Indian Institute of Information Technology, Nagpur
Department of Basic Sciences
Calculus for Data Science (MAL 105)
Sessional -II Examination
B.Tech. 1st Semester – CSE- Data Science, CSE-AIML

Duration: 1 hour
Date: December 19th, 2023 (Thursday)

Max. Marks: 15
Time: 09:00 am - 10:00 am

Important Instructions:
(i) This is a closed book, closed notes examination.
(ii) This question paper comprises total 6 questions printed on one page. **Attempt any five questions.** Maximum marks for a particular question are indicated in the brackets [ ] on the extreme right of the corresponding question.
(iii) Use of non-programming calculators are permitted.
(iv) Please indicate the important steps of reasoning/calculations carefully.
(v) Assume suitable data wherever necessary. Please mention the assumptions made, if any.

Q. 1: Show that the whole length of the curve $8a^2y^2 = x^2(a^2 - x^2)$ is $\pi a\sqrt{2}$.

[CO2] **[3 Marks]**

Q. 2: Find the area [CO2] **[3 Marks]**

(a) of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$.

(b) of the portion bounded by the curve and its asymptotes.

Q. 3 Find the surface of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ or $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis. [CO2] **[3 Marks]**

Q. 4: Find the volume of the solid formed by the revolution of the curve $r = a(1+\cos\theta)$ about the initial line. [CO 2] **[3 Marks]**

Q. 5: If $\theta = t^n e^{-r^2/4t}$, what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$ [CO 3] **[3 Marks]**

Q. 6: If $u = \sin^{-1}\left(\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right)^{1/2}$, prove that [CO 3] **[3 Marks]**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}\left(13 + \tan^2 u\right)$$