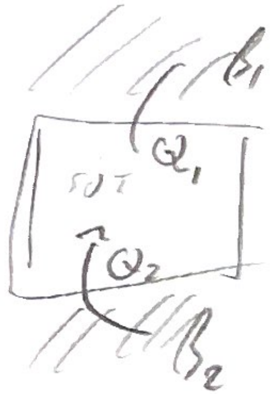
$e^{-\beta W}$

$\langle e^{+\Delta F - \beta W} \rangle = 1$ 1

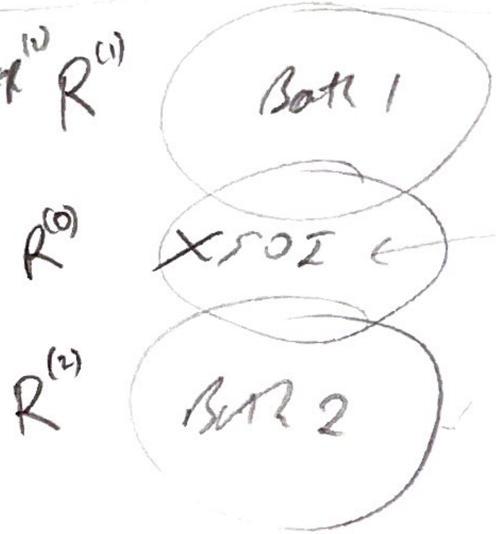
e ~~$Q = \dots$~~

#21



$R = R^{(0)} + R^{(1)} + R^{(2)}$

Rate matrices



no overlap

$e^{+\beta Q_1 + \beta Q_2}$

$\Sigma^{env} = -\beta_1 Q_1 - \beta_2 Q_2$

$$\frac{P(x|x_a)}{P(\tilde{x}|x_b)} = e^{+\Sigma^{env}} = e^{-\beta Q_1 - \beta Q_2}$$

$\Sigma = \Sigma^{sys} + \Sigma^{env}$

$\downarrow$

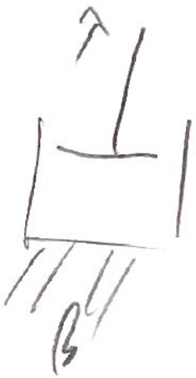
ΔS

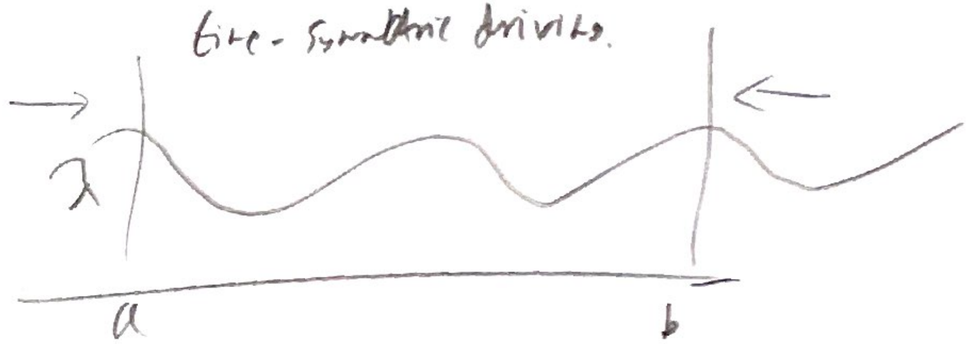
$-\ln p_{ss}(x_a) + \ln p_{ss}(x_b)$

Steady state FT

$$\frac{P(+\Sigma)}{P(-\Sigma)} = e^{+\Sigma}$$

↑ no distinction forward & back





$$\Sigma = \Sigma^{sys} + \Sigma^{env}$$

$$-\beta Q_1 \quad -\beta Q_2 \quad \ldots$$

(Hatano, throw away boundary term)

$$-\ln P_{ss}(x_t) + \ln P_{ss}(x_0)$$

Not canonical. Dynamics not detailed balanced.

Refs: Stochastic Thermodynamics Peliti & Pigolotti c26

Hugo Touchette 2009

Large Deviations

Rain today? Probability (rain) = r . independent

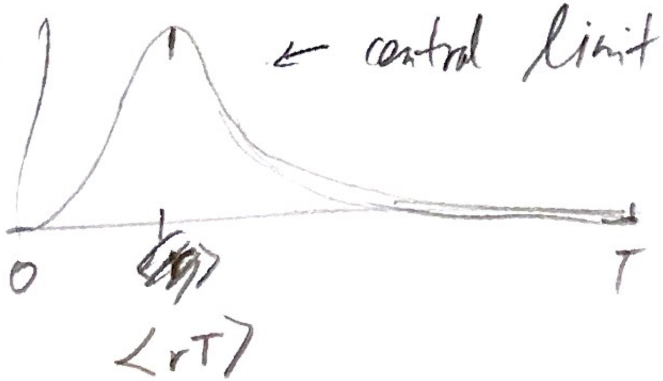
τ days rain in T days to

Binomial coefficient $\begin{pmatrix} a \\ b \end{pmatrix} = \frac{a!}{b!(a-b)!}$

$$P(\tau ; T) = \begin{pmatrix} T \\ r \end{pmatrix} r^{\tau} (1-r)^{T-\tau}$$

mean binomial np

var $npq = T(1-r)r$



← central limit theorem, around mean ~ Gaussian

[If finite cumulants!]

