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①

Quantum Master Equation / Lindblad Dynamics

↑ GORAN LINDBLAD, Stockholm

	CLASSICAL	QUANTUM	
Discrete Time Markov Chain	$p' = M p$	Quantum Channel / CPTP Map Quantum Operation	$\rho' = S \rho$. (Quantum Information Theory Wilde 2017)
Continuous Time Master Equation	$\frac{\partial p}{\partial t} = Q p$	Quantum Master Eq.	$\frac{\partial \rho}{\partial t} = -i \mathcal{L} \rho$

↑ Lindbladian Super operator

Embeded Markov chain

for thermal equilibrium, energy eigenbasis is

$$\rho = p_i |i\rangle\langle i| \quad , \quad \pi = \frac{e^{-\beta H}}{\mathrm{Tr}\, e^{-\beta H}} = e^{-\beta E_i} |i\rangle\langle i|$$

$$S\rho = \sum_\alpha A_\alpha \rho A_\alpha^\dagger \qquad A_{fi} = \sqrt{M_{fi}}\, |f\rangle\langle i|$$

$$p' = Mp \quad \equiv \mathrm{diag}(p') = S[\mathrm{diag}(p)]$$

$$S_{ff\,ii} = M_{fi} \quad \text{or} \quad S = \mathrm{diag}(M)$$

$$\mathrm{diag}(A)\,\mathrm{diag}(B) = \mathrm{diag}(AB)$$

matrix

von Neumann Equation

[(Liouville) - von Neumann Equation]

Unitary

$$\rho(t) = e^{-iHt/\hbar} \rho(0) e^{iHt/\hbar}$$

$$\frac{\partial}{\partial t}\rho(t) = \left(\frac{-i}{\hbar}H\right) e^{-iHt/\hbar} \rho(0) e^{iHt/\hbar} + e^{-iHt/\hbar} \rho(0) e^{+iHt/\hbar}\left(\frac{iH}{\hbar}\right)$$

$t \to 0$

$$\frac{\partial}{\partial t}\rho = \left(\frac{-i}{\hbar}H\right)\rho(0) + \rho(0)\left(\frac{i}{\hbar}H\right)$$

$$= -\frac{i}{\hbar}[H, \hat{\rho}] = -i\mathcal{L}\rho$$

$$\uparrow$$

$$\equiv \frac{1}{\hbar}[H, \cdot\,]$$

$[A,B] = AB - BA$

Quantum Master Equation / Lindbladian

Lindblad 1976

$$\frac{\partial\rho}{\partial t} = -i\mathcal{L}\rho = -i[H,\rho] + \sum_i \gamma_i \left[L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\}\right]$$

γ_i: positive real "damping coefficients"

L_i: Jump operators

$\{A,B\} = AB + BA$

All $\gamma_i = 0$, $\rightarrow$ von Neumann equation

Superficially more general: GKSL

diagonalize h (change of basis)

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + \sum_{n,m} h_{nm}\left(A_n \rho A_m^\dagger - \frac{1}{2}\{A_m^\dagger A_n, \rho\}\right)$$

h_{nm}: positive semidefinite

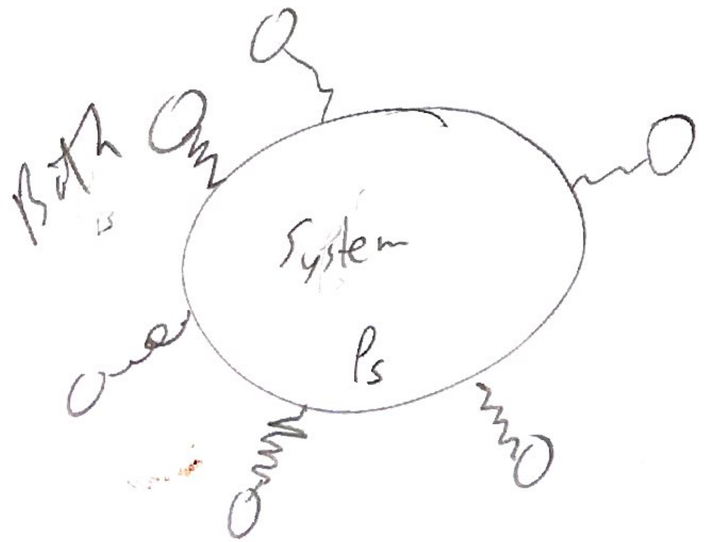
Redfield

Ref: David Limmer 220B Lecture 16

Redfield (1956) Nuclear spin Relaxation in Magnetic Resonance

$$\mathcal{H} = H_S + H_E + V$$

← weak coupling



← Trace out Harmonic bath modes

Large Thermal environment

Weak coupling

Continuously measure environment

$$\rho(t) = \rho_s(t) \otimes \rho_E \approx \rho_s(t) \otimes \underbrace{\frac{e^{-\beta H_E}}{Tr_E\, e^{-\beta H_E}}}_{\text{Thermal equilibrium bath}}$$

