



15.415x Foundations of Modern Finance

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Lecture 10: Capital Budgeting I

Key concepts

- NPV rule
- Cash flows from capital investments
- Discount rates and project interaction
- Alternative capital budgeting rules
- Additional considerations

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NPV rule

A firm's business involves capital investments (capital budgeting), e.g., the acquisition of real assets. The objective is to increase the firm's current market value. Decision reduces to valuing real assets, i.e., their cash flows.

Let the expected cash flow of an investment (a project) be:

$$\{CF_0, CF_1, \dots, CF_T\}$$

Its current market value is:

$$NPV = CF_0 + \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_2)^2} + \dots + \frac{CF_T}{(1+r_T)^T}$$

This is the increase in the firm's current market value by the project.

NPV rule

Investment Criteria:

- For a single project, take it if and only if its NPV is positive.
- For many independent projects, take all those with positive NPV.
- For **mutually exclusive** projects, take the one with positive and highest NPV.

In order to compute the NPV of a project, we need to analyze

1. Cash flows,
2. Discount rates,
3. Strategic options.

We will focus mostly on 1 here and return to 2 and 3 later in Part II.

Cash flow calculations

Important Points:

1. Use cash flows, not accounting earnings.
2. Use after-tax cash flows.
3. Use cash flows attributable to the project (compare firm value with and without the project):
 - Use incremental cash flows.
 - Forget sunk costs: bygones are bygones.
 - Include investment in working capital as capital expenditure.
 - Include opportunity costs of using existing facilities.

Cash flow calculations

In what follows, all cash flows are attributable to the project.

We can write a project's CF as follows:

$$\begin{aligned} \text{CF} &= [\text{Project Cash Inflows}] - [\text{Project Cash Outflows}] \\ &= [\text{Operating Revenues}] - [\text{Operating Expenses without depreciation}] \\ &\quad - [\text{Taxes}] \\ &\quad - [\text{Capital Expenditures}] \end{aligned}$$

Cash flow calculations

Defining operating profit by:

$$\text{Operating Profit} = \text{Operating Revenues} \\ - \text{Operating Expenses w/o Depreciation}$$

Let τ be the “effective” tax rate. The income taxes are:

$$[\text{Taxes}] = (\tau)[\text{Operating Profit}] - (\tau)[\text{Depreciation}]$$

Accounting depreciation affects CF because it reduces firm's tax bill.

$$\text{CF} = (1 - \tau)[\text{Operating Profit}] - [\text{Capital Expenditures}] \\ + (\tau)[\text{Depreciation}]$$

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Cash flow calculations

Example. Accounting earnings vs. cash flows. A machine purchased for \$1,000,000 with a life of 10 years generates annual revenue of \$300,000 and operating expense of \$100,000. Assume that the machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%.

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	-1,000,000
1	$300,000 - 100,000 - 100,000$ $= 100,000$	$(1 - 0.4)(100,000)$ $= 60,000$	$(1 - 0.4)(300,000 - 100,000) +$ $40,000 = 160,000$
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

Accounting earnings may not accurately reflect the actual CF timing.

Cash flow calculations

Example. Use after-tax cash flows. Consider the following project (the cash flow is in thousands of dollars and tax rate is 50%):

Year	0	1	2	3	4	5
Investment	500					
Operating CF		0	100	300	300	300
Depreciation		100	100	100	100	100
Income		-100	0	200	200	200
Tax		-50	0	100	100	100
After-tax CF	-500	50	100	200	200	200
PV at 10%	-500	45.45	82.64	150.26	136.60	124.18

NPV = +39.13.

Cash flow calculations

Example. Inventory. You run a chain of shoe stores.

- This quarter, you buy 1,000,000 pairs of shoes at a price of \$30.00 each.
- For the next two quarters, you sell 500,000 pairs each quarter for \$60.00 each.
- The effective corporate tax rate is 40%.

