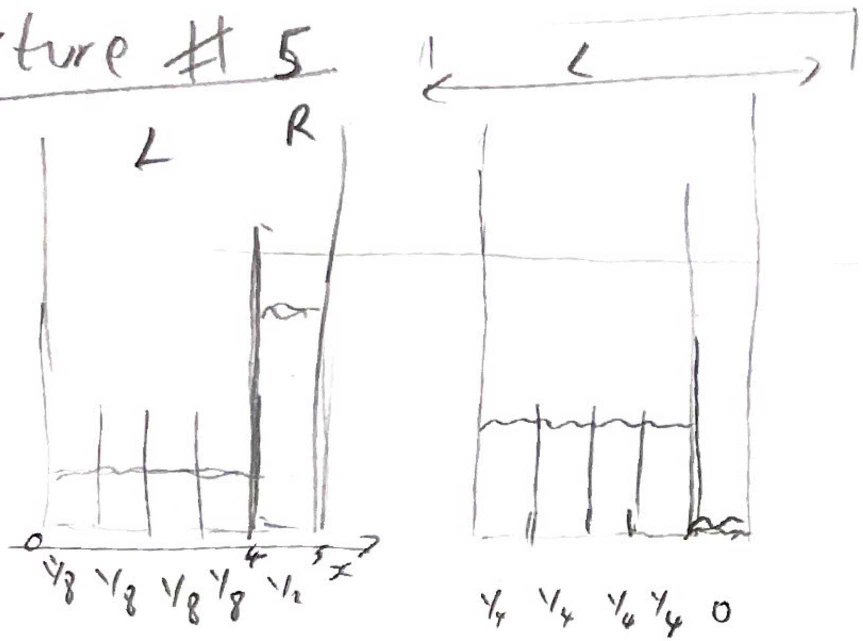
Lecture #5



Review entropy of decisions

Continuous

$$-\int p \ln \frac{p}{q}\, dx$$

$$S = 4 \times \frac{1}{8} \times 3 + \frac{1}{2} \qquad S = 4 \times \frac{1}{4} \times 2 =$$

$-\lg 2^{-3} = 3$

2 bits $\qquad = 2$ bits

$$S(A,B) = S(A) + S(B|A)$$

$$= S(A) + \sum P(a,b) \log P(b|a)$$

$$\sum_a P(a) \underbrace{\sum_b P(b|a) \log P(b|a)}_{\langle S(B|a) \rangle}$$

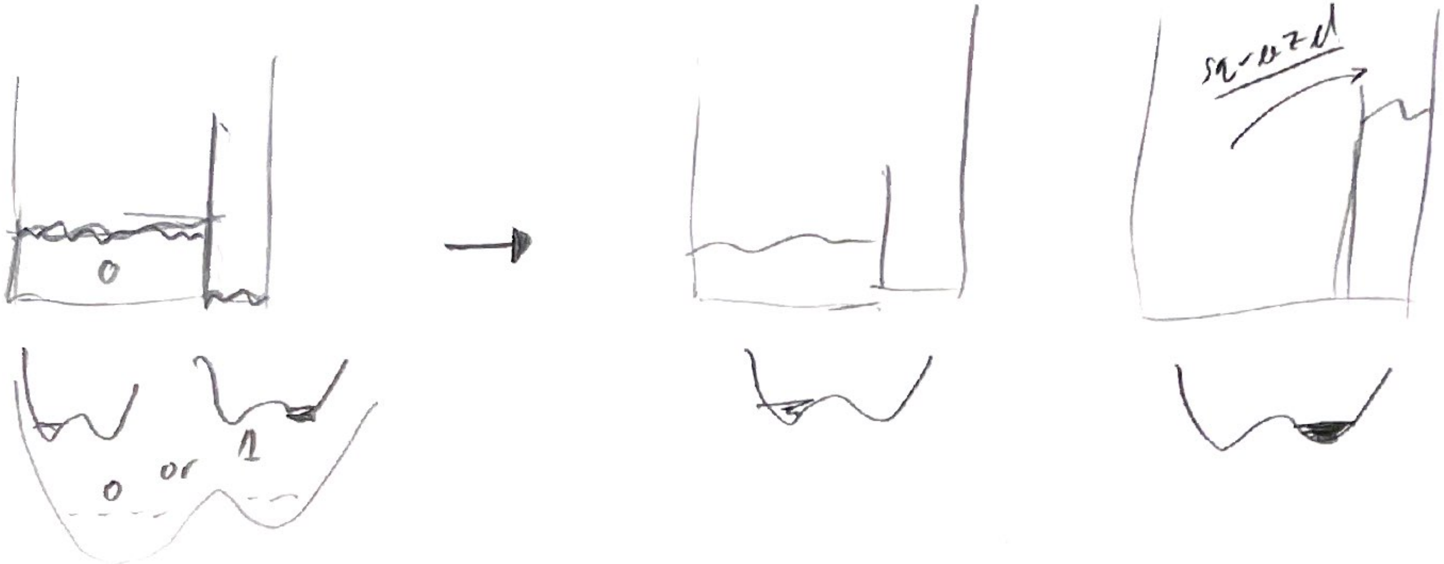
$$S(A) = 1 \text{ bit} + \frac{1}{2}\underbrace{S(B|l)}_{2} + \frac{1}{2}\underbrace{S(B|r)}_{0}$$

