



# 15.415x Foundations of Modern Finance

Leonid Kogan and Jiang Wang  
MIT Sloan School of Management

## **Lecture 5: Stock Valuation**

## Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Introduction to common stocks

- **Common stock** represents equity or an ownership position in a corporation.
- Payments to common stock are in the form of
  - dividends;
  - share repurchases.
- Contrary to payments to bondholders, payments to stockholders are uncertain in both magnitude and timing.

## Basic characteristics of common stocks

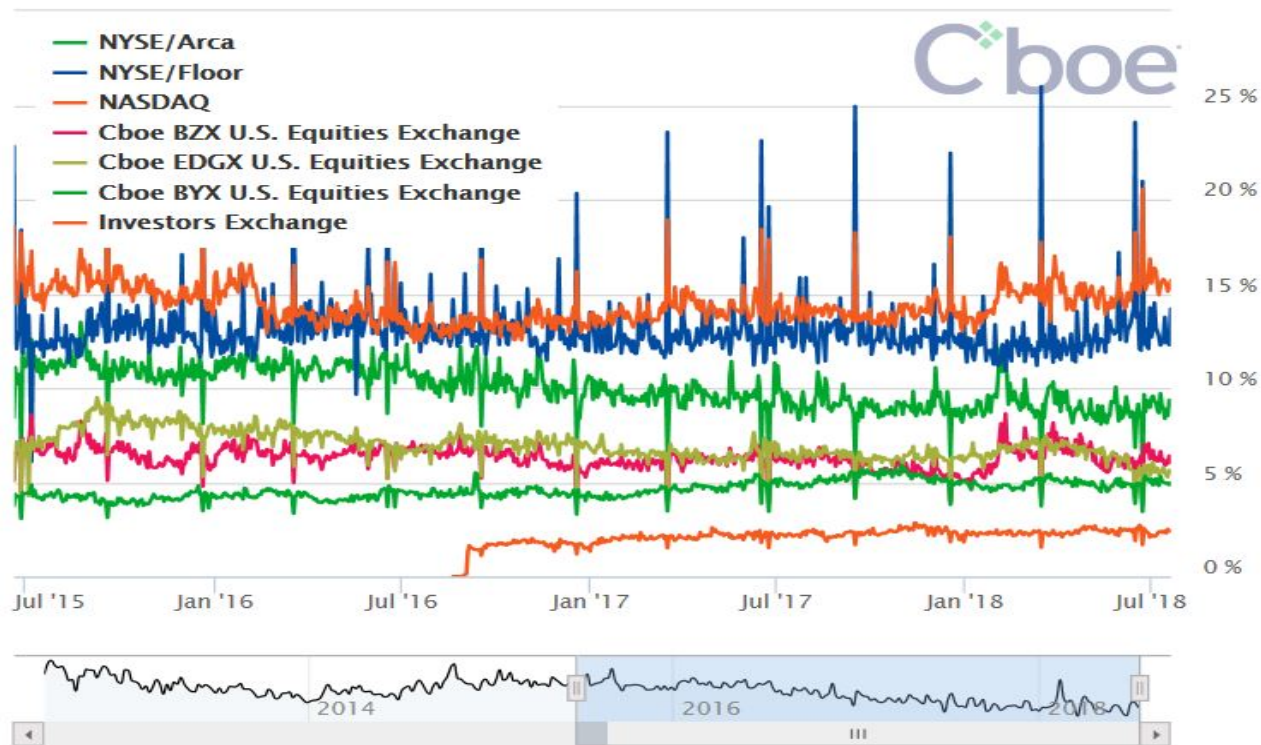
- Residual claim — stockholders have claim to firm's cash flows/assets after all obligations to creditors are met.
- Limited liability — stockholders may lose their investments, but no more.
- Voting rights — stockholders are entitled to vote on major corporate matters, e.g., elections for the board of directors; mergers and acquisitions; executive compensation, etc.

## Primary market: underwriting

- Venture capital: A company issues shares to investment partnerships, investment institutions and wealthy individuals.
- Initial public offering (IPO): A company issues shares to general public for the first time (i.e., going public).
- Secondary (seasoned) offerings (SEO): A public company issues additional shares.
- Stock issuing to the public is usually organized by investment banks who act as underwriters.

