Refs

Mathematical Physics Geroch

Why do we call numbers Scalars?

#17

(1)

Nielsen & Chuang 2000

"Mike & Ike" Quantum Computing & Quantum Information

threeplusone.com/gates

Hilbert Space

(1) Complex vector space, H x, y, z

(2) A Rule "Inner Product" maps
Pairs of vector → scalars

$$\langle x, y \rangle = c \quad (\text{scalar } a, b, c)$$

Complex Vector Space.

Contains
1) A set of Vectors V
2) Sum Rule

$$x = y + z$$

3) Product Rule Scalars

$$x = c\, z$$

a, b, c Scalars.

Given Conditions

a) V abelian under addition

commute $x + y = y + x$

identity $0 + y = y$

$(-x) + x = 0$ Inverses

b $(a+b)x = ax + bx$

c $(ab)x = a(bx)$

d $1x = x$

Inner Product

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Inner Product

$$\langle x, y \rangle = c$$

Conditions

a) $\langle c x, y \rangle = c^* \langle x, y \rangle$

b) $\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

c) $\langle x, y \rangle^* = \langle y, x \rangle$

d) $\langle x, x \rangle$ = Real & positive (non-zero vectors)

e) The Topological Vector Space V is complete

Topology → sense of which vectors are near each other

Complete →

why? So that inf Dimension Hilbert Space well behaved

Norm = $\sqrt{\langle x, x \rangle}$

$$d = \frac{\sqrt{\langle x-y, x-y \rangle}}{\sqrt{\langle x, x \rangle \langle y, y \rangle}}$$

$$\cos^2 D_{FS} = F = \frac{|\langle x, y \rangle|^2}{\langle x, x \rangle \langle y, y \rangle}$$

Fidelity

Fubini-Study angle

QM states vectors in Hilbert space

DIRAC NOTATION

$|x\rangle$ "ket" x^a "column vector"

Inner Product $\langle x | y \rangle$

Dual vector space "bra" $\langle x | = |x\rangle^\dagger$

x_a "Row Vector"

Vectors Normalized

$\langle x | x \rangle = 1$

(direction not length)

phase irrelevant $|x\rangle \equiv c\,|x\rangle$ $|c|=1$

Half Job ③

$\Downarrow$

Dynamics (closed system)

$$|x(t)\rangle = U\,|x(0)\rangle$$

Unitary operator $U^\dagger = U^{-1}$

$$= e^{-it\mathcal{H}}$$

$H^\dagger = H$ Hermitian (all Real eigenvectors)

$|\det U| = 1$

$\det U = e^{i\psi}$

if $|\det U| = 1 \rightarrow SU$

Measurement Von-Neumann projective

$$\langle x|H|x\rangle = \langle H\rangle$$

$$H = \sum_m a_m P_m$$

$\leftarrow$ Projection

$\uparrow$ eigenvalues

$P_m P_m = P_m$

$P_m^\dagger = P_m$

$\sum_m P_m = I$

e.g. $P_m = |m\rangle\langle m|$

$P = $ e.g. $|m\rangle\langle m| + |n\rangle\langle n|$

$$p(m) = \langle x|P_m|x\rangle$$

New state $= \dfrac{P_m|x\rangle}{\sqrt{p(m)}}$

Mixed States

$$\rho = \sum p_m |m\rangle\langle m|$$

$$\rho^{a}_{b}$$

Pure

Dynamics

$$\rho(t) = U \rho U^\dagger$$

$$= \sum p_m U|m\rangle\langle m| U^\dagger$$

Measurement

$$\rho_m = \frac{P_m \rho P_m}{\mathrm{Tr}\, P_m \rho P_m}$$

$$\langle H \rangle = \mathrm{Tr}\, H\rho \quad \sum_m \mathrm{Tr} \quad E_m P_m \rho P_m$$

Quantum Eq. Thermodynamics

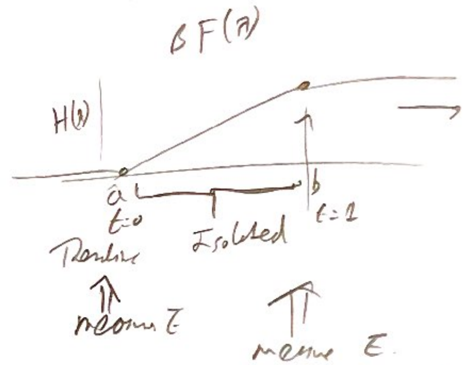
$$\rho = \frac{e^{-\beta H}}{\mathrm{Tr}\, e^{-\beta H}}$$

H diagonal, Eigenvalues E_i

$$\rho = \frac{1}{Z} \sum e^{-\beta E_i} |i\rangle\langle i|$$

$$Z = -\ln \beta F$$

Quantum Jarzynski /FT   2 time Energy Measmt.



