^{0.99}) = 0.01$$

  In statistical terms, `VaR` is the 0.01 quantile of the probability distribution of $R_{t+1}$
- For a portfolio value of $W_t$ the $`VaR` is calculated as $\$VaR_{t+1}^{0.99} = W_t * (exp(|VaR_{t+1}^{0.99}|/100) - 1)$

# VaR III

- **VaR** at $100 * (1 - \alpha)\%$ represents the $100 * \alpha\%$ quantile of the profit/loss distribution



# VaR IV

## B1. VaR under normality

- Assume that $R_{t+1} \sim N(\mu, \sigma^2)$ with constant mean and variance
- Based on this assumption the $P(R_{t+1} \leq VaR^{1-\alpha}) = \alpha$, with $1 - \alpha = 0.99$ is

$$VaR_{t+1}^{0.99} = \mu - 2.33 * \sigma$$

  and, more generally,

$$VaR_{t+1}^{1-\alpha} = \mu + z_\alpha * \sigma$$

  where $z_\alpha$ represents the quantile from the standard normal distribution at level $\alpha$

- Example: we hold a portfolio that replicates the S&P 500 Index and we obtain historical data for the Index from Jan 03, 1990, to May 08, 2017. R implementation:

```
mu    = mean(sp500daily)
sigma = sd(sp500daily)
var   = mu + qnorm(0.01) * sigma ## qnorm(0.01) = -2.33
```

```
[1] -2.569
```

# VaR V

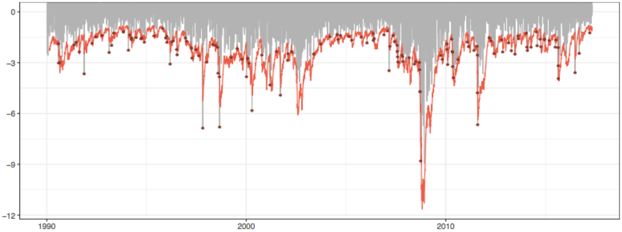
## B2. Time-varying VaR

- The assumption that $R_{t+1} \sim N(\mu, \sigma^2)$ restricts the mean and the variance of the portfolio return to be constant over time
- This is an unrealistic assumption given the evidence that daily returns show significant variation of volatility over time
- We can thus assume that $R_{t+1} \sim N(\mu_{t+1}, \sigma^2_{t+1})$ where:
  - *Conditional mean* $\mu_{t+1}$: an AR(1) process, i.e., $\mu_{t+1} = \phi_0 + \phi_1 R_t$
  - *Conditional variance* $\sigma_{t+1}$: MA, EMA, GARCH(1,1), or GJR-GARCH(1,1)
- *Value-at-Risk* is then calculated as: $VaR^{0.99}_{t+1} = \mu_{t+1} - 2.33\sigma_{t+1}$

# VaR VI

- Model: $R_{t+1} = \mu + \sigma_{t+1}\epsilon_{t+1}$
  - $\sigma_{t+1}$ estimated by EMA(0.06)
  - $\epsilon_{t+1} \sim N(0, 1)$
- For 1.074% of the 6891 days the return was smaller relative to **VaR**

```
library(TTR)
mu       <- mean(sp500daily)
sigmaEMA <- EMA((sp500daily-mu)^2, ratio=0.06)^0.5
var      <- mu + qnorm(0.01) * sigmaEMA
```