## Secondary market (resale market)

- Exchanges: NYSE, CBOE, NASDAQ.
- Alternative Trading Systems: Electronic Communication Networks, dark pools.

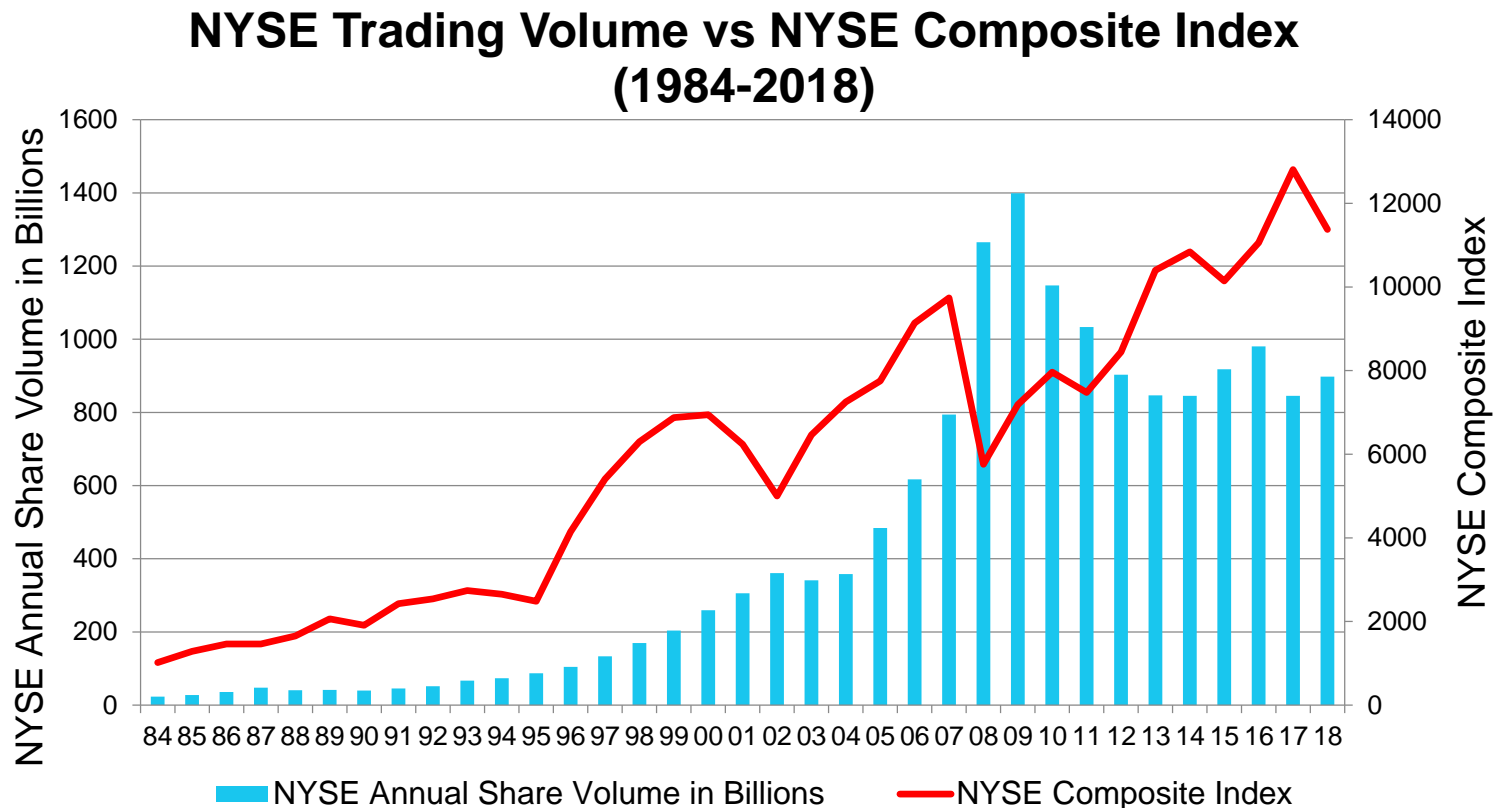


Data is believed reliable but not guaranteed.

