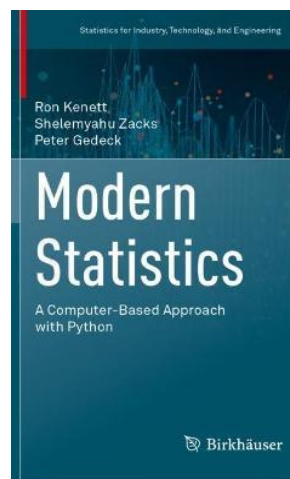


# **A Biomed Data Analyst Training Program**

**Supervised learning**

**Professor Ron S. Kenett**



# Chapter 7

## Modern Analytic Methods: Part I



**Preview** This chapter is a door opener to computer age statistics. It covers a range of supervised and unsupervised learning methods and demonstrates their use in various applications.

### 7.1 Introduction to Computer Age Statistics

Big data and data science applications have been facilitated by hardware developments in computer science. As data storage began to increase, more advanced software was required to process it. This led to the development of cloud computing and distributed computing. Parallel machine processing was enhanced by the development of Hadoop, based on off-the-shelf Google File System (GFS) and Google MapReduce, for performing distributed computing.

# Chapter 7

Modern Statistics: A Computer Based Approach with Python  
by Ron Kenett, Shelemyahu Zacks, Peter Gedeck

Publisher: Springer International Publishing; 1st edition (September 15, 2022)  
ISBN-13: 978-3031075650

(c) 2022 Ron Kenett, Shelemyahu Zacks, Peter Gedeck

The code needs to be executed in sequence.

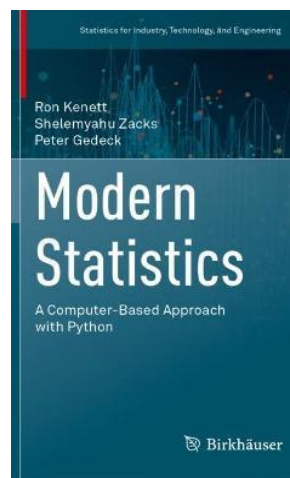
```
In [1]: import warnings
import os
os.environ['OUTDATED_IGNORE'] = '1'
from outdated import OutdatedPackageWarning
warnings.filterwarnings('ignore', category=FutureWarning)
warnings.filterwarnings('ignore', category=OutdatedPackageWarning)
```

## Modern Analytic Methods: Part I

Ron Kenett, Shelemyahu Zacks, Peter Gedeck  
Pages 361-393

## Modern analytic methods: Part I

```
In [2]: import warnings
import random
import pandas as pd
import numpy as np
from sklearn.ensemble import RandomForestClassifier
from sklearn.naive_bayes import MultinomialNB
from sklearn.metrics import accuracy_score
from sklearn.impute import SimpleImputer
from sklearn.neural_network import MLPClassifier
from sklearn.preprocessing import MinMaxScaler
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay
```



# Chapter 7

## Modern Analytic Methods: Part I



### 7.5 Decision Trees

Partition models, also called decision trees, are non-parametric tools used in supervised learning in the context of classification and regression. In supervised learning you observe multiple covariate and one or more target variables. The goal is to predict or classify the target using the values of covariates. Decision trees are based on splits in covariates or predictors that create separate but homogeneous groups. Splits are not sensitive to outliers but are based on a “greedy” one -step look ahead, without accounting for overall performance. Breiman et al. (1984) implement a decision tree procedure called CART (Classification And Regression Trees). Other procedures are C4.5 and CHAID (Chi-square Automatic Interaction

## Chapter 7

# Modern Analytic Methods: Part I



*Example 7.2* Data set **SENSORS.csv** consists of 174 measurements from 63 sensors tracking performance of a system under test. Each test generates values for these 63 sensors and a status determined by the automatic test equipment. The distribution of the test results is presented in Fig. 7.5. Our goal is to predict the outcome recorded by the testing equipment, using sensor data. The test results are coded as Pass (corresponding to “Good,” 47% of the observations) and Fail (all other categories, marked in grey). The column **Status** is therefore a dichotomized version of the column **Test result**.

---

```
sensors = mistat.load_data('SENSORS.csv')
dist = sensors['testResult'].value_counts()
dist = dist.sort_index()
ax = dist.plot.bar(color='lightgrey')
ax.patches[dist.index.get_loc('Good')].set_facecolor('black')
plt.show()
```

---

## Chapter 7

# Modern Analytic Methods: Part I



The goal is to predict the outcome recorded by the testing equipment, using sensor data. We can use `scikit-learn` for this. It has decision tree implementations for classification and regression. Here, we create a classification model for Pass-Fail using the 67 sensors.

---

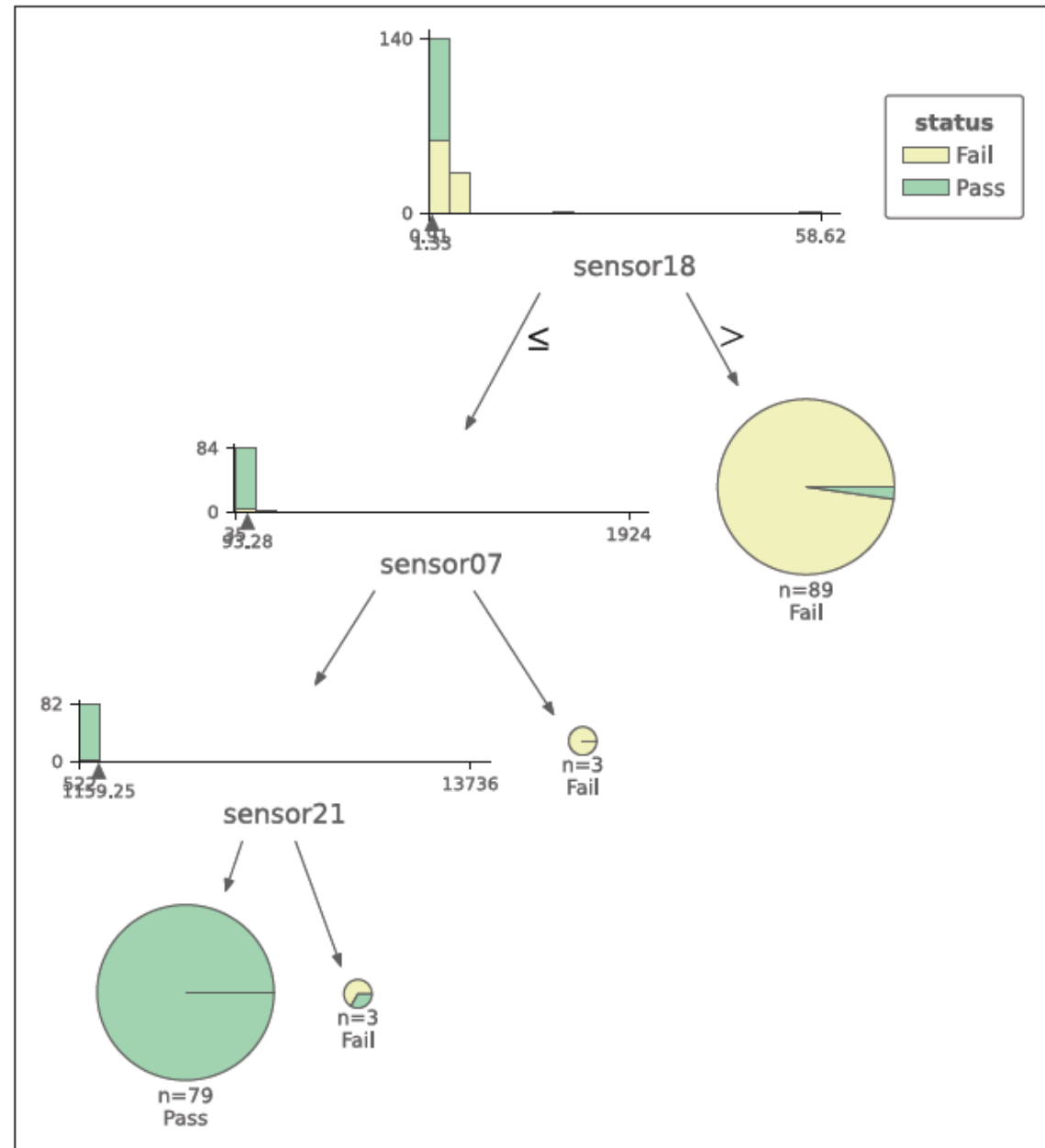
```
from sklearn.tree import DecisionTreeClassifier, plot_tree, export_text

predictors = [c for c in sensors.columns if c.startswith('sensor')]
outcome = 'status'
X = sensors[predictors]
y = sensors[outcome]

# Train the model
clf = DecisionTreeClassifier(ccp_alpha=0.012, random_state=0)
clf.fit(X, y)

# Visualization of tree
plot_tree(clf, feature_names=list(X.columns))
plt.show()
```

---

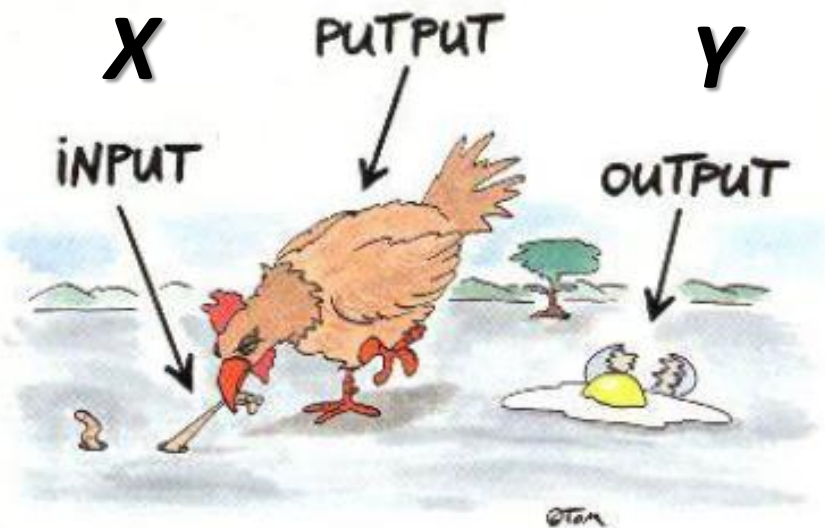


**Fig. 7.7** Decision tree visualization of classification tree



← **X** → **Y**

	A	B	C	D	E	F
1	Country	Salesperson	Order Date	OrderID	Units	Order Amount
2	USA	Fuller	1/01/2011	10392	13	1,440.00
3	UK	Gloucester	2/01/2011	10397	17	716.72
4	UK	Bromley	2/01/2011	10771	18	344.00
5	USA	Finchley	3/01/2011	10393	16	2,556.95
6	USA	Finchley	3/01/2011	10394	10	442.00
7	UK	Gillingham	3/01/2011	10395	9	2,122.92
8	USA	Finchley	6/01/2011	10396	7	1,903.80
9	USA	Callahan	8/01/2011	10399	17	1,765.60
10	USA	Fuller	8/01/2011	10404	7	1,591.25
11	USA	Fuller	9/01/2011	10398	11	2,505.60
12	USA	Coghill	9/01/2011	10403	18	855.01
13	USA	Finchley	10/01/2011	10401	7	3,868.60
14	USA	Callahan	10/01/2011	10402	11	2,713.50
15	UK	Rayleigh	13/01/2011	10406	15	1,830.78
16	USA	Callahan	14/01/2011	10408	10	1,622.40
17	USA	Farnham	14/01/2011	10409	19	319.20
18	USA	Farnham	15/01/2011	10410	16	802.00