# VaR VII

## B3. The $\sqrt{K}$ rule

- So far we discussed how to calculate **VaR** at the 1-day horizon but regulators ask financial institutions to report their potential losses over a 10 day horizon
- Basel rules allow banks to scale-up the one-day risk measures by $\sqrt{10}$
- Where does this $\sqrt{10}$ rule come from?
- The return of holding the portfolio in the following $K$ days is

$$R_{t+1:t+k} = \sum_{k=1}^{K} R_{t+k} = R_{t+1} + \cdots + R_{t+K}$$

# VaR VIII

- Assuming that the daily returns are *independent* and *identically distributed* (*i.i.d.*) with mean $\mu$ and variance $\sigma^2$, then we have that the expected value of the multi-period return is

$$E\left(\sum_{k=1}^{K} R_{t+k}\right) = \sum_{k=1}^{K} \mu = K\mu$$

and its variance is

$$Var\left(\sum_{k=1}^{K} R_{t+k}\right) = \sum_{k=1}^{K} \sigma^2 = K\sigma^2$$

- So that the standard deviation of the $R_{t+1:t+k}$ is equal to $\sqrt{K}\sigma$
- Expected return for unit of risk: $E(R_{t+1:t+K})/\sqrt{Var(R_{t+1:t+K})} = \frac{K\mu}{\sqrt{K}\sigma} = \sqrt{K}\frac{\mu}{\sigma}$
- *VaR* at 1% of the multi-period return $R_{t+1:t+K}$ is given by

$$VaR^{1-\alpha}_{t+1:t+K} = K\mu - 2.33\sqrt{K}\sigma$$

# VaR IX

- The $\sqrt{10}$ rule relies on the assumption that daily returns are independent over time
- What happens if this assumption fails to hold?
- Assume that $K = 2$ so that $R_{t+1:t+2} = R_{t+1} + R_{t+2}$
- the variance of the multiperiod return $Var(R_{t+1} + R_{t+2})$ is equal to

$$Var(R_{t+1}) + Var(R_{t+2}) + 2Cov(R_{t+1}, R_{t+2})$$

$$= \sigma^2 + \sigma^2 + 2\rho\sigma^2 = 2\sigma^2(1 + \rho)$$

- If $\rho > 0$ the variance (and thus $VaR$) of the multiperiod return is larger relative to the case with no correlation since $2\sigma^2(1 + \rho) > 2\sigma^2$
- Bottom line: assuming independence we under-estimate the risk in our portfolio if returns are actually positively correlated

# VaR X

## B4. VaR under non-normality

- *Fat tails* is the characteristic of daily financial returns to have events 3 (or larger) standard deviations away from the mean more often relative to the normal distribution
- Is time-varying volatility a possible explanation for the *non-normality* in the data?
- This could be the case if returns are switching between two regimes of high and low volatility
- The sample skewness of the *standardized residuals* is **-0.431** and the sample excess kurtosis is **2.261**
- In this case, there is still some skewness and, more generally, there might still be some non-normality in the standardized residuals
- We can depart from the assumption of normality in two ways:
  - *Cornish-Fisher* approximation
  - $t$ distribution

# VaR XI

## B4.1 Cornish-Fisher approximation

- The **Cornish-Fisher** (**CF**) approximation consists of a Taylor expansion of the normal density which has the effect of making the approximate density a function of skewness and kurtosis
- **VaR** is calculated by $VaR_{t+1}^{1-\alpha} = z_\alpha^{CF}\sigma_{t+1}$ where the value $z_\alpha^{CF}$ is given by:

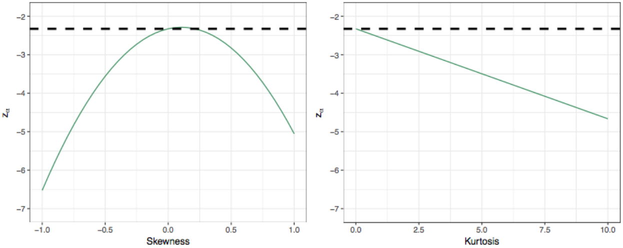
$$z_\alpha^{CF} = z_\alpha + \frac{SK}{6}\left((z_\alpha)^2 - 1\right) + \frac{EK}{24}\left((z_\alpha)^3 - 3z_\alpha\right) + \frac{SK^2}{36}\left(2(z_\alpha)^5 - 5z_\alpha\right)$$

