

8.Circles

EE24BTECH11025 - GEEDI HARSHA VARDHAN

- 1) Let a circle be given by $2x(x - a) + y(2y - b) = 0, (a \neq 0, b \neq 0)$. Find the condition on a and b if two chords, each bisected by the x-axis, can be drawn to the circle from $(a, \frac{b}{2})$.
(1992- 6 Marks)
- 2) Consider a family of circles passing through two fixed points A(3, 7) and B(6, 5). Show that chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point.
(1993- 5 Marks)
- 3) Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y = -4$ and $x^2 + y^2 - 12x - 8y = -36$ touch each other. Also find equations common tangents touching the circles in the distinct points.
(1993- 5 Marks)
- 4) Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2})$ to the circle $2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$.
(1996- 5 Marks)
- 5) A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and midpoint of the line segment DC is d , prove that the area of the circle is $\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta \cos (\beta - \alpha)}$.
(1996- 5 Marks)
- 6) Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C . (A rational point is a point both of whose coordinates are rational numbers)
(1997- 5 Marks)
- 7) C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1 . From a point P on C_2 , tangents PA and PB are drawn to C_1 . Prove that the centroid of the triangle PAB lies on C_1 . (1998- 8 Marks)
- 8) Let T_1, T_2 be two tangents drawn from $(2, 0)$ onto the circle $C: x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time.
(1999- 10 Marks)
- 9) Let $2x^2 + y^2 - 3xy = 0$ be the equation of pair of tangents drawn from the origin O to a circle of radius 3 with the centre in the first quadrant. If A is one of the points of contact, find the length of OA .
(2001- 5 Marks)
- 10) Let C_1 and C_2 be two circles with C_2 lying inside C_1 . A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of centre of C .
(2001- 5 Marks)

- 11) For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point $P(6, 8)$ to the circle and the chord of contact is maximum. (2003- 2 Marks)
- 12) Find the equation of circle touching the line $2x + 3y + 1 = 0$ at $(1, -1)$ and cutting orthogonally the circle having line segment joining $(0, 3)$ and $(-2, -1)$ as diameter. (2004- 4 Marks)
- 13) Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. (2005- 2 Marks)

MATCH THE FOLLOWING

- 1) Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y . Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following condition:

- (i) Centre of C_3 is collinear with the centres of C_1 and C_2
- (ii) C_1 and C_2 both lie inside C_3 , and
- (iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W , and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8ay$.

There are some expressions given in the List-1 whose values are given in List -2 below

Column 1

- (A) $2h + k$
- (B) $\frac{\text{Length of } ZW}{\text{length of } XY}$
- (C) $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$

Column 2

- (p) 6
- (q) $\sqrt{6}$
- (r) $\frac{5}{4}$
- (s) $\frac{21}{5}$
- (t) $2\sqrt{6}$
- (u) $\frac{10}{3}$

Which of the following is the only CORRECT combination ?

- (a) (I),(U)
 - (b) (I),(S)
 - (c) (II),(T)
 - (d) (II),(Q)
- 2) Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y . Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:
- (i) Centre of C_3 is collinear with centres of C_1 and C_2
 - (ii) C_1 and C_2 both lie inside C_3 , and
 - (iii) C_3 touches C_1 at M and C_2 at N
- Let the line through X and Y intersect C_3 at Z and W , and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8ay$. There are some expressions given in the List- 1 whose values are given in List - 2 below

Column1

- (A) $2h + k$
 (B) $\frac{\text{Length of ZW}}{\text{length of XY}}$
 (C) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$

Column2

- (p) 6
 (q) $\sqrt{6}$
 (r) $\frac{5}{4}$
 (s) $\frac{21}{5}$
 (t) $2\sqrt{6}$
 (u) $\frac{10}{3}$

Which of the following is the only INCORRECT combination?

- (a) (IV),(S)
 (b) (I),(P)
 (c) (III),(R)
 (d) (IV),(U)