# **Sorting Algorithms**

# Sorting Algorithms







#### Function: BUBBLE\_SORT (K,N)

```
1. [Initialize]
        LAST ← N (entire list assumed unsorted at this point)
   [Loop on pass index]
        Repeat thru step 5 for PASS = 1, 2, ..., N - 1
    [Initialize exchanges counter for this pass]
        EXCHS ← 0
   [Perform pairwise comparisons on unsorted elements]
        Repeat for I = 1, 2, ..., LAST - 1
           If K[I] > K[I + 1]
           then K[I] \longleftrightarrow K[I+1]
                  EXCHS ← EXCHS + 1
[Were any exchanges made on this pass ?]
        If EXCHS = 0
        then Return (mission accomplished; return early)
        else LAST ← LAST - 1 (reduce size of unsorted list)
6. [Finished]
        Return (maximum number of passes required)
```

#### **Function: BUBBLESORT (A)**

```
BUBBLESORT(A)

1 for i = 1 to A.length - 1

2 for j = A.length down to i + 1

3 if A[j] < A[j - 1]

4 exchange A[j] with A[j - 1]
```

#### Function: BUBBLESORT (A)

i	Unsorted K <sub>j</sub>	Pass Number (i)					Sorted
		1	2	3	4	5	6
1	42	23	23	<b>1</b> 1	11	11	11
2	23	42	<b>~</b> 11	23	23	23	23
3	74	ساالد	42	42	42	-36	36
4	11	65	58	58	36	42	<u>42</u>
5	65	×58	65	36.	58	<u>58</u>	58
6	58	74	36	65	<u>65</u>	65	65
7	94	36	74	<u>74</u>	74	74	74
8	36	94	<u>87</u>	87	87	87	87
9	99	▶87	94	94	94	94	94
10	87	99	99	99	99	99	99

FIGURE Trace of a bubble sort.

#### **Insertion Sort**

#### **Insertion Sort**



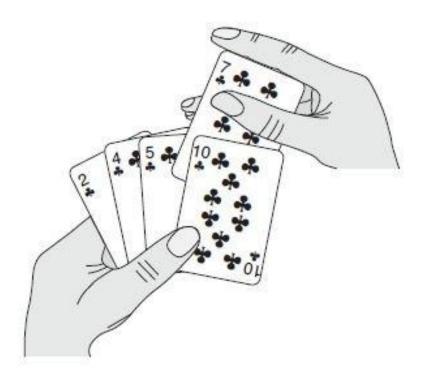


Figure Sorting a hand of cards using insertion sort.

#### **Insertion Sort**

#### **Function : INSERTION-SORT (A)**

```
INSERTION-SORT (A)

1 for j = 2 to A.length

2 key = A[j]

3 // Insert A[j] into the sorted sequence A[1..j-1].

4 i = j-1

5 while i > 0 and A[i] > key

6 A[i+1] = A[i]

7 i = i-1

8 A[i+1] = key
```

# **Selection Sort**

#### Selection Sort

• Consider sorting n numbers stored in array A by first finding the smallest element of A and exchanging it with the element in A[1]. Then find the second smallest element of A, and exchange it with A[2]. Continue in this manner for the first n-1 elements of A. Write pseudo-code for this algorithm, which is known as selection sort.

# Merge Sort

# Merge Sort

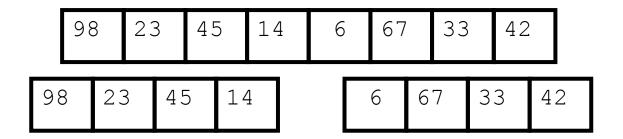
```
MergeSort(low,high)
  if(low<high)
       mid = (low + high)/2
       MergeSort(low,mid)
       MergeSort(mid+1,high)
       Merge(low,mid,high)
```

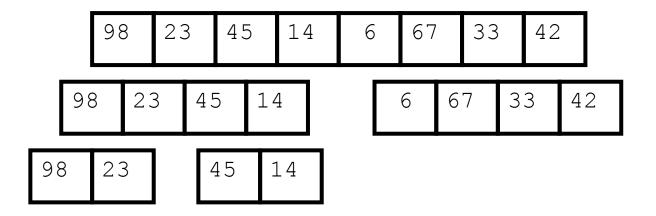
#### Merge Sort

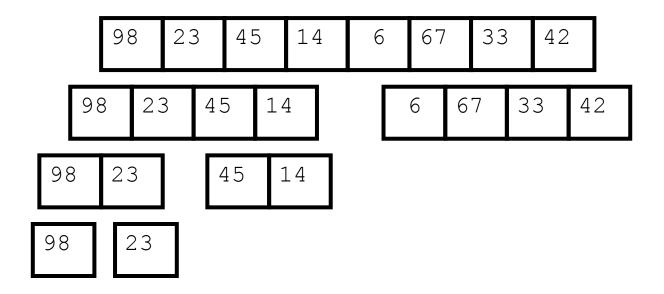
```
Merge(low,mid,high) {
   h=low; i=low; j=mid+1;
   while ( (h \le mid) and (j \le high) )
          if(a[h] \le a[j])
                     b[i]=a[h]
                     h=h+1
          else
                     b[i]=a[j]
                     j=j+1
          i=i+1
   }//End of while
```

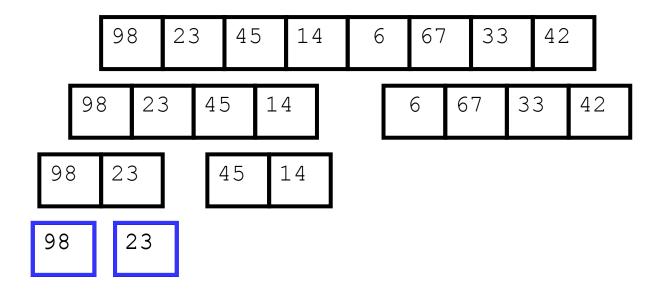
```
If(h>mid)
          for k=j to high
                     b[i]=a[k]
                     i=i+1
  else
          for k=h to mid
                     b[i]=a[k]
                     i=i+1
   for k=low to high
          a[k]=b[k]
}//End of Merge
```

