AVL Trees

Introduction to AVL Tree

- AVL tree is a self-balancing Binary Search Tree (BST) where the difference between heights of left and right subtrees cannot be more than one for all nodes.
- Improved efficiency in search operation as compared to BST

AVL tree Insertion

- To make sure that the given tree remains AVL after every insertion, we must augment the standard BST insert operation to perform some re-balancing.
- Two basic operations that can be performed to re-balance a BST without violating the BST property (keys(left) < key(root) < keys(right)).
  - Left Rotation
  - Right Rotation

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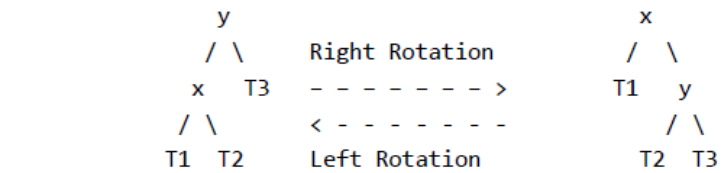
AVL tree Insertion…

T1, T2 and T3 are subtrees of the tree rooted with y (on left side) or x (on right side)



Keys in both of the above trees follow the following order

keys(T1) < key(x) < keys(T2) < key(y) < keys(T3)

So BST property is not violated anywhere.

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AVL tree Insertion…

Let the newly inserted node be w

1) Perform standard BST insert for w.

2) Starting from w, travel up and find the first unbalanced node.

Let z be the first unbalanced node, y be the child of z that comes on the path from w to z and x be the grandchild of z that comes on the path from w to z.

3) Re-balance the tree by performing appropriate rotations on the subtree rooted with z.

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AVL tree Insertion…

- There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways:
  a) y is left child of z and x is left child of y (Left Left Case)
  b) y is left child of z and x is right child of y (Left Right Case)
  c) y is right child of z and x is right child of y (Right Right Case)
  d) y is right child of z and x is left child of y (Right Left Case)

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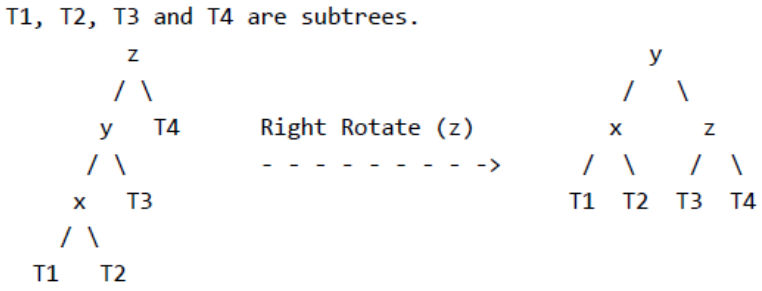
AVL tree Insertion…

- Operations to be performed

a) Left Left Case



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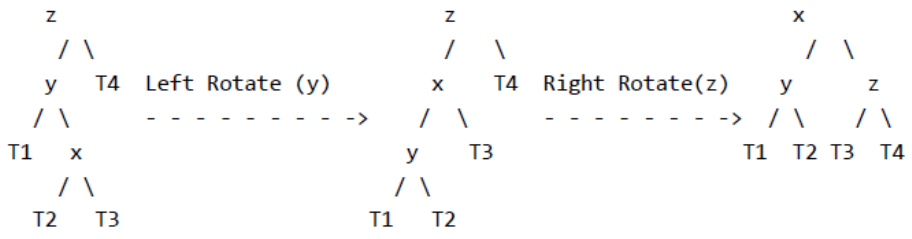
AVL tree Insertion…

- Operations to be performed

a) Left Right Case



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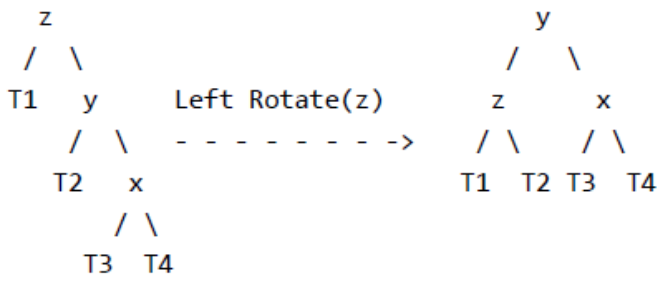
AVL tree Insertion…

- Operations to be performed

a) Right Right Case



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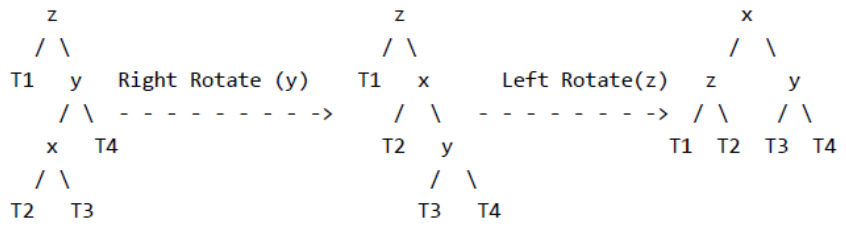
AVL tree Insertion…

- Operations to be performed

a) Right Left Case



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AVL tree Deletion

- Let w be the node to be deleted
  1) Perform standard BST delete for w.
  2) Starting from w, travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the larger height child of z, and x be the larger height child of y.
  3) Re-balance the tree by performing appropriate rotations on the subtree rooted with z.

