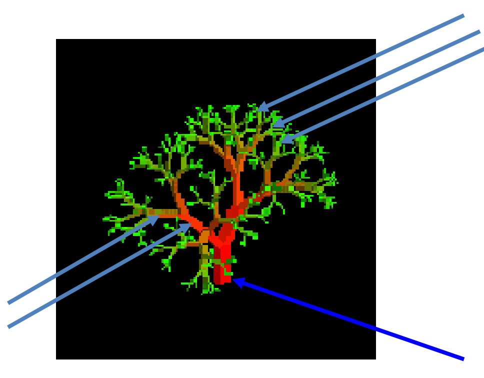
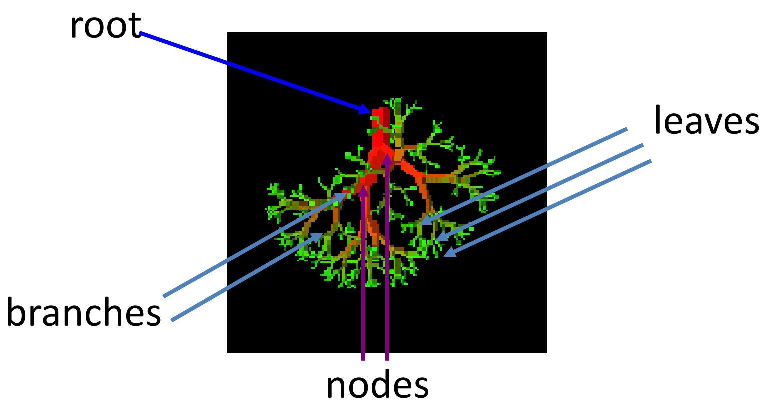
Trees and
Binary Trees

Nature View of a Tree

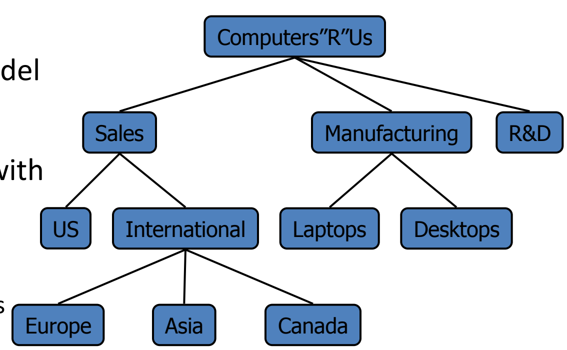


Computer Scientist’s View



What is a Tree

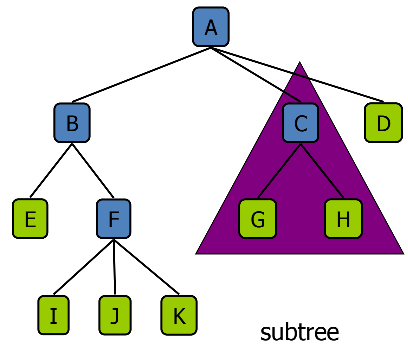
- A tree is a finite nonempty set of elements.
- It is an abstract model of a hierarchical structure.
- Consists of nodes with a parent-child relation.
- Applications:
  - Organization charts
  - File systems
  - Programming environments



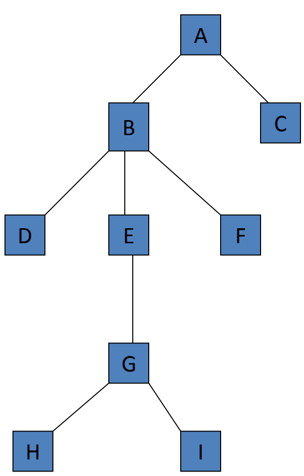
Tree Terminology

- **Root**: node without parent (A)
- **Siblings**: nodes share the same parent
- **Internal node**: node with at least one child (A, B, C, F)
- **External node** (leaf): node without children (E, I, J, K, G, H, D)
- **Ancestors** of a node: parent, grandparent, grand-grandparent, etc.
- **Descendant** of a node: child, grandchild, grand-grandchild, etc.
- **Depth** of a node: number of ancestors
- **Height** of a tree: maximum depth of any node (3)
- **Degree** of a node: the number of its children
- **Degree** of a tree: the maximum degree of its node.