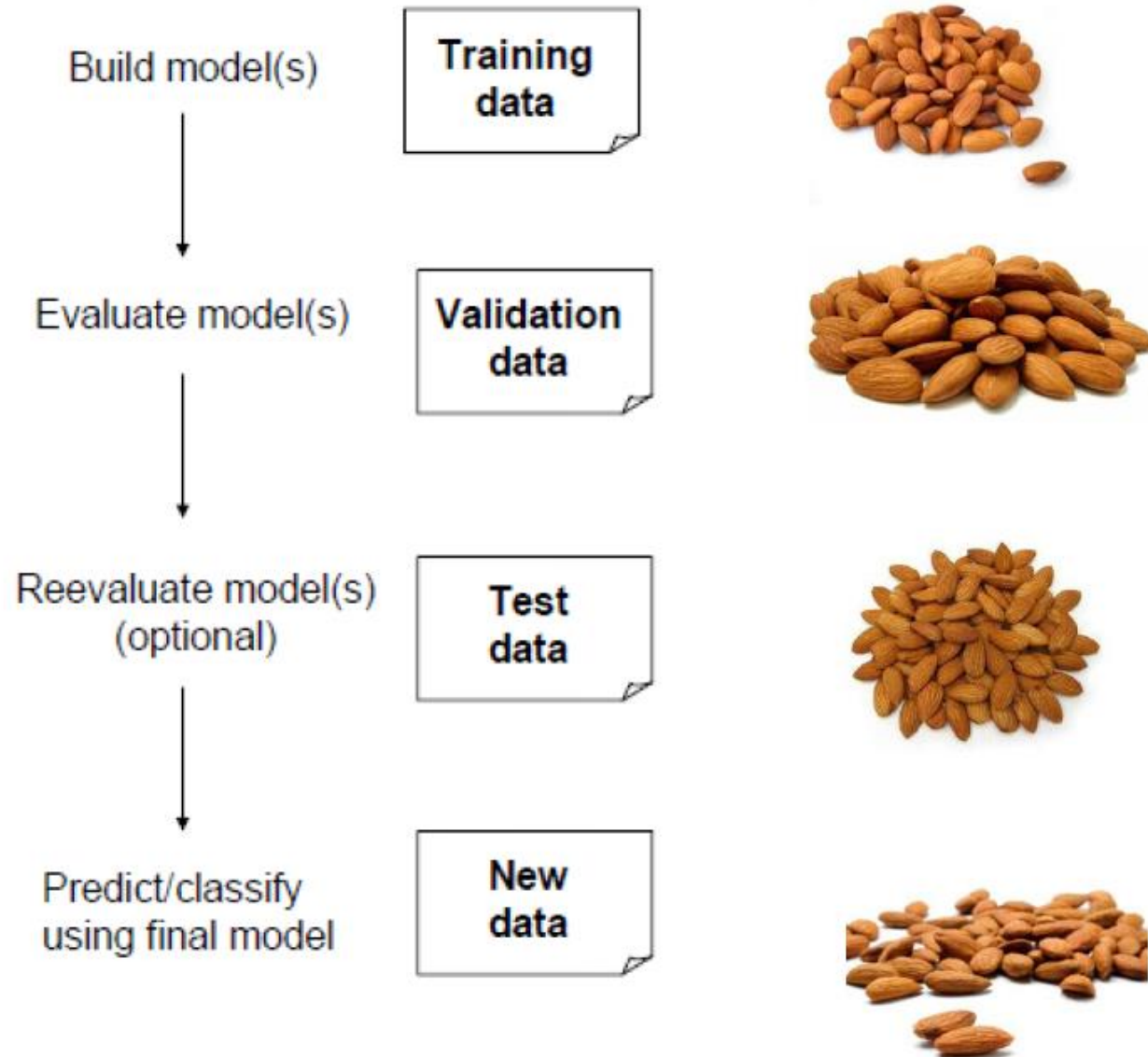


## Supervised Learning

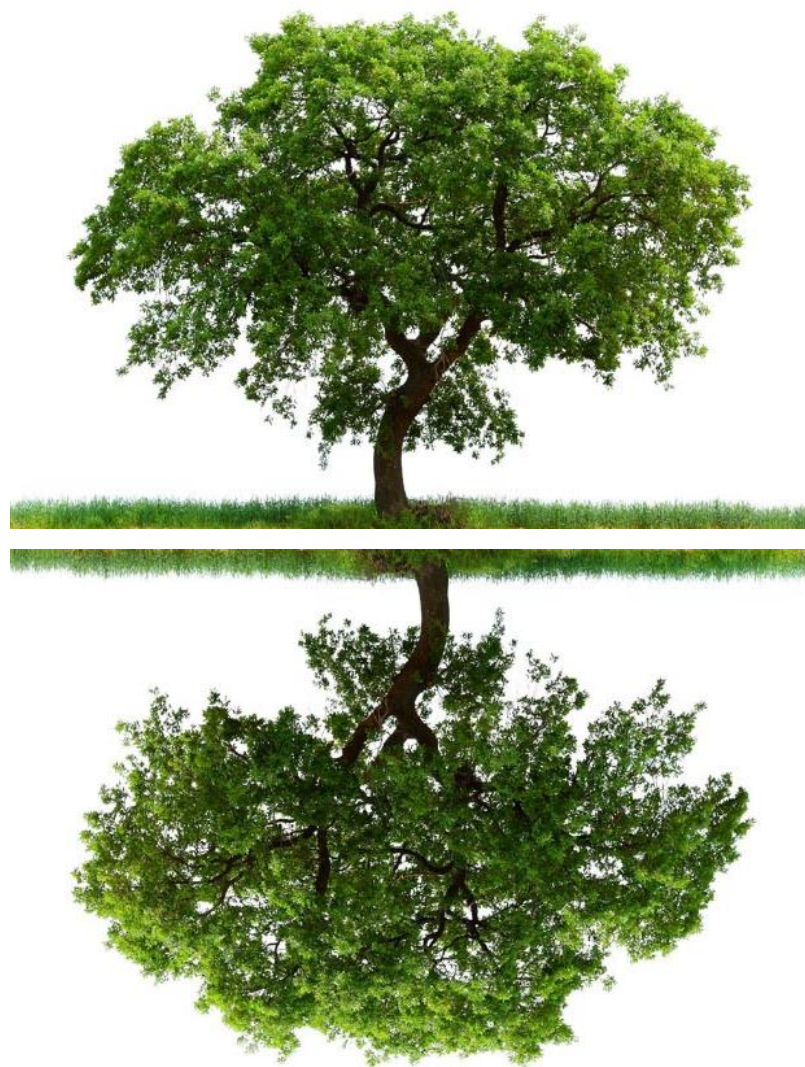




# Data Stratification



# Decision Trees





Income



Lot size



	Income	Lot_Size	Ownership
1	60	18.4	owner
2	85.5	16.8	owner
3	64.8	21.6	owner
4	61.5	20.8	owner
5	87	23.6	owner
6	110.1	19.2	owner
7	108	17.6	owner
8	82.8	22.4	owner
9	69	20	owner
10	93	20.8	owner
11	51	22	owner
12	81	20	owner
13	75	19.6	non-owner
14	52.8	20.8	non-owner
15	64.8	17.2	non-owner
16	43.2	20.4	non-owner
17	84	17.6	non-owner
18	49.2	17.6	non-owner
19	59.4	16	non-owner
20	66	18.4	non-owner
21	47.4	16.4	non-owner
22	33	18.8	non-owner
23	51	14	non-owner
24	63	14.8	non-owner

# Splitting on Continuous Variables

- Order records according to one variable, say lot size
- Split at the first value
- Measure the dissimilarity between the two subsets
- Split at the next value, and continue
- Repeat for the other variable(s)
- For all variables, the split value that drives the greatest dissimilarity in propensities (or probabilities) is selected as the split point

# Splitting on Categorical Variables

- Examine all possible ways in which the categories can be split.
- E.g., nominal categories A, B, C can be split 3 ways
  - {A} and {B, C}
  - {B} and {A, C}
  - {C} and {A, B}
- With many categories, # of potential splits becomes huge

# Splitting on Categorical Variables

- For ordinal data (ordered categories) there is an option for the splits to respect ordering
- Example: An ordinal predictor takes on the values 1, 2, 3, or 4
- The data can be split 3 ways:
  - {1} and {2, 3, 4}
  - {1, 2} and {3, 4}
  - {1, 2, 3} and {4}



# Gini Index

Gini Index for rectangle  $A$  containing  $m$  records

$$I(A) = 1 - \sum_{k=1}^m p_k^2$$

$p$  = proportion of cases in rectangle  $A$  that belong to class  $k$

- $I(A) = 0$  when all cases belong to same class
- Max value when all classes are equally represented (= 0.50 in binary case)

# Entropy

$p$  = proportion of cases (out of  $m$ ) in rectangle  $A$  that belong to class  $k$

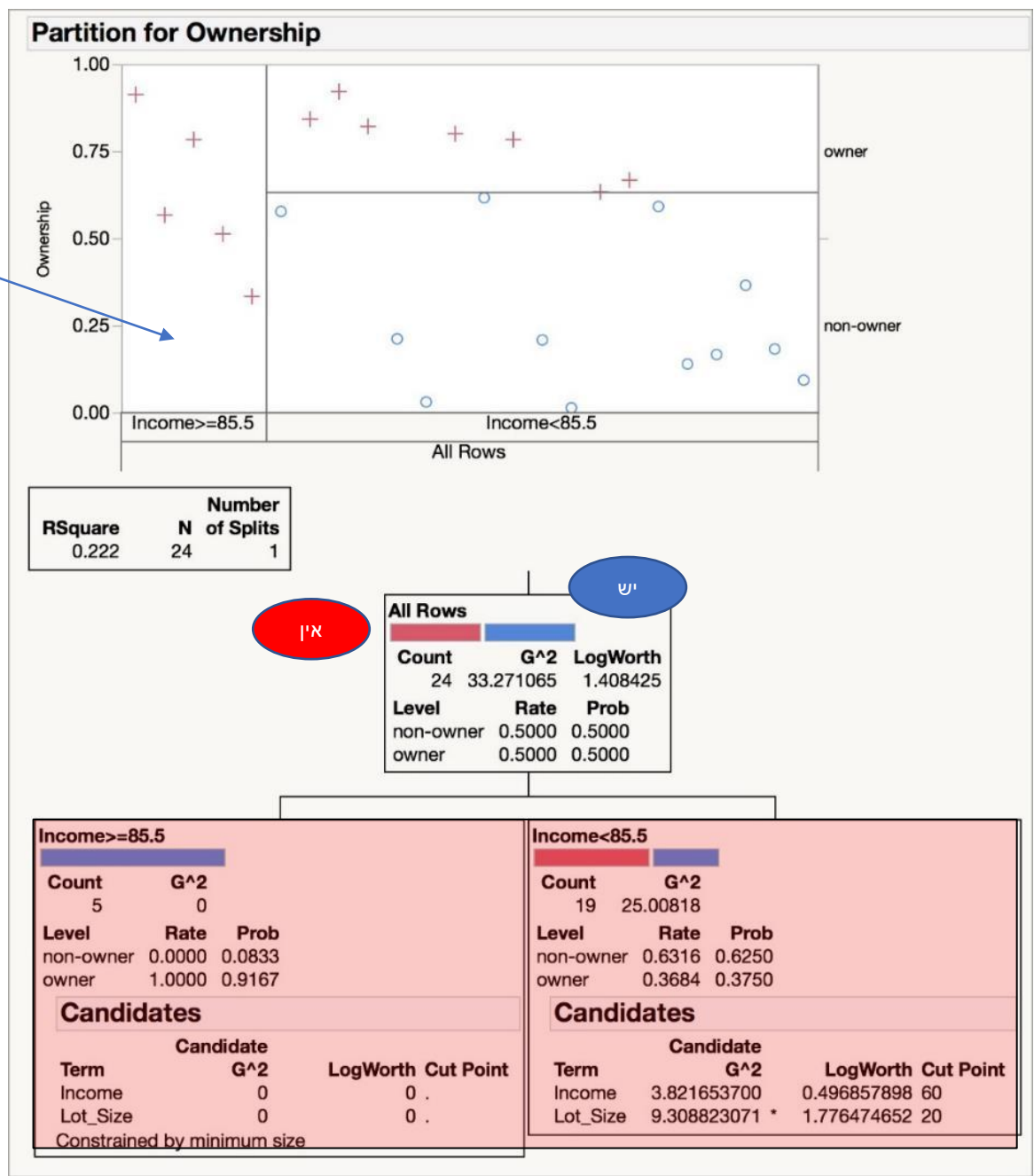
$$\text{entropy}(A) = - \sum_{k=1}^m p_k \log_2(p_k)$$

- Entropy ranges between 0 (most pure) and  $\log_2(m)$  (equal representation of classes)

# Impurity and Recursive Partitioning

- Obtain overall impurity measure (weighted avg. of individual rectangles)
- At each successive stage, compare this measure across all possible splits in all variables
- Choose the split that reduces impurity the most
- Chosen split points become nodes on the tree

Rectangle A



*Gini and Entropy measures are not available in JMP. JMP uses measures of dissimilarity (G<sup>2</sup> and Sum of Squares) rather than measures of impurity*



Income



▼ All Rows		
Count	G^2	LogWorth
24	33.271065	1.408425



▼ Income >= 85.5		
Count	G^2	
5	0	
► Candidates		

▼ Income < 85.5		
Count	G^2	LogWorth
19	25.00818	1.7764747

▼ Lot_Size >= 20		
Count	G^2	LogWorth
8	8.9973623	0.8810527



▼ Lot_Size < 20		
Count	G^2	LogWorth
11	6.7019941	0.1154386

▼ Income >= 61.5		
Count	G^2	
5	0	
► Candidates		

▼ Income < 61.5		
Count	G^2	LogWorth
3	3.819085	1.0511517

▼ Lot_Size >= 17.6		
Count	G^2	LogWorth
6	5.4067345	0.1999259

▼ Lot_Size < 17.6		
Count	G^2	
5	0	
► Candidates		

▼ Lot_Size >= 22		
Count	G^2	
1	0	
► Candidates		

▼ Lot_Size < 22		
Count	G^2	
2	0	
► Candidates		

▼ Income < 66		
Count	G^2	LogWorth
3	3.819085	1.0511517

▼ Income >= 66		
Count	G^2	
3	0	
► Candidates		

▼ Income >= 60		
Count	G^2	
1	0	
► Candidates		

▼ Income < 60		
Count	G^2	
2	0	
► Candidates		

## ▼ Leaf Report

Response Prob

### Leaf Label

	non-owner	.2	.4	.6	.8	owner
Income $\geq$ 85.5	0.0833					0.9167
Income $<$ 85.5&Lot_Size $\geq$ 20&Income $\geq$ 61.5	0.0815					0.9185
Income $<$ 85.5&Lot_Size $\geq$ 20&Income $<$ 61.5&Lot_Size $\geq$ 22	0.2512					0.7488
Income $<$ 85.5&Lot_Size $\geq$ 20&Income $<$ 61.5&Lot_Size $<$ 22	0.8342					0.1658
Income $<$ 85.5&Lot_Size $<$ 20&Lot_Size $\geq$ 17.6&Income $<$ 66&Income $\geq$ 60	0.2901					0.7099
Income $<$ 85.5&Lot_Size $<$ 20&Lot_Size $\geq$ 17.6&Income $<$ 66&Income $<$ 60	0.8601					0.1399
Income $<$ 85.5&Lot_Size $<$ 20&Lot_Size $\geq$ 17.6&Income $\geq$ 66	0.8933					0.1067
Income $<$ 85.5&Lot_Size $<$ 20&Lot_Size $<$ 17.6	0.9248					0.0752





# Tree Structure

- Split points become nodes on the tree
- Leaves are the terminal nodes (there are no further splits)
- Read down tree to derive the decision rule

E.g.,  $\text{Income} < 85.5$ ,  $\text{Lot Size is } \geq 20$ , and  $\text{Income } \geq 61.5$ , the probability that a household is an owner is 0.9185.
- Records within each node are from the training data (validation data are not used in building the tree)
- Default cutoff = 0.5 is used for classification

In the previous example, the record would be classified as an owner.

# The Riding Mowers

The leaf report provides a summary the splits

It displays the rules for classifying outcomes

For example, If Income < 85.5, Lot Size is < 17.6, the probability that a household is an owner is 0.0752. This record will be classified as a non-owner.

## ▼ Leaf Report

Response Prob

### Leaf Label

Income>=85.5

Income<85.5&Lot\_Size>=20&Income>=61.5

Income<85.5&Lot\_Size>=20&Income<61.5&Lot\_Size>=22

Income<85.5&Lot\_Size>=20&Income<61.5&Lot\_Size<22

Income<85.5&Lot\_Size<20&Lot\_Size>=17.6&Income<66&Income>=60

Income<85.5&Lot\_Size<20&Lot\_Size>=17.6&Income<66&Income<60

Income<85.5&Lot\_Size<20&Lot\_Size>=17.6&Income>=66

Income<85.5&Lot\_Size<20&Lot\_Size<17.6

non-owner .2 .4 .6 .8 owner

0.0833



0.9167



0.0815



0.9185



0.2512



0.7488



0.8342



0.1658



0.2901



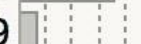
0.7099



0.8601



0.1399



0.8933



0.1067



0.9248



0.0752



$$\text{Prob}_i = \frac{n_i + \text{prior}_i}{\sum (n_i + \text{prior}_i)}$$

where the summation is across all response levels;  $n_i$  is the number of observations at the node for the  $i^{\text{th}}$  response level; and  $\text{prior}_i$  is the prior probability for the  $i^{\text{th}}$  response level, calculated as follows:

$$\text{prior}_i = \lambda p_i + (1 - \lambda) P_i$$

where  $p_i$  is the  $\text{prior}_i$  from the parent node,  $P_i$  is the  $\text{Prob}_i$  from the parent node, and  $\lambda$  is a weighting factor currently set at 0.9.

## ▼ Leaf Report

Response Prob

### Leaf Label

Income $\geq$ 85.5

Income $<$ 85.5&Lot\_Size $\geq$ 20&Income $\geq$ 61.5

Income $<$ 85.5&Lot\_Size $\geq$ 20&Income $<$ 61.5&Lot\_Size $\geq$ 22

Income $<$ 85.5&Lot\_Size $\geq$ 20&Income $<$ 61.5&Lot\_Size $<$ 22

Income $<$ 85.5&Lot\_Size $<$ 20&Lot\_Size $\geq$ 17.6&Income $<$ 66&Income $\geq$ 60

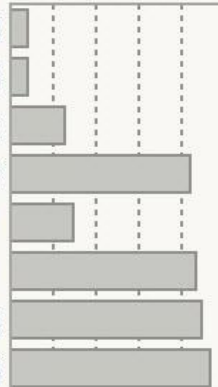
Income $<$ 85.5&Lot\_Size $<$ 20&Lot\_Size $\geq$ 17.6&Income $<$ 66&Income $<$ 60

