Hashing

Hash Table

- As we saw with binary search, certain data structures such as a binary search tree can help improve the efficiency of searches.
- From linear search to binary search, we improved our search efficiency from O(n) to O(logn) .
- We now present a new data structure, called a hash table, that will increase our efficiency to O(1) , or constant time.

Hash Table

- A hash table is made up of two parts: an array (the actual table where the data to be searched is stored) and a mapping function, known as a hash function.
- The hash function is a mapping from the input space to the integer space that defines the indices of the array.
- In other words, the hash function provides a way for assigning numbers to the input data such that the data can then be stored at the array index corresponding to the assigned number.

Hash Table

- Let's take a simple example. First, we start with a hash table array of strings (we'll use strings as the data being stored and searched in this example).
- Let's say the hash table size is 12:
- Next we need a hash function.

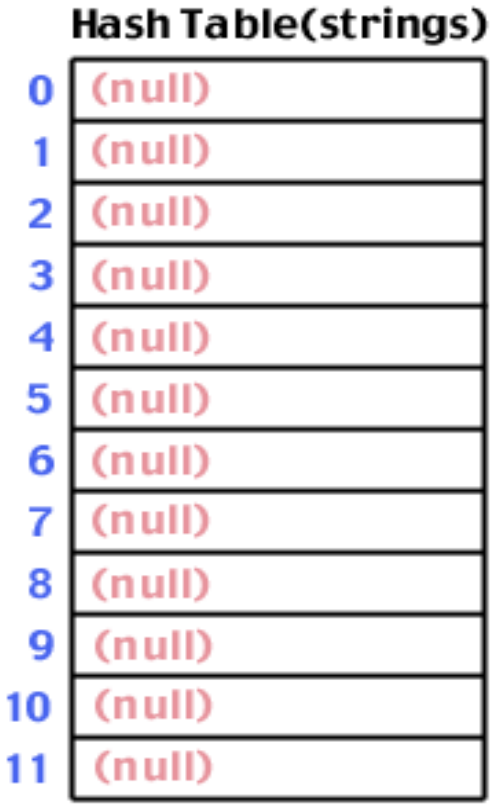


Figure : The empty hash table of strings

Hash Function

- There are many possible ways to construct a hash function.
- For now, let's assume a simple hash function that takes a string as input.
- The returned hash value will be the sum of the ASCII characters that make up the string mod the size of the table.

```
int hash(char *str, int table_size)
    {       int sum;
            if (str==NULL) return -1;
            for( ; *str!=null; str++) sum += *str;
            return (sum % table_size); }
```

Hash Function

- As mentioned briefly in the previous section, there are multiple ways for constructing a hash function.
- Remember that hash function takes the data as input (often a string), and return s an integer in the range of possible indices into the hash table.
- Every hash function must do that, including the bad ones. So what makes for a good hash function?

Characteristics of a Good Hash Function

- The hash value is fully determined by the data being hashed.
- The hash function uses all the input data.
- The hash function "uniformly" distributes the data across the entire set of possible hash values.
- The hash function generates very different hash values for similar strings.

Hashing Function

- Division Method
- Mid-Square Method
- Multiplicative Hash
- Folding Method
- Digit Analysis

Division Method

- Defined as:
- $H(X) = X \bmod m + 1$

for some integer value m. The operator mod denotes the modulo arithmetic system.

e.g.

$H(35) = 35 \bmod 11 + 1 = 2 + 1 = 3$

The division method yields a "hash value" which belongs to the set $\{1,2,\ldots,m\}$.

Mid-Square Method

- In this method a key is multiplied by itself and the address is obtained by selection an appropriate number of bits or digits from the middle of the square.

- Usually, the number of bits or digits chosen depends on the table size

- e.g.
- Consider a six digit key 123456.
- Squaring the key results in the value 15241383936.
- If the three digit address is required, position 5 to 7 could be chosen, giving address 138.

Multiplicative Hash

- The given record is multiplied by some constant value.
- The formula for computing the hash key is

H(Key) = floor(p*(fractional part of (key * A))

where, p is integer constant and A is constant real number.

