Lecture 8   Fluctuation Theorems, Intro   [redacted] |

①

JARZYNSKI IDENTITY

[We have discussed Probability, Fluctuation Theory
Renewed foundations of Thermodynamics, (Fluids, [illegible] (F.E)
Dynamics]

→ Today Jarzynski Equation
- Simplest, most profound & beautiful result in S.M. That's not in L&L]

"Integrated Fluctuation Theorem"

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

Jarzynski PRL 1997
[Letter]

"25-years of Nano-scale Thermodynamics"
Nature 2022
Broedersz & Ronceray

[redacted]
Nature of Physical World

T.R. & Thermodynamic Equilibrium

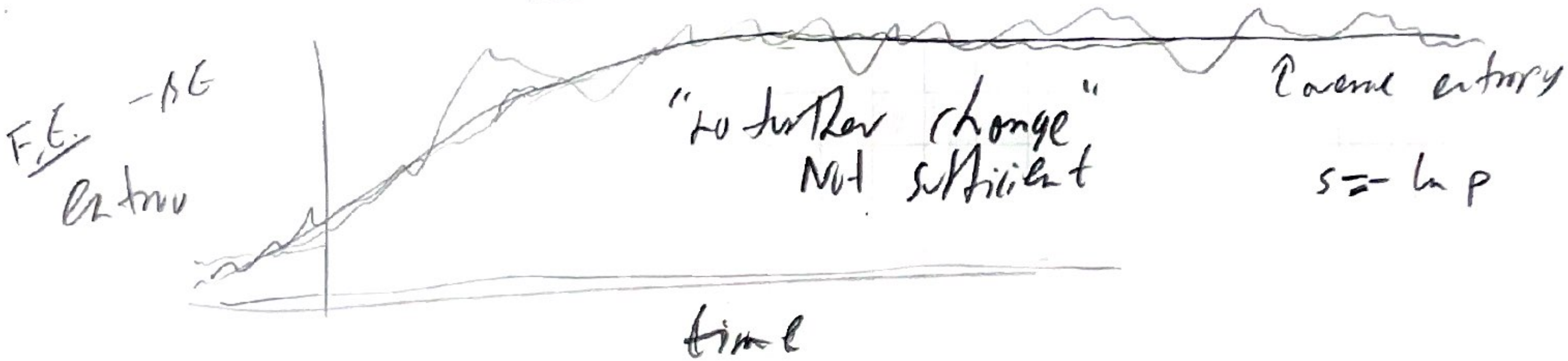
Eddington 1928
"The Nature of The Physical World" ②

ONLY Entropy determines arrow of time.

Thermodynamic Equilibrium – statistically time reversal ~~invariant~~ symmetric

Time has no orientation nor origin

Homogenious & isotropic



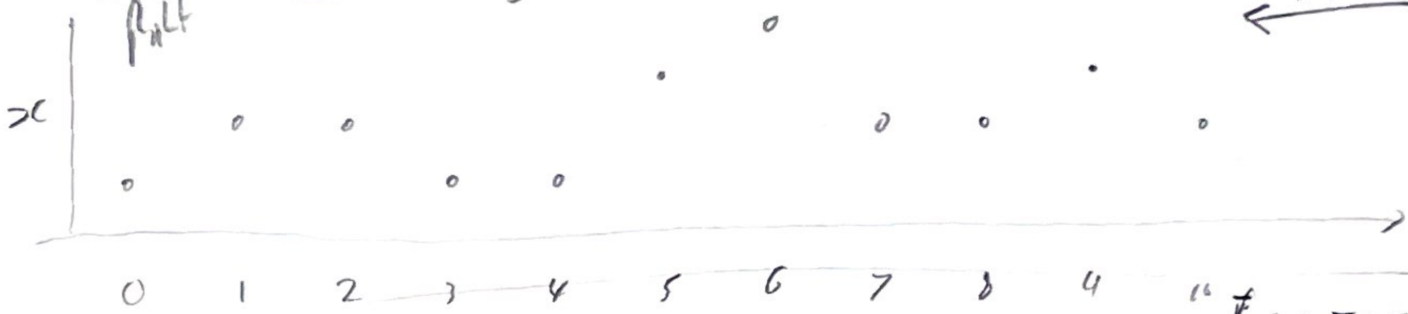
[clocks are highly out eq. devices
NO clocks at Thermodynamic equilibrium]
Time has extent, but no orientation]

[This is very profound statement!]

