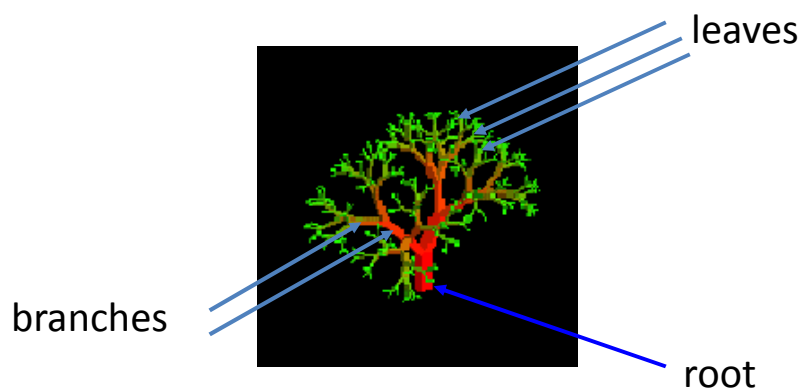


Trees and Binary Trees

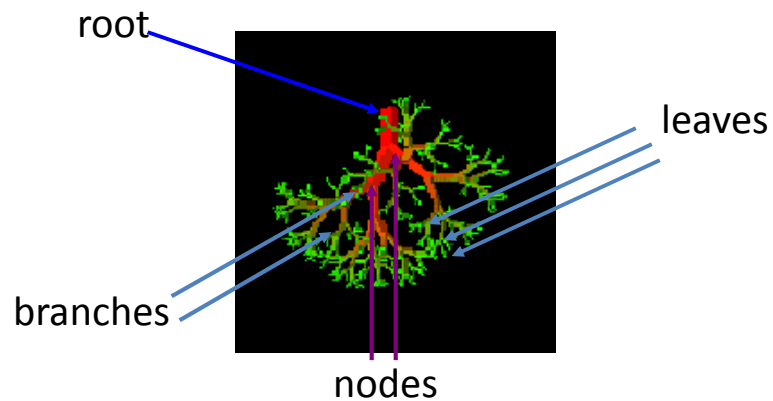
1

Nature View of a Tree



2

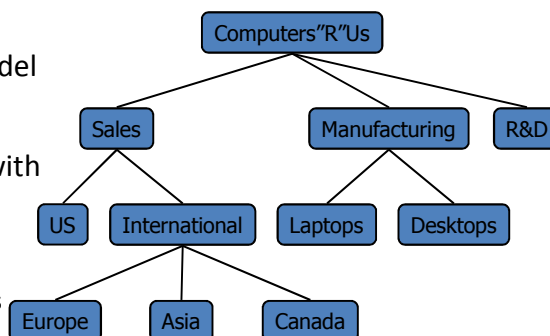
Computer Scientist's View



3

What is a Tree

- A tree is a finite nonempty set of elements.
- It is an abstract model of a hierarchical structure.
- Consists of nodes with a parent-child relation.
- Applications:
 - Organization charts
 - File systems
 - Programming environments

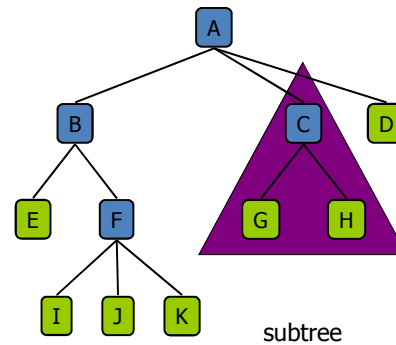


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Tree Terminology

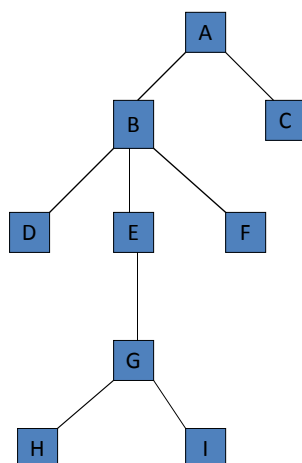
- **Root:** node without parent (A)
- **Siblings:** nodes share the same parent
- **Internal node:** node with at least one child (A, B, C, F)
- **External node (leaf):** node without children (E, I, J, K, G, H, D)
- **Ancestors** of a node: parent, grandparent, grand-grandparent, etc.
- **Descendant** of a node: child, grandchild, grand-grandchild, etc.
- **Depth** of a node: number of ancestors
- **Height** of a tree: maximum depth of any node (3)
- **Degree** of a node: the number of its children
- **Degree** of a tree: the maximum degree of its node.