Income $<$ 85.5&Lot\_Size $<$ 20&Lot\_Size $\geq$ 17.6&Income $\geq$ 66

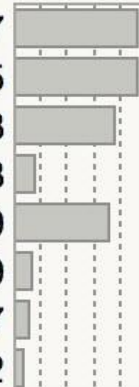
Income $<$ 85.5&Lot\_Size $<$ 20&Lot\_Size $<$ 17.6

non-owner .2 .4 .6 .8 owner

0.0833



0.9167



0.0815

0.9185

0.2512

0.7488

0.8342

0.1658

0.2901

0.7099

0.8601

0.1399

0.8933

0.1067

0.9248

0.0752

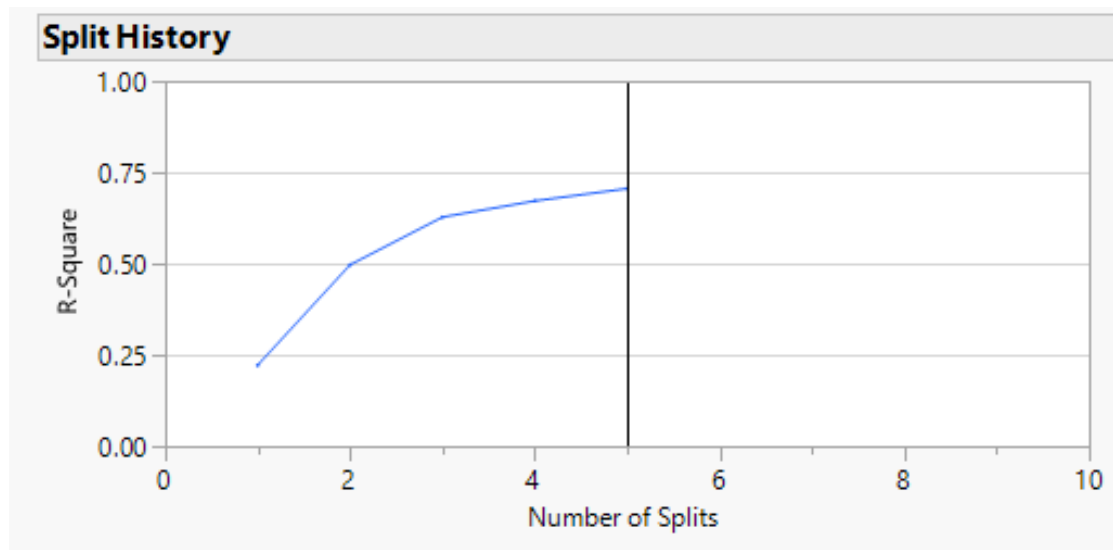
# Stopping Tree Growth

Natural end of process is 100% purity in each leaf

This **overfits** the data, which end up fitting noise in the data

Overfitting leads to low predictive accuracy of new data

Past a certain point, the error rate for the validation data starts to increase



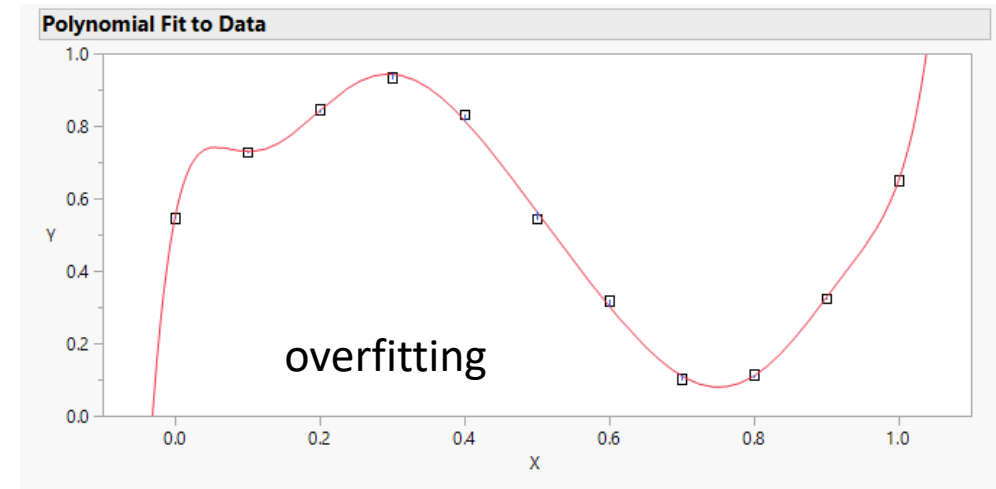
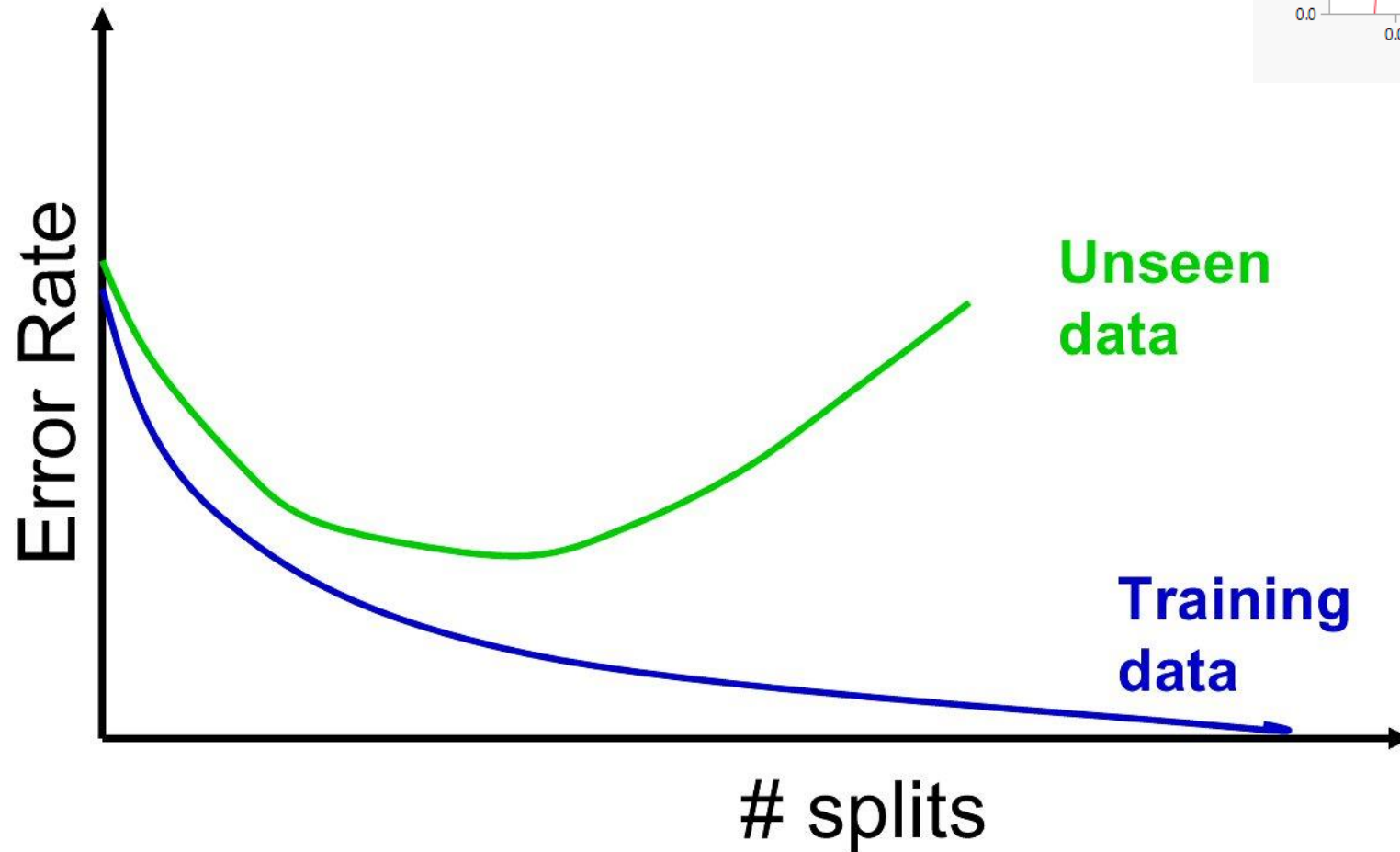
**Fit Details**

Measure	Training	Definition
Entropy RSquare	0.7060	$1 - \text{Loglike}(\text{model}) / \text{Loglike}(0)$
Generalized RSquare	0.8323	$(1 - (L(0) / L(\text{model}))^{2/n}) / (1 - L(0)^{2/n})$
Mean -Log p	0.2038	$\sum -\text{Log}(p[j]) / n$
RASE	0.2333	$\sqrt{\sum (y[j] - p[j])^2 / n}$
Mean Abs Dev	0.1620	$\sum  y[j] - p[j]  / n$
Misclassification Rate	0.0833	$\sum (p[j] \neq p\text{Max}) / n$
N	24	n

**Confusion Matrix**

Training		
Actual	Predicted Count	
Ownership	non-owner	owner
non-owner	12	0
owner	2	10

# Full Tree Error Rate



# CART - Classification and regression trees

- CART lets tree grow to full extent, then prunes it back
- Idea is to find that point at which the validation error begins to rise
- Generate successively smaller trees by pruning leaves
- At each pruning stage, multiple trees are possible
- Use *cost complexity* to choose the best tree at that stage



# Cost Complexity

$$CC(T) = Err(T) + \alpha L(T)$$

$CC(T)$  = cost complexity of a tree

$Err(T)$  = proportion of misclassified records

$L(T)$  – size of tree

$\alpha$  = penalty factor attached to tree size (set by user)

Among trees of given size, choose the one with lowest CC

Do this for each size of tree

# CART - Classification and regression trees

- Nonparametric (no probabilistic assumptions)
- Automatically performs variable selection
- Uses any combination of continuous/discrete variables
  - Very nice feature: ability to automatically bin massively categorical variables into a few categories (zip code, business class, make/model...)
- Invariant to monotonic transformations of predictive variable
- Unlike regression, not sensitive to outliers in predictive variables

# CART Overview

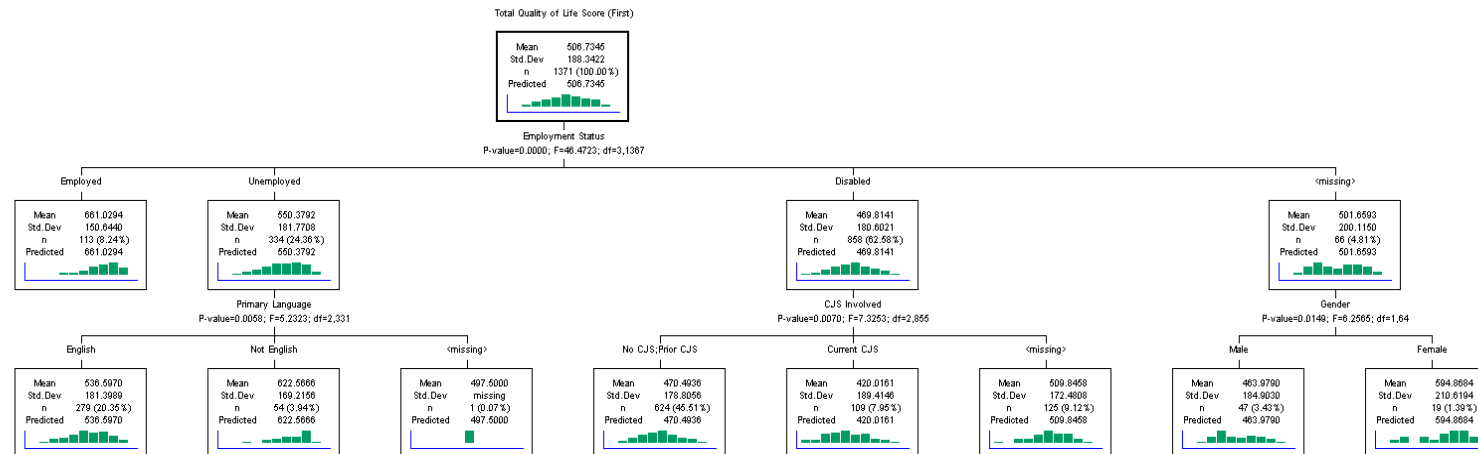
- Classification and Regression Trees are an easily understandable and transparent method for predicting or classifying new records
- A tree is a graphical representation of a set of rules
- Trees must be pruned to avoid over-fitting of the training data
- As trees do not make any assumptions about the data structure, they usually require large samples

# CHAID - Chi-squared automatic interaction detector

- CHAID, older than CART, uses chi-square statistical test to limit tree growth
- Splitting stops when purity improvement is not statistically significant

# CHAID - Chi-squared automatic interaction detector

- CHAID is a non-binary decision tree.
- The decision or split made at each node is still based on a single variable, but can result in multiple branches.
- The split search algorithm is designed for categorical variables.



# Classification Trees: CART versus CHAID

- At each split, the CHAID algorithm looks for the predictor variable that if split, most "explains" the category response variable. In order to decide whether to create a particular split based on this variable, the CHAID algorithm tests a hypothesis regarding dependence between the split variable and the categorical response (using the chi-squared test for independence). Using a pre-specified significance level, if the test shows that the split variable and the response are independent, the algorithm stops the tree growth. Otherwise, the split is created, and the next best split is searched. In contrast, the CART algorithm decides on a split based on the amount of homogeneity within class that is achieved by the split. The split is reconsidered based on considerations of over-fitting.
- CHAID is most useful for analysis, whereas CART is more suitable for prediction. In other words, CHAID should be used when the goal is to describe or understand the relationship between a response variable and a set of explanatory variables, whereas CART is better suited for creating a model that has high prediction accuracy of new cases.



# Limiting Tree Size

JMP uses a combination of limiting tree growth and pruning the tree after it has grown

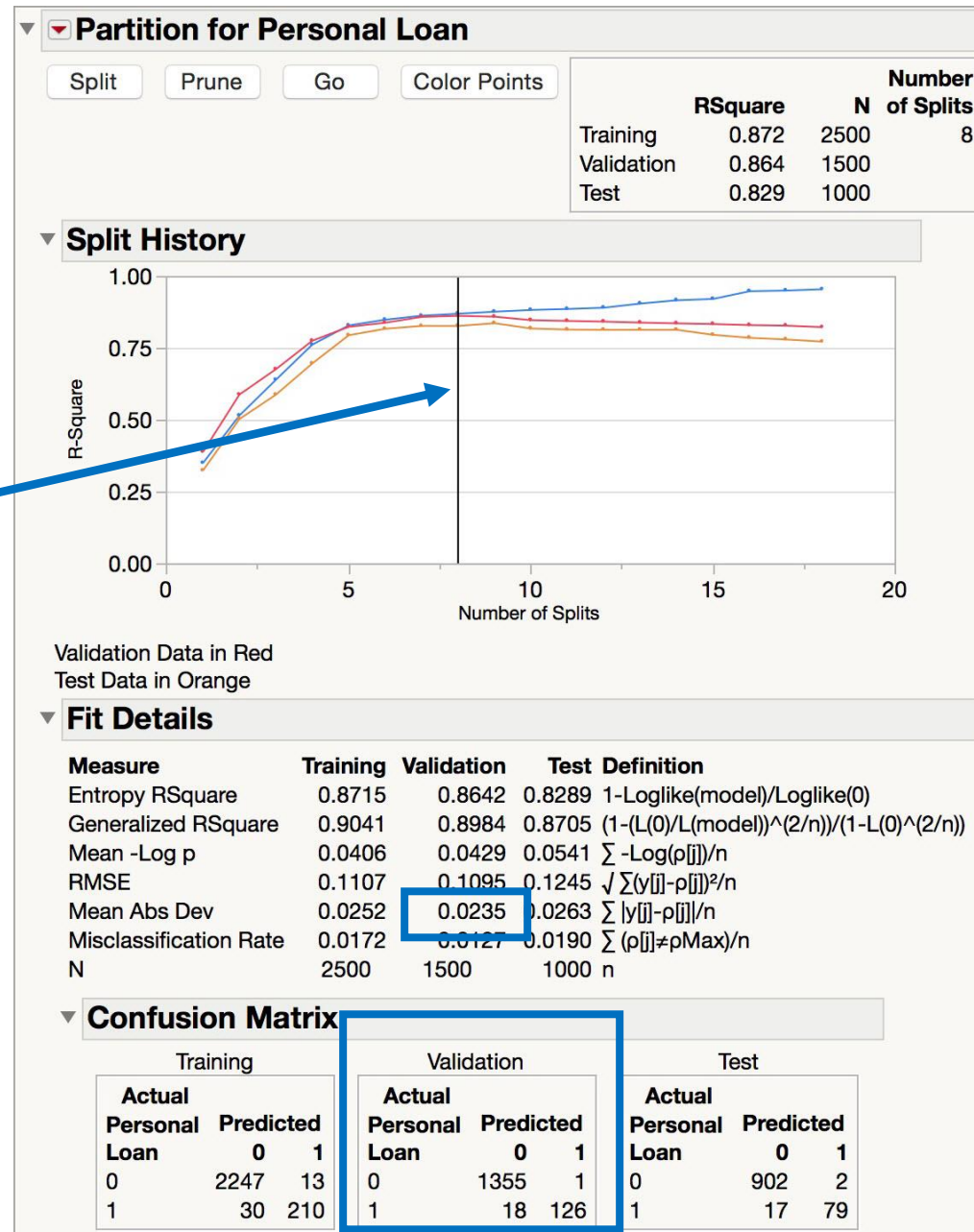
- **Minimum Split Size:** Controls the minimum number of records in terminal nodes
- **Validation:** The tree is grown, and pruned back to maximize the RSquare on the validation data

When validation is used, the “Go” option automates tree growth and pruning

The tree with the maximum Validation Rsquare has 8 splits

The tree is grown to 18 splits, and is pruned back to 8 splits

Validation error rate and confusion matrix for the final tree (cutoff for classification = 0.50)



# Validation

**Generalized RSquare** A measure that can be applied to general regression models. It is based on the likelihood function  $L$  and is scaled to have a maximum value of 1. The value is 1 for a perfect model, and 0 for a model no better than a constant model. The Generalized RSquare measure simplifies to the traditional RSquare for continuous normal responses in the standard least squares setting. Generalized RSquare is also known as the Nagelkerke or Craig and Uhler  $R^2$ , which is a normalized version of Cox and Snell's pseudo  $R^2$ . See Nagelkerke (1991).