  where $SK$ and $EK$ represent the skewness and excess kurtosis, respectively, and $z_\alpha$ represents the $\alpha$ quantile
- If $SK = EK = 0$ then $z_\alpha^{CF} = z_\alpha = -2.33$ (normal)
- For $\alpha = 0.01$ we have that $z_\alpha = -2.33$ and $z_\alpha^{CF}$ is then equal to:

$$z_\alpha^{CF} = -2.33 + 0.7 * SK - 0.23 * EK - 3.46 * SK^2$$

# VaR XII

- Effect of changing skewness and kurtosis on $z_\alpha^{CF}$

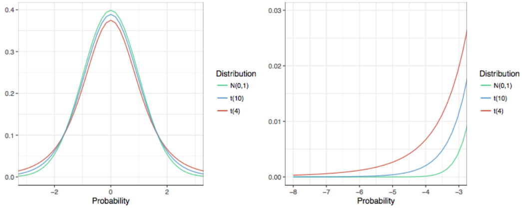


# VaR XIII

## B4.2 *t*-distribution (Fat tails)

- An alternative approach is to discard the normal distribution in favor of a distribution with fat tails such as the $t$ distribution with $d$ degrees-of-freedom
- If we assume that $\epsilon_{t+1} \sim t_d$ in $R_{t+1} = \sigma_{t+1}\epsilon_{t+1}$ we can estimate the volatility parameters and $d$ by ML

```
q1 <- ggplot(data.frame(x = c(-8, 8)), aes(x = x)) +
       stat_function(fun = dnorm, aes(colour = "N(0,1)")) +
       stat_function(fun = dt, args = list(df=4),  aes(colour = "t(4)")) +
       stat_function(fun = dt, args = list(df=10),  aes(colour = "t(10)")) +
       theme_bw() + labs(x = "Probability", y= NULL) + coord_cartesian(x=c(-3,3)) +
       scale_colour_manual("Distribution", values = c("seagreen3", "steelblue3", "tomato3"))
q2 <- q1 + coord_cartesian(x=c(-8, -3), y=c(0,0.03))
```



# VaR XIV

- A simple way to obtain a rough estimate of the degrees of freedom is $d = 6/EK + 4$, where $EK$ is the excess kurtosis of the returns
- This rule is based on the fact that the $t_d$ distribution (for $d > 4$) has excess kurtosis equal to $6/(d-4)$ (and it is infinite for $2 < d < 4$)
- The standardized returns have sample excess kurtosis of 2.26 so that the estimate of $d$ is equal to 6.65 which can be rounded to 7
- This value confirms the evidence that the returns standardized by the volatility forecast still deviate from normality, in particular on the left tail

When assumption of Student's t-distribution with $\nu$ parameter is introduced VaR can be calculated as:

$$VaR_{t+1}^{1-\alpha} = \mu + t_{\alpha}^{\nu}\sqrt{\frac{\nu-2}{\nu}}\sigma$$

where $t_{\alpha}^{\nu}$ is critical value of t-distribution depending on given probability and estimated degrees of freedom, while $\sqrt{\frac{\nu-2}{\nu}}$ is correction factor for unbiased standard deviation estimation from sample.

# VaR XV

## B5. Portfolio VaR

- In practice, the portfolio of large financial institutions is composed of several assets and its return can be express as

$$R_{p,t} = \sum_{j=1}^{J} w_{j,t} R_{j,t}$$

where:

  - $R_{p,t}$ represents the portfolio return in day $t$
  - $w_{j,t}$ is the weight of asset $j$ in day $t$ (and there is a total of $J$ assets)
  - $R_{j,t}$ is the return of asset $j$ in day $t$

- Let's assume that the bank holds only 2 assets (i.e., $J = 2$)

- The expected portfolio return is given by: $E(R_{p,t}) = \mu_{p,t} = w_{1,t}\mu_1 + w_{2,t}\mu_2$