98 23 45 14 6 67 33 42

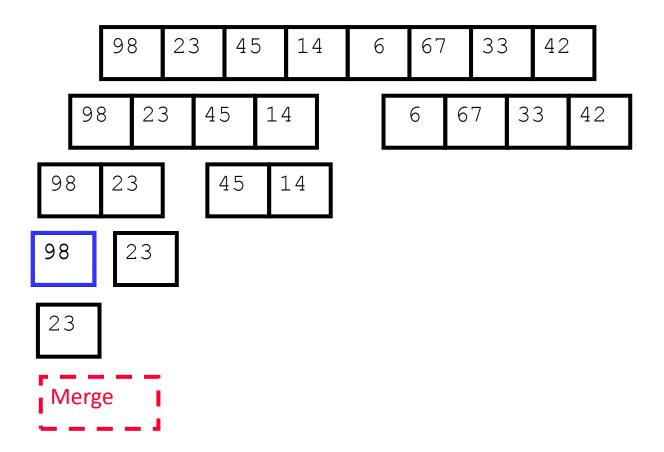


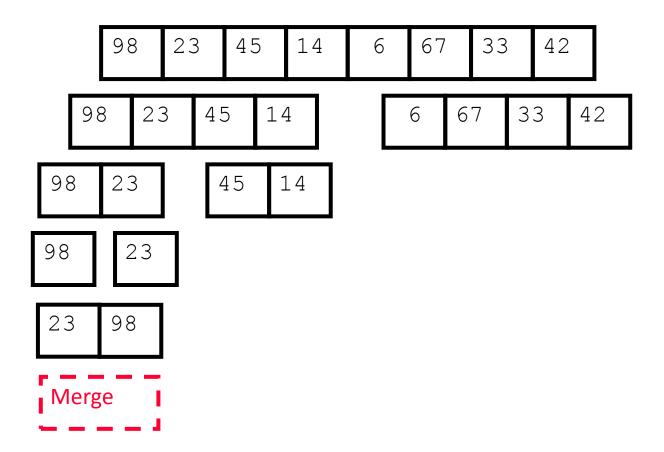


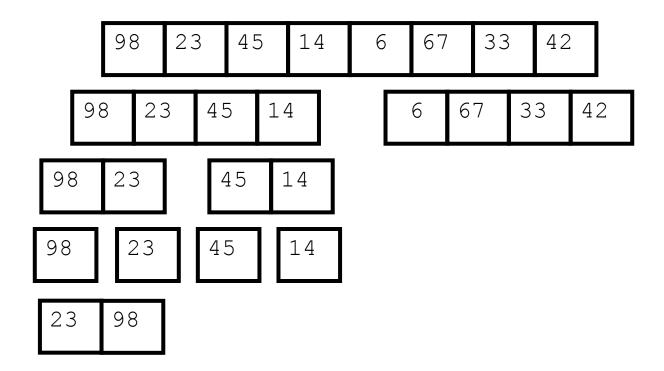


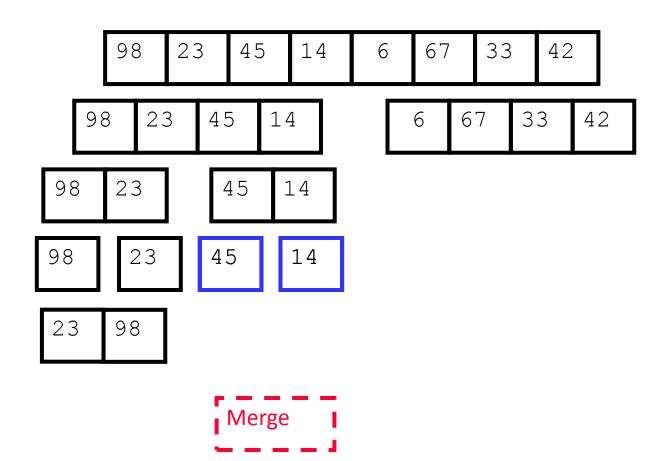


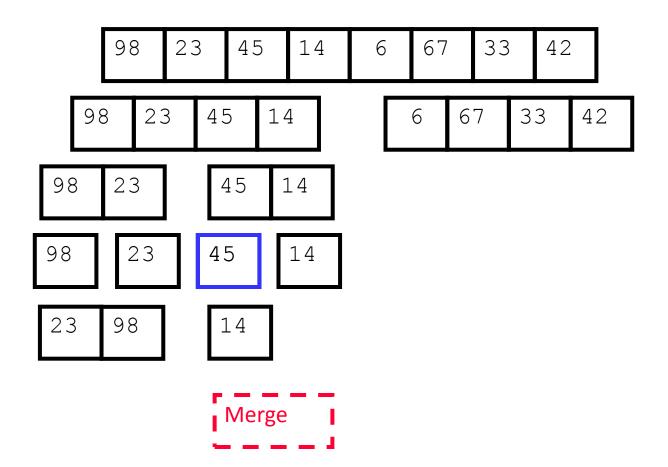


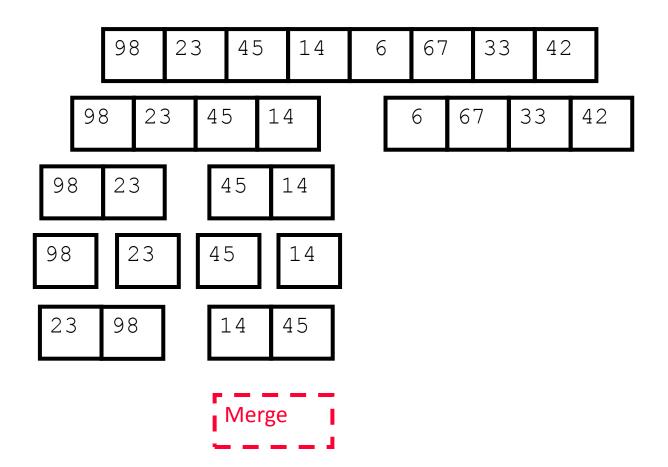