**Entropy RSquare** (Appears only when the response is nominal or ordinal.) A measure of fit that compares the log-likelihoods from the fitted model and the constant probability model. Entropy RSquare ranges from 0 to 1, where values closer to 1 indicate a better fit. See “Entropy RSquare” on page 501 in the “Statistical Details” chapter.

**RSquare** Gives the RSquare for the model.

# Validation

**RMSE** Gives the root mean square error. When the response is nominal or ordinal, the differences are between 1 and  $p$  (the fitted probability for the response level that actually occurred).

**Mean Abs Dev** The average of the absolute values of the differences between the response and the predicted response. When the response is nominal or ordinal, the differences are between 1 and  $p$  (the fitted probability for the response level that actually occurred).

**Misclassification Rate** The rate for which the response category with the highest fitted probability is not the observed category. Appears only when the response is nominal or ordinal.

**-LogLikelihood** Gives the negative of the log-likelihood. See *Fitting Linear Models*.

**SSE** Gives the error sums of squares. Available only when the response is continuous.

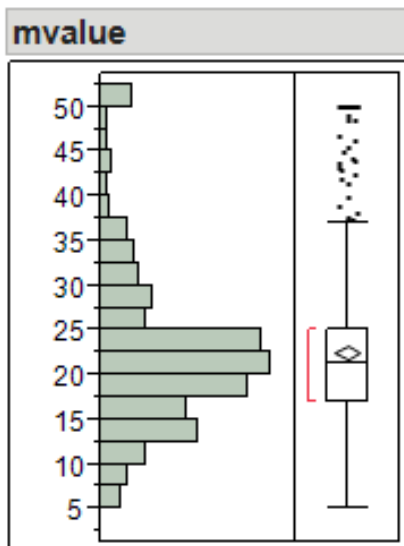
**Sum Freq** Gives the number of observations that are used. If you specified a Freq variable in the Neural launch window, Sum Freq gives the sum of the frequency column.

If there are multiple responses, fit statistics are given for each response, and an overall Generalized RSquare and negative Log-Likelihood is given.

# Regression Trees for Prediction

- Used with continuous outcome variable
- Procedure like classification tree
- Many splits attempted, choose the one that maximizes the difference between subgroup means
- Difference measured as the sum of squared deviations
- Prediction is the **average** of the numerical target variable (rather than a probability)

# Regression Trees



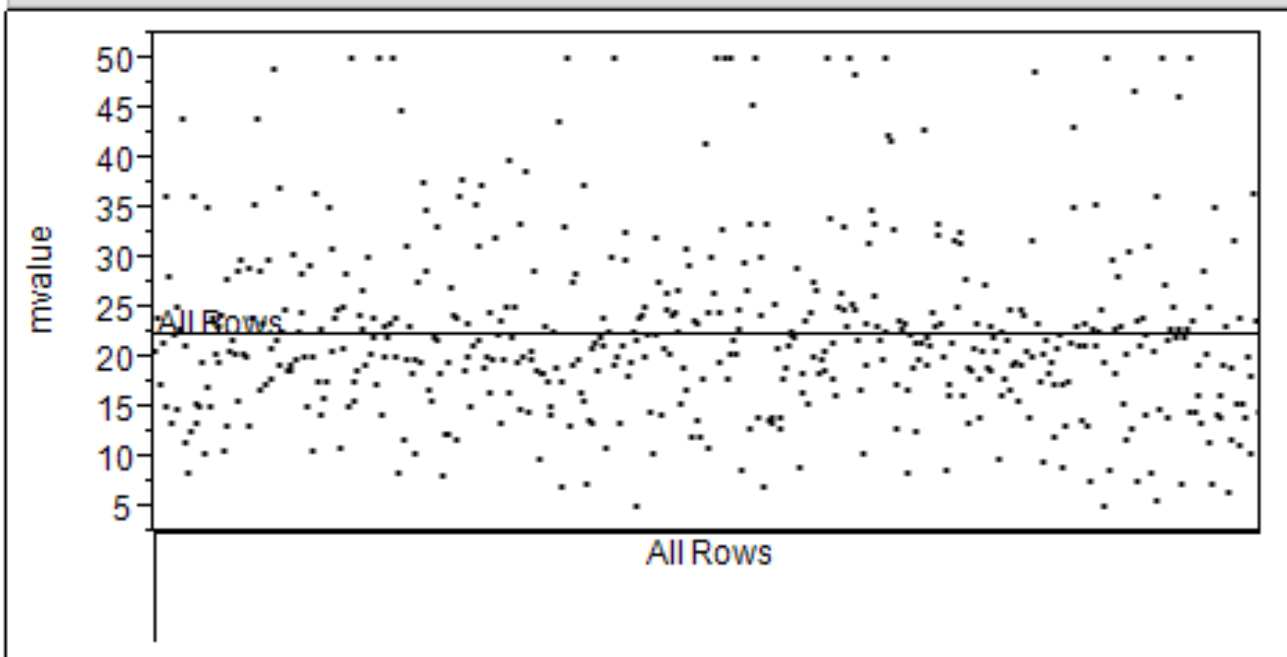
## Quantiles

100.0%	maximum	50
99.5%		50
97.5%		50
90.0%		34.9
75.0%	quartile	25
50.0%	median	21.2
25.0%	quartile	16.95
10.0%		12.7
2.5%		8.235
0.5%		5.321
0.0%	minimum	5

## Summary Statistics

Mean	22.532806
Std Dev	9.1971041
Std Err Mean	0.4088611
Upper 95% Mean	23.336085
Lower 95% Mean	21.729528
N	506

## Partition for mvalue

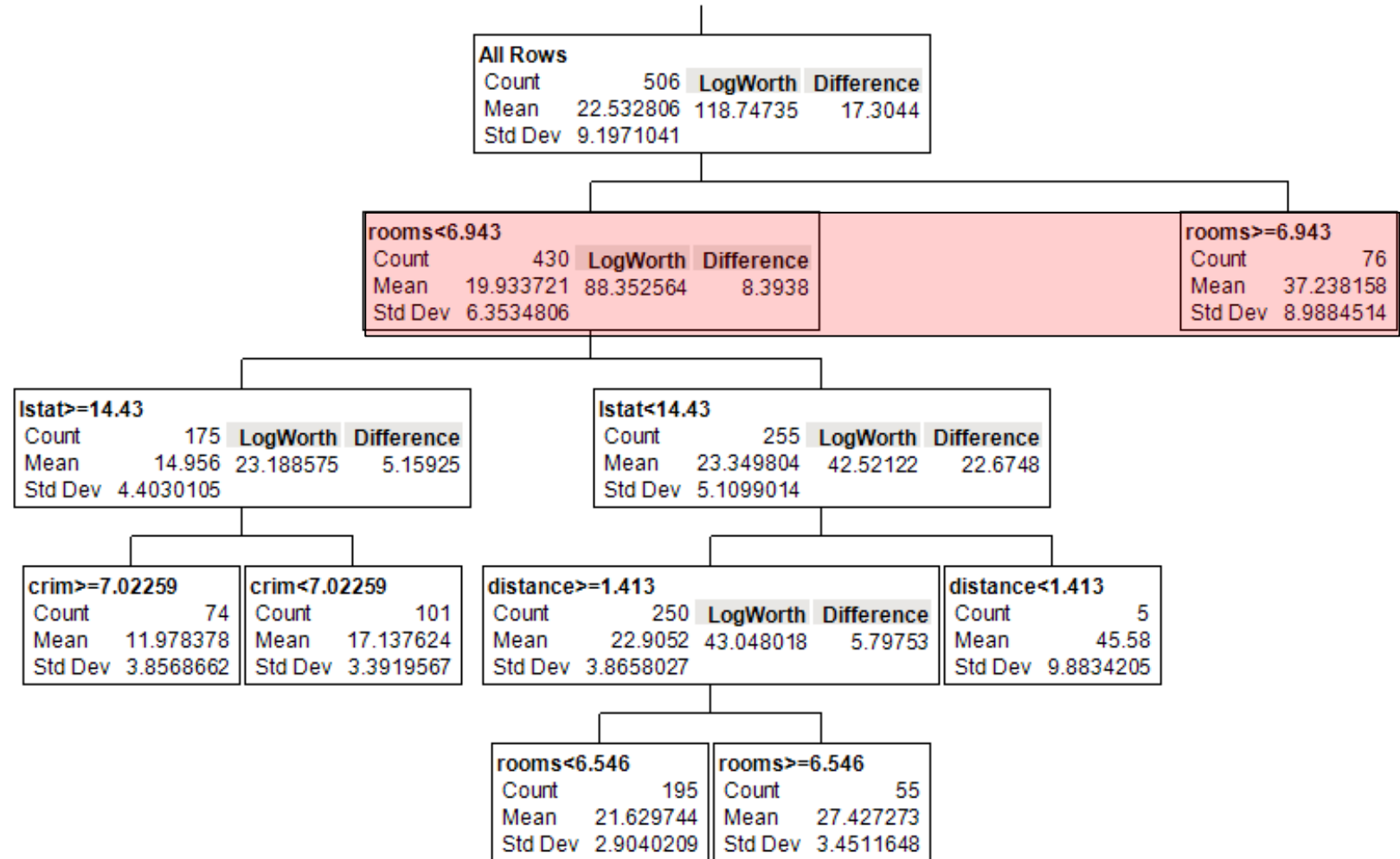


R Square	RMSE	N	Number of Splits	AICc
0.000	.	506	0	0

<b>All Rows</b>	
Count	506
Mean	22.532806
Std Dev	9.1971041



# Regression Trees





# Boston Housing Data

## All Rows

Count 506  
Mean 22.532806  
Std Dev 9.1971041

## Candidates

Term	Candidate SS	LogWorth
crim	8266.17273	32.6638216
zn	6669.06251	24.9773486
indus	11083.22547	48.7519537
chas	1312.07927	4.1110954
nox	9536.22405	39.5670978
rooms	19339.55503 *	118.7473483
age	5573.64765	19.6751451
distance	4994.54054	17.1453361
radial	6708.64333	24.6205659
tax	8618.08428	34.5266980
pt	10438.69478	44.8775094
b	5259.31980	18.2910466
lstat	18896.19401	113.7427626

## All Rows

Count 506  
Mean 22.532806  
Std Dev 9.1971041

LogWorth 118.74735  
Difference 17.3044

rooms<6.943			rooms>=6.943		
Count	430		Count	76	
Mean	19.933721		Mean	37.238158	
Std Dev	6.3534806		Std Dev	8.9884514	
Candidates			Candidates		
Term	Candidate SS	LogWorth	Term	Candidate SS	LogWorth
crim	4300.967311	38.57528016	crim	1296.353462	4.24150833
zn	1961.912781	13.93948488	zn	154.894267	0.16015922
indus	3552.756728	29.65539469	indus	650.180018	1.45829879
chas	533.165511	3.56806955	chas	97.802924	0.53155728
nox	4806.344267	45.22939006	nox	510.976998	0.97911866
rooms	2498.676569	18.68959899	rooms	3060.957502 *	19.65116632
age	3618.341104	30.39395326	age	106.820174	0.05293436
distance	3526.248005	29.35482815	distance	210.835800	0.20608146
radial	2778.264622	21.29849865	radial	1296.353462	4.68218182
tax	3487.174824	28.92548472	tax	1296.353462	4.30278667
pt	3808.647013	32.66254455	pt	1514.119195	5.52903675
b	2454.655577	18.26837433	b	750.759998	1.79989185
lstat	7311.852356 *	88.35256425	lstat	2011.069265	8.73682304

$-\log_{10}(p\text{-value})$

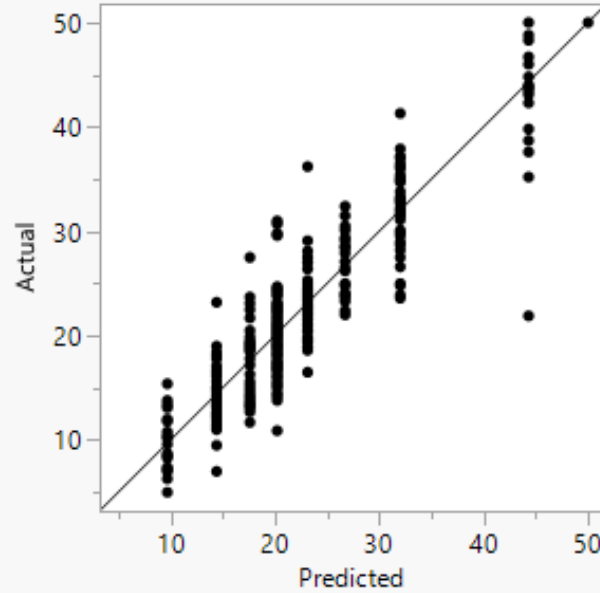


### Leaf Report

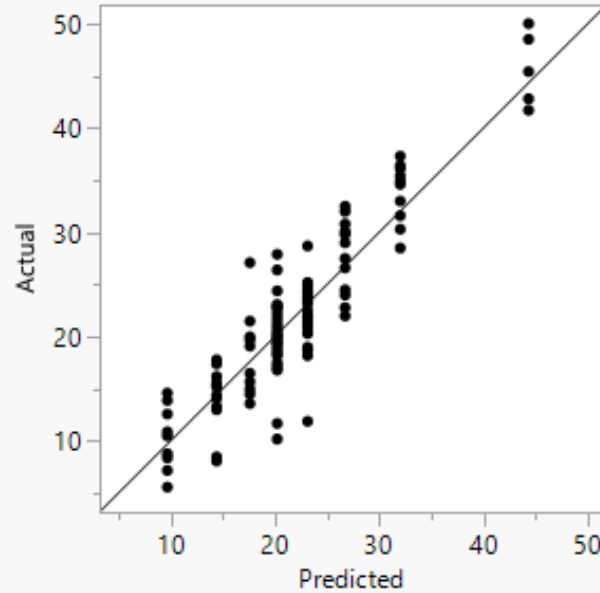
Leaf Label	Mean	Count
Istat>=9.97&Istat<=16.14&nox>=0.609&crim>=9.91655	9.6866667	30
Istat>=9.97&Istat<=16.14&nox>=0.609&crim<9.91655	14.41	40
Istat>=9.97&Istat<=16.14&nox<0.609	17.5947368	38
Istat>=9.97&Istat<16.14	20.2123894	113
Istat<9.97&rooms<7.454&distance>=1.6132&rooms<6.8&rooms<6.546	23.1253968	63
Istat<9.97&rooms<7.454&distance>=1.6132&rooms<6.8&rooms>=6.546	26.7185185	27
Istat<9.97&rooms<7.454&distance>=1.6132&rooms>=6.8	32.002381	42
Istat<9.97&rooms<7.454&distance<1.6132	50	5
Istat<9.97&rooms>=7.454	44.2954545	22

### Actual by Predicted Plot

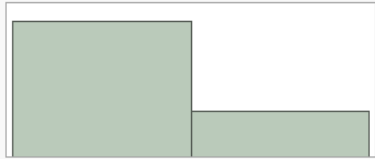
Training Set



Validation Set



### Validation



### Frequencies

Level	Count	Prob
Training	380	0.75099
Validation	126	0.24901
Total	506	1.00000
N Missing	0	

2 Levels

### Specify rates or relative rates

	Adjusted Rates	Row Counts
Training Set	0.75	0.75099 380
Validation Set	0.25	0.24901 126
Test Set	0	0 0
Excluded Rows		0 0
Total Rows		506