Data is based on UTDf and CTS consolidated data feeds. Data displayed represents volume of shares matched.

## Secondary market: volume

- Trading volume trends up with the size of the markets and the economy.
- Over the last two decades, trading activity has been rising sharply due to developments in electronic trading technologies.





# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Stock price and fundamentals

Company Name	Price	Earnings Per Share	P/E ratio	Price-to-Sales Ratio	Market Cap(\$B)	Dividend Yield(%)
JPMorgan Chase & Co.	97.62	9.04	10.80	2.43	319.78	3.16
Bank of America	24.64	2.64	9.33	2.16	238.25	2.89
Wells Fargo & Co	46.08	4.31	10.69	2.09	211.10	4.41
Morgan Stanley	39.65	4.81	8.24	1.34	67.40	3.52
U.S. Bancorp	45.70	4.15	11.01	2.85	73.50	3.22
PNC Financial	116.91	10.79	10.84	2.67	53.43	3.44
Truist Financial	43.32	3.96	10.94	2.54	33.07	4.17
State Street	63.07	6.46	9.76	1.83	23.96	3.46

The table (gathered from COMPUSTAT) shows information on the stocks of several companies in the financial industry (end of the 4<sup>th</sup> quarter, 2018).

## Stock price and fundamentals

Company Name	Price	Earnings Per Share	P/E ratio	Price-to-Sales Ratio	Market Cap(\$B)	Dividend Yield(%)
Toyota Motor Corp	116.08	12.76	9.10	0.81	165.24	3.56
Volkswagen	15.90	2.70	5.89	0.30	79.71	3.41
Daimler AG	52.77	7.76	6.80	0.29	56.46	7.92
Bayer Motoren Werke AG	61.71	9.49	6.50	0.44	37.15	6.18
Honda Motor Co.	26.45	4.08	6.48	0.44	47.04	2.81
Ford	7.65	0.93	8.23	0.19	30.44	9.54
Nissan Motor Co.	16.05	2.46	6.52	0.40	31.39	6.25
Fiat Chrysler Auto	14.46	2.46	5.88	0.18	22.42	0.00

The table (gathered from COMPUSTAT) shows information on the stocks of several companies in the automobile industry (end of the 4<sup>th</sup> quarter, 2018).

## Discounted cash flow model

- Basic PV formula applies to the valuation of stocks. Need to know
  - Expected future dividends;
  - Discount rates for dividends.
  
- Notation:
  - $P_t$ : expected stock price at  $t$  (ex-dividend);
  - $D_t$ : expected cash dividend at  $t$ ;
  - $r_t$ : expected stock return over period  $t$ .

## Discounted cash flow model

- Start with the definition of the expected stock return – an accounting identity:

$$1 + r_1 = \frac{D_1 + P_1}{P_0}$$

- Express the initial stock price as the discounted value of  $D_1$  and  $P_1$ :

$$P_0 = \frac{D_1 + P_1}{1 + r_1}$$

- Conclusion: discount factor = expected return.

## Discounted cash flow model

$$P_0 = \frac{D_1 + P_1}{1 + r_1}$$

- Iterate forward, substitute out  $P_1$ :

$$P_0 = \frac{D_1}{1 + r_1} + \frac{P_2 + D_2}{(1 + r_1)(1 + r_2)}$$

- Simplification:  $r_t = r$ .
- Continue to obtain a general discounted cash flow formula

$$P_0 = \sum_{t=1}^T \frac{D_t}{(1 + r)^t} + \frac{P_T}{(1 + r)^T}$$

## Discounted cash flow model

$$P_0 = \sum_{t=1}^T \frac{D_t}{(1+r)^t} + \frac{P_T}{(1+r)^T}$$

- The price of the stock is a sum of
  - The discounted value of future dividends,  $t = 1, \dots, T$ ; and
  - The discounted value of the terminal price,  $P_T$ .
- The discount factor equals the expected stock return.
- If we assume that  $\lim_{T \rightarrow \infty} \frac{P_T}{(1+r)^T} = 0$ , then

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses



## Constant Growth Model

- **The Gordon Model.** Two key assumptions:

