A Biomed Data Analyst Training Program

Time series

Professor Ron S. Kenett

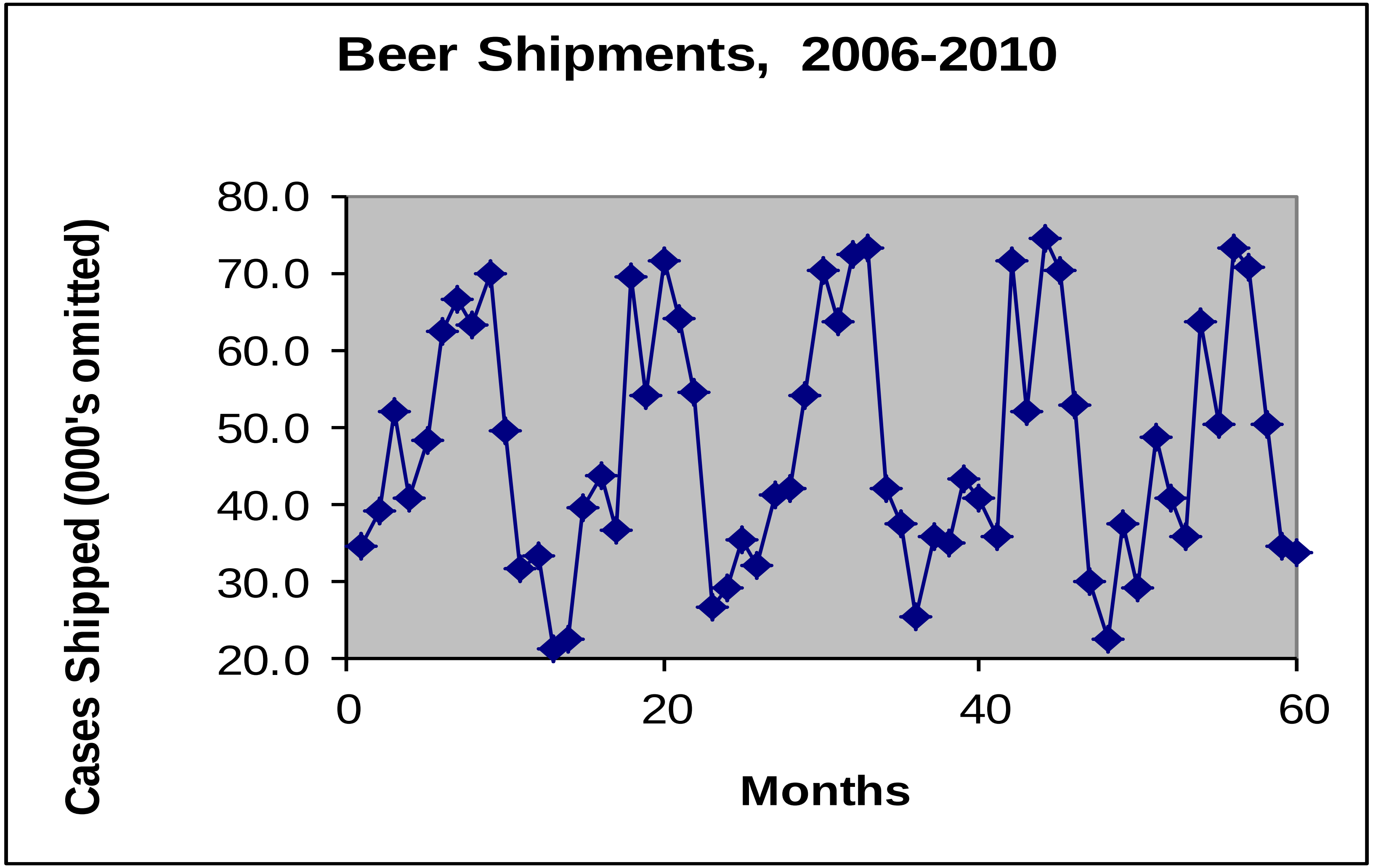
The Beer Deliveries Example

Deliveries of beer by a beer distributor over five years, the sixty months from January 2006 to December 2010

The data is measured as the number of cases distributed (000’s omitted)

	January	February	March	April	May	June	July	August	September	October	November	December
2006	34.6	39.3	52.4	40.7	48.5	62.7	66.8	63.5	70.3	49.9	31.8	33.3
2007	21.3	22.6	39.5	43.9	36.7	69.7	54.2	71.8	64.4	54.7	26.7	29.1
2008	35.6	32.2	41.4	42.2	54.3	70.5	63.9	72.6	73.6	42.3	37.5	25.3
2009	35.9	35.2	43.6	41.0	35.8	71.8	52.0	74.7	70.6	53.0	29.9	22.5
2010	37.4	29.1	48.9	40.9	36.1	64.0	50.4	73.4	71.1	50.4	34.8	33.8

Run chart of sales



Three months moving average

$$\bar{X}_t = \frac{X_{t-1} + X_{t-2} + X_{t-3}}{3}$$

$$= \frac{34.6 + 39.3 + 52.4}{3} = \frac{126.3}{3} = 42.1$$

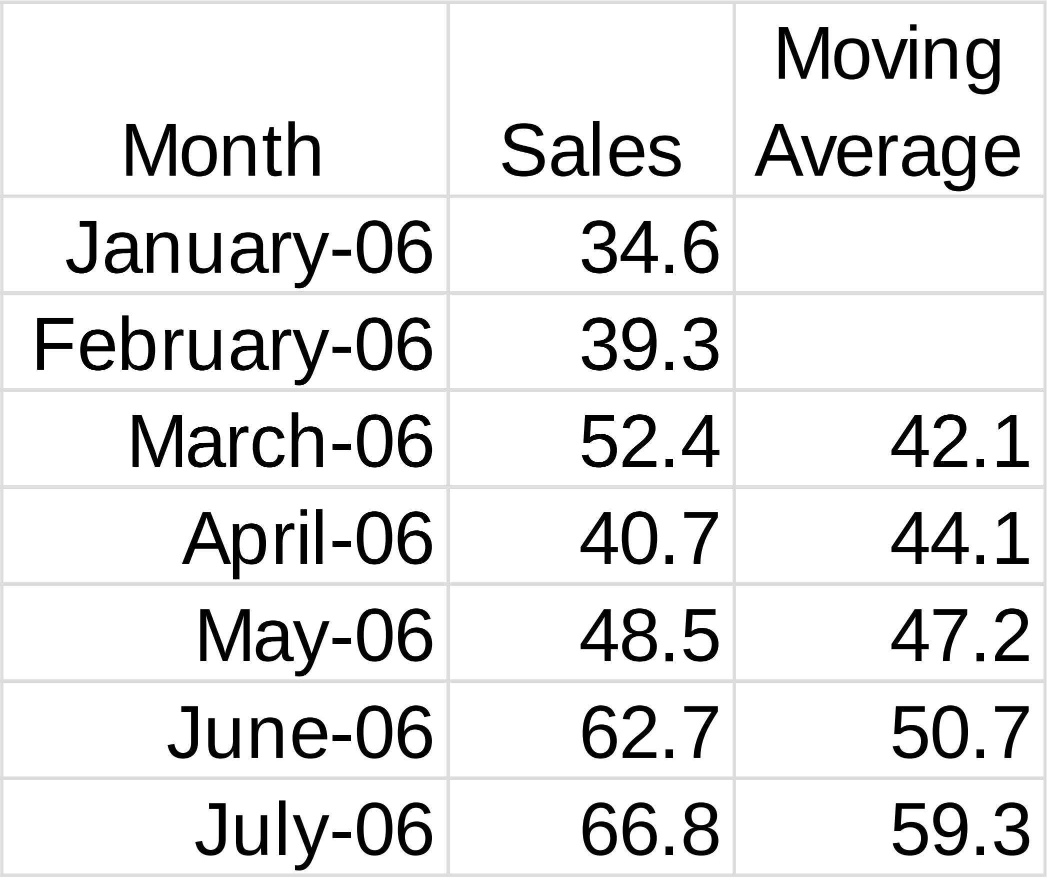
Three months average

Three months moving average

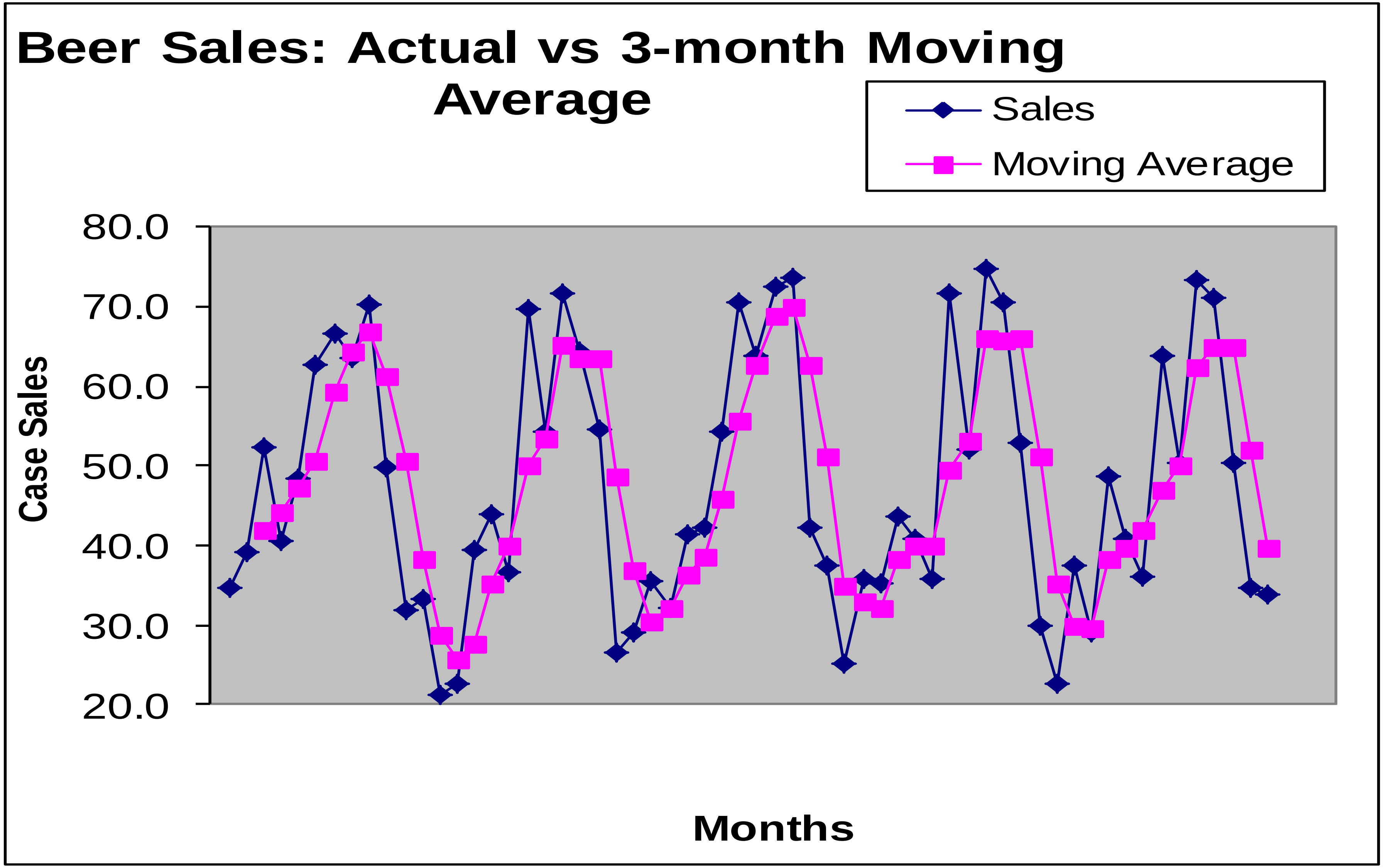
$$\bar{X}_t = \frac{39.3+52.4+40.7}{3} = \frac{132.4}{3} = 44.1$$

Average of next three months

Three months moving average

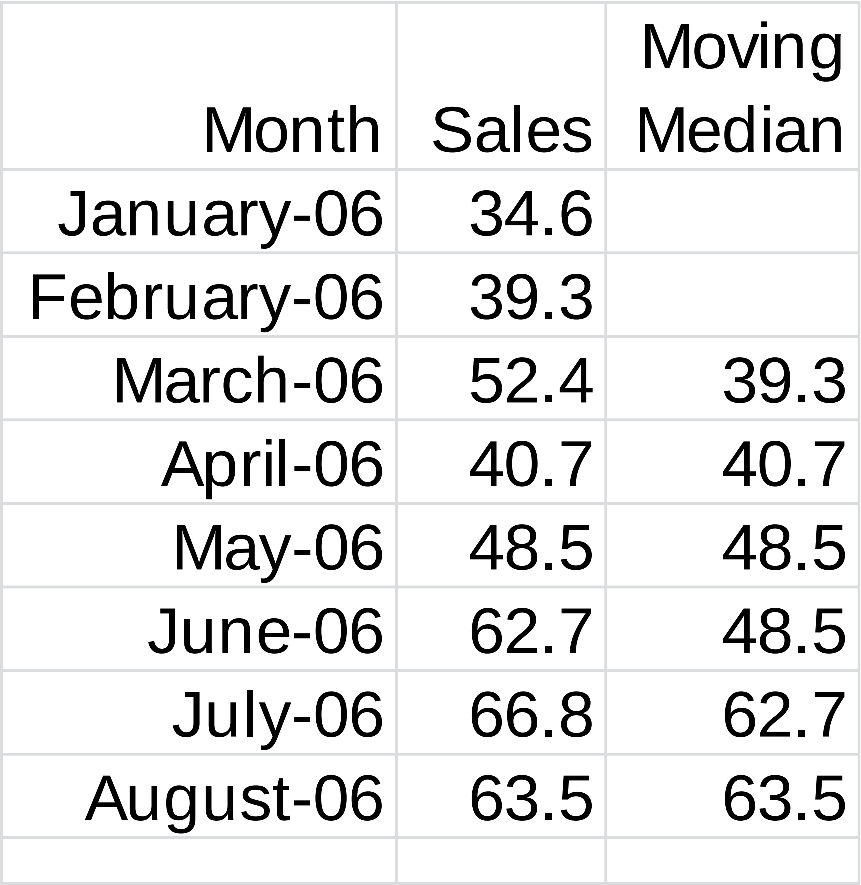


Month	Sales	Moving Average
January-06	34.6	
February-06	39.3	
March-06	52.4	42.1
April-06	40.7	44.1
May-06	48.5	47.2
June-06	62.7	50.7
July-06	66.8	59.3

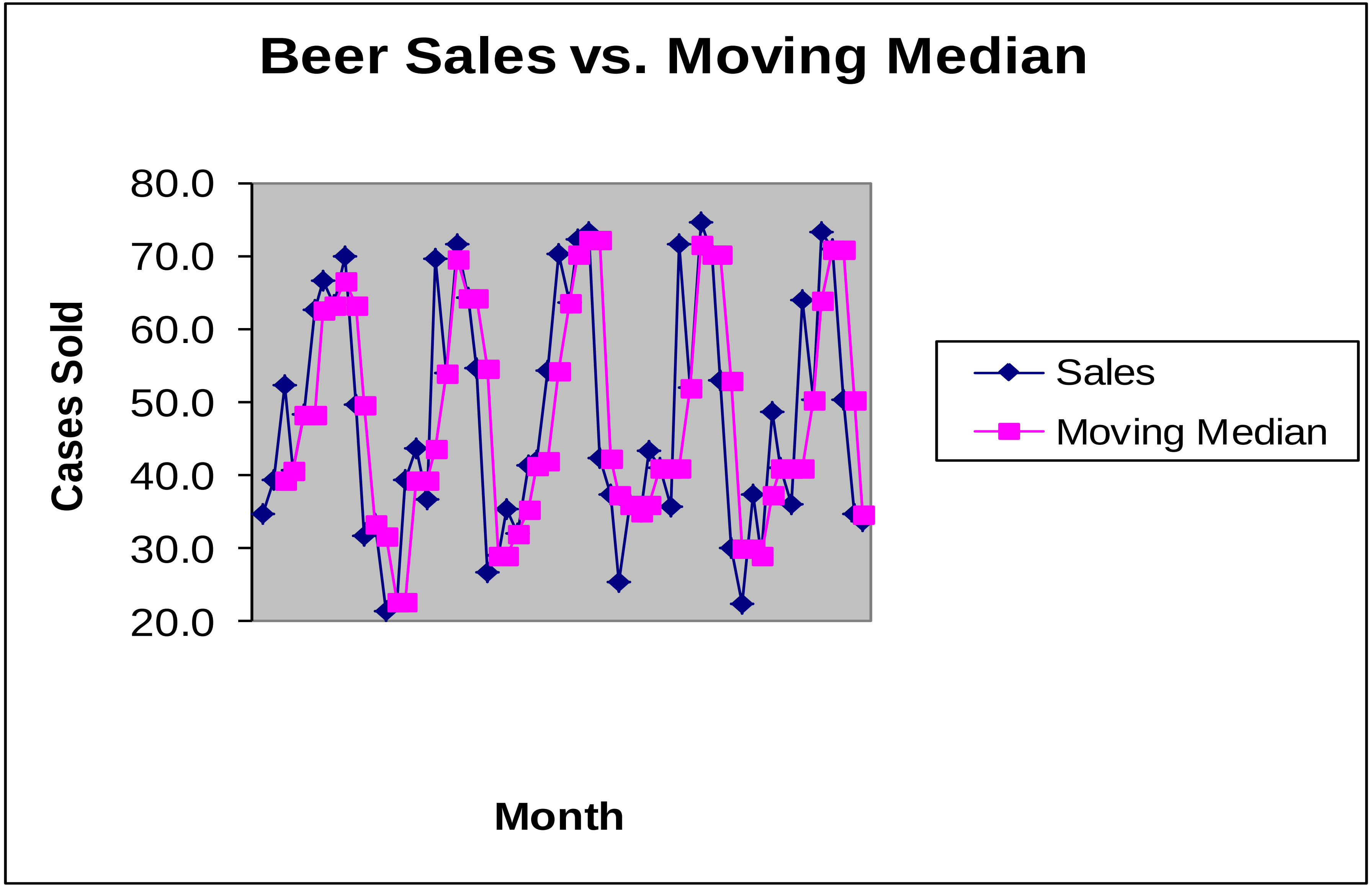


Beer Sales: Actual vs 3-month Moving Average

Three months moving median



Month	Sales	Moving Median
January-06	34.6	
February-06	39.3	
March-06	52.4	39.3
April-06	40.7	40.7
May-06	48.5	48.5
June-06	62.7	48.5
July-06	66.8	62.7
August-06	63.5	63.5


Beer Sales vs. Moving Median
80.0
70.0
60.0
50.0
40.0
30.0
20.0
Cases Sold
Month
Sales
Moving Median

Exponential smoothing

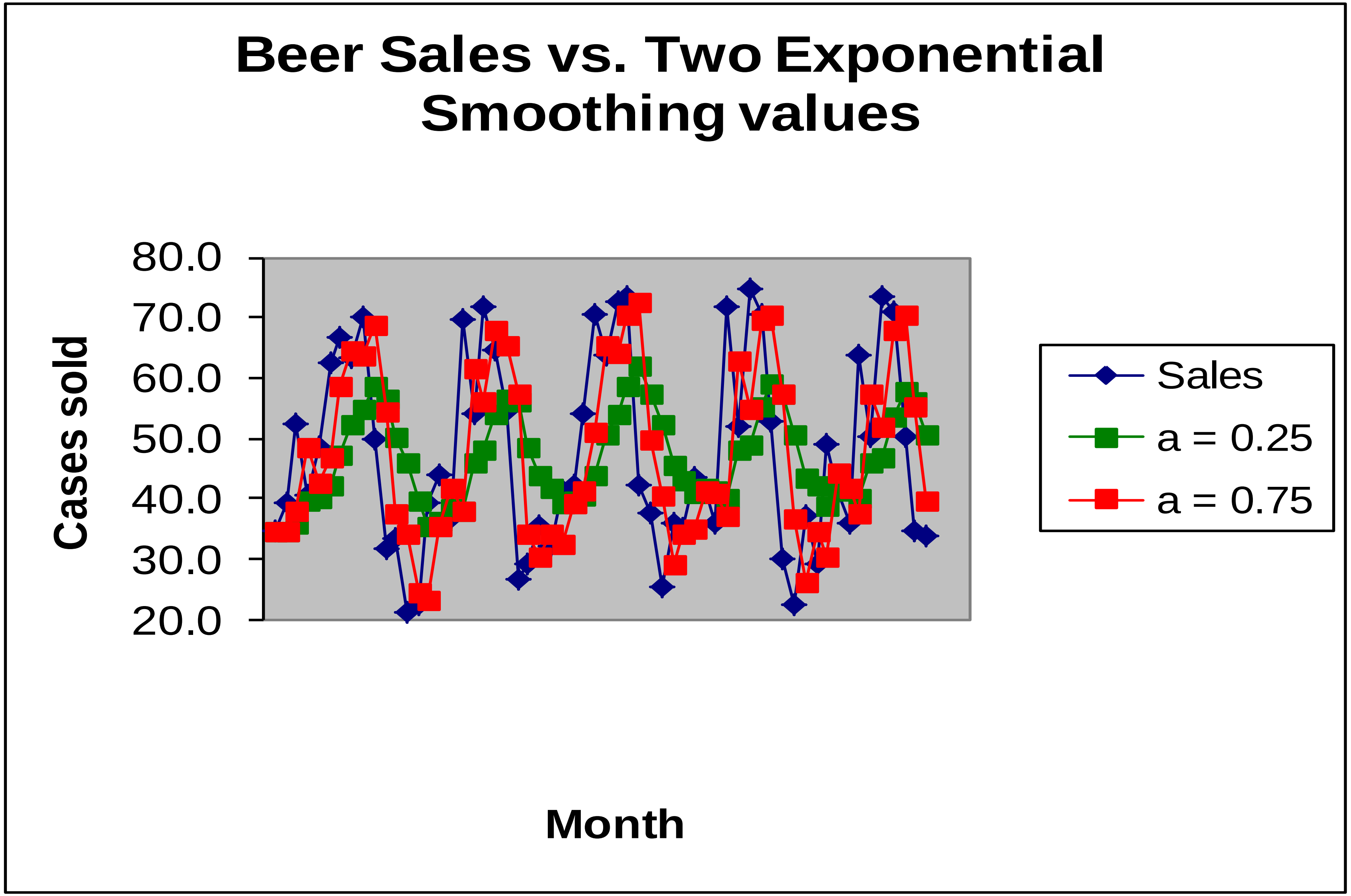
$$\hat{y}_{t+1} = ay_t + a(1-a)y_{t-1} + a(1-a)^2 y_{t-2} + \ldots$$

Generally, for data that is highly variable, a higher **a** is chosen

In practice, however, **a** seldom exceeds 0.5

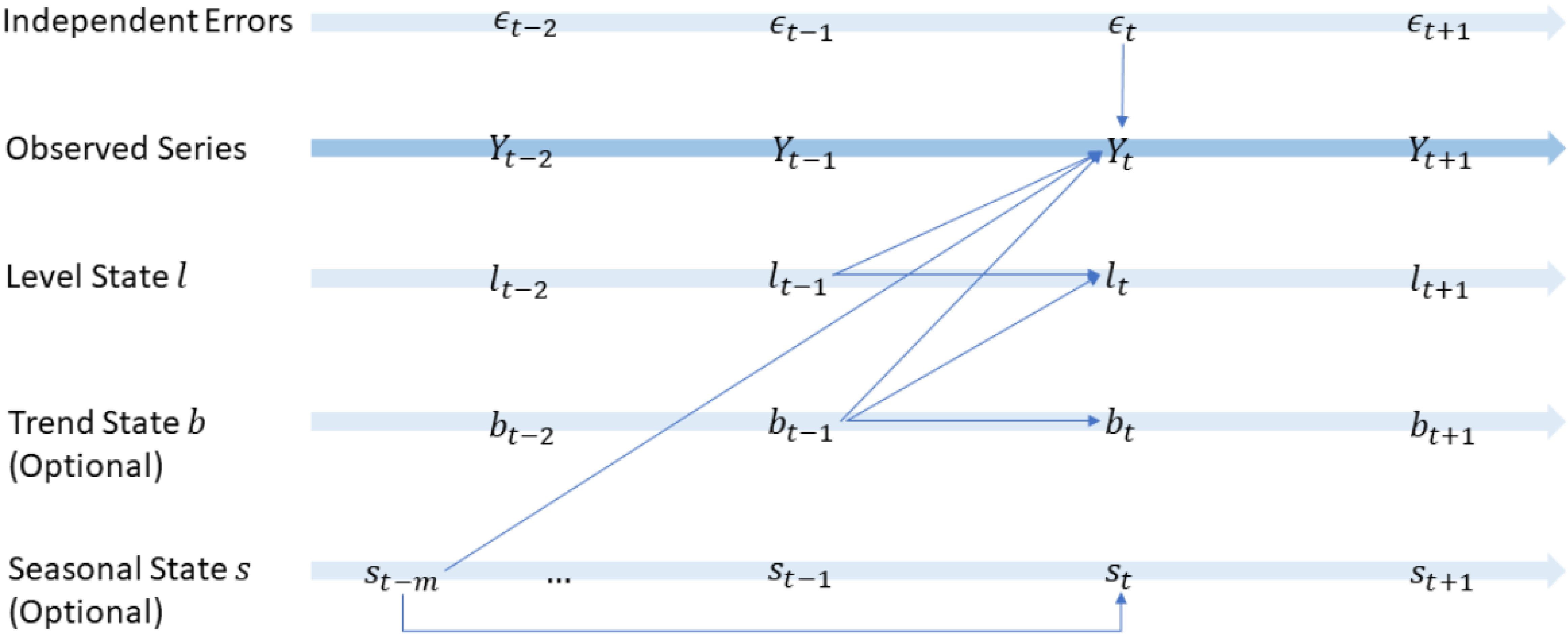
For data which has more stability, a lower value of **a** is chosen

Typical **a** values range for 0.2 to 0.3


Beer Sales vs. Two Exponential Smoothing values
Cases sold
80.0
70.0
60.0
50.0
40.0
30.0
20.0
Month
Sales
a = 0.25
a = 0.75

State space model

$$Y_t = \left(\left(l_{t-1} \begin{matrix}+\\ \times\end{matrix} b_{t-1} \right) \begin{matrix}+\\ \times\end{matrix} s_{t-m} \right) \begin{matrix}+\,\epsilon_t \\ \times\,(1+\epsilon_t)\end{matrix}$$



Independent Errors ϵ_{t-2} ϵ_{t-1} ϵ_t ϵ_{t+1}

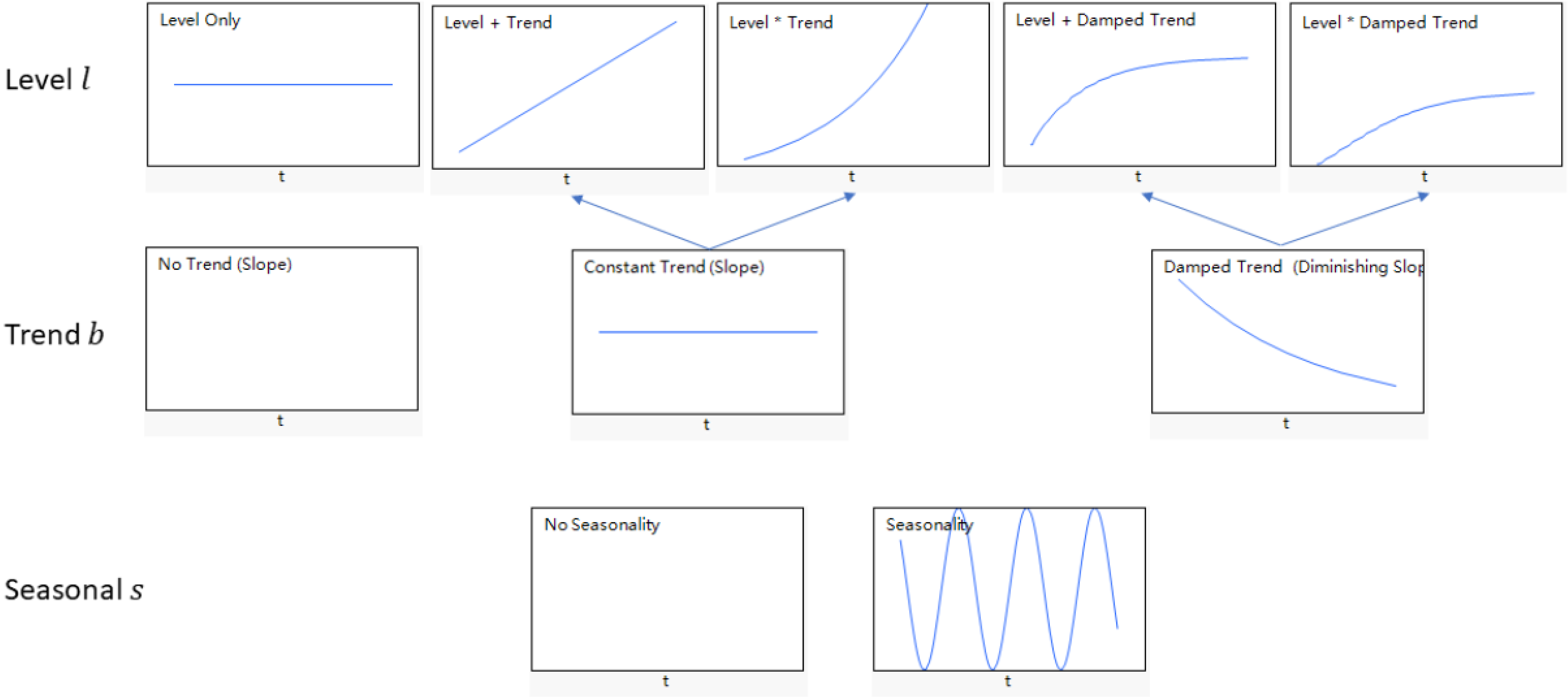
Observed Series Y_{t-2} Y_{t-1} Y_t Y_{t+1}

Level State l l_{t-2} l_{t-1} l_t l_{t+1}

Trend State b (Optional) b_{t-2} b_{t-1} b_t b_{t+1}

Seasonal State s (Optional) s_{t-m} … s_{t-1} s_t s_{t+1}

$$Y_t = \left(\left(l_{t-1} \begin{matrix} + \\ \times \end{matrix} b_{t-1} \right) \begin{matrix} + \\ \times \end{matrix} s_{t-m} \right) \begin{matrix} + \epsilon_t \\ \times (1 + \epsilon_t) \end{matrix}$$



Level l

Level Only

Level + Trend

Level * Trend

Level + Damped Trend

Level * Damped Trend

Trend b

No Trend (Slope)

Constant Trend (Slope)

Damped Trend (Diminishing Slop

Seasonal s

No Seasonality

Seasonality

The Multiplicative Model

$$y_t = T_t C_t S_t I_t$$

T_t = trend factor

C_t = cyclic factor

S_t = seasonal factor

I_t = random factor

The Multiplicative Model

Example:

