Radar Cross Section Engineering Homework 3

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ECE-5973-004

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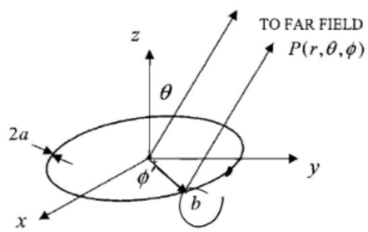
Part 1:

Circular Wire Example



P1: Loop made from resistive film + surfaces resistivity R_s.
Find MoM expression for load impedances $(Z_L)_{mn}$.

$\mathbf{Z}$: (3.68) (3.69) $\quad$ $\mathbf{R}$ Or $\mathbf{V}$: (3.72)

$$\left.\begin{aligned}\hat{k}\cdot\hat{r}' &= b\sin\theta\cos(\phi-\phi') \\ \hat{\phi}\cdot\hat{\phi}' &= \cos(\phi-\phi')\end{aligned}\right\}$$ Impedance & Excitation Matrices

$$\vec{J}_s(\phi) = \hat{\phi}\sum_{a=-\infty}^{+\infty} I_n \cdot \frac{e^{jn\phi}}{2\pi a}$$ } Entire Domain Base Function (Mode)

$$\vec{W}_n(\phi) = \hat{\phi}\cdot\frac{e^{-jn\phi}}{2\pi a}$$ } Conjugate of original base function.

Impedance and Excitation Matrices

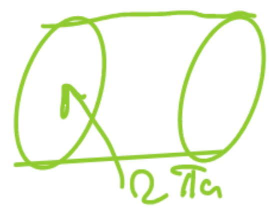
$$Z_{mn} = \iint_{S_m} ds \iint_{S_n} ds' \left[j\omega\mu_o \vec{W}_m(\vec{r}) \bullet \vec{J}_n(\vec{r}\,') - \frac{j}{\omega\varepsilon_o} \nabla' \bullet \vec{J}_n(\vec{r}\,') \nabla \bullet \vec{W}_m(r') \right] G(\vec{r}, \vec{r}\,')$$

$$V_m = \iint_{S_m} \vec{W}_m(\vec{r}) \bullet \vec{E}_i(\vec{r})\Big|_{\tan} ds \quad \rightarrow \quad \begin{cases} V_m^{\theta}, & \vec{E}_i = \hat{\theta} E_{i\theta} \text{ (TM)} \\ V_m^{\phi}, & \vec{E}_i = \hat{\phi} E_{i\phi} \text{ (TE)} \end{cases}$$

$$Z_m = \iint_{S_m} ds \iint_{S_n} ds' \Big[j\omega\mu \bar{W}_m(r) \cdot \bar{J}_n(\bar{r}\,') - \frac{j}{\omega\varepsilon}\left(\frac{j}{\omega\varepsilon}\right)$$

$$(\nabla' \cdot J_n(\bar{r}\,'))(\nabla \cdot \bar{W}_m(\bar{r}))\Big] G.T.$$

$$= \frac{\hat{\phi} \cdot \hat{\phi}'}{4\pi a^2} \cdot e^{j(n\phi' - m\phi)}$$



$$= \iint_S ds = 2\pi a \int_0^{2\pi} b\, d\phi \qquad \nabla = \hat{\phi} \frac{1}{b} \frac{\partial}{\partial \phi}$$

$$\vec{J}_n(\phi') = \hat{\phi}\, \frac{e^{jn\phi'}}{2\pi a} \quad \nabla' \cdot \vec{J}_n(\phi') = \frac{n \cdot e^{jn\phi'}}{2\pi \cdot a \cdot b}$$

$$\vec{W}_m(\phi) = \hat{\phi} \cdot \frac{e^{-jn\phi}}{2\pi a} \cdot \nabla \cdot \vec{W}_m(\phi) = -m \cdot \frac{e^{-jm\phi m}}{2\pi ab}$$

$$R = b\sqrt{4\sin^2\left(\frac{\Delta}{2}\right) + \frac{a}{b}} \quad \Delta = \phi - \phi' \quad \therefore \text{Not zero only when } m=n$$

$$Z_{mn} = \int_0^{2\pi} 2\pi a \cdot b\, d\phi \cdot \int_0^{2\pi} 2\pi\, a\, b\, d\phi'$$

$$= \left[\frac{\hat{\phi}' \cdot \hat{\phi}}{4\pi^2 \cdot a^2} \cdot e^{-j(m\phi - m\phi')} \cdot j\omega\mu + \frac{jm \cdot n}{\omega\varepsilon b^2} \cdot e^{-j(m\phi - n\phi')} \cdot \frac{1}{4\pi^2 a^2}\right] \cdot \frac{e^{-jkR}}{4\pi R}$$