- The portfolio variance is given by:
$Var(R_{p,t}) = \sigma^2_{p,t} = w^2_{1,t}\sigma^2_1 + w^2_{2,t}\sigma^2_2 + 2w_{1,t}w_{2,t}\rho_{12}\sigma_1\sigma_2$
which is a function of the individual variances and the correlation between the two assets, $\rho_{12}$

# VaR XVI

- The portfolio Value-at-Risk is then given by

$$\begin{aligned} VaR_{p,t}^{1-\alpha} &= \mu_{p,t} + z_\alpha \sigma_{p,t} \\ &= w_{1,t}\mu_1 + w_{2,t}\mu_2 + z_\alpha \sqrt{w_{1,t}^2\sigma_1^2 + w_{2,t}^2\sigma_2^2 + 2w_{1,t}w_{2,t}\rho_{12}\sigma_1\sigma_2} \end{aligned}$$

- If we assume that $\mu_1 = \mu_2 = 0$, the $VaR_{p,t}^{1-\alpha}$ formula can be expressed as follows:

$$\begin{aligned} VaR_{p,t}^{1-\alpha} &= +z_\alpha \sqrt{w_{1,t}^2\sigma_1^2 + w_{2,t}^2\sigma_2^2 + 2w_{1,t}w_{2,t}\rho_{12}\sigma_1\sigma_2} \\ &= -\sqrt{z_\alpha^2 w_{1,t}^2\sigma_1^2 + z_\alpha^2 w_{2,t}^2\sigma_2^2 + 2 * z_\alpha^2 w_{1,t}w_{2,t}\rho_{12}\sigma_1\sigma_2} \\ &= -\sqrt{(VaR_{1,t}^{1-\alpha})^2 + (VaR_{2,t}^{1-\alpha})^2 + 2 * \rho_{12}\, VaR_{1,t}^{1-\alpha}\, VaR_{2,t}^{1-\alpha}} \end{aligned}$$

and:

1. if $\rho_{12} = 1$: $VaR_{1,t}^{1-\alpha} + VaR_{2,t}^{1-\alpha}$
2. if $\rho_{12} = -1$: $-|VaR_{1,t}^{1-\alpha} - VaR_{2,t}^{1-\alpha}|$
3. if $-1 < \rho_{12} < 1$: $VaR_{1,t}^{1-\alpha} + VaR_{2,t}^{1-\alpha} < VaR_{p,t}^{1-\alpha} < -|VaR_{1,t}^{1-\alpha} - VaR_{2,t}^{1-\alpha}|$

# VaR XVII

- If we assume that the mean and the variance vary over time then $VaR_{p,t}^{1-\alpha}$ is:

$$VaR_{p,t}^{1-\alpha} = w_{1,t}\mu_{1,t} + w_{2,t}\mu_{2,t} + z_\alpha \sqrt{w_{1,t}^2\sigma_{1,t}^2 + w_{2,t}^2\sigma_{2,t}^2 + 2w_{1,t}w_{2,t}\rho_{12,t}\sigma_{1,t}\sigma_{2,t}}$$

  where:

  - $\mu_{1,t} = \beta_0 + \beta_1 R_{1,t-1}$ and $\mu_{2,t} = \beta_0 + \beta_1 R_{2,t-1}$
  - $\sigma_{1,t}^2$, $\sigma_{2,t}^2$ MA, EMA, and a GARCH
  - $\rho_{12,t}$ ??