1. The long-term trend for a specific product is +1.02
2. The cyclic factor at time *t* is 1.03
3. The seasonal factor at time *t* is 0.96
4. We cannot state the random factor

The Multiplicative Model

$$\hat{y}_t = T_t C_t S_t = 1.02 \cdot 1.03 \cdot 0.96$$

$$= 1.0086$$

- This predicts a slight increase (~ 1%) for period *t*
- If the actual increase was 1.0125, then I_t = 1.0125 / 1.0086 = 1.0039

The Multiplicative Model

We can also take away the effect of the long-term data

This is termed a *detrended series*

Recall that y_t = 1.0125 and that the trend factor was T_t = 1.02

$$\frac{y_t}{T_t} = \frac{T_t C_t S_t I_t}{T_t} = \frac{1.0125}{1.02} = 0.9927$$

- Thus, without the long-term trend, we would have shipped less units this period

The Additive Model

$$y_t = T_t + C_t + S_t + I_t$$

T_t = trend factor

C_t = cyclic factor

S_t = seasonal factor

I_t = random factor

The Additive Model

Say, we know that overtime, we ship 50 additional units per period, and in this business cycle we are shipping an additional 200 units

However, during this period, we typically ship 175 fewer units:

$$\hat{y}_t = T_t + C_t + S_t$$

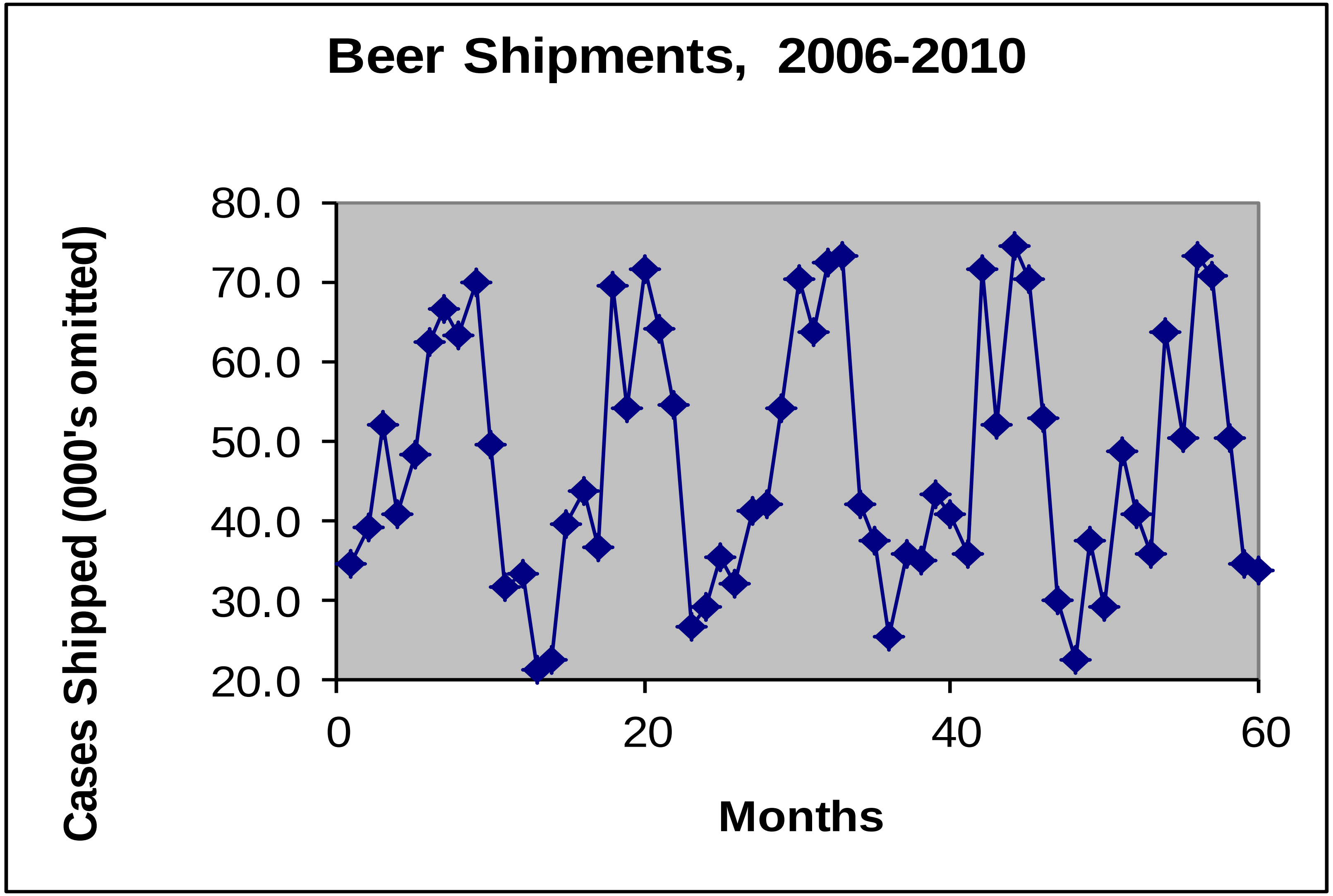
$$= 50 + 200 - 175 = 75$$

The Additive Model

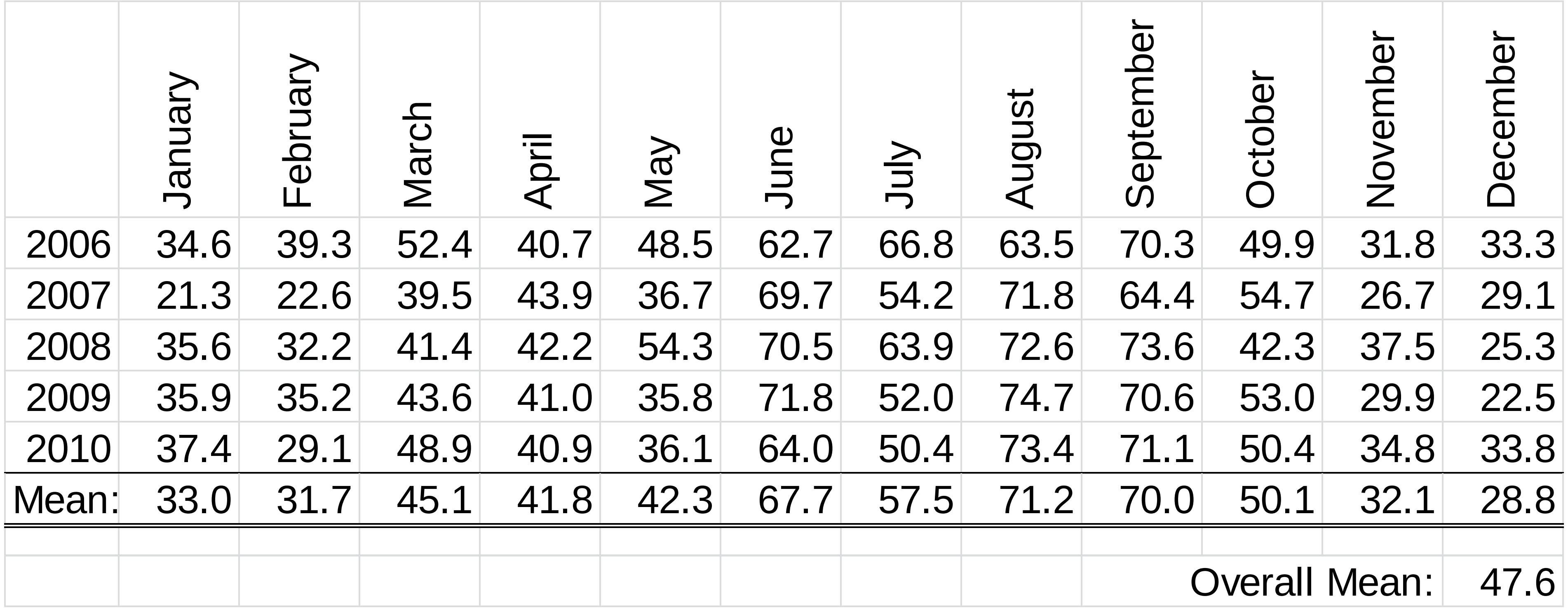
Thus, we could forecast shipping an additional 75 units this period

If we actually shipped 91 additional units

Then, I_t = (91 - 75) = 16

Beer Shipments, 2006-2010
Cases Shipped (000's omitted)
80.0
70.0
60.0
50.0
40.0
30.0
20.0
0
20
40
60
Months

Analysis of seasonal effects



	January	February	March	April	May	June	July	August	September	October	November	December
2006	34.6	39.3	52.4	40.7	48.5	62.7	66.8	63.5	70.3	49.9	31.8	33.3
2007	21.3	22.6	39.5	43.9	36.7	69.7	54.2	71.8	64.4	54.7	26.7	29.1
2008	35.6	32.2	41.4	42.2	54.3	70.5	63.9	72.6	73.6	42.3	37.5	25.3
2009	35.9	35.2	43.6	41.0	35.8	71.8	52.0	74.7	70.6	53.0	29.9	22.5
2010	37.4	29.1	48.9	40.9	36.1	64.0	50.4	73.4	71.1	50.4	34.8	33.8
Mean:	33.0	31.7	45.1	41.8	42.3	67.7	57.5	71.2	70.0	50.1	32.1	28.8
											Overall Mean:	47.6

Analysis of seasonal effects

For January, the average is 33.0 versus an overall mean of 47.6. The seasonal adjustment is thus:

$$S_{Jan} = \frac{\overline{X}}{\overline{X}_{Jan}} = \frac{47.6}{33.0} = 1.442$$

- As a result, we will multiple actual January sales by 1.442 to adjust for the fact that January is a low sales month

Analysis of seasonal effects

July, however, is a usually high sales month

As a result, its seasonal adjustment will bring its value down:

$$S_{Jul} = \frac{\overline{X}}{\overline{X}_{Jul}} = \frac{47.6}{57.5} = 0.828$$

- As a result, to "seasonally adjust", we multiple each July value by 0.828

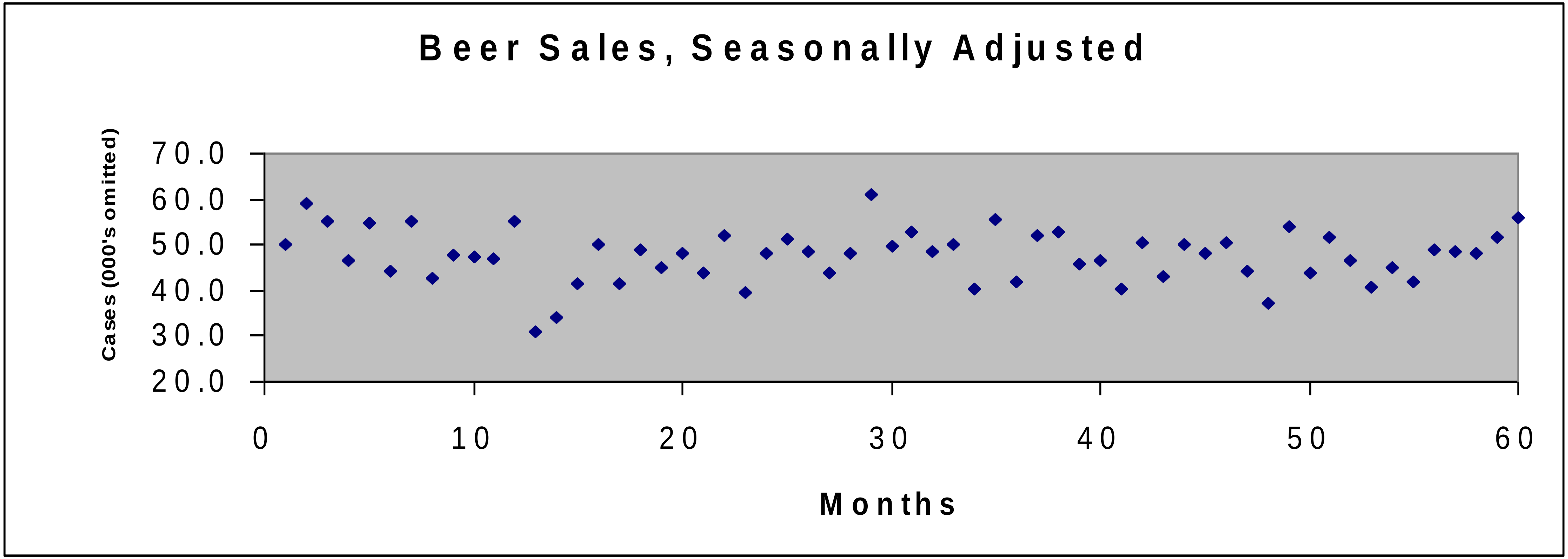
Analysis of seasonal effects

In the table below, the seasonal adjustment factors are applied to all the original values

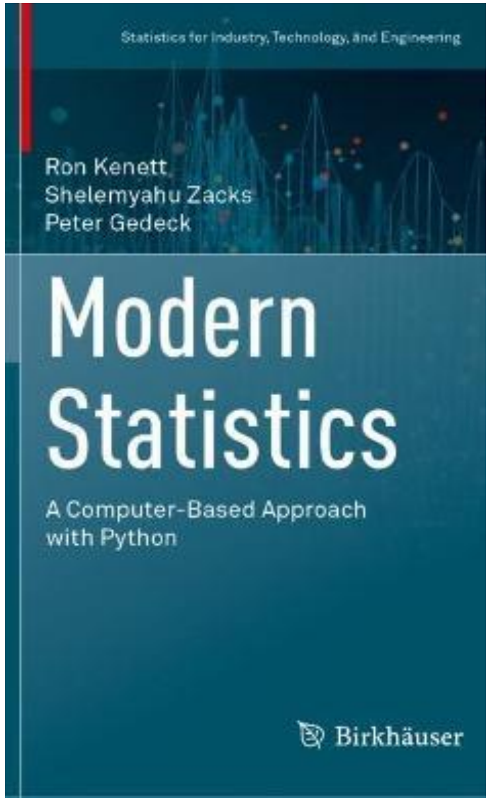
They are “deseasonalized”:

	January	February	March	April	May	June	July	August	September	October	November	December
2006	50.0	59.0	55.2	46.4	54.6	44.1	55.3	42.5	47.8	47.4	47.1	55.1
2007	30.8	34.0	41.6	50.0	41.4	49.0	44.9	48.0	43.8	52.0	39.5	48.1
2008	51.3	48.4	43.7	48.1	61.1	49.5	52.9	48.5	50.0	40.2	55.5	41.8
2009	51.9	52.9	45.9	46.7	40.3	50.4	43.1	49.9	48.0	50.4	44.3	37.2
2010	54.0	43.7	51.5	46.7	40.6	45.0	41.8	49.1	48.4	47.9	51.5	55.8

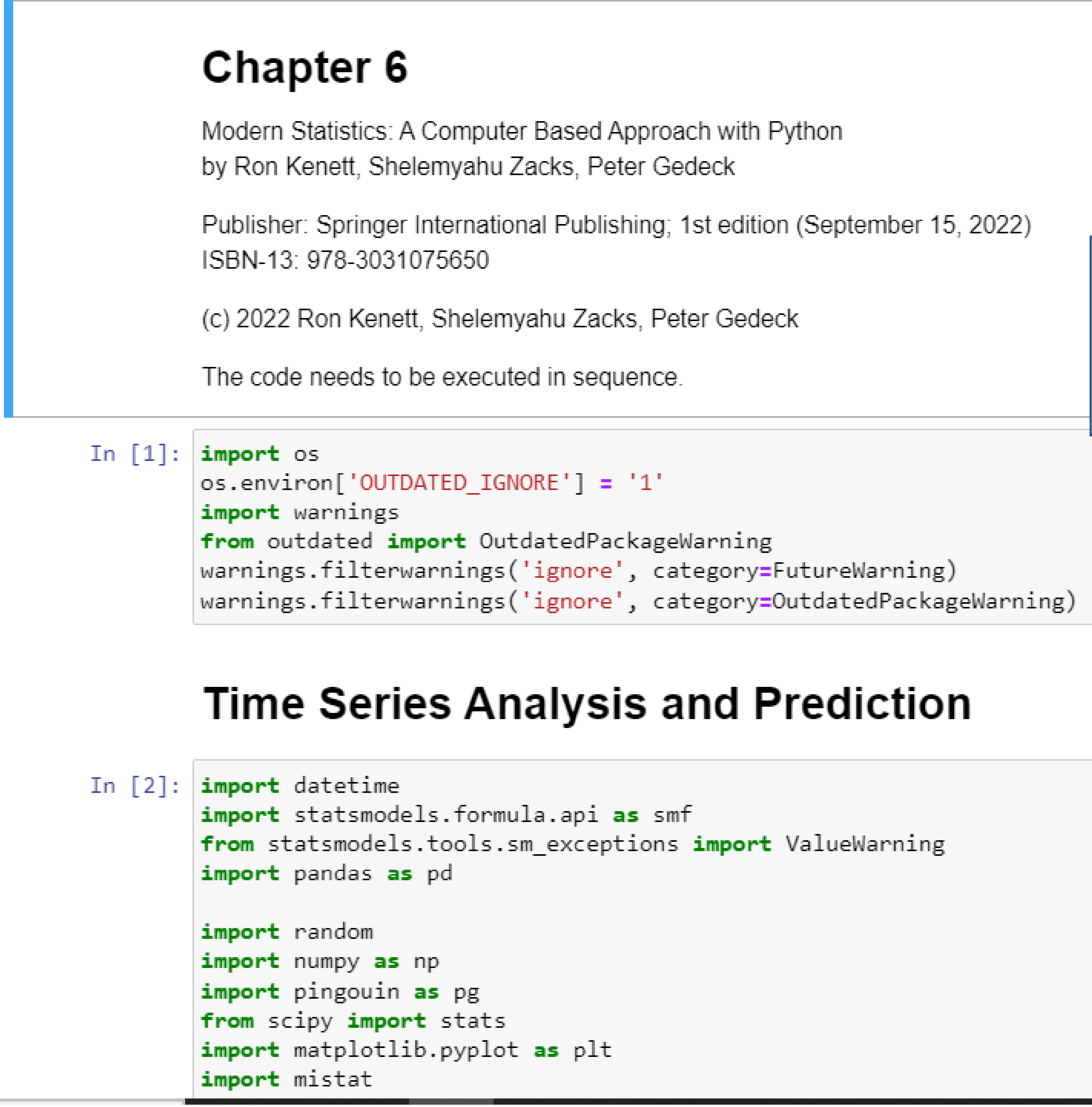
Seasonality Adjusted Series



Chapter 6
Time Series Analysis and Prediction


Statistics for Industry, Technology, and Engineering
Ron Kenett
Shelemyahu Zacks
Peter Gedeck
Modern Statistics
A Computer-Based Approach with Python
Birkhäuser

Preview In this chapter, we present essential parts of time series analysis, with the objective of predicting or forecasting its future development. Predicting future behavior is generally more successful for stationary series, which do not change their stochastic characteristics as time proceeds. We develop and illustrate time series which are of both types, namely, covariance stationary and non-stationary.



Chapter 6

Modern Statistics: A Computer Based Approach with Python
by Ron Kenett, Shelemyahu Zacks, Peter Gedeck

Publisher: Springer International Publishing; 1st edition (September 15, 2022)
ISBN-13: 978-3031075650

The code needs to be executed in sequence.

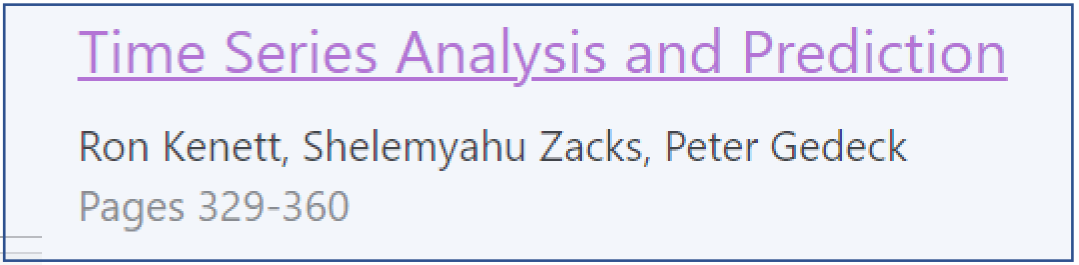
In [1]:

```
import os
os.environ['OUTDATED_IGNORE'] = '1'
import warnings
from outdated import OutdatedPackageWarning
warnings.filterwarnings('ignore', category=FutureWarning)
warnings.filterwarnings('ignore', category=OutdatedPackageWarning)
```

Time Series Analysis and Prediction

In [2]:

```
import datetime
import statsmodels.formula.api as smf
from statsmodels.tools.sm_exceptions import ValueWarning
import pandas as pd

import random
import numpy as np
import pingouin as pg
from scipy import stats
import matplotlib.pyplot as plt
import mistat
```

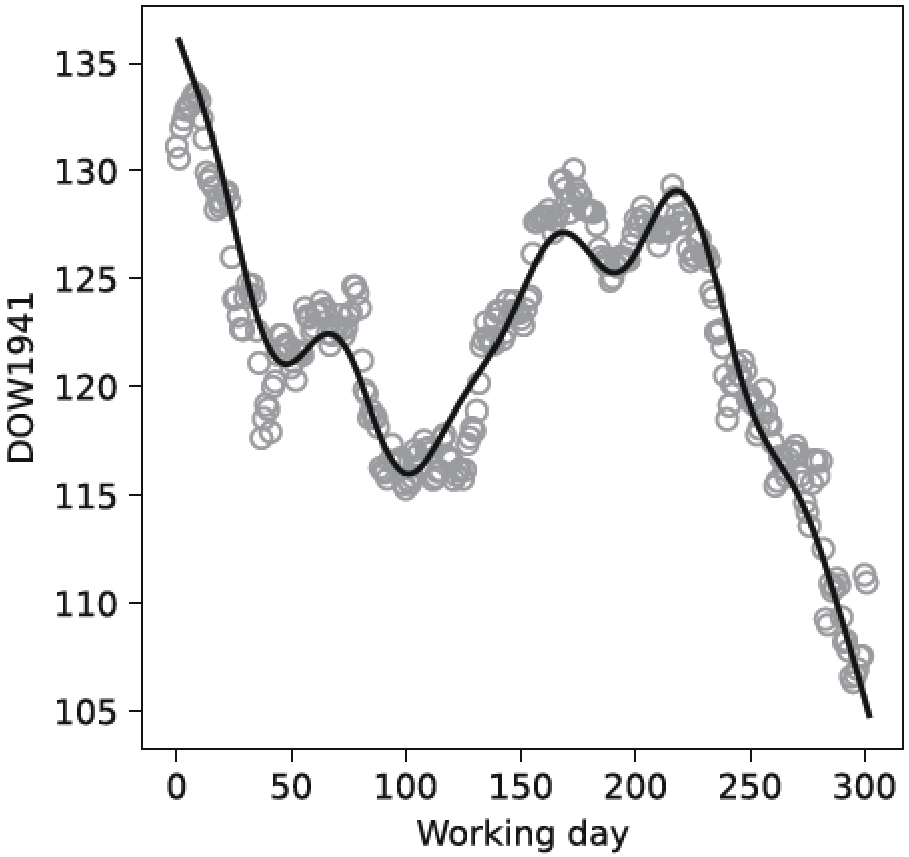


Time Series Analysis and Prediction

Ron Kenett, Shelemyahu Zacks, Peter Gedeck

Pages 329-360

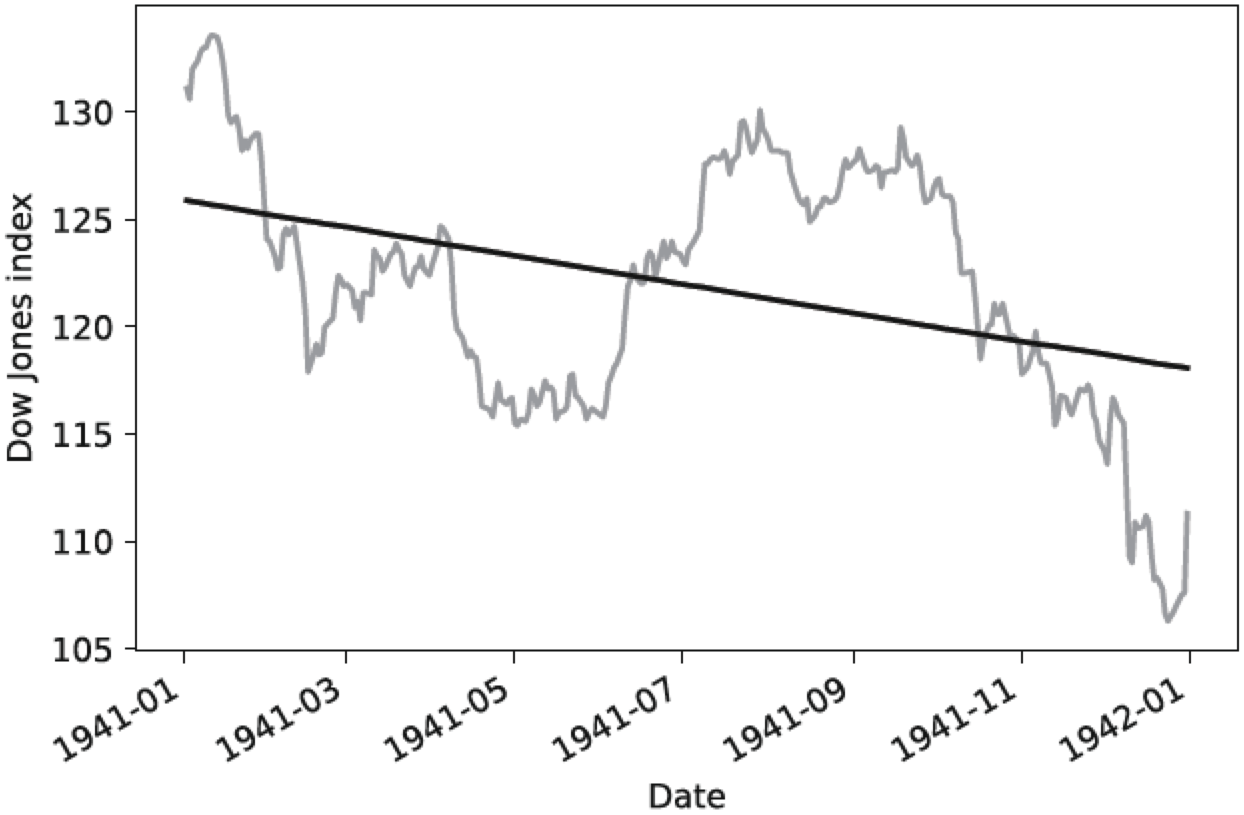
The Components of a Time Series

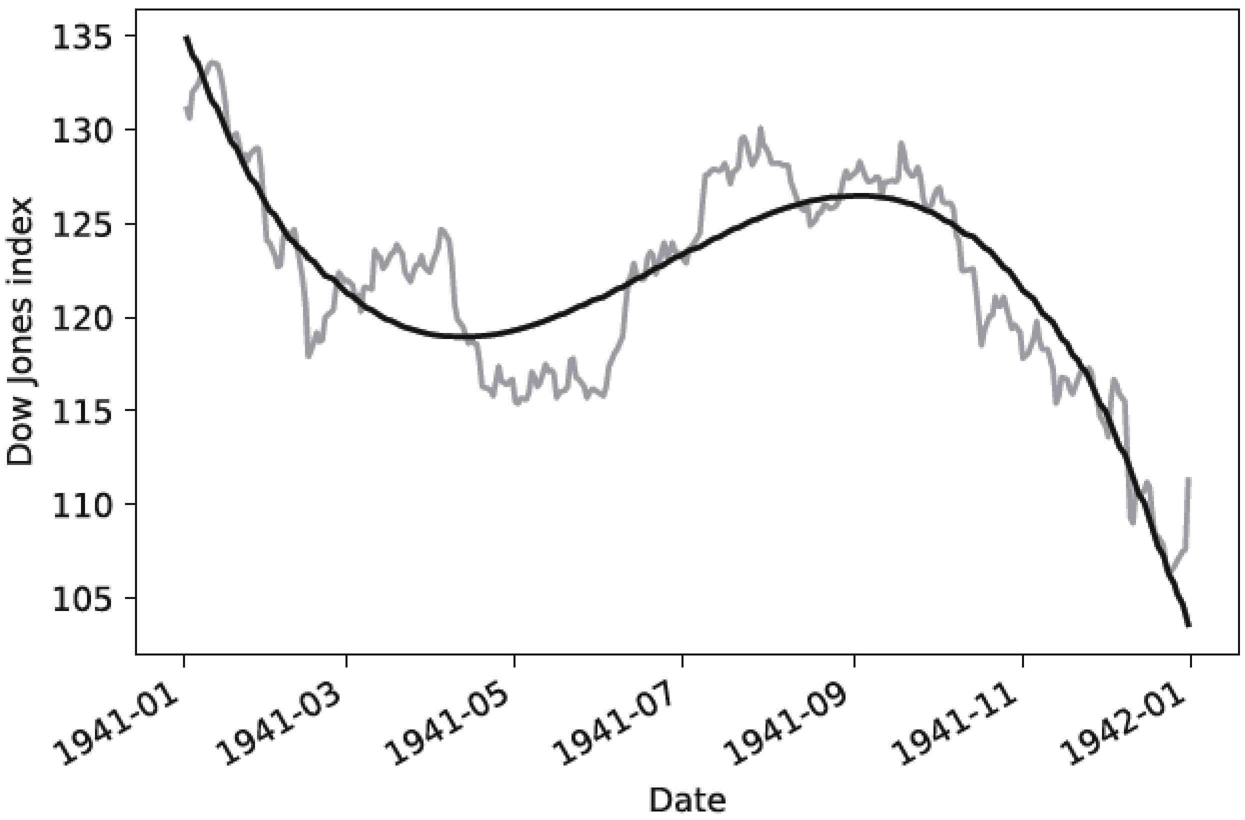


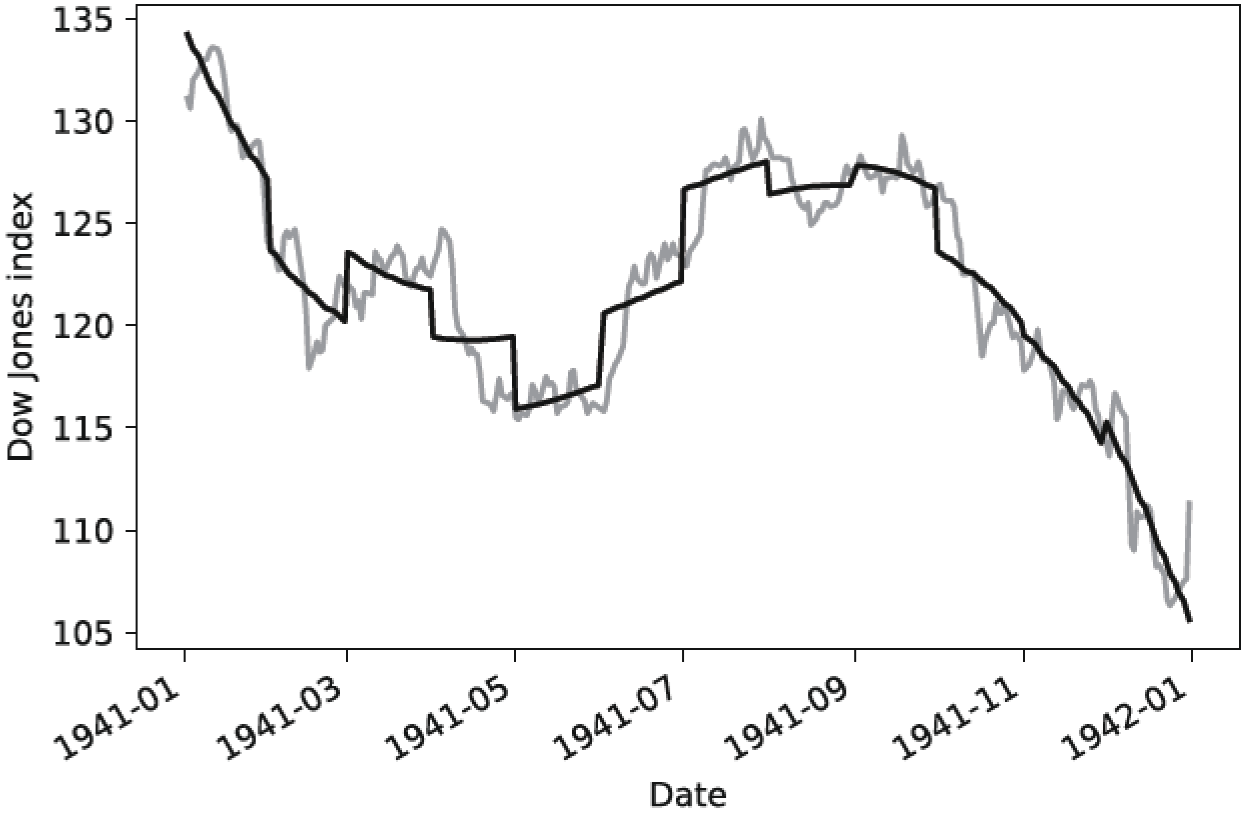
$$
\begin{aligned}
(t) =& 123.34 + 27.73\frac{t-151}{302} - 15.83\left(\frac{t-151}{302}\right)^2 - 237.00\left(\frac{t-151}{302}\right)^3 \\
&+ 0.1512\cos\frac{4\pi t}{302} + 1.738\sin\frac{4\pi t}{302} + 1.770\cos\frac{8\pi t}{302} - 0.208\sin\frac{8\pi t}{302} \\
&- 0.729\cos\frac{12\pi t}{302} + 0.748\sin\frac{12\pi t}{302}.
\end{aligned}
$$

```
dow1941 = mistat.load_data('DOW1941')
t = np.arange(1, len(dow1941) + 1)
x = (t - 151) / 302
omega = 4 * np.pi * t / 302
ft = (123.34 + 27.73 * x - 15.83* x ** 2 - 237.00 * x**3
     + 0.1512 * np.cos(omega) + 1.738 * np.sin(omega)
     + 1.770 * np.cos(2 * omega) - 0.208 * np.sin(2 * omega)
     - 0.729 * np.cos(3 * omega) + 0.748 * np.sin(3 * omega))

fig, ax = plt.subplots(figsize=[4, 4])
ax.scatter(dow1941.index, dow1941, facecolors='none', edgecolors='grey')
ax.plot(t, ft, color='black')
ax.set_xlabel('Working day')
ax.set_ylabel('DOW1941')
plt.show()
```

