Mistat bootstrap analysis

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1 Bootstrap approaches

Bootstrap Analysis (BA): For each resample, create a new dataset by randomly sampling rows from the original dataset with replacement

Befitting Bootstrap Analysis (BBA): For each resample, create a new dataset by randomly sampling rows from each group with replacement

Parametric Bootstrap Analysis (pBA): Create a model using the original dataset. For each resample, replace the outcome with adding randomly sampled residuals (with replacement) to the predicted values from the original model.

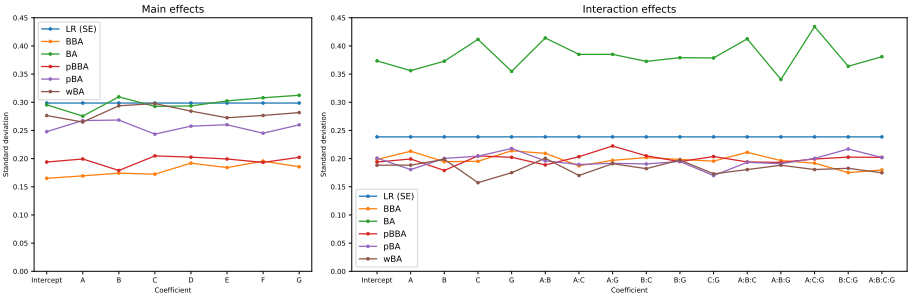
Parametric Befitting Bootstrap Analysis (pBBA): For each resample, replace the outcome values for each group with a random values sampled from a normal distribution with mean and standard deviation as the original group.

Wild Bootstrap Analysis (wBA): Create a model using the original dataset. For each resample, multiply the residuals with a random value sampled from a normal distribution $N(0, 1)$ and add to the fitted values from the original model.

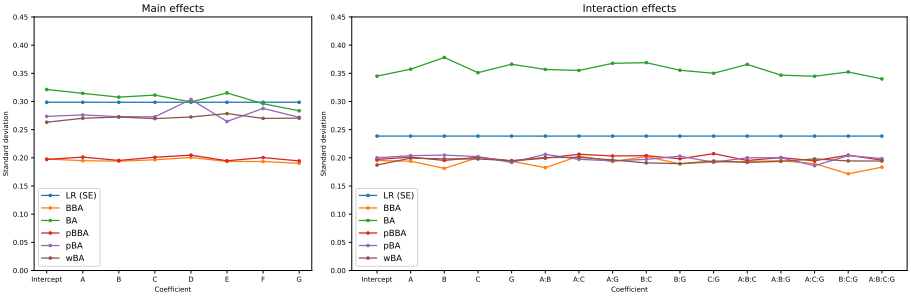
2 Wave soldering dataset

2.1 Main effects

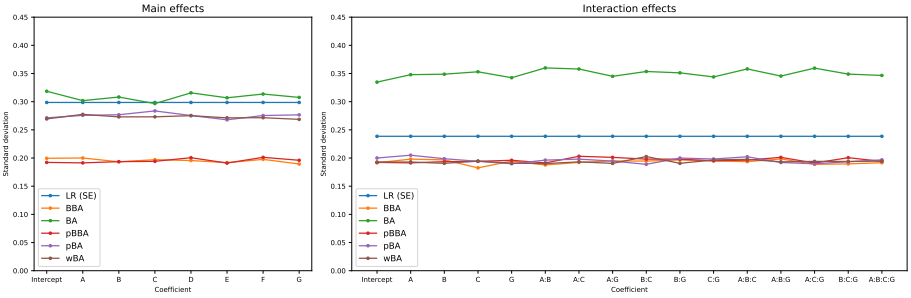
- Formula: $Data \sim A + B + C + D + E + F + G$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 2