# VaR XVIII

## Modeling correlations

- A simple approach to modeling correlations consists of using MA and EMA smoothing as in the case of modeling volatility
- Denote the returns of asset 1 by $R_{1,t}$ and of asset 2 by $R_{2,t}$
- The `MA(M)` estimate of the covariance of the two assets is:

$$\sigma_{12,t+1} = \frac{1}{M} \sum_{m=1}^{M} R_{1,t-m+1} R_{2,t-m+1}$$

- The correlation is then given by: $\rho_{12,t+1} = \sigma_{12,t+1} / \left( \sigma_{1,t+1} * \sigma_{2,t+1} \right)$
- If the portfolio is composed of $J$ assets there are $J * (J-1)/2$ correlations to estimate
- An alternative approach is to use EMA smoothing which can be implemented using the recursive formula discussed earlier:

$$\sigma_{12,t+1} = (1-\lambda)\sigma_{12,t} + \lambda R_{1,t} R_{2,t}$$

  and the correlation can be obtained as earlier by dividing with the standard deviation forecasts.

## Example in R

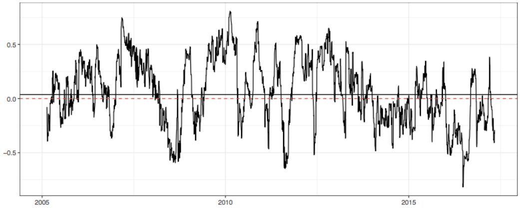
# VaR XIX

- We hold a portfolio that is invested for a fraction $w_1$ in a gold ETF (ticker: GLD) and the remaining fraction $1 - w_1$ in the S&P 500 ETF (ticker: SPY)
- The data are from January 02, 1970, January 02, 1970 until January 03, 1970, January 02, 1970 at the daily frequency
- I will forecast volatility and correlation using the EMA approach with $\lambda = 0.94$

```
data      <- getSymbols(c("GLD", "SPY"), from="2005-01-01")
R         <- 100 * merge(ClCl(GLD), ClCl(SPY))
names(R) <- c("GLD","SPY")
# EMA for the product of returns
prod          <- R[,1] * R[,2]
cov           <- EMA(prod, ratio=0.06)
# EMA for the squared returns
sigma         <- do.call(merge, lapply(R^2, FUN = function(x) EMA(x, ratio=0.06)))^0.5
names(sigma) <- names(R)
# correlation given by covariance divided by product of the st. dev.
corr          <- cov / (sigma[,1] * sigma[,2])
```

# VaR XX

```
autoplot(corr) + geom_hline(yintercept=0, color="tomato3", linetype="dashed") +
  geom_hline(yintercept = as.numeric(cor(R[,1], R[,2], use="pairwise.complete"), color="seagreen4", linety
  theme_bw() + labs(x=NULL, y=NULL)
```



# VaR XXI

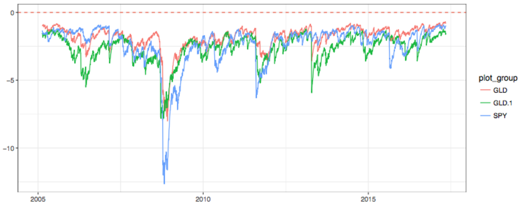
- VaR for a portfolio investing $w_1$ in GLD and $1 - w_1$ in SPY

```
w1  = 0.5     # weight of asset 1
w2  = 1 - w1  # weight of asset 2
VaR = -2.33 * ( (w1*sigma[,1])^2 + (w2*sigma[,2])^2 +
                  2*w1*w2*corr*sigma[,1]*sigma[,2] )^0.5
autoplot(VaR) + geom_hline(yintercept=0, color="tomato3", linetype="dashed") +
  theme_bw() + labs(x=NULL, y=NULL)
```

# VaR XXII

- A comparison of portfolio **VaR** with **VaR** for a portfolio that is fully invested in **GLD** or **SPY**

```
VaRGLD = -2.33 * sigma[,1]
VaRSPY = -2.33 * sigma[,2]
mydata <- merge(VaR, VaRSPY, VaRGLD)
autoplot(mydata, facets=FALSE) + theme_bw() +
  geom_hline(yintercept=0, color="tomato2", linetype="dashed")
```



