prove $Fib(n)$ closest to $\phi^n/\sqrt{5}$, $\phi = (1+\sqrt{5})/2$:

$$\begin{aligned}
Fib(0) &= (\phi^0 - \psi^0)/\sqrt{5} = 0 \\
Fib(1) &= (\phi^1 - \psi^1)/\sqrt{5} = 1 \\
Fib(n) &= Fib(n-1) + Fib(n-2) \\
&= (\phi^{n-1} - \psi^{n-1})/\sqrt{5} - (\phi^{n-2} - \psi^{n-2})/\sqrt{5} \\
&= (\phi^{n-1} + \phi^{n-2} - (\psi^{n-1} + \psi^{n-2}))/\sqrt{5} \\
&= (\phi^{n-2}(\phi+1) - \psi^{n-2}(\psi+1))/\sqrt{5} \\
&\because \phi^2 = \phi + 1, \psi^2 = \psi + 1 \\
Fib(n) &= (\phi^n - \psi^n)/\sqrt{5} \\
&\because |\psi| < 1 \therefore |\frac{\psi^n}{\sqrt{5}}| < 1 \\
&\therefore Fib(n) \ \mathit{closest\ to}\ \phi^n/\sqrt{5}
\end{aligned}$$