Open Quantum Dynamics

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Quantum Channels / CPTP Map

Ref: Nielsen & Chuang 2000/2010 (PDF Online)

Mixed Quantum state : Density matrix

$\rho = \sum_i p_i |i\rangle\langle i|$

or $\sum_i p_i P_i \leftarrow$ Projection operator.

e.g. $|0\rangle$ [illegible]

$P^2 = P$

$P^\dagger = P$

Dynamics Open System.

"Quantum Markov Chain"

$\rho' = S(\rho) \quad (\text{or } S\rho)$

$\uparrow$ superoperator

$\langle a | S(|c\rangle\langle b|) | b\rangle = S_{abcd}$

or $S^{a\ \ d}_{\ bc}\ \rho^{c}{}_{d} = \rho^{a}{}_{b}$

$S\rho = \sum_\alpha A_\alpha \rho A^\dagger_\alpha$ $\leftarrow$ Kraus operators

$\sum_\alpha A_\alpha^\dagger A_\alpha = I$

(number of Kraus operators $\leq d^2$)

Bloch Sphere

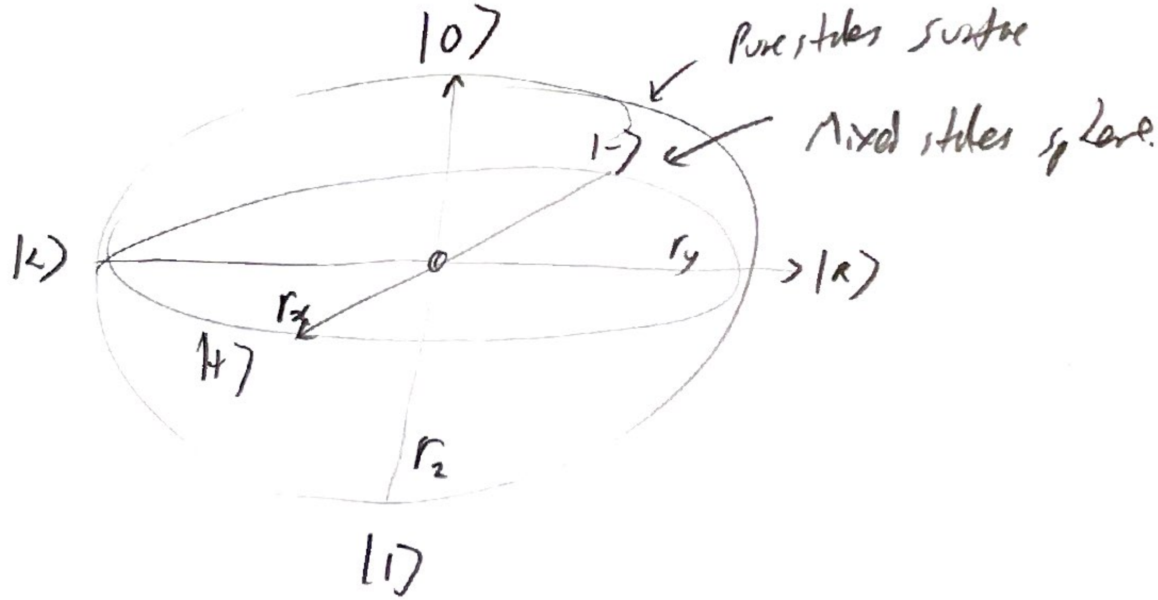
$$\rho = \frac{I + r_x \sigma_x + r_y \sigma_y + r_z \sigma_z}{2}$$

$\sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Bit flip

$\sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

[Euler is the ex e!]

$\sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ phase flip



[standard convention]
Good for Qubits, but doesn't generalize.