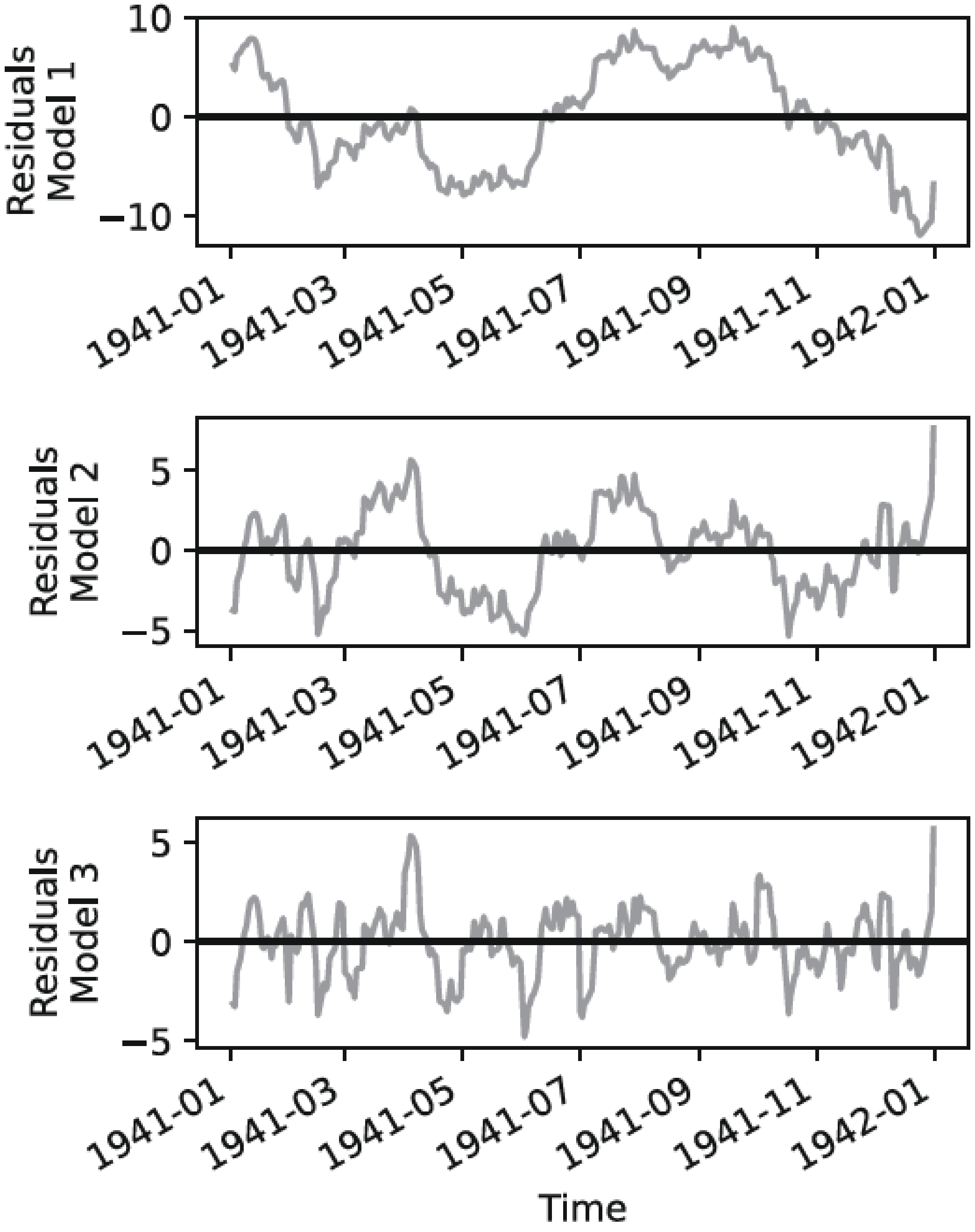
$$
\begin{aligned}
f(t) =& 123.34 + 27.73 \frac{t-151}{302} - 15.83 \left(\frac{t-151}{302}\right)^2 - 237.00 \left(\frac{t-151}{302}\right)^3 \\
& + 0.1512 \cos \frac{4\pi t}{302} + 1.738 \sin \frac{4\pi t}{302} + 1.770 \cos \frac{8\pi t}{302} - 0.208 \sin \frac{8\pi t}{302} \\
& - 0.729 \cos \frac{12\pi t}{302} + 0.748 \sin \frac{12\pi t}{302}.
\end{aligned}
$$


130
125
120
115
110
105
Dow Jones index
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Date

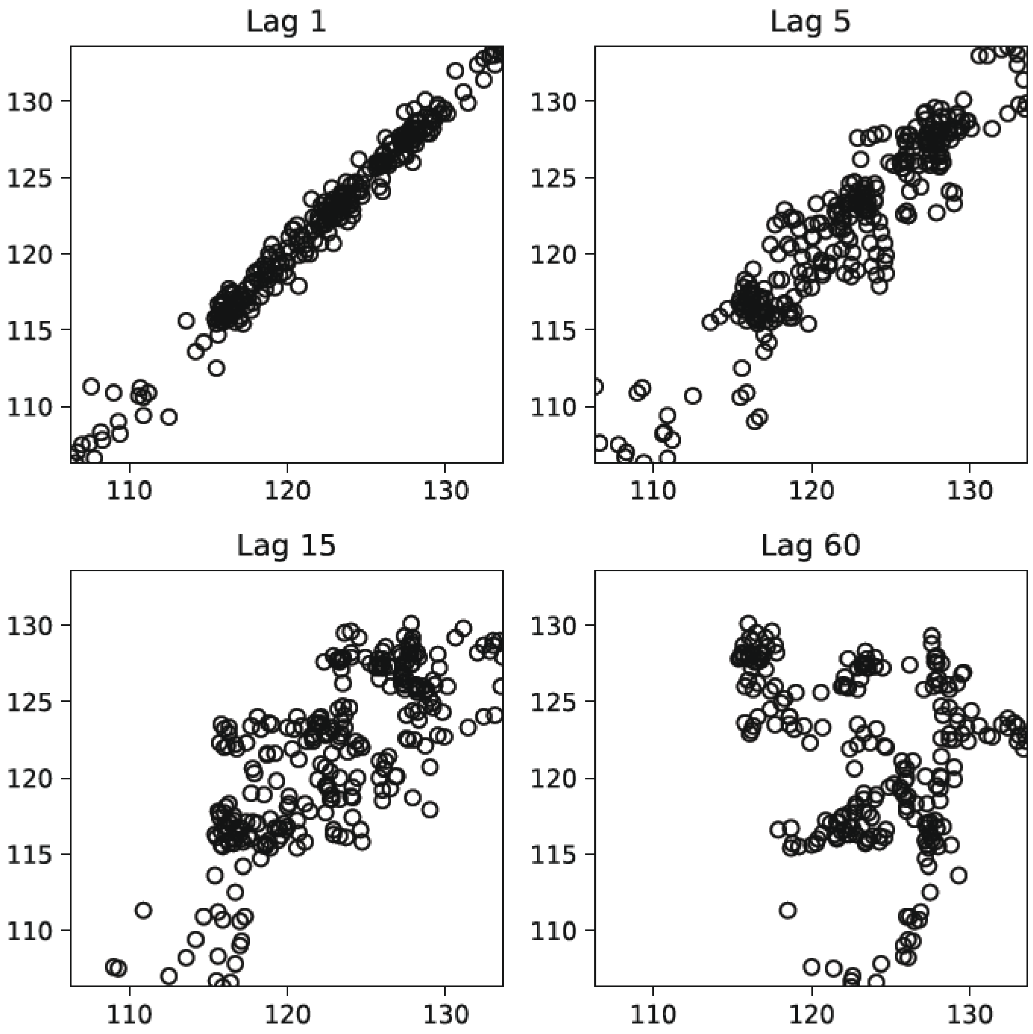

135
130
125
120
115
110
105
Dow Jones index
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Date

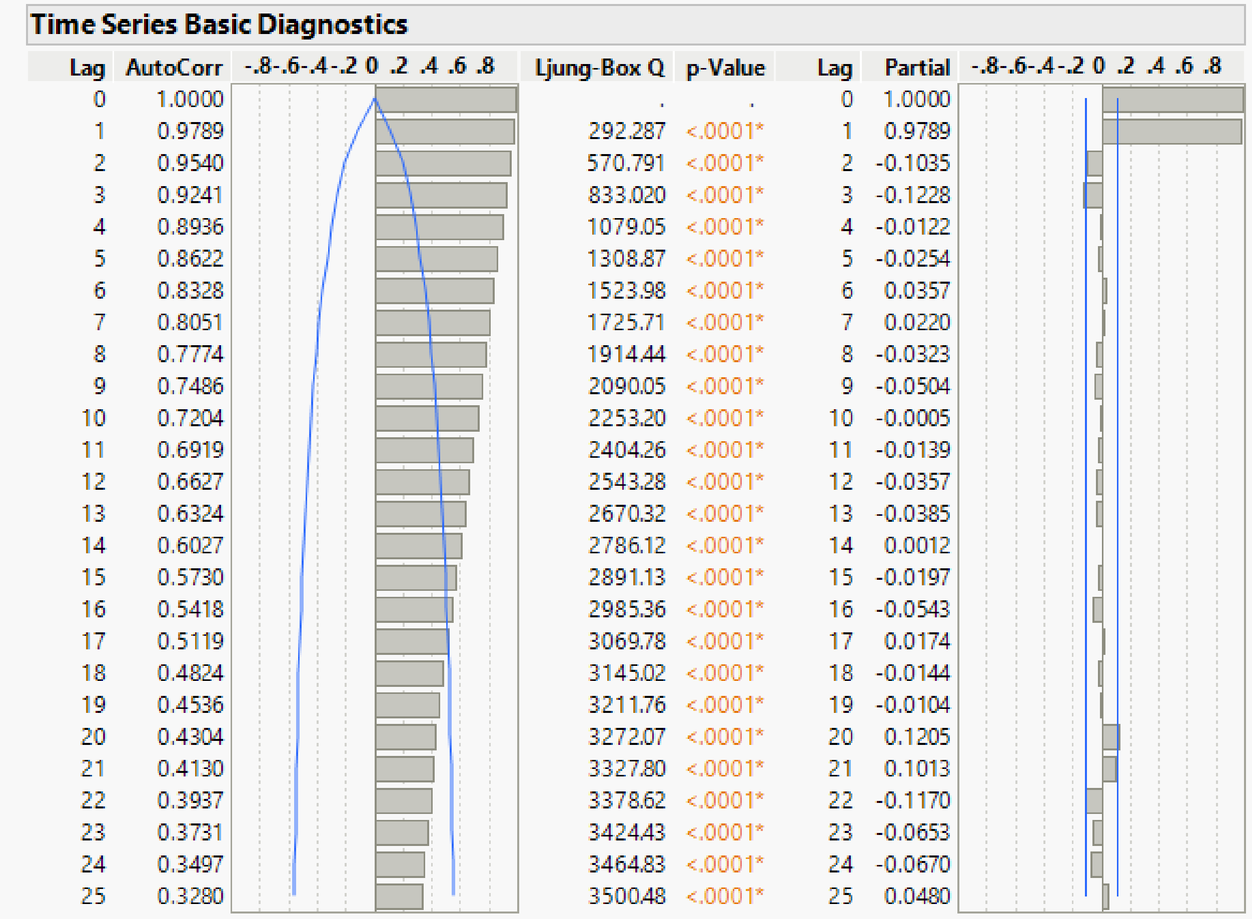


Dow Jones index
135
130
125
120
115
110
105
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Date


Residuals
Model 1
10
0
−10
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Residuals
Model 2
5
0
−5
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Residuals
Model 3
5
0
−5
1941-01
1941-03
1941-05
1941-07
1941-09
1941-11
1942-01
Time

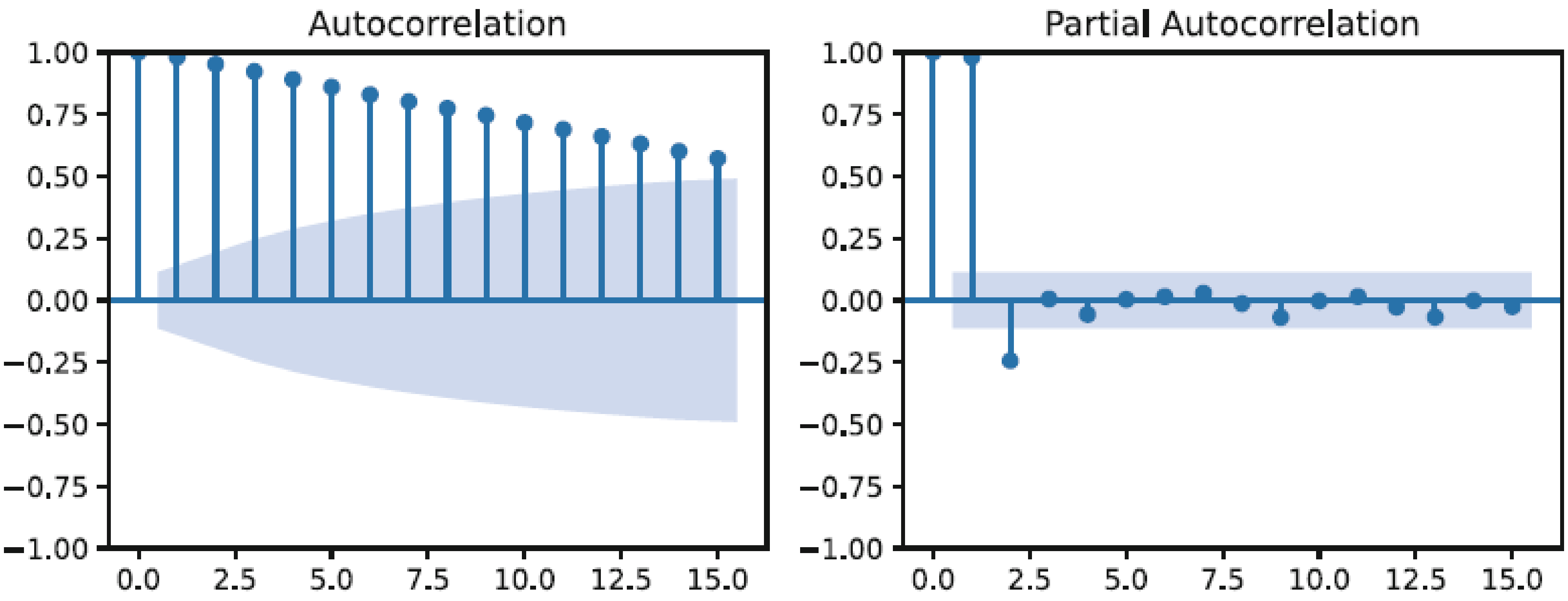
Autocorrelations



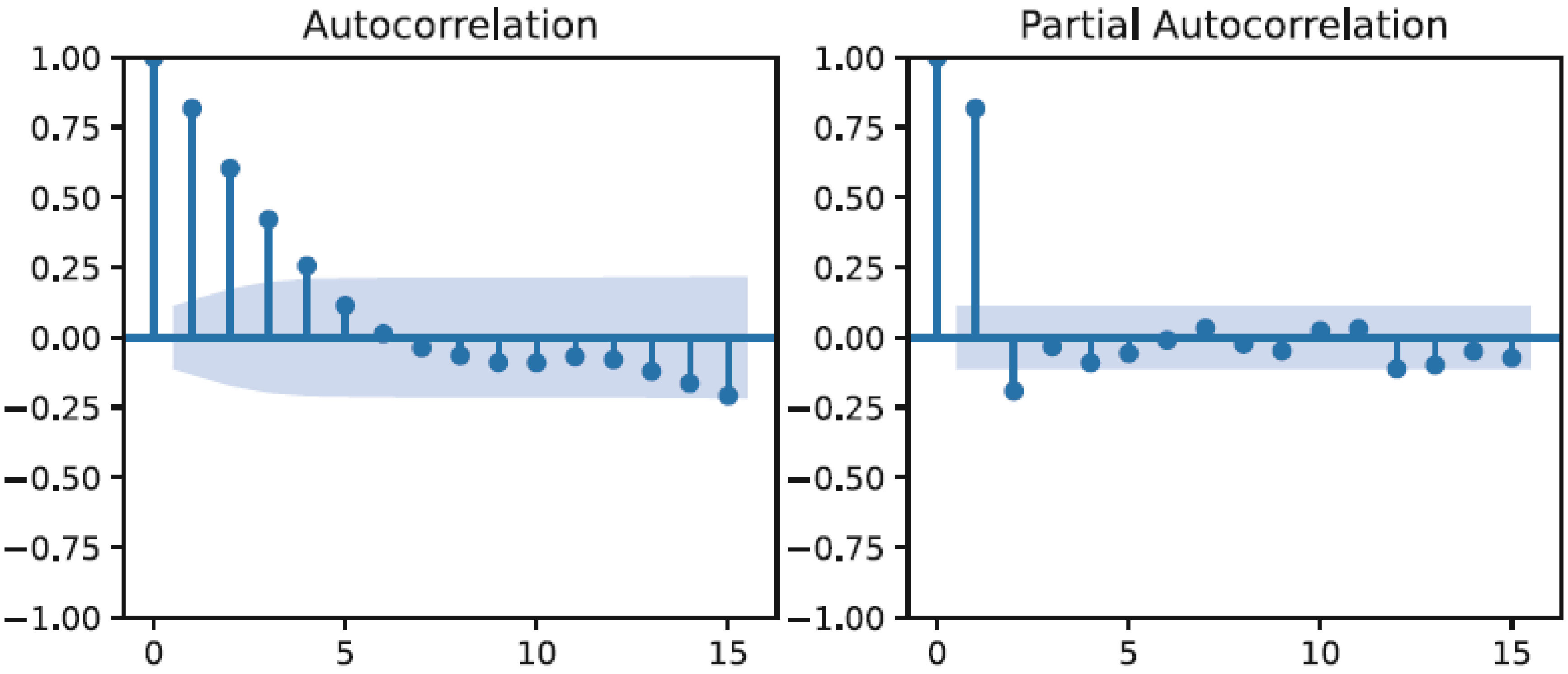

Time Series Basic Diagnostics

Lag	AutoCorr	-.8-.6-.4-.2 0 .2 .4 .6 .8	Ljung-Box Q	p-Value	Lag	Partial	-.8-.6-.4-.2 0 .2 .4 .6 .8
0	1.0000		.	.	0	1.0000	
1	0.9789		292.287	<.0001*	1	0.9789	
2	0.9540		570.791	<.0001*	2	-0.1035	
3	0.9241		833.020	<.0001*	3	-0.1228	
4	0.8936		1079.05	<.0001*	4	-0.0122	
5	0.8622		1308.87	<.0001*	5	-0.0254	
6	0.8328		1523.98	<.0001*	6	0.0357	
7	0.8051		1725.71	<.0001*	7	0.0220	
8	0.7774		1914.44	<.0001*	8	-0.0323	
9	0.7486		2090.05	<.0001*	9	-0.0504	
10	0.7204		2253.20	<.0001*	10	-0.0005	
11	0.6919		2404.26	<.0001*	11	-0.0139	
12	0.6627		2543.28	<.0001*	12	-0.0357	
13	0.6324		2670.32	<.0001*	13	-0.0385	
14	0.6027		2786.12	<.0001*	14	0.0012	
15	0.5730		2891.13	<.0001*	15	-0.0197	
16	0.5418		2985.36	<.0001*	16	-0.0543	
17	0.5119		3069.78	<.0001*	17	0.0174	
18	0.4824		3145.02	<.0001*	18	-0.0144	
19	0.4536		3211.76	<.0001*	19	-0.0104	
20	0.4304		3272.07	<.0001*	20	0.1205	
21	0.4130		3327.80	<.0001*	21	0.1013	
22	0.3937		3378.62	<.0001*	22	-0.1170	
23	0.3731		3424.43	<.0001*	23	-0.0653	
24	0.3497		3464.83	<.0001*	24	-0.0670	
25	0.3280		3500.48	<.0001*	25	0.0480	

Autocorrelations and Partial Autocorrelations



Autocorrelations and Partial Autocorrelations

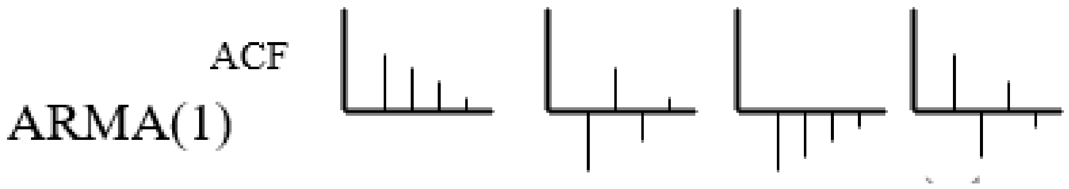


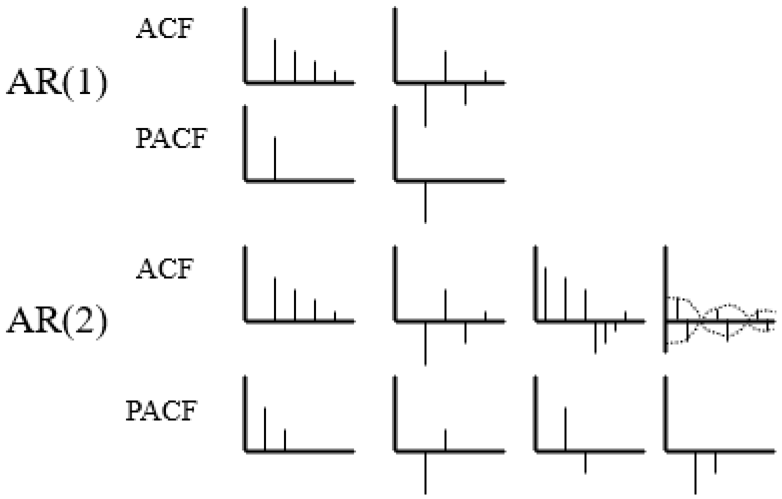
ARIMA Models

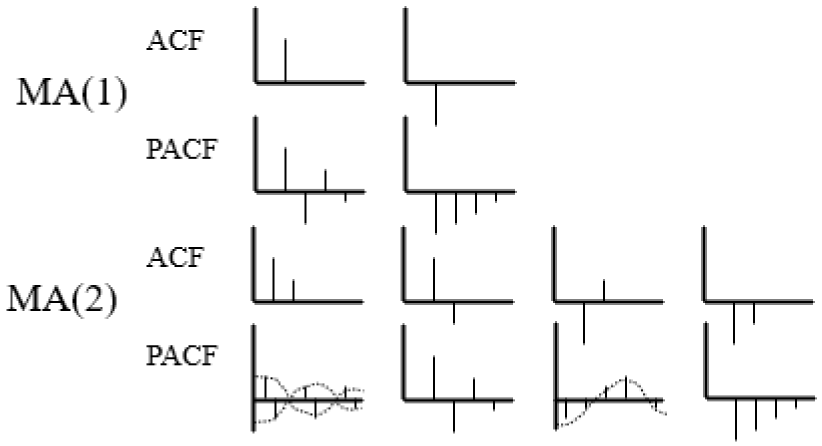
1. Auto Regressive (AR)

2. Moving Average (MA)

3. Auto Regressive Integrated Moving Average (ARIMA)







First-Order Autoregressive Processes, AR(1):

$$y_t = \delta + \theta_1 y_{t-1} + e_t, \quad t = 1, 2, \ldots, T.$$

δ is the intercept.

θ_1 is parameter generally between -1 and +1.

e_t is an uncorrelated random error with mean zero and variance σ_e^2.

Autoregressive Process of order p, AR(p) :

$$y_t = \delta + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_p y_{t-p} + e_t$$

δ is the intercept.

θ_i's are parameters generally between -1 and +1.

e_t is an uncorrelated random error with mean zero and variance σ_e^2 .

AR(2) model of U.S. unemployment rates

$$y_t = \underset{(0.1267)}{0.5051} \underset{\underset{\text{positive}}{\uparrow}}{+} \underset{(0.0707)}{1.5537}\, y_{t-1} \underset{\underset{\text{negative}}{\uparrow}}{-} \underset{(0.0708)}{0.6515}\, y_{t-2}$$

Using AR Model for Forecasting:

unemployment rate: $y_{T-1} = 6.63$ and $y_T = 6.20$

$$y_{T+1} = \delta + \theta_1 y_T + \theta_2 y_{T-1} = 0.5051 + (1.5537)(6.2) - (0.6515)(6.63)$$

$$= 5.8186$$

$$y_{T+2} = \delta + \theta_1 y_{T+1} + \theta_2 y_T = 0.5051 + (1.5537)(5.8186) - (0.6515)(6.2)$$

$$= 5.5062$$

$$y_{T+1} = \delta + \theta_1 y_T + \theta_2 y_{T-1} = 0.5051 + (1.5537)(5.5062) - (0.6515)(5.8186)$$

$$= 5.2693$$

Choosing the lag length, p, for AR(p):

The Partial Autocorrelation Function (PAF)

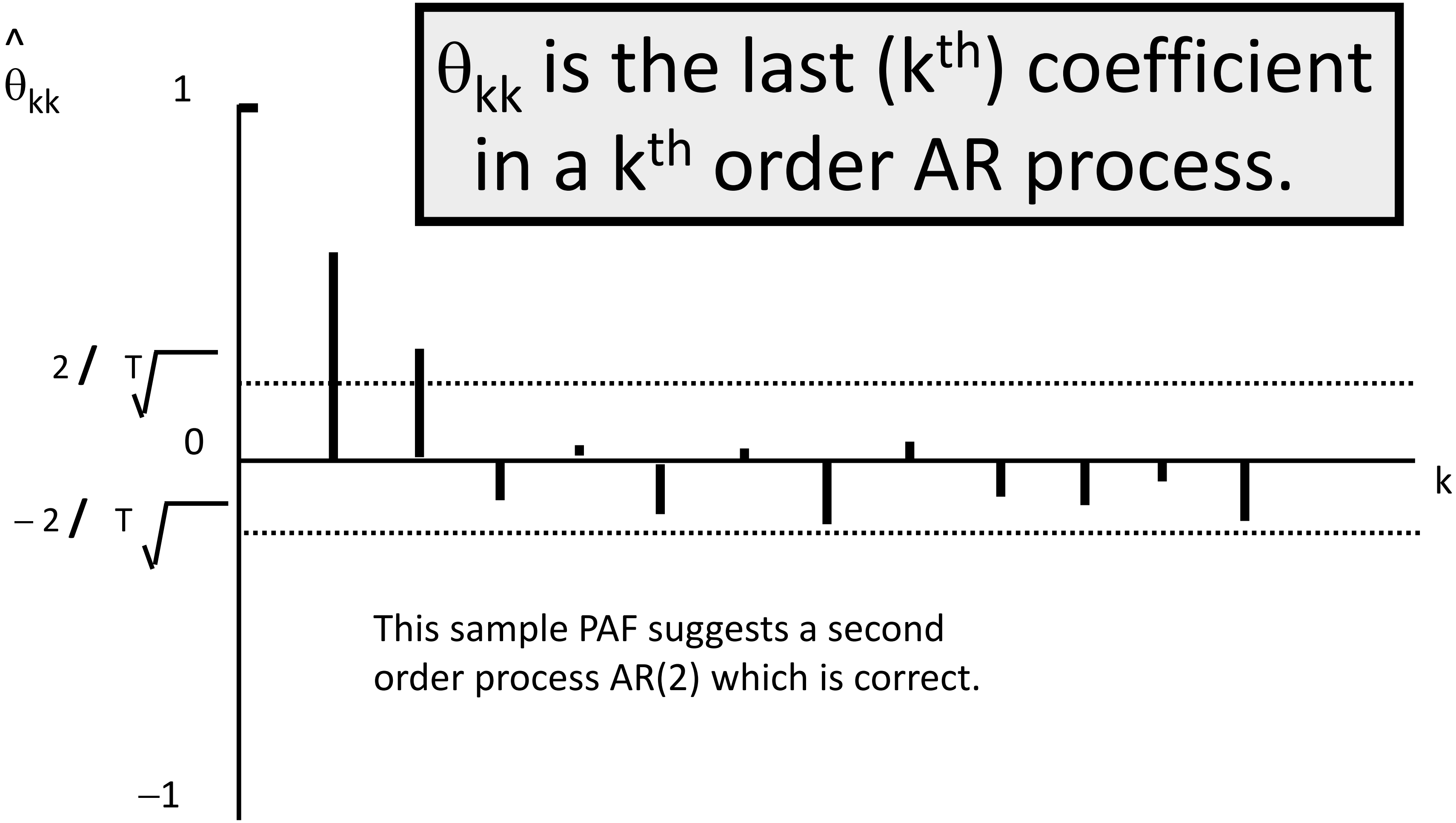
The PAF is the sequence of correlations between (y_t and y_{t-1}), (y_t and y_{t-2}), (y_t and y_{t-3}), and so on, given that the effects of earlier lags on y_t are held constant.