- **Subtree**: tree consisting of a node and its descendants

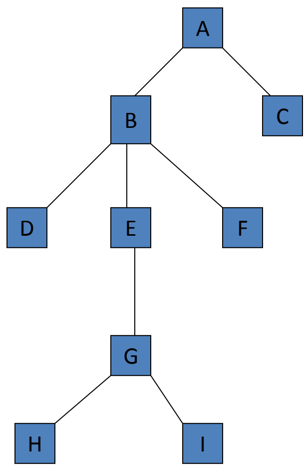


Tree Properties



Property	Value
Number of nodes	
Height	
Root Node	
Leaves	
Interior nodes	
Ancestors of H	
Descendants of B	
Siblings of E	
Right subtree of A	
Degree of this tree	

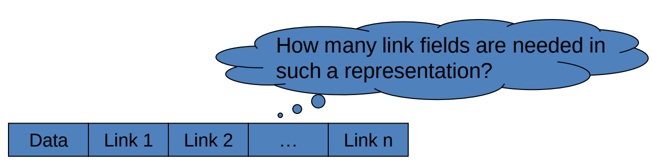
Tree Properties



Property	Value
Number of nodes :	9
Height :	4
Root Node :	A
Leaves :	5
Interior nodes :	4
Ancestors of H :	G, E,B, A
Descendants of B :	E, G, H, I
Siblings of E :	D, F
Right subtree of A :	C
Degree of this tree :	3

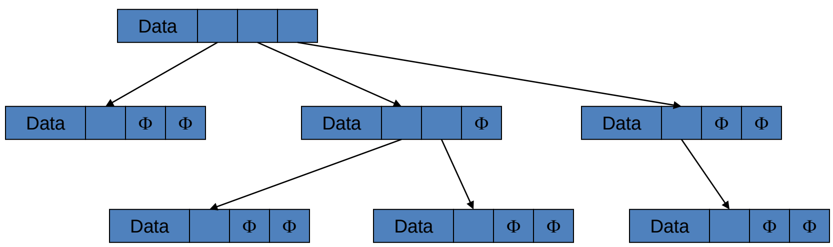
Intuitive Representation of Tree Node

- List Representation
  - (A (B (E (K, L), F), C (G), D (H (M), I, J)))
  - The root comes first, followed by a list of links to sub-trees



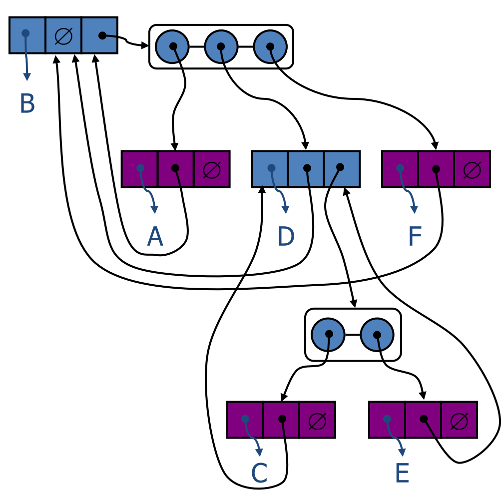
Trees

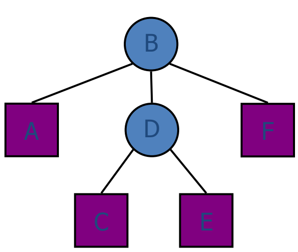
- Every tree node:
  - object – useful information
  - children – pointers to its children



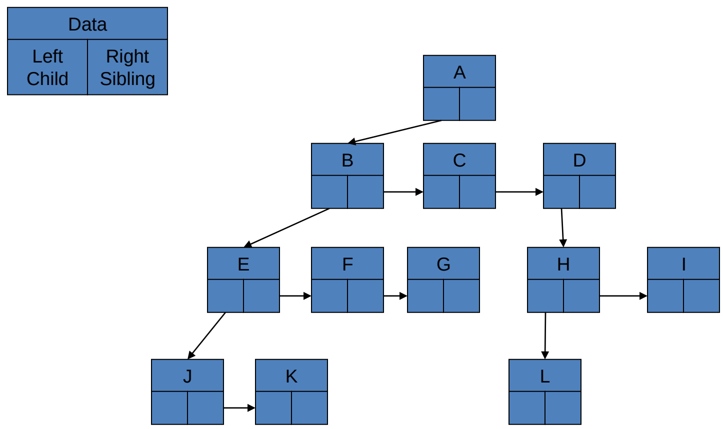
A Tree Representation

- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes





Left Child, Right Sibling Representation



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TREE ADT: Operations

- Operations
  - Traversal
  - Insertion
  - Deletion
  - Search
  - Copy
  - ….

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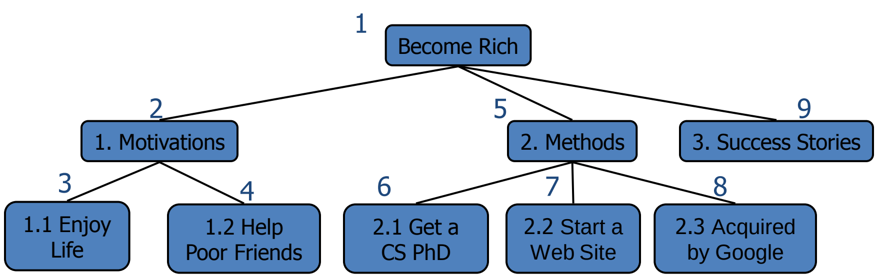
Tree Traversal

- Two main methods:
  - Preorder
  - Postorder
- Recursive definition
- Preorder:
  - visit the root
  - traverse in preorder the children (subtrees)
- Postorder
  - traverse in postorder the children (subtrees)
  - visit the root