- **Subtree:** tree consisting of a node and its descendants



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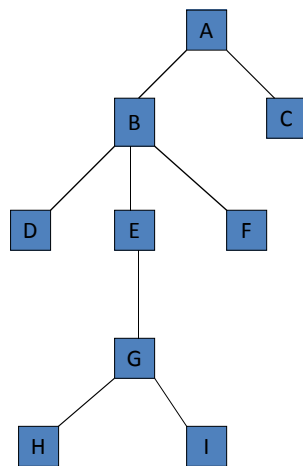
Tree Properties



Property	Value
Number of nodes	
Height	
Root Node	
Leaves	
Interior nodes	
Ancestors of H	
Descendants of B	
Siblings of E	
Right subtree of A	
Degree of this tree	

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Tree Properties



Property	Value
Number of nodes :	9
Height :	4
Root Node :	A
Leaves :	5
Interior nodes :	4
Ancestors of H :	G, E, B, A
Descendants of B :	E, G, H, I
Siblings of E :	D, F
Right subtree of A :	C
Degree of this tree :	3

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Intuitive Representation of Tree Node

● List Representation

- (A (B (E (K, L), F), C (G), D (H (M), I, J)))
- The root comes first, followed by a list of links to sub-trees

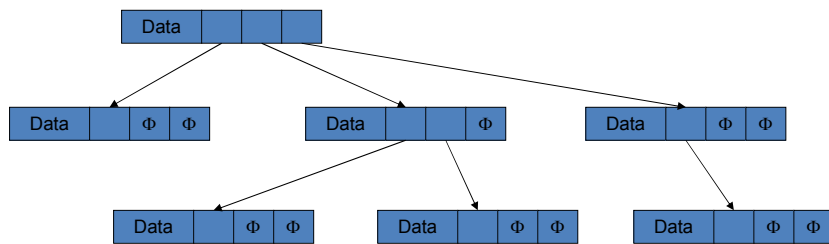
How many link fields are needed in such a representation?



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Trees

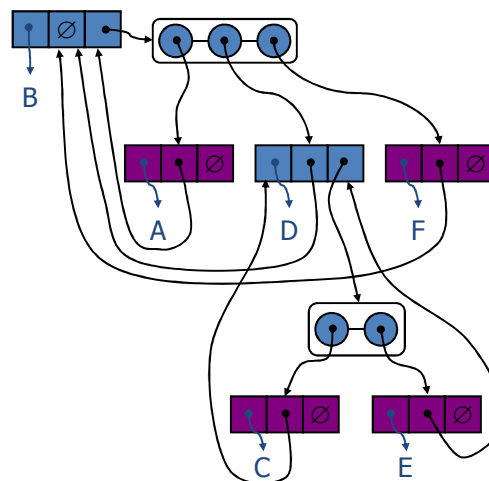
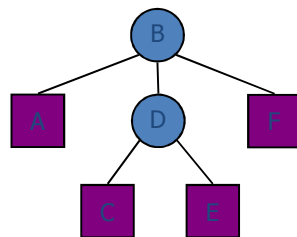
- Every tree node:
 - object – useful information
 - children – pointers to its children