Donald Knuth suggested to use constant A=0.61803398987

if key = 107 and p=50 then
H(Key) = floor(50 * fractional part of(107 * 0.61803398987))
= 6

Folding Method

- In the folding method a key is partitioned into number of parts, each of which has the same length as required address with the possible exception of the last part. Parts are then added together, ignoring the final carry, to form an address.

- e.g.
- Assume that key 356942781 is to be transformed into a three digit address.
- 356, 942 and 781 are added to yield 079.

Digit Analysis

- The digit analysis is used in a situation when all the identifiers are known in advance.
- We first transform the identifiers into numbers using some radix r. Then we examine the digits of each identifiers. Some digits having most skewed distribution are deleted.
- This deleting of digits is continued until the number of remaining digits is small enough to give an address in the range of the Hash table.
- Then these digits are used to calculate the hash address.

Digit Analysis

e.g.

Keys: 2234, 3452, 2784
Distribution:

Digit	position 0	1	2	3 (from right to left)
2	1	0	1	2
3	0	1	0	1
4	2	0	1	0
5	0	1	0	0
7	0	0	1	0
8	0	1	0	0

Positions 1 and 2 : best distribution. Hence we have:

Addresses: 23, 45, 78

- Now that we have a framework in place, let's try using it.
- First, let's store a string into the table: "Steve".
- We run "Steve" through the hash function, and find that hash("Steve",12) yields 3:

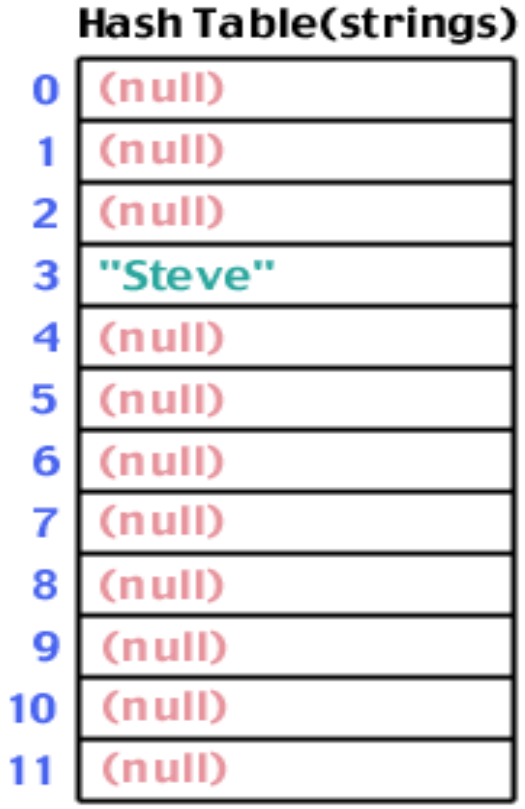


Figure : The hash table after inserting "Steve"

- Let's try another string: "Spark". We run the string through the hash function and find that hash("Spark",12) yields 6. Fine. We insert it into the hash table:

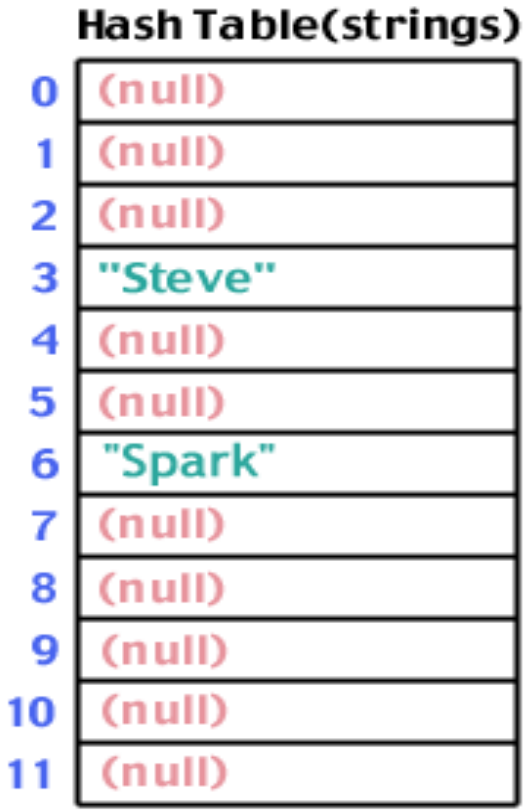


Figure : The hash table after inserting "Spark"