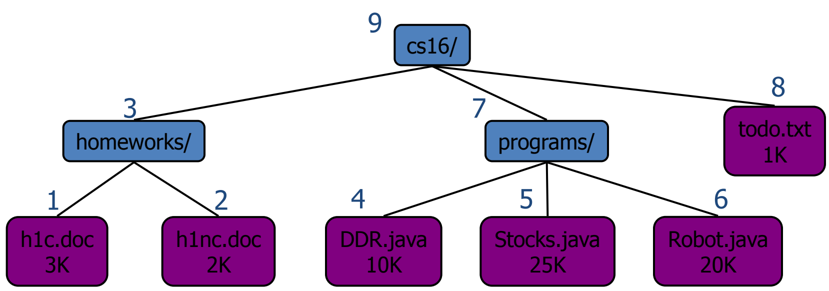
Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document



Postorder Traversal

- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories



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Binary Trees

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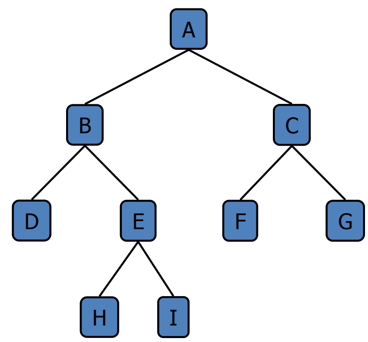
Binary Tree

- A binary tree is a tree with the following properties:
  - Each internal node has at most two children (degree of two)
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, OR
  - a tree whose root has an ordered pair of children, each of which is a binary tree

- Applications:
  - arithmetic expressions
  - decision processes
  - searching



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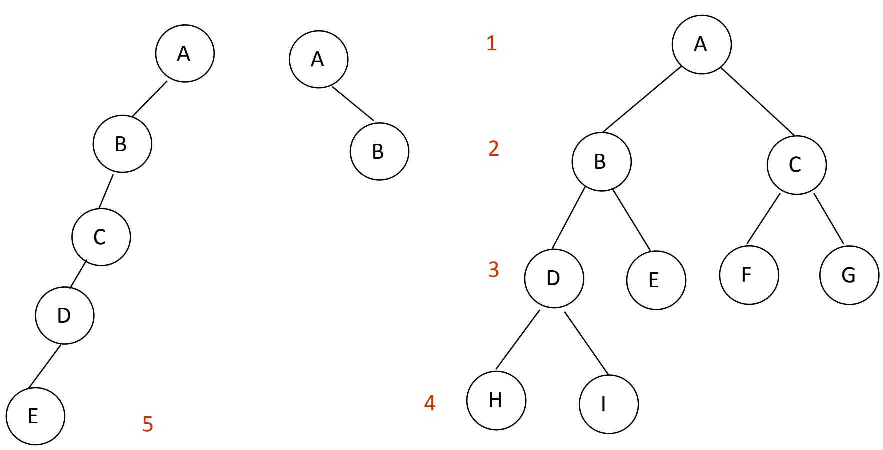
Examples of the Binary Tree



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Difference Between A Tree and A Binary Tree

- The subtrees of a binary tree are ordered; those of a tree are not ordered.

- Are different when viewed as binary trees.
- Are the same when viewed as trees.

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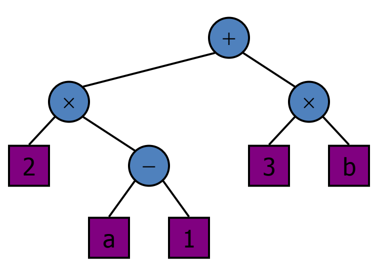
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



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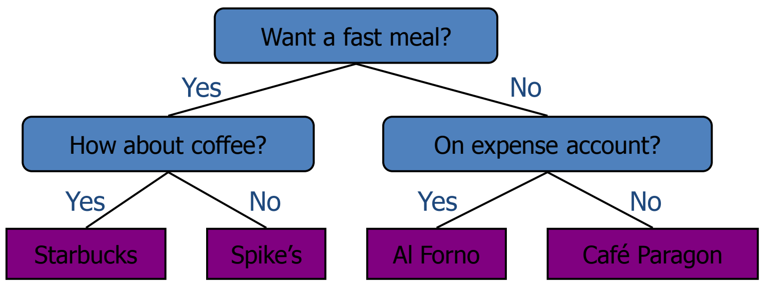
Decision Tree

- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision

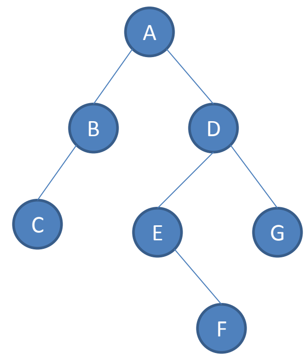


Binary Tree Traversal

- Traversal
  - Each node in a tree is processed exactly once in a systematic manner
- Three main ways of tree traversal
  - Preorder
  - Inorder
  - Postorder