$= 2$ bits.

$$-\int_0^4 \frac{1}{8} \lg \frac{\frac{1}{8}}{1/L}\, dx - \int_0^5 \frac{1}{2} \ln \frac{\frac{1}{2}}{1/L}\, dx \qquad \underbrace{-\int_0^4 \frac{1}{4} \lg \frac{\frac{1}{4}}{L}\, dx}$$

$= 2$ bits $- \lg L \qquad = 2 - \lg L$?

Relative



NOTE: Quantum Different! No Cloning Theo

Can't copy Quantum information.

Intro to Jarzyski ③

CLAUSIUS 1864

The energy of the world is constant

The entropy of the world tends to a maximum.

$$\Delta S^{tot} \geq 0 \qquad \Delta E = Q + W$$

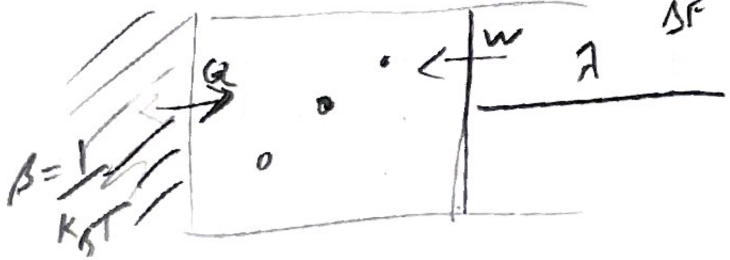
$$\Delta S_s + \underbrace{\Delta S^{env}}_{-\beta Q}$$

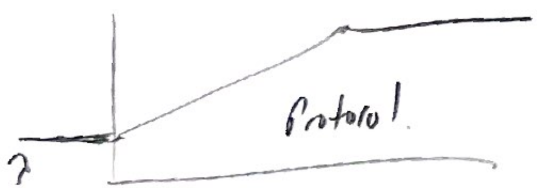
$$\Delta \quad \underbrace{-\beta Q}_{+\beta W - \beta \Delta E}$$

$$\underbrace{\Delta S - \beta \Delta E}$$

$$-\beta \Delta F + \beta W \geq 0$$

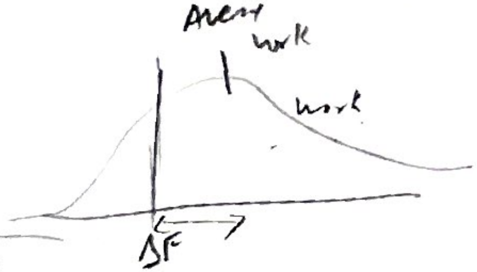
$$\Delta F \leq W$$





Jarzynski equality 1997

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F} \quad !$$



↓ Jensen

$$\Delta F \leq \langle W \rangle$$

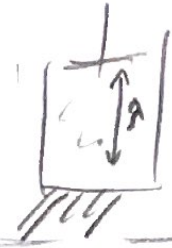
[Surprise in 1997!
Not a near-equilibrium result]