DTMC

[note, not flipping time index]

$X \longrightarrow$ $\qquad$ $\xleftarrow{\tilde{X}}$



$\Delta E = W + \overset{\text{heat}}{Q}$

$e^{\cancel{+\beta\Delta E}} = e^{-\beta Q}$

$$\frac{P(X)}{P(\tilde{X})} = 1 \qquad \frac{P(X \mid x_0)}{P(\tilde{X} \mid x_t)} = \frac{P(x_t)}{P(x_0)} = e^{-\beta[E(x_t) - E(x_0)]}$$

Conter

$\overrightarrow{\text{Bond}}$ 3

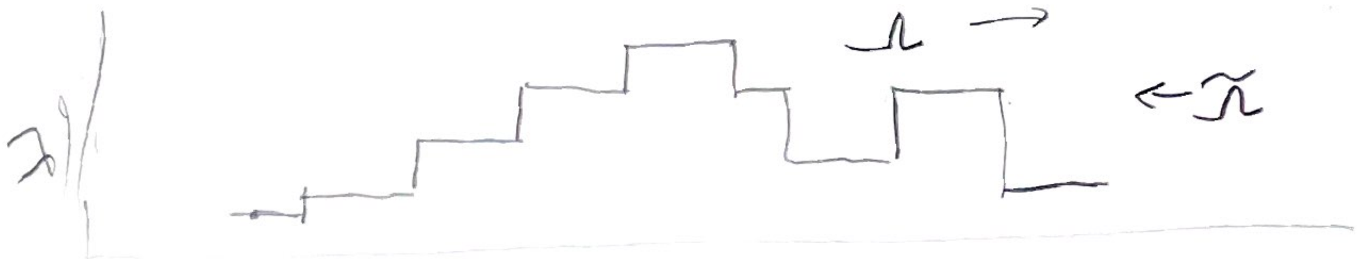
$$= e^{-\beta Q} = e^{+\Delta S_{env}}$$

40

[Crook 1998]

[Right]

Microscopic Fluctuation Theorem



[Center]

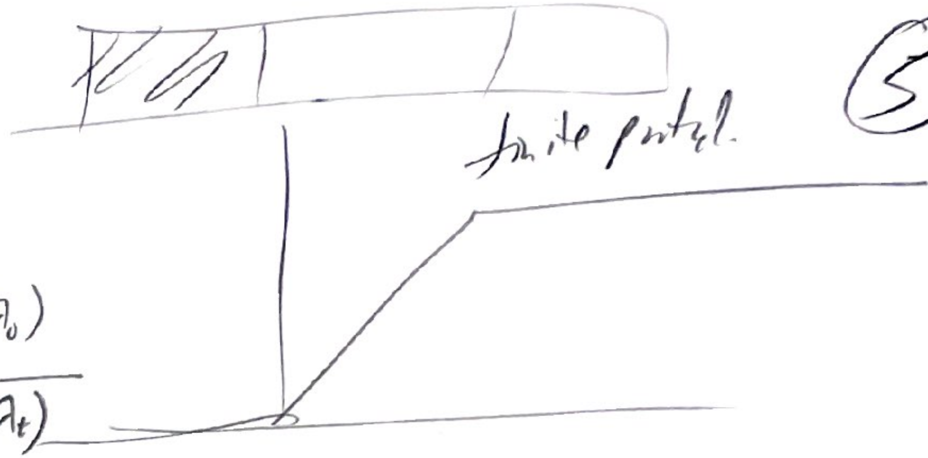
$$\frac{P(X|x_0, \Lambda)}{P(\tilde{X}|x_t, \tilde{\Lambda})} = \prod \frac{P(x_{t+1}|x_t, \Lambda_t)}{\tilde{P}(x_t|x_{t+1}, \tilde{\Lambda})} = \prod_t e^{-\beta[E(x_{t+1}) - E(x_t)]}$$

$$= e^{-\beta Q} = e^{+\beta \Delta S^{en}}$$

(center)

[D.B Property of the ensemble

$$\frac{P(x)}{P(\tilde{x})} = \frac{P(x|x_0,\Lambda)}{P(\tilde{x}|x_t,\tilde{\Lambda})} \frac{P(x_0|\lambda_0)}{\underbrace{P(x_t|\lambda_t)}}$$



$$= e^{-\beta Q} \underbrace{\frac{e^{-\beta E(x_0,\lambda_0)+\beta F(\lambda_0)}}{e^{-\beta E(x_t,\lambda_t)+\beta F(\lambda_t)}}}_{e^{+\beta\Delta E-\beta\Delta F}}$$

$$= e^{+\beta W[x,\Lambda]-\beta\Delta F} = e^{+\beta W^{ex}}$$

$$\Delta F = F(\lambda_t) - F(\lambda_0)$$

$$\Delta E = Q + W$$

$$\Delta E - Q = W$$

$$\left\langle e^{-\beta W[x,\Lambda]} \right\rangle = \sum_x P(x)\, e^{-\beta W[x,\Lambda]}$$

$$= \sum_{\tilde{x}} P(\tilde{x}) \frac{P(x)}{P(\tilde{x})} e^{-\beta W} = \underbrace{\sum_{\tilde{x}} P(\tilde{x})\, e^{+\beta W-\beta W-\beta\Delta F}}_{1}$$

$$= e^{-\beta\Delta F}$$