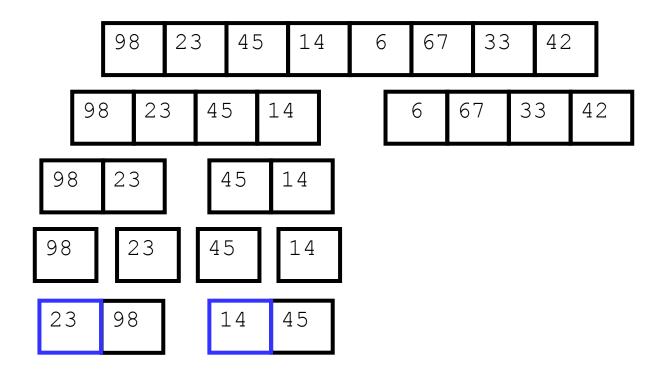




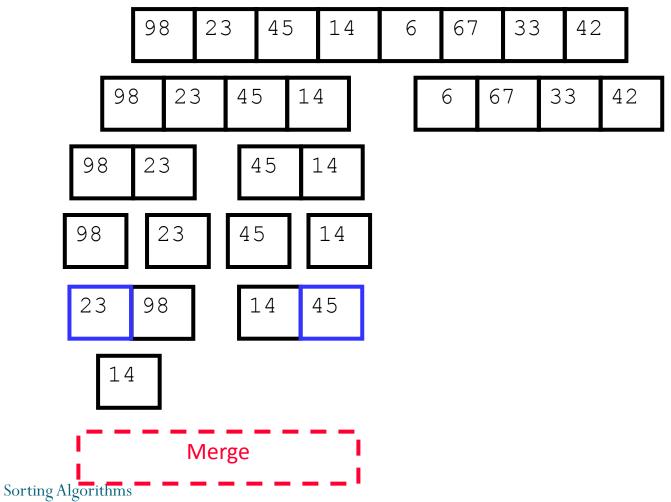


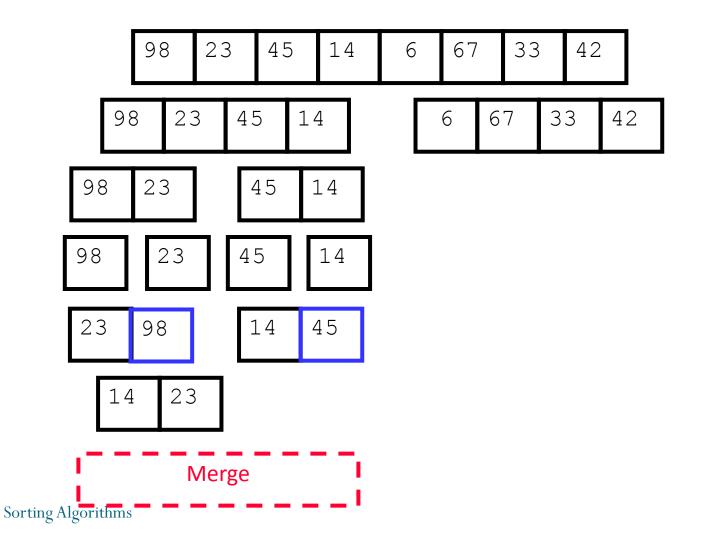


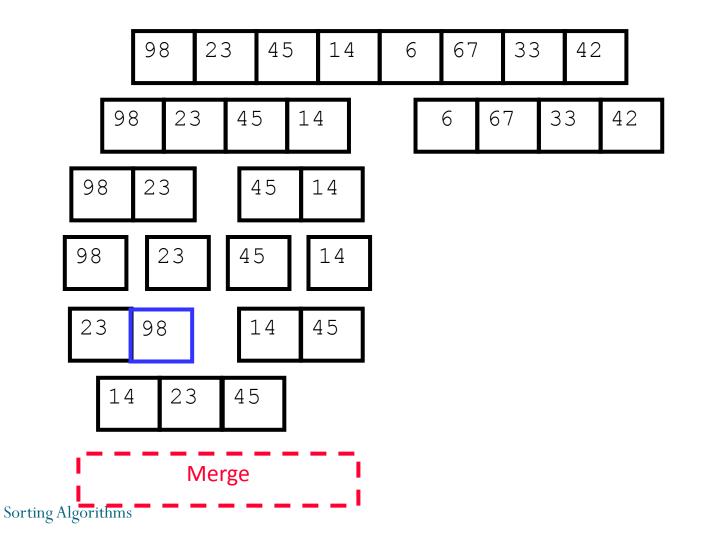


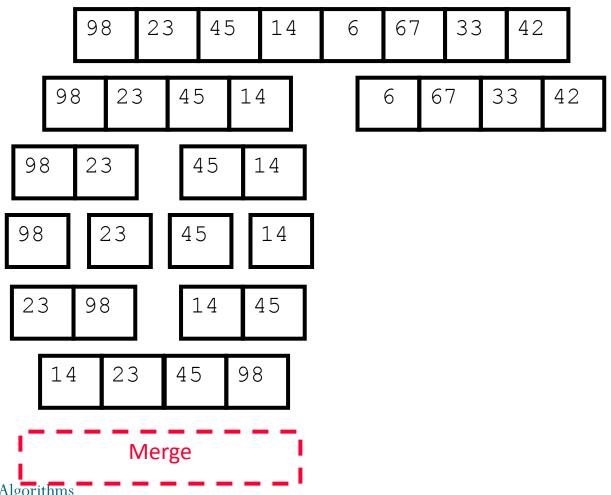


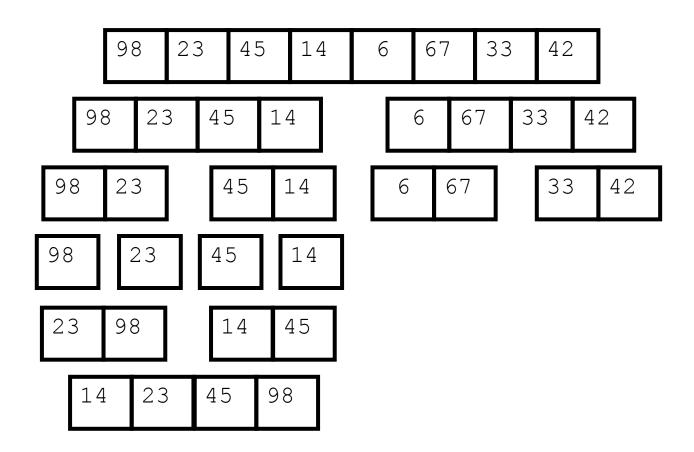
Merge Sorting Algorithms