Binary Tree Traversal…

- The easiest way to define each order is by using recursion
- Preorder traversal (Rlr)
  - Process the root node
  - Traverse the left subtree in preorder
  - Traverse the right subtree in preorder
- Preorder traversal: ABCDEFG



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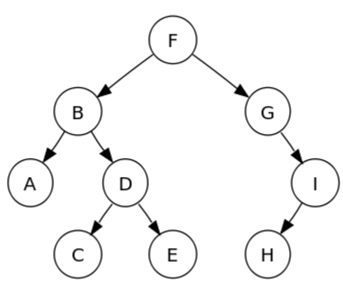
Assignment: Preorder Traversal



Preorder: FBADCEGIH

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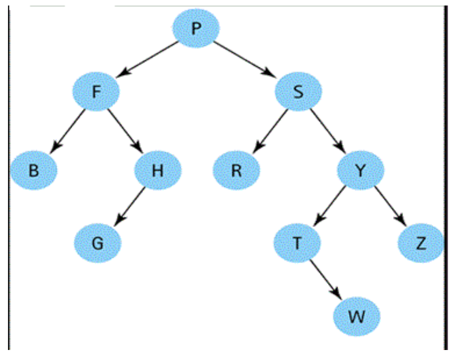
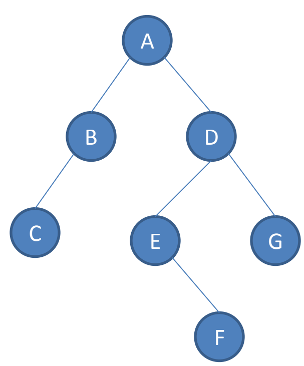
Assignment: Preorder Traversal



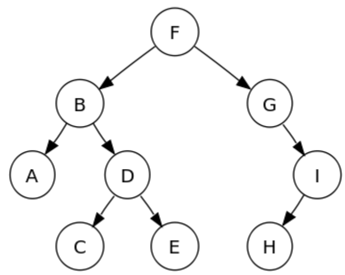
Preorder: PFBHGSRYTWZ

Binary Tree Traversal…

- Inorder traversal (lRr)
  - Traverse the left subtree in Inorder
  - Process the root node
  - Traverse the right subtree in Inorder
- Inorder traversal: CBAEFDG

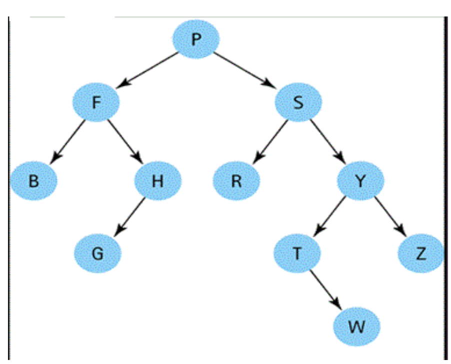


Assignment: Inorder Traversal



Inorder: ABCDEFGHI

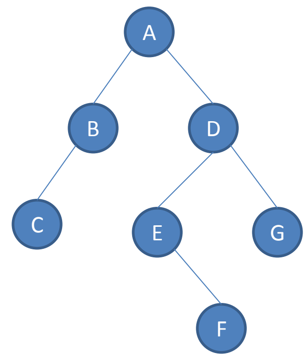
Assignment: Inorder Traversal



Inorder: BFGHPRSTWYZ

Binary Tree Traversal…

- Postorder traversal (lrR)
  - Traverse the left subtree in postorder
  - Traverse the right subtree in postorder
  - Process the root node
- Postorder traversal: CBFEGDA



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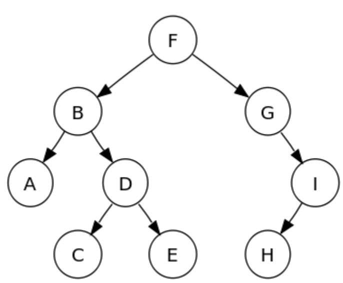
Assignment: Postorder Traversal



Postorder: ACEDBHIGF

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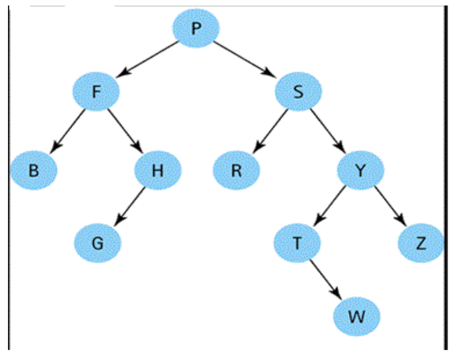
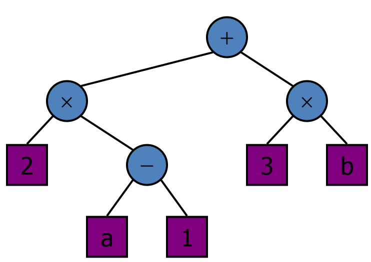
Assignment: Postorder Traversal