[Journey → more important than destination
Not going to follow Chris' original derivation.
→ come back to that later]

"Fluctuation Theorems" (Then models, experiments, consequences)

Canonical Ensemble

"Boltzman distribution"



Gibbs 1902
~~Boltzmann~~

$$P_A(x) = \frac{e^{-\beta E(x,\lambda)}}{Z} = e^{\beta F - \beta E(x,\lambda)}$$

$$Z = \sum_x e^{-\beta E(x,\lambda)}$$

$$\beta F = -S + \beta\langle E\rangle$$

$$= \sum P(x)\ln P(x) + \beta\langle E\rangle$$

$$= \sum P(x)\ln \frac{e^{-\beta E(x)}}{Z} + \beta\langle E\rangle$$

$$= -\ln Z + \underbrace{\sum P(x)\left(-\beta E(x)\right) + \beta\langle E\rangle}_{0}$$

$$\beta F \overset{eq}{=} -\ln Z$$

$$\frac{\partial \beta F}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$$

$$= -\frac{1}{Z}\sum_x[-E(x)]\,e^{-\beta E(x)}$$

$p(x)$

$$= \langle E\rangle$$

Maximum Entropy

Entropy is maximized at Thermodynamic equilibrium.

Calculus of variations / Lagrange multipliers

Maximize $S = -\sum P(x) \ln P(x)$ subject to constraints $\sum P_x = 1$ & $\sum P_x E(x) = \langle E \rangle$

$J(P) = S - \lambda_0 \sum P_x - \lambda_1 \sum P_x E(x)$

variation $\frac{\delta J}{\delta P} = 0$ $= -1 - \ln P(x) - \lambda_0 - \lambda_1 E(x)$

$$P(x) = e^{-1-\lambda_0 - \lambda_1 E(x)} = \frac{e^{-\beta E(x)}}{Z}$$

Max entropy Jaynes [whats wrong?
1963 → Not a general approach to reasoning
Max entropy property of equilibrium
Need to know constraints!]

Tsallis Entropy $S_q = \frac{-\sum p_i^q - 1}{q-1}$ $\langle E \rangle_q = \frac{\sum p_i^q E_i}{\sum_i p_i^q}$

Maximize $\rightarrow P_i = \frac{1}{Z}\left[1+ \beta(q-1) E_i\right]^{-1/q - 1}$

"Power laws"

JUNK

Don't waste Your Time!

"Maximum Caliber"

Temperature

<u>Thermodynamics</u> S function of E, V, N.... extensive

$$cS(E,V,N) \overset{T.L.}{\approx} S(cE, cV, cN) \quad | \quad S \simeq$$

Euler's Theorem for 1st order Homogeneous function.

$$f(x,y,z) = ax + by + cz$$

$$a = \partial_x f = \partial_y f + \partial_z f$$

$$\lambda f(x,y,z) = f(\lambda x, \lambda y, \lambda z)$$

$$\frac{\partial}{\partial \lambda} \qquad f(x,y,z) = \frac{\partial f}{\partial \lambda x}\underbrace{\frac{\partial \lambda x}{\partial \lambda}}_{x} + \frac{\partial f}{\partial \lambda y}\underbrace{\frac{\partial \lambda y}{\partial \lambda}}_{y} + \frac{\partial f}{\partial \lambda z}\underbrace{\frac{\partial \lambda z}{\partial \lambda}}_{z}$$