# ES I

## C. Expected Shortfall (or Conditional VaR):

- **VaR** represents the **maximum (minimum) loss that is expected with 99% (1%) probability**
- However, it does not provide a measure of how large the losses are likely to be in case an extreme event happens
- An alternative risk measure is **Expected Shortfall** (**ES**) that is defined as

$$ES_{t+1}^{1-\alpha} = E(R_{t+1} | R_{t+1} \leq VaR_{t+1}^{1-\alpha})$$

and represents the expected return conditional on the return being smaller than 100*$(1-\alpha)$% **VaR**

# ES II

- If we assume that returns are normally distributed, **ES** is given by

$$ES_{t+1}^{1-\alpha} = -\sigma_{t+1} \frac{\phi\left[VaR_{t+1}^{1-\alpha}/\sigma_{t+1}\right]}{\alpha} = -\sigma_{t+1} \frac{\phi\left[z_\alpha\right]}{\alpha}$$

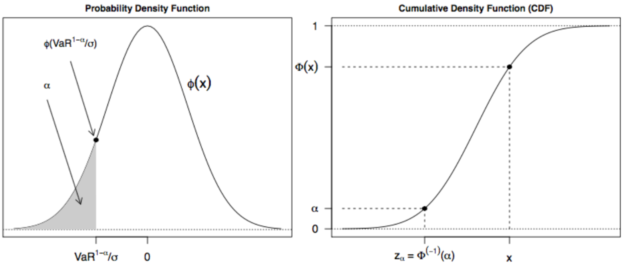
  where $\phi(\cdot)$ represents the PDF of the standard normal distribution

- If we are using $1-\alpha = 0.99$ then we have that (for $\mu = 0$):
  - $VaR_{t+1}^{0.99} = -2.33\sigma_{t+1}$
  - $\Phi^{-1}(VaR_{t+1}^{0.99}/\sigma_{t+1}) = \phi(-2.33) = 0.026$
  - $ES_{t+1}^{0.99} = -\sigma_{t+1} \frac{\phi(-2.33)}{0.01} = -\ 2.643\sigma_{t+1}$

- Given the same level $\alpha$, **ES** provides a more conservative risk measure relative to **VaR**

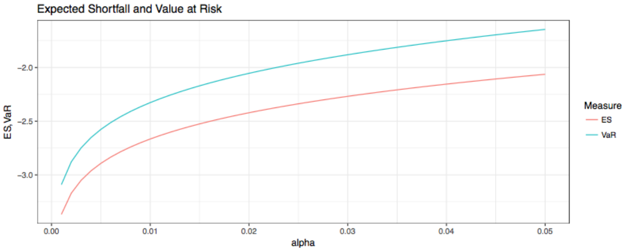
# ES III

- Density and Distribution functions for the standard normal distribution $x \sim N(0,1)$
  - $\Phi(-2.33) =$
  - $\Phi(2.33) =$
  - $\Phi^{-1}(0.01) =$
  - $\Phi^{-1}(0.99) =$



# ES IV

```
sigma = 1
alpha = seq(0.001, 0.05, by=0.001)
ES    =  - dnorm(qnorm(alpha)) / alpha * sigma
VaR   = qnorm(alpha) * sigma
```



# Value at Risk vs Expected Shortfall

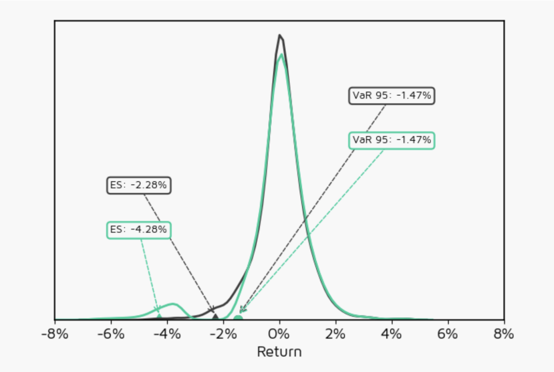
There are four properties required by any risk measure:

- *Monotonicity:* if a portfolio achieves higher returns than another in every state of the world, then it will have lower risk.
- *Invariance:* if an amount of cash is added to our portfolio, the risk will be reduced by that amount.
- *Homogeneity:* maintaining the weights, if the size of a portfolio is increased by a factor, the risk will be multiplied by the same factor.
- *Subadditivity:* the risk measure of two merged portfolios should be lower than the sum of their risk measures individually.