Postorder: BGHFRWTZYSP

Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print operand or operator when visiting node
  - print “(“ before traversing left subtree
  - print “)“ after traversing right subtree



$((2 \times (a - 1)) + (3 \times b))$

Tutorial: Algorithm to Print Arithmetic Expression using Binary Tree

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Linked storage representation for binary tree

LPTR	DATA	RPTR

- DATA
  - Information associated with node
- LPTR
  - Address of location of left subtree
- RPTR
  - Address of location of right subtree

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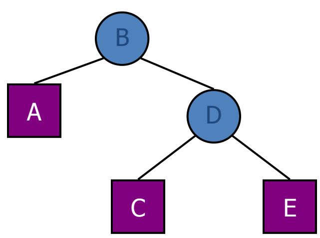
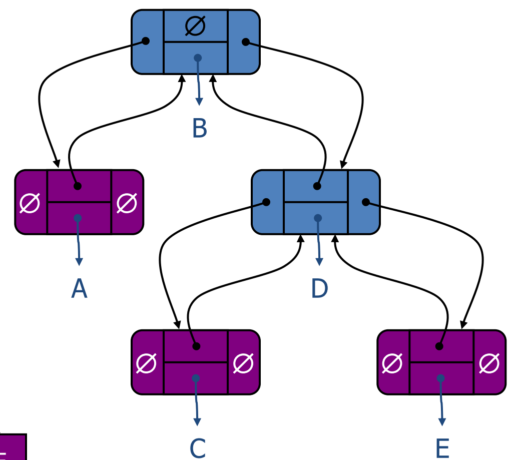
Linked storage representation for binary tree: option 2

- A node is represented by an object storing
  - Data
  - Parent
  - Left child
  - Right child





Binary tree traversal: algorithms

- Recursive
- Iterative

Preorder traversal: iterative algorithm

Algorithm PREORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Preorder traversal of a tree
[Initialize]
if T=NULL **then**
    return false {Empty Tree}
else
    TOP← -1
    push(S, T)
[Process each stacked branch address]
Repeat while TOP>=0
[Get stored address and branch left]
    P← pop(S)
    Repeat while P != NULL
        write DATA(P)
        if RPTR(P) != NULL **then**
            push(S,RPTR(P)) *[Store address of nonempty right subtree]*
            P← LPTR(P) *[branch left]*
[Finished]
return

Preorder traversal: recursive algorithm

Algorithm RPREORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Preorder traversal of a tree
[Process the root node]
if T != NULL **then**
    write DATA(T)
else
    return false {Empty Tree}
[Process the left subtree]
if LPTR(T) != NULL **then**
RPREORDER(LPTR(T))
[Process the right subtree]
if RPTR(T) != NULL **then**
RPREORDER(RPTR(T))
[Finished]
return

Postorder traversal: iterative algorithm

Algorithm POSTORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Postorder traversal of a tree
[Initialize]
if T=NULL **then**
    return false {Empty Tree}
else
    TOP← -1
    P←T
[Traverse in postorder]
Repeat while true
[Descend left]
    Repeat while P != NULL
        push(S,P)
        P← LPTR(P)
[Process a node whose left and right subtree have been traversed]
    Repeat while S[TOP] <0
        P← POP(S)
        write DATA(P)
        if TOP = -1 **then**
        return
[Branch right and then mark node from which we branched]
    P← RPTR(S[TOP])
    S[TOP] ← -S[TOP]
[Finished]
return

Postorder traversal: recursive algorithm

Algorithm RPOSTORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Postorder traversal of a tree
[Initialize]
if T=NULL **then**
    return false {Empty Tree}
[Traverse in postorder the left subtree]
if LPTR(T) != NULL **then**
    RPOSTORDER(LPTR(P))
[Traverse in postorder the left subtree]
if RPTR(T) != NULL **then**
    RPOSTORDER(RPTR(P))
[Process the root node]
write DATA(P)
[Finished]
return

Inorder traversal: iterative algorithm

Algorithm INORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Inorder traversal of a tree

Inorder traversal: recursive algorithm

Algorithm RINORDER(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: Inorder traversal of a tree

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Making a duplicate copy of a tree

Algorithm COPY(T)
Input: A binary tree whose root node address is given by a pointer variable T
Output: address of the root node of a new tree which is the copy of the tree with root T

```
[Null pointer?]
if T = NULL then
        return false {Empty tree}
[Create a new node]
        NEW ← NODE
[Copy information field]
        DATA(NEW)← DATA(T)
[Set structural links]
        LPTR(NEW) ← COPY(LPTR(T))
        RPTR(NEW) ← COPY(RPTR(T))
[Return address of a new node]
return NEW
```

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Delete a node from Binary tree

- If a node to be deleted has no offspring, it can simply be deleted
- If a node has either a right or left empty subtree, the nonempty subtree can be appended to its grandparent node
- If the node has both a right and left subtree, the deletion strategy…

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Delete a node from Binary tree…