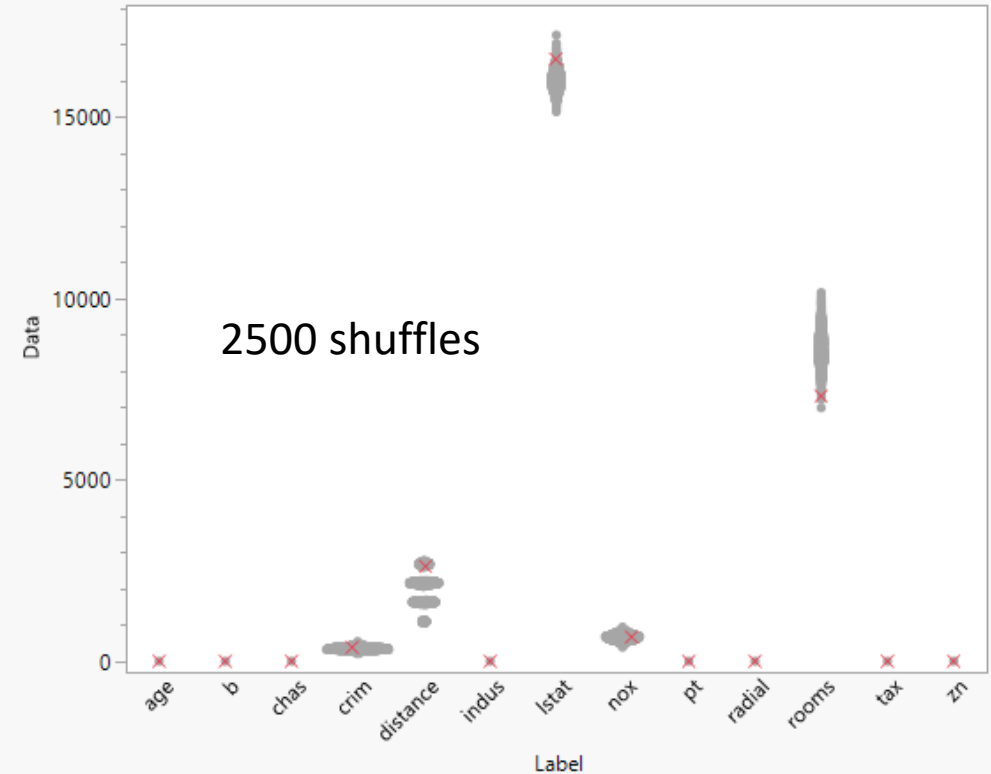
### Options

New Column Name	Validation
Validation Column Type	Formula

### Column Contributions

Term	Number of Splits	SS	Portion
Istat	2	16598.194	0.6019
rooms	3	7310.3095	0.2651
distance	1	2618.78505	0.0950
nox	1	668.298301	0.0242
crim	1	382.455048	0.0139
zn	0	0	0.0000
indus	0	0	0.0000
chas	0	0	0.0000
age	0	0	0.0000

### Data vs. Label



# Advantages of Trees

- Easy to use, understand
- Produce rules that are easy to interpret & implement
- Variable selection & reduction is automatic
- Do not require the assumptions of statistical models
- Can work without extensive handling of missing data (this is an option in the Partition dialog in JMP)

# Disadvantages of Trees

- May not perform well where there is structure in the data that is not well captured by horizontal or vertical splits
- Since the process deals with one variable at a time, no way to capture interactions between variables

# Improving Trees

- Single trees may not have good predictive ability.
- Results from multiple trees can be combined to improve performance
- The resulting model is an “ensemble” model
- Two multi-tree approaches in JMP Pro:
  - **Bootstrap Forests** (a variant of Random Forests)
  - **Boosted Trees**

# Ensemble Tree Methods

- Bootstrap Forests (Random Forrest)

Grow many trees to bootstrapped versions of the training data and average them

- Boosted Trees (Boosting)

Repeatedly grow shallow trees to the residuals and build up an additive model consisting of a sum of trees



# Ensemble Tree Methods

## Bootstrap Forests

1. A random sample is drawn with replacement from the data set (bootstrapping)
2. Predictors are randomly drawn from the candidate list of predictors
3. A small tree is fit (a “weak learner”)
4. The process is repeated
5. The final model is the average of all of the trees, producing a “Bootstrap aggregated” (or “bagged”) model

# Ensemble Tree Methods

## Boosted Trees

1. A simple (small) tree is fit to the data with a random sample of the predictors
2. The scaled residuals from this tree are calculated
3. A new simple tree is fit to these scaled residuals with another random sample of predictors
4. This process continues
5. The final boosted model is the sum of the models for the individual trees

## Chapter 7

# Modern Analytic Methods: Part I



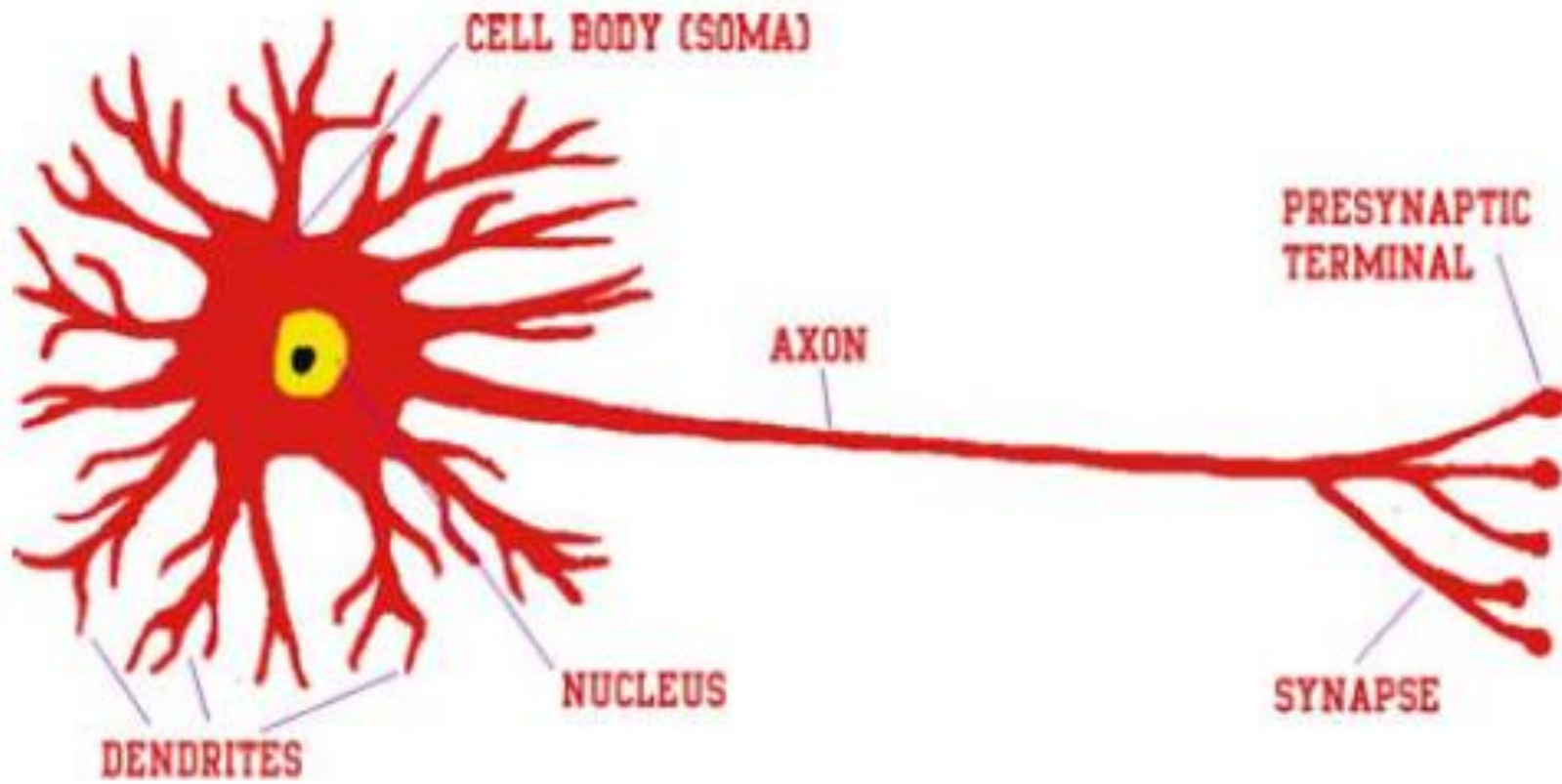
### 7.8 Neural Networks

A neural network is composed of a set of computational units, called neurons, connected together through weighted connections. Neurons are organized in layers so that every neuron in a layer is exclusively connected to the neurons of the preceding layer and the subsequent layer. Every neuron, also called a node, represents an autonomous computational unit and receives inputs as a series of signals that dictate its activation. Following activation, every neuron produces an output signal. All the input signals reach the neuron simultaneously, so the neuron receives more than one input signal, but it produces only one output signal. Every input signal is associated with a connection weight. The weight determines the relative importance the input signal can have in producing the final impulse transmitted by

# Neural networks: Basic Idea

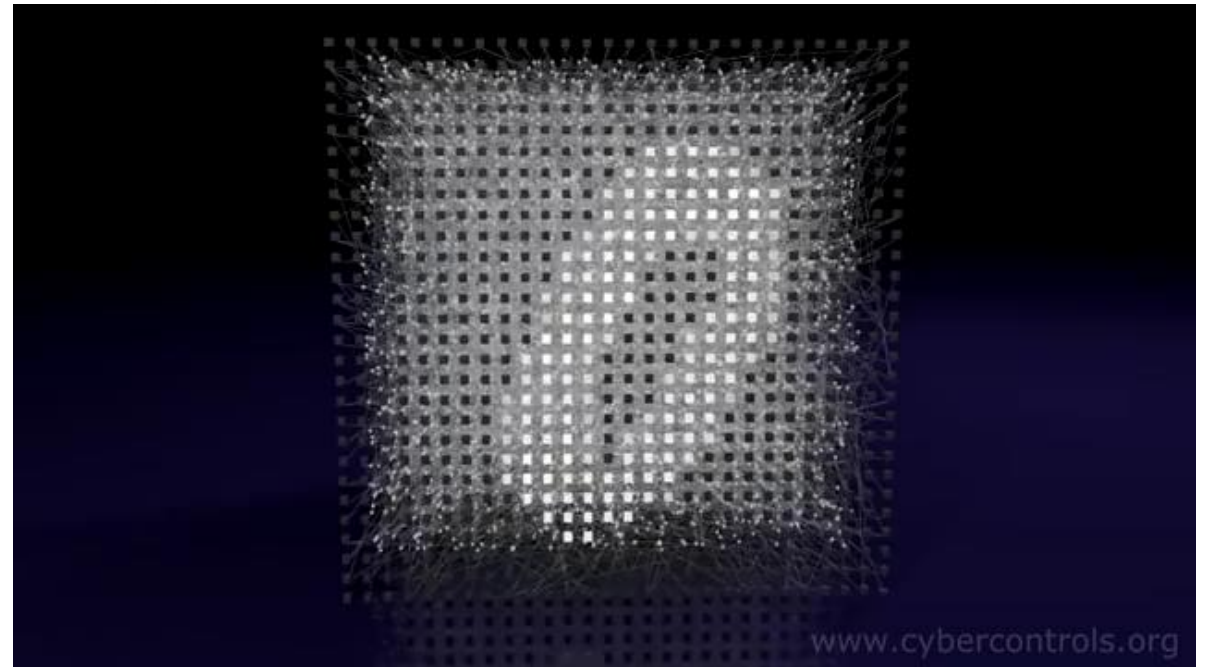
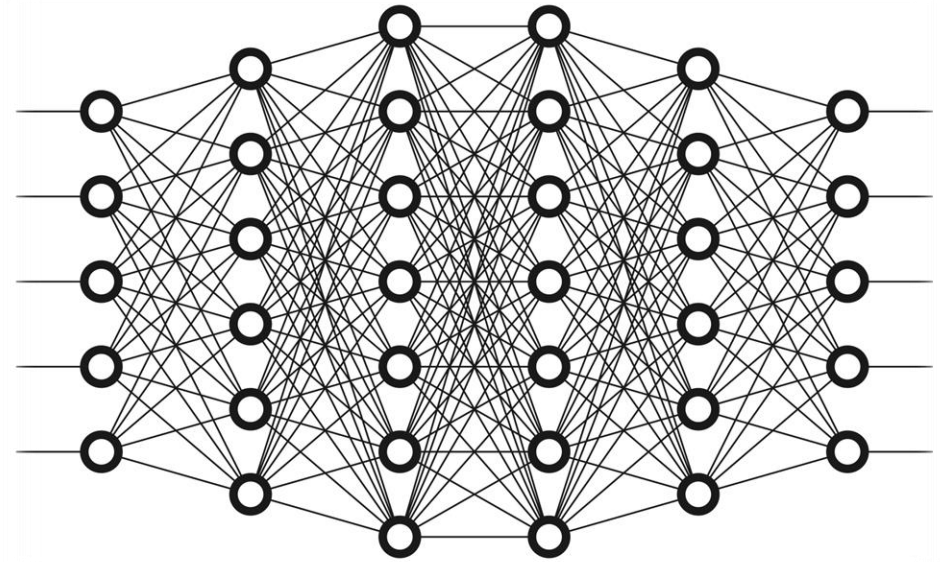
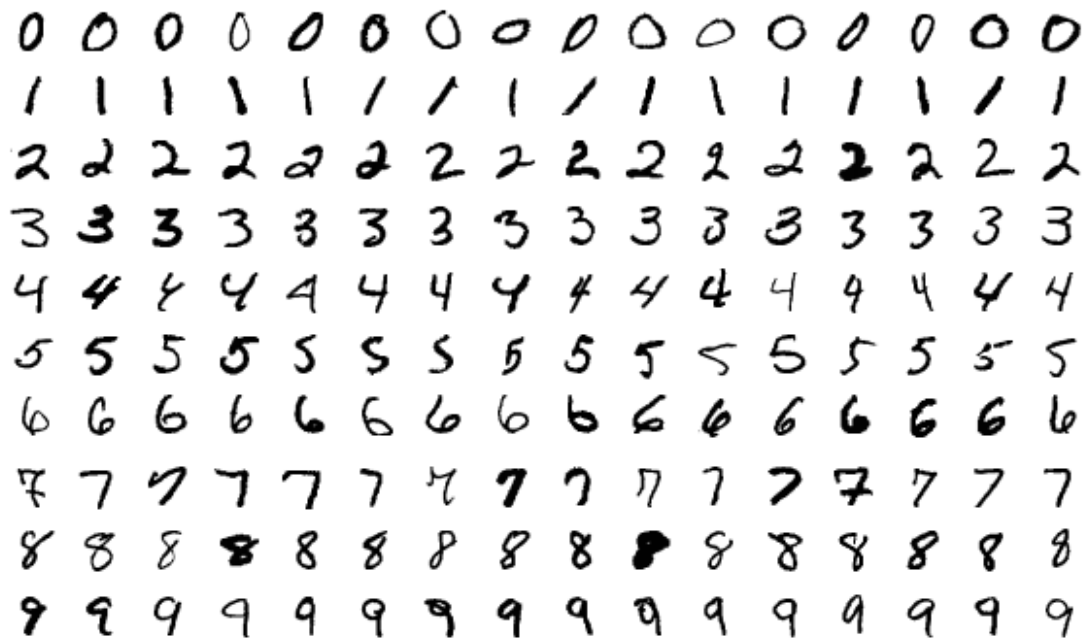
- Combine input information in a complex and flexible neural net “model”
- Model “coefficients” are continually tweaked in an iterative process
- The network’s interim performance in classification and prediction informs successive tweaks

# Neural Networks



# MNIST

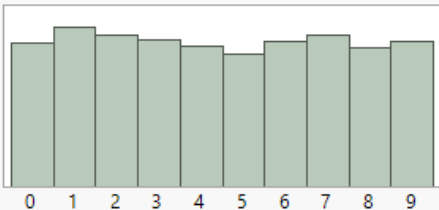
Modified National Institute of  
Standards and Technology





[illegible]