In million dollars, your cash flows are:

Date	After Tax Profit	Inventory	Cash Flow
0	0	$(1)(30)=30$	-30
1	$(0.5)(60-30)(1-0.4)=9$	$(0.5)(30)=15$	$(0.5)(60)-(0.5)(60-30)(0.4)=24$
2	$(0.5)(60-30)(1-0.4)=9$	0	$(0.5)(60)-(0.5)(60-30)(0.4)=24$

Notice:

$$\text{Cash Flow} = \text{Profit (after tax)} - \text{Change in Inventory}$$

Cash flow calculations

Typically, there are timing differences between the accounting measure of earnings (Sales - Cost of Goods Sold) and cash flows.

$$\text{Working Capital (WC)} = \text{Inventory} + \text{A/R} - \text{A/P}$$

Changes in Working Capital

- Inventory: Cost of goods sold includes only the cost of items sold. When inventory is rising, the cost of goods sold understates cash outflows. When inventory is falling, cost of goods sold overstates cash outflows.
- Accounts Receivable (A/R): Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.
- Accounts Payable (A/P) -- conceptually the reverse of A/R.

$$\begin{aligned} \text{CF} = & (1-\tau)[\text{Operating Profits}] + (\tau)[\text{Depreciation}] \\ & - [\text{Capital Expenditures}] - \Delta[\text{Working Capital}] \end{aligned}$$

Putting all together

Example. MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

- TW were developed at an R&D cost of \$1M over past 3 years.
- New machine to produce TW would cost \$2M.
- New machine lasts for 15 years, with salvage value of \$50,000.
- New machine can be depreciated linearly to \$0 over 10 years.
- TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of \$30,000 (regardless of how much it is used).
- Operating cost: \$40,000 per year.
- Sales: \$400,000, but cannibalization would lead existing sales of regular widgets to decrease by \$20,000.
- Working Capital (WC): \$250,000 needed over the life of the project.
- Tax rate: 34%.
- Opportunity cost of capital: 10%.

Should MSW go ahead to produce TW?

Putting all together

- R&D expense is sunk cost.
- Initial investment includes capital expenditure.
- Depreciation is $\$2\text{M}/10 = \0.2M for first 10 years.
- Salvage value is fully taxable since the book value at the end of year 10 is \$0 (the machine cost has been fully depreciated).
- Project should not be charged for painting-machine time.
- Project should be charged for cannibalization of regular widget sales.
- Working capital is an initial investment, but recovered at the end.

The cash flows (in thousand dollars) are calculated as follows:

Year	Cash Flow
0	$-(2,000 + 250) = -2,250.0$
1-10	$(400 - 40 - 20)(1 - 0.34) + (200)(0.34) = 292.4$
11-14	$(400 - 40 - 20)(1 - 0.34) = 224.4$
15	$224.4 + (50)(1 - 0.34) + 250 = 507.4$

$\Rightarrow \text{NPV} = -\$57,617$

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Discount rates

So far, we have shown that:

- A project's discount rate (required rate of return or cost of capital) is the expected rate of return demanded by investors for the project.
- Discount rate(s) in general depend on the timing and risk of the cash flow(s).
- Discount rate is usually different for different projects.
- It is in general incorrect to use a company-wide “cost of capital” to discount cash flows of all projects.

What is the required rate of return on a project?

- Simple case: a single discount rate used for all cash flows of a project.
- General case: different discount rates for different cash flows.
 - Different timing and risk.

We will return to the determination of discount rates after developing more asset pricing tools.

Project interaction

Often we have to decide on more than one project.

- For mutually independent projects, apply NPV rule to each project.
- For projects dependent of each other (e.g., mutually exclusive), we have to compare their NPVs.

Example. Potential demand for your product is projected to increase over time. If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity cost of capital is 10%. Denoting by FPV the project's NPV at the time of introduction, we have:

Year to start	Future PV	% Change in FPV	NPV
1	100		91
2	120	20	99
3	138	15	104
4	149	8	102

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Alternatives to NPV

In practice, investment rules other than NPV are also used:

- Payback Period,
- Internal Rate of Return (IRR),
- Profitability Index (PI) ...

Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.

These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.

The bottom line:

The NPV rule dominates the alternative rules.

Payback period

Payback period is the minimum length of time s such that the sum of net cash flows from a project becomes positive:

$$CF_1 + CF_2 + \dots + CF_s \geq -CF_0 = I_0$$

Decision Criterion Using Payback Period

- For independent projects: Accept if s is less than or equal to some fixed threshold t^* : $s \leq t^*$.
- For mutually exclusive projects: Among all the projects having $s \leq t^*$, accept the one that has the minimum payback period.

Payback period

Example. Let $t^* = 3$. Consider the two independent projects, 1 and 2, with the following cash flows (in thousand dollars):

	CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6	s
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	10	80	5	10	10	3

Decision: Accept Project 2.

Payback period

- Payback period rule ignores cash flows after the payback period;
- It ignores discounting.

Example (cont'd). Suppose that the appropriate discount rate is a constant 10% per period. Then,

$$NPV_1 = 39,315 \quad \text{and} \quad NPV_2 = -7,270$$

But we accepted project 2 and not project 1!