- Let's try another: "Notes". We run "Notes" through the hash function and find that hash("Notes",12) is 3. Ok. We insert it into the hash table:

- What happened? :: Collision

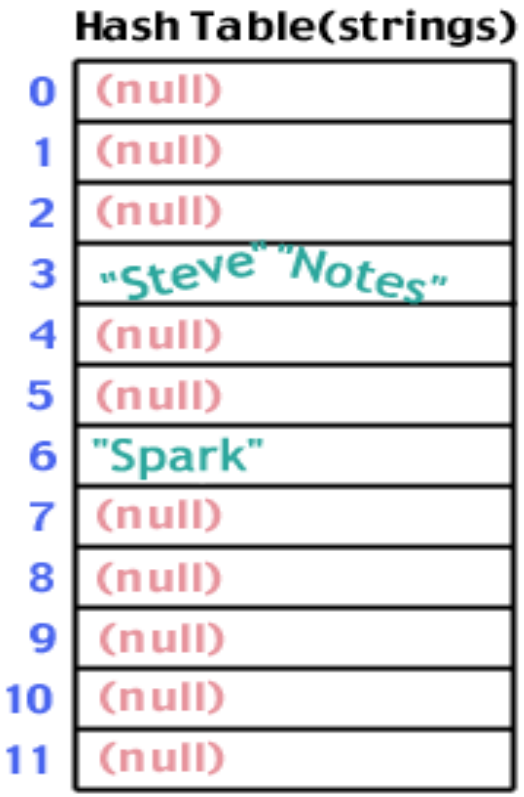


Figure : A hash table collision

Collision

- A hash function doesn't guarantee that every input will map to a different output.
- There is always the chance that two inputs will hash to the same output.
- This indicates that both elements should be inserted at the same place in the array, and this is impossible. This phenomenon is known as a collision.
- There are many algorithms for dealing with collisions.

Collision Resolution

- We now return to the problem of collisions. When two items hash to the same slot, we must have a systematic method for placing the second item in the hash table. This process is called collision resolution. As we stated earlier, if the hash function is perfect, collisions will never occur. However, since this is often not possible, collision resolution becomes a very important part of hashing.

Collision Resolution Techniques

- Open Addressing (Liner Probing).
- Chaining

Liner Probing

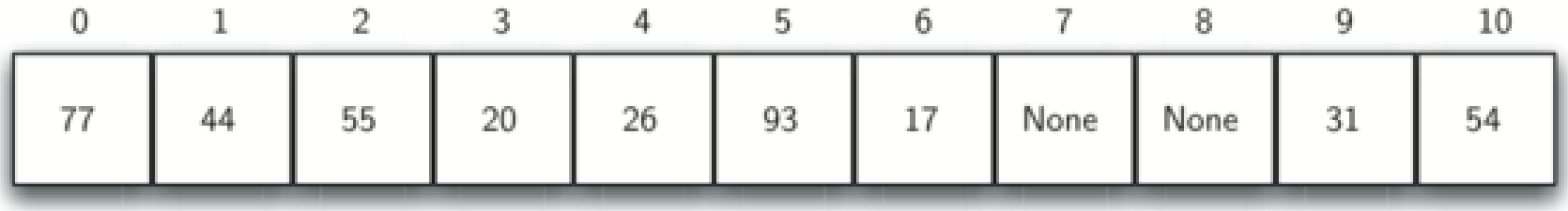
- One method for resolving collisions looks into the hash table and tries to find another open slot to hold the item that caused the collision. A simple way to do this is to start at the original hash value position and then move in a sequential manner through the slots until we encounter the first slot that is empty.

- Note that we may need to go back to the first slot (circularly) to cover the entire hash table.

- This collision resolution process is referred to as **open addressing** or **linear probing**.

Liner Probing

- Figure shows an extended set of integer items under the simple remainder method hash function (54,26,93,17,77,31,44,55,20).