(a) 100 bootstrap samples



(b) 500 bootstrap samples



(c) 1000 bootstrap samples

Figure 1: Std.dev. of coefficients for wave soldering dataset

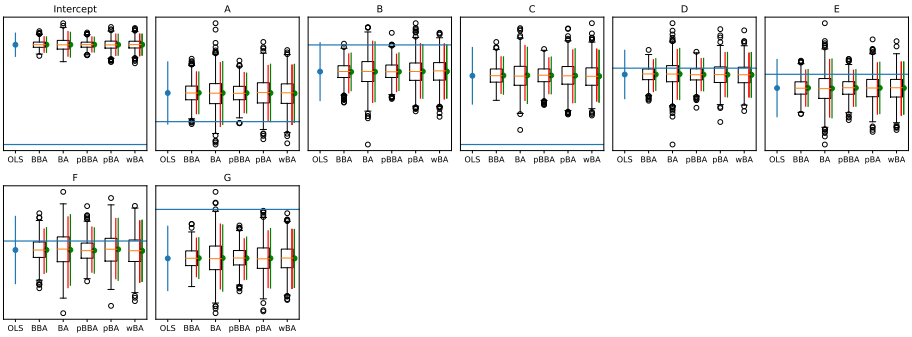


Figure 2: Distribution of main effect coefficient estimates for the wave soldering dataset. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

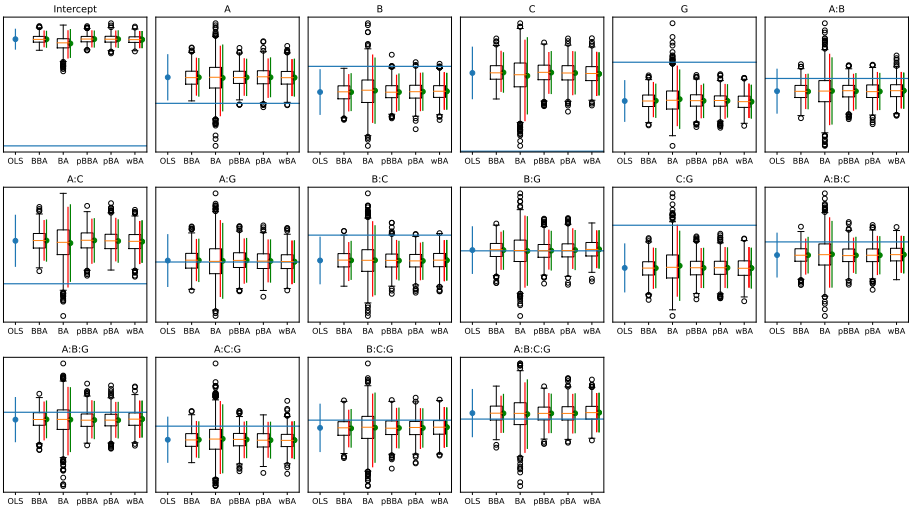


Figure 3: Distribution of main effects and interaction coefficient estimates for the wave soldering dataset. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

Table 1: BBA std. deviation of the regression coefficients for the wave soldering data.

	Main			Interaction		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.299	0.200	-33.2	0.239	0.192	-19.7
A	0.299	0.200	-33.0	0.239	0.198	-17.0
B	0.299	0.193	-35.3	0.239	0.197	-17.5
C	0.299	0.197	-34.0	0.239	0.183	-23.5
D	0.299	0.196	-34.5			
E	0.299	0.192	-35.8			
F	0.299	0.198	-33.8			
G	0.299	0.189	-36.6	0.239	0.195	-18.4
A:B				0.239	0.188	-21.3
A:C				0.239	0.192	-19.4
A:G				0.239	0.195	-18.3
B:C				0.239	0.195	-18.5
B:G				0.239	0.197	-17.6
C:G				0.239	0.195	-18.3
A:B:C				0.239	0.194	-18.9
A:B:G				0.239	0.198	-16.8
A:C:G				0.239	0.189	-20.9
B:C:G				0.239	0.190	-20.5
A:B:C:G				0.239	0.192	-19.7

2.2 Interactions

- Formula: $Data \sim A + B + C + G + A:B + A:C + A:G + B:C + B:G + C:G + A:B:C + A:B:G + A:C:G + B:C:G + A:B:C:G$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 3

2.3 Comparisons

Table 2: BA std. deviation of the regression coefficients for the wave soldering data.