let $\lambda = 1$

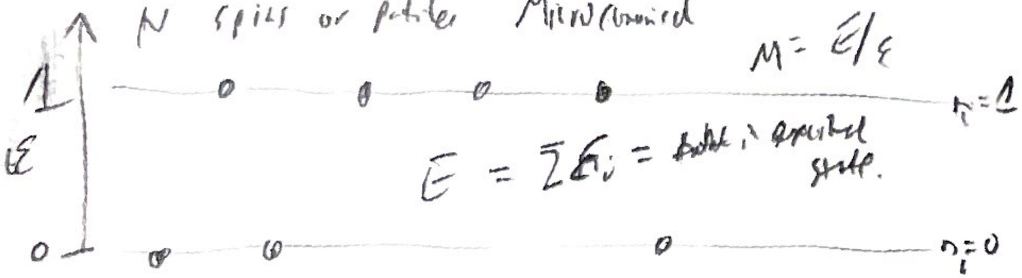
MC → Canonical

(9)

1950s

N spins or particles Microcanonical

$M = E/\varepsilon$



$E = \sum E_i$ = total # excited state.

N spins, Energy E

$$S = \ln \Omega = \ln \frac{N!}{\frac{E}{\varepsilon}!\left(N - \frac{E}{\varepsilon}\right)!}$$

Microcanonical

Spins not independent

Average $\langle E \rangle$ Canonical

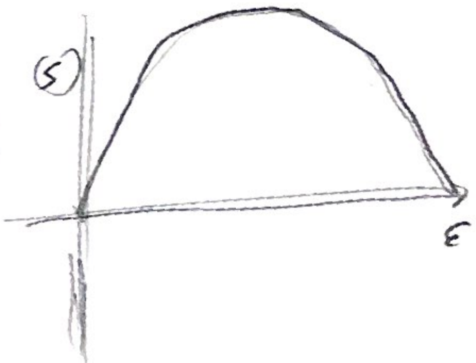
P(1)=

$\langle E \rangle$

$$Z = \sum_{E=0}^{N} \binom{N}{E} e^{-\beta E} = \sum_{n_1, n_2, n_3 \ldots n_N} e^{-\beta E}$$