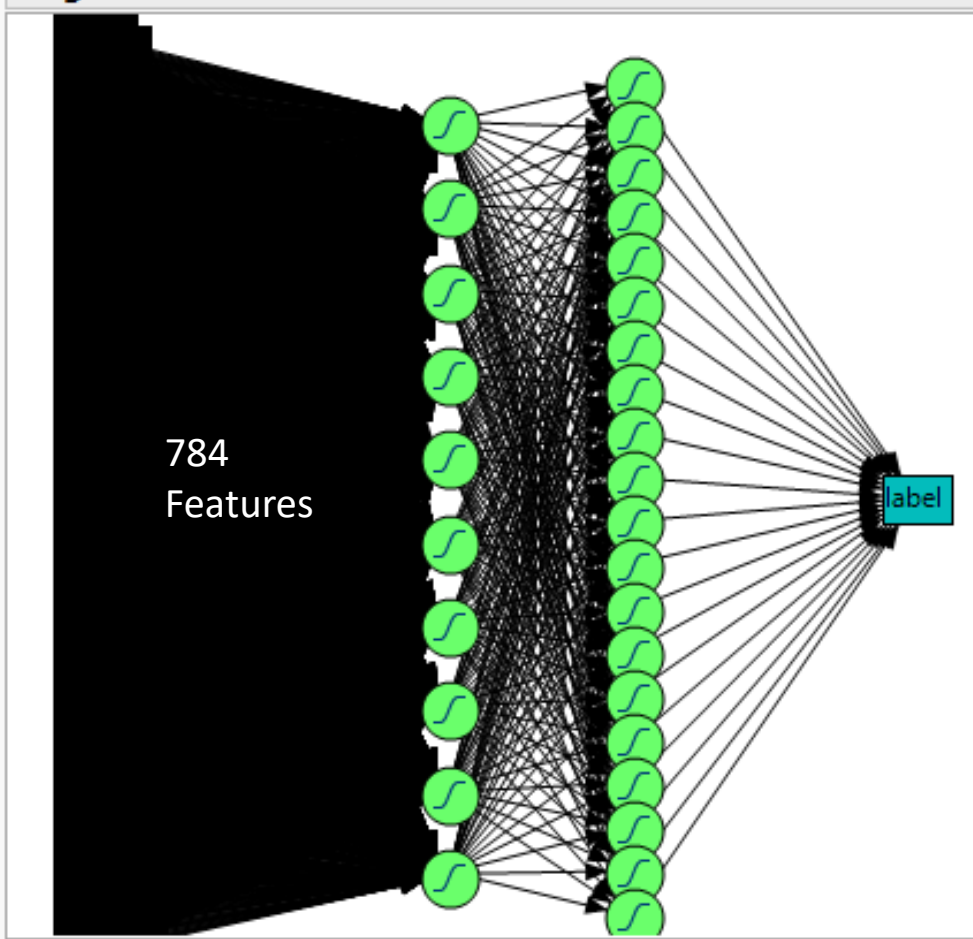
Measures	Value	label
Generalized RSquare	0.9800862	
Entropy RSquare	0.7637519	
RASE	0.3642818	
Mean Abs Dev	0.1918127	
Misclassification Rate	0.146	
-LogLikelihood	1359.2096	
Sum Freq	2500	
Confusion Ma		

Frequencies		
Level	Count	Prob
0	742	0.09893
1	822	0.10960
2	783	0.10440
3	757	0.10093
4	725	0.09667
5	680	0.09067
6	752	0.10027
7	780	0.10400
8	713	0.09507
9	746	0.09947
Total	7500	1.00000
N Missing	0	
10 Levels		

Frequencies		
Level	Count	Prob
0	742	0.09893
1	822	0.10960
2	783	0.10440
3	757	0.10093
4	725	0.09667
5	680	0.09067
6	752	0.10027
7	780	0.10400
8	713	0.09507
9	746	0.09947
Total	7500	1.00000
N Missing	0	
10 Levels		

Actual	Predicted Count									
label	0	1	2	3	4	5	6	7	8	9
0	222	0	1	6	1	7	4	1	7	0
1	0	259	2	3	0	1	4	0	3	1
2	5	5	227	5	4	0	1	5	9	1
3	2	1	7	213	0	10	0	4	11	4
4	3	0	6	1	197	1	3	5	0	26
5	6	2	5	18	2	172	3	4	13	1
6	5	0	8	0	2	5	230	0	1	0
7	0	3	6	6	4	2	0	227	1	10
8	2	6	9	12	1	10	3	3	188	3
9	2	1	1	4	17	4	0	17	3	200

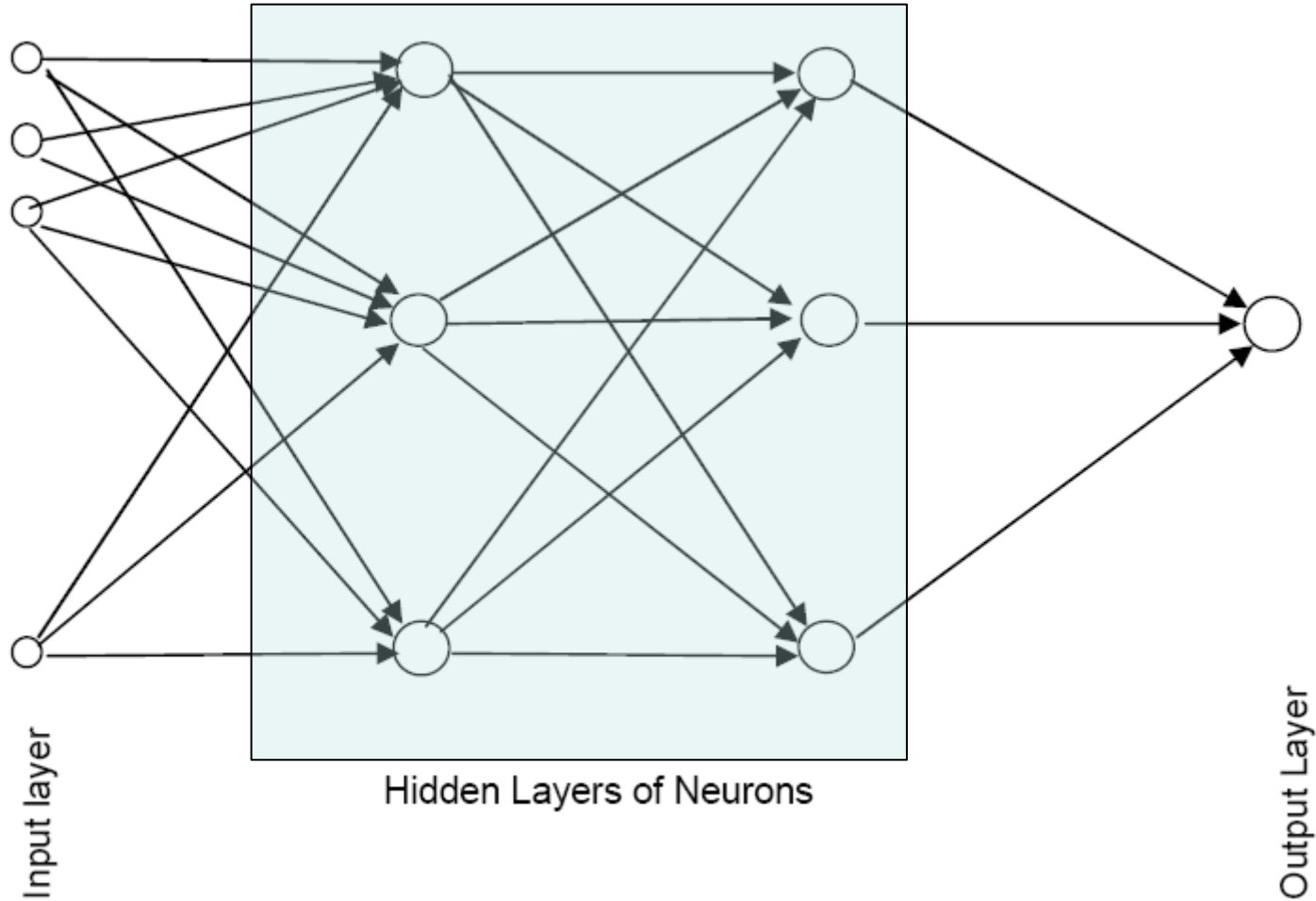
Diagram



# Network Structure

- Multiple layers
  - Input layer (raw observations)
  - Hidden layers
  - Output layer
- Nodes
- Weights (like coefficients, subject to iterative adjustment)
- Bias values (also like coefficients, but not subject to iterative adjustment)

# Schematic Diagram





Tiny Dataset.jmp

# Tiny Example

- Using fat and salt content to predict consumer acceptance of cheese

	Obs	Fat Score	Salt Score	Acceptance
1	1	0.2	0.9	like
2	2	0.1	0.1	dislike
3	3	0.2	0.4	dislike
4	4	0.2	0.5	like
5	5	0.4	0.5	like
6	6	0.3	0.8	like

# Tiny Example Neural Network

Neural - JMP Pro

Predicts one or more response variables using a flexible function of the input variables.

Select Columns

▼ 5 Columns

- ▲ Obs
- ▲ Fat Score
- ▲ Salt Score
- Acceptance
- Validation

☐ Informative Missing

Set Random Seed

Cast Selected Columns into Roles

Y, Response	■ Acceptance <i>optional</i>
X, Factor	▲ Fat Score ▲ Salt Score <i>optional</i>
Freq	<i>optional numeric</i>
Validation	<i>optional numeric</i>
By	<i>optional</i>

Action

OK

Cancel

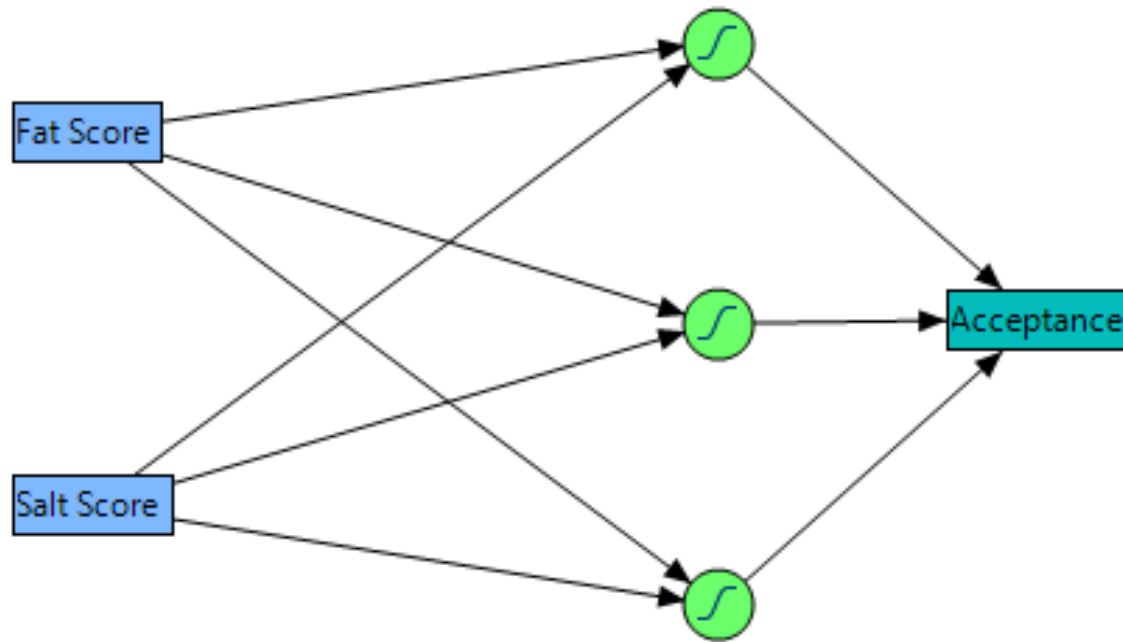
Remove

Recall

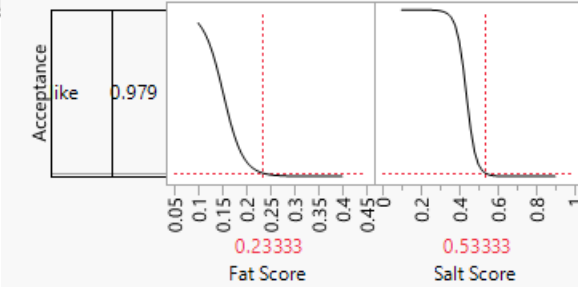
Help

# Tiny Example Neural Network

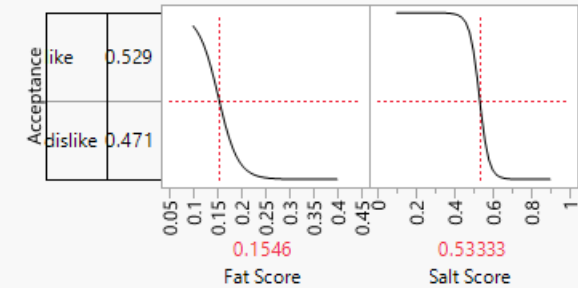
Diagram



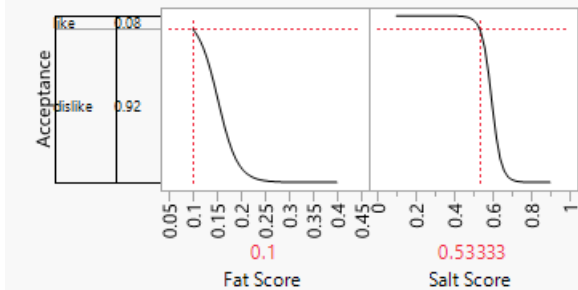
Prediction Profiler



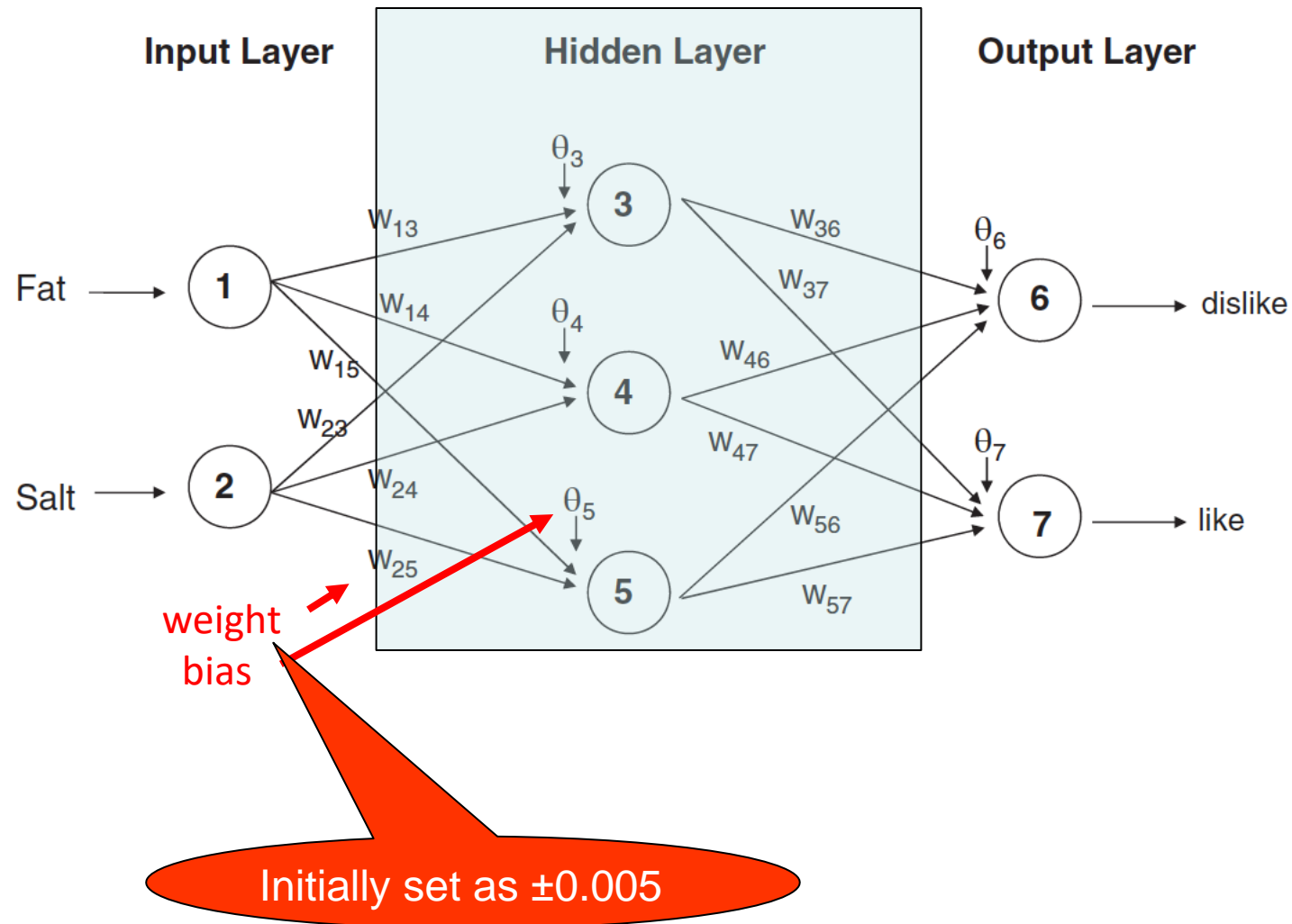
Prediction Profiler



Prediction Profiler



# Tiny Example Neural Network





# The Input Layer

For input layer, input = output

E.g., for record #1:

Fat input = output = 0.2

Salt input = output = 0.9

Output of input layer = input into hidden layer

# The Hidden Layer

In this example, hidden layer has 3 nodes

Each node receives as input the output of all input nodes

Output of each hidden node is a function of the weighted sum of inputs

$$output_j = g(\Theta_j + \sum_{i=1}^p w_{ij} x_i)$$

(The hidden layer function is also called an "activation function".)