	Main			Interaction		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.299	0.319	6.7	0.239	0.335	40.3
A	0.299	0.302	1.1	0.239	0.348	45.8
B	0.299	0.308	3.2	0.239	0.349	46.2
C	0.299	0.297	-0.7	0.239	0.353	48.0
D	0.299	0.316	5.7			
E	0.299	0.307	2.8			
F	0.299	0.314	5.0			
G	0.299	0.308	3.0	0.239	0.343	43.6
A:B				0.239	0.360	50.9
A:C				0.239	0.358	50.1
A:G				0.239	0.345	44.6
B:C				0.239	0.354	48.2
B:G				0.239	0.351	47.2
C:G				0.239	0.344	44.2
A:B:C				0.239	0.358	50.1
A:B:G				0.239	0.345	44.8
A:C:G				0.239	0.360	50.7
B:C:G				0.239	0.349	46.3
A:B:C:G				0.239	0.347	45.3

Table 3: pBBA std. deviation of the regression coefficients for the wave soldering data.

	Main			Interaction		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.299	0.192	-35.6	0.239	0.192	-19.4
A	0.299	0.191	-35.9	0.239	0.191	-19.8
B	0.299	0.194	-35.2	0.239	0.194	-18.9
C	0.299	0.194	-35.0	0.239	0.194	-18.6
D	0.299	0.201	-32.8			
E	0.299	0.191	-36.0			
F	0.299	0.201	-32.6			
G	0.299	0.196	-34.4	0.239	0.196	-17.8
A:B				0.239	0.191	-19.8
A:C				0.239	0.203	-14.9
A:G				0.239	0.201	-15.7
B:C				0.239	0.198	-17.0
B:G				0.239	0.198	-17.0
C:G				0.239	0.195	-18.2
A:B:C				0.239	0.196	-17.7
A:B:G				0.239	0.201	-15.7
A:C:G				0.239	0.191	-19.9
B:C:G				0.239	0.201	-15.9
A:B:C:G				0.239	0.194	-18.7

Table 4: pBA std. deviation of the regression coefficients for the wave soldering data.

	Main			Interaction		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.299	0.271	-9.2	0.239	0.200	-16.2
A	0.299	0.276	-7.7	0.239	0.205	-14.1
B	0.299	0.277	-7.3	0.239	0.199	-16.7
C	0.299	0.284	-5.0	0.239	0.194	-18.6
D	0.299	0.275	-7.8			
E	0.299	0.268	-10.3			
F	0.299	0.275	-7.8			
G	0.299	0.277	-7.4	0.239	0.190	-20.4
A:B				0.239	0.196	-17.7
A:C				0.239	0.198	-17.0
A:G				0.239	0.195	-18.5
B:C				0.239	0.189	-20.8
B:G				0.239	0.200	-16.2
C:G				0.239	0.198	-16.9
A:B:C				0.239	0.202	-15.2
A:B:G				0.239	0.192	-19.5
A:C:G				0.239	0.190	-20.4
B:C:G				0.239	0.193	-19.0
A:B:C:G				0.239	0.197	-17.5

Table 5: wBA std. deviation of the regression coefficients for the wave soldering data.

	Main			Interaction		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.299	0.269	-9.8	0.239	0.193	-19.2
A	0.299	0.277	-7.1	0.239	0.193	-19.1
B	0.299	0.273	-8.6	0.239	0.191	-20.1
C	0.299	0.273	-8.5	0.239	0.195	-18.5
D	0.299	0.275	-7.9			
E	0.299	0.271	-9.1			
F	0.299	0.272	-9.1			
G	0.299	0.269	-10.0	0.239	0.191	-20.0
A:B				0.239	0.190	-20.3
A:C				0.239	0.193	-19.0
A:G				0.239	0.191	-20.1
B:C				0.239	0.203	-15.1
B:G				0.239	0.191	-20.1
C:G				0.239	0.197	-17.6
A:B:C				0.239	0.198	-17.1
A:B:G				0.239	0.193	-19.0
A:C:G				0.239	0.194	-18.7
B:C:G				0.239	0.194	-18.7
A:B:C:G				0.239	0.195	-18.5