Ref for notes threeplusone/gates

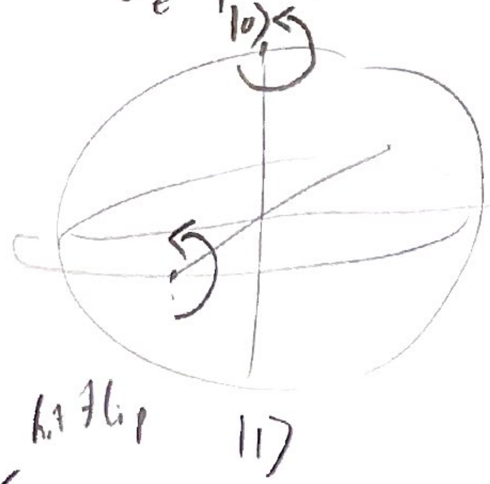
Unitary dynamics

$S\rho = U \rho U^\dagger$ (One unitary Kraus operator)

→ rigid rotations of sphere

—$\boxed{Z}$—

σ_z phase flip

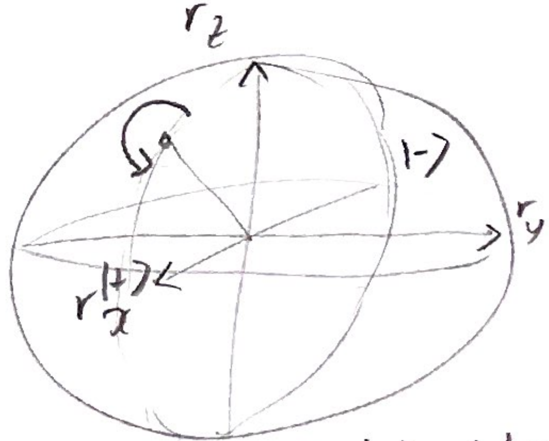


σ_x bit flip

X

—$\boxed{X}$—

Hadamard —$\boxed{H}$— $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$



$H|0\rangle =$

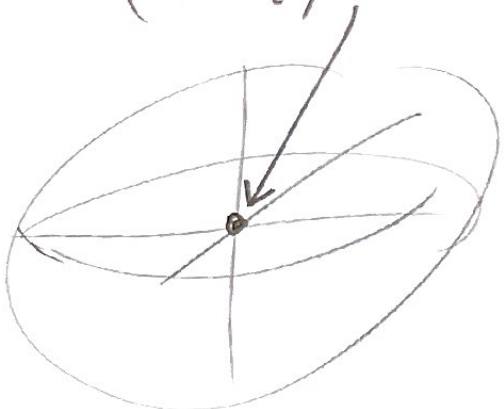
$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$

$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = |-\rangle$

$H|0\rangle\langle 0|H$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\rightarrow \rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Unital $\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ fixed point $S\rho = \rho$



e.g. $S\rho = \sum_\alpha U_\alpha \rho U_\alpha^\dagger$

Mixtures of Unitaries are Unital

(Not all Unitals are Mixtures of Unitaries)

(Equivalent to doubly stochastic matrix)

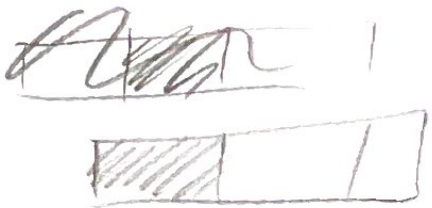
$p' = Mp$ $M^T \quad p'' = M^T p$

[Adjoint]
$S^\times = S'$ N...

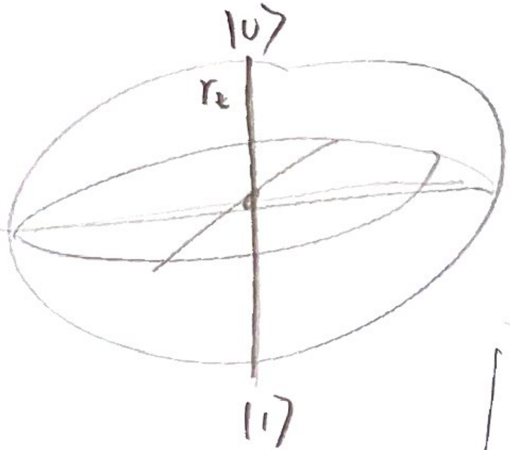
For Unital ... $S^\times = S$

Iff Unital Then $S^\times = S^{-1}$

Iff Perm Then $M^T = M^{-1}$



Measurement in computational Basis.