Partial Autocorrelation Function

Data simulated from this model:

$$y_t = 0.5\, y_{t-1} + 0.3\, y_{t-2} + e_t$$

θ_{kk} is the last (k^{th}) coefficient in a k^{th} order AR process.



This sample PAF suggests a second order process AR(2) which is correct.

Moving Average Process of order q, MA(q):

$$y_t = \mu + e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2} + \ldots + \alpha_q e_{t-q}$$

μ is the intercept.

α_i's are unknown parameters.

e_t is an uncorrelated random error with mean zero and variance $\sigma_e{}^2$.

An MA(1) process:

$$y_t = \mu + e_t + \alpha_1 e_{t-1}$$

Minimize sum of least squares deviations:

$$S(\mu,\alpha_1) = \sum_{t=1}^{T} e_t^2 = \sum_{t=1}^{T} (y_t - \mu - \alpha_1 e_{t-1})^2$$

Choosing the lag length, q, for MA(q):

The Autocorrelation Function (AF)

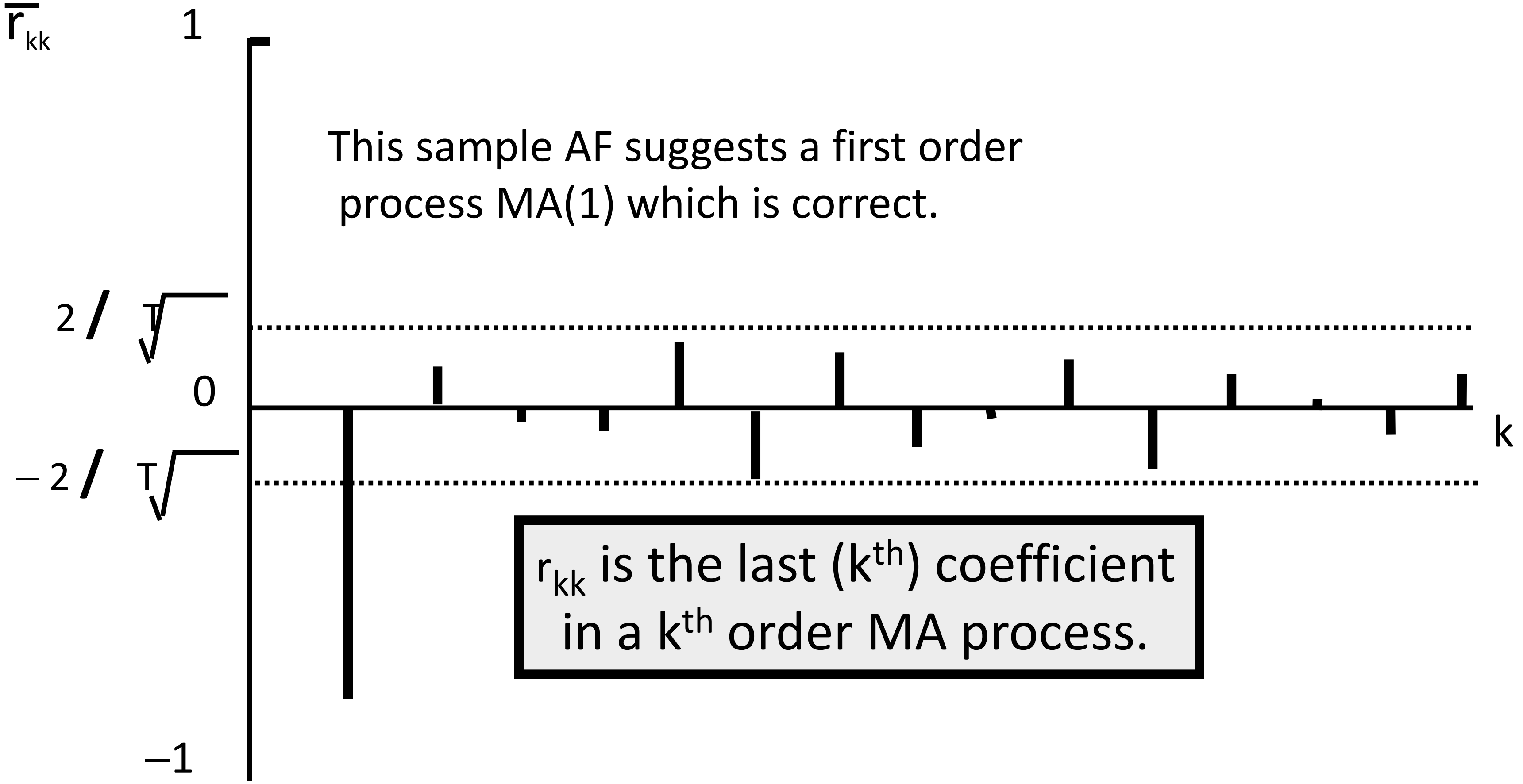
The AF is the sequence of correlations between (y_t and y_{t-1}), (y_t and y_{t-2}), (y_t and y_{t-3}), and so on, without holding the effects of earlier lags on y_t constant.

The PAF controlled for the effects of previous lags but the AF does not control for such effects.

Autocorrelation Function

Data simulated
from this model:

$$y_t = e_t - 0.9\, e_{t-1}$$



This sample AF suggests a first order process MA(1) which is correct.

r_{kk} is the last (k^{th}) coefficient in a k^{th} order MA process.

Autoregressive Moving Average ARMA(p,q)

An ARMA(1,2) has one autoregressive lag and two moving average lags:

$$\mathbf{y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1} + \alpha_2 e_{t-2}}$$

Goodness of fit criteria

- Akaike's Information Criterion [AIC]
- Schwartz's Bayesian Criterion [BIC]
- -2LogLikelihood

The smaller the better….

Stationary vs. Nonstationary

Stationary:

A stationary time series is one whose mean, variance, and autocorrelation function do not change over time.

Nonstationary:

A nonstationary time series is one whose mean, variance or autocorrelation function change over time.

First Differencing is often used to transform a nonstationary series into a stationary series:

$$y_t = z_t - z_{t-1}$$

where z_t is the original nonstationary series and y_t is the new stationary series.

Auto Regressive Integrated Moving Average, ARIMA(p,d,q)

An ARIMA(p,d,q) model represents an AR(p) - MA(q) process that has been differenced (integrated, I(d)) d times

$$y_t = \delta + \theta_1 y_{t-1} + \ldots + \theta_p y_{t-p} + e_t + \alpha_1 e_{t-1} + \ldots + \alpha_q e_{t-q}$$

The Box-Jenkins approach:

1. Identification

   determining the values of p, d, and q

2. Estimation

   linear or nonlinear least squares

3. Diagnostic Checking

   model fits well with no autocorrelation?

4. Forecasting

   short-term forecasts of future y_t values

A Case Study: Series F

The series consists of 70 observations on the yield of a batch chemical process.

Series FJMP

A Case Study: Series F

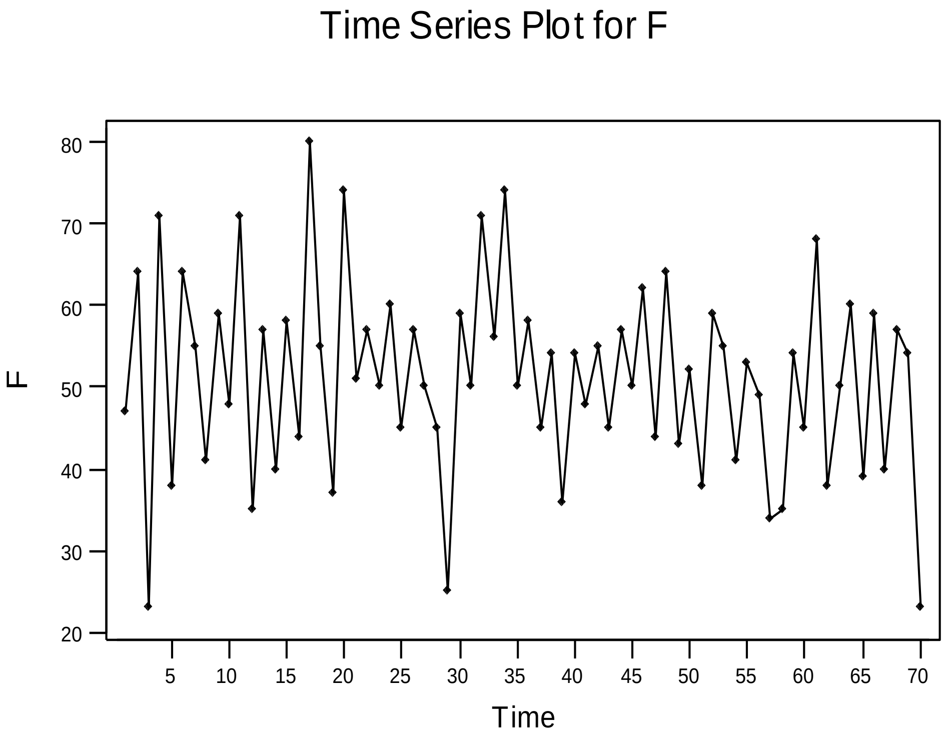
First we plot the series to check for trend, periodicity, etc. which will need the application of differencing.

We inspect the correlogram and partial correlogram to help in identifying an ARMA model for the stationary series we obtain.

The Autocorrelation Function (AF)

The Partial Autocorrelation Function (PAF)

Time Plot of Series F



No obvious non-stationarity in the form of trend or periodic effects.

No apparent need to difference the series.

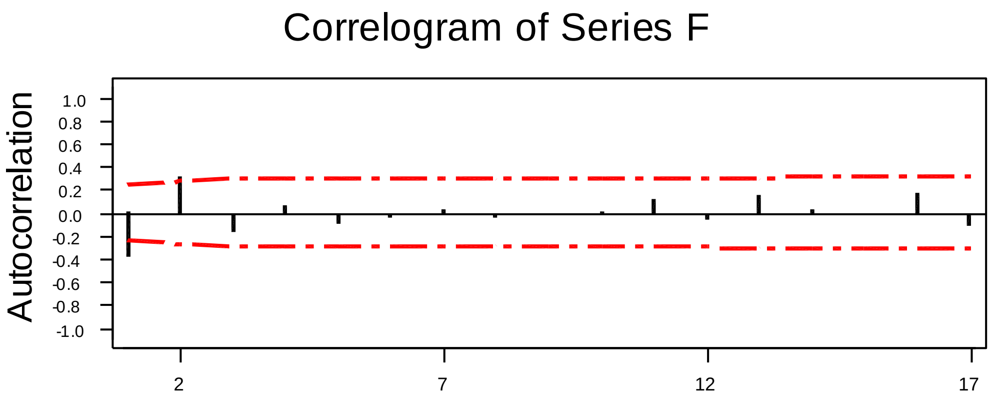
Model Identification.

- The theoretical acf of a MA(q) series shows a cutoff after lag q.
- The acf of an AR(p) series theoretically shows a geometric decline after lag p.
- The pacf of an AR(p) series theoretically shows a cutoff at lag p.

More Identification.

- Often several models look plausible .
- We can try to identify the order of an AR process by fitting several models of orders in the region of p which we think is plausible. Plotting the residual sum of squares against p may show a “flattening” for values beyond the “true” order.

Sample ACF of Series F



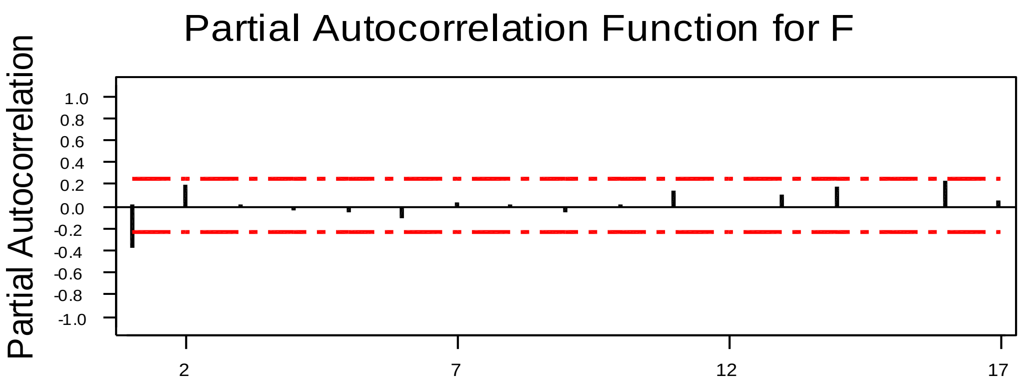
```
Lag  Corr    T    LBQ    Lag  Corr    T    LBQ    Lag  Corr    T    LBQ

  1 -0.39 -3.26  11.10     8 -0.04 -0.29  21.57    15 -0.01 -0.04  25.09
  2  0.30  2.23  17.97     9 -0.00 -0.03  21.57    16  0.17  1.12  27.89
  3 -0.17 -1.14  20.03    10  0.01  0.10  21.59    17 -0.11 -0.71  29.06
  4  0.07  0.48  20.41    11  0.11  0.73  22.62
  5 -0.10 -0.65  21.14    12 -0.07 -0.45  23.03
  6 -0.05 -0.31  21.32    13  0.15  0.97  24.97
  7  0.04  0.24  21.42    14  0.04  0.23  25.09
```

Let’s look at a correlogram of the series.

The only large values are at lags 1 and 2.

Maybe AR(2) or AR(1)?

Partial Correlogram



```
Lag  PAC       T       Lag  PAC       T       Lag  PAC       T

  1  -0.39  -3.26        8   0.00   0.04       15  -0.00  -0.01
  2   0.18   1.50        9  -0.06  -0.47       16   0.22   1.85
  3   0.00   0.02       10   0.00   0.03       17   0.05   0.44
  4  -0.04  -0.37       11   0.14   1.19
  5  -0.07  -0.58       12  -0.01  -0.08
  6  -0.12  -1.01       13   0.09   0.77
  7   0.02   0.16       14   0.17   1.40
```

The partial correlogram also only has appreciable values at lags 1 and 2.

Try an AR(2) model.

i.e. p=2, d=0, q=0

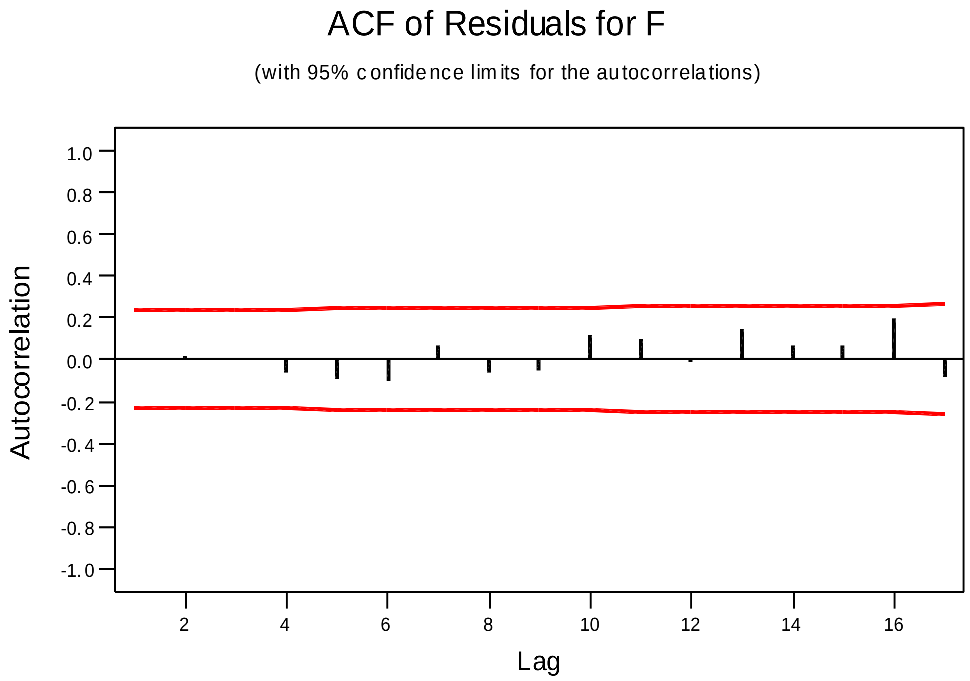
Fitting an AR(2) model

```
Final Estimates of Parameters
Type            Coef            StDev       T
AR   1           -0.3461        0.1259    -2.75
AR   2          0.1934          0.1259     1.54
Constant        59.047          1.298     45.50
Mean            51.227          1.126
```

Model Checking

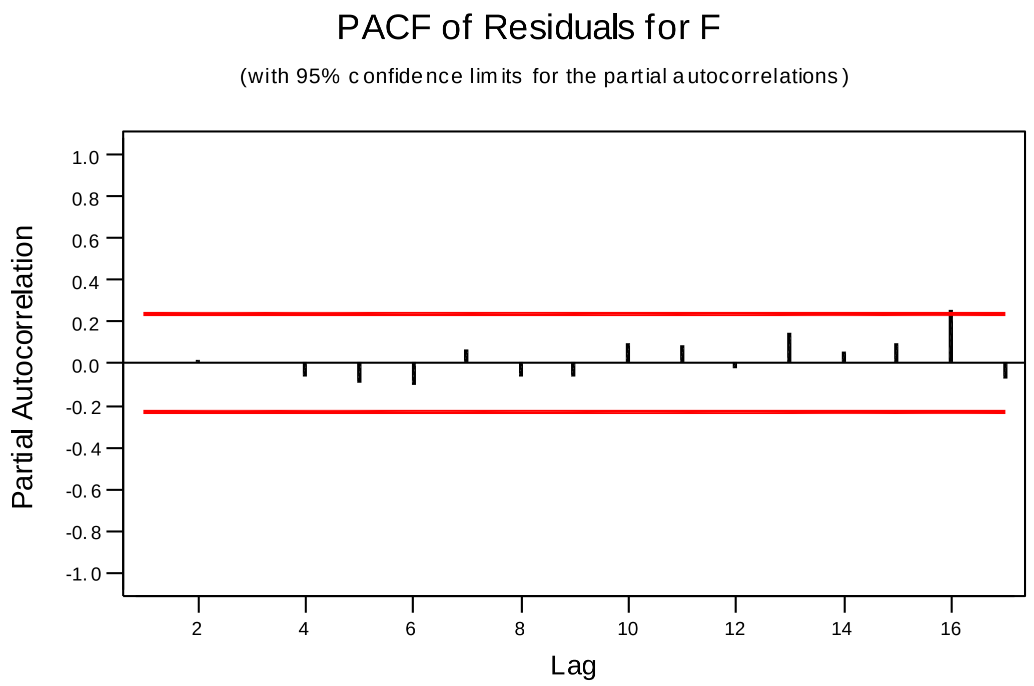
- Examine various aspects of the residuals to evaluate the adequacy of our chosen model.
- Use the acf and pacf to look for any remaining times series structure in the residuals which we have not removed.
- Check for normality and constant variance.

Correlogram of Residuals



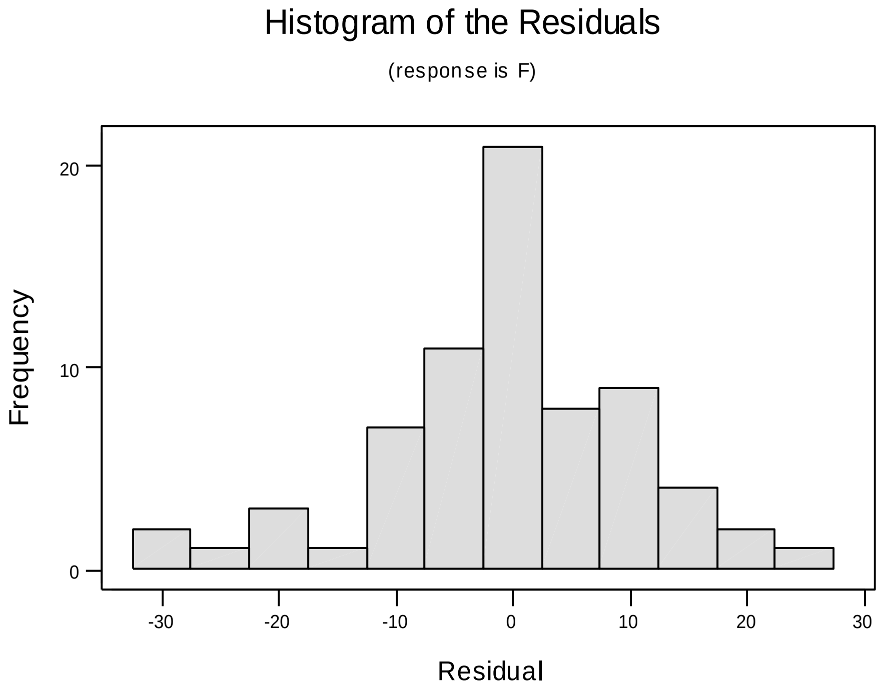
No obvious autocorrelation left after we have fitted our AR(2) model to the original series.

Partial Correlogram of Residuals



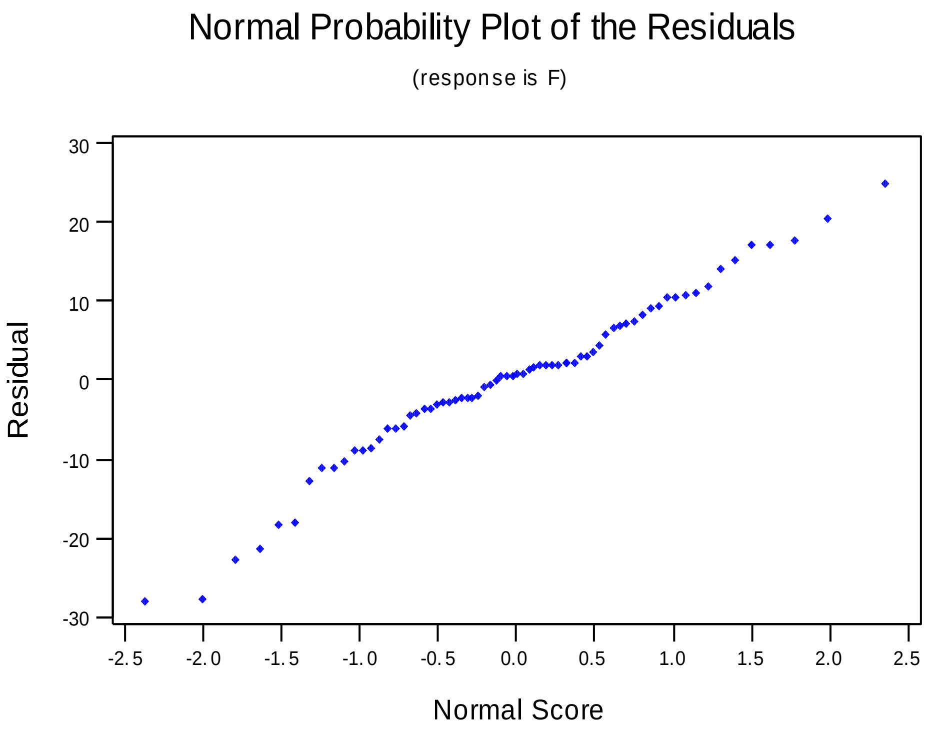
As we expect from the **correlogram**, there is no indication of residual partial autocorrelation.

Histogram of Residuals



The residuals are symmetric about zero and die away fairly rapidly. A normal distribution with mean zero looks plausible.

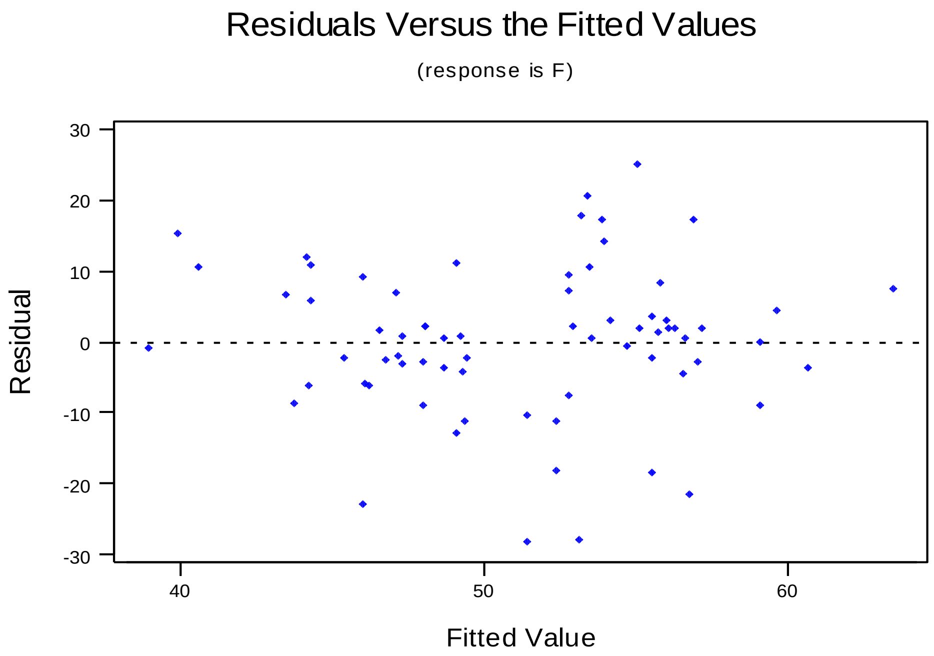
Normal Probability Plot



The graph should approximate a straight line if the residuals are normally distributed.

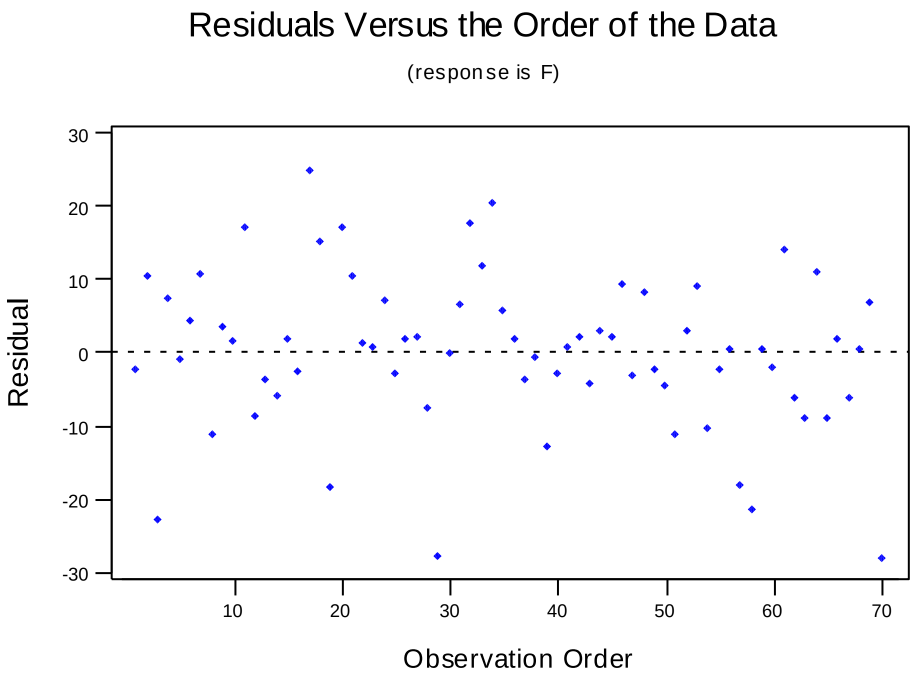
A normal distribution looks OK.

Residuals v. Fitted Values

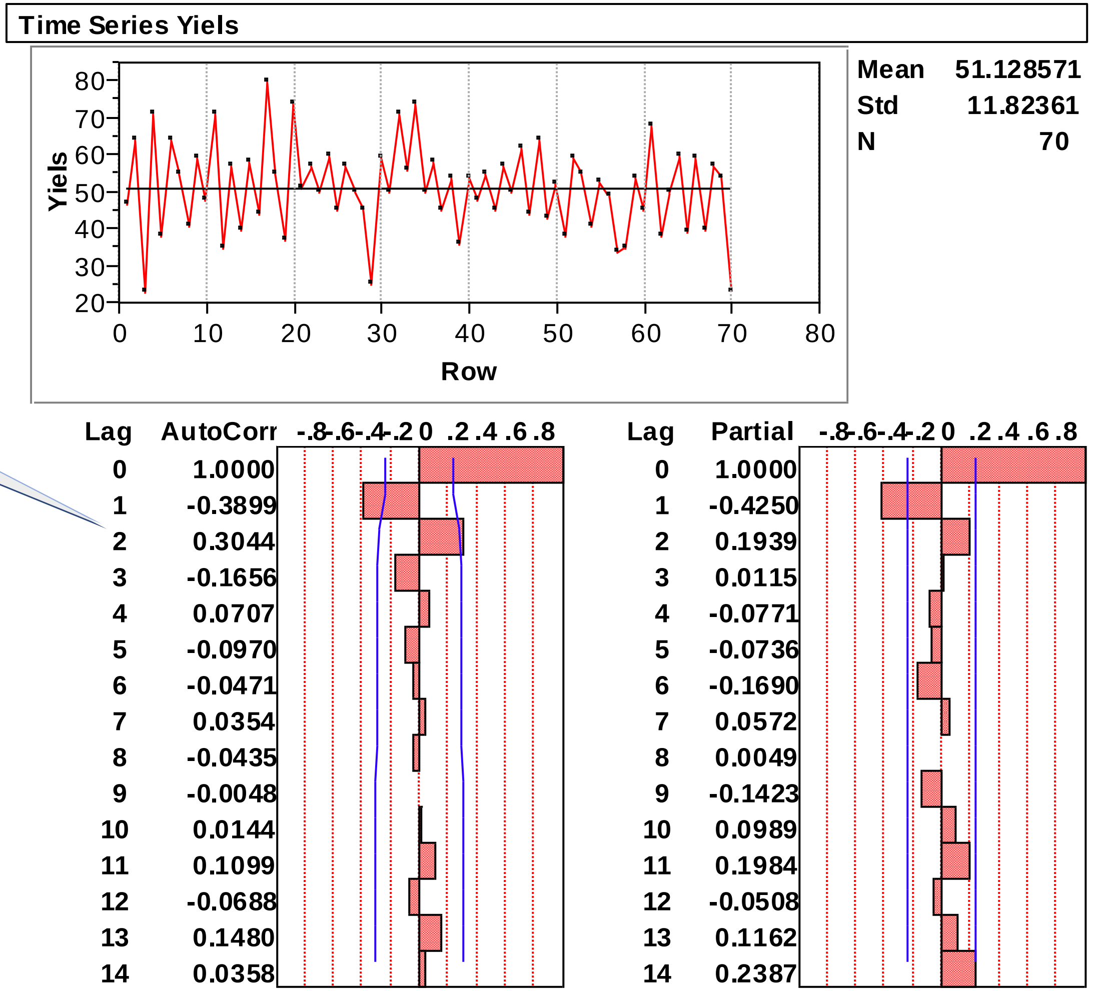


No obvious problems such as the variance depending on the size of the observation.