3 Piston simulator

3.1 Main effects

- Formula: $seconds \sim x1 + x2 + x3 + x4$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 5

3.2 Interactions

- Formula: $seconds \sim x1 + x2 + x3 + x1 : x2 + x1 : x3 + x2 : x3$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 6

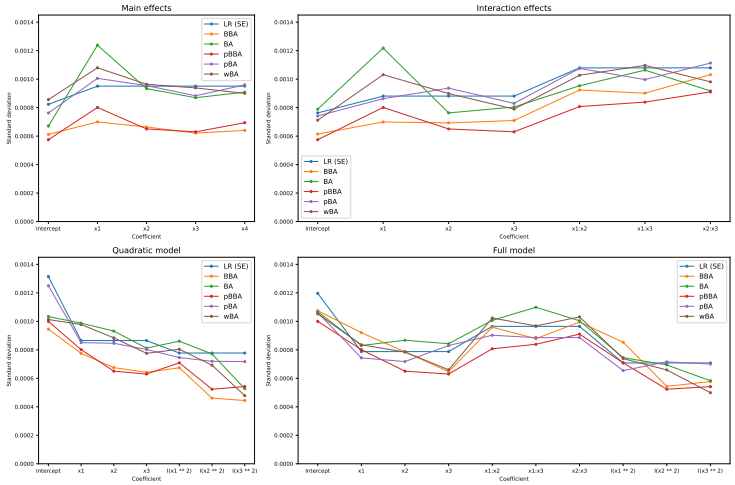
3.3 Quadratic model

- Formula: $seconds \sim x1 + x2 + x3 + x1^2 + x2^2 + x3^2$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 7

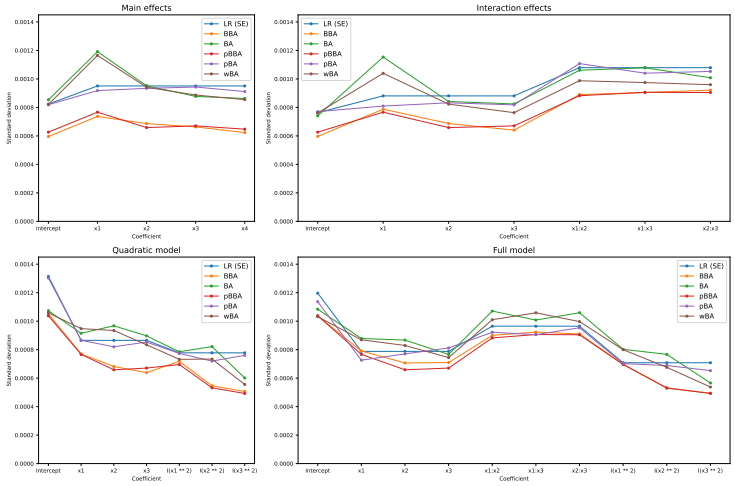
3.4 Full model

- Formula: $seconds \sim x1+x2+x3+x1 : x2+x1 : x3+x2 : x3+x1^2+x2^2+x3^2$
- Number of bootstrap samples: 100
- Distribution of bootstrap sampled coefficients: Figure 8

