1.4.9.d

EE24BTECH11025 - GEEDI HARSHA

Question: Which of the following differential equations has $y = C_1e^x + C_2e^{-x}$ as the general solution?

$$\text{(D)}\ \frac{d^2y}{dx^2} - 1 = 0 \tag{0.1}$$

Solution:

First, Plot the graph of $y = C_1e^x + C_2e^{-x}$

Consider,

$$\frac{d^2y}{dx^2} = 1 \tag{0.2}$$

On integrating both sides w.r.to x

$$\int \frac{d^2y}{dx^2}dx = \int 1dx \tag{0.3}$$

$$\frac{dy}{dx} = x + C_1 \tag{0.4}$$

Again integrating on both sides w.r.to x

$$\int \frac{dy}{dx}dx = \int (x + C_1)\,dx \tag{0.5}$$

$$y = \frac{x^2}{2} + C_1x + C_2 \tag{0.6}$$

Where,C_1, C_2 are integration constants. Now lets plot $y = \frac{x^2}{2} + C_1x + C_2$ in the same graph.

Solution using Laplace Transform:
Laplace Transform definition

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st}f(t)\,dt \tag{0.7}$$

Properties of Laplace transform

$$\mathcal{L}(y'') = s^2\mathcal{L}(y) - sy(0) - y'(0) \tag{0.8}$$

$$\mathcal{L}(1) = \frac{1}{s} \tag{0.9}$$

$$\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) = x^2u(x) \tag{0.10}$$

$$\mathcal{L}(cf(t)) = c\mathcal{L}(f(t)) \tag{0.11}$$

Applying the properties to the given equation

$$y'' = 1 \tag{0.12}$$

$$\mathcal{L}(y'') = \mathcal{L}(1) \tag{0.13}$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) = \frac{1}{s} \tag{0.14}$$

Substituting the initial conditions gives

$$s^2\mathcal{L}(y) = \frac{1}{s} \tag{0.15}$$

$$\mathcal{L}(y) = \frac{1}{s^3} \tag{0.16}$$

$$y = \mathcal{L}^{-1}\left(\frac{1}{s^3}\right) \tag{0.17}$$

$$y = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) \tag{0.18}$$

$$y = \frac{1}{2}x^2u(x) \tag{0.19}$$

The theoretical solution is

$$f(x) = \frac{x^2}{2}u(x) \tag{0.20}$$

Using difference equation to find solution for this differential equation This method is used to find the approximate solution for the differential equation by plotting the discret points of the function.

From defination of second order differentiation,

$$\frac{d^2y}{dx^2} = \lim_{h\to 0}\frac{y(x_i+h) - 2y(x_i) + y(x_i-h)}{h^2} \tag{0.21}$$

$$\frac{d^2y}{dx^2} \approx \frac{y(x_i+h) - 2y(x_i) + y(x_i-h)}{h^2} \tag{0.22}$$

From (0.2)

$$\frac{d^2y}{dx^2} = 1 \tag{0.23}$$

$$1 = \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \tag{0.24}$$

$$h^2 = y_{n+1} - 2y_n + y_{n-1} \tag{0.25}$$

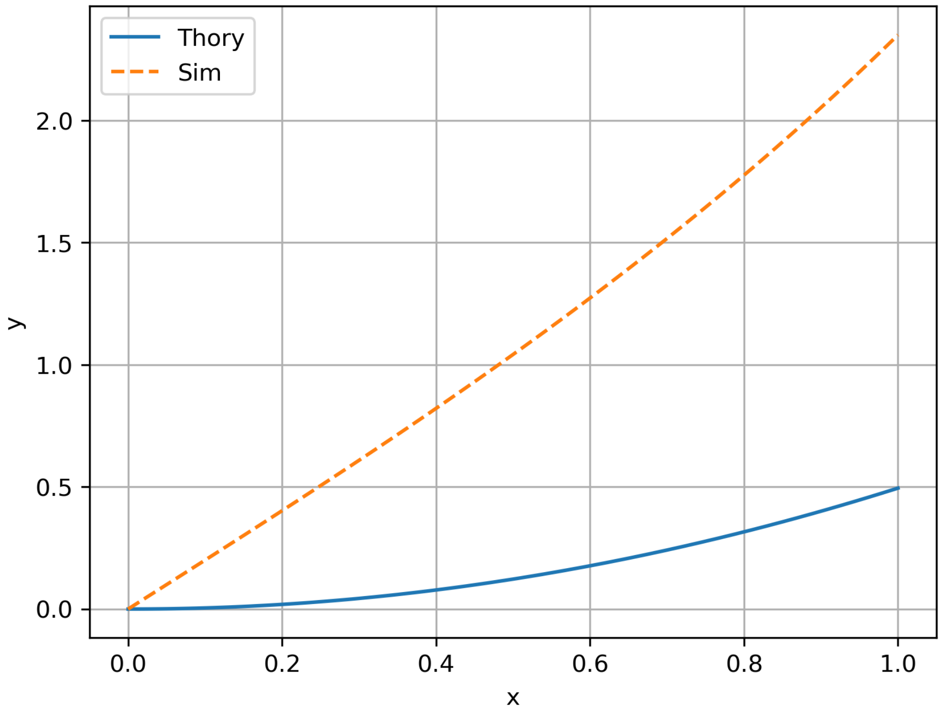
$$y_{n+1} = 2y_n - y_{n-1} + h^2 \tag{0.26}$$

Let (x_0, y_0), be points on the function

$$x_{n+1} = x_n + h \tag{0.27}$$

$$y_{n+1} = 2y_n - y_{n-1} + h^2 \tag{0.28}$$

By taking initial conditions as $y_0 = 0$ at $x_0 = 0$, $h = 0.1$ and running this algorithm,We can get the approximatet plot of the solution of the given differential equation.
This is done by plotting all the (x.y) points at different n,and joining all these points.



As both the plots are not same, Option D is not the solution.