Residuals in Time Order



No obvious failures to model dependence of residuals over time .



Time Series Yiels

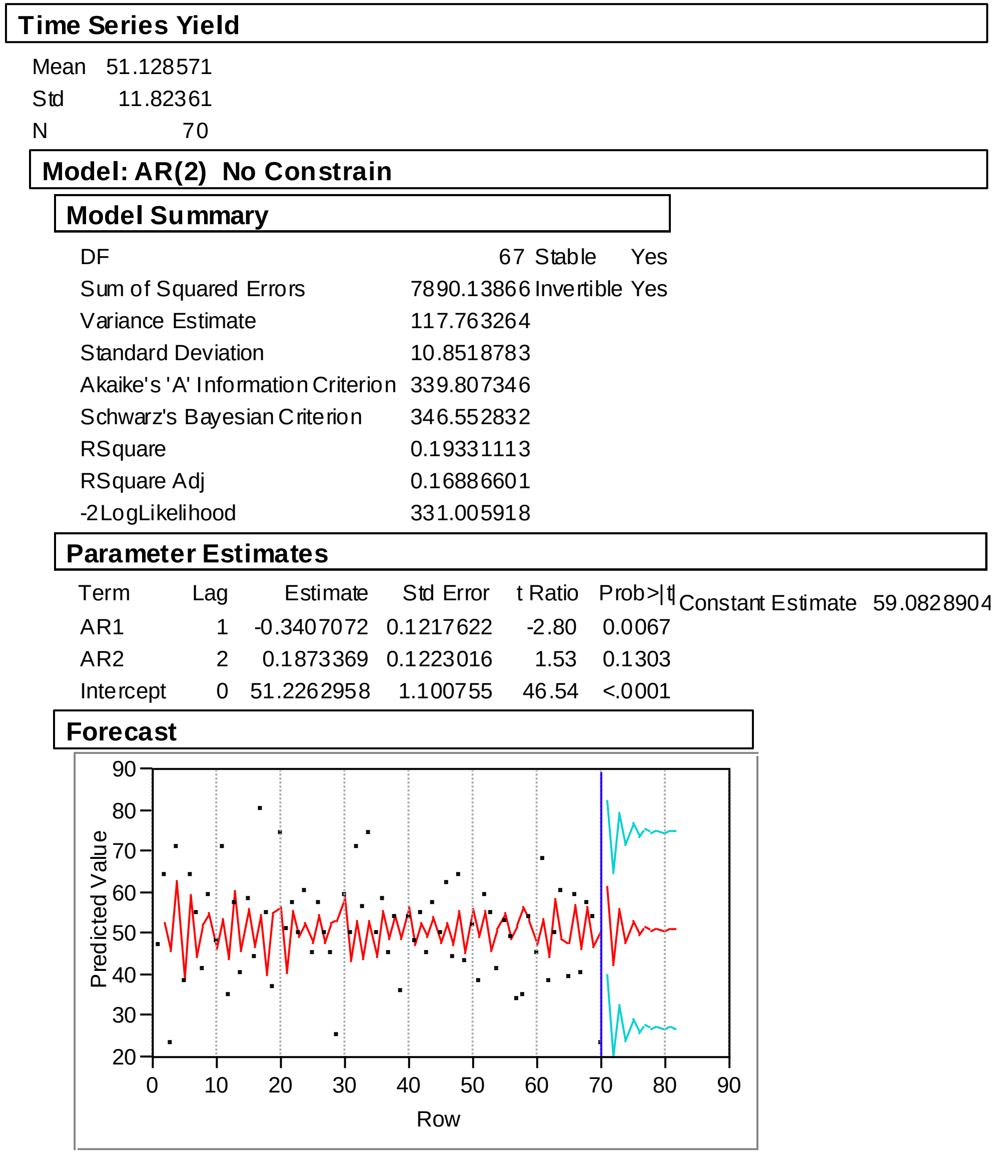
Mean	51.128571
Std	11.82361
N	70

Lag	AutoCorr
0	1.0000
1	-0.3899
2	0.3044
3	-0.1656
4	0.0707
5	-0.0970
6	-0.0471
7	0.0354
8	-0.0435
9	-0.0048
10	0.0144
11	0.1099
12	-0.0688
13	0.1480
14	0.0358

Lag	Partial
0	1.0000
1	-0.4250
2	0.1939
3	0.0115
4	-0.0771
5	-0.0736
6	-0.1690
7	0.0572
8	0.0049
9	-0.1423
10	0.0989
11	0.1984
12	-0.0508
13	0.1162
14	0.2387

MA(2) ?

AR(1),
AR(2) ?



Time Series Yield

Mean	51.128571
Std	11.82361
N	70

Model: AR(2) No Constrain

Model Summary

DF	67	Stable	Yes
Sum of Squared Errors	7890.13866	Invertible	Yes
Variance Estimate	117.763264		
Standard Deviation	10.8518783		
Akaike's 'A' Information Criterion	339.807346		
Schwarz's Bayesian Criterion	346.552832		
RSquare	0.19331113		
RSquare Adj	0.16886601		
-2LogLikelihood	331.005918		

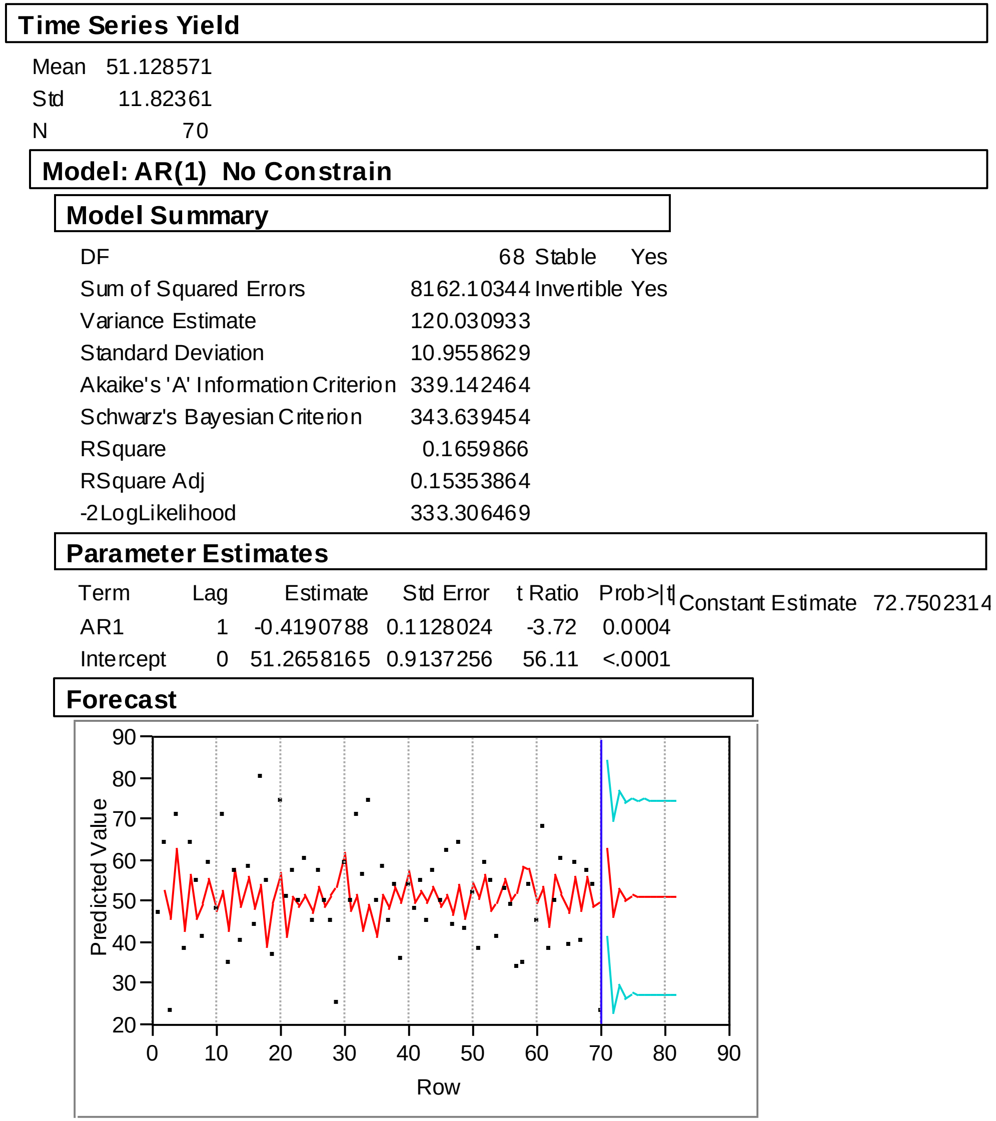
Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|
AR1	1	-0.3407072	0.1217622	-2.80	0.0067
AR2	2	0.1873369	0.1223016	1.53	0.1303
Intercept	0	51.2262958	1.100755	46.54	<.0001

Constant Estimate 59.0828904

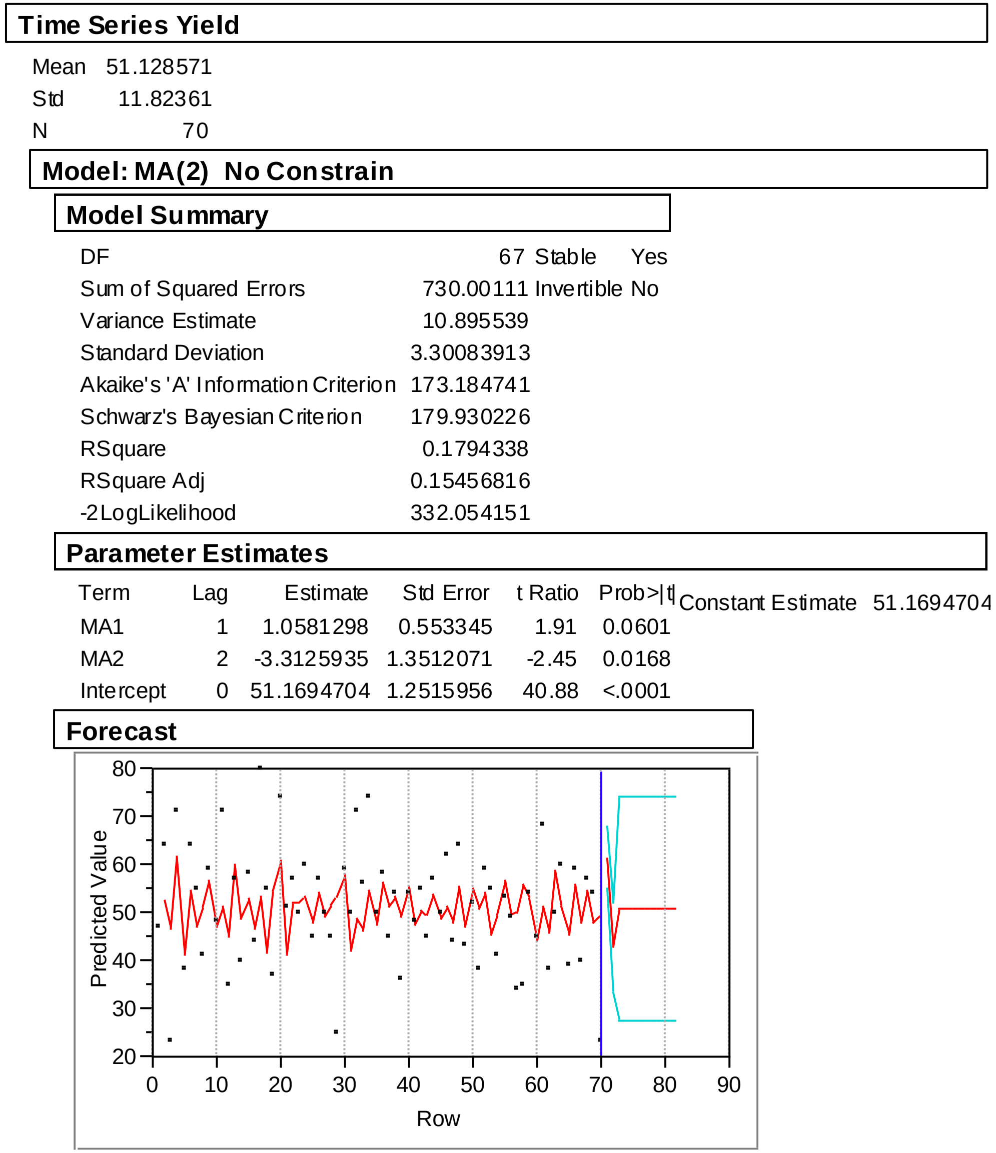
Forecast

AR (2)



```
Time Series Yield
  Mean   51.128571
  Std     11.82361
  N             70
 Model: AR(1)  No Constrain
  Model Summary
    DF                                        68  Stable      Yes
    Sum of Squared Errors             8162.10344  Invertible  Yes
    Variance Estimate                 120.030933
    Standard Deviation                10.9558629
    Akaike's 'A' Information Criterion 339.142464
    Schwarz's Bayesian Criterion      343.639454
    RSquare                            0.1659866
    RSquare Adj                       0.15353864
    -2LogLikelihood                   333.306469
  Parameter Estimates
    Term       Lag    Estimate   Std Error  t Ratio  Prob>|t|  Constant Estimate  72.7502314
    AR1          1  -0.4190788   0.1128024    -3.72    0.0004
    Intercept    0  51.2658165   0.9137256    56.11    <.0001
  Forecast
```

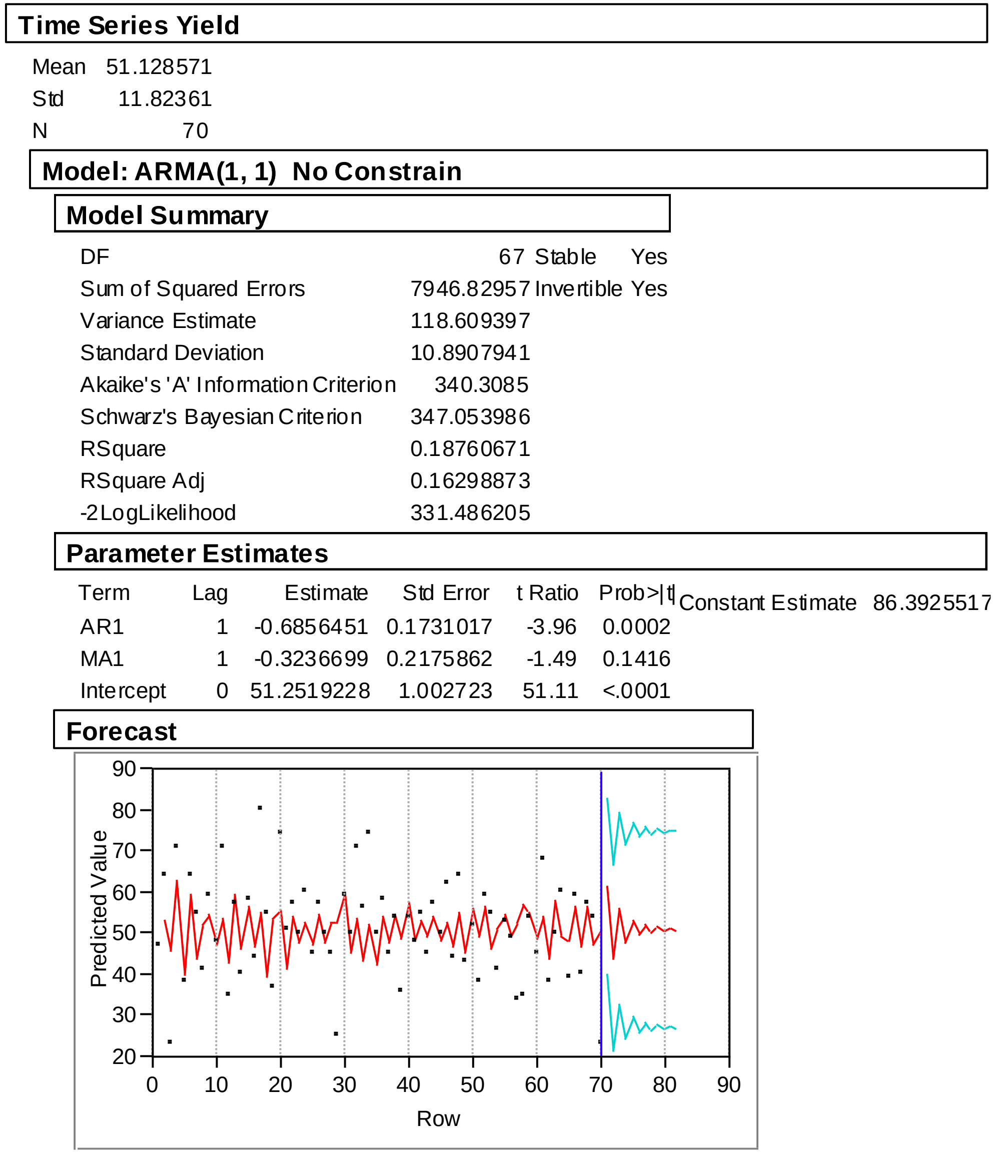
AR (1)



```
Time Series Yield
 Mean   51.128571
 Std     11.82361
 N             70

Model: MA(2)  No Constrain

Model Summary
 DF                                  67  Stable      Yes
 Sum of Squared Errors        730.00111  Invertible  No
 Variance Estimate            10.895539
 Standard Deviation          3.30083913
 Akaike's 'A' Information Criterion  173.184741
 Schwarz's Bayesian Criterion  179.930226
 RSquare                      0.1794338
 RSquare Adj                 0.15456816
 -2LogLikelihood             332.054151

Parameter Estimates
 Term        Lag    Estimate   Std Error   t Ratio   Prob>|t|   Constant Estimate   51.1694704
 MA1           1   1.0581298    0.553345      1.91     0.0601
 MA2           2  -3.3125935   1.3512071     -2.45     0.0168
 Intercept     0  51.1694704   1.2515956     40.88     <.0001

Forecast
```

MA (2)

ARMA (1,1)



Time Series Yield

Mean	51.128571
Std	11.82361
N	70

Model: ARMA(1, 1) No Constrain

Model Summary

DF	67	Stable	Yes
Sum of Squared Errors	7946.82957	Invertible	Yes
Variance Estimate	118.609397		
Standard Deviation	10.8907941		
Akaike's 'A' Information Criterion	340.3085		
Schwarz's Bayesian Criterion	347.053986		
RSquare	0.18760671		
RSquare Adj	0.16298873		
-2LogLikelihood	331.486205		

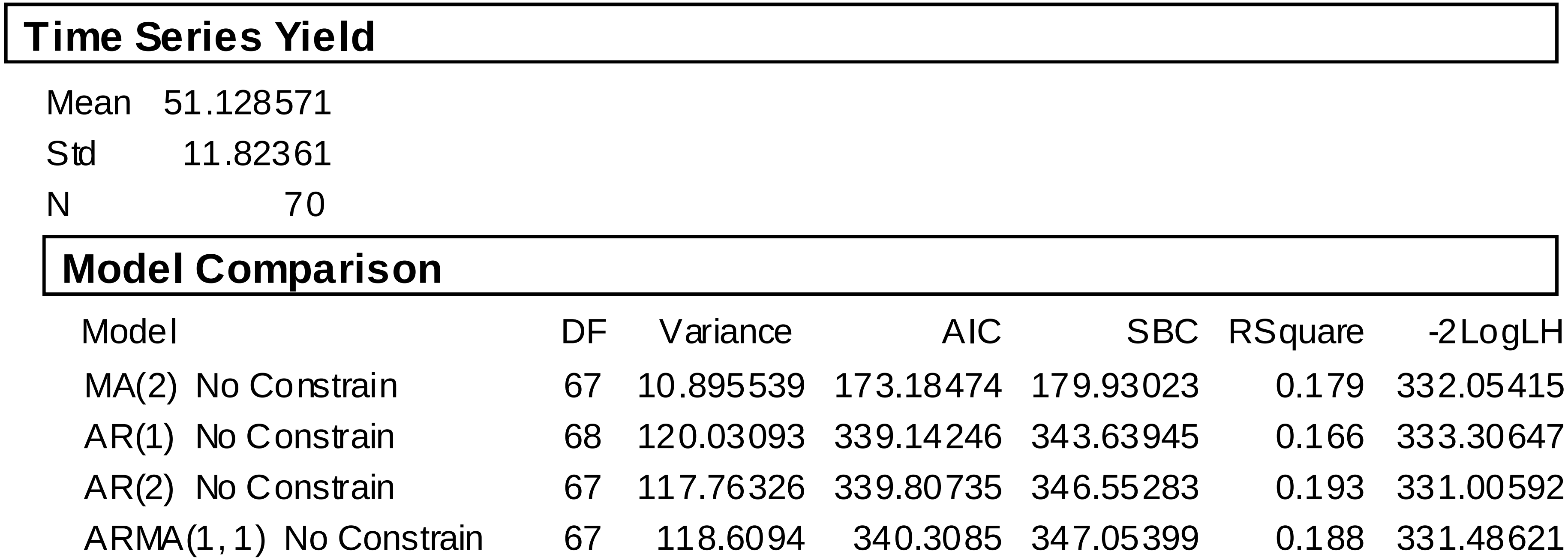
Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|
AR1	1	-0.6856451	0.1731017	-3.96	0.0002
MA1	1	-0.3236699	0.2175862	-1.49	0.1416
Intercept	0	51.2519228	1.002723	51.11	<.0001

Constant Estimate 86.3925517

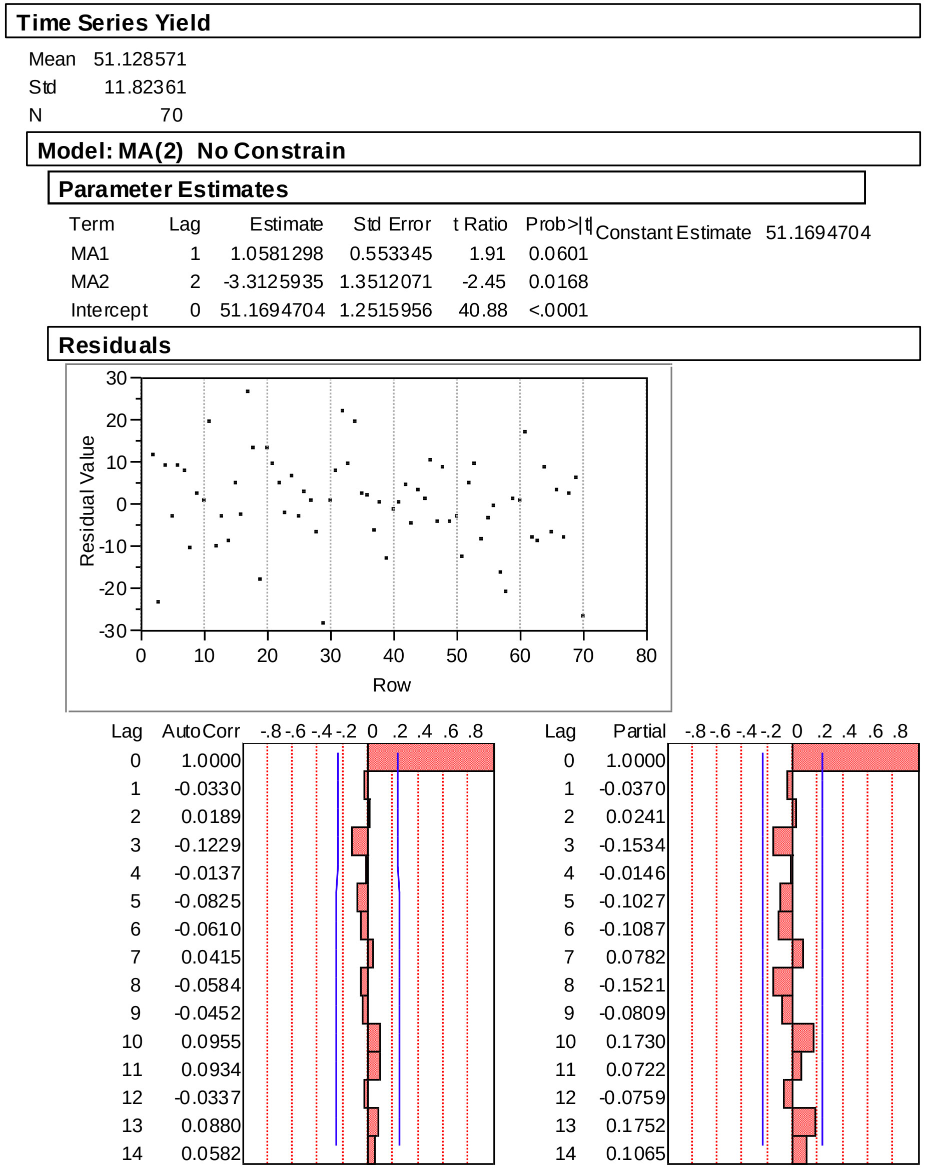
Forecast

Which Model ?



```
Time Series Yield

 Mean   51.128571
 Std     11.82361
 N             70

 Model Comparison

  Model                           DF    Variance         AIC          SBC   RSquare     -2LogLH
  MA(2)  No Constrain             67   10.895539   173.18474    179.93023     0.179   332.05415
  AR(1)  No Constrain             68   120.03093   339.14246    343.63945     0.166   333.30647
  AR(2)  No Constrain             67   117.76326   339.80735    346.55283     0.193   331.00592
  ARMA(1,1)  No Constrain         67    118.6094    340.3085    347.05399     0.188   331.48621
```

MA(2)



Time Series Yield

Mean	51.128571
Std	11.82361
N	70

Model: MA(2) No Constrain

Parameter Estimates

Term	Lag	Estimate	Std Error	t Ratio	Prob>|t|
MA1	1	1.0581298	0.553345	1.91	0.0601
MA2	2	-3.3125935	1.3512071	-2.45	0.0168
Intercept	0	51.1694704	1.2515956	40.88	<.0001

Constant Estimate 51.1694704

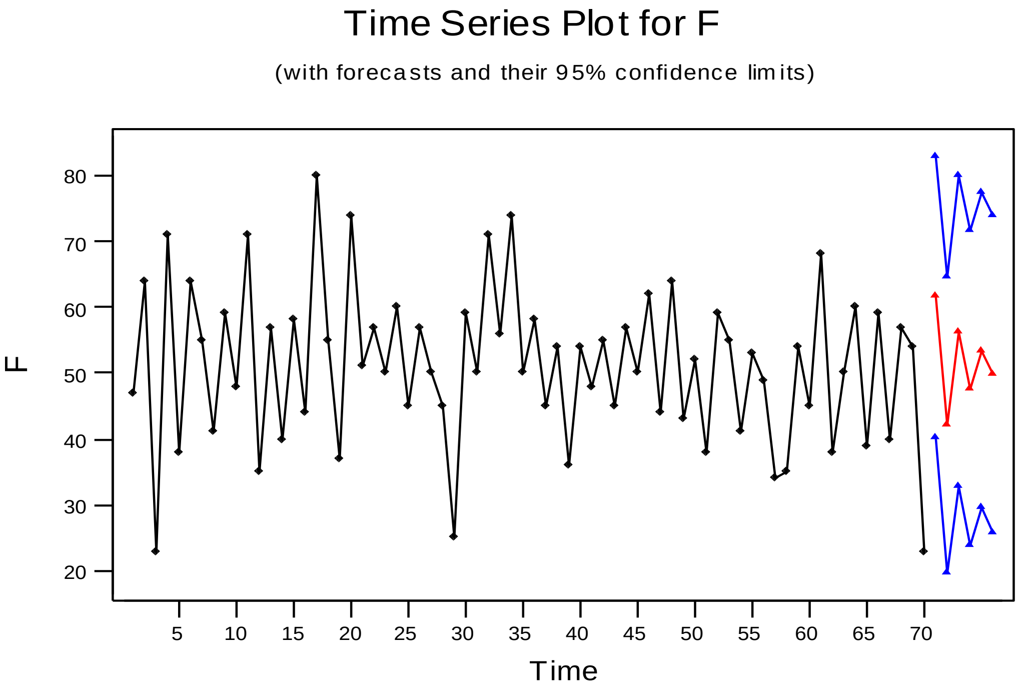
Residuals

Lag	AutoCorr
0	1.0000
1	-0.0330
2	0.0189
3	-0.1229
4	-0.0137
5	-0.0825
6	-0.0610
7	0.0415
8	-0.0584
9	-0.0452
10	0.0955
11	0.0934
12	-0.0337
13	0.0880
14	0.0582

Lag	Partial
0	1.0000
1	-0.0370
2	0.0241
3	-0.1534
4	-0.0146
5	-0.1027
6	-0.1087
7	0.0782
8	-0.1521
9	-0.0809
10	0.1730
11	0.0722
12	-0.0759
13	0.1752
14	0.1065

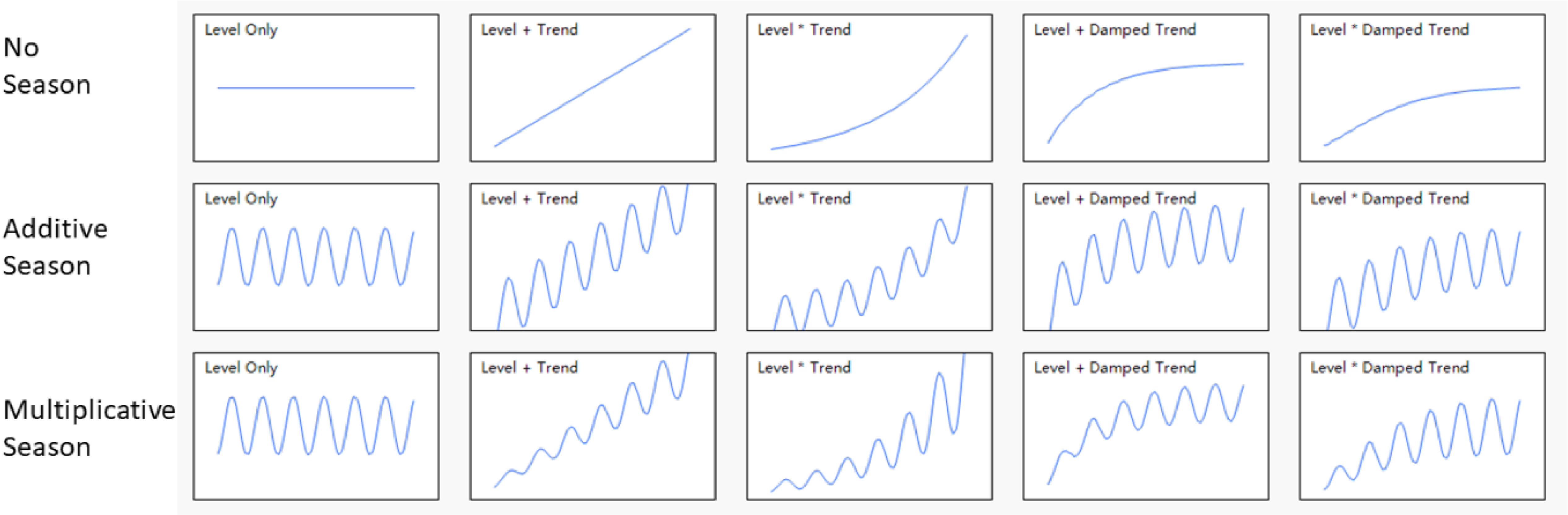
$$y_t = 51.17 + 1.06e_{t-1} - 3.31e_{t-2}$$

Forecasting 6 periods ahead



Opposite is a time series plot with forecasts up to 6 periods ahead added.