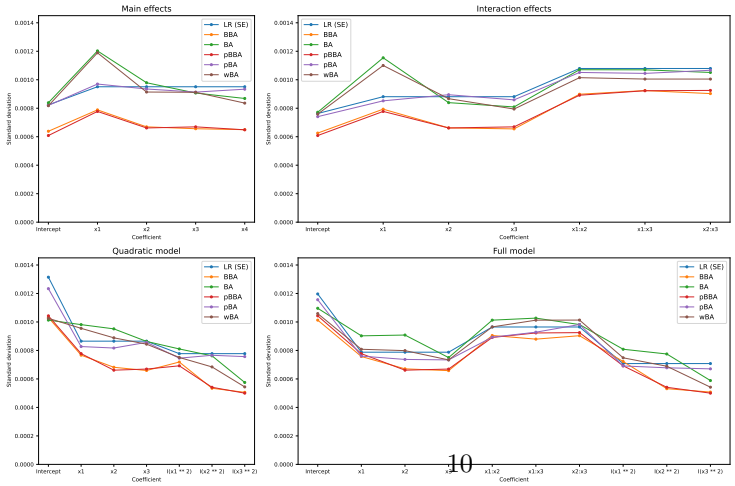
3.5 Comparisons



(a) 100 bootstrap samples



(b) 500 bootstrap samples



(c) 1000 bootstrap samples

Figure 4: Std.dev. of coefficients for piston simulator

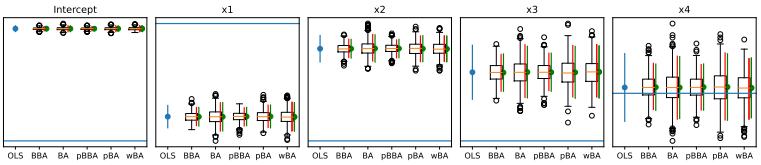


Figure 5: Distribution of main effect coefficient estimates for the piston simulation. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

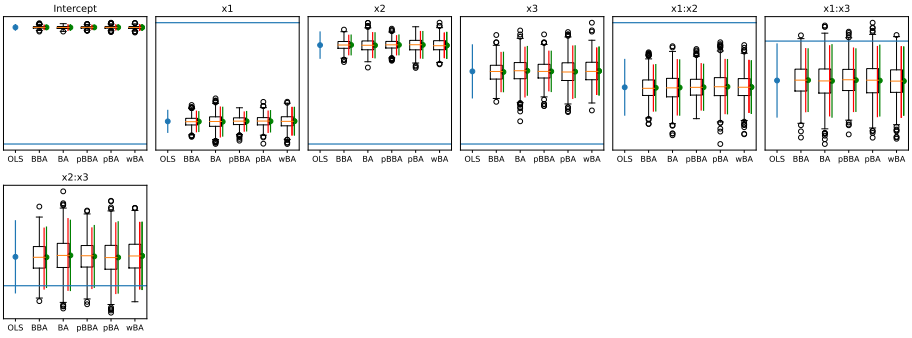


Figure 6: Distribution of main effects and interaction coefficient estimates for the piston simulation. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

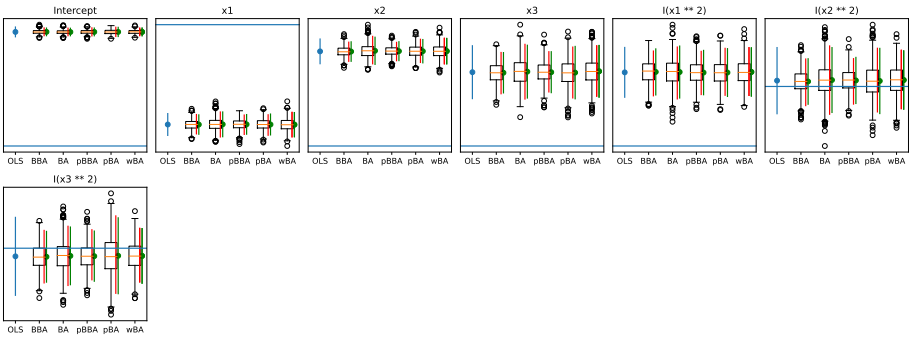


Figure 7: Distribution of main effects and quadratic coefficient estimates for the piston simulation. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

