30 时变电磁场的应用-均匀传输线（2）

-无损耗均匀传输线传播特性

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主要内容

- 无损耗均匀传输线的瞬态解
- 无损耗均匀传输线的正弦稳态解
- 含无损耗均匀传输线电路的分析

无损耗均匀传输线的传播特性

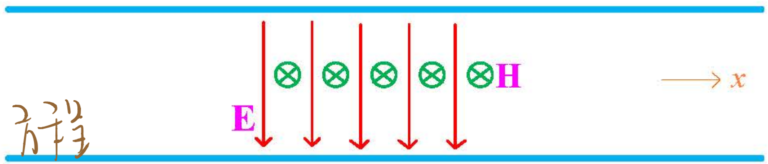
- **瞬态解**
  - 思路：根据对偶原理，由理想介质平面电磁波的方程和瞬态解直接得到无损耗均匀传输线的瞬态解
  - 对偶元素
    - $H_z \leftrightarrow I \quad E_y \leftrightarrow U \quad \varepsilon \leftrightarrow C_0 \quad \mu \leftrightarrow L_0$
  - 瞬态解及波阻抗
    - $\frac{U^+\left(t-\frac{x}{v}\right)}{I^+\left(t-\frac{x}{v}\right)} = Z_0 \quad \frac{U^-\left(t-\frac{x}{v}\right)}{I^-\left(t-\frac{x}{v}\right)} = -Z_0 \quad Z_0 = \sqrt{\frac{L_0}{C_0}}$
- **正弦稳态解**
  - 思路：对偶原理+相量法
  - 瞬态解及空间角频率
    - $\begin{cases} \dot{U}(x) = \dot{U}^+ e^{-j\beta x} + \dot{U}^- e^{j\beta x} \\ \dot{I}(x) = \dfrac{\dot{U} e^{-j\beta x} - \dot{U}^- e^{j\beta x}}{Z_0} \end{cases} \qquad \beta = w\sqrt{L_0 C_0}$
- **电路分析**
  - 已知始端（x=-l）
    - $\dot{U}(x) = \dot{U}_1 cos(\beta x + \beta l) - jZ_0 \dot{I}_1 sin(\beta x + \beta l)$
    - $\dot{I}(x) = \dot{I}_1 cos(\beta x + \beta l) - j\frac{\dot{U}_1}{Z_0} sin(\beta x + \beta l)$
  - 已知终端（x=0）
    - $\dot{U}(x) = \dot{U}_2 cos(\beta x) - jZ_0 \dot{I}_2 sin(\beta x)$
    - $\dot{I}(x) = \dot{I}_2 cos(\beta x) - j\frac{\dot{U}_2}{Z_0} sin(\beta x)$

无损耗均匀传输线的瞬态解

与平面电磁波所满足的基本方程相同

↓

通过对偶直接得到传输线以U、I为变量的方程



$$\frac{\partial^2 E_y}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 H_z}{\partial x^2} = \mu\varepsilon \frac{\partial^2 H_z}{\partial t^2}$$

$$E_y(x,t) = E_y^+(t-\frac{x}{v}) + E_y^-(t+\frac{x}{v})$$

$$H_z(x,t) = H_z^+(t-\frac{x}{v}) + H_z^-(t+\frac{x}{v})$$

$$\frac{E_y^+(t-\frac{x}{v})}{H_z^+(t-\frac{x}{v})} = Z_0 \qquad \frac{E_y^-(t-\frac{x}{v})}{H_z^-(t-\frac{x}{v})} = -Z_0$$

$$H_z(x,t) = \frac{E_y^+(t-\frac{x}{v}) - E_y^-(t+\frac{x}{v})}{Z_0}$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon}}$$