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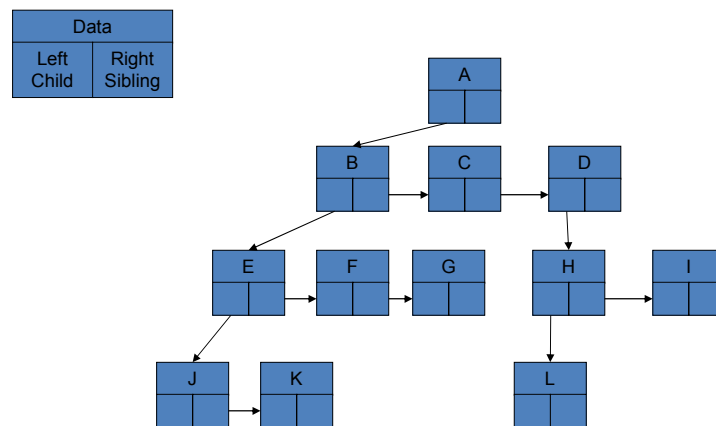
A Tree Representation

- A node is represented by an object storing
 - Element
 - Parent node
 - Sequence of children nodes



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Left Child, Right Sibling Representation



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TREE ADT: Operations

- Operations
 - Traversal
 - Insertion
 - Deletion
 - Search
 - Copy
 -

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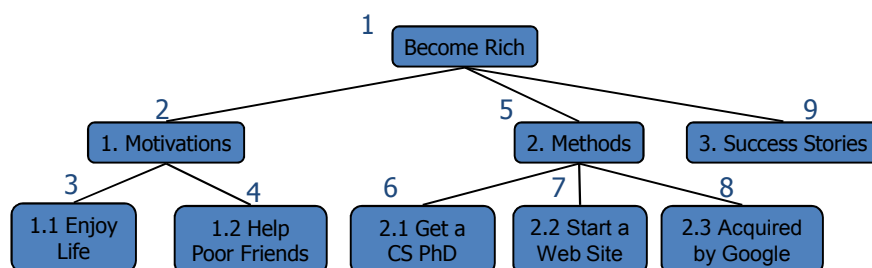
Tree Traversal

- Two main methods:
 - Preorder
 - Postorder
- Recursive definition
- Preorder:
 - visit the root
 - traverse in preorder the children (subtrees)
- Postorder
 - traverse in postorder the children (subtrees)
 - visit the root

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Preorder Traversal

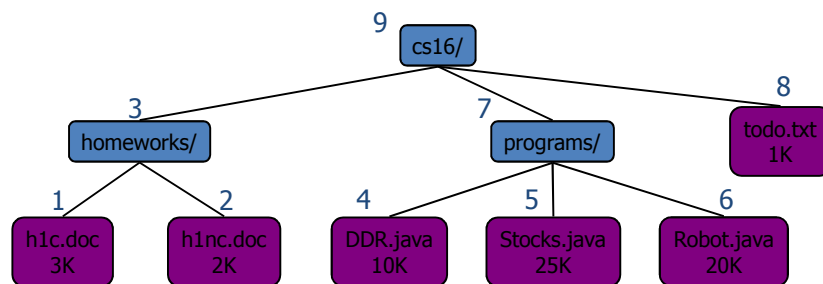
- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document



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Postorder Traversal

- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories



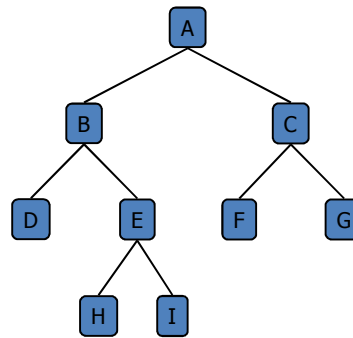
15

Binary Trees

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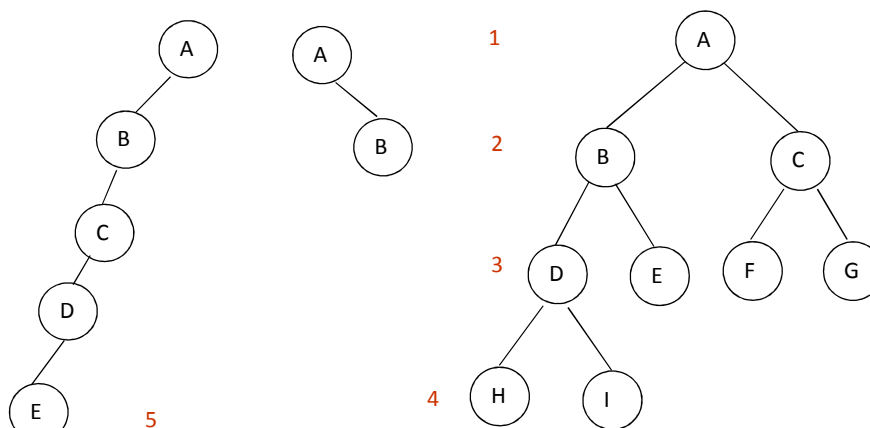
Binary Tree