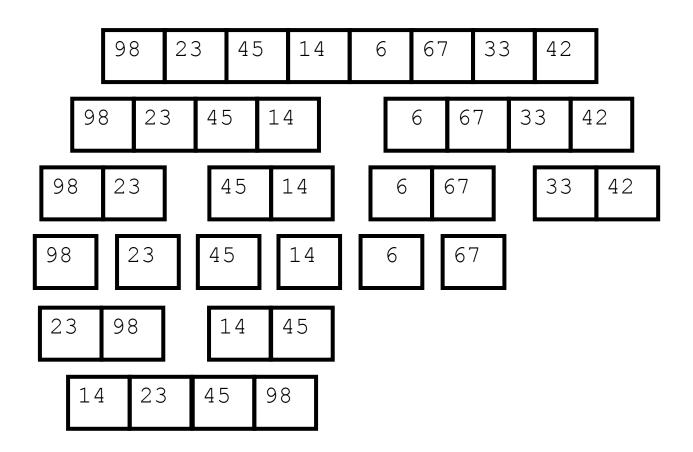


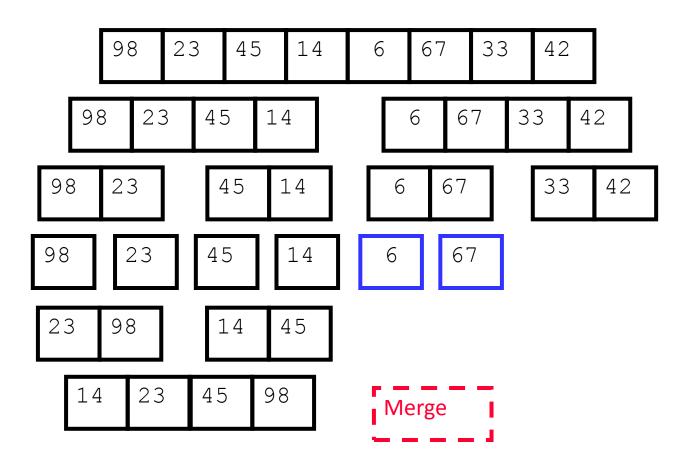


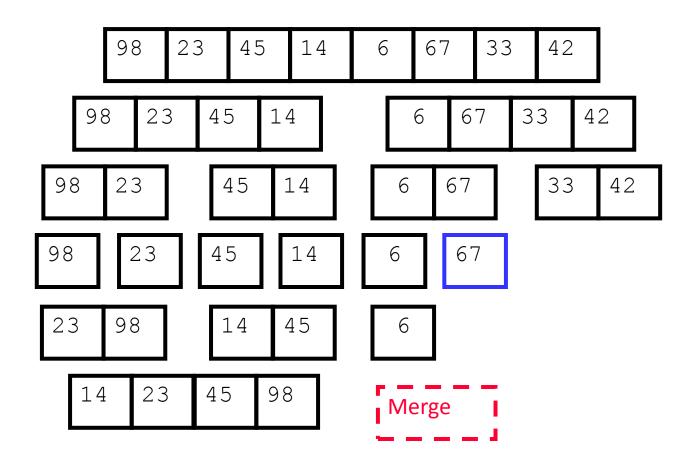


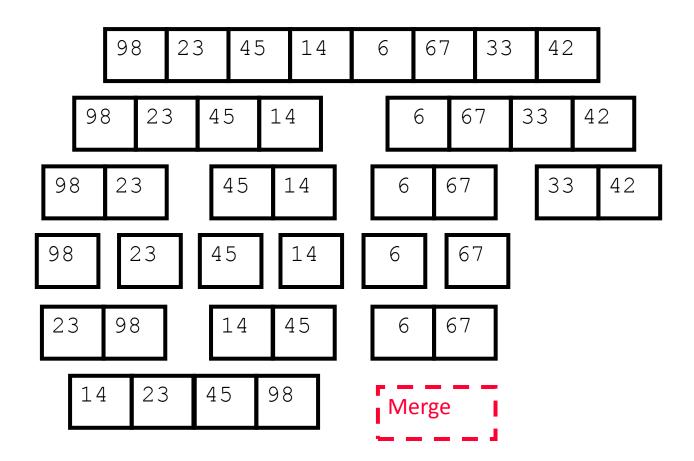


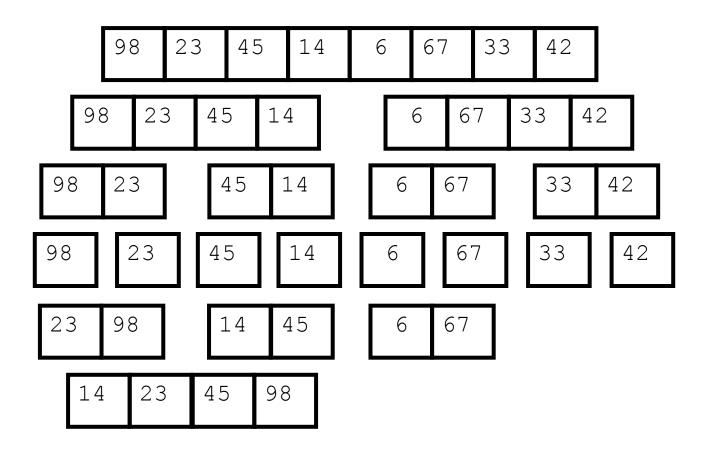








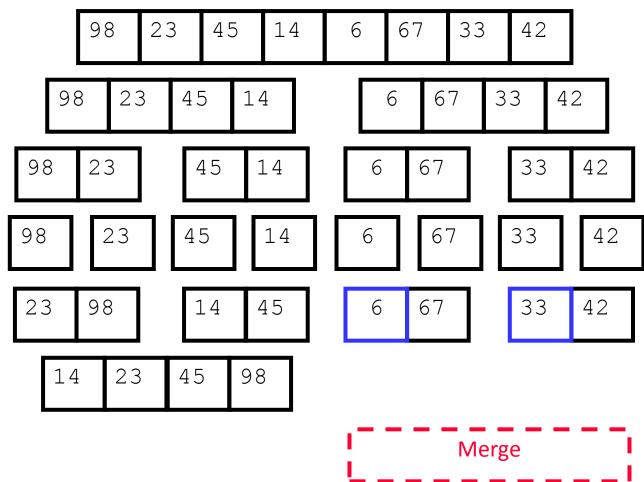


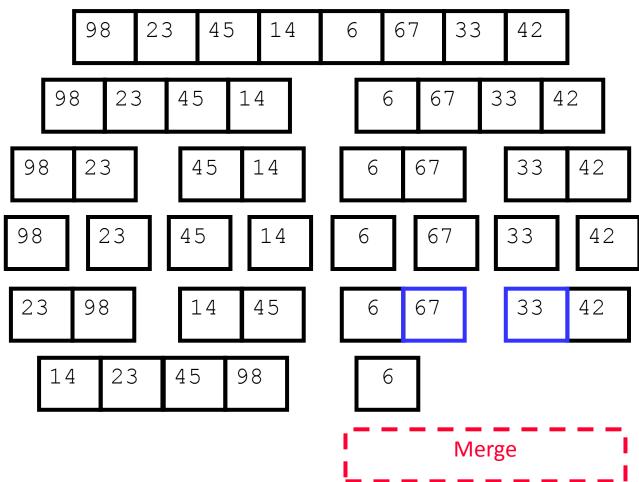