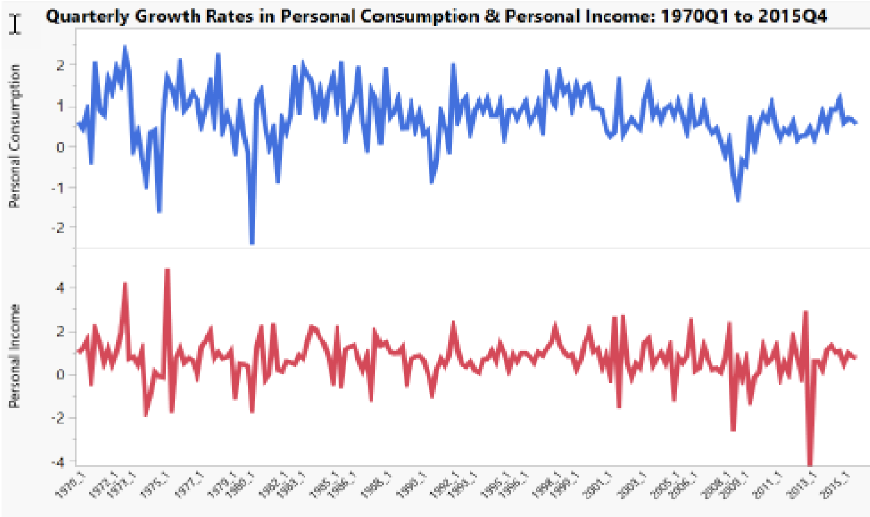
You can see that the interval estimates in blue are quite wide.

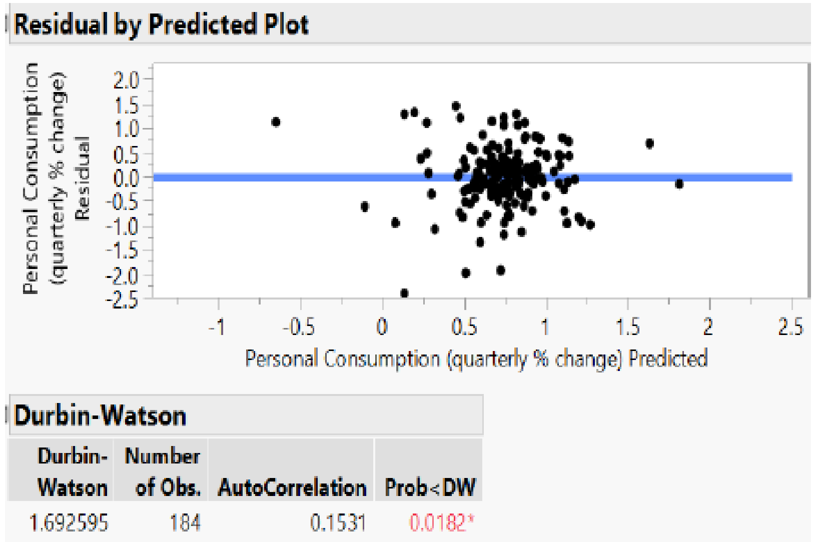

No Season
Additive Season
Multiplicative Season
Level Only
Level + Trend
Level * Trend
Level + Damped Trend
Level * Damped Trend
Level Only
Level + Trend
Level * Trend
Level + Damped Trend
Level * Damped Trend
Level Only
Level + Trend
Level * Trend
Level + Damped Trend
Level * Damped Trend

Transfer Functions

y_t: **Quarterly US Personal Consumption Growth Rate**

x_t: **Quarterly US Personal Income Growth Rate**




Residual by Predicted Plot
Personal Consumption (quarterly % change) Residual
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-1
-0.5
0
0.5
1
1.5
2
2.5
Personal Consumption (quarterly % change) Predicted

Durbin-Watson

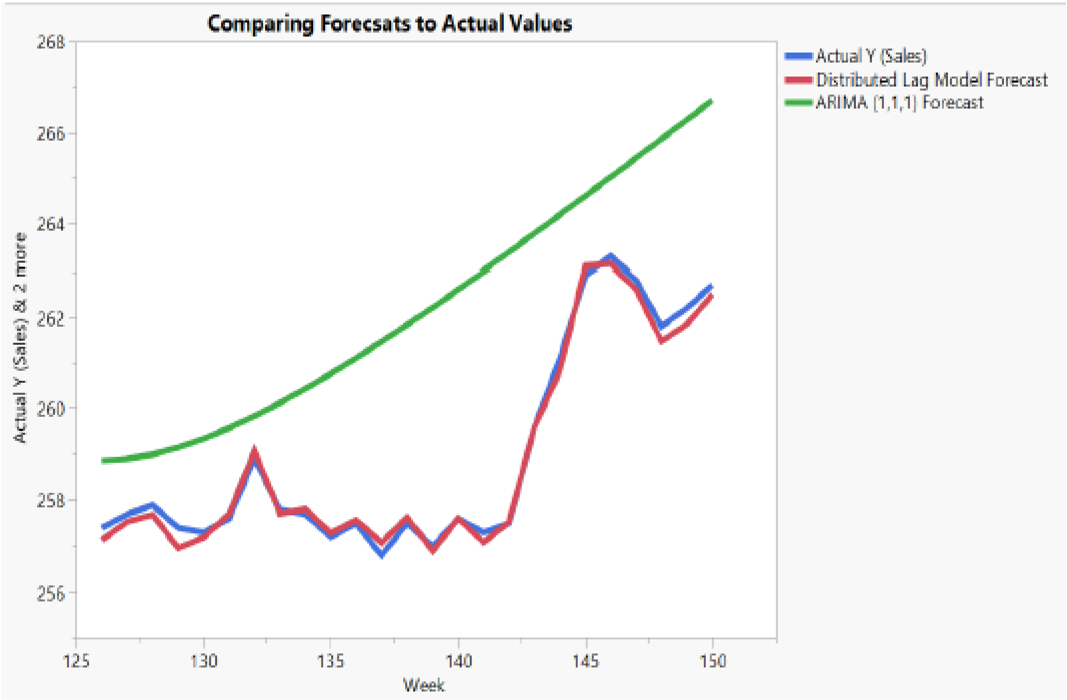
Durbin-Watson	Number of Obs.	AutoCorrelation	Prob<DW
1.692595	184	0.1531	0.0182*

- *"Can I just run a standard least squares regression model to predict y(t)?"* ($\widehat{y_t} = \mathbf{0.55 + 0.28} * x_t$)

  --Key assumption that the regression errors are independent is likely violated => Misleading inferences.

Transfer Functions

--A pure ARIMA model uses the past values of an variable to predict its future values. If there is a good predictor bring it in.



Blue line: Actual Sales
Green Line: Forecast from Pure ARIMA
Red Line: Forecast from Transfer Function Model

Transfer Functions

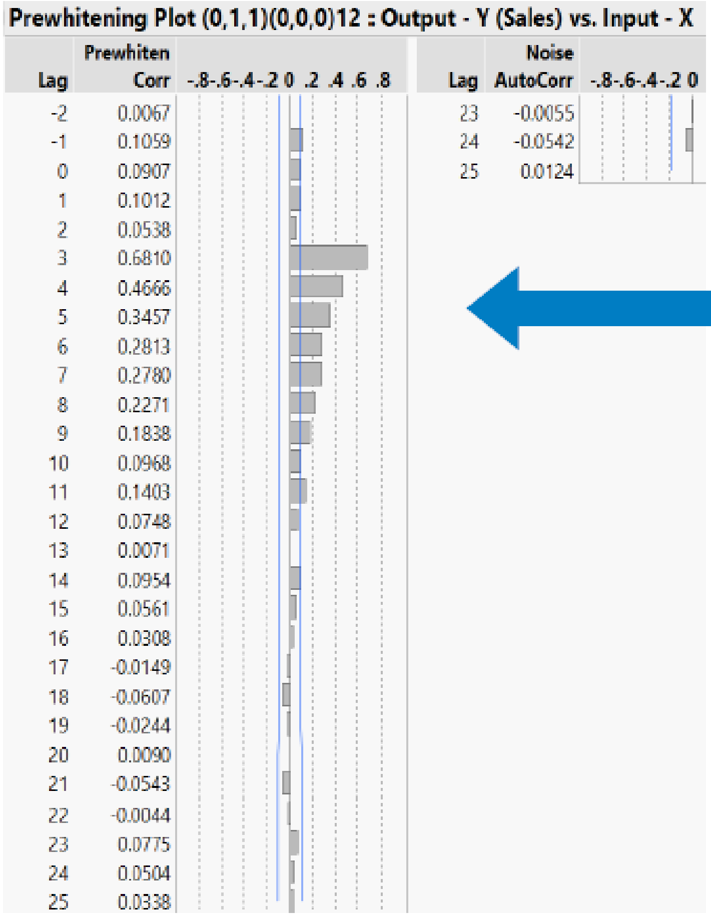
$$y_t = \mu + x_t\beta + e_t \qquad \text{(OLS)}$$

$$y_t = \mu + \frac{\theta(B)}{\varphi(B)} e_t \qquad \text{(Pure ARIMA)}$$

$$y_t = \mu + \overset{\substack{\text{Transfer Function} \\ \Downarrow}}{\frac{\omega(B)}{\delta(B)}} x_{t-l} + \overset{\substack{\text{ARIMA Error Term} \\ \Downarrow}}{\frac{\theta(B)}{\varphi(B)}} e_t$$

Prewhitening

Identifying *when* the input effect takes place, *how long* it lasts and in *what* shape it decays.



Prewhitening Plot (0,1,1)(0,0,0)12 : Output - Y (Sales) vs. Input - X

Lag	Prewhiten Corr
-2	0.0067
-1	0.1059
0	0.0907
1	0.1012
2	0.0538
3	0.6810
4	0.4666
5	0.3457
6	0.2813
7	0.2780
8	0.2271
9	0.1838
10	0.0968
11	0.1403
12	0.0748
13	0.0071
14	0.0954
15	0.0561
16	0.0308
17	-0.0149
18	-0.0607
19	-0.0244
20	0.0090
21	-0.0543
22	-0.0044
23	0.0775
24	0.0504
25	0.0338

Lag	Noise AutoCorr
23	-0.0055
24	-0.0542
25	0.0124

Ex2: Forecasting Sales using Lagged Predictors

Input series x_t is delayed by 3 lags and then exponentially decreasing:

$$\omega_0(x_{t-3} + \delta x_{t-4} + \delta^2 x_{t-5} + \delta^3 x_{t-6} + \delta^4 x_{t-7} + \cdots)$$

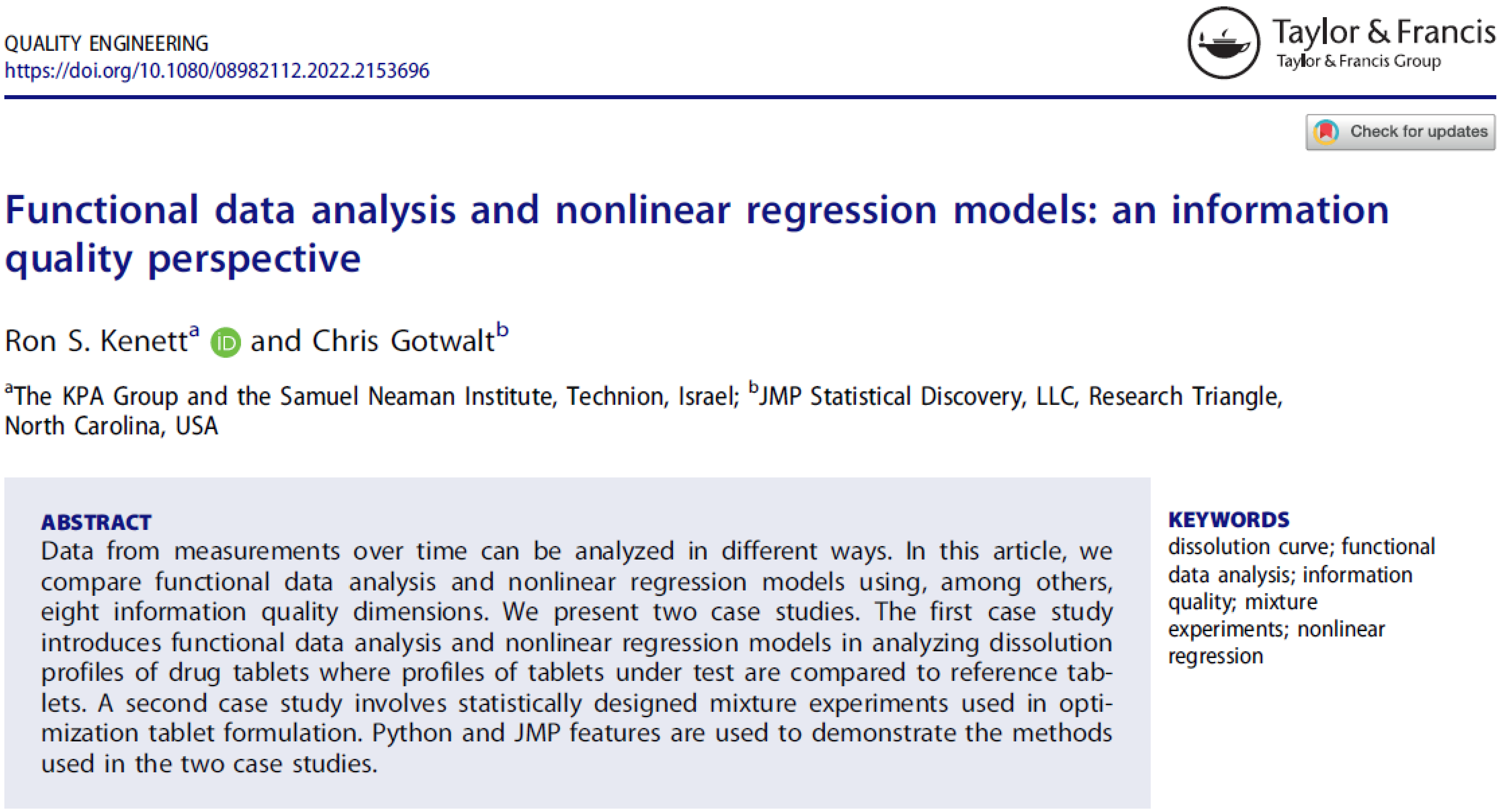
try the transfer function $\boldsymbol{\frac{\omega_0}{1-\delta B} x_{t-3}}$ to approximate the input-output relationships

Forecasting: Principles and Practice (2nd ed)

Rob J Hyndman and George Athanasopoulos

Monash University, Australia

https://otexts.com/fpp2/



QUALITY ENGINEERING
https://doi.org/10.1080/08982112.2022.2153696

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Check for updates

Functional data analysis and nonlinear regression models: an information quality perspective

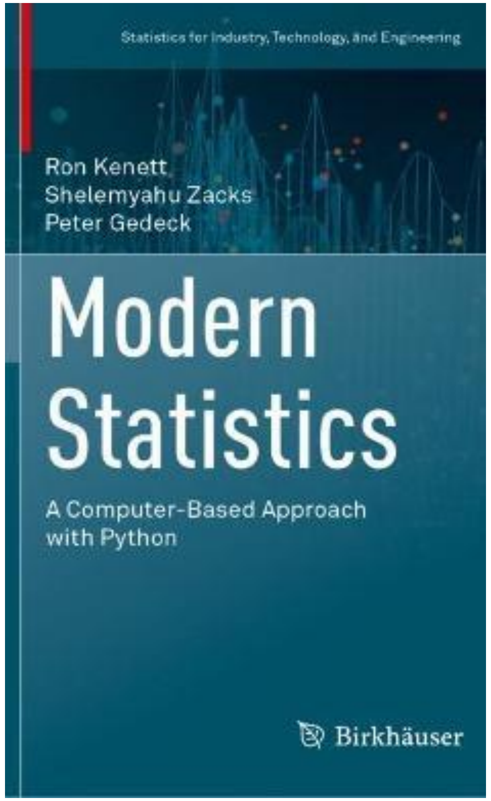
Ron S. Kenett[a] and Chris Gotwalt[b]

[a]The KPA Group and the Samuel Neaman Institute, Technion, Israel; [b]JMP Statistical Discovery, LLC, Research Triangle, North Carolina, USA

ABSTRACT
Data from measurements over time can be analyzed in different ways. In this article, we compare functional data analysis and nonlinear regression models using, among others, eight information quality dimensions. We present two case studies. The first case study introduces functional data analysis and nonlinear regression models in analyzing dissolution profiles of drug tablets where profiles of tablets under test are compared to reference tablets. A second case study involves statistically designed mixture experiments used in optimization tablet formulation. Python and JMP features are used to demonstrate the methods used in the two case studies.

KEYWORDS
dissolution curve; functional data analysis; information quality; mixture experiments; nonlinear regression

Chapter 8
Modern Analytic Methods: Part II



Statistics for Industry, Technology, and Engineering

Ron Kenett
Shelemyahu Zacks
Peter Gedeck

Modern Statistics

A Computer-Based Approach with Python

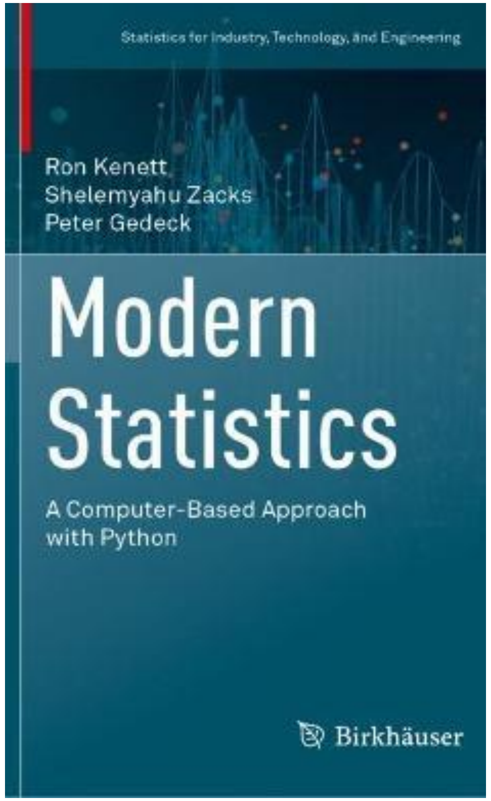
Birkhäuser

Preview Chapter 8 includes the tip of the iceberg examples with what we thought were interesting insights, not always available in standard texts. The chapter covers functional data analysis, text analytics, reinforcement learning, Bayesian networks, and causality models.

Chapter 8
Modern Analytic Methods: Part II

8.1 Functional Data Analysis

When you collect data from tests or measurements over time or other dimensions, we might want to focus on the functional structure of the data. Examples can be chromatograms from high-performance liquid chromatography (HPLC) systems, dissolution profiles of drug tablets over time, distribution of particle sizes, or measurement of sensors. Functional data is different using individual measurements recorded at different sets of time points. It views functional observations as continuously defined so that an observation is the entire function. With functional data consisting of a set of curves representing repeated measurements, we characterize the main features of the data, for example, with a functional version of principal component analysis (FPCA). The regular version of principal component analysis (PCA) is presented in detail in Chap. 4 (Industrial Statistics book) on Multivariate Statistical Process Control. With this background, let us see an example of functional data analysis (FDA).



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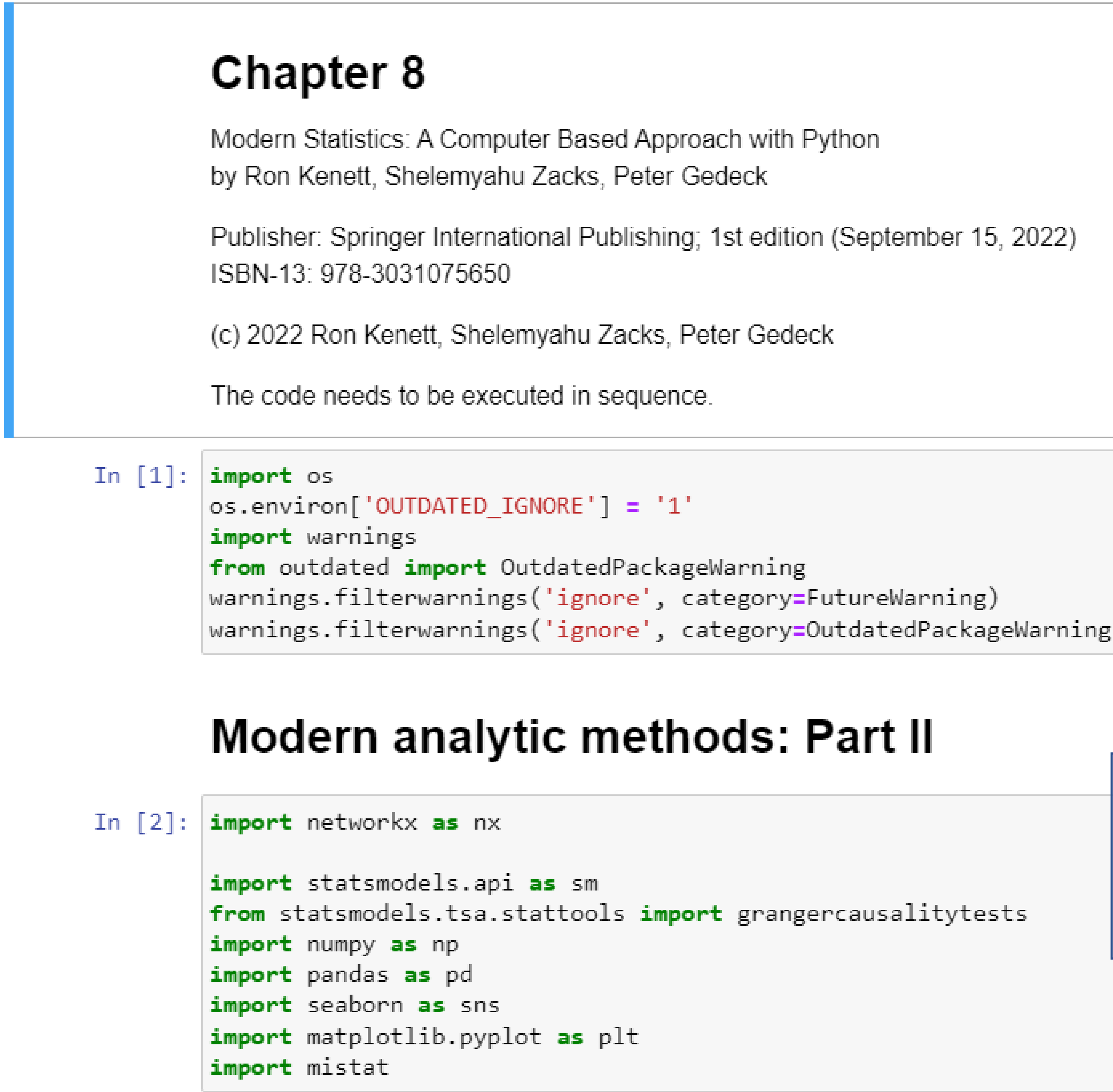


Chapter 8

Modern Statistics: A Computer Based Approach with Python
by Ron Kenett, Shelemyahu Zacks, Peter Gedeck

Publisher: Springer International Publishing; 1st edition (September 15, 2022)
ISBN-13: 978-3031075650

The code needs to be executed in sequence.

In [1]:

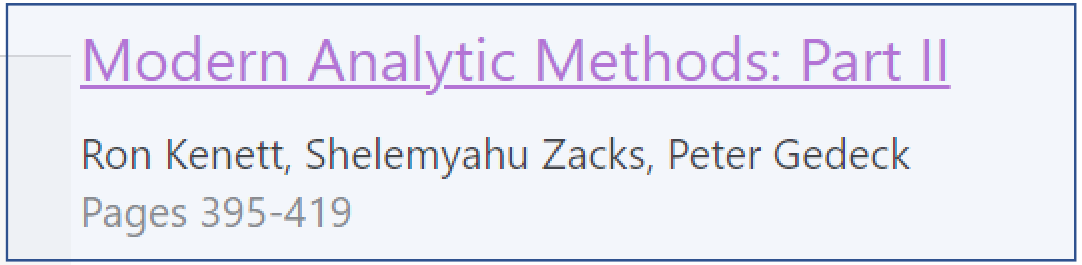
```
import os
os.environ['OUTDATED_IGNORE'] = '1'
import warnings
from outdated import OutdatedPackageWarning
warnings.filterwarnings('ignore', category=FutureWarning)
warnings.filterwarnings('ignore', category=OutdatedPackageWarning)
```

Modern analytic methods: Part II

In [2]:

```
import networkx as nx

import statsmodels.api as sm
from statsmodels.tsa.stattools import grangercausalitytests
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import mistat
```



Modern Analytic Methods: Part II

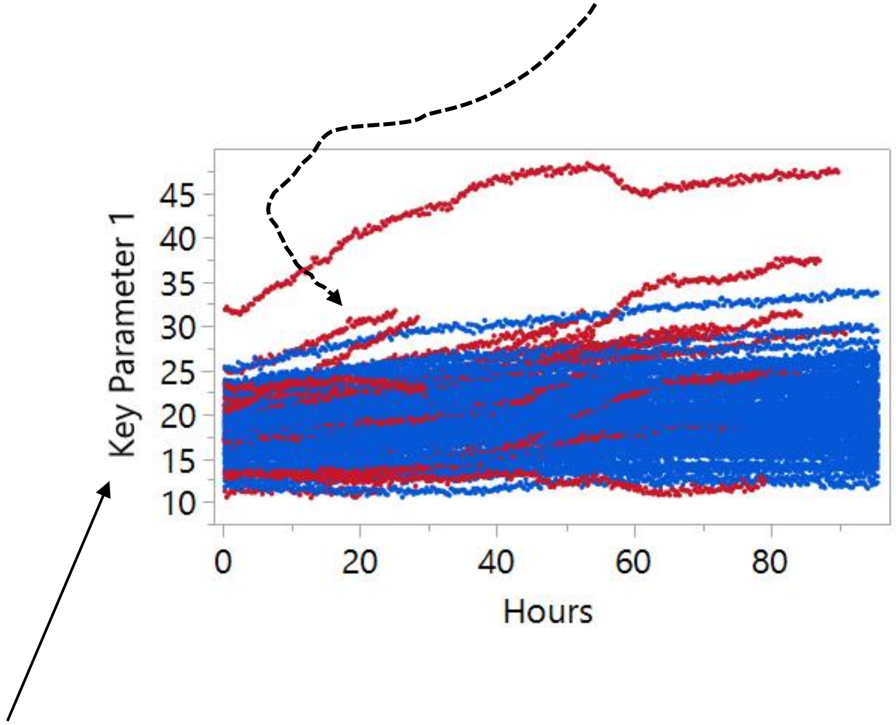
Ron Kenett, Shelemyahu Zacks, Peter Gedeck
Pages 395-419

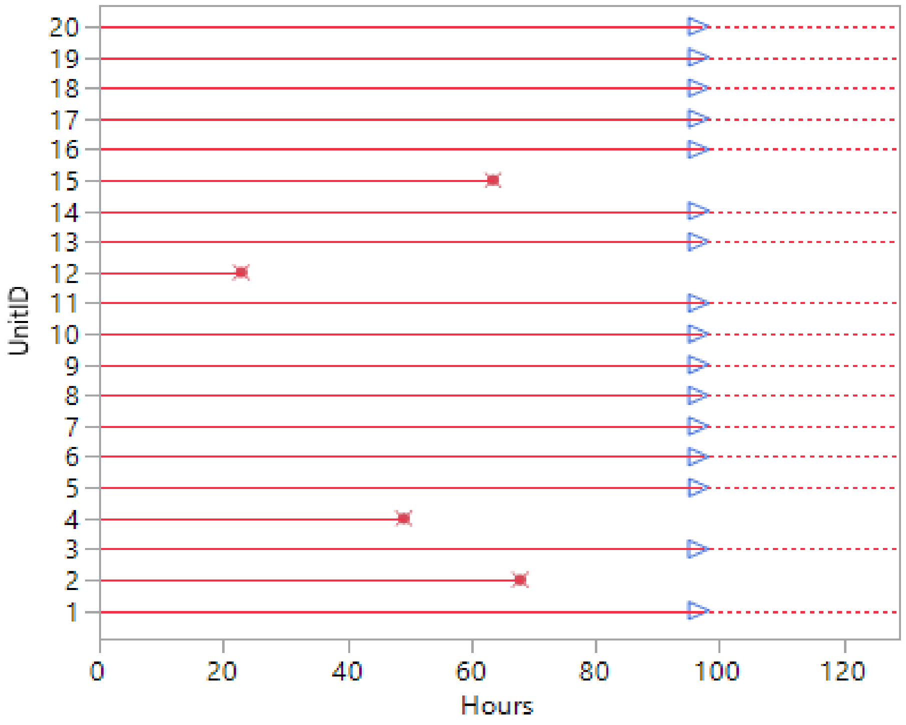
A Case Study

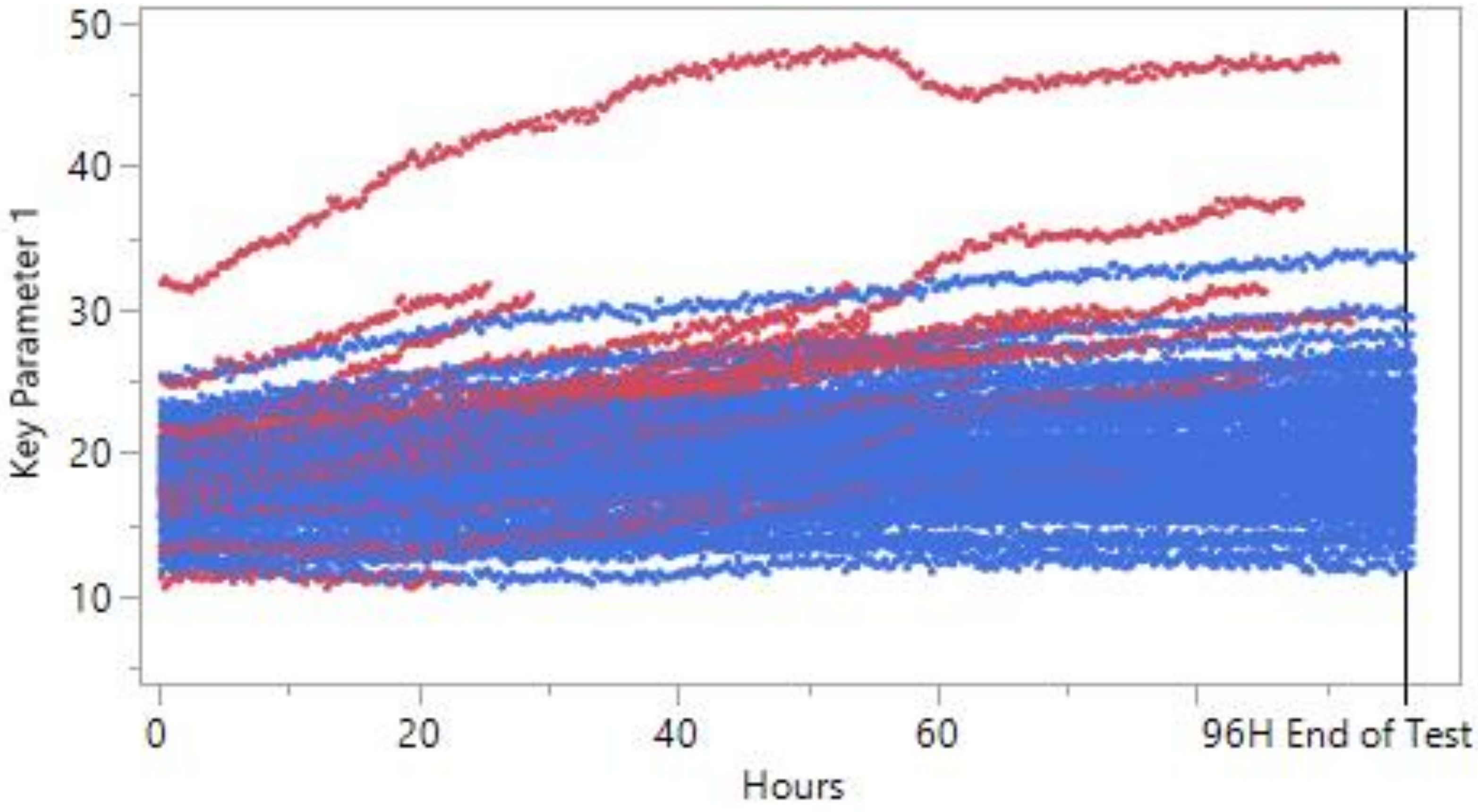
- A manufacturer of electro-mechanical devices
- 22% of units fail early
- The company screens units via burn-in under accelerated conditions
- 23 measurements are collected in real time (2 key parameters)
- Can we use these measurements to predict which units will fail in less time than the current protocol?

