MIT Sloan School of Management
Kogan and Wang

15.415.1x Sample Exam

Grade Sheet

1.	_____	/	10
2.	_____	/	12
3.	_____	/	11
4.	_____	/	16
6.	_____	/	18
6.	_____	/	18
7.	_____	/	15
8.	_____	/	20
Total	_____	/	120

1. (10 points) True or false?

   (a) (2 points) When evaluating investment projects, for the projects with higher variance of sales forecasts, firms should apply higher discount rates.

   (b) (2 points) Consider a firm maximizing its market value. When selecting among two mutually exclusive projects with the same initial investment, this firm may optimally select the project with the lower internal rate of return.

   (c) (2 points) When computing the NPV of potential new investments, a firm should discount expected future cash flows attributed to the project using the firm's cost of capital as the discount rate.

   (d) (2 points) If the yield curve is flat, then yield to maturity on a risk-free coupon bond is equal to the expected return (in annualized terms) the investor would collect by holding the bond to its maturity.

   (e) (2 points) Consider a nominal risk-free cash flow. When computing the present value in the presence of inflation risk, it is generally wrong to discount the expected real cash flows at the real risk-free interest rates.

2. (12 points) Consider a state-space model with two periods and three states at time $t = 1$: 1, 2, and 3. All three states are equally likely. Primitive state-contingent claims on each state are traded in the market, and their time-0 prices are:

$$\phi_1 = 0.4; \ \phi_2 = 0.3; \ \phi_3 = 0.2.$$

In addition to the primitive claims, the risk-free asset is traded.

   (a) (4 points) Based on absence of arbitrage, what is the risk-free interest rate in this market?

   (b) (4 points) Consider a cash flow C_1 equal to $1, $2, and $4, in states 1, 2, and 3, respectively. What is the expected value of this cash flow (as of time 0)?

   (c) (4 points) Compute the time-0 market value of the cash flow C_1.

3. (11 points) Alice is taking out a bank loan to pay for a new addition to her house. She is comparing two options: a 10-year loan with an annual APR of 6%, compounded monthly; and a 5-year loan, with an annual APR of 6.7%, compounded monthly. The market interest rate is 4% (EAR), and is the same for all maturities. Alice needs to borrow $50,000.

   (a) (3 points) Compute the EAR on each of the two loans.

   (b) (4 points) Compute the monthly payments on each of the two loans.

   (c) (3 points) Compute the present value of payments on each of the two loans.

   (d) (1 point) Is the ten-year loan a better deal, judging by the present value of the payments that Alice would need to make between now and the maturity of the loan? "Yes" or "no."

4. (17 points) Consider a frictionless market. Several Treasury bonds (with face values of $100) are traded in the market. Their coupon rates and yield-to-maturity are given in the following table:

Bond name	Maturity	Coupon rate	Yield to maturity
A	1-year	0%	3%
B	2-year	6%	4%
C	3-year	3%	5%

The coupons are paid annually. Now is year 0.

(a) (2 points) What is the 1-year spot interest rate?

(b) (2 points) What is the 2-year spot interest rate?

(c) (2 points) What is the 3-year spot interest rate?

(d) (3 points) Suppose that a new Treasury bond is introduced to the market. It is a zero-coupon bond with 3 years to maturity, and it trades at the 4% yield to maturity. What is the no-arbitrage price of the new bond?

(e) (7 points) Describe explicitly the arbitrage trading strategy involving bonds A, B, C, and the new bond, which pays $1 at time $t = 0$ and nothing afterwards.

5. (18 points) You are advising a local municipal treasury on a bridge construction project. The project requires an upfront investment (at time/year 0) of $10M, with an additional investment in year 1 of $5M. The bridge will become operational two years from now, and will start generating toll revenue. Specifically, the bridge produces no cash flows in year 1, and produces a perpetual stream of cash flows of $1M per year in subsequent years. Assume that all cash flows are risk-free.

The treasury is financing this project with a ten-year zero-coupon bond. The current term structure of risk-free interest rates is flat at 2%. Assume that the treasury is able to finance this project at the risk-free interest rate.