- A binary tree is a tree with the following properties:
 - Each internal node has at most two children (degree of two)
 - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
 - a tree consisting of a single node, OR
 - a tree whose root has an ordered pair of children, each of which is a binary tree
- Applications:
 - arithmetic expressions
 - decision processes
 - searching



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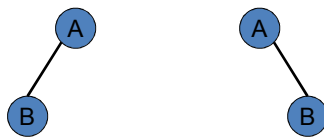
Examples of the Binary Tree



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Difference Between A Tree and A Binary Tree

- The subtrees of a binary tree are ordered; those of a tree are not ordered.

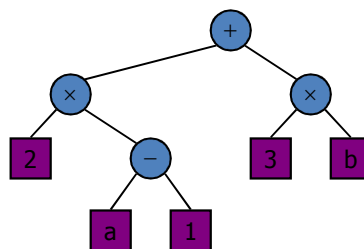


- Are different when viewed as binary trees.
- Are the same when viewed as trees.

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Arithmetic Expression Tree

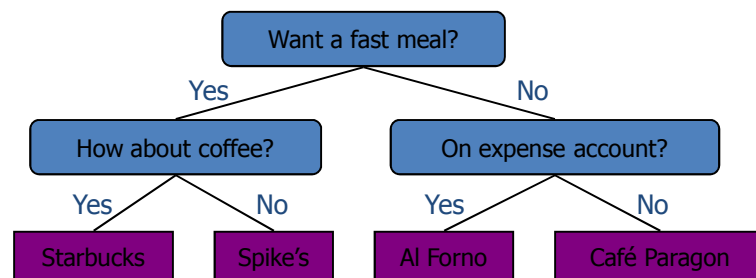
- Binary tree associated with an arithmetic expression
 - internal nodes: operators
 - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



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Decision Tree

- Binary tree associated with a decision process
 - internal nodes: questions with yes/no answer
 - external nodes: decisions
- Example: dining decision



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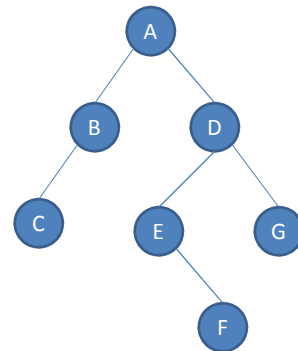
Binary Tree Traversal

- Traversal
 - Each node in a tree is processed exactly once in a systematic manner
- Three main ways of tree traversal
 - Preorder
 - Inorder
 - Postorder

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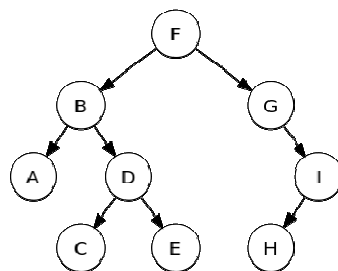
Binary Tree Traversal...

- The easiest way to define each order is by using recursion
- Preorder traversal (Rlr)
 - Process the root node
 - Traverse the left subtree in preorder
 - Traverse the right subtree in preorder
- Preorder traversal:
ABCDEFGG



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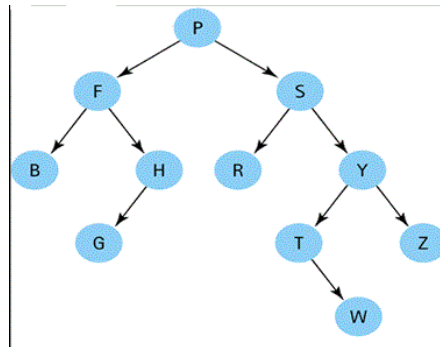
Assignment: Preorder Traversal