Continuously Measure the system

Born approximation (Markov ~~essentially~~)

→ environment starts ~~remains~~ ~~in~~ remains in thermal equilibrium

i.e. Assume bath relaxes much faster than system

Trace out bath degrees Keep only up to 2nd Order (Linear Response)

relaxation, dissipation, decoherence ⇒ come from bath correlations

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + \sum_i \gamma_i \left[L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\} \right]$$

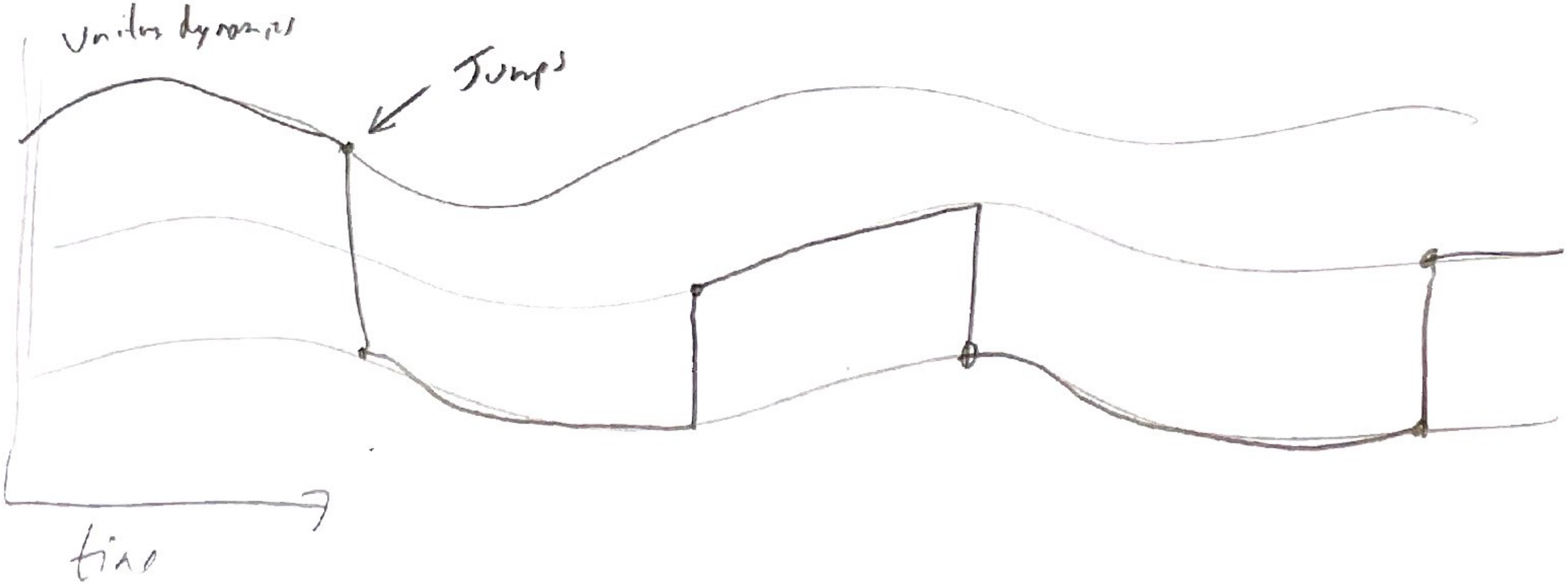
Unitary dynamics

Jumps

Normalization, no Jump

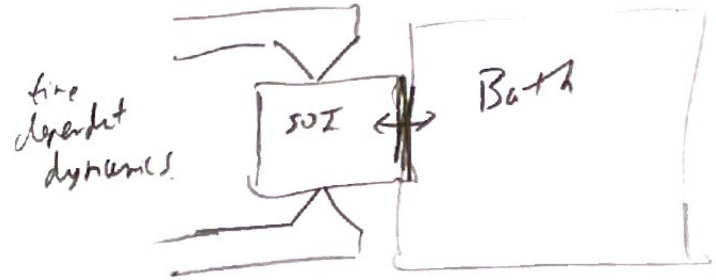
Continuous measurement of the environment