$$S = N \ln \xi \qquad N\left(1 + e^{-\beta \varepsilon}\right)$$

$$P(n_i = 0,1) = \frac{e^{-\beta \varepsilon n_i}}{1 + e^{-\beta \varepsilon}}$$



9½

$\ln \frac{N!}{E!\,(N-E)!}$ $\qquad$ $\ln N! = N\ln N - N$

$N\ln(N-E) + E\ln(N-E)]$

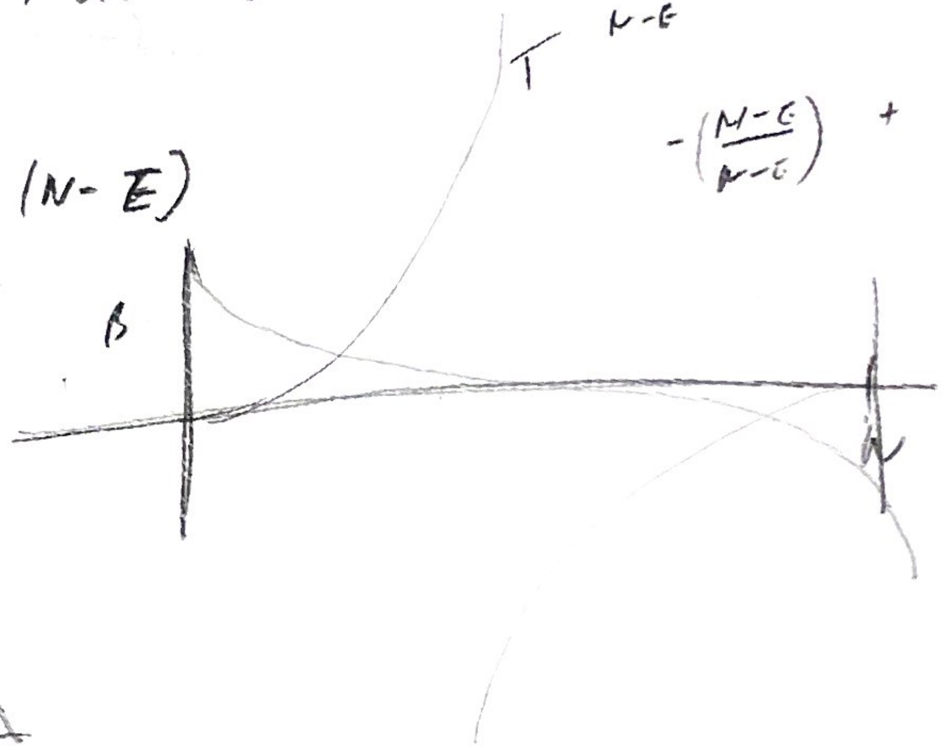
$S = N\ln N - E\ln E - (N-E)\ln(N-E)$

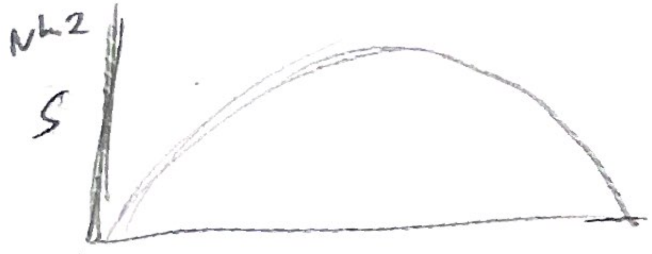
$\frac{N}{N-E}$

$-\left(\frac{N-E}{N-E}\right) + \ln N-E$

$\frac{\partial S}{\partial E} = -1 - \ln E \overset{?}{+1} + \ln(N-E)$

$\frac{\partial S}{\partial E} = \ln\left(\frac{N-E}{E}\right)$





(Dunkel et al 2014)

Temp Cont Inventing Temperature, Chang 2004

why not neg? Even not generally have upper bound

Not stable.

Temperature → well defined for Macroscopic eq. Systems.

Temperature of environment

$$P(x) \quad \frac{e^{-\beta E}}{Z}$$

Reserve, Both convert energy to entropy

e.g. Literm $W = k_B T \ln 2$

Does not generalize! to neg.

e.g. Kinetic Temperature every Harmonic Degree of freedom $\langle E \rangle = \frac{1}{2} k_B T$

$$\langle K \rangle = \frac{3}{2} k_B T N$$

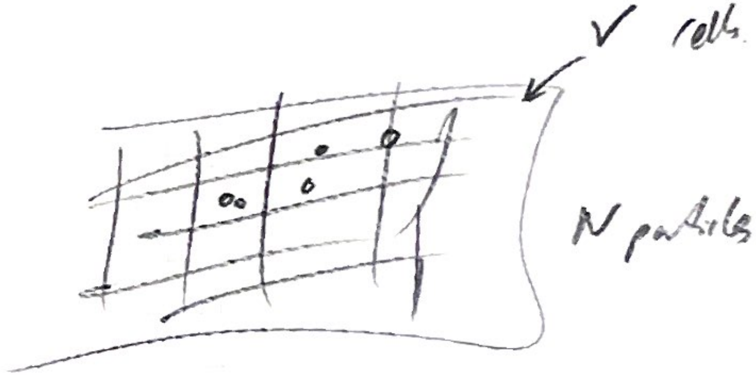
$$k_B T^{kinetic} \neq \frac{\langle K \rangle}{N} \frac{2}{3}$$

Ideal Gas

$$pV = nRT$$

$$pV = N k_B T$$

$$\underbrace{\beta p}_{\frac{\partial S}{\partial V}} = \frac{N}{V}$$



$$S = \ln \Omega = \ln \frac{V^N}{N!}$$

$$\boxed{\frac{\partial S}{\partial V} = \frac{N}{V}}$$