- Constant discount rate  $r$ ;
- Dividends are expected to grow at a constant rate  $g < r$  in perpetuity:

$$D_{t+1} = (1 + g)D_t$$

- Then, using the DCF formula,

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1 + r)^t} = \frac{D_1}{r - g}$$

## Example

- The current dividend is \$1 per share:  $D_0 = 1$ .
- Dividends are expected to grow at 6% per year:  $g = 0.06$ .
- The expected rate of return is 20%:  $r = 0.20$ .
- Then, expected dividend next year is

$$D_1 = (1 + g)D_0 = \$1.06$$

- The current stock price should be

$$P_0 = \frac{D_1}{r - g} = \frac{\$1.06}{0.20 - 0.06} = \$7.57$$

## Using market information

- Gordon model relates the valuation ratio,  $P_0/D_1$ , to expected returns and expected dividend growth

$$P_0 = \frac{D_1}{r - g}$$

- In applications, we may be interested in using market prices to infer market forecasts of future dividend growth, or expected future stock returns.

## Example: forecasting returns

- Northern Co. is a utility firm. It is expected to grow its dividends at a steady rate in the future.
- The current dividend/price ratio (dividend yield) is  $D_0/P_0 = 0.04$ .
- Analysts forecast long-run growth at the rate of 5%.
- From the Gordon model,

$$r = \frac{D_1}{P_0} + g = (1 + g) \frac{D_0}{P_0} + g$$

- Expected return combines dividend yield and expected dividend growth.
- Plug in problem data:

$$r = 1.05 \times 0.04 + 0.05 = 0.092$$

Expected stock return = Dividend yield + Dividend growth

## Example: forecasting firm growth

- Kendall Square Cuisine (KSC) is a new health-food franchise.
- It has 10 restaurants as of now.
- Each restaurant is expected to generate a profit (paid out in dividends) of \$1M every year, forever.
- KSC is expected to grow its market share going forward by opening new restaurants.
- Each new restaurant is also expected to generate \$1M profit every year, forever.
  - New restaurants start generating profits a year after opening.
- KSC's total market cap is now \$200M, and its discount rate is 10% annually.
- At what rate does the market expect KSC to grow its number of restaurants?

## Example: forecasting firm growth

- For KSC, this the Gordon model implies

$$g = r - \frac{D_1}{P_0} = 0.10 - \frac{\$10M}{\$200M} = 0.05$$

- For the company to grow its profits at 5% per year, it must grow its restaurant count at the expected rate of 5%.

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Multi-stage growth

Firms evolve through different stages in their lifecycles. For example,

1. Growth stage — rapidly expanding sales, high profit margins, and abnormally high growth in earnings per share, many new investment opportunities, low dividend payout ratio.
2. Transition stage — growth rate and profit margin reduced by competition, fewer new investment opportunities, high payout ratio.
3. Maturity stage — earnings growth, payout ratio and average return on equity stabilizes for the remaining life of the firm.



## Example

- Northern Co. expects to pay a dividend of \$1 per share next year, which will grow at 10% for the following 5 years.
- Afterwards, the growth becomes 2%.
- The cost of capital for Northern Co. is 8%.
- What is today's share price of Northern Co.?

## Example

- Dividends grow at 10%, then at 2%

Time	0	1	2	3	4	5	6	7
Dividend		1.00	1.10	1.21	1.33	1.46	1.61	1.64

- Compute stock price at  $t = 6$ :

$$P_6 = \frac{D_7}{r - g} = \frac{1.64}{0.08 - 0.02} = 27.38$$

- Compute time-0 price

$$P_0 = \sum_{t=1}^6 \frac{D_t}{(1 + 0.08)^t} + \frac{P_6}{(1 + 0.08)^6} = 23.07$$

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- **Forecasting Dividends**
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Forecasting dividends

- Forecast of dividends involves many practical factors.
- Relate dividends to firm profits.
- Assume all equity firm for simplicity – no interest payments.
- Terminology:
  - **Earnings (E or EPS)**: total profit net of depreciation and taxes (assuming all equity firm – no interest payments)
  - **Payout ratio**:  $\text{dividend/earnings} = \text{DPS/EPS} = p$
- Express dividends as

$$DPS = p \times EPS$$

## Forecasting earnings

- **Retained earnings:** (earnings – dividends).
- **Plowback ratio:** retained earnings/total earnings =  $b = 1 - p$ .

$$BVPS_t = BVPS_{t-1} + EPS_t \times b$$

- Firm's investments generate incremental cash flows in the future.
- We assume that investment at time  $t$  generates a constant perpetual stream of expected future cash flows starting with time  $t + 1$ .
- Also, assume that without additional investments, firm's expected future earnings remain constant.