Preorder: FBADCEGIH

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Assignment: Preorder Traversal

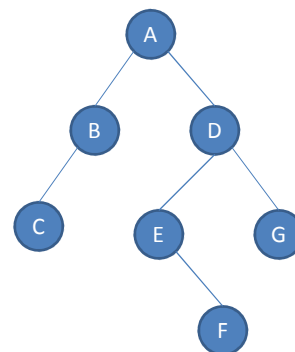


Preorder: PFBHGSRYTWZ

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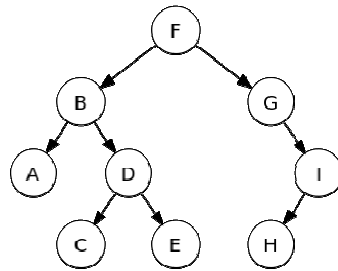
Binary Tree Traversal...

- Inorder traversal (IRr)
 - Traverse the left subtree in Inorder
 - Process the root node
 - Traverse the right subtree in Inorder
- Inorder traversal: **CBAEFDG**



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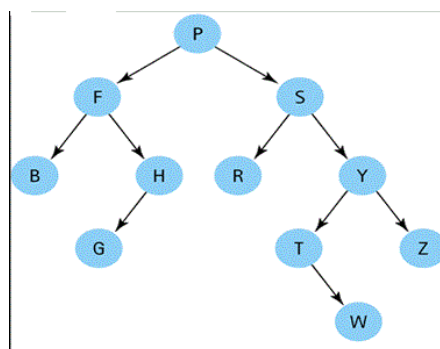
Assignment: Inorder Traversal



Inorder: ABCDEFGHI

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Assignment: Inorder Traversal

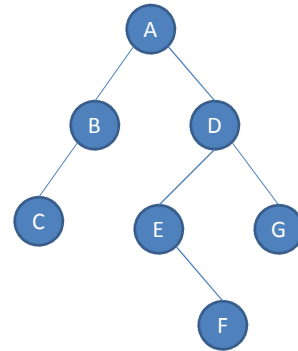


Inorder: BFGHPRSTWYZ

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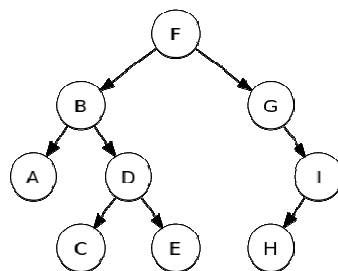
Binary Tree Traversal...

- Postorder traversal (lrR)
 - Traverse the left subtree in postorder
 - Traverse the right subtree in postorder
 - Process the root node
- Postorder traversal:
CBFEGDA



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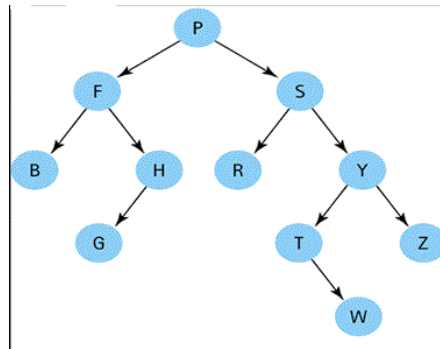
Assignment: Postorder Traversal



Postorder: ACEDBHIGF

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Assignment: Postorder Traversal

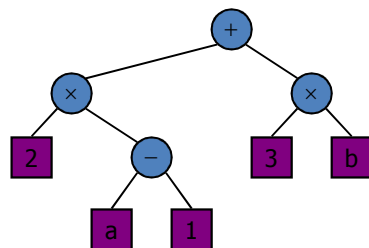


Postorder: BGHFRWTZYSP

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Print Arithmetic Expressions

- Specialization of an inorder traversal
 - print operand or operator when visiting node
 - print "(" before traversing left subtree
 - print ")" after traversing right subtree



$((2 \times (a - 1)) + (3 \times b))$

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Tutorial: Algorithm to Print Arithmetic Expression using Binary Tree

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