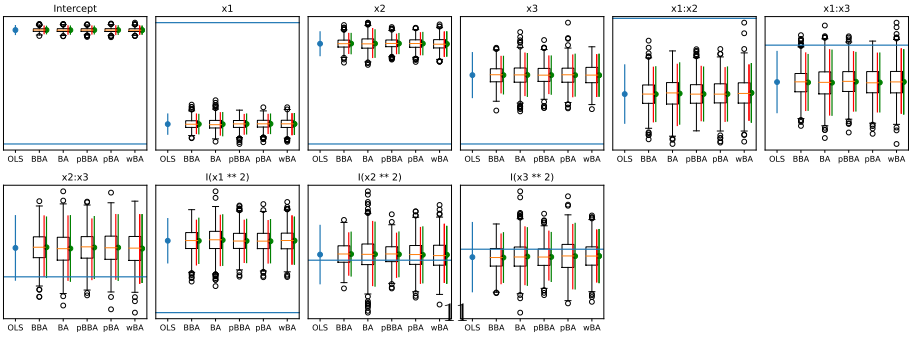


Figure 8: Distribution of main effects, interactions, and quadratic coefficient estimates for the piston simulation. Blue: ols estimate ± std.dev. For each bootstrap approach, boxplot: distribution, red: interquartile range, green: mean ± std.dev.

Table 6: BBA std. deviation of the regression coefficients for the Piston simulation.

	Main			Interaction			Quadratic			Full		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.00082	0.00064	-22.5	0.00076	0.00063	-18.0	0.00131	0.00103	-21.5	0.00120	0.00101	-15.3
x1	0.00095	0.00079	-17.1	0.00088	0.00079	-9.9	0.00087	0.00077	-11.4	0.00079	0.00076	-3.8
x2	0.00095	0.00067	-29.6	0.00088	0.00066	-24.9	0.00087	0.00068	-21.2	0.00079	0.00067	-14.8
x3	0.00095	0.00066	-31.0	0.00088	0.00065	-25.7	0.00087	0.00066	-23.8	0.00079	0.00066	-16.4
x4	0.00095	0.00065	-31.7									
x1:x2				0.00108	0.00090	-16.7				0.00096	0.00091	-6.2
x1:x3				0.00108	0.00092	-14.4				0.00096	0.00088	-8.8
x2:x3				0.00108	0.00090	-16.3				0.00096	0.00090	-6.3
I(x1 ** 2)							0.00078	0.00072	-7.6	0.00071	0.00072	2.0
I(x2 ** 2)							0.00078	0.00053	-31.3	0.00071	0.00053	-24.9
I(x3 ** 2)							0.00078	0.00051	-34.9	0.00071	0.00051	-28.3

Table 7: BA std. deviation of the regression coefficients for the Piston simulation.

	Main			Interaction			Quadratic			Full		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.00082	0.00084	1.8	0.00076	0.00077	1.2	0.00131	0.00101	-22.9	0.00120	0.00110	-8.4
x1	0.00095	0.00120	26.5	0.00088	0.00115	31.0	0.00087	0.00098	13.4	0.00079	0.00090	14.6
x2	0.00095	0.00098	3.0	0.00088	0.00084	-4.7	0.00087	0.00095	10.0	0.00079	0.00091	15.3
x3	0.00095	0.00091	-4.6	0.00088	0.00081	-8.2	0.00087	0.00086	-0.2	0.00079	0.00075	-4.8
x4	0.00095	0.00087	-8.6									
x1:x2				0.00108	0.00107	-0.8				0.00096	0.00101	5.0
x1:x3				0.00108	0.00107	-0.9				0.00096	0.00103	6.5
x2:x3				0.00108	0.00105	-2.6				0.00096	0.00098	1.8
I(x1 ** 2)							0.00078	0.00081	4.2	0.00071	0.00081	14.2
I(x2 ** 2)							0.00078	0.00076	-2.0	0.00071	0.00078	9.5
I(x3 ** 2)							0.00078	0.00058	-26.0	0.00071	0.00059	-16.9

Table 8: pBBA std. deviation of the regression coefficients for the Piston simulation.

