Assignment - 4

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Tutorial - 04

Ques 1: $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$C = \log_b a = \log_2 3 = n^{\log_2 3}$

$n^{\log_b a} < f(n)$

$\therefore \boxed{T(n) = \theta(n^2)}$

Ques 2: $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2$

$C = n^{\log_b a} = n^{\log_2 4} = n^{2\log_2 2} = n^2$

$n^{\log_b a} = f(n)$

$T(n) = \theta(n^2 \log n)$

Ques 3: $T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$\therefore n^{\log_b a} < 2^n$

$\therefore T(n) = \theta(2^n)$

Ques4 $T(n) = 2^n + T(n/2) + n^2$

$\therefore$ a is a function

$\Rightarrow$ Master's theorem is not possible.

Ques5: $T(n) = 16\,T(n/4) + n$

$a = 16 \quad b = 4 \quad f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$\therefore n^{\log_b a} > n$

$T(n) = \Theta(n^2)$

Ques6: $T(n) = 2\,T(n/2) + n\log n$

$a = 2 \quad b = 2 \quad f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n.$

$n\log n > n$

$\boxed{T(n) = \Theta(n\log n)}$

Ques7: $T(n) = 2\,T(n/2) + n/\log n$

$a = 2, \quad b = 2 \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$n^{\log_b a} \not> f(n)$

$T(n) = \Theta(n \not\log n)$

Ques8: $T(n) = 2\,T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{\log_b a} < f(n)$

$\therefore T(n) = \Theta(n^{0.51})$

Ques9: $T(n) = 0.5\,T(n/2) + 1/n$

$\because a < 1$

$\therefore$ Masters theorem not applicable

Ques10: $T(n) = 16\,T(n/4) + n!$

$a = 16 \quad b = 4, \quad f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$\because n^2 < n!$

$T(n) = \Theta(n!)$

Ques11: $T(n) = 4\,T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n^2)$

Ques12: $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$\because a$ is not constant

$\therefore$ Master's theorem not applicable.

Ques13: $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2 \quad f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n^{1.58})$

Ques14: $T(n) = 4T(n/2) + cn$

$a = 4 \quad b = 2 \quad f(n) = cn$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n^2)$

Ques 16: $T(n) = 3T(n/4) + n \log n$

$a = 3, b = 4, f(n) = n \log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$\because n^{\log_b a} < f(n)$

$\therefore T(n) = \Theta(n \log n)$

Ques 17: $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3, f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\because n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n)$

Ques 18: $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \quad b = 3 \quad f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{\log_b a} < f(n) \Rightarrow T(n) = \Theta(n^2 \log n)$

Ques 19: $T(n) = 4T(n/2) + n/\log n$

$a = 4 \quad b = 2 \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^{\log_b a} > f(n)$

$\therefore T(n) = \Theta(n^2)$

Ques 20: $T(n) = 64\,T(n/8) - n^2 \log n$

master's theorem is not applicable as $f(n)$ is not increasing function

Ques 21: $T(n) = 7\,T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.3}$

$n^{\log_b a} < f(n)$

$\Rightarrow T(n) = \Theta(n^2)$

Ques 22: $T(n) = T(n/2) + n(2 - \cos n)$

→ Master's Theorem is not applied

∵ regularity condition is isolated in case 3.