because variance ~ width T
but rescale $\frac{1}{T} \to \frac{1}{T^2}$

---

fraction of days

$f = \frac{x}{T}$ $\quad \langle f \rangle = r \quad \sigma_f^2 = \frac{r(1-r)}{T}$

$$p(f:T) \approx \sqrt{\frac{T}{2\pi r(1-r)}}\; e^{-T\frac{(f-r)^2}{2r(1-r)}}$$

↑

Central limit theorem

Only applies to "center", not wings.

Cramér 1944

Rate function (Cramér function)

Rate function
(not mutual information)

"Large deviation principle" $\quad \lim_{T\to\infty} \frac{1}{T} \ln p(f;T) = -I(f)$

or restated as $\quad p(f;T) \asymp e^{-T I(f)}$ $\quad$ (unless strong long range correlations)

For binomial $\quad I(f) = f \ln \frac{f}{r} + (1-f) \ln \frac{1-f}{1-r})$

Gaussian $\quad I(f) = \frac{(f - \langle f \rangle)^2}{2\tilde{\sigma}_f^2}$ $\quad$ Scaled Variance $\quad \tilde{\sigma}_f^2 = \lim_{T\to\infty} T\sigma_f^2$

Supremum - smallest upper bound

[illegible]

Maximum - largest

(1, 5) - Max ill defined, Sup = 5

Scaled Cumulant Generating function

$$\psi^{(f)}(q) = \lim_{T\to\infty} \frac{1}{T} \ln \left\langle e^{qTf} \right\rangle$$

Gärtner-Ellis theorem
1977 1984

Legendre-Fenchel transform

$$I(f) = \sup_{q} \left[q f - \psi^{(f)}(q) \right)$$

$$\psi^{(f)}(q) = \sup_{f} \left[q f - I(f) \right]$$

Involution

Heuristic Proof

$$\psi^{(t)}(q) = \lim_{T\to\infty} \frac{1}{T} \ln \int_{-\infty}^{\infty} dt \; \underbrace{P(t;T)}_{\simeq\, e^{-T I(t)}} \, e^{Tqt}$$

$$= \lim_{T\to\infty} \frac{1}{T} \ln \int_{-\infty}^{\infty} dt \; e^{+T(-I(t)+q(t))}$$

(Roughly) Replace by maximum value "saddle point"

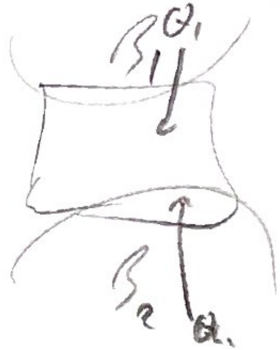
$$= \lim_{T\to\infty} \frac{1}{T} \ln \; e^{+T \sup(qt - I(t))}$$

$$= \sup(qt - I(t))$$

"Gallavotti-Cohen" Symmetry 1995

Lebowitz & Spohn 1998

Gallavotti-Cohen FT



$$\Sigma^{tot} = \Delta s^{sys} + \Sigma^{env}$$

intensive with time extensive

Entropy Production Rate

$$j = \frac{\Sigma}{T} \leftarrow \text{time}$$

$$\frac{P(+j_{ss};T)}{P(-j_{ss};T)} = e^{Tj}$$

Large Deviations $P(j_{ss};T) \asymp e^{-T I(j_{ss})}$

$$I(j_{ss}) - I(-j_{ss}) = -j_{ss}$$

(generalize for forward/Reverse)

$$\psi(q) = \psi(-1-q) \qquad \langle e^{-qTj} \rangle = \langle e^{+(1+q)Tj} \rangle$$

Ref: Field Guide to Continuous Probability Distributions

threeplusone.com/fieldguide 10

Extreme Value Distributions

$P(x)$, $x = \max\{x\}$, over many realizations at time T?

Generalized Fisher-Tippett Distribution

$$P(x) = \frac{n^n}{\Gamma(n)} \left|\frac{\beta}{\omega}\right| \left(\frac{x-a}{\omega}\right)^{n\beta-1} e^{-n\left(\frac{x-a}{\omega}\right)^{\beta}}$$

(special case of Amoroso)