# The Hidden Layer

**TanH** The hyperbolic tangent function is a sigmoid function. TanH transforms values to be between -1 and 1, and is the centered and scaled version of the logistic function. The hyperbolic tangent function is:

$$\frac{e^{2x} - 1}{e^{2x} + 1}$$

where  $x$  is a linear combination of the  $X$  variables.

**Linear** The identity function. The linear combination of the  $X$  variables is not transformed.

The Linear activation function is most often used in conjunction with one of the non-linear activation functions. In this case, the Linear activation function is placed in the second layer, and the non-linear activation functions are placed in the first layer. This is useful if you want to first reduce the dimensionality of the  $X$  variables, and then have a nonlinear model for the  $Y$  variables.

For a continuous  $Y$  variable, if only Linear activation functions are used, the model for the  $Y$  variable reduces to a linear combination of the  $X$  variables. For a nominal or ordinal  $Y$  variable, the model reduces to a logistic regression.

# The Hidden Layer

**Gaussian** The Gaussian function. Use this option for radial basis function behavior, or when the response surface is Gaussian (normal) in shape. The Gaussian function is:

$$e^{-x^2}$$

where  $x$  is a linear combination of the  $X$  variables.

Use the Boosting panel in the Model Launch control panel to specify the number of component models and the learning rate. Use the Hidden Layer Structure panel in the Model Launch control panel to specify the structure of the base model.

The learning rate must be  $0 < r \leq 1$ . Learning rates close to 1 result in faster convergence on a final model, but also have a higher tendency to overfit data. Use learning rates close to 1 when a small Number of Models is specified.

# Options

Method	Penalty Function	Description
Squared	$\sum \beta_i^2$	Use this method if you think that most of your X variables are contributing to the predictive ability of the model.
Absolute	$\sum  \beta_i $	Use either of these methods if you have a large number of X variables, and you think that a few of them contribute more than others to the predictive ability of the model.
Weight Decay	$\sum \frac{\beta_i^2}{1 + \beta_i^2}$	
NoPenalty	none	Does not use a penalty. You can use this option if you have a large amount of data and you want the fitting process to go quickly. However, this option can lead to models with lower predictive performance than models that use a penalty.

# The Weights

The weights  $\theta$  (theta) and  $w$  are typically initialized to random values in the range -0.05 to +0.05

➤ JMP uses random normal starting weights

Equivalent to a model with random prediction (in other words, no predictive value)

These initial weights are used in the first round of training

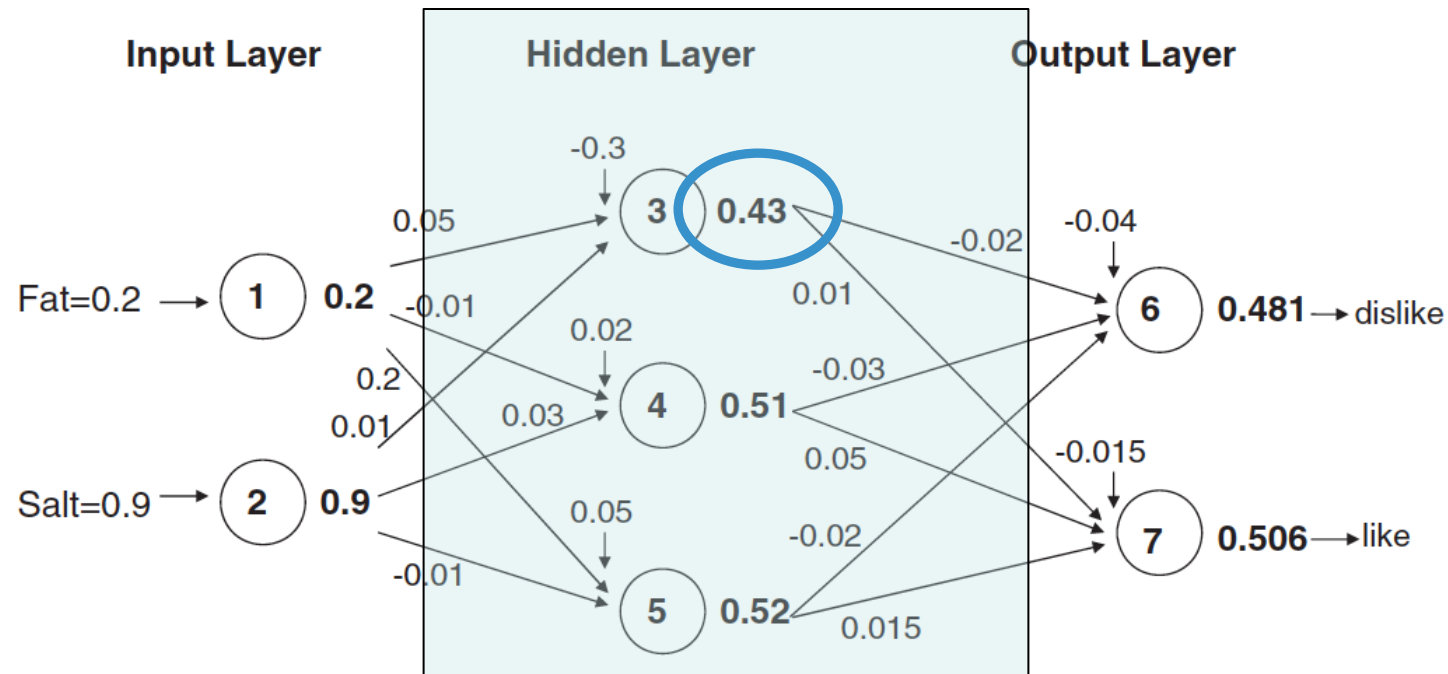
Output of Node 3, if  $g$  is a Logistic Function

$$output_j = g(\theta_j + \sum_{i=1}^p w_{ij} x_i)$$

$$Output_j = g\left(\theta_j + \sum_{i=1}^p w_{ij} x_i\right) = \frac{1}{1 + e^{-(\theta_j + \sum_{i=1}^p w_{ij} x_i)}}.$$

$$output_3 = \frac{1}{1 + e^{-[-0.3 + (0.05)(0.2) + (0.01)(0.9)]}} = 0.43$$

# Tiny Example Neural Weights





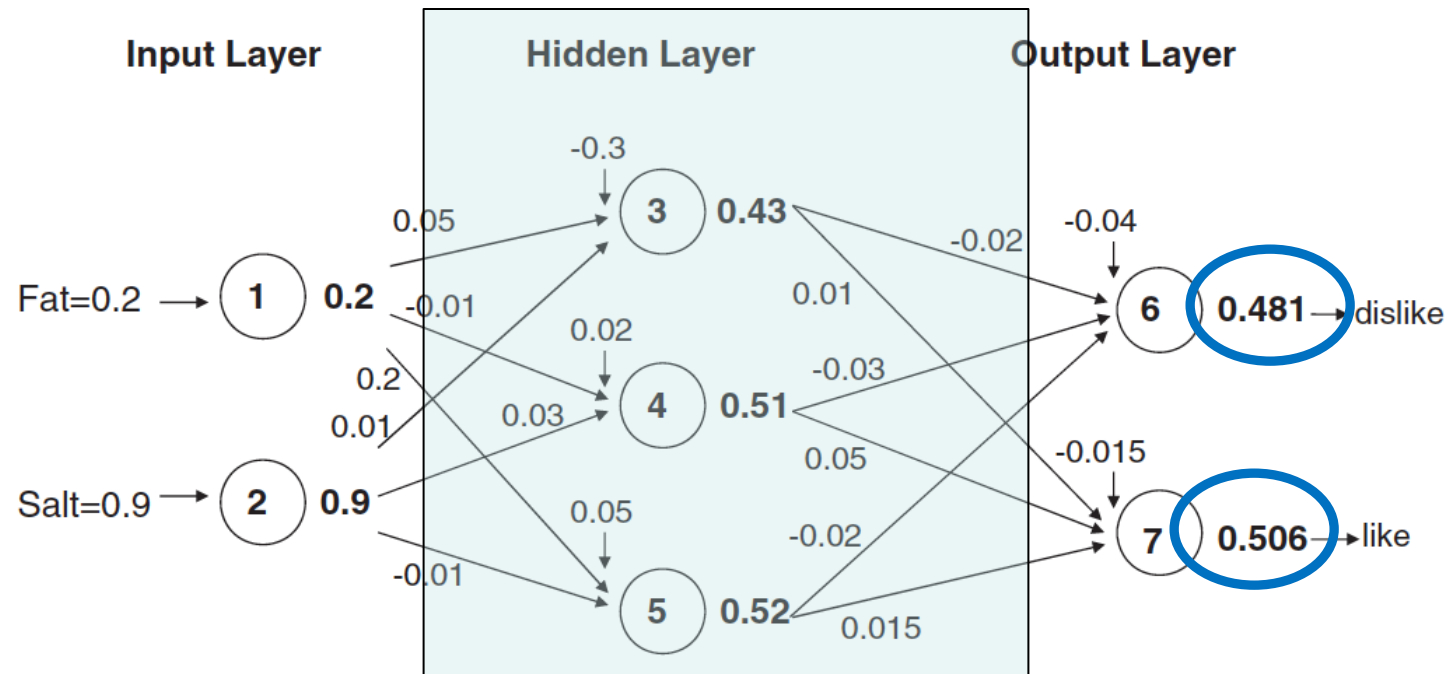
# Output Layer

The output of the last hidden layer becomes input for the output layer

Uses same function as above, i.e. a function  $g$  of the weighted average

$$\text{Output}_6 = \frac{1}{1 + e^{-[-0.04 + (-0.02)(0.43) + (-0.03)(0.51) + (0.015)(0.52)]}} = 0.481$$
$$\text{Output}_7 = \frac{1}{1 + e^{-[-0.015 + (0.01)(0.430) + (0.05)(0.507) + (0.015)(0.511)]}} = 0.506$$

# Tiny Example Output Layer



# Mapping the output to a classification

These values are normalized so they are propensities (which add up to 1.0).

$$\begin{aligned}P(Y = \text{Dislike}) &= \text{Output}_6 / (\text{Output}_6 + \text{Output}_7) \\&= 0.481 / (0.481 + 0.506) = 0.49 \\P(Y = \text{Like}) &= 1 - P(Y = \text{Dislike}) \\&= 0.506 / (0.481 + 0.506) = 0.51\end{aligned}$$

The default cutoff for classification is 0.5.

This first record would be classified as a Like.

# Relation to Linear Regression

A net with a single output node and no hidden layers, where  $g$  is the identity function, takes the same form as a linear regression model

$$\hat{y} = \Theta + \sum_{i=1}^p w_i x_i$$

# Initial Pass-Through Network

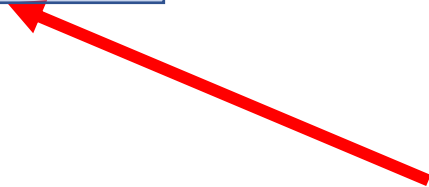
**Goal:** Find weights that yield best predictions

- The process we described above is repeated for all records
- At each record, compare prediction to actual
- Difference is the error for the output node
- Error is propagated back and distributed to all the hidden nodes and used to update their weights

## Training the Model

# Back Propagation of Error

- Output from output node k:  $\hat{y}_k$
- Error associated with that node:

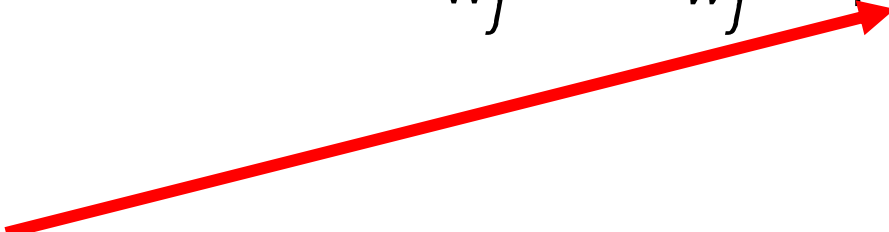
$$err_k = \hat{y}_k(1 - \hat{y}_k)(y_k - \hat{y}_k)$$