Unravel

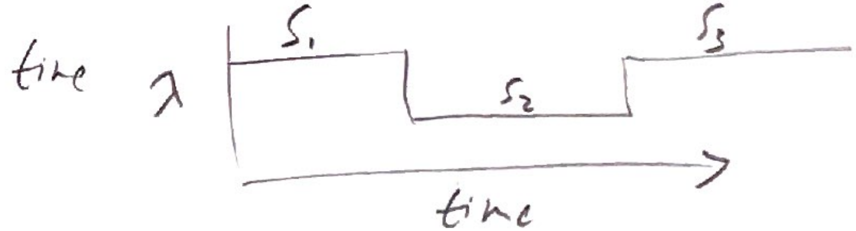


Thermodynamics

$$\rho(3) = S_3\, S_2\, S_1\, \rho(0)$$



$$S\rho_s = tr_B\, U_{SB} \left[\rho \otimes \pi_B \right] U_{SB}^\dagger$$



$$U_{SB} = e^{-i H_{SB}^{s} t/\hbar}$$

$$= \sum_{if} \frac{e^{-\beta E_i^B}}{Z_B} \langle b_f | U_{SB} | b_i \rangle \rho_s \langle b_i | U_{SB}^\dagger | b_f \rangle$$

$$A_{if} = \frac{e^{-\frac{1}{2}\beta E_i^B}}{\sqrt{Z_B}} \langle b_f | U_{SB} | b_i \rangle \qquad [d^2!]$$

Open System Quantum FT

$U^{\dagger}_{SB} = U^{-1}_{SB}$ inverse dynamics.

time Reversed dynamics! Can't observe system. Can observe environment

Crooks 2008

$$\tilde{A}_{if} = \frac{e^{-\frac{1}{2}\beta E^B_f}}{\sqrt{Z_B}} \langle b_i | U^{\dagger}_{SB} | b_j \rangle$$

$$P(A^3_\gamma A^2_\beta A^1_\alpha | e_0, e_\tau) = \mathrm{Tr}\ A^{(3)}_\gamma A^{(2)}_\beta A^{(1)}_\alpha |0\rangle\langle 0| A^{(1)}_\alpha A^{(2)}_\beta A^{(3)}_\gamma\ | e_\tau \rangle \langle e_\tau |$$

$$\tilde{P}(\tilde{A}^1_\alpha \tilde{A}^2_\beta \tilde{A}^3_\gamma | e_\tau, e_0) = \mathrm{Tr}\ A^{(1)}_\alpha A^{(2)}_\beta A^{(3)}_\gamma | e_\tau \rangle\langle e_\tau)\ A^{(3)}_\gamma A^{(2)}_\beta A^{(1)}_\alpha\ | e_0 \rangle \langle e_0 |$$

$$\ln \frac{P(\quad)}{\tilde{P}(\quad)} = -\sum_t \beta \Delta E = -\beta Q$$

Problems!
Assumptions
Experiments

$$\tilde{\mathcal{L}} = D_\pi \mathcal{L}^\times D_\pi^{-1}$$

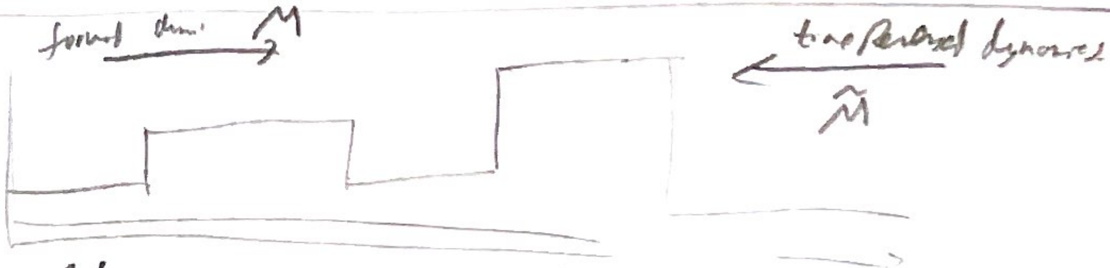
(10)

Super-Duper Operators

Quantum Information Theory - Wilde

time Reversal

$$\tilde{M} = \mathrm{diag}(p^{eq}) M^T \mathrm{diag}(p^{eq})^{-1}$$



Detailed Balanced if $M = \tilde{M}$

Quantum Channel

$\cancel{\rho}\pi = S\pi$

$$\tilde{S} = D_\pi S^\times D_\pi^{-1} \qquad D_\pi = \sqrt{\pi}\, \rho \sqrt{\pi}$$

Diagonal super operator.

Detailed balanced if $\tilde{S} = S$

$$\tilde{S}\pi = D_\pi S^\times D_\pi^{-1} \pi = D_\pi S^\times \mathcal{I} = D_\pi \mathcal{I} = \pi$$

Petz Recover Map

Super-Duper Operator!

$$\tilde{S}^\times \mathcal{I} = \hat{\hat{D}}_\pi^{-1} \hat{\hat{S}} \hat{\hat{D}}_\pi \hat{\mathcal{I}} = D_\pi^{-1} S \pi = D_\pi^{-1} \pi = \mathcal{I}$$