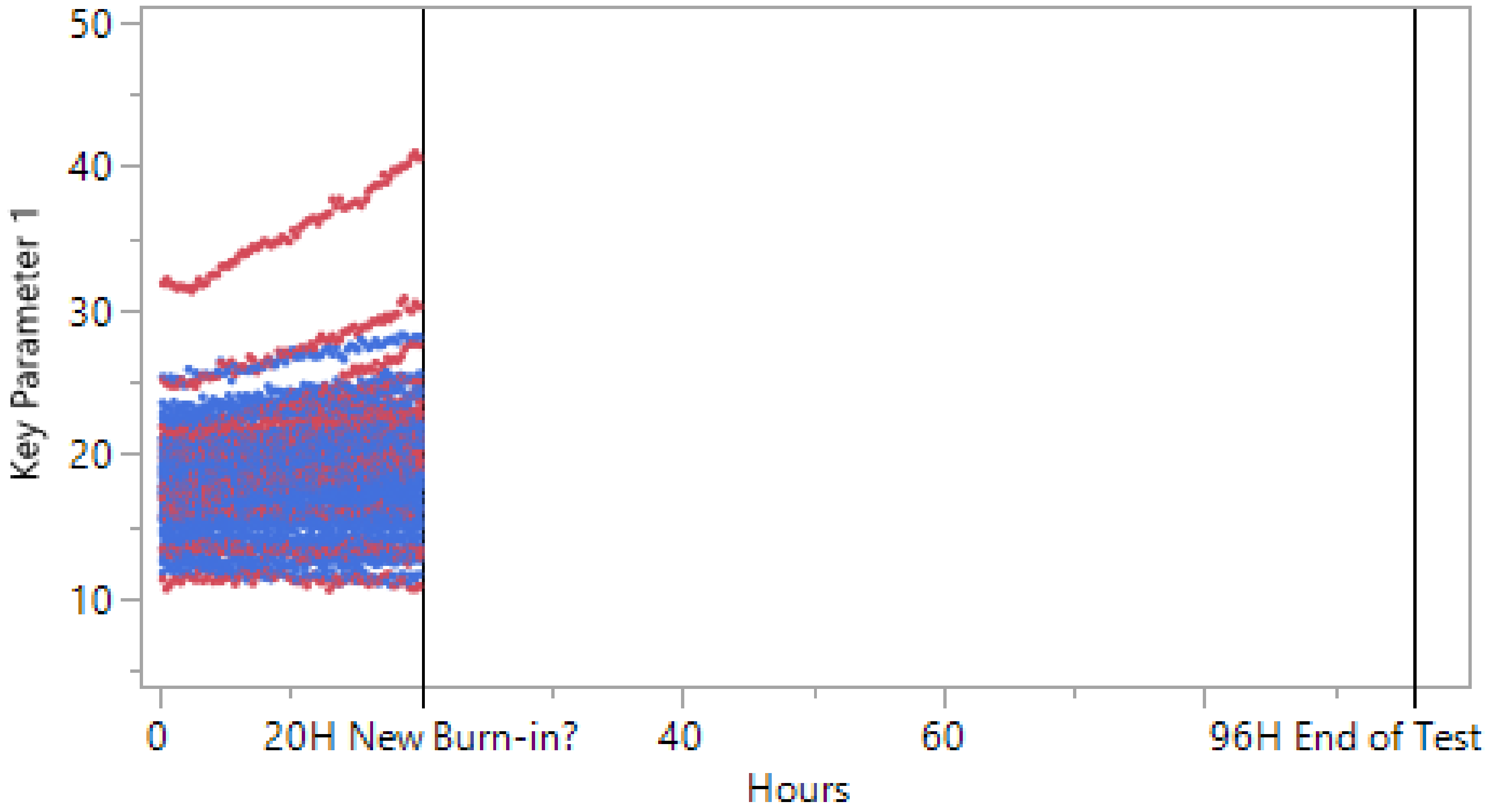
Key parameter 1

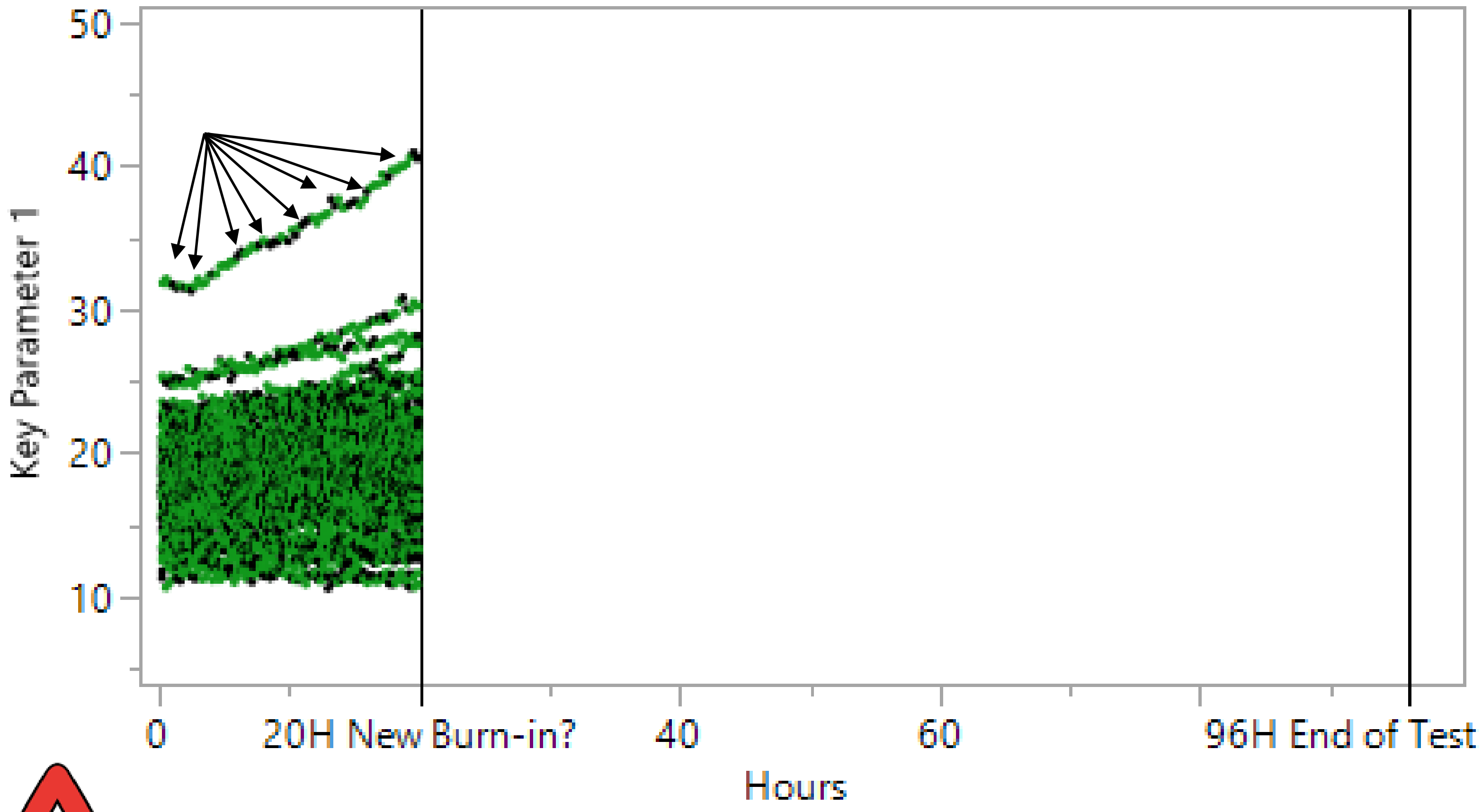
Data consists of 92 units. If a unit didn't fail, there would be around 25,000 measurements. Failed series can be much shorter





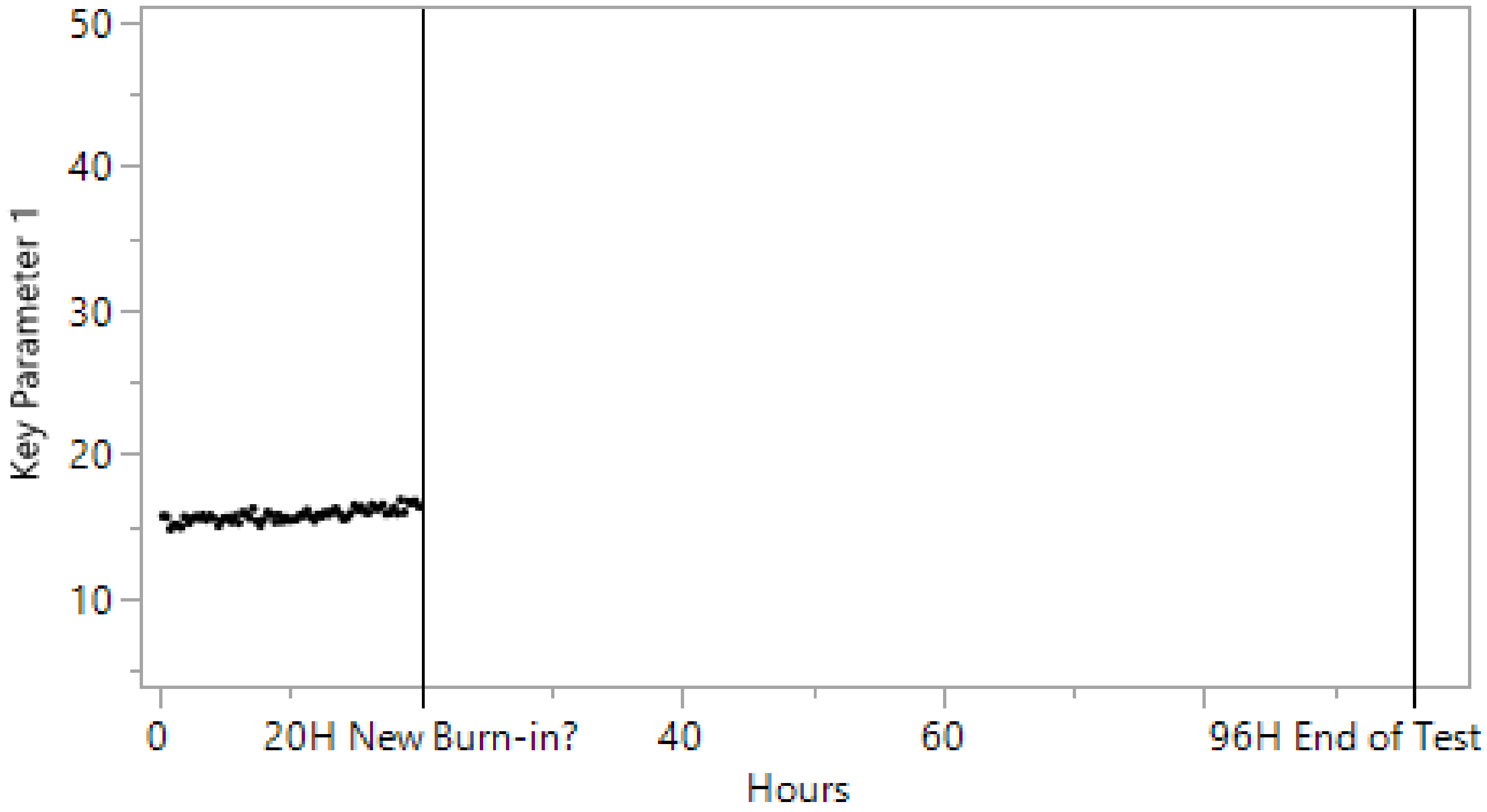

Key Parameter 1
50
40
30
20
10
0
20
40
60
96H End of Test
Hours


50
40
30
20
10
Key Parameter 1
0
20H New Burn-in?
40
60
96H End of Test
Hours

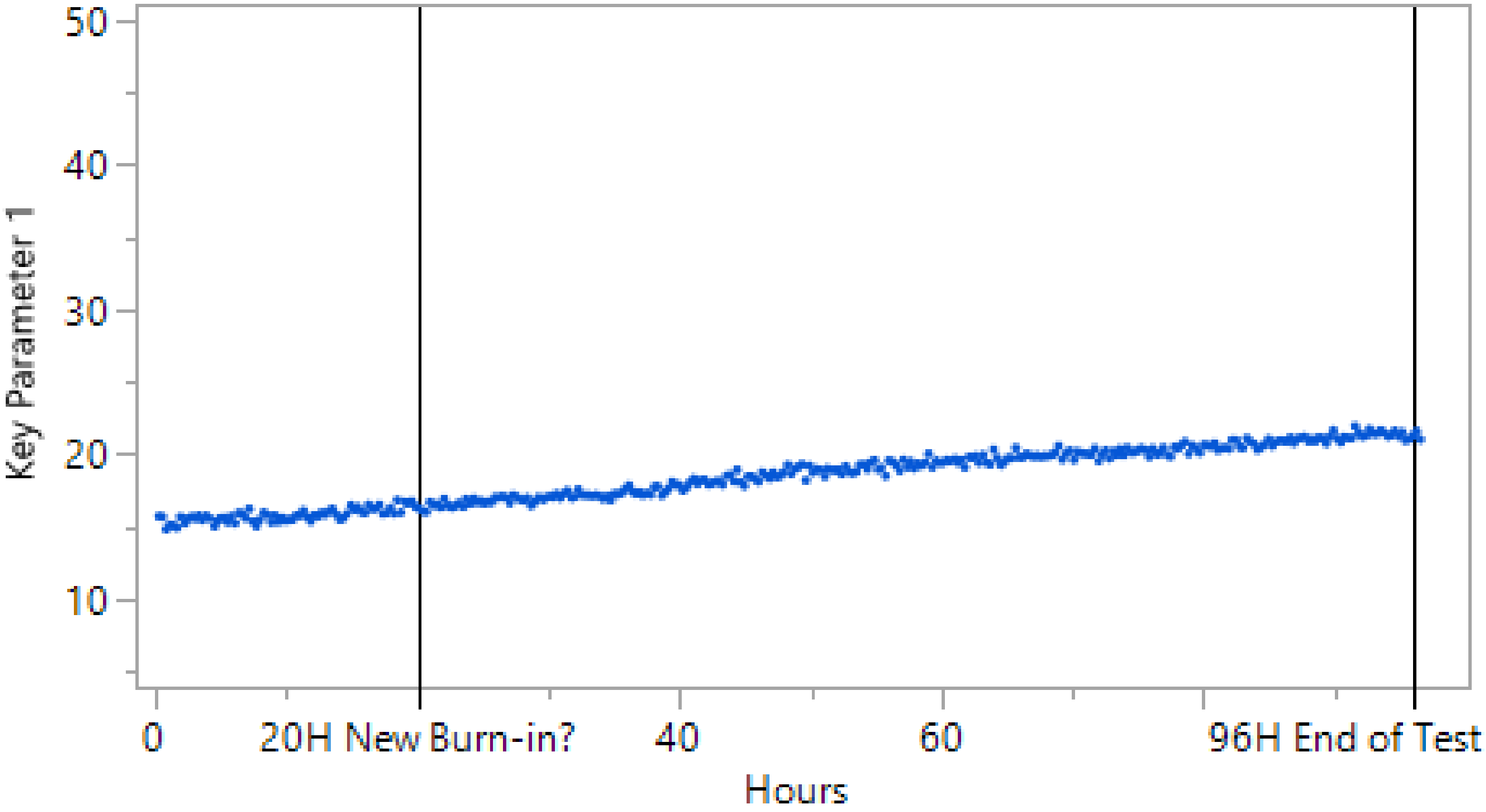

50
40
30
20
10
Key Parameter 1
0
20H New Burn-in?
40
60
96H End of Test
Hours

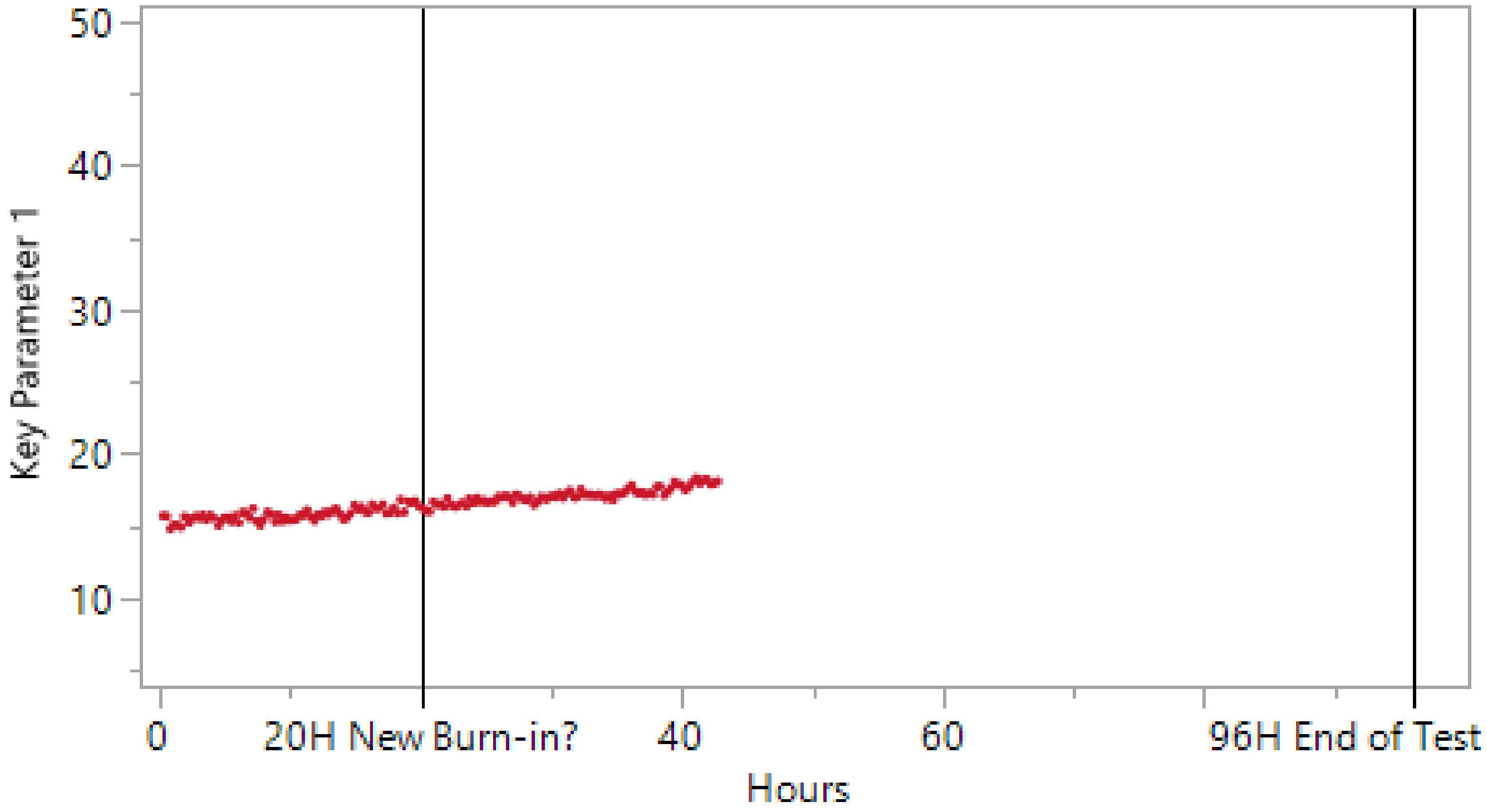
DANGER

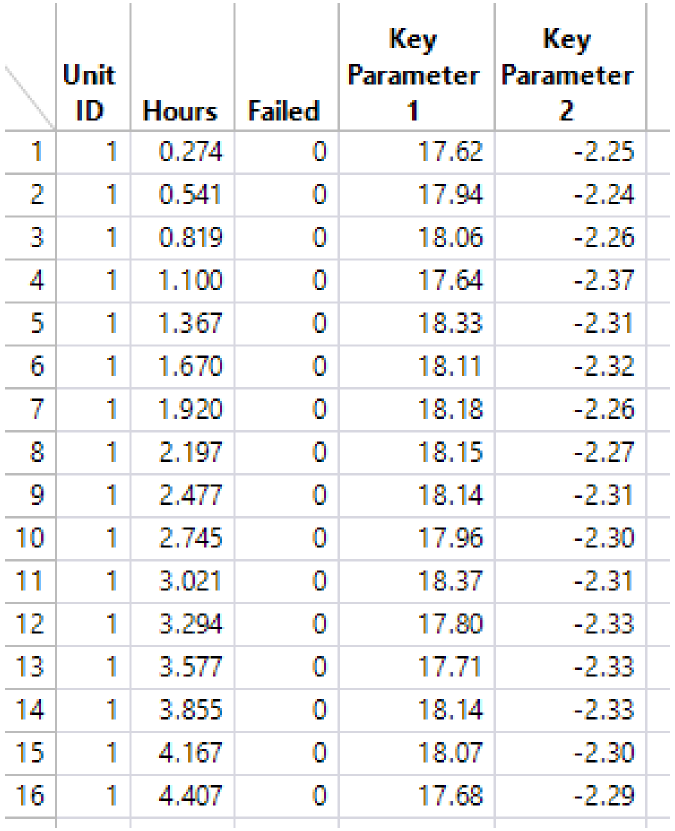
Without BCV



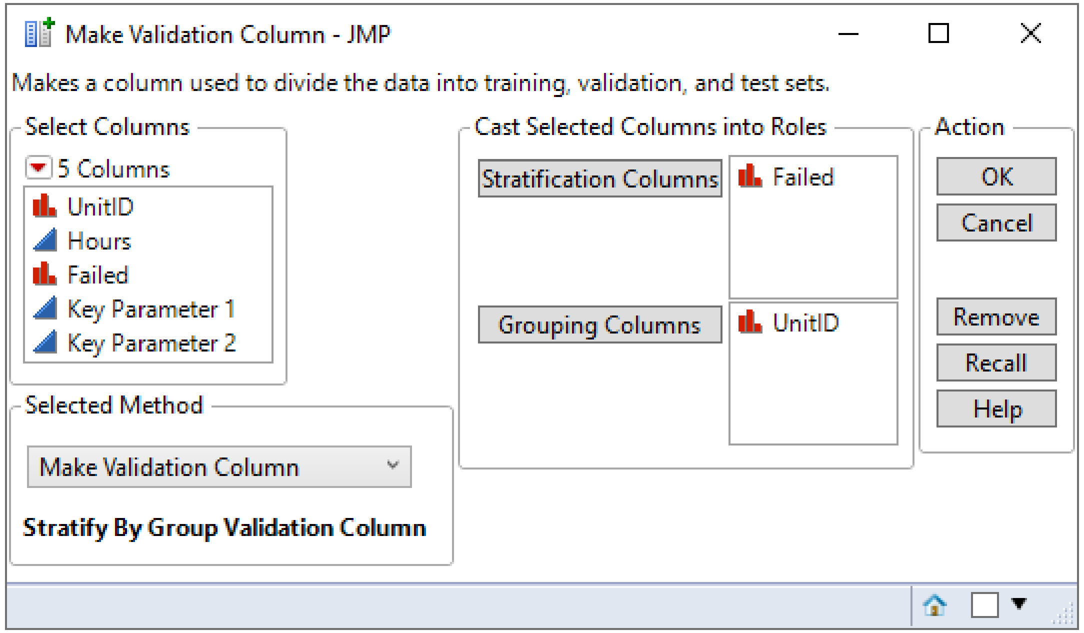
50
40
30
20
10
Key Parameter 1
0
20H New Burn-in?
40
60
96H End of Test
Hours


50
40
30
20
10
Key Parameter 1
0
20H New Burn-in?
40
60
96H End of Test
Hours


Key Parameter 1
50
40
30
20
10
0
20H New Burn-in?
40
60
96H End of Test
Hours



	Unit ID	Hours	Failed	Key Parameter 1	Key Parameter 2
1	1	0.274	0	17.62	-2.25
2	1	0.541	0	17.94	-2.24
3	1	0.819	0	18.06	-2.26
4	1	1.100	0	17.64	-2.37
5	1	1.367	0	18.33	-2.31
6	1	1.670	0	18.11	-2.32
7	1	1.920	0	18.18	-2.26
8	1	2.197	0	18.15	-2.27
9	1	2.477	0	18.14	-2.31
10	1	2.745	0	17.96	-2.30
11	1	3.021	0	18.37	-2.31
12	1	3.294	0	17.80	-2.33
13	1	3.577	0	17.71	-2.33
14	1	3.855	0	18.14	-2.33
15	1	4.167	0	18.07	-2.30
16	1	4.407	0	17.68	-2.29



Make Validation Column - JMP

Makes a column used to divide the data into training, validation, and test sets.

Select Columns

5 Columns

- UnitID
- Hours
- Failed
- Key Parameter 1
- Key Parameter 2

Cast Selected Columns into Roles

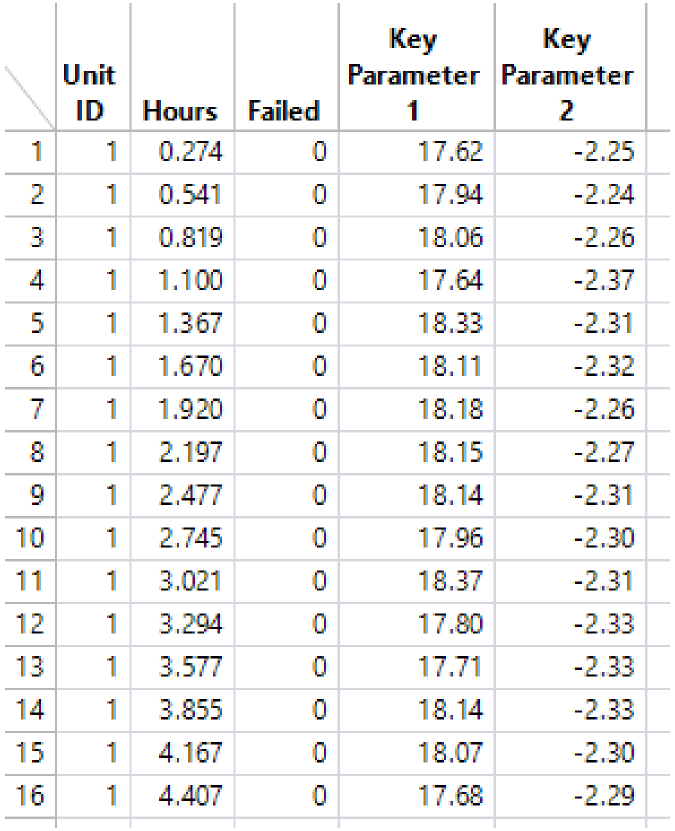
Stratification Columns: Failed

Grouping Columns: UnitID

Selected Method

Make Validation Column

Stratify By Group Validation Column



	Unit ID	Hours	Failed	Key Parameter 1	Key Parameter 2
1	1	0.274	0	17.62	-2.25
2	1	0.541	0	17.94	-2.24
3	1	0.819	0	18.06	-2.26
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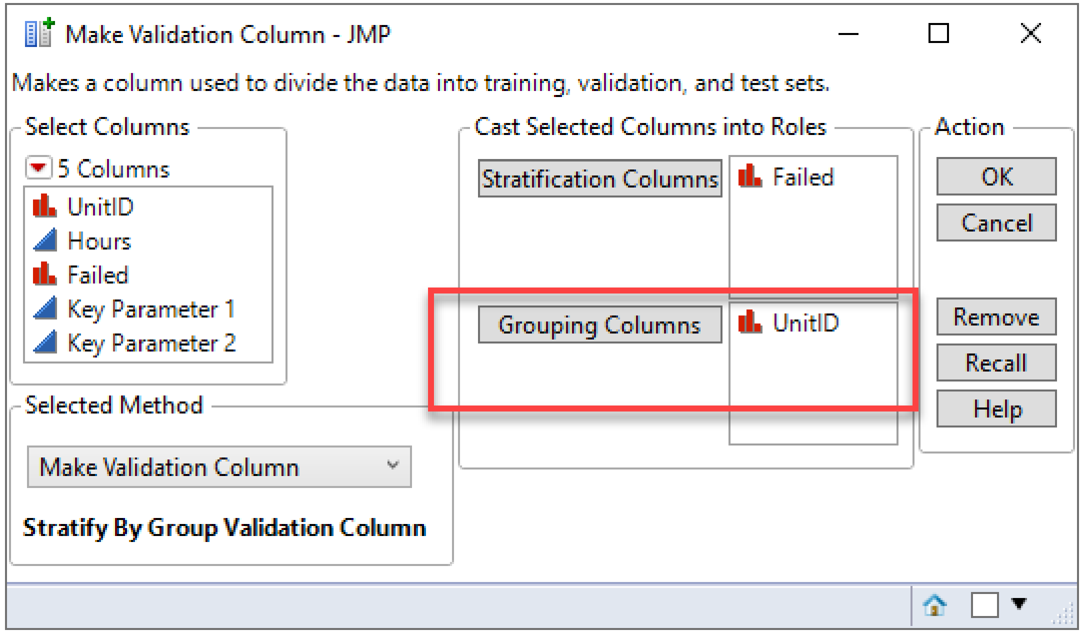
15	1	4.167	0	18.07	-2.30
16	1	4.407	0	17.68	-2.29



Make Validation Column - JMP

Makes a column used to divide the data into training, validation, and test sets.