$$Z_{mn} = Z_{nn} = \frac{1}{2\pi}\int_0^{2\pi} d\Delta \left[j\omega\mu b\cos\Delta - \frac{jn^2}{\omega\varepsilon b}\, e^{jn\Delta}\right] \cdot \frac{e^{-jkR}}{R}$$

$$R_n^{\phi} = 2\pi a \int_0^{2\pi} \vec{E}_n \cdot \vec{J}_n\, b\, d\phi = \pi b j^{n+1} e^{jn\phi}\left[J_{n+1}(kb\sin\theta) - J_{n-1}(kb\sin\theta)\right]$$

\# Can now calculate MoM solution using looped or straight wires.

$$(Z_L)_{mn} = \iint_S \vec{W}_m \cdot \left[R_s(\vec{r})\vec{J}_n(\vec{r})\right] ds$$

Thus:

$$\boxed{\begin{aligned} Z_{Lmn} &= (2\pi a) R_s \int_{-L/2}^{L/2} \left(\hat{z}\frac{P_m(z')}{2\pi a}\right) \cdot \left(\hat{z}\frac{P_n(z')}{2\pi a}\right) dz \\ &= \begin{cases} R_s \Delta / (2\pi a), & m = n \\ 0, & m \neq n \end{cases} \end{aligned}}$$

Part 2:

a. Installed FEKO
b. Using FEKO standard MoM solver, compute mono-static RCS of the rectangular PEC plate. Assume aa=bb=3$\lambda\lambda$. Generating results similar to Fig.2.13 in text book.