- If the node has both a right and left subtree…
  - First obtain the inorder successor of the node to be deleted
  - Then the right subtree of this successor node is appended to its grandparent node
  - Then the node to be deleted is replaced by its inorder successor

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```
Algorithm DELETE(HEAD,X)
Input: A binary tree whose root node address is given by a pointer variable HEAD, X is the
data value of the node marked for deletion
Output: address of the root node of a new tree in which node with data value X is deleted
[Initialize]
if LPTR(HEAD) != HEAD then
        CUR ← LPTR(HEAD)
        PARENT ← HEAD
        D = 'L'
else
        return false {node not found}
[Search for the node marked for deletion]
FOUND← false
Repeat while not FOUND and CUR != NULL
        if DATA(CUR)=X then
                FOUND=true
        else
                if X < DATA(CUR) then
                        PARENT← CUR
                        CUR ← LPTR(CUR)
                        D = 'L'
                else
                        PARENT← CUR
                        CUR← RPTR(CUR)
                        D= 'R'
```

```
if FOUND = false then
        return false {node not found}
[perform the indicated deletion and restructure the tree]
if LPTR(CUR) = NULL then
        Q← RPTR(CUR)
else
        if RPTR(CUR) = NULL then
                Q← LPTR(CUR)
        else   [check the right child of successor]
                SUC ← RPTR(CUR)
                if LPTR(SUC) = NULL then
                        LPTR(SUC) ← LPTR(CUR)
                        Q ← SUC
                else    [search for the successor of CUR]
                        PRED ← RPTR(CUR)
                        SUC ← LPTR(PRED)
                        Repeat while LPTR(SUC) != NULL
                                PRED ← SUC
                                SUC ← LPTR(PRED)
                        [connect successor]
                        LPTR(PRED) ← RPTR(SUC)
                        LPTR(SUC)← LPTR(CUR)
                        RPTR(SUC)← RPTR(CUR)
                        Q← SUC
```

Delete a node from Binary tree…

```
[connect parent of X to its replacement]
if D = 'L' then
        LPTR(PARENT)  ← Q
else
        RPTR(PARENT) ← Q
```

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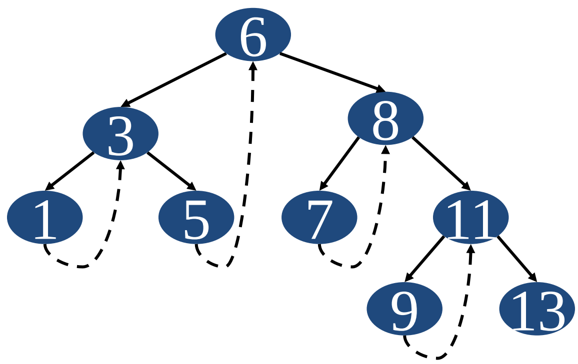
Threaded Storage Representation for Binary Trees

- Binary trees have a lot of wasted space: the leaf nodes each have 2 null pointers
- We can use these pointers to help us in inorder traversals
- We have the pointers reference the next node in an inorder traversal; called threads
- We need to know if a pointer is an actual link or a thread, so we keep a boolean for each pointer

Threaded tree node structure