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AVL tree Deletion…

- There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways. Following are the possible 4 arrangements:
  a) y is left child of z and x is left child of y (Left Left Case)
  b) y is left child of z and x is right child of y (Left Right Case)
  c) y is right child of z and x is right child of y (Right Right Case)
  d) y is right child of z and x is left child of y (Right Left Case)

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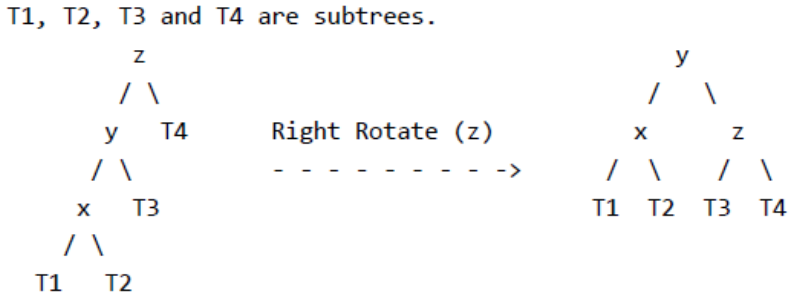
AVL tree Deletion…

a)Left left case



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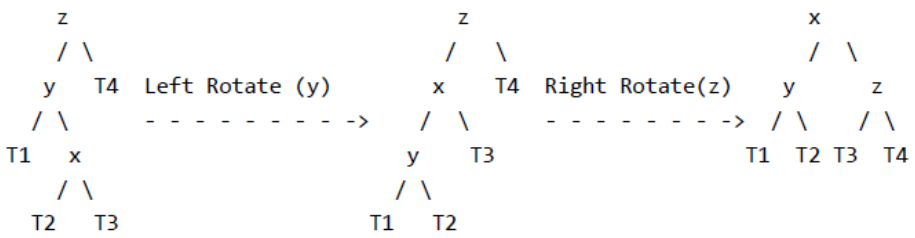
AVL tree Deletion…

b)Left right case



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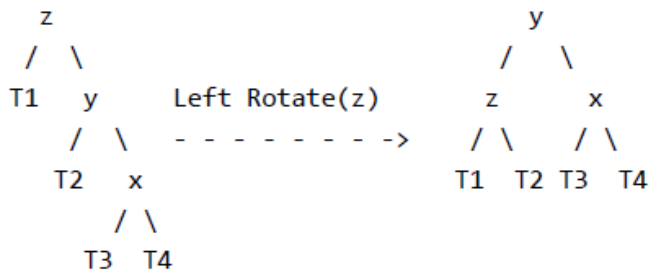
AVL tree Deletion…

c)Right right case



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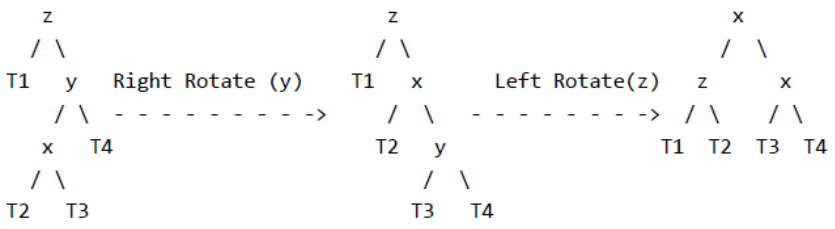
AVL tree Deletion…

d)Right left case



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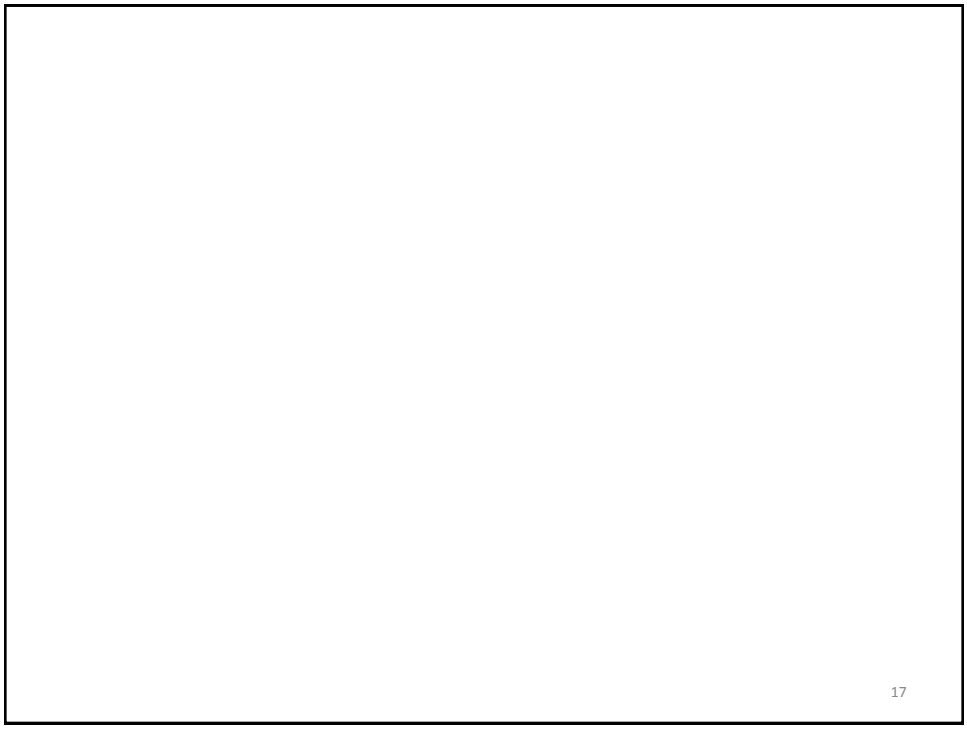
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