## Example: investment and growth

- Texas Western (TW) is expected to earn \$1.00 per share next year.
- Plans an expansion to increase earnings by 8% per year.
- Without additional investments, earnings are expected to remain constant forever.
- Each \$1 of investment at  $t$  generates incremental perpetual cash flow of \$0.10, starting at  $t + 1$ .
- New investment is financed by retained earnings.
- Discount rate is 10%, applied to all cash flows.
- Determine the price of TW's shares if
  - TW pays out all of its earnings forever and does not invest; or
  - TW expands (grows earnings) at 8% forever.

## Solution: no investment

- 100% payout.
- The firm does not invest, and earnings remain at the same expected level forever:

$$DPS_t = EPS_t = \$1$$

- Then

$$P_0 = \frac{EPS_1}{r} = \frac{\$1.0}{0.10} = \$10.00$$

## Solution: 8% growth

- At what rate should TW plow back earnings to achieve 8% growth?
- At time  $t$ , TW reinvests  $EPS_t \times b$ .
  - This investment raises future earnings by  $0.1 \times EPS_t \times b$ , starting at  $t + 1$ .
  - As a result,  $EPS_{t+1} = EPS_t(1 + 0.1 \times b)$ .
  - Conclude that

$$1 + 0.08 = \frac{EPS_{t+1}}{EPS_t} = 1 + 0.1 \times b$$

- The firm needs to reinvest with the plow-back ratio of  $b = 0.8$ .
- The payout ratio is then  $p = 0.2$ .



## Solution: 8% growth

- Under permanent expansion at the 8% rate:

$$D_1 = EPS_1 \times p = \$1 \times 0.2 = \$0.2$$

$$P_0 = \frac{D_1}{r - g} = \frac{\$0.2}{0.10 - 0.08} = \$10.00$$

- Why are the values the same under these scenarios?
- Because new investments do not add to the firm value. If the firm invests \$1, it generates a perpetual cash flow stream of \$0.10, starting a period later.
  - The NPV of this investment with the 10% discount rate is zero – no growth opportunities:

$$-1 + \frac{\$0.10}{0.10} = 0$$

- New investments are zero-NPV.

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Growth opportunities and growth stocks

- **Growth opportunities** are future investment opportunities that generate a positive NPV.
- Stocks of companies that have access to growth opportunities are considered **growth stocks**.
- The following may not be growth stocks:
  - A stock with growing sales;
  - A stock with growing assets;
  - A stock with growing dividends.

## Example: Value of growth opportunities

- ABC Software has the following data:
  - Expected EPS next year is \$8.33;
  - Payout ratio is 0.6;
  - New investments generate a perpetual expected cash flow of \$0.25 per \$1 invested;
  - Discount rate for all cash flows is  $r = 15\%$ .
- What portion of firm value reflects the value of its growth opportunities?
  - Compute stock price in two ways: with and without growth.
  - Present value of growth opportunities (PVGO) is the difference between actual stock price and the price without growth (100% payout, no investment):

$$PVGO_0 = P_0 - P_0^{\text{no growth}}$$

## Stock price with growth

- Compute stock price with growth:

$$DPS_1 = p \times EPS_1 = 0.6 \times \$8.33 = \$5.00$$

- Earnings and dividends grow at the 10% rate:

$$INV_1 = (1 - p) \times EPS_1 = 0.4 \times \$8.33 = \$3.332$$

$$EPS_2 = EPS_1 + INV_1 \times 0.25 = \$8.33 + \$0.833 = \$9.163$$

$$g = \frac{EPS_2}{EPS_1} - 1 = 10\%$$

- Stock price is

$$P_0 = \frac{DPS_1}{r - g} = \frac{\$5.00}{0.15 - 0.10} = \$100$$

## Present value of growth opportunities (PVGO)

- Following a no-growth policy ( $g = 0$ ,  $p = 1$ ):

$$DPS_1^{\text{no growth}} = EPS_1 = \$8.33$$

$$P_0^{\text{no growth}} = \frac{DPS_1^{\text{no growth}}}{r - g} = \frac{\$8.33}{0.15 - 0} = \$55.56$$

- Stock price with growth:

$$P_0 = \$100$$

- The difference of  $\$100 - \$55.56 = \$44.44$  comes from the growth opportunities

$$PVGO = \$44.44$$

- Investment increases firm value.