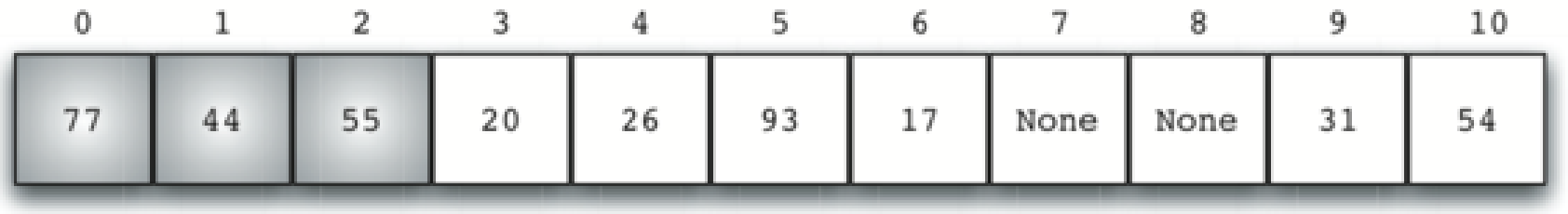


- When we attempt to place 44 into slot 0, a collision occurs. Under linear probing, we look sequentially, slot by slot, until we find an open position. In this case, we find slot 1.

Liner Probing

- A disadvantage to linear probing is the tendency for clustering, items become clustered in the table.

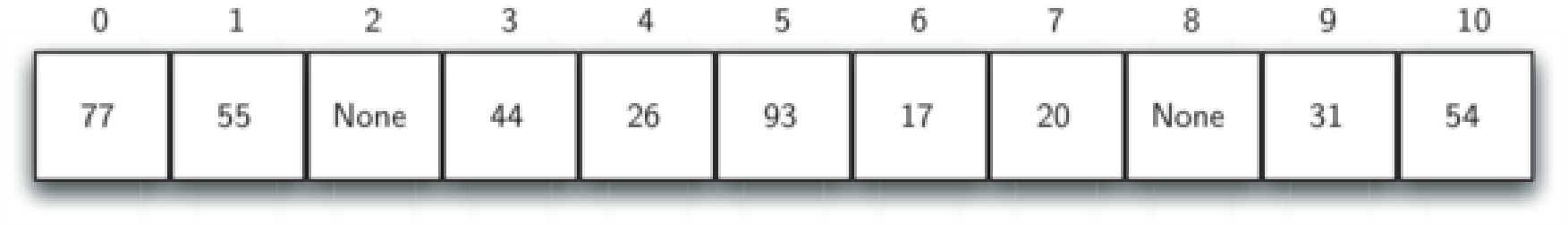
- This means that if many collisions occur at the same hash value, a number of surrounding slots will be filled by the linear probing resolution.



Liner Probing

- One way to deal with clustering is to extend the linear probing technique so that instead of looking sequentially for the next open slot, we skip slots, thereby more evenly distributing the items that have caused collisions.

- Figure shows the items when collision resolution is done with a "plus 3" probe. This means that once a collision occurs, we will look at every third slot until we find one that is empty.



Liner Probing

- The general name for this process of looking for another slot after a collision is rehashing. With simple linear probing, the rehash function is *newhashvalue = rehash (oldhashvalue)*.

  Where *rehash(pos) = (pos + 1) % sizeoftable*

  rehash(pos) = (pos + 3) % sizeoftable

  in general, *rehash(pos) = (pos + skip) % sizeoftable*
- A variation of the linear probing idea is called quadratic probing.
- Instead of using a constant "skip" value, we use a rehash function that increments the hash value by 1, 3, 5, 7, 9, and so on.