```
struct node
{
    struct node *lptr, *rptr;
    bool lthread, rthread;
};
```

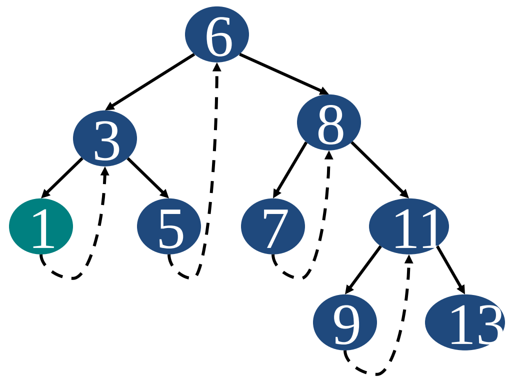
Threaded Tree Example



Threaded Tree Traversal

- We start at the leftmost node in the tree, print it, and follow its right thread
- If we follow a thread to the right, we output the node and continue to its right
- If we follow a link to the right, we go to the leftmost node, print it, and continue

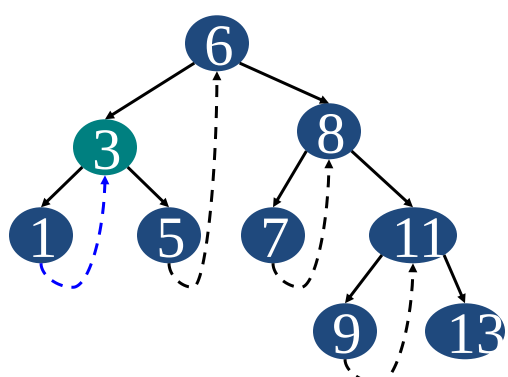
Threaded Tree Traversal



Output
1

Start at leftmost node, print it

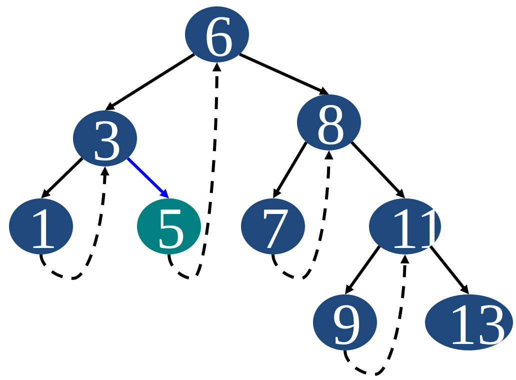
Threaded Tree Traversal



Output
1
3

Follow thread to right, print node

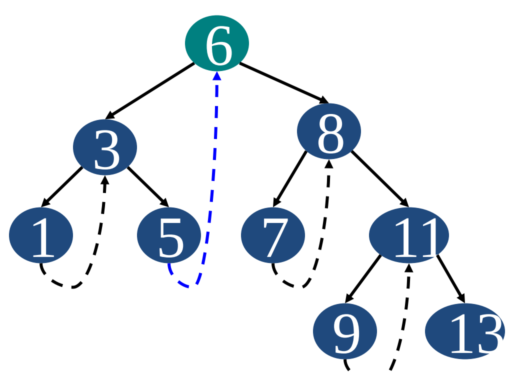
Threaded Tree Traversal



Output
1
3
5

Follow link to right, go to leftmost node and print

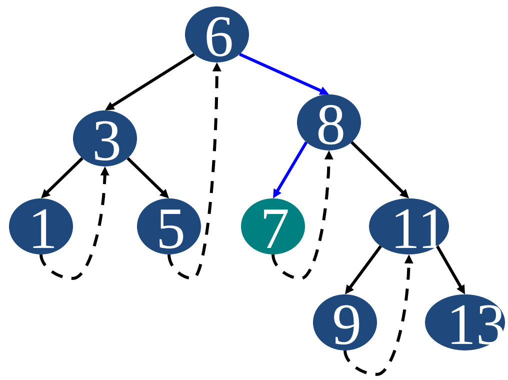
Threaded Tree Traversal



Output
1
3
5
6

Follow thread to right, print node

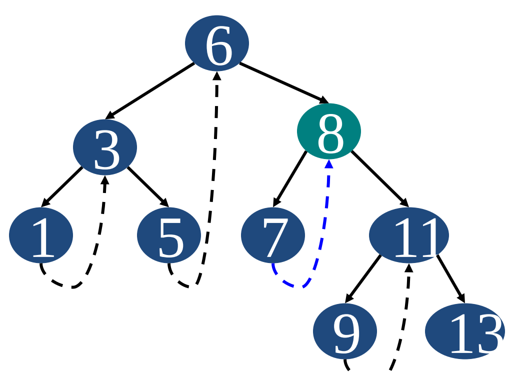
Threaded Tree Traversal



Output
1
3
5
6
7

Follow link to right, go to leftmost node and print

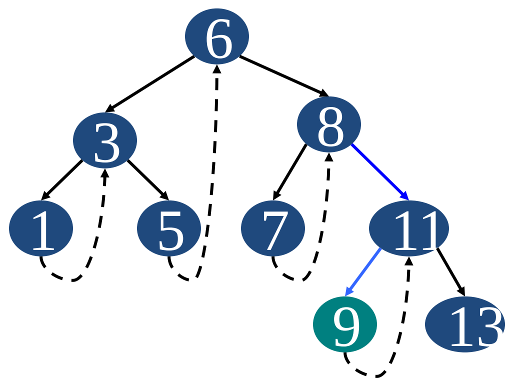
Threaded Tree Traversal



Output
1
3
5
6
7
8

Follow thread to right, print node

Threaded Tree Traversal



Output
1
3
5
6
7
8
9

Follow link to right, go to leftmost node and print

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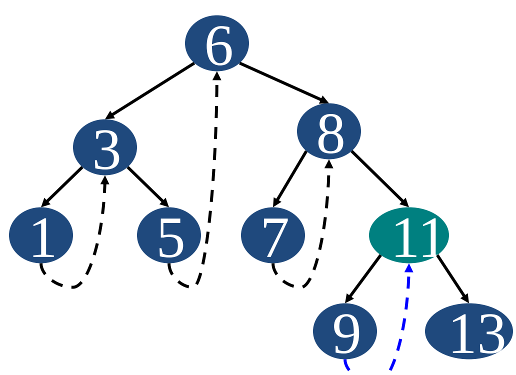
Threaded Tree Traversal



Output
1
3
5
6
7
8
9
11

Follow thread to right, print node

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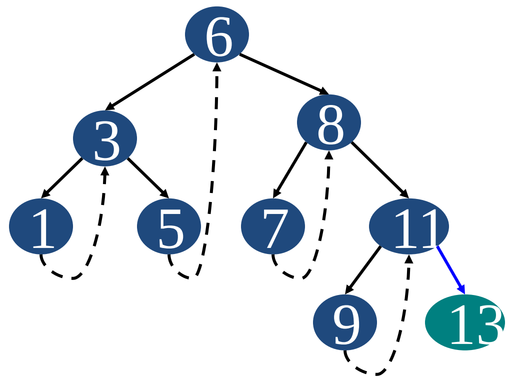
Threaded Tree Traversal



Output
1
3
5
6
7
8
9
11
13

Follow link to right, go to leftmost node and print

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Threaded Tree modification

- We're still wasting pointers, since half of our leafs' pointers are still null
- We can add threads to the previous node in an inorder traversal as well, which we can use to traverse the tree backwards or even to do postorder traversals

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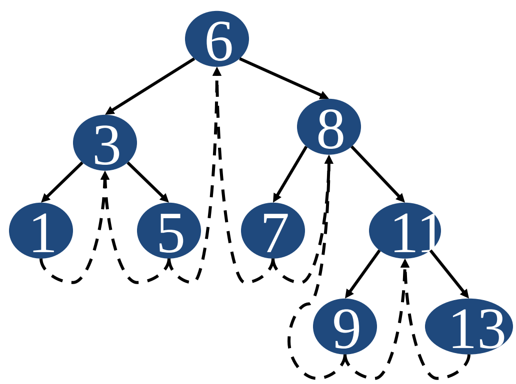
Threaded Tree Modification



Find Inorder Successor

Algorithm INS(X)
Input: X, the address of a node in a threaded binary tree
Output: address of the inorder successor of X
[Return the right pointer of the given node if a thread]
P ← RPTR(X)
if RTHREAD(X) = true
then return P
[Branch left repeatedly until a left thread]
repeat while LTHREAD(P) != true
    P ← LPTR(P)
[Return address of successor]
return P

Find Inorder Predecessor

Algorithm INP(X)
Input: X, the address of a node in a threaded binary tree
Output: address of the inorder predecessor of X
[Return the left pointer of the given node if a thread]