(a) (3 points) What is the NPV of this project?

(b) (3 points) Suppose that the treasury wants to issue enough bonds to cover the present value of construction costs of this project. Let the face value of each bond be $1,000. What is the total number of bonds that need to be issued?

(c) (5 points) Compute the modified duration of the bond issued by the treasury.

(d) (5 points) Suppose the treasury goes ahead with your suggestion in (b) and starts the project. Right after its start (at time 0), the yield curve unexpectedly rises by 1% across all maturities. What is the resulting change in the NPV of the project, following the change in interest rates?

(e) (2 points) Using the duration-based approximation, what would be the change in the value of the outstanding bonds following a 1% rise in interest rates?

6. (18 points) A private equity investment fund has firm XYZ in its portfolio. Your task is to estimate the value of this firm, which does not trade publicly. XYZ is 100% equity financed. It is now year 0, and you have the following data on XYZ:

Full-year earnings over year 0	$100M
Payout ratio in year 0	0%
Cost of capital	10%

Based on your market analysis, you forecast that without any new investments, XYZ is expected to generate $100M in earnings per year in perpetuity. Investments made in year 0 and 1 are expected to generate $0.20 per year in perpetuity for each $1 invested, starting in a year following the investment. Starting in year 2, new investments are expected to generate $0.10 per year in perpetuity for each $1 invested, starting in a year following the investment.

The payout ratio of XYZ will stay at zero in year 1, rising permanently to 60% afterward. Assume the cost of capital in the above table applies to all future cash flows generated by XYZ (including its future investments and earnings), and will remain constant.

(a) (2 points) Compute the expected earnings of XYZ in year 1.

(b) (3 points) Compute the expected earnings of XYZ in year 2.

(c) (10 points) Compute the market value of XYZ as of year 0.

(d) (3 points) What is the net present value of growth opportunities (PVGO) of XYZ as of year 0? Do not include the net present value of year-0 investment into the PVGO.

7. (15 points) Suppose that asset returns are described by a 2-factor APT model, which applies exactly to all assets:

$$\tilde{r}_i = \bar{r}_i + b_{i1}\tilde{f}_1 + b_{i2}\tilde{f}_2 + \tilde{u}_i, \quad i = 1, 2, \ldots$$

where both factors have unit variance and are uncorrelated with each other.

The risk premia associated with factors 1 and 2 are 20% and 30%, respectively. The risk-free rate is 2%.

(a) (3 points) We are contemplating investing in a stock, which has the following factor loadings: $b_1 = 0.20$ and $b_2 = 0.10$. According to APT, what should be the expected return on this stock?

(b) (6 points) Consider two stocks, 1 and 2, with the following parameters. For stock 1: $b_{11} = 0.2$, $b_{12} = 0.1$, $SD(\tilde{u}_1) = 0.20$. For stock 2: $b_{21} = 0.1$, $b_{12} = 0.4$, $SD(\tilde{u}_2) = 0.25$. Compute the correlation between returns on stocks 1 and 2.

(c) (6 points) Construct a portfolio with stocks 1 and 2 above, with weights w_1 and w_2, with the expected return equal to the risk-free rate. What is w_1?

8. (20 points) A semiconductor company is considering a purchase of a silicon measurement system costing $500,000. By reducing wasted Silicon, the system is expected to save approximately $100,000 per year in raw material costs. The system costs $25,000 to install. The device would reduce approximately 40 hours of work per week provided by an outsourcing service charged at $40 per hour. The company is expecting to upgrade the entire plant in 3 years at which time they expect to dispose of the equipment. They estimate that they would receive $25,000 for the equipment. The equipment is to be depreciated on a straight line basis over 3 years. The discount rate of the firm is 10%.

(a) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 0?

(b) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 1?

(c) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 3?

(d) (2 points) Assume no taxes. What is the Net Present Value of the measurement system?

(e) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 0?

(f) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 1?

(g) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 3?

(h) (3 points) Suppose now the corporate tax rate is 35%. What is the Net Present Value of the measurement system?