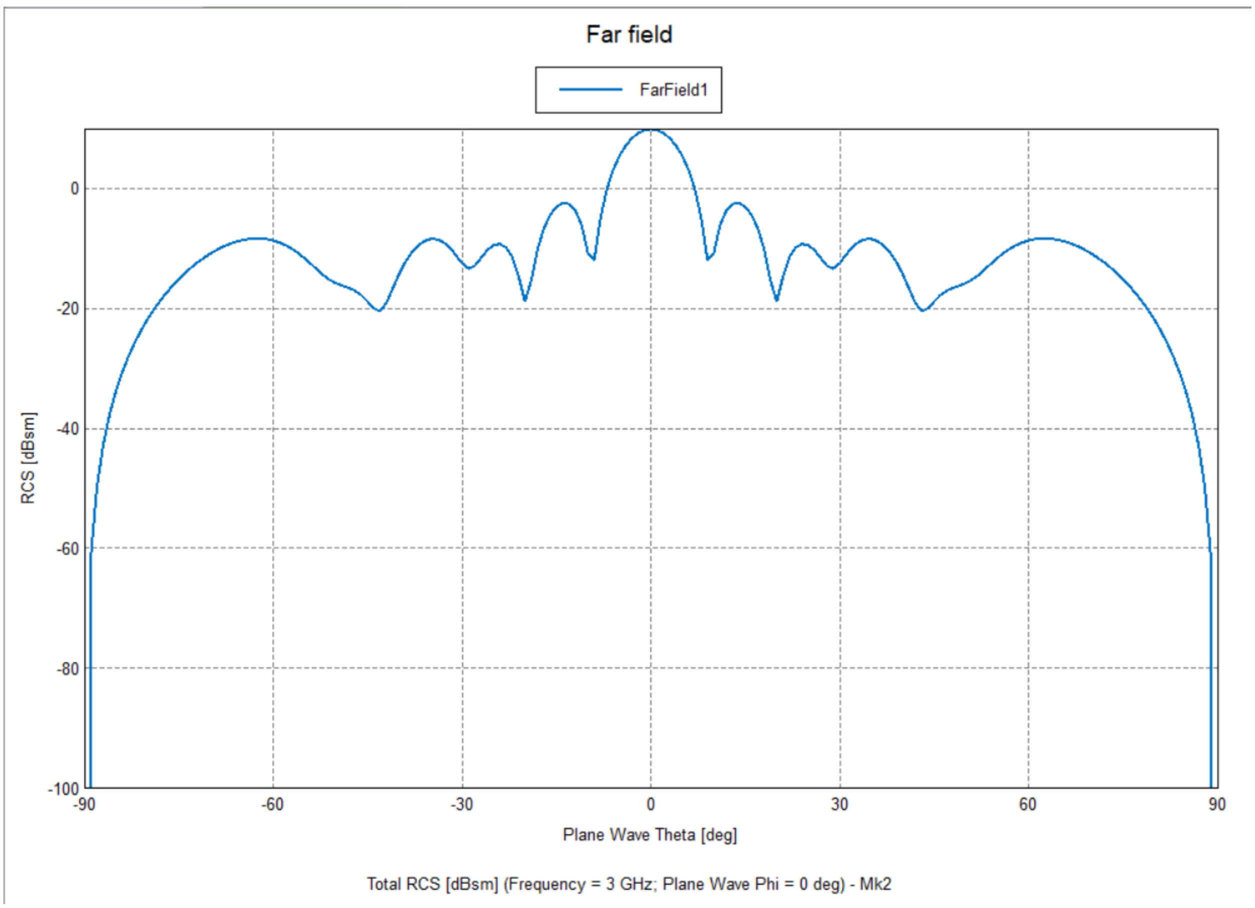


Figure 1: MoM Far Field RCS

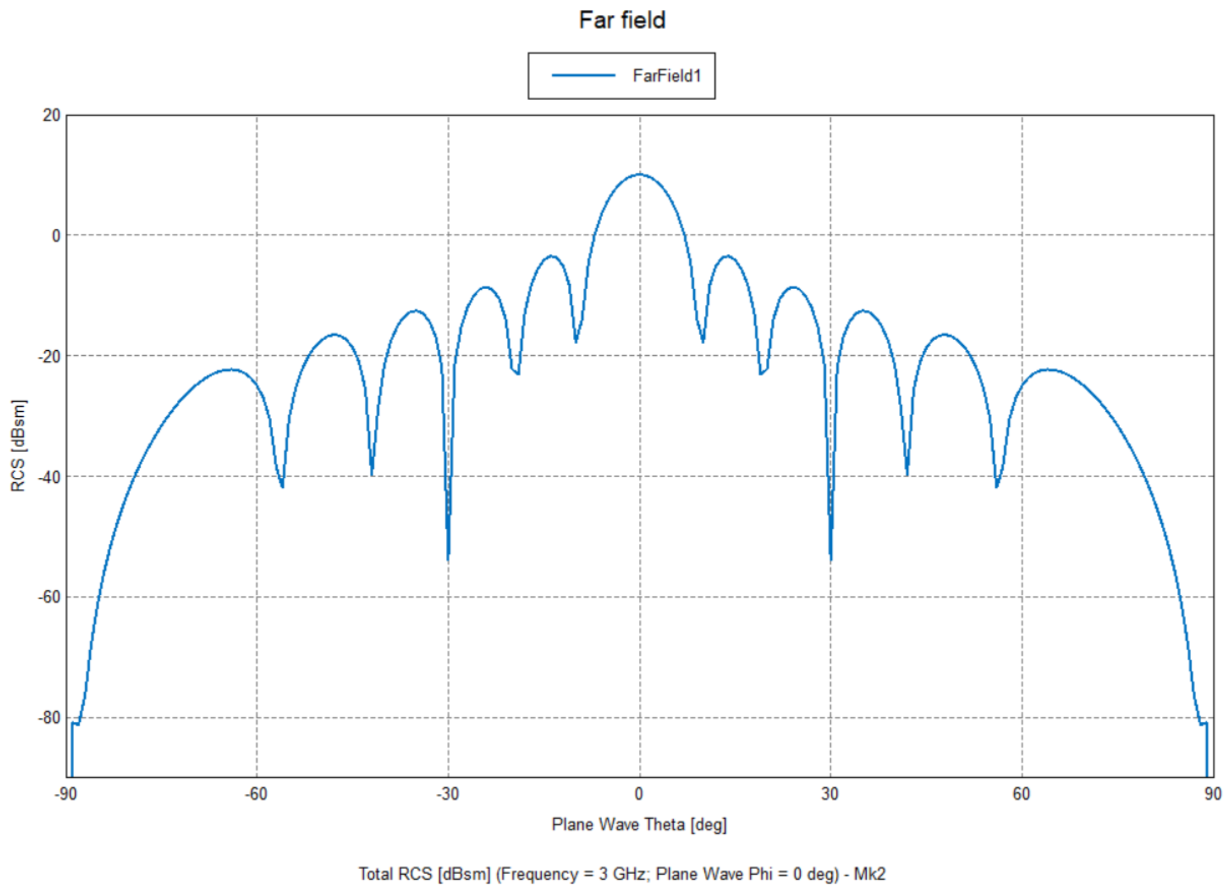


Figure 2: Physical Optics Far Field RCS.

c. Compare the results with PO solution and discuss the difference.
    a. The PO solution and the MoM solutions a quite similar, however, the PO solution does not lose shape continuity after 60/-60 degrees. Also, the PO solution is computed much faster.