Kraus : $|0\rangle\langle 0|$ & : $|1\rangle\langle 1|$ $\quad \sum A^\dagger A = I$

$$\rho' = |0\rangle\langle 0|\,\rho\,|0\rangle\langle 0| + |1\rangle\langle 1|\,\rho\,|1\rangle\langle 1|$$

system output

$|00\rangle\langle 00|$

$|00\rangle\langle 01|$

$|11\rangle\langle 10|$

$|11\rangle\langle 11|$

Kraus operators for Measurement &

$|a\rangle \otimes |b\rangle \equiv |a\ b\rangle$

Opd.

POVMs

Each Kraus operator is an effect on system that could be measured.

General Measurements POVMs "Positive Operator Value Measures"

Projective Measurement (von Neumann)

$$p(m) = \langle i | P_m | i \rangle \quad \text{or} \quad \mathrm{Tr}\, P_m \rho$$

$$H = \sum_m m P_m \qquad \langle m \rangle = \mathrm{Tr}\, H \rho$$

Kraus $S_\rho = \rho'$

$$p(m) = \mathrm{Tr}\, A_m \rho A_m^\dagger = \mathrm{Tr}\, \underbrace{A_m^\dagger A_m}_{\text{positive operator } E_m} \rho$$

state after measurement α

$$\rho_\alpha = \frac{A_\alpha \rho A_\alpha^\dagger}{\mathrm{Tr}\, A_\alpha \rho A_\alpha}$$

Note $\mathrm{tr}\, \mathcal{A} b_\rho$ also called due to correlation function
Arise in perturbative expansions

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Crooks 2007

Hermitian Map

measurement outcomes

$$\mathcal{A}\rho = \sum_\alpha a_\alpha A_\alpha \rho A_\alpha^\dagger$$

value-operator-sum

$$[\mathcal{A} H]^\dagger = \sum_\alpha m_\alpha A_\alpha H^\dagger A_\alpha^\dagger = \mathcal{A} H^\dagger = \mathcal{A} H$$

$$\langle a \rangle = \mathrm{tr}\, \mathcal{A}\rho = \sum_\alpha a_\alpha \,\mathrm{tr}\, A_\alpha \rho A_\alpha^\dagger = \sum_\alpha a_\alpha\, p(\alpha)$$

Correlation function.

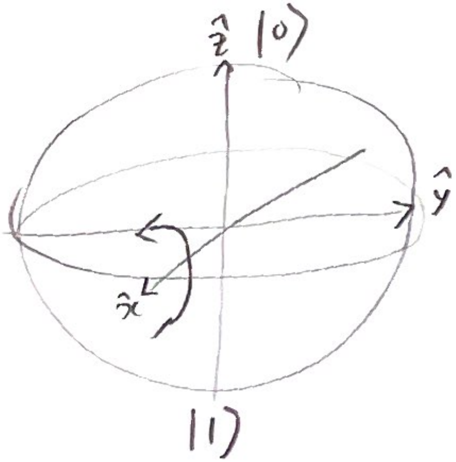
$$\langle b(t)\, a(0) \rangle = \mathrm{tr}\, \mathcal{B} S_t \mathcal{A} \rho(0)$$

$$= \sum_{\substack{\alpha\beta \\ s}} a_\alpha b_\beta \,\mathrm{tr}\, B_\beta K_s A_\alpha \rho(0) A_\alpha K_s B_\beta = \sum_{\alpha\beta} p(\alpha,\beta)\, a_\alpha b_\beta$$

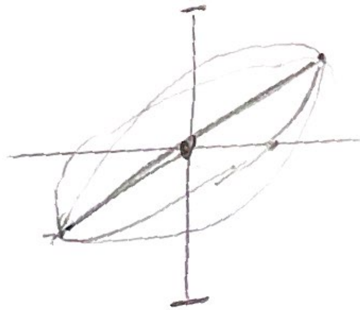
Channels

Bit flip Channel

$A_0 = \sqrt{p}\, I = \sqrt{p}\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $A_1 = \sqrt{(1-p)}\, X = \sqrt{(1-p)}\begin{pmatrix}0&1\\1&0\end{pmatrix}$

