Assignment-2

Name - Vibha Rani
Rollno - 45
Section - CE [Computer Engineering]
University Rollno - 2017395

Tutorial-3

Ques1 → Find time complexity

```
void func (int n)
{
    int j = 1 ; i = 0;
    while ( i < n)
    {
        i = i + j;
        j++;
    }
}
```

j	i
j = 1	i = 1
j = 2	i = 1+2 = 3
j = 3	i = 3+3 = 6
j = 4	i = 6+4 = 10
	!
j = k	i = (k-1)+k

$$S = 1 + 3 + 6 + 10 + \cdots (k-1) + k$$

$$S = \underline{1} + \underline{3} + \underline{6} + \cdots \underline{+(k-1)} \underline{+k}$$

$$0 = 1 + 2 + 3 + 4 + \cdots n - k$$

$$k = 1 + 2 + 3 + 4 + \cdots k$$

$$T(n) = \frac{k(k+1)}{2}$$

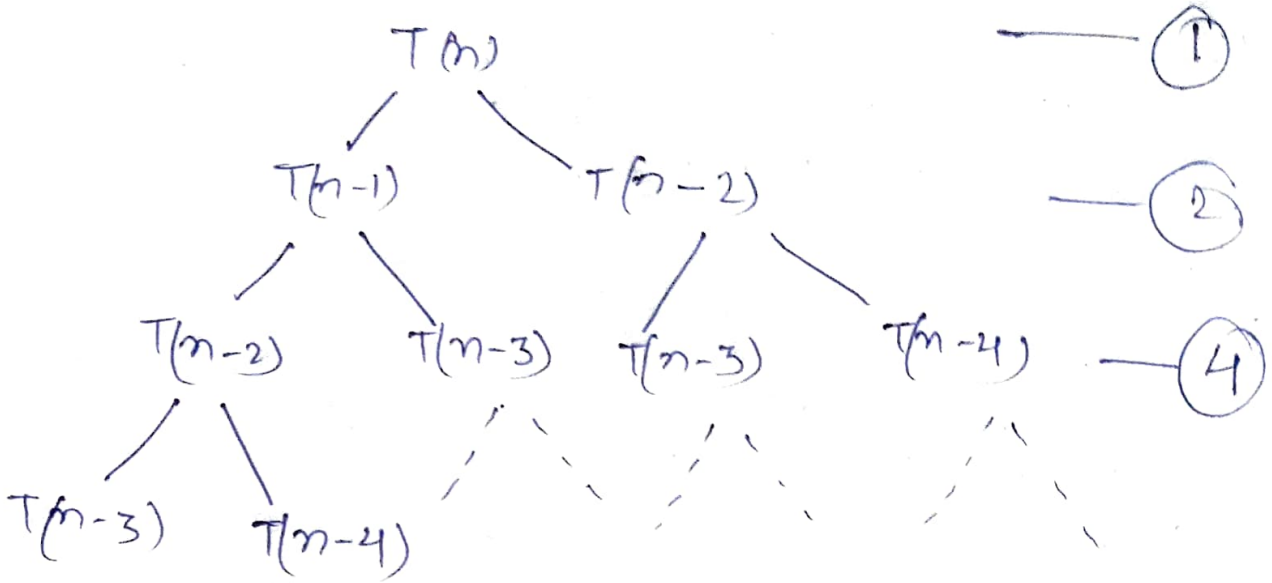
$$\frac{k^2 + k}{2} < n$$

$$k < \sqrt{n}$$

$$T(n) = O(\sqrt{n})$$

Ques 3 Recursive relation for fibonacci series:

$T(n) = T(n-1) + T(n-2)$



$\Rightarrow 1 + 2 + 4 + 8 + \cdots$

$a = 1 \quad r = 2$

$= \frac{a(r^n - 1)}{r - 1} = \frac{1(2^n - 1)}{1}$

$T(n) = O(2^n)$

Ques 3: Write programs having following complexity

(i) $n(\log n)$

```
void quick_sort(int a[], int lb, int ub)
{
    int i = lb, j = ub;
    int key = a[lb];
    int t = 0;
    if (lb >= ub)
        return;
    while (i < j)
    {
        while (key >= a[i] && i < j)
            i++;
```

```
while (key < a[j])
    j--;
if (i < j)
{
    t = a[i];
    a[i] = a[j];
    a[j] = t;
}
}
a[lb] = a[j];
a[j] = key;
quick_sort(a, 0, j-1);
quick_sort(0, j+1, ub);
}
```

(ii) $T(n) = O(n^3)$

```
for (int i=0; i<n; i++)
{
    for (int j=0; j<n; j++)
    {
        for (int k=0; k<n; k++)
        {
            sum += k;
        }
    }
}
```

(iii) O log(log n)

```
int p=0;
for(int i=1; i<n; i=i*2)
{
    p++;
}
for(int j=1; j<p; j=j*2)
{
    //O(1) operation.
}
```

Ques 4- $T(n) = T(n/4) + T(n/2) + n^2$

$2T(n/2) + n^2$

Using Master's Method

$T(n) = aT(n/b) + f(n)$

$a = 2$

$b = 2$

$c = \log_b a = 1$

$\boxed{T(n) = f(n) = O(n^2)}$

Ques 5 i = 1 2 3 4 ... n

j = 1, 2, 3, ... n times

$T(n) = n + n/2 + n/3 + n/4 + \cdots + 1$

$n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\right]$

$T(n) = O(n \log n)$

Ques 6 $T(n) = 2, 2^k, 2^{k^2}, 2^{k^3} \ldots 2^{k \log_k(\log_k n)}$

So $2^{k \log_k \log_k n} = 2^{\log n} = n$

So Total Time complexity

$T(n) = O \log_k (\log n)$

Ques 7

(i) $100 < \log(\log n) < \log n < \log^2 n < \sqrt{n} < n <$
$n \log n < n^2 < 2^n < 4^n < 2^{2^n} < \log(n!) < n!$

(ii) $1 < \log(\log n) < \sqrt{\log n} < \log n < \log 2n < 2 \log n$
$< n < 2n < 4n < n \log n < n^2 < \log(n!) < n! <$
$2(2^n)$

(iii) $96 < \log_2(n) < \log_2 n < 5n < n \log_2 n < n \log_2 n$
$n! < \log n! < 8^{2^n}$.