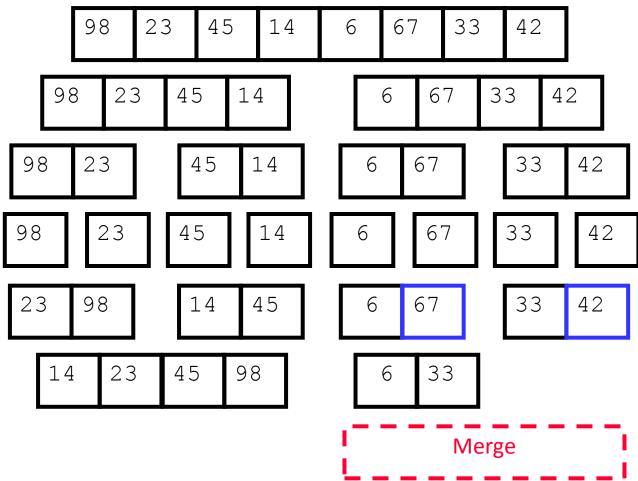


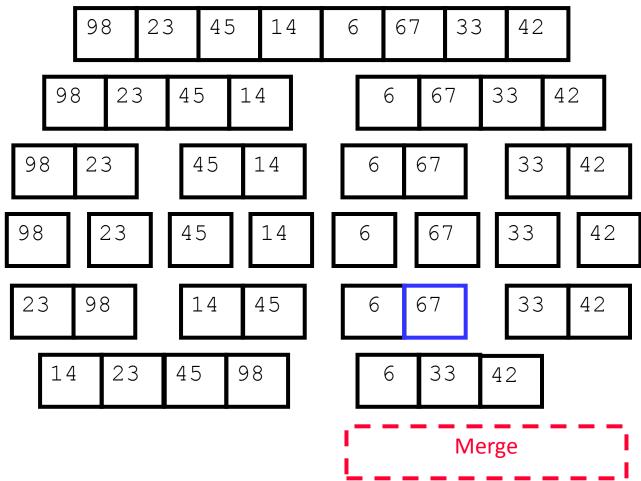


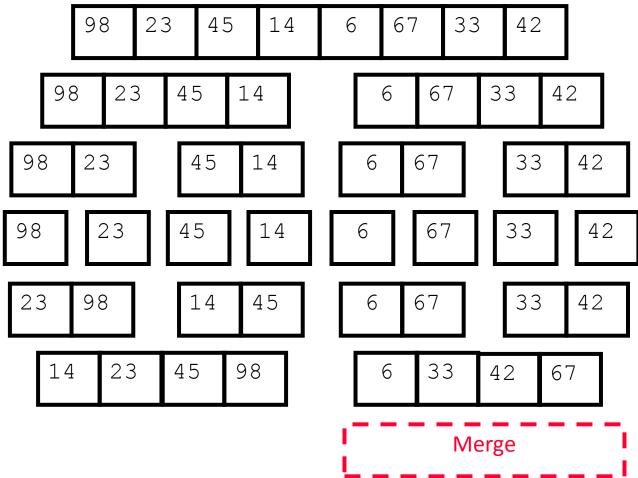


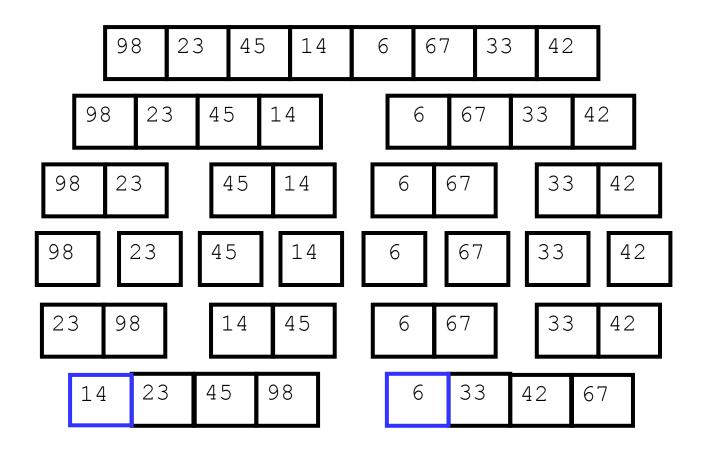


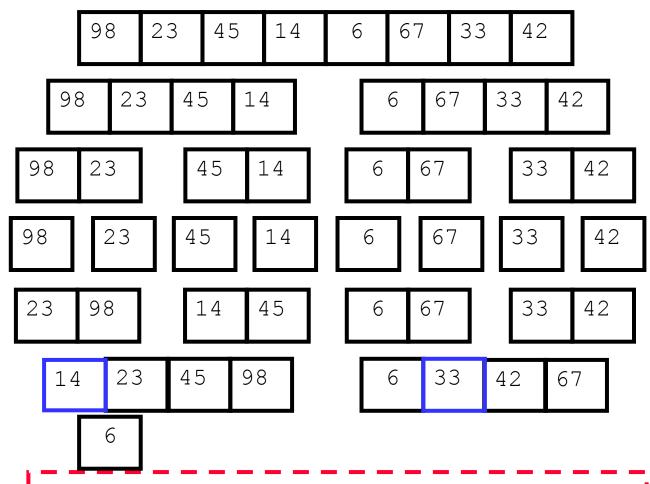


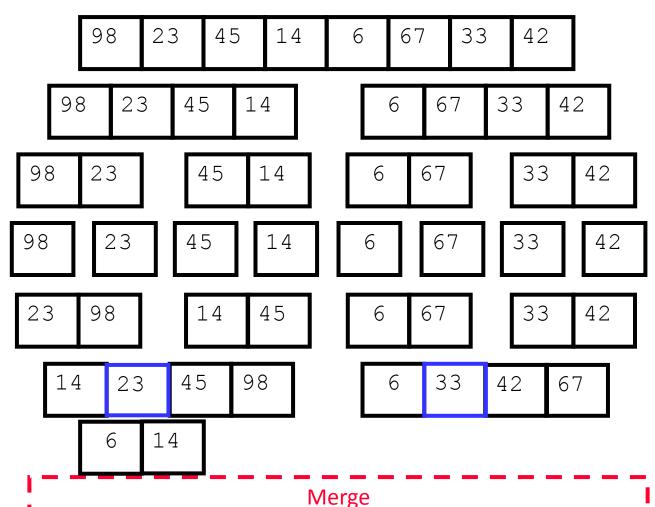


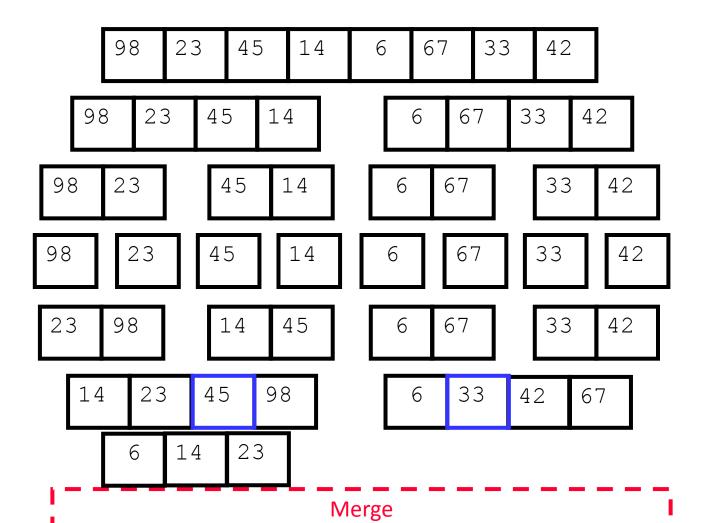