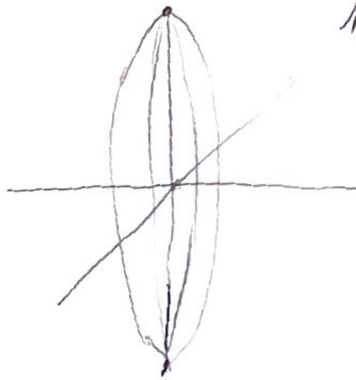


$\Longrightarrow$



Unital

$\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ phase flip Z instead of X $\Longrightarrow$



Doesn't change $|0\rangle$ & $|1\rangle$

But lose Quantum Coherences

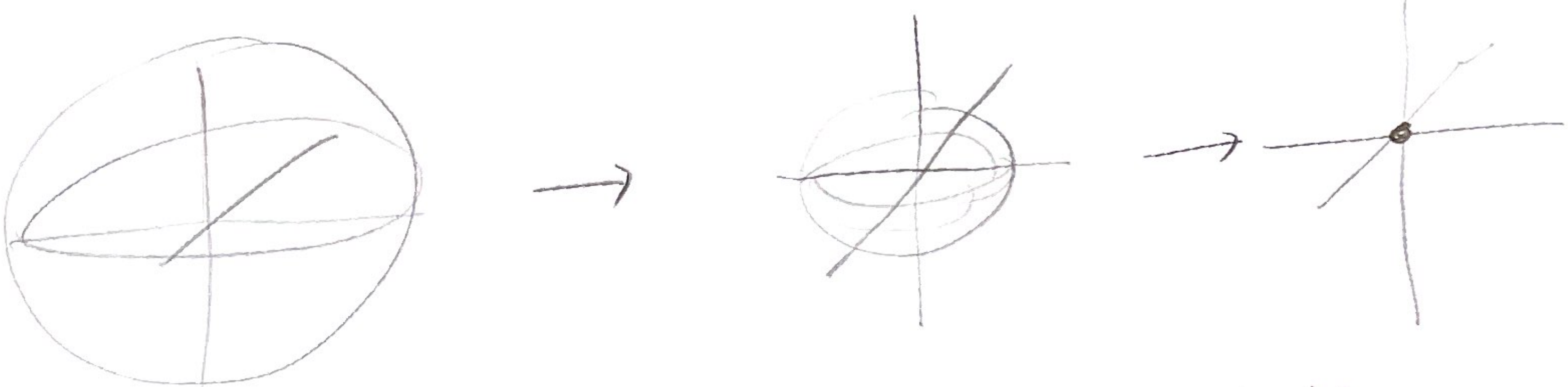
Bit-phase flip $Y = iXZ$

Same for Y-axis.

Depolarization Channel

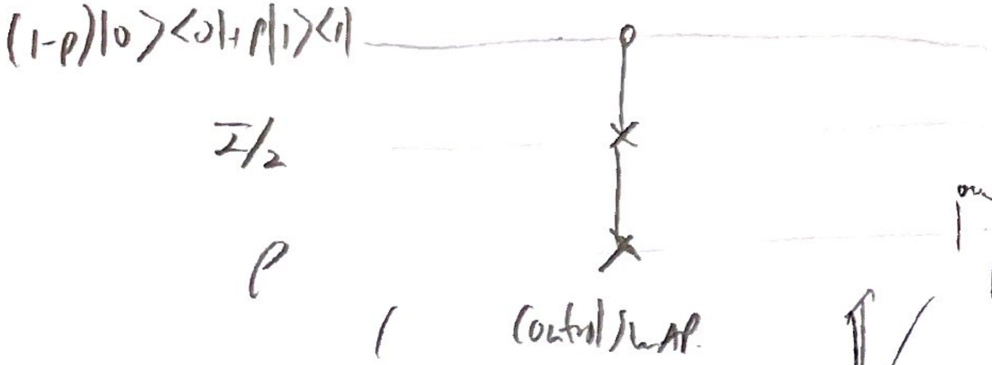
$$\mathcal{E}(\rho) = \frac{pI}{2} + (1-p)\rho$$