Note: this is like ordinary error, multiplied by a correction factor

# Error is Used to Update Weights

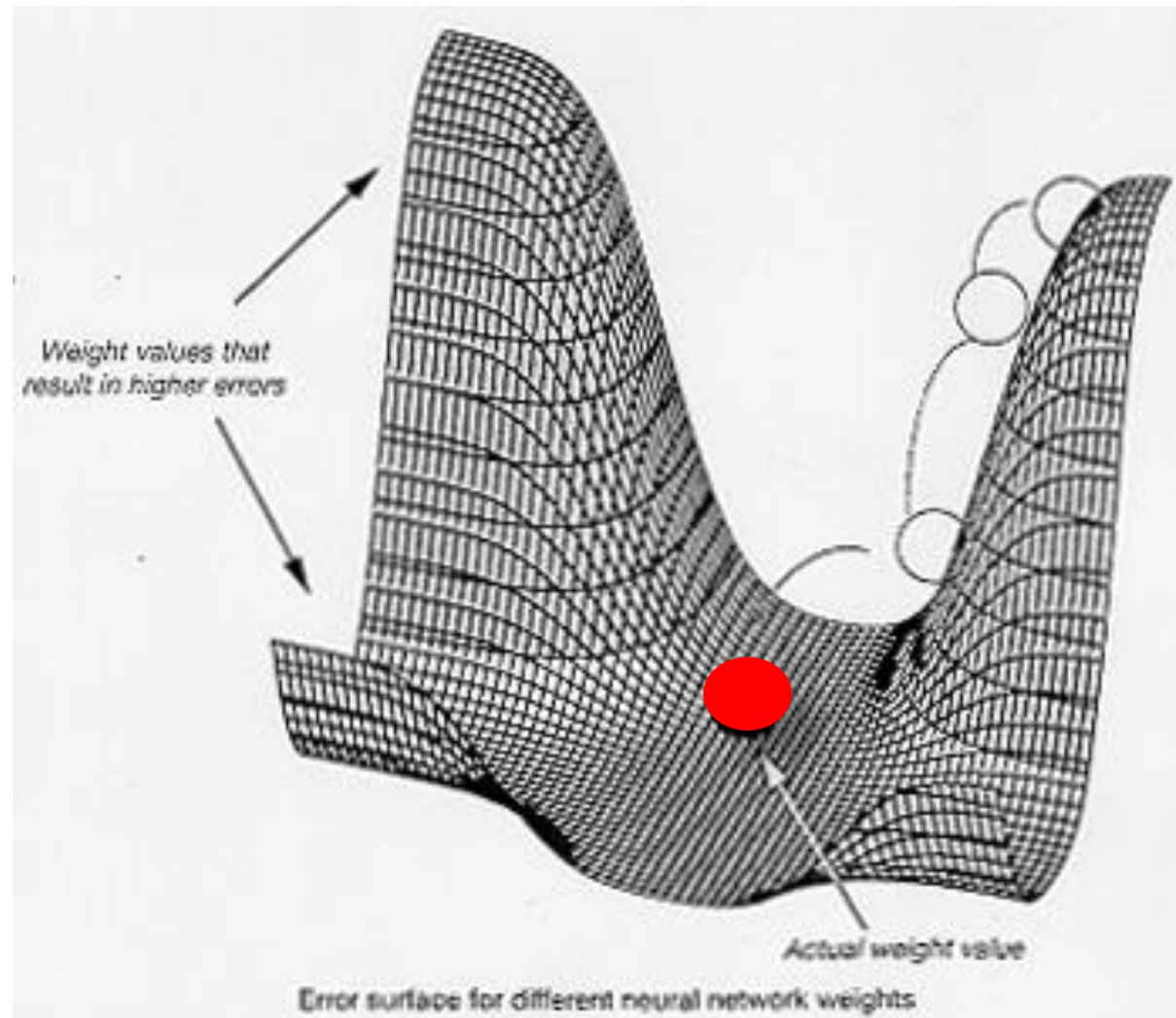
$$\theta_j^{new} = \theta_j^{old} + l(err_j)$$

$$w_j^{new} = w_j^{old} + l(err_j)$$



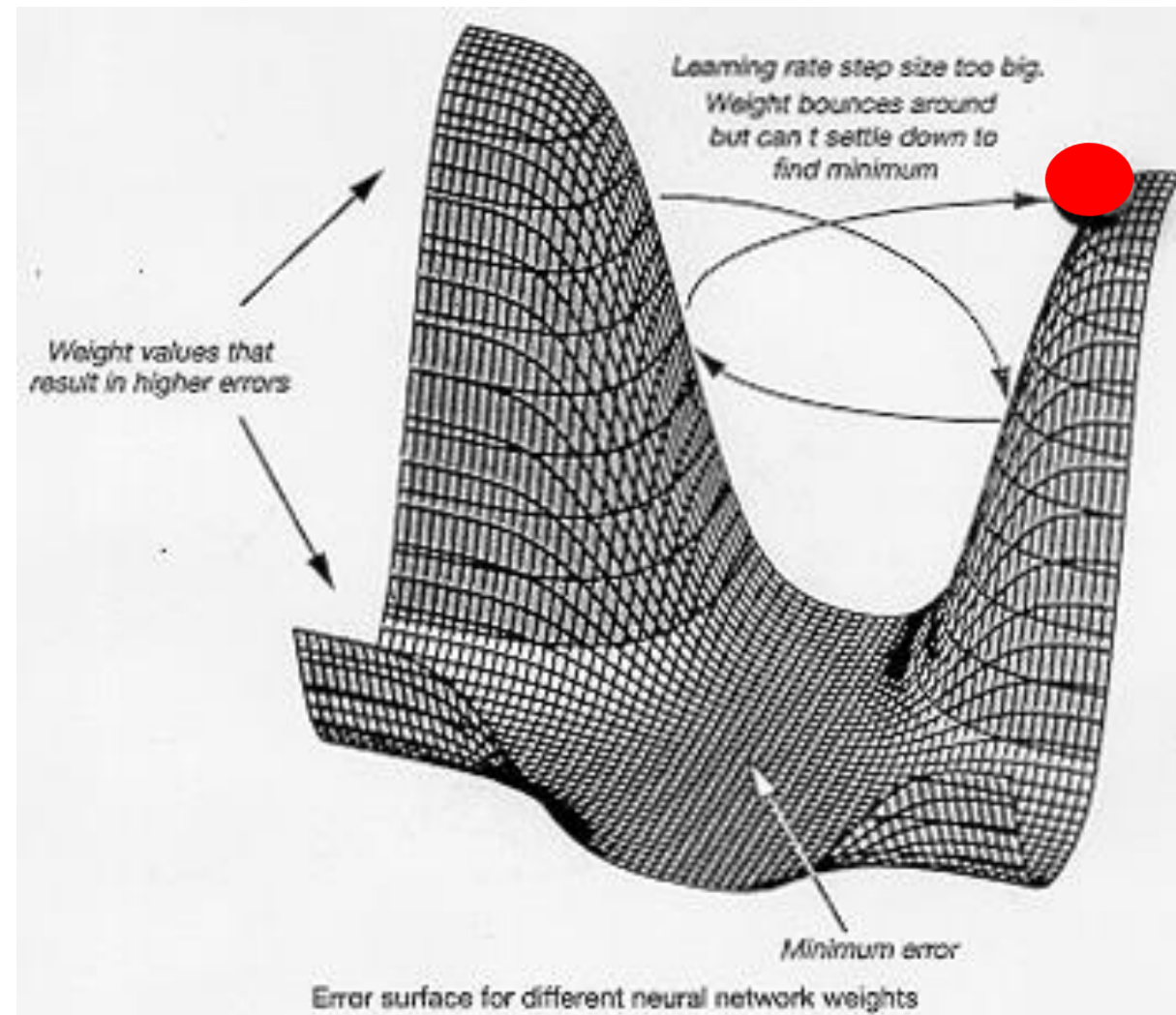
$l$  = constant between 0 and 1, reflects the “learning rate” or “weight decay parameter”

# Error is Used to Update Weights





# Error is Used to Update Weights



# Case Updating

- Weights are updated after each record is run through the network
- Completion of all records through the network is one *epoch* (also called *sweep* or *iteration*)
- After one epoch is completed, return to first record and repeat the process

# Case Updating

In case updating, the weights are updated after each record is run through the network (called a *trial*). For example, if we used case updating in the tiny example, the weights would first be updated after running record 1 as follows: Using a learning rate of 0.5, the weights  $\theta_7$ ,  $w_{3,7}$ ,  $w_{4,7}$ , and  $w_{5,7}$  are updated to

$$\begin{aligned}\theta_7 &= -0.015 + (0.5)(0.123) = 0.047 \\ w_{3,7} &= 0.01 + (0.5)(0.123) = 0.072 \\ w_{4,7} &= 0.05 + (0.5)(0.123) = 0.112 \\ w_{5,7} &= 0.015 + (0.5)(0.123) = 0.077\end{aligned}$$

Similarly, we obtain updated weights  $\theta_6 = 0.025$ ,  $w_{3,6} = 0.045$ ,  $w_{4,6} = 0.035$ , and  $w_{5,6} = 0.045$ . These new weights are next updated after the second record is run through the network, the third, and so on, until all records are used. This is called one *epoch*, *sweep*, or *iteration* through the data. Typically, there are many iterations.

# Batch Updating

- All records in the training set are fed to the network before updating takes place
- In this case, the error used for updating is the sum of all errors from all records

# Batch Updating

In batch updating, the entire training set is run through the network before each updating of weights takes place. In that case, the errors  $\text{err}_k$  in the updating equation is the sum of the errors from all records. In practice, case updating tends to yield more accurate results than batch updating, but requires a longer run time. This is a serious consideration, since even in batch updating, hundreds or even thousands of sweeps through the training data are executed.

When does the updating stop? The most common conditions are one of the following:

1. When the new weights are only incrementally different from those of the preceding iteration
2. When the misclassification rate reaches a required threshold
3. When the limit on the number of runs is reached

# Why It Works

- Big errors lead to big changes in weights
- Small errors leave weights relatively unchanged
- Over thousands of updates, a given weight keeps changing until the error associated with that weight is negligible, at which point weights change little

# Common Criteria to Stop the Updating

- When weights change very little from one iteration to the next
- When the misclassification rate reaches a required threshold
- When a limit on runs is reached

# Neural Model fitting in JMP

- JMP uses an algorithm that finds optimal values of weights and bias values that minimize a function of the combined errors (maximum likelihood)
- This approach produces similar results to back propagation, but
  - Its generally much faster
  - It can be used for both continuous and categorical responses



# Neural Model fitting in JMP

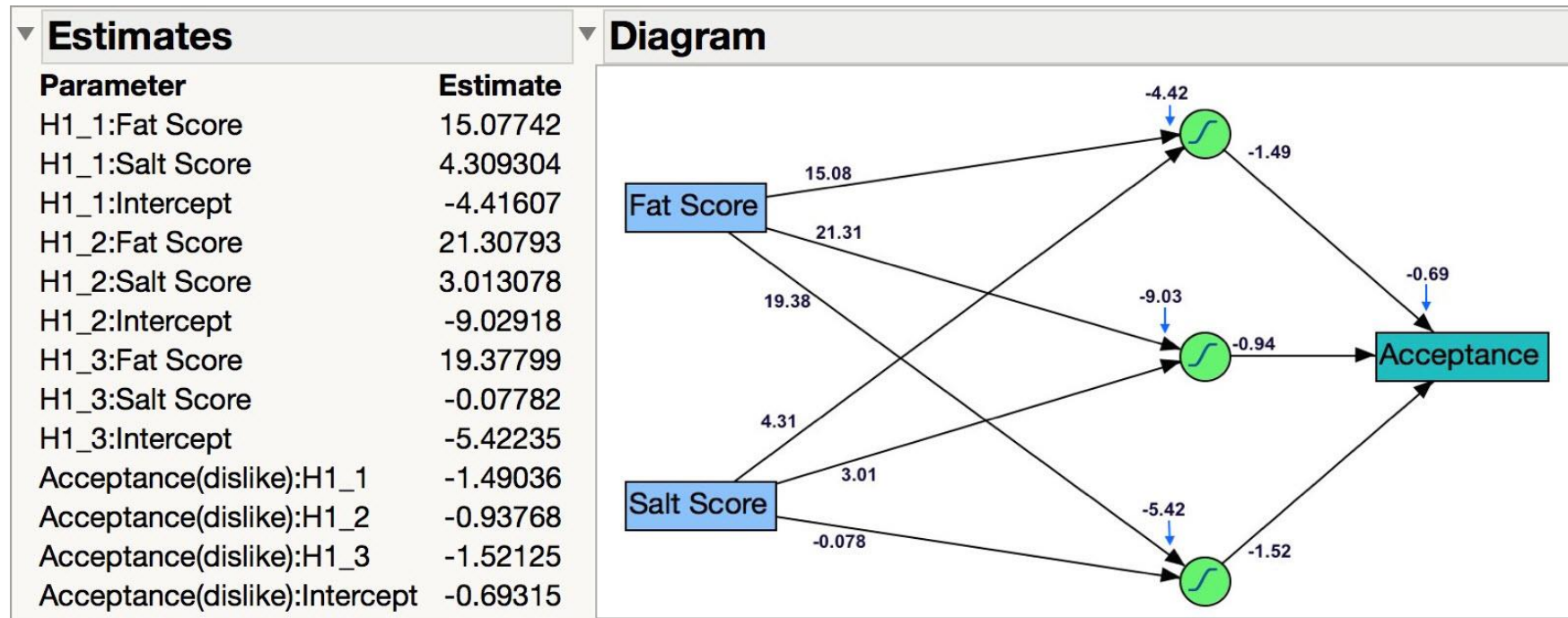
Neural models tend to overfit the data.

To avoid overfitting, JMP uses a **penalty parameter** and requires **crossvalidation**

The JMP Neural fitting process:

1. Set the penalty to 0
2. Use random normal weights for the starting values
3. Vary the penalty parameter
4. For each value of the penalty parameter, search for weights that minimize error
5. Select the model with the lowest crossvalidation error

# Tiny Example: Final Weights



# Tiny Example: Fit Statistics

▼ 

Neural

Validation Column: Validation

▶ 

Model Launch

▼ 

Model NTanH(3)

▼ 

Training

▼ 

Acceptance

Measures	Value
Generalized RSquare	0.3370917
Entropy RSquare	0.2296459
RMSE	0.3977493
Mean Abs Dev	0.2938792
Misclassification Rate	0.25
-LogLikelihood	1.7327887
Sum Freq	4

Confusion Matrix

Actual	Predicted	
Acceptance	dislike	like
dislike	1	0
like	1	2

Confusion Rates

Actual	Predicted Rate	
Acceptance	dislike	like
dislike	1.000	0.000
like	0.333	0.667

▼ 

Validation

▼ 

Acceptance

Measures	Value
Generalized RSquare	0.8226929
Entropy RSquare	0.6923289
RMSE	0.2079112
Mean Abs Dev	0.1869488
Misclassification Rate	0
-LogLikelihood	0.4265227
Sum Freq	2

Confusion Matrix

Actual	Predicted	
Acceptance	dislike	like
dislike	1	0
like	0	1

Confusion Rates

Actual	Predicted Rate	
Acceptance	dislike	like
dislike	1.000	0.000
like	0.000	1.000

# Tiny Example: Classifications

Estimated propensities and classifications

- One record in the training set was misclassified
- Both records in the validation set were correctly classified

	Obs	Fat Score	Salt Score	Acceptance	Validation	Probability (Acceptance =dislike)	Probability (Acceptance =like)	H1_1	H1_2	H1_3	Most Likely Acceptance
1	1	0.2	0.9	like	Training	0.447556	0.552444	0.84514	-0.77307	-0.66871	like
2	2	0.1	0.1	dislike	Training	0.949447	0.050553	-0.84508	-0.99728	-0.94094	dislike
3	3	0.2	0.4	dislike	Validation	0.722072	0.277928	0.160176	-0.94482	-0.65781	dislike
4	4	0.2	0.5	like	Training	0.655339	0.344661	0.360128	-0.92614	-0.66001	dislike
5	5	0.4	0.5	like	Training	0.022069	0.977931	0.954915	0.462325	0.816079	like
6	6	0.3	0.8	like	Validation	0.09597	0.90403	0.944404	-0.11269	0.162926	like

# Specify Network Architecture in JMP

## **Number of hidden layers**

- Most popular – one hidden layer

## **Number of nodes in hidden layer(s)**

- More nodes capture complexity, but increase chances of overfit

## **Hidden Layer Activation Functions**

- Combinations of three functions (TanH, Linear and Gaussian) can be applied in the hidden layers to add model complexity

User Inputs

# JMP Network Architecture, cont.

## **Number of tours**

- How many times JMP restarts the model-fitting algorithm

## **“Learning Rate”**

- Low values “downweight” the new information from errors at each iteration
- This slows learning, but reduces tendency to overfit to local structure

# Advantages

- Good predictive ability
- Can capture complex relationships
- No need to specify a model

# Disadvantages

- Considered a “black box” prediction machine, with no insight into relationships between predictors and outcome
- No variable-selection mechanism, so you have to exercise care in selecting variables
- Heavy computational requirements if there are many variables



# Addressing Disadvantages in JMP

- Considered a “black box” prediction machine, with no insight into relationships between predictors and outcome
  - **JMP Prediction Profiler allows you to explore the model**
- No variable-selection mechanism, so you have to exercise care in selecting variables
  - **JMP Variable Importance can help identify most important variables**
- Heavy computational requirements if there are many variables
  - **The JMP Neural algorithm is more efficient than back propagation**

# A journey in random forests and penalized regression in an industrial classification problem: How to use models to sharpen your questions

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## Abstract

The mathematician and population geneticist Sam Karlin considered that “The purpose of models is not to fit the data but to sharpen the question”. Motivated by this, we describe here a journey through questions, models and data analysis, to reach specific goals. Specifically, we consider random forests, ridge regression, lasso and elastic nets in a case study of 63 sensors collected in functional testing of an electronic system. The paper lists a sequence of questions and how they were tackled by statistical analysis, sometimes unsuccessfully. Eventually we were able to provide a robust, parsimonious and effective model for predicting the system condition, using a small subset of the 63 sensors. In handling this problem, we combine several innovative methods and insights that can prove useful also in other contexts. The underlying objective is enhancing the awareness to the journey representing the modeling process. We conclude with an assessment of the information quality provided by the analysis.