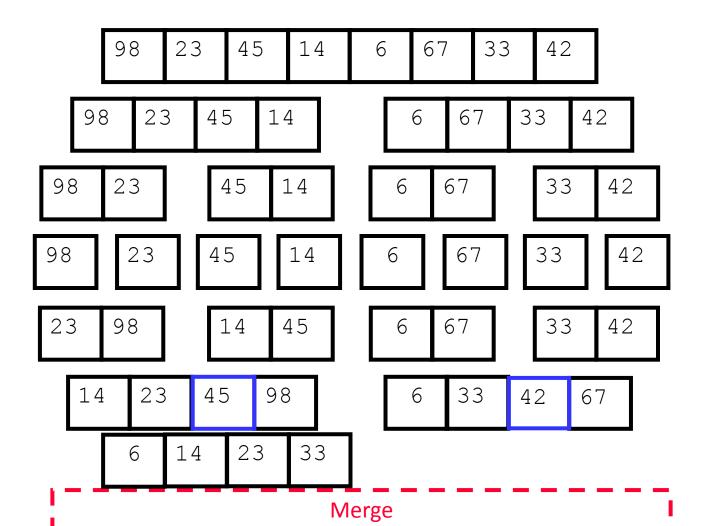


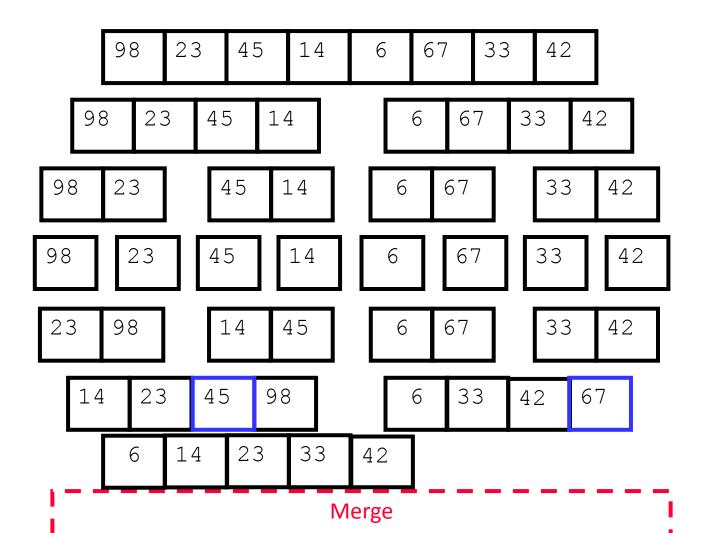


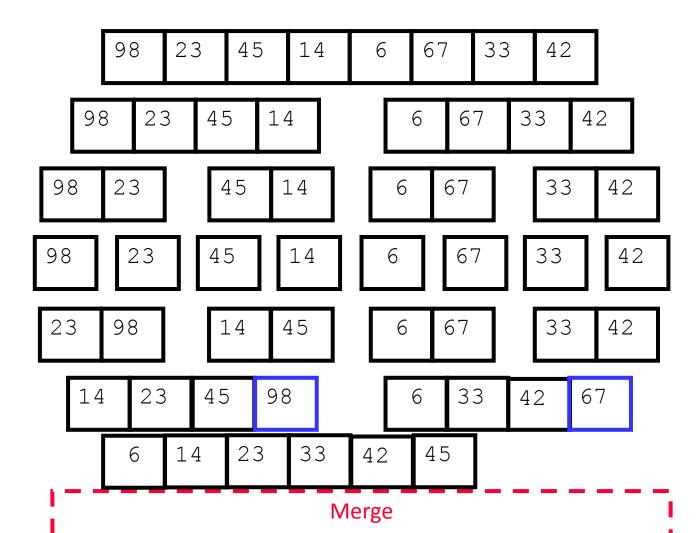


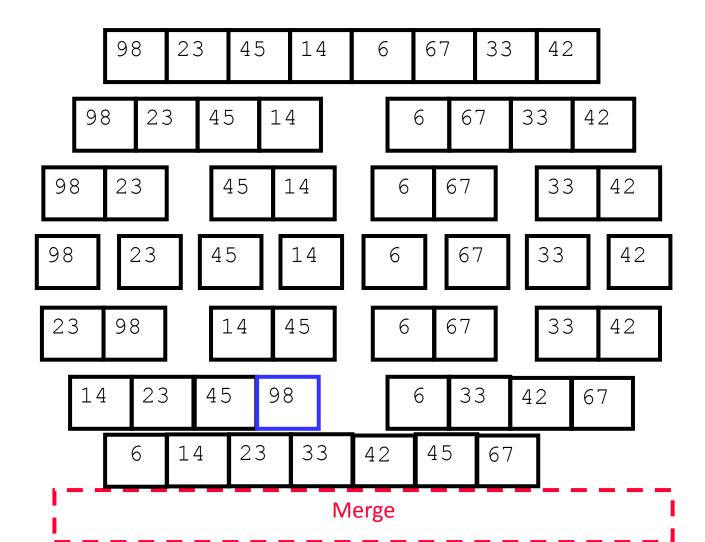


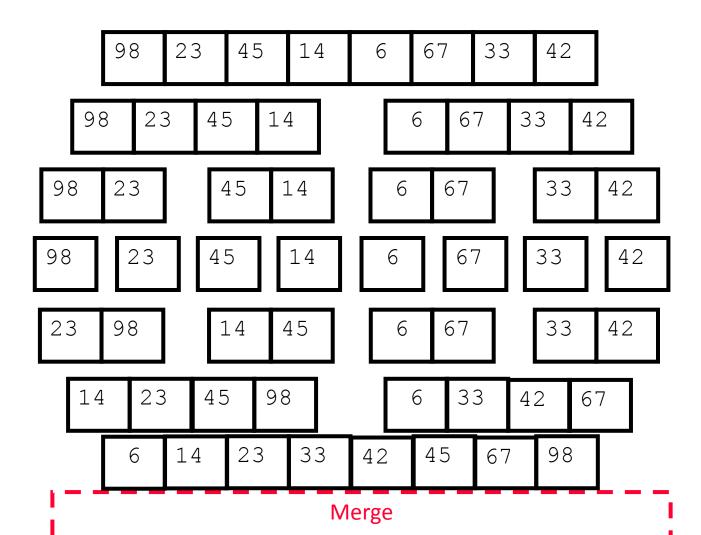












```
QUICKSORT(A, p, r)

1 if p < r

2 q = \text{PARTITION}(A, p, r)

3 QUICKSORT(A, p, q - 1)

4 QUICKSORT(A, q + 1, r)
```

To sort an entire array A, the initial call is QUICKSORT (A, 1, A.length).

```
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```

To sort an entire array A, the initial call is QUICKSORT (A, 1, A.length).