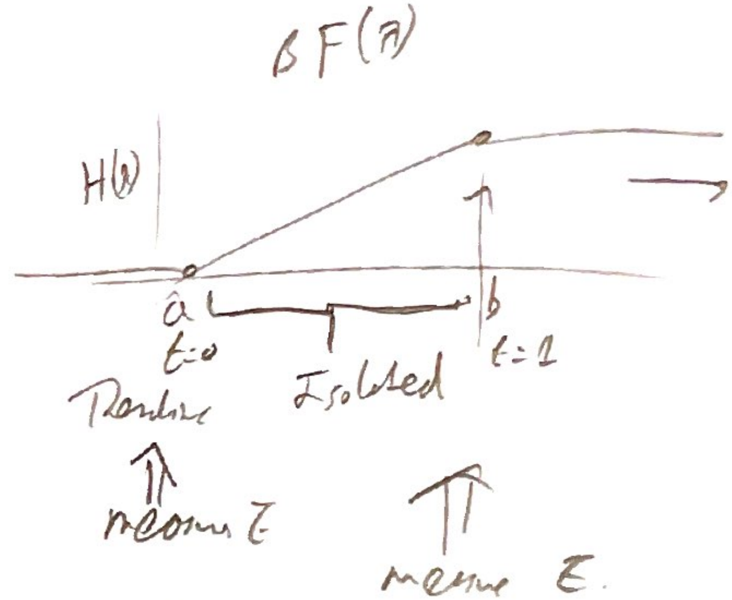
$$\rho = \frac{e^{-\beta H}}{\operatorname{Tr} e^{-\beta H}} \qquad \rho = \frac{1}{Z} \sum e^{-\beta \epsilon_i} \underbrace{|i\rangle\langle i|}_{P_i}$$

$$Z = -\ln \beta F$$

$$P(i, f) = \operatorname{Tr} \; P_f \, U \, P_i \, \rho_a^{eq} P_i \, U^\dagger P_f = e^{+\beta F^a - \beta \epsilon_i^a} \operatorname{Tr} P_f \, U \, P_i \, U^\dagger$$

$$\tilde{P}(f, i) = \operatorname{Tr} \; P_i \underset{\substack{\uparrow \\ \text{backwards in time}}}{U^\dagger} P_f \, \rho_b^{eq} P_f \, U \, P_i = e^{\beta F^b - \beta \epsilon_f^b} \operatorname{Tr} P_i \, U^\dagger P_f \, U$$

Quantum Jarzynski / FT 2 time Energy Measurement.



$$\rho = \frac{e^{-\beta H}}{\mathrm{Tr}\, e^{-\beta H}} \qquad \rho = \frac{1}{Z} \sum e^{-\beta E_i} \underbrace{|i\rangle\langle i|}_{P_i}$$

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$$P(i, f) = \mathrm{Tr} \quad P_f \; U \; P_i \; \rho_a^{eq} P_i \; U^\dagger \; P_f \quad = e^{+\beta F_i^a - \beta E_i^a} \, \mathrm{Tr}\, P_f \, U \, P_i \, U^\dagger$$

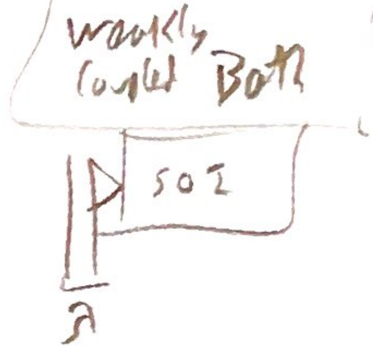
$$\tilde{P}(f, i) = \mathrm{Tr} \quad P_i \; U^\dagger \; P_f \; \rho_b^{eq} P_f \; U \; P_i \quad = e^{\beta F^b - \beta E_f^b} \, \mathrm{Tr}\, P_i \, U^\dagger \, P_f \, U$$

backwards in time

$$\frac{P(i,f)}{P(f,i)} = e^{-\beta\Delta F + \underbrace{\beta\Delta E}}$$

$$E_f^a - E_f^b = \delta W$$

$$\Delta F = \beta F^a - F^b$$



$$\langle e^{-\beta W}\rangle = e^{-\beta\Delta F}$$

Jorzynski 1999

Quantum Work → Hard concept

(Implicit - Unitary Rigid Rotation det = 1)

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Generalized Dynamics

superoperator $S\rho = \rho'$

$S^{a\ \ d}_{\ bc}\,\rho^{c}_{\ d} = \rho^{a}_{\ b}$

Hilbert Space

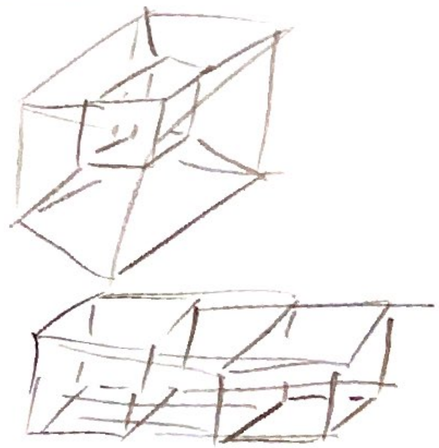
Operators form a Vector Sp $C = A + B$

$C = cA$

$\langle a | S(|c\rangle\langle d|)|b\rangle = S_{abcd}$

Hilbert - Schmidt Inner Product $\langle A, B\rangle = Tr(A^{\dagger} B)$

Hypercube



Superoperator $S(\rho) = \rho'$ operator of an operator

Super duper operators $\langle A, B\rangle = Tr(A^{\times} B)$

$\cancel{(A\, BB^\dagger A^\dagger)}$

(for any operator A, $A^\dagger A$ is positive

~~Hermitian A~~ $\cancel{\langle e | A | e \rangle \geq e'e}$ $\rho = BB^\dagger$

~~Then~~

CPTP Map completly Positive, Trace Preserving.

$$S\rho = \sum_\alpha A_\alpha \rho A^\dagger \qquad \sum A_\alpha^\dagger A_\alpha = I$$

↑ Kraus ops.

Trace Preserving

$$\operatorname{Tr} S\rho = \sum_\alpha \operatorname{Tr} A_\alpha \rho A_\alpha^\dagger = \operatorname{Tr}\left(\sum A_\alpha^\dagger A_\alpha\right)\rho = \operatorname{Tr} I\rho = \operatorname{Tr} \rho$$

Positive Map positive operators $\longrightarrow$ positive operators

Complete $\rightarrow$