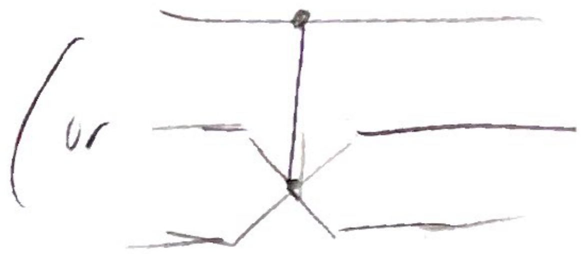
$$= \sqrt{\left(1-\frac{3p}{4}\right)} I\rho I \sqrt{1-\frac{3p}{4}} + \frac{p}{4}\left(X\rho X + Y\rho Y + Z\rho Z\right)$$

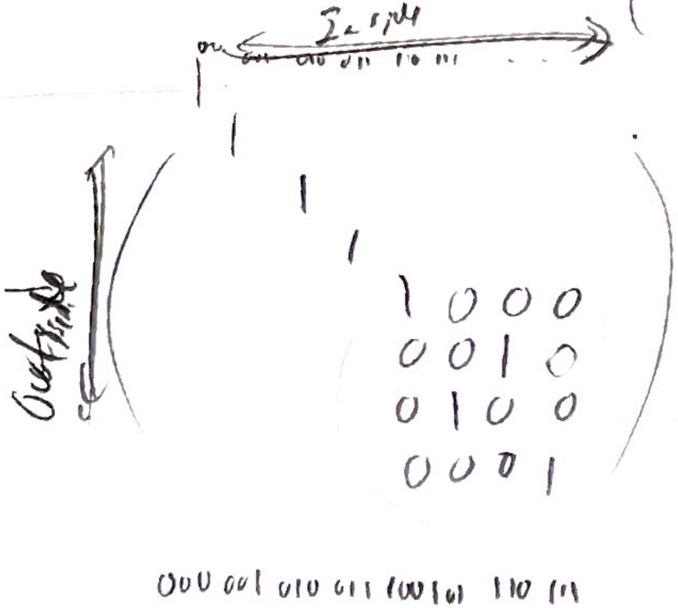


Unital

Depolarize channel   Circuit Model   Need 2 environment Qubits







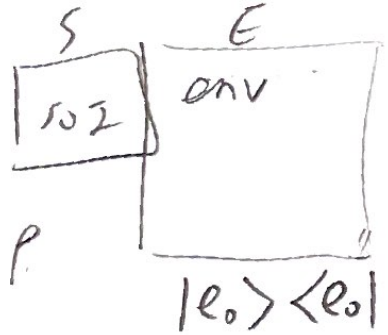
Controlled SWAP

(Circuit Model)

Fredkin Gate.

Any Channel is Unitary on larger Space. [Many Worlds]

Stinespring's Dilation Theorem



Partial Trace over environment

$$S_\rho = \mathrm{Tr}_E\, U_{SE}(\rho \otimes |e_0\rangle\langle e_0|)\, U_{SE}^\dagger$$

$$= \sum_\alpha A_\alpha \rho A_\alpha^\dagger$$

dimension $\quad d \quad\quad d^2$ (i.e. twice as many Qubits)

"Church of the Larger Hilbert Space" John Smolin

- every mixed state is a pure state of a bigger space
- every mixed dynamics is a pure dynamics of the larger space.

→ Measurement is not a separate dynamics in QM

→ But you do need low entropy environment $|e_0\rangle\langle e_0|$

Quantum Process Tomography (Hard) n Qubits $d = 2^n$

d	State space Hilbert Space dimension d	$2d-1$
ρ	mixed state	d^2-1
	Unitary dynamics	d^2-1
	Mixed Dynamics	d^4-d^2

(Also SPAM errors)

State Preparation & Measurement)

$$\rho' = S\rho = \sum_\alpha A_\alpha \rho A_\alpha^\dagger$$

Superoperator Adjoint

$$S^\times \rho = \sum_\alpha A_\alpha^\dagger \rho A_\alpha$$

$$S^\times I = \sum_\alpha A_\alpha^\dagger I A_\alpha = \sum_\alpha A_\alpha^\dagger A_\alpha = I$$

Inner Product $\langle S_1, S_2 \rangle = \mathrm{Tr}\, S_1^\times S_2$ → Hilbert Space.

Hilbert-Schmit Inner Product