理想介质平面电磁波

$$H_z \leftrightarrow I$$

$$E_y \leftrightarrow U$$

$$\varepsilon \leftrightarrow C_0$$

$$\mu \leftrightarrow L_0$$

对偶原理

$$\frac{\partial^2 U}{\partial x^2} = L_0 C_0 \frac{\partial^2 U}{\partial t^2} \qquad \frac{\partial^2 I}{\partial x^2} = L_0 C_0 \frac{\partial^2 I}{\partial t^2}$$

$$U(x,t) = U^+(t-\frac{x}{v}) + U^-(t+\frac{x}{v})$$

$$I(x,t) = I^+(t-\frac{x}{v}) + I^-(t+\frac{x}{v})$$

$$\boxed{\frac{U^+(t-\frac{x}{v})}{I^+(t-\frac{x}{v})} = Z_0} \qquad \boxed{\frac{U^-(t-\frac{x}{v})}{I^-(t-\frac{x}{v})} = -Z_0}$$

$$I(x,t) = \frac{U^+(t-\frac{x}{v}) - U^-(t+\frac{x}{v})}{Z_0}$$

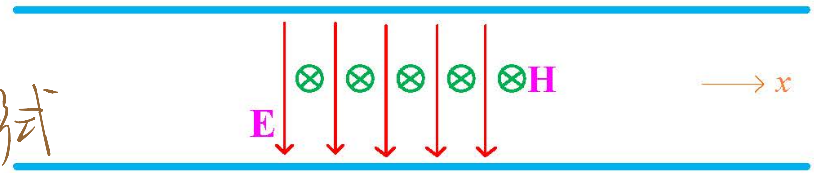
传输线的特性阻抗

$$\boxed{Z_0 = \sqrt{\frac{L_0}{C_0}}}$$

无损耗均匀传输线

无损耗均匀传输线的正弦稳态解

U、I实质上由电磁波产生，故其解与E和H一样具有入射波+反射波的形式



$$\frac{d^2\dot{E}_y}{dx^2}+\omega^2\mu\varepsilon\dot{E}_y=0$$

$$\frac{d^2\dot{H}_z}{dx^2}+\omega^2\mu\varepsilon\dot{H}_z=0$$

$$\dot{E}_y(x)=\dot{E}_y^{+}e^{-j\beta x}+\dot{E}_y^{-}e^{j\beta x}$$

$$\dot{H}_z(x)=\frac{\dot{E}_y^{+}e^{-j\beta x}-\dot{E}_y^{-}e^{j\beta x}}{Z_0}$$

$$\beta=\omega\sqrt{\mu\varepsilon}\qquad Z_0=\sqrt{\frac{\mu}{\varepsilon}}$$

理想介质平面电磁波

$$H_z\leftrightarrow I$$

$$E_y\leftrightarrow U$$

$$\varepsilon\leftrightarrow C_0$$

$$\mu\leftrightarrow L_0$$

对偶原理

$$\frac{d^2\dot{U}}{dx^2}+\omega^2L_0C_0\dot{U}=0$$

$$\frac{d^2\dot{I}}{dx^2}+\omega^2L_0C_0\dot{I}=0$$

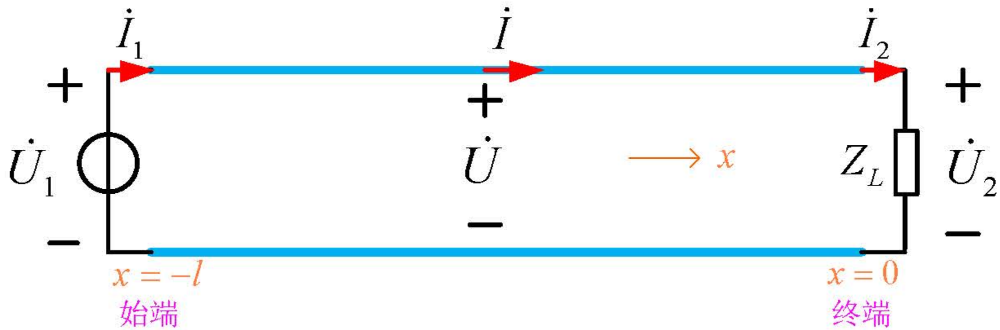
$$\dot{U}(x)=\dot{U}^{+}e^{-j\beta x}+\dot{U}^{-}e^{j\beta x}$$