	Main			Interaction			Quadratic			Full		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.00082	0.00061	-26.1	0.00076	0.00061	-20.3	0.00131	0.00104	-20.6	0.00120	0.00104	-12.8
x1	0.00095	0.00078	-18.2	0.00088	0.00078	-11.7	0.00087	0.00078	-10.1	0.00079	0.00078	-1.3
x2	0.00095	0.00066	-30.4	0.00088	0.00066	-24.9	0.00087	0.00066	-23.5	0.00079	0.00066	-16.0
x3	0.00095	0.00067	-29.6	0.00088	0.00067	-24.0	0.00087	0.00067	-22.6	0.00079	0.00067	-15.0
x4	0.00095	0.00065	-31.8									
x1:x2				0.00108	0.00089	-17.4				0.00096	0.00089	-7.6
x1:x3				0.00108	0.00092	-14.5				0.00096	0.00092	-4.3
x2:x3				0.00108	0.00093	-14.3				0.00096	0.00093	-4.1
I(x1 ** 2)							0.00078	0.00069	-11.0	0.00071	0.00069	-2.2
I(x2 ** 2)							0.00078	0.00054	-30.4	0.00071	0.00054	-23.5
I(x3 ** 2)							0.00078	0.00050	-35.7	0.00071	0.00050	-29.4

Table 9: pBA std. deviation of the regression coefficients for the Piston simulation.

	Main			Interaction			Quadratic			Full		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.00082	0.00082	-0.4	0.00076	0.00074	-2.9	0.00131	0.00123	-6.1	0.00120	0.00116	-3.4
x1	0.00095	0.00097	2.0	0.00088	0.00085	-3.3	0.00087	0.00083	-4.4	0.00079	0.00076	-3.5
x2	0.00095	0.00094	-1.6	0.00088	0.00089	1.6	0.00087	0.00082	-5.6	0.00079	0.00074	-6.5
x3	0.00095	0.00091	-3.9	0.00088	0.00086	-2.5	0.00087	0.00086	-1.0	0.00079	0.00073	-6.9
x4	0.00095	0.00093	-1.8									
x1:x2				0.00108	0.00105	-2.6				0.00096	0.00089	-7.4
x1:x3				0.00108	0.00104	-3.2				0.00096	0.00093	-3.8
x2:x3				0.00108	0.00107	-1.2				0.00096	0.00098	1.8
I(x1 ** 2)							0.00078	0.00075	-4.1	0.00071	0.00069	-2.4
I(x2 ** 2)							0.00078	0.00077	-1.5	0.00071	0.00068	-4.2
I(x3 ** 2)							0.00078	0.00076	-2.7	0.00071	0.00067	-5.2

Table 10: wBA std. deviation of the regression coefficients for the Piston simulation.

	Main			Interaction			Quadratic			Full		
	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta	Regr.	Bootstrap	Delta
Intercept	0.00082	0.00082	-0.7	0.00076	0.00076	-0.6	0.00131	0.00103	-22.0	0.00120	0.00106	-11.5
x1	0.00095	0.00119	25.1	0.00088	0.00110	24.9	0.00087	0.00096	10.4	0.00079	0.00081	2.6
x2	0.00095	0.00091	-3.9	0.00088	0.00087	-1.6	0.00087	0.00089	2.7	0.00079	0.00080	1.4
x3	0.00095	0.00091	-4.1	0.00088	0.00079	-9.8	0.00087	0.00084	-2.4	0.00079	0.00073	-6.9
x4	0.00095	0.00084	-12.1									
x1:x2				0.00108	0.00101	-6.0				0.00096	0.00097	0.1
x1:x3				0.00108	0.00100	-6.9				0.00096	0.00101	5.0
x2:x3				0.00108	0.00100	-6.9				0.00096	0.00101	5.0
I(x1 ** 2)							0.00078	0.00075	-3.4	0.00071	0.00075	5.7
I(x2 ** 2)							0.00078	0.00068	-12.0	0.00071	0.00069	-2.7
I(x3 ** 2)							0.00078	0.00054	-30.0	0.00071	0.00054	-23.4