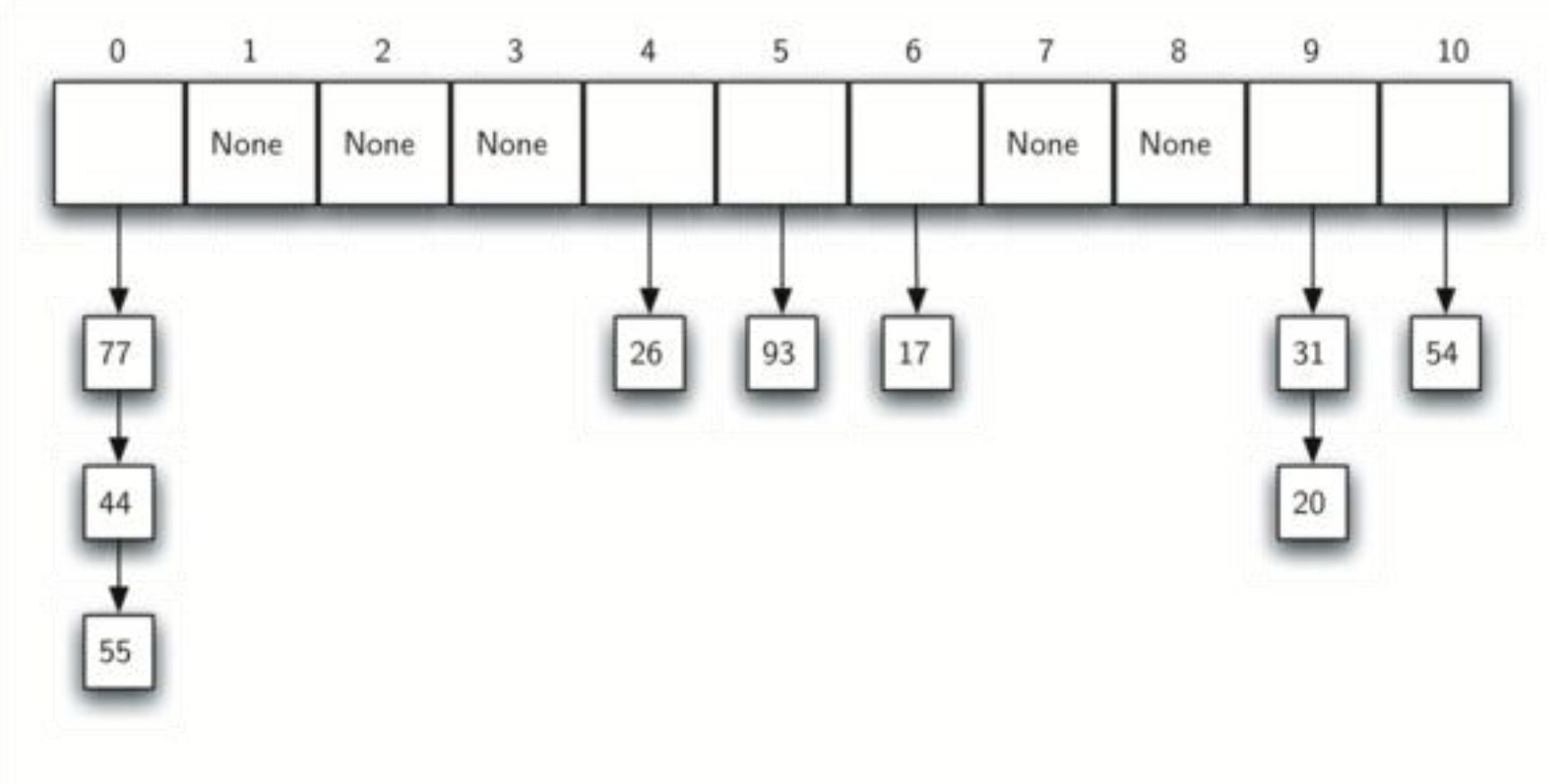
0	1	2	3	4	5	6	7	8	9	10
77	44	20	55	26	93	17	None	None	31	54

Chaining

- An alternative method for handling the collision problem is to allow each slot to hold a reference to a collection (or chain) of items.
- Chaining allows many items to exist at the same location in the hash table. When collisions happen, the item is still placed in the proper slot of the hash table.
- As more and more items hash to the same location, the difficulty of searching for the item in the collection increases.

Chaining

- Figure shows the items as they are added to a hash table that uses chaining to resolve collisions.



Chaining

- When we want to search for an item, we use the hash function to generate the slot where it should reside. Since each slot holds a collection, we use a searching technique to decide whether the item is present. The advantage is that on the average there are likely to be many fewer items in each slot, so the search is perhaps more efficient.

End of Chapter