$$\dot{I}(x)=\frac{\dot{U}^{+}e^{-j\beta x}-\dot{U}^{-}e^{j\beta x}}{Z_0}$$

$$\beta=\omega\sqrt{L_0C_0}\qquad Z_0=\sqrt{\frac{L_0}{C_0}}$$

无损耗均匀传输线

含无损耗均匀传输线电路的分析——已知始端的分析



$$\dot U(x)=\dot U^+e^{-j\beta x}+\dot U^-e^{j\beta x}$$

$$\dot I(x)=\frac{\dot U^+e^{-j\beta x}-\dot U^-e^{j\beta x}}{Z_0} \Rightarrow \dot I_1=\frac{\dot U^+e^{j\beta l}-\dot U^-e^{-j\beta l}}{Z_0}$$

$$\dot U(-l)=\dot U^+e^{j\beta l}+\dot U^-e^{-j\beta l}$$

$$\dot U(-l)=\dot U_1 \qquad \dot U_1=\dot U^+e^{j\beta l}+\dot U^-e^{-j\beta l}$$

$$\dot U^+=0.5\left(\dot U_1+Z_0\dot I_1\right)e^{-j\beta l}$$

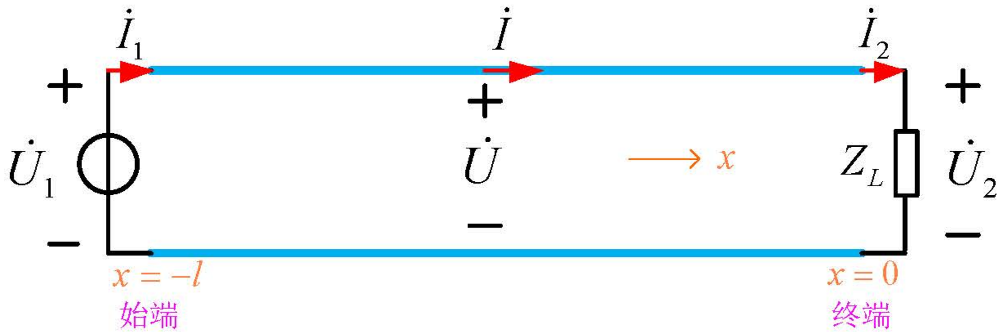
$$\dot U^-=0.5\left(\dot U_1-Z_0\dot I_1\right)e^{j\beta l}$$

$$\boxed{\dot U(x)=\dot U_1\cos\left(\beta x+\beta l\right)-jZ_0\dot I_1\sin\left(\beta x+\beta l\right)}$$

$$\boxed{\dot I(x)=\dot I_1\cos\left(\beta x+\beta l\right)-j\frac{\dot U_1}{Z_0}\sin\left(\beta x+\beta l\right)}$$

key: 求 $\dot U^+$、$\dot U^-$

含无损耗均匀传输线电路的分析——已知终端的分析



$$\dot{U}(x)=\dot{U}^{+}e^{-j\beta x}+\dot{U}^{-}e^{j\beta x}$$

$$\dot{I}(x)=\frac{\dot{U}^{+}e^{-j\beta x}-\dot{U}^{-}e^{j\beta x}}{Z_0} \Rightarrow \dot{I}_2=\frac{\dot{U}^{+}-\dot{U}^{-}}{Z_0}$$

$$\dot{U}(0)=\dot{U}^{+}+\dot{U}^{-}$$

$$\dot{U}(0)=\dot{U}_2 \qquad \dot{U}_2=\dot{U}^{+}+\dot{U}^{-}$$

$$\dot{U}^{+}=0.5\left(\dot{U}_2+Z_0\dot{I}_2\right)$$

$$\dot{U}^{-}=0.5\left(\dot{U}_2-Z_0\dot{I}_2\right)$$

$$\dot{U}(x)=\dot{U}_2\cos\beta x-jZ_0\dot{I}_2\sin\beta x$$

$$\dot{I}(x)=\dot{I}_2\cos\beta x-j\frac{\dot{U}_2}{Z_0}\sin\beta x$$

作业二十八

1. 写出含无损传输线电路的传输线上的电压、电流表达式

（分两种情况：已知始端电压电流和已知终端电压电流）