## PVGO: a closer look

### ■ At time 1

- ABC invests  $0.4 \times \$8.33 = \$3.33$ .
- This investment generates a permanent cash flow stream of  $0.25 \times \$3.33 = \$0.83$  per year starting in year 2.
- The NPV of investment at time 1 is

$$NPV_1 = -\$3.33 + \frac{\$0.83}{0.15} = \$2.22$$

### ■ At time 2

- Investment increases by 10%, and resulting cash flow too: invest \$3.66 and generate cash flow of \$0.91:

$$NPV_2 = 1.1 \times NPV_1 = \$2.44$$

## PVGO: a closer look

- Repeating the valuation process for all future years,  $t = 1, 2, 3, \dots$ , NPV of future investment grows at 10% per year.
- The total PV of the growth opportunities (PVGO) is

$$PVGO = \frac{NPV_1}{r - g} = \frac{\$2.22}{0.15 - 0.10} = \$44.44$$

- This gives the difference in stock valuation between the growth and no-growth policies.



# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## P/E and PVGO

- Why do firms trade at different P/E ratios? This is partly due to differences in growth opportunities.
- To illustrate the issue, maintain the assumption that expected earnings remain constant without investment.
- Stock price has two components:
  - Present value of earnings under a no-growth policy ( $EPS_1/r$ )
  - Present value of growth opportunities ( $PVGO$ )

$$P_0 = \frac{EPS_1}{r} + PVGO$$

- Terminology: P/E ratio is

$$P/E = \frac{P_0}{EPS_1}$$

- In business media,  $P/E$  is often quoting  $P_0/EPS_0$  rather than  $P_0/EPS_1$ .

## P/E and PVGO

$$P_0 = \frac{EPS_1}{r} + PVGO$$

- If  $PVGO = 0$ ,  $P/E$  ratio equals inverse of cost of capital

$$P/E = \frac{1}{r}$$

- If  $PVGO > 0$ ,  $P/E$  ratio becomes higher:

$$P/E = \frac{1}{r} + \frac{PVGO}{EPS_1} > \frac{1}{r}$$

# Key Concepts

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses

## Valuation of an operating business

- We developed methodology for pricing stocks.
- The same techniques can be used to value the entire business.
- Start with the DCF formula

$$PV = \sum_{t=1}^T \frac{FCF_t}{(1+r)^t} + \frac{PV_T}{(1+r)^T}$$

- FCF is Free Cash Flow – net cash flows attributed to the business under evaluation.

## Valuation of an operating business

$$PV = \sum_{t=1}^T \frac{FCF_t}{(1+r)^t} + \frac{PV_T}{(1+r)^T}$$

- Need to
  - Forecast cash flows for  $t = 1, 2, \dots, T$ .
  - Determine horizon value  $PV_T$ .
- $PV_T$  can be computed using several alternative methods:
  - $P/E$  multiple;
  - $P/B$  multiple;
  - $DCF$ .

## Valuation of an operating business

- Multipliers: this is a relative valuation approach – similar firms should trade at similar valuation ratios.
- Challenge: it is hard to find firms that are similar enough.
- We forecast cash flows for  $t = 1, 2, \dots, T$  directly and apply multiples at  $T$ .
  - We expect that firms that look different now will converge in the future, on average.
  - Apply future multiples similar to comparable firms.
- Pure DCF approach: choose  $T$  to be when  $PVGO$  goes to zero. Then

$$PV_T = \frac{FCF_{T+1}(\text{No Growth})}{r}$$

## Example

- Forecasts of cash flows in \$ millions

Time	0	1	2	3	4	5	6
Asset value	5.00	8.00	9.50	10.75	11.75	12.55	13.15
Earnings		1.00	1.25	1.50	1.60	1.75	1.88
Net investment		3.00	1.50	1.25	1.00	0.80	0.60
Free cash flow		-2.00	-0.25	0.25	0.60	0.95	1.28
Discount factor		0.87	0.76	0.66	0.57	0.50	0.43
FCF*Disc. factor		-1.74	-0.19	0.16	0.34	0.47	0.55