P ← LPTR(X)
if LTHREAD = true
then **return** P
[Branch right repeatedly until a right thread]
repeat while RTHREAD(P) != false
    P ← RPTR(P)
[Return address of successor]
return P

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Traverse Threaded Binary Tree in Inorder

Algorithm TINORDER(HEAD)
Input: address of a list head (HEAD) in a threaded binary tree, a subalgorithm INS previously discussed
Output: inorder traversal of a tree
[Initialize]
P ← HEAD
[Traverse threaded tree in inorder]
repeat while true
    P ← INS(P)
    if P=HEAD
    then **return**
    else Write(DATA(P))

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Inserting a node into a Threaded Binary Tree

Algorithm LEFT(X, INFO)
Input: address of a designated node X in an inorder threaded binary tree and the information associated with the new node (INFO)
Output: Inserts a new node to the left of X
[Create a new node]
P ← NODE
DATA(P) ← INFO
[Adjust pointer fields]
    LPTR(P) ← LPTR(X)
    LPTR(X) ← P
    RPTR(P) ← -X
[Reset predecessor thread if required]
    if LPTR(P) > 0
    then RPTR(INP(P)) ← -P
    return

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Threaded Binary Tree:Advantages

- Inorder traversal is somewhat faster than that of its unthreaded version
  - Because stack is not required in the former
- Efficient determination of the predecessor and successor nodes for any node P
- What are the disadvantages????

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Conversion of General tree forest to Binary Tree

ALGORITHM CONVERT
[Given a forest of trees in preorder sequence, it is required to convert this forest into an equivalent binary tree with a list head (HEAD)].

1. [*Initialize*]

HEAD ← NODE
LPTR(HEAD) ← NULL
RPTR(HEAD) ← HEAD
LEVEL[1] ← 0
LOCATION TOP ← 1

2. [*Process the input*]

Repeat thru step 6 while input is there.

3. [*Input a node*]

Read(LEVEL,INFO).

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Conversion of General tree forest to Binary Tree

4. [*Create a tree node*]
NEW <-- NODE
LPTR(NEW) <-- RPTR(NEW) <-- NULL
DATA(NEW) <-- INFO.

5. [*Compare levels*]

PRED_LEVEL ← LEVEL[TOP]
PRED_LOC ←LOCATION[TOP]
if LEVEL > PRED_LEVEL
then LPTR(PRED_LOC) ← NEW
else if LEVEL = PRED_LEVEL
RPTR(PRED_LOC) ← NEW
TOP ← TOP – 1
else
Repeat while LEVEL != PRED_LEVEL
TOP ← TOP – 1
PRED_LEVEL ← LEVEL[TOP]
PRED_LOC ← LOCATION[TOP]
if PRED_LEVEL ← LEVEL
then write (“Invalid Input”)
return

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Conversion of General tree forest to Binary Tree

RPTR(PRED_LOC) <-- NEW
TOP <-- TOP – 1.

6. [*Pushing values in stack*]

TOP <-- TOP + 1
LEVEL[TOP] <-- LEVEL
LOCATION[TOP] <-- NEW.

7. [*FINISH*]
return.