

MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p, i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2, 12), (3, 12)]$.

The reduce function is addition. That is, $\text{reduce}(p, [i_1, i_2, \dots, i_k])$ is $(p, i_1 + i_2 + \dots + i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Output of map()-

$\text{map}(15) = [(3, 15), (5, 15)]$

$\text{map}(21) = [(3, 21), (7, 21)]$

$\text{map}(24) = [(2, 24), (3, 24)]$

$\text{map}(30) = [(2, 30), (3, 30), (5, 30)]$

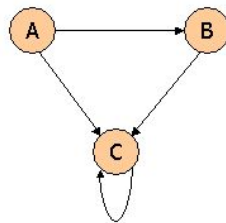
$\text{map}(49) = [(7, 49)]$

Output of reduce()-

$\text{reduce}(2, 54), \text{reduce}(3, 90), \text{reduce}(5, 45), \text{reduce}(7, 70)$

Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a , b , and c of the three pages A, B, and C, respectively.

$$\textcircled{2} \beta = 0.7, a+b+c=3$$

$$a = \beta(a) + (1-\beta)$$

$$= 0.7(a) + (1-0.7)$$

$$= 0.3$$

$$b = \beta(a/2) + (1-\beta)$$

$$= 0.7(0.3/2) + (1-0.7)$$

$$= 0.405$$

$$c = \beta(a/2 + b + c) + (1-\beta)$$

$$= 0.7(0.3/2 + 0.405 + c) + (1-0.7)$$

$$= 0.7 \times \frac{0.3}{2} + 0.7 \times 0.405 + 0.7 \times c + 0.3$$

$$= 0.105 + 0.2835 + 0.7c + 0.3$$

$$= 0.6885 + 0.7c$$

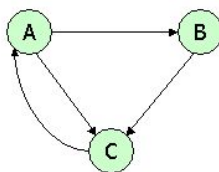
$$c - 0.7c = 0.6885$$

$$c(1-0.7) = 0.6885$$

$$c = 0.6885 / 0.3$$

$$c = 2.295$$

Question 3:



Suppose we compute PageRank with $\beta=0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively.

③ $\beta = 0.85$

$$a = \beta \times c + (1-\beta)/3$$

$$= 0.85c + (1-0.85)/3$$

$$= 0.85c + 0.05$$

$$b = \beta \times a/2 + (1-\beta)/3$$

$$= 0.85 \times 0.5 \times a + 0.05$$

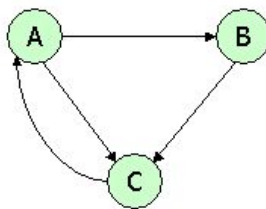
$$= 0.425a + 0.05$$

$$c = \beta \times (a/2 + b) + (1-\beta)/3$$

$$= 0.85(0.5a + b) + 0.05$$

$$= 0.425a + 0.85b + 0.05$$

Question 4:



Assuming no "taxation," compute the PageRanks a , b , and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

④ 0th iteration - $a=1, b=1, c=1$

1st iteration - $a=1, b=1/2, c=1/2+1=3/2$

2nd iteration - $a=c=3/2, b=a/2=1/2, c=a/2+b=1$

3rd iteration - $a=1, b=3/4, c=5/4$

4th iteration - $a=5/4, b=1/2, c=5/4$

5th iteration - $a=5/4, b=5/8, c=9/8$