Taking into account appropriate discounting, we have the **discounted payback period**, which is the minimum s so that:

$$\frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_s}{(1+r)^s} \geq -CF_0$$

where r is the discount rate (cost of capital). (It still ignores the cash flows after the discounted payback period.)

Internal rate of return (IRR)

A project's **internal rate of return (IRR)** is the number that satisfies:

$$0 = CF_0 + \frac{CF_1}{(1 + IRR)} + \frac{CF_2}{(1 + IRR)^2} + \cdots \frac{CF_t}{(1 + IRR)^t}$$

Decision Criteria Using IRR

- For independent projects: Accept a project if its IRR is greater than some fixed IRR^* , the threshold rate/hurdle rate.
- For mutually exclusive projects: Among the projects having IRR's greater than IRR^* , accept one with the highest IRR.

Internal rate of return (IRR)

Example (IRR). Consider the following mutually exclusive projects:

	CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6
Project 1	-100	20	40	30	10	40	60
Project 2	-100	10	10	80	5	10	10

Then, $IRR_1 = 21\%$ and $IRR_2 = 7\%$.

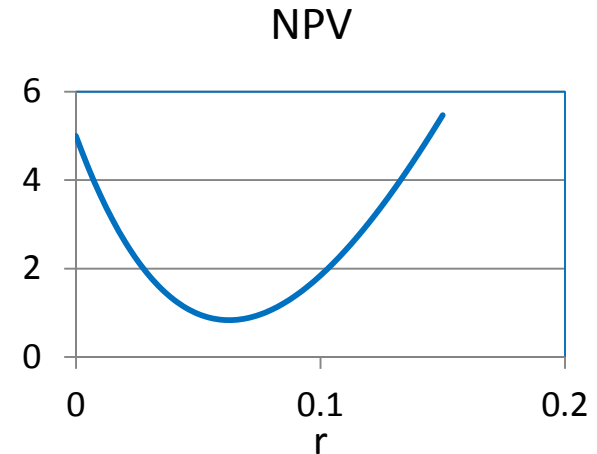
IRR rule leads to the same decisions as NPV if:

- Cash outflow occurs only at time 0,
- Only one project is under consideration,
- Opportunity cost of capital is the same for all periods,
- Threshold rate is set equal to opportunity cost of capital.

Internal rate of return (IRR)

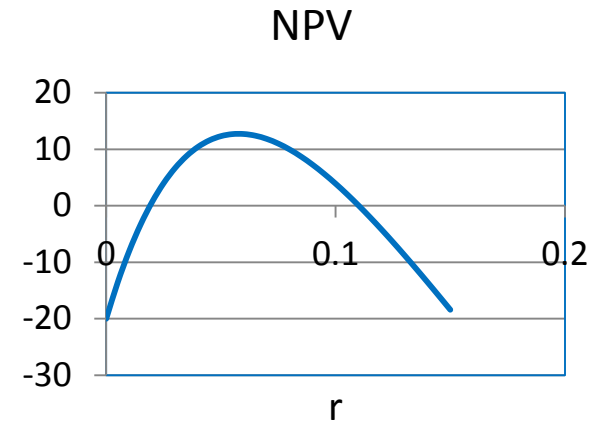
■ Non-existence of IRR

Period	0	3	6
CF	105	-250	150



■ Multiple IRR's

Period	0	3	6	9	12
CF	-200	100	300	280	-500



Internal rate of return (IRR)

Project ranking using IRR for mutually exclusive projects:

- Projects of different scales

CF_t	CF_0	CF_1	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27

- Project 1 has higher IRR.
- Project 2 has higher NPV.
- Project 2 has a larger scale than Project 1.

Profitability index (PI)

Profitability index (PI) is the ratio of the present value of future cash flows and the initial cost of a project:

$$PI = \frac{PV}{-CF_0} = \frac{PV}{I_0}$$

Decision criterion using PI

- For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule).
- For mutually exclusive projects: Among the projects with PI greater than one, accept the one with the highest PI.