Select Columns

5 Columns

- UnitID
- Hours
- Failed
- Key Parameter 1
- Key Parameter 2

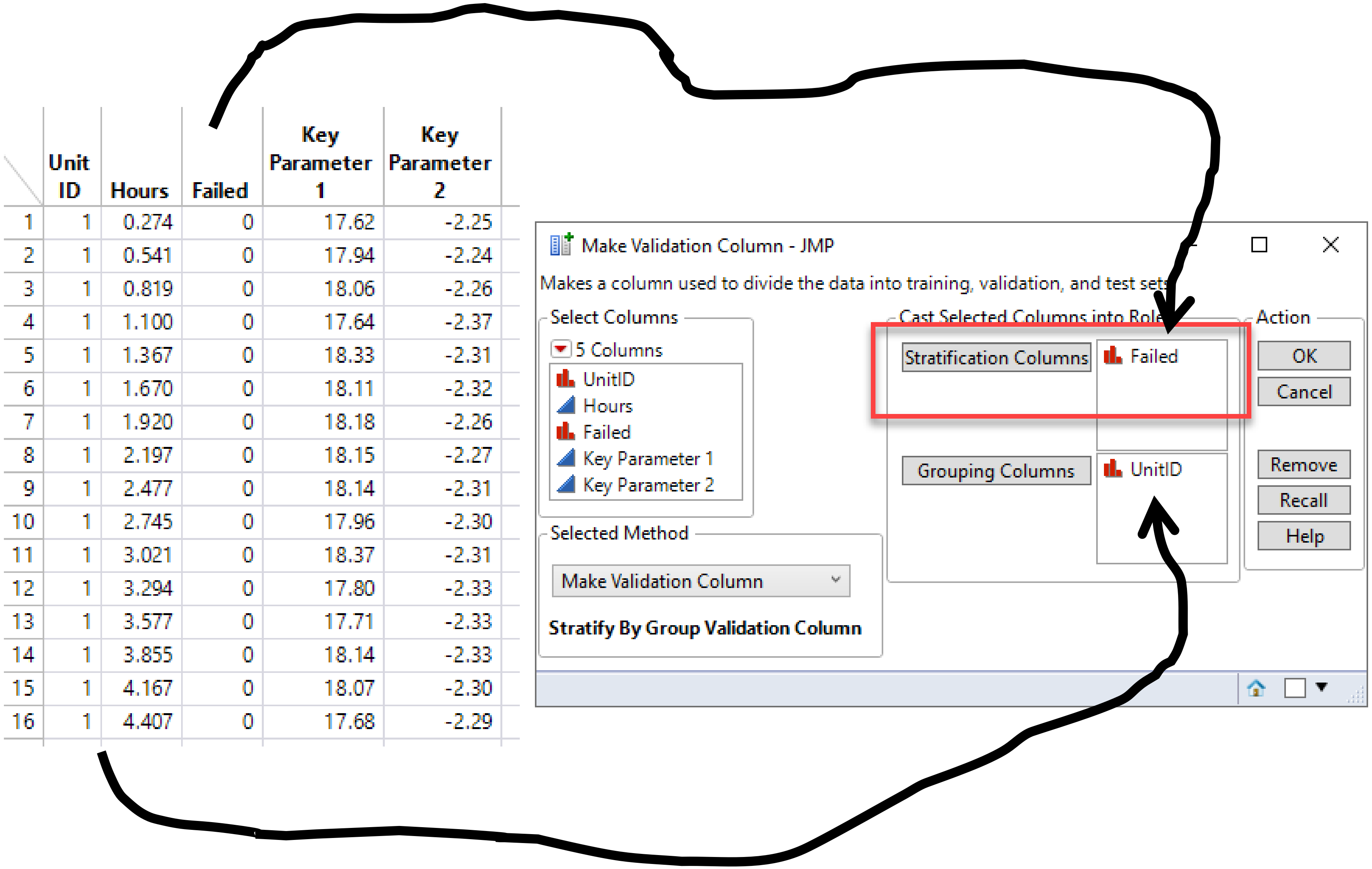
Selected Method: Make Validation Column

Stratify By Group Validation Column

Cast Selected Columns into Roles

Stratification Columns: Failed

Grouping Columns: UnitID



	Unit ID	Hours	Failed	Key Parameter 1	Key Parameter 2
1	1	0.274	0	17.62	-2.25
2	1	0.541	0	17.94	-2.24
3	1	0.819	0	18.06	-2.26
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Make Validation Column - JMP

Makes a column used to divide the data into training, validation, and test sets

Select Columns

5 Columns

- UnitID
- Hours
- Failed
- Key Parameter 1
- Key Parameter 2

Cast Selected Columns into Roles

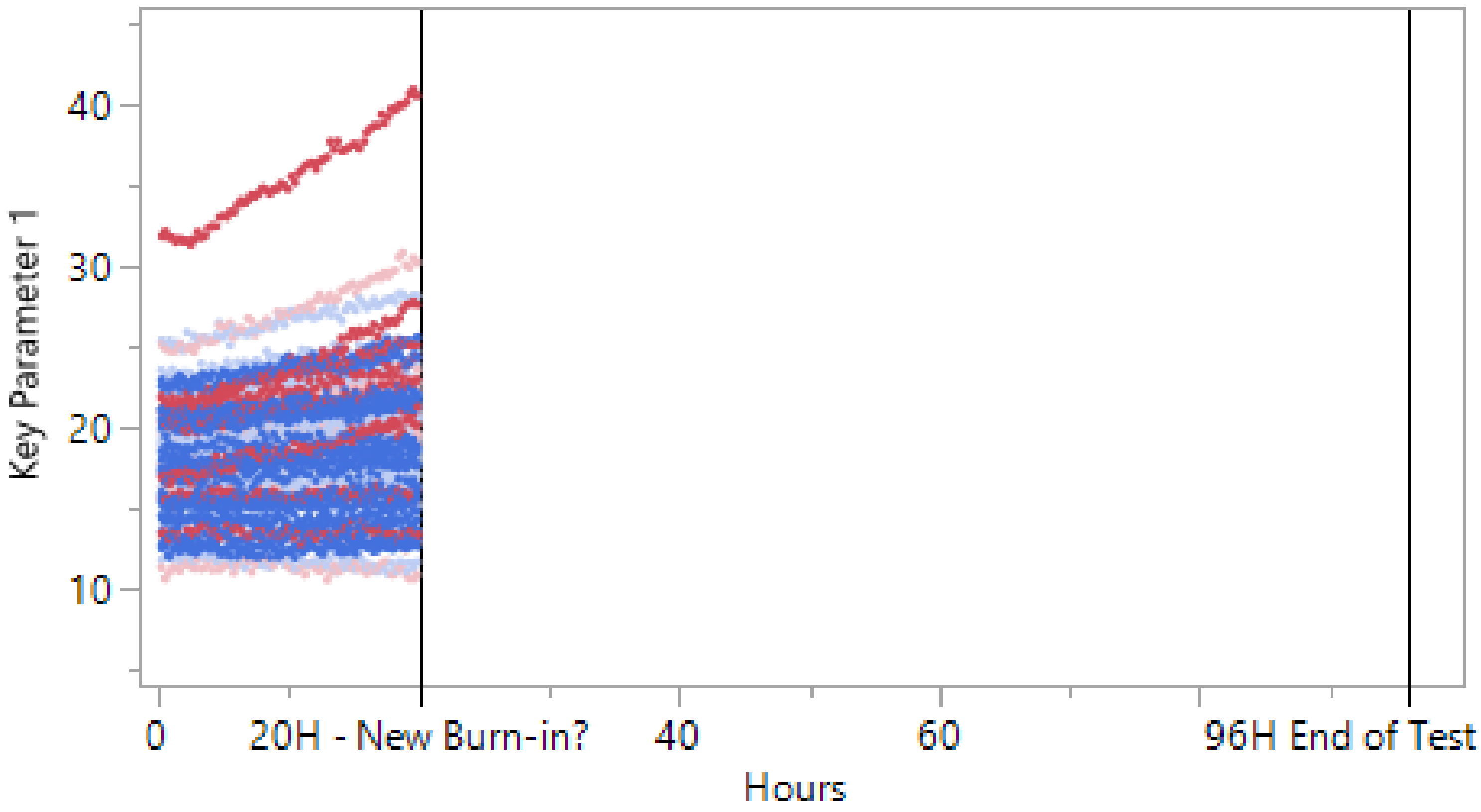
Stratification Columns: Failed

Grouping Columns: UnitID

Selected Method

Make Validation Column

Stratify By Group Validation Column


Key Parameter 1
40
30
20
10
0
20H - New Burn-in?
40
60
96H End of Test
Hours

With BCV

JMP FDE platform

- B-Splines
  - Piecewise polynomials with an underlying mean model and variance components on the spline coefficients.
  - These often work the best.
  - Try these first and customize the #Knots as needed.

- P-Splines
  - P is for “Penalized”. These tend to have lots of knots and are often slower to fit but similar in properties to B Splines.
  - Worth trying if B Spline do not fit well.

- Fourier Basis
  - Uses a sine/cosine expansion as the basis.
  - Good for periodic data (like vibration/sound signals).
  - Usually the spline models work better on other types of functional data.

P-Spline Functional Model Fit

$$y_i(t_{i,j}) = \sum_k \beta_k b(t_{i,j}) + \sum_k \gamma_{i,k} b(t_{i,j}) + \varepsilon_{i,j,k}$$

- $b(t_{i,j})$: basis functions, these form X, and Z.
- β_k: mean function coefficients, fixed effects
- $\gamma_{i,k} \sim N(0, \sigma_k^2)$: random effects
- $\varepsilon_{i,j,k} \sim N(0, \sigma_\varepsilon^2)$: errors

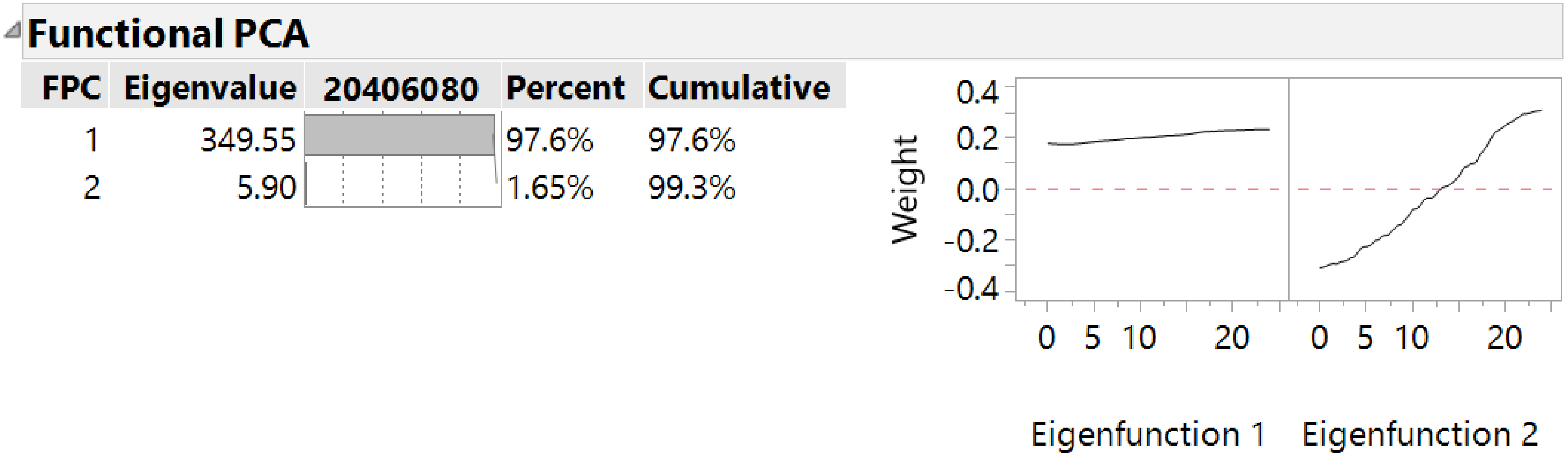
Scoring New P-Spline Functions

Training

$$\hat{\gamma}_{Val} = \left(Z_{Val}^{T} Z_{Val} + G(\hat{\sigma}_{Tr}^{2})\right)^{-1} Z_{Val}^{T}(y_{Val} - X_{Val}\hat{\beta}_{Tr})$$

- $\hat{\beta}_{Tr}, \hat{\sigma}_{Tr}^{2}$ estimated from training data
- Use BLUP formula to score $\hat{\gamma}_{Val}$ or any new units

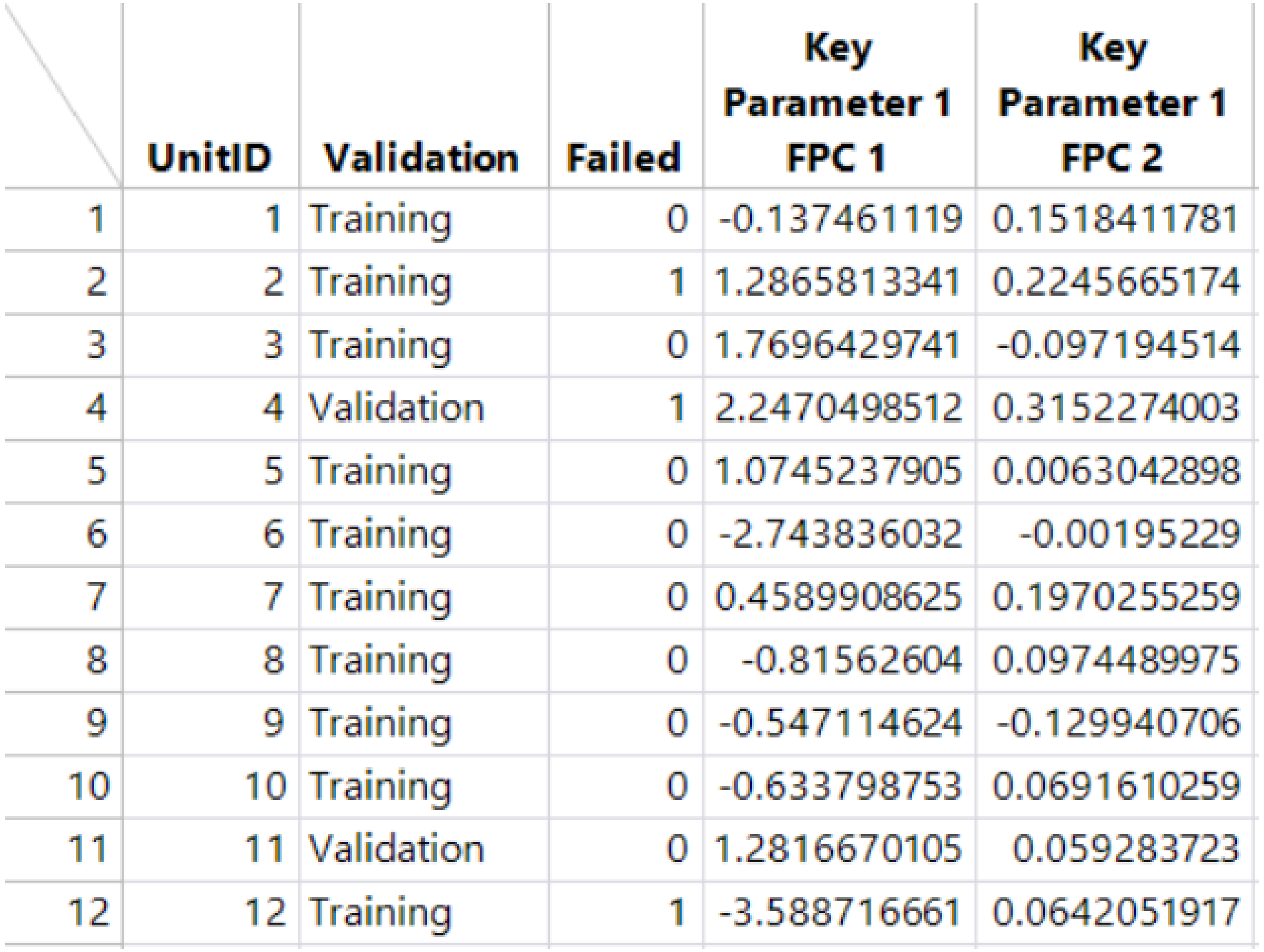
A best linear unbiased prediction (BLUP) estimate of realized values of a random variable are linear in the sense that they are linear functions of the data. They are unbiased in the sense that the average value of the estimate is equal to the average value of the quantity being estimated and best in the sense that they have minimum sum of squared error within the class of linear unbiased estimators. Estimators of random effects are called **predictors**, to distinguish them from **estimators** of fixed effects called estimators. BLUP estimates are solutions to mixed model equations and are usually different from generalized linear regression estimates used for fixed effects.



Functional PCA

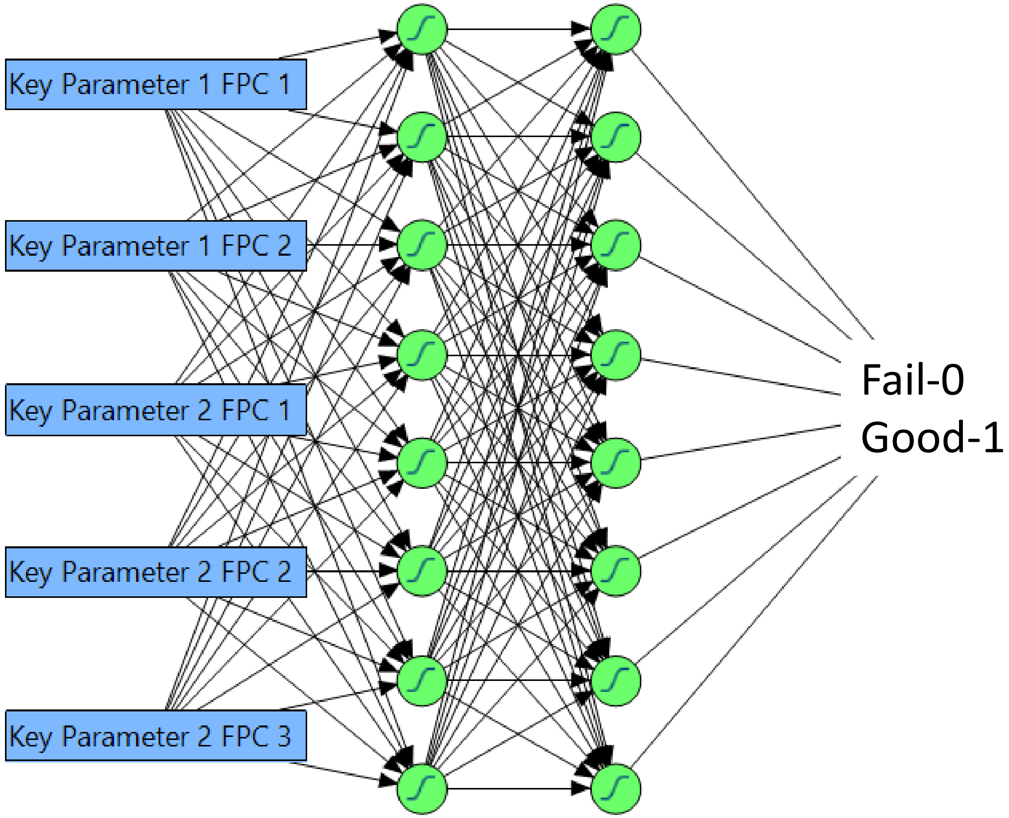
FPC	Eigenvalue	20406080	Percent	Cumulative
1	349.55		97.6%	97.6%
2	5.90		1.65%	99.3%

- Functional PCA amounts to rotating the P Spline basis functions
- FPC Scores are the induced rotation of the coefficients
- Operationally, FPC is derived from a weighted SVD of the training $\hat{\gamma}_{Tr}$



	UnitID	Validation	Failed	Key Parameter 1 FPC 1	Key Parameter 1 FPC 2
1	1	Training	0	-0.137461119	0.1518411781
2	2	Training	1	1.2865813341	0.2245665174
3	3	Training	0	1.7696429741	-0.097194514
4	4	Validation	1	2.2470498512	0.3152274003
5	5	Training	0	1.0745237905	0.0063042898
6	6	Training	0	-2.743836032	-0.00195229
7	7	Training	0	0.4589908625	0.1970255259
8	8	Training	0	-0.81562604	0.0974489975
9	9	Training	0	-0.547114624	-0.129940706
10	10	Training	0	-0.633798753	0.0691610259
11	11	Validation	0	1.2816670105	0.059283723
12	12	Training	1	-3.588716661	0.0642051917

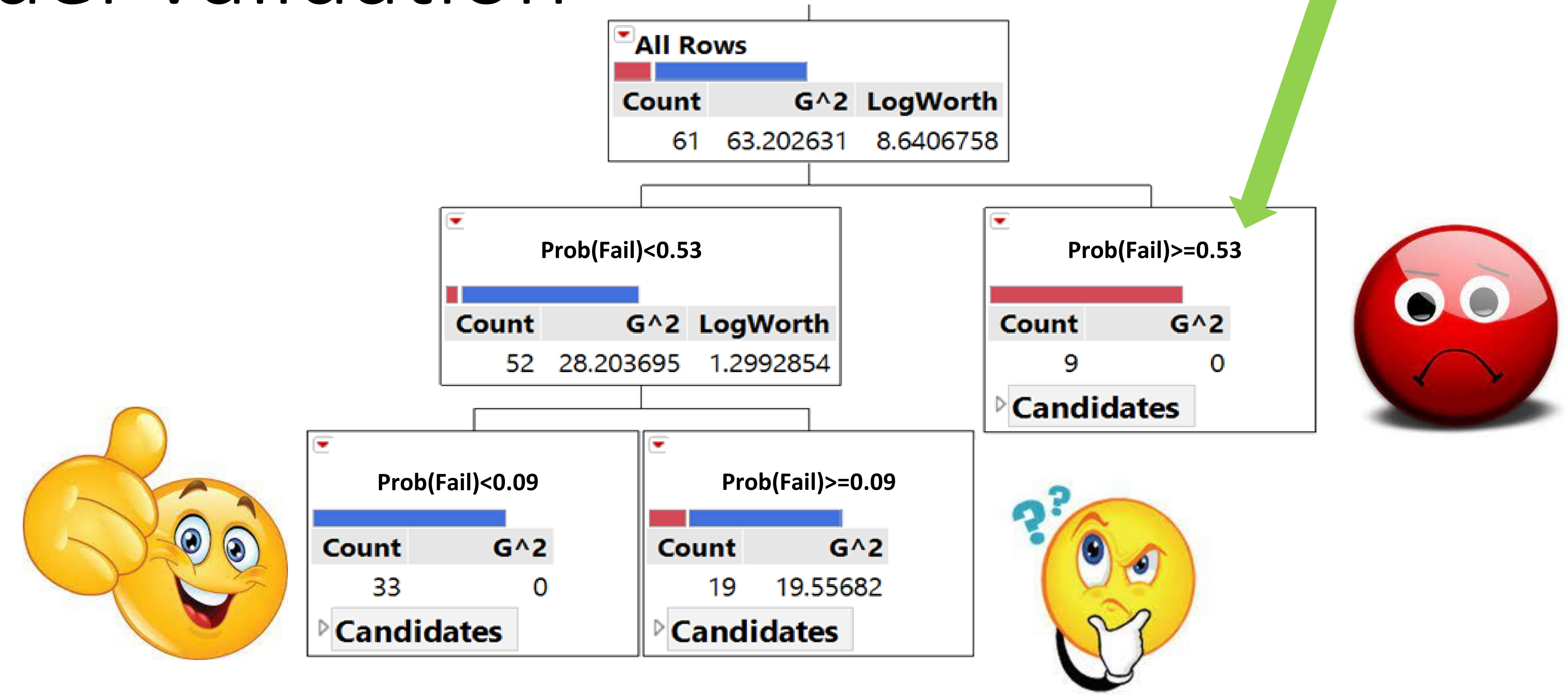
- Data summarized into one row per unit
- Model failure probability



A two-layer Neural Network predicting failure probability

Model Validation

Cut off



All Rows

Count	G^2	LogWorth
61	63.202631	8.6406758

Prob(Fail)<0.53

Count	G^2	LogWorth
52	28.203695	1.2992854

Prob(Fail)>=0.53

Count	G^2
9	0

Candidates

Prob(Fail)<0.09

Count	G^2
33	0

Candidates

Prob(Fail)>=0.09

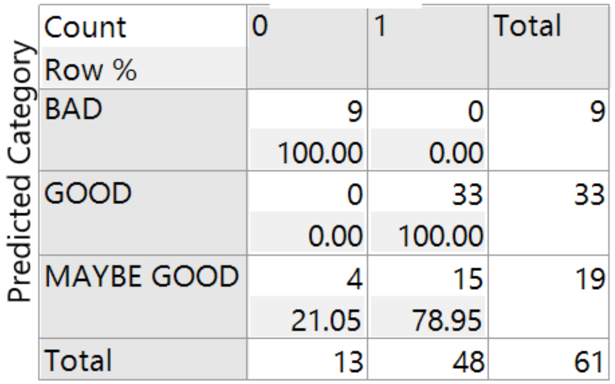
Count	G^2
19	19.55682

Candidates

A Regression Tree was used to predict “Censor” at 20 hours using the Neural probability as input, ***using only the Training subset of the data***.

Model Validation

Fail

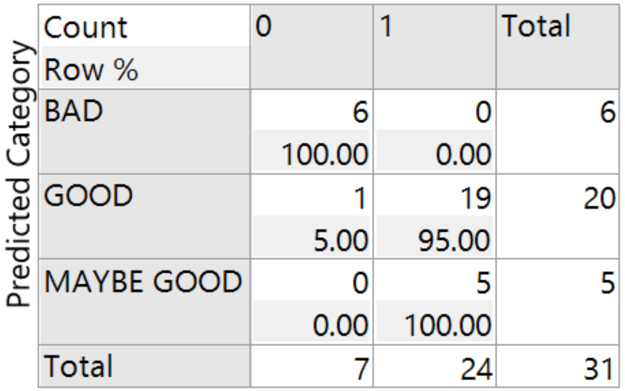


Predicted Category

Count Row %	0	1	Total
BAD	9 100.00	0 0.00	9
GOOD	0 0.00	33 100.00	33
MAYBE GOOD	4 21.05	15 78.95	19
Total	13	48	61

Training

Fail



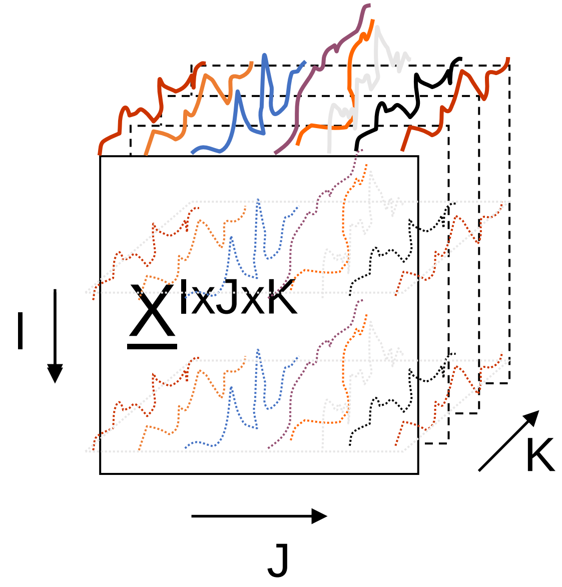
Predicted Category

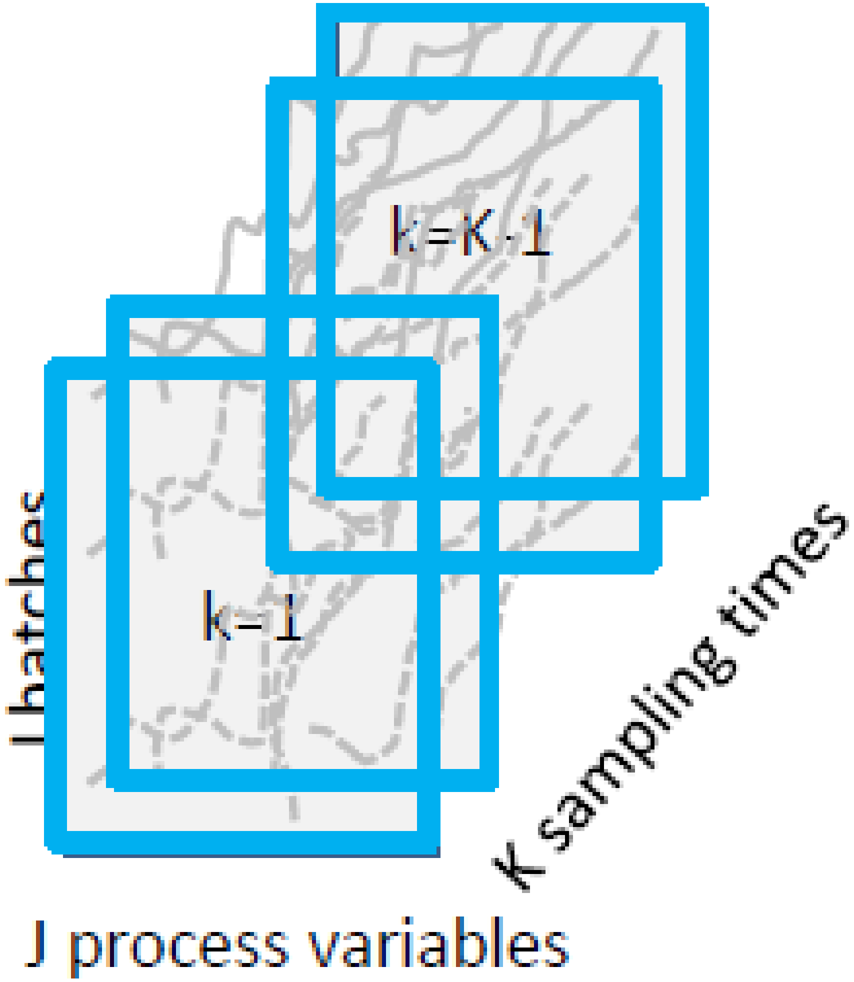
Count Row %	0	1	Total
BAD	6 100.00	0 0.00	6
GOOD	1 5.00	19 95.00	20
MAYBE GOOD	0 0.00	5 100.00	5
Total	7	24	31

Validation

Yes/No/Maybe decision rule was developed from the neural network prediction

Multiple Time Series





More challenges