The Value at Risk measure always satisfies the first three properties but it will only satisfy the fourth one if portfolio returns follow a normal distribution. On the other hand, the Expected Shortfall measure satisfies the four properties in any circumstance.

# Value at Risk vs Expected Shortfall

We have two assets and we have to decide which is the riskiest investment.




To that end, we calculate the VaR with 5 % of confidence level (VaR 95) and the Expected Shortfall with the same confidence level (5 %). Using this measure, we conclude the turquoise distribution is riskier than the black one.

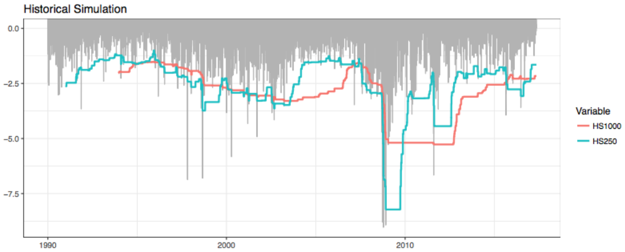
# Historical simulation I

## D.1. Historical Simulation:

- **Historical Simulation** (HS) is an alternative approach that calculates **VaR** at $\alpha$ level as the $1-\alpha$ quantile of the most recent $M$ days
- HS does not make any assumptions about the mean/volatility model and/or the error distribution
- In this sense, it can be considered a *non*-parametric approach because of the lack of parametric assumptions
- A typical value for $M$ is 250 days (one trading year)

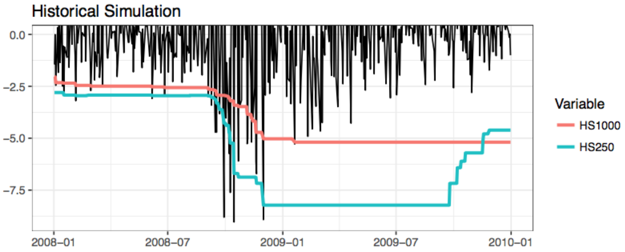
# Historical simulation II

```
M1 = 250
M2 = 1000
alpha = 0.01
hs1 <- rollapply(sp500daily, M1, quantile, probs=alpha, align="right")
hs2 <- rollapply(sp500daily, M2, quantile, probs=alpha, align="right")
```



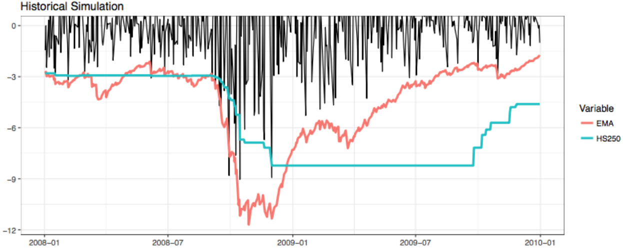
# Historical simulation III

- Zoom on the financial crisis period 2008-2009



# Historical simulation IV

- Comparison of **VaR** based on **EMA** (with normal errors) and **HS** during the financial crisis
- **EMA** is clearly faster in adapting to changes in volatility
- Percentage of violations from January 03, 1990 to May 08, 2017:
  - **HS**: 1.3061%
  - **EMA**: 1.0013%



# Historical simulation V

- Pros of HS:
  - No model required for the volatility dynamics and error distribution
  - Easy and fast to calculate, no estimation required
- Cons of HS:
  - Choice of the estimation window `M`
  - Transforming the 1-day `VaR` to 10-day `VaR` (practice: multiply by $\sqrt{10}$)