- Net investment = CAPX – Depreciation.
- Free cash flow = Earnings – Net Investment.
- Discount rate  $r = 15\%$ . Starting in period 5, return on new investments is 15%.



## Valuation of operating business

- Stages of growth:
  - Fast growth in early years, with high levels of investment and negative free cash flow.
  - Slowing growth, with investment declining and return on investment converging to the discount rate.

$$PV(\text{Business}) = PV(\text{FCF from years 1 to } T) + PV(\text{Horizon Value at } T).$$

## Horizon value using multiples

- Discount free cash flow for  $t = 1, \dots, 4$ , and estimate horizon value at  $T = 4$ .

$$PV(\text{FCF from years 1 to 4}) = -\$1.42$$

- Estimate  $PV_4$  using P/E multiples:

- Suppose the P/E ratio for comparable firms is on average 6.0 (price over expected earnings next year).

Time	0	1	2	3	4	5	6
Asset value	5.00	8.00	9.50	10.75	11.75	12.55	13.15
Earnings		1.00	1.25	1.50	1.60	1.75	1.88

- Apply this multiple to  $E_5$ :  $PV_4 = \$1.75 \times 6.0 = \$10.50$ .

$$PV_0(PV_4) = \frac{\$10.50}{(1.15)^4} = \$6.00$$

## Horizon value using multiples

- Estimate  $PV_4$  using P/B multiples:
  - Suppose the P/B ratio for comparable firms is on average 1.2 (price over contemporaneous book value).

Time	0	1	2	3	4	5	6
Asset value	5.00	8.00	9.50	10.75	11.75	12.55	13.15

- Apply this multiple to asset value at  $T = 4$  :  $PV_4 = \$11.75 \times 1.2 = \$14.10$ .

$$PV_0(PV_4) = \frac{\$14.10}{(1.15)^4} = \$8.06$$

## Horizon value using DCF

- Suppose that starting at  $T = 5$ , PVGO converges to zero.
  - Competition catches up and eliminates positive NPV investment opportunities.
- Discount free cash flow for  $t = 1, \dots, 5$ , and estimate horizon value at  $T = 5$ :

$$PV(\text{FCF from years 1 to 5}) = -\$0.95$$

Time	0	1	2	3	4	5	6
Earnings		1.00	1.25	1.50	1.60	1.75	1.88

- Horizon value at  $T = 5$ :  $PV_5 = E_6/r = \$1.88/0.15 = \$12.55$

$$PV_0(PV_5) = \frac{\$12.55}{(1.15)^5} = \$6.24$$

## Alternative estimates

- We now have three alternative estimates

$$PV = \sum_{t=1}^T \frac{FCF_t}{(1+r)^t} + \frac{PV_T}{(1+r)^T}$$

- Using  $P/E$  multiples to estimate  $PV_4$ :

$$PV_0 = -\$1.42 + \$6.00 = \$4.58$$

- Using  $P/B$  multiples to estimate  $PV_4$ :

$$PV_0 = -\$1.42 + \$8.06 = \$6.64$$

- Using DCF to estimate  $PV_5$ :

$$PV_0 = -\$0.95 + \$6.24 = \$5.29$$

## Conclusion

- The approach using  $P/E$  multiples generates the most conservative estimate *in this case*.
- These are estimates based on a number of simplifying assumptions.
- We get a sense of the range of possible values – it can be wide!
- Must study sensitivity to assumptions.
- Valuation is not a simple problem, but we now have the tools.

# Summary

- Introduction
- Discounted Cash Flow Model
- Constant Growth Model
- Multistage Growth Model
- Forecasting Dividends
- Growth Opportunities and Growth Stocks
- Growth Opportunities and Stock Valuation
- Valuation of Operating Businesses