8.Circles

EE24BTECH11025 - GEEDI HARSHA VARDHAN

1) Let a circle be given by $2x(x-a)+y(2y-b)=0$,$(a \neq 0, b \neq 0)$.Find the condition on a and b if two chords, each bisected by the x-axis,can be drawn to the circle from $\left(a, \frac{b}{2}\right)$.
(1992- 6 Marks)

2) Consider a family of circles passing through two fixed points A$(3,7)$ and B$(6,5)$. Show that chords in which the circle $x^2+y^2-4x-6y-3=0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point.
(1993- 5 Marks)

3) Find the coordinates of the point at which the circles $x^2+y^2-4x-2y=-4$ and $x^2+y^2-12x-8y=-36$ touch each other. Also find equations common tangents touching the circles in the distinct points.
(1993- 5 Marks)

4) Find the intervals of values of a for which the line $y+x=0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2+2y^2-\left(1+\sqrt{2}a\right)x-\left(1-\sqrt{2}a\right)y=0$.
(1996- 5 Marks)

5) A circle passes through three points A,B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and midpoint of the line segment DC is d, prove that the area of the circle is $\frac{\pi d^2 \cos^2\alpha}{\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta\cos(\beta-\alpha)}$
(1996- 5 Marks)

6) Let C be any circle with centre $\left(0, \sqrt{2}\right)$. Prove that at the most two rational points can be there on C.(A rational point is a point both of whose coordinates are rational numbers)
(1997- 5 Marks)

7) C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. (1998- 8 Marks)

8) Let T_1, T_2 be two tangents drawn from $(2,0)$ onto the circle C:$x^2+y^2=1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. (1999- 10 Marks)
â[1]

9) Let $2x^2+y^2-3xy=0$ be the equation of pair of tangents drawn from the origin O to a circle of radius 3 with the centre in the first quadrant. If A is one of the points of contact, find the length of OA. (2001- 5 Marks)

10) Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of centre of C. (2001- 5 Marks)

11) For the circle $x^2+y^2=r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P$(6,8)$ to the circle and the chord of contact is maximum.
(2003- 2 Marks)

12) Find the equation of circle touching the line $2x+3y+1=0$ at $(1,-1)$ and cutting orthogonally the circle having line segment joining $(0,3)$ and $(-2,-1)$ as diameter.
(2004- 4 Marks)

13) Circles with radii 3,4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. (2005- 2 Marks)

MATCH THE FOLLOWING

1) Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following condition:

(i) Centre of C_3 is collinear with the centres of C_1 and C_2
(ii) C_1 and $C2$ both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List-1 whose values are given in List -2 below

Column 1	**Column 2**
(A) $2h + k$	(p) 6
(B) $\frac{\text{Length of ZW}}{\text{length of XY}}$	(q) $\sqrt{6}$
(C) $\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(r) $\frac{5}{4}$
	(s) $\frac{21}{5}$
	(t) $2\sqrt{6}$
	(u) $\frac{10}{3}$

Which of the following is the only CORRECT combination?

(a) (I),(U)
(b) (I),(S)
(c) (II),(T)
(d) (II),(Q)

2) Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x-3)^2 + (y-4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions:

(i) Centre of C_3 is collinear with centres of C_1 and C_2
(ii) C_1 and C_2 both lie inside C_3, and
(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$. There are some expressions given in the List- 1 whose values are given in List - 2 below

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	(s) $\frac{21}{5}$
	(t) $2\sqrt{6}$
	(u) $\frac{10}{3}$

Which of the following is the only INCORRECT combination?

(a) (IV),(S)
(b) (I),(P)
(c) (III),(R)
(d) (IV),(U)