Problems with PI

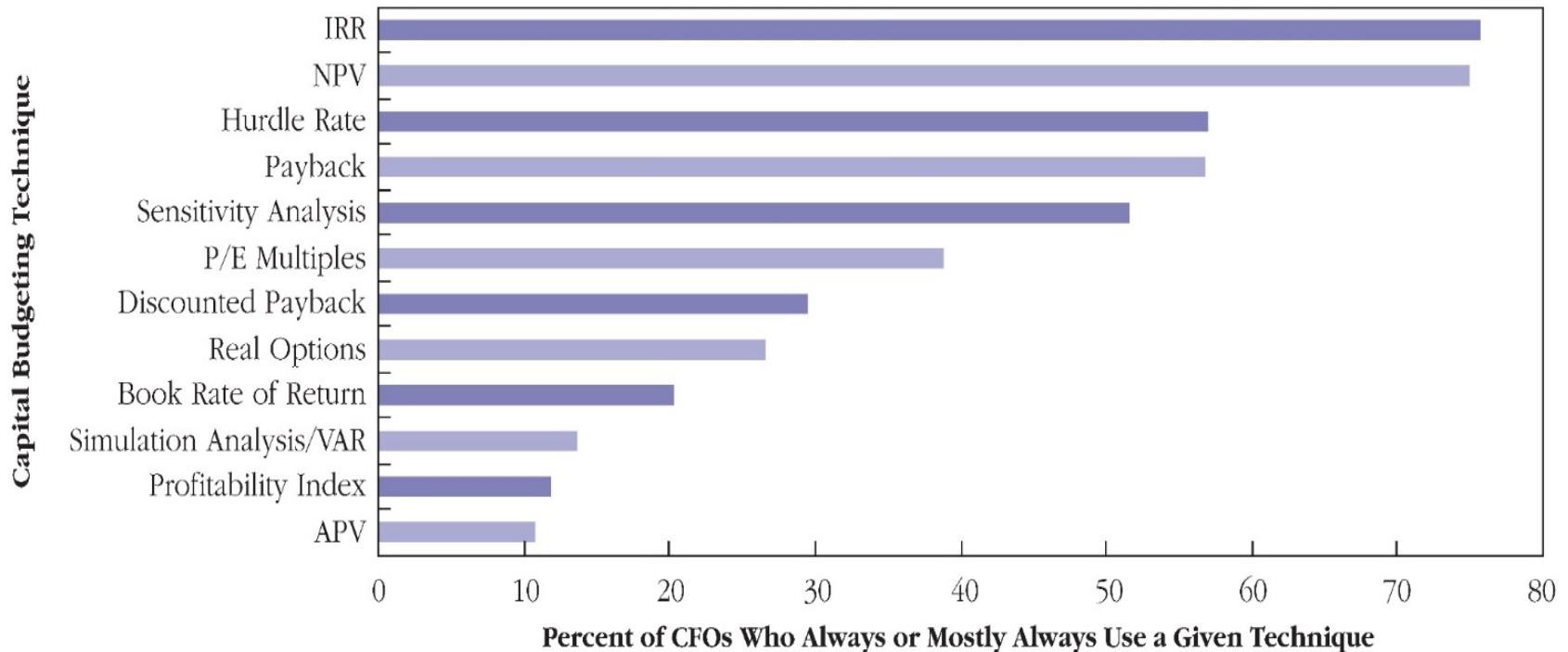
- PI gives the same answer as NPV when
 - There is only one cash outflow, which is at time 0,
 - Only one project is under consideration.

- PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.

	CF_0	CF_1	IRR	NPV at 10%	PI at 10%
Project 1	-1,000	2,000	100%	818.18	1.82
Project 2	-2,000	3,600	80%	1,272.73	1.64

Capital budgeting in practice

FIGURE 1 ■ SURVEY EVIDENCE ON THE POPULARITY OF DIFFERENT CAPITAL BUDGETING METHODS*



Source: John Graham and Campbell Harvey, 2002, "How do CFOs make capital budgeting and capital structure decisions?" *Journal of Applied Corporate Finance*.

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Additional considerations

1. Competitive Response:

- CF forecasts should consider responses of competitors.

2. Capital Rationing.

3. Sources of Positive-NPV Projects:

- Short-run competitive advantage (right place at right time)
- Long-run competitive advantage
 - ❖ Patent
 - ❖ Technology
 - ❖ economies of scale, etc.
- Noise.

Additional considerations

Capital rationing

- N independent projects,
- All with positive NPVs. For $i = 1, 2, \dots, N$:
 - Initial investment I_i
 - Net present value $NPV_i > 0$,
- Total capital available I .

We should choose the set of projects which maximizes the total NPV, subject to the capital constraint:

$$\begin{aligned} \text{Maximize } NPV &= \sum_i NPV_i \\ \text{subject to: } &\sum I_i \leq I \end{aligned}$$

Summary

- NPV rule
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- Alternative capital budgeting rules
- Additional considerations