```
PARTITION (A, p, r)

1 x = A[r]

2 i = p - 1

3 for j = p to r - 1

4 if A[j] \le x

5 i = i + 1

6 exchange A[i] with A[j]

7 exchange A[i + 1] with A[r]

8 return i + 1
```

```
QUICKSORT(A, p, r)
                                                                      (a)
   if p < r
       q = PARTITION(A, p, r)
                                                                      (b)
                                                                                        3
        QUICKSORT(A, p, q - 1)
       QUICKSORT(A, q + 1, r)
4
                                                                     (c)
To sort an entire array A, the initial call is QUICKSORT (A, 1, A.length).
                                                                     (d)
PARTITION (A, p, r)
                                                                     (e)
   x = A[r]
  i = p - 1
                                                                      (f)
   for j = p to r - 1
        if A[j] \leq x
5
            i = i + 1
                                                                     (g)
            exchange A[i] with A[j]
6
   exchange A[i + 1] with A[r]
                                                                     (h)
   return i+1
                                                                     (i)
 Sorting Algorithms
```

• Radix sort is a non-comparative integer sorting algorithm that sorts data with integer keys by grouping keys by the individual digits which share the same significant position and value.

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- There are two classification of Radix sort
- 1. Least significant digit (LSD)
- 2. Most significant digit (MSD)

### Least significant digit (LSD)

**Definition:** Each key is first figuratively dropped into one level of buckets corresponding to the value of the rightmost digit. Each bucket preserves the original order of the keys as the keys are dropped into the bucket. There is a one-to-one correspondence between the buckets and the values that can be represented by the rightmost digit. Then, the process repeats with the next neighboring more significant digit until there are no more digits to process.

#### In other words

- 1. Take the least significant digit of each key
- 2. Group the keys based on that digit, but otherwise keep the original order of keys.
- 3. Repeat the grouping process with each more significant digit.

### **Example**

Original, unsorted list: 170, 45, 75, 90, 802, 2, 24, 66

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Sorting by least significant digit (1s place) gives: 170, 90, 802, 2, 24, 45, 75, 66

#### **Example**

Original, unsorted list:

170, 45, 75, 90, 802, 2, 24, 66

Sorting by least significant digit (1s place) gives:

17<u>0</u>, 9<u>0</u>, 80<u>2</u>, <u>2</u>, 2<u>4</u>, 4<u>5</u>, 7<u>5</u>, 6<u>6</u>

Notice that we keep 802 before 2, because 802 occurred before 2 in the original list, and similarly for pairs 170 & 90 and 45 & 75.

#### Example

Original, unsorted list: 170, 45, 75, 90, 802, 2, 24, 66

Sorting by least significant digit (1s place) gives: 170, 90, 802, 2, 24, 45, 75, 66

Sorting by next digit (10s place) gives: 802, 2, 24, 45, 66, 170, 75, 90

#### Example

Original, unsorted list:

170, 45, 75, 90, 802, 2, 24, 66

Sorting by least significant digit (1s place) gives:

17<u>0</u>, 9<u>0</u>, 80<u>2</u>, <u>2</u>, 2<u>4</u>, 4<u>5</u>, 7<u>5</u>, 6<u>6</u>

Sorting by next digit (10s place) gives:

8<u>0</u>2, 2, <u>2</u>4, <u>4</u>5, <u>6</u>6, 1<u>7</u>0, <u>7</u>5, <u>9</u>0

Notice that 802 again comes before 2 as 802 comes before 2 in the previous list.

#### Example

Original, unsorted list: 170, 45, 75, 90, 802, 2, 24, 66

Sorting by least significant digit (1s place) gives: 170, 90, 802, 2, 24, 45, 75, 66

Sorting by next digit (10s place) gives: 802, 2, 24, 45, 66, 170, 75, 90

Sorting by most significant digit (100s place) gives: 2, 24, 45, 66, 75, 90, <u>1</u>70, <u>8</u>02

### Iterative version using queues

A simple version of an LSD radix sort can be achieved using queues as buckets.

#### Iterative version using queues

1. The integers are enqueued into an array of ten separate queues based on their digits from right to left. Computers often represent integers internally as fixed-length binary digits. Here, we will do something analogous with fixed-length decimal digits. So, using the numbers from the previous example, the queues for the 1st pass would be:

0: 17<u>0</u>, 09<u>0</u>

1: none

2: 802, 002

3: none

4: 024

5: 04<u>5</u>, 07<u>5</u>

6: 06<u>6</u>

7-9: none

### Iterative version using queues

2. The queues are dequeued back into an array of integers, in increasing order. Using the same numbers, the array will look like this after the first pass:

170, 090, 802, 002, 024, 045, 075, 066

#### Iterative version using queues

2. The queues are dequeued back into an array of integers, in increasing order. Using the same numbers, the array will look like this after the first pass:

170, 090, 802, 002, 024, 045, 075, 066

3. For the second pass:

#### Queues:

0: 8<u>0</u>2, 0<u>0</u>2

1: none

2: 0<u>2</u>4

3: none

4: 045

5: none

6: 066

7: 1<u>7</u>0, 0<u>7</u>5

 $_{Sorting\,Algorithms} \textbf{8:} \textbf{none}$ 

9: 0<u>9</u>0

#### Array:

802, 002, 024, 045, 066, 170, 075, 090 (note that at this point only 802 and 170 are out of order)

#### Iterative version using queues

4. For the third pass:

#### Queues:

0: <u>0</u>02, <u>0</u>24, <u>0</u>45, <u>0</u>66, <u>0</u>75, <u>0</u>90

1: <u>1</u>70

2-7: none

8: <u>8</u>02

9: none

#### Array:

002, 024, 045, 066, 075, 090, 170, 802 (sorted)

#### Iterative version using queues

4. For the third pass:

#### Queues:

0: <u>0</u>02, <u>0</u>24, <u>0</u>45, <u>0</u>66, <u>0</u>75, <u>0</u>90

1: <u>1</u>70

2-7: none

8: <u>8</u>02

9: none

#### Array:

002, 024, 045, 066, 075, 090, 170, 802 (sorted)

#### General algorithm for Radix sort

- 1. Repeat thru step 6 for each digit in the key
- 2. Initialize the pockets
- 3. Repeat thru step 5 until the end of the linked list
- 4. Obtain the next digit of the key
- 5. Insert the record in the appropriate pocket
- 6. Combine the pockets to form a new linked list

Input: 42, 23, 74, 11, 65, 57, 94, 36, 99, 87, 70, 81, 61

Input: 42, 23, 74, 11, 65, 57, 94, 36, 99, 87, 70, 81, 61

After the first pass on the unit digit position of each number we have: