భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Deep Learning

08 Training DNNs - I

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Dept. of Artificial Intelligence
IIT Hyderabad
Jan-May 2025

So far...

- Artificial Neuron models - MP neuron, Perceptron

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- Universality

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- Artificial Neuron models - MP neuron, Perceptron
- Universality
- Gradient Descent (Backprop)

Issues with SGD

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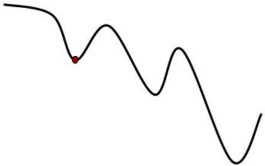
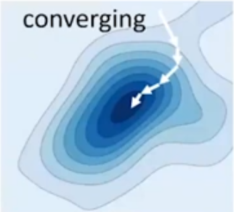
  - May have local minima
  - May have saddle points



Stuck at a local minimum

Stuck at a saddle point

Convergence of Gradient Descent



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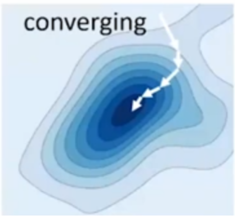

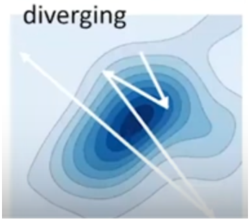

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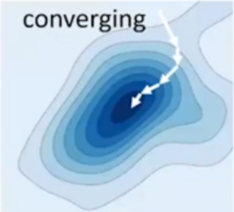
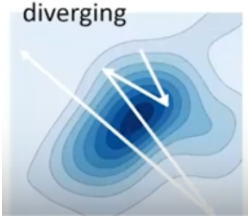
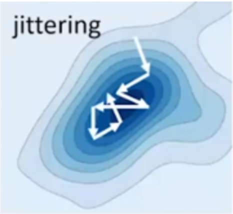






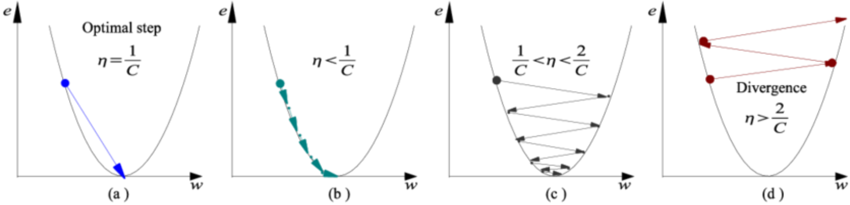
Convergence of Gradient Descent

- When does it diverge?

Convergence of Gradient Descent

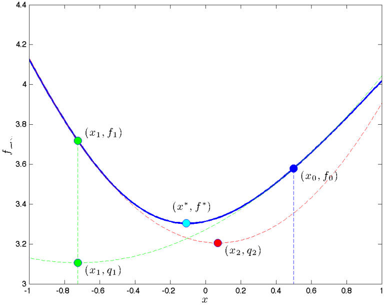
- When does it diverge?
- How to ensure smooth convergence? (Conditions for convergence)

Convergence for Quadratic functions



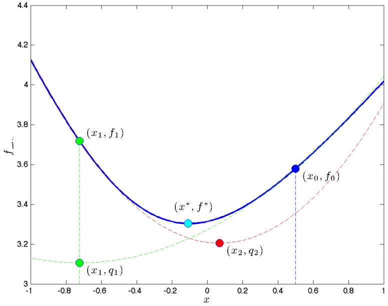
In case of generic and convex functions

- Perform a quadratic approximation



In case of generic and convex functions

- Perform a quadratic approximation
- $\eta_{opt} = \frac{1}{f''}$ (Newton's Method)



Multivariate functions

- $f(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{b} + \mathbf{c}$

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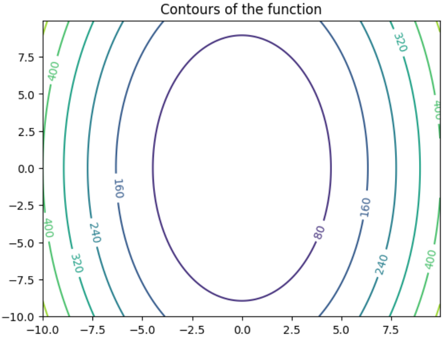
Multivariate functions

- $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x} + \mathbf{x}^{\mathbf{T}}\mathbf{b} + \mathbf{c}$
- For convex functions, A is positive definite
- (For simplicity) If A is diagonal ($+$ve entries for convex f), then f is sum of multiple quadratic functions

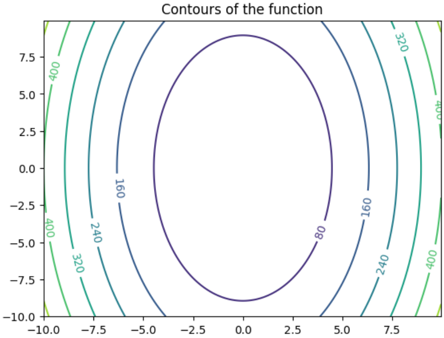
Multivariate functions

- Optimization gets decoupled (each component can be optimized independently)



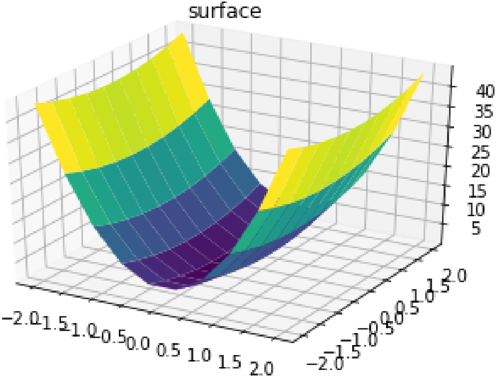
Multivariate functions

- Optimization gets decoupled (each component can be optimized independently)
- Optimal Learning rate is different for different components



Issues with SGD

- DNNs are trained via SGD: $w_{t+1} = w_t - \eta \cdot \nabla_w J(w)$
- Loss is a high dimensional function
  - May vary swiftly in one direction and slowly in the other



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- Learning rate must be smaller than the twice the smallest optimal learning rate $\eta < 2 \cdot \eta_{min}$

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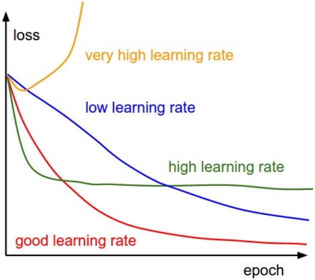
- Learning rate must be smaller than the twice the smallest optimal learning rate $\eta < 2 \cdot \eta_{min}$
- Else, it may diverge
- This makes the convergence slow (and oscillate in some directions)

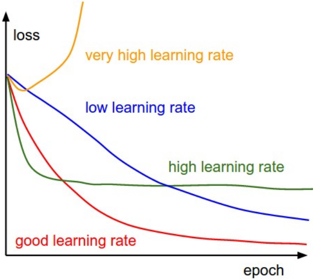
Learning rate (lr)



- What lr to use?

Figure credits: CS231n-Standford

Learning rate (lr)



- What lr to use?
- Different lr at different stages of the training!

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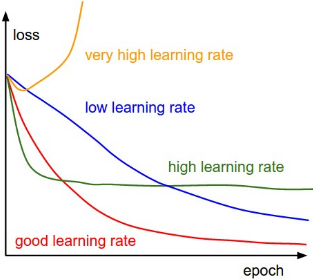
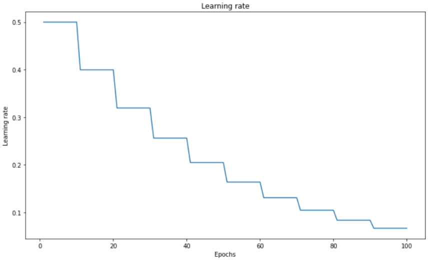


- What lr to use?
- Different lr at different stages of the training!
- Start with high lr and reduce it with time

Figure credits: CS231n-Standford

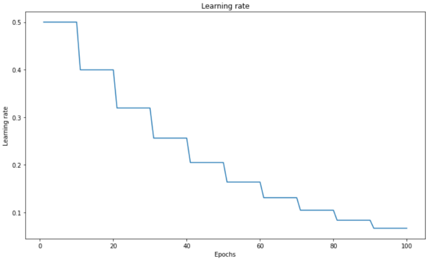
Learning Rate decay: Step



1. Reduce the lr after regular intervals

Figure credits: Katherine Li

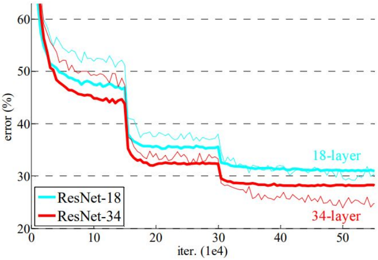
Learning Rate decay: Step



1. Reduce the lr after regular intervals
2. E.g. after every 30 epochs, $\eta* = 0.1 \cdot \eta$

Figure credits: Katherine Li

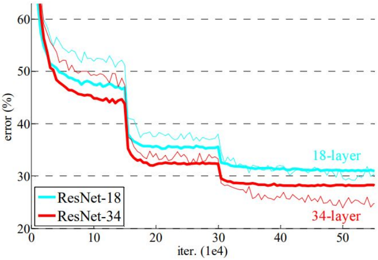
Learning Rate decay: Step



1. Characteristic loss curve: different phases for ''stage'

Figure credits: Kaiming He et al. 2015, ResNets

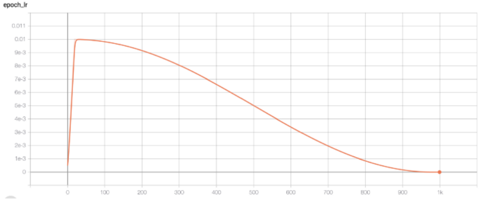
Learning Rate decay: Step



1. Characteristic loss curve: different phases for ''stage'
2. Issues: annoying hyper-params (when to reduce, by how much, etc.)

Figure credits: Kaiming He et al. 2015, ResNets

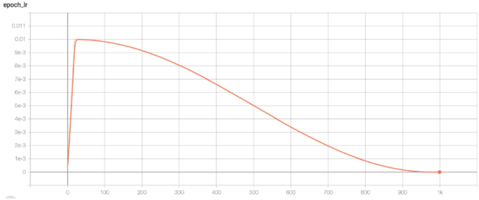
Learning Rate decay: Cosine



1. Reduces the lr continuously
$\eta_t = \frac{1}{2}\eta_0(1+cos(t\pi/T))$

Figure credits: Sebastian Correa and Medium.com

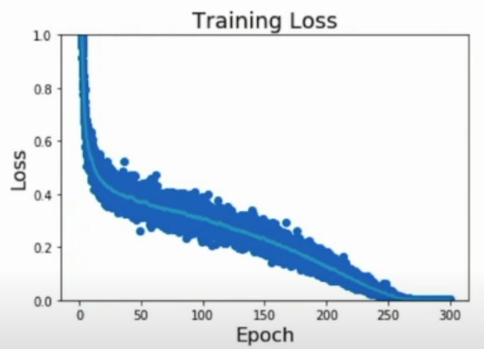
Learning Rate decay: Cosine



1. Reduces the lr continuously $\eta_t = \frac{1}{2}\eta_0(1+cos(t\pi/T))$
2. Less number of hyper-parameters

Figure credits: Sebastian Correa and Medium.com

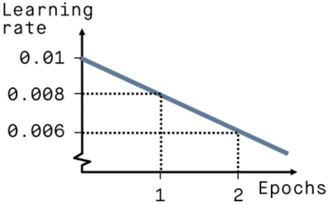
Learning Rate decay: Cosine



1. Training longer tends to work, but initial lr is still a tricky one

Figure credits: Dr Justin Johnson, U Michigan

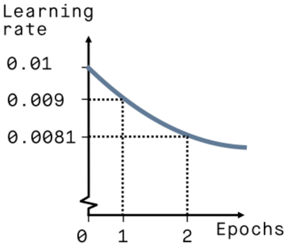
Learning Rate decay: Linear



1. $\eta_t = \eta_0(1 - t/T)$

Figure credits: peltarion.com

Learning Rate decay: Exponential



1. $\eta_t = \eta_0 \cdot (1 - \alpha/100)^t$

Figure credits: peltarion.com

Learning Rate decay: Constant lr

1. No change in the learning rate
   $\eta_t = \eta_0$

Learning Rate decay: Constant lr

1. No change in the learning rate
   $\eta_t = \eta_0$
2. Works for prototyping of ideas (other schedules may be better for squeezing in those 1-2% of gains in the performance)

Issues with SGD

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- SGD leads to jitter along the deep dimension and slow progress along the shallow one

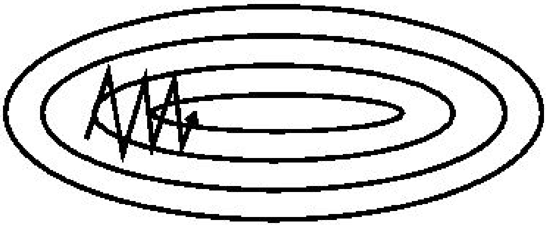

Figure credits: Sebastian Ruder

SGD+Momentum

SGD

$$w_{t+1} = w_t - \eta \cdot \nabla_w J(w)$$

SGD+Momentum

$$\begin{aligned} v_0 &= 0 \\ v_{t+1} &= \rho \cdot v_t + \nabla_w J(w) \\ w_{t+1} &= w_t - \eta \cdot v_{t+1} \end{aligned}$$

I Sutskever et al., ICML 2013

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- Aggregates velocity: exponential moving average over gradients

I Sutskever et al., ICML 2013

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- Aggregates velocity: exponential moving average over gradients
- ρ is the friction (typically set to 0.9 or 0.99)

I Sutskever et al., ICML 2013

SGD+Momentum

SGD

$$w_{t+1} = w_t - \eta \cdot \nabla_w J(w)$$

```
for i in range(num_iters):
→dw = grad(J, W, x, y)
→ w -= η · dw
```

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$$v_0 = 0$$
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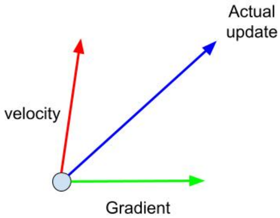
$$w_{t+1} = w_t - \eta \cdot v_{t+1}$$

```
v0 = 0
for i in range(num_iters):
→dw = grad(J, W, x, y)
→ v = ρ · v + dw
→ w -= η · v
```

I Sutskever et al., ICML 2013

SGD+Momentum

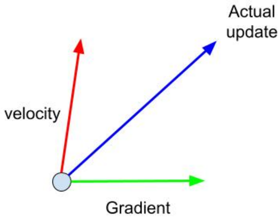


Momentum Update

1. How can momentum help?

I Sutskever et al., ICML 2013

SGD+Momentum



Momentum Update

1. How can momentum help?
   - Optimization proceeds even at the local minimum or saddle point (because of the accumulated velocity)

I Sutskever et al., ICML 2013

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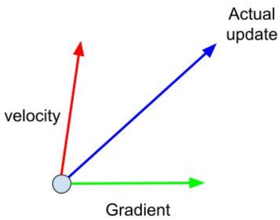

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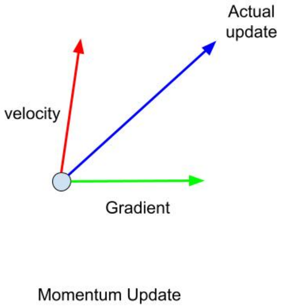
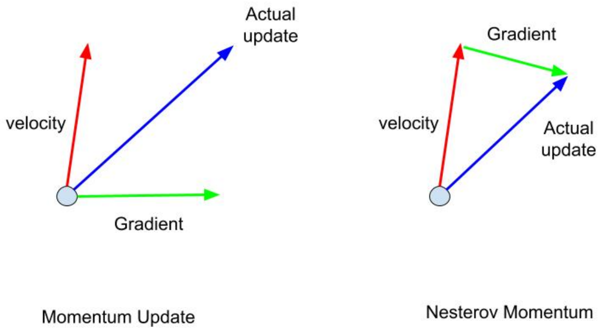


1. How can momentum help?
   - Optimization proceeds even at the local minimum or saddle point (because of the accumulated velocity)
   - Jitter is reduced in ravine like loss surfaces
   - Updates are more smoothed out (less noisy because of the exponential averaging)

I Sutskever et al., ICML 2013

Nesterov Momentum

1. Look ahead with the velocity, then take a step in the gradient's direction



I Sutskever et al., ICML 2013

Nesterov Momentum

$v_0 = 0$

```
for i in range(num_iters):
```

$\rightarrow$`dw = grad(`$J, W + \rho \cdot v, x, y$`)`

$\rightarrow v = \rho \cdot v + dw$

$\rightarrow w- = \eta \cdot v$

NAG allows to change velocity in a faster and more responsive way (particularly for large values of ρ)

I Sutskever et al., ICML 2013

Ada Grad

1. Goal: Adaptive (or, per-parameter) learning rates are introduced

---

Duchi et al. 2011, JMLR

Ada Grad

1. Goal: Adaptive (or, per-parameter) learning rates are introduced
2. Parameter-wise scaling of the learning rate by the aggregated gradient

Duchi et al. 2011, JMLR

Ada Grad


```
 grad_sq = 0
for i in range(max_iters):
→ dw = grad(J,w,x,y)
→grad_sq += dw ⊙ dw
→ w -= η · dw/(sqrt(grad_sq) + ϵ)
```

Duchi et al. 2011, JMLR

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- Smaller (larger) updates for parameters associated with frequently (infrequently) occurring features

Duchi et al. 2011, JMLR

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- Smaller (larger) updates for parameters associated with frequently (infrequently) occurring features
- well-suited for dealing with sparse data

Duchi et al. 2011, JMLR

RMS Prop

1. If Ada Grad is run for too long
   - the gradients accumulate to a big value
   - $\rightarrow$ update becomes too small (or, learning rate is reduced continuously)

RMS Prop

1. If Ada Grad is run for too long
    - the gradients accumulate to a big value
    - $\rightarrow$ update becomes too small (or, learning rate is reduced continuously)
2. RMS prop (a leaky version of Ada Grad) addresses this using a friction coefficient (ρ)

RMS Prop

grad_sq = 0

for i in range(max_iters):

$\rightarrow$ dw = grad(J,w,x,y)

$\rightarrow$grad_sq = $\rho\cdot$ grad_sq + $(1-\rho)\cdot$ dw $\odot$ dw

$\rightarrow w -= \eta \cdot dw/($sqrt(grad_sq) $+ \epsilon)$

Adam

1. Inculcates both the good things: momentum and the adaptive learning rates
   Adam = RMSProp + Momentum

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1. Inculcates both the good things: momentum and the adaptive learning rates
   Adam = RMSProp + Momentum
2. $m1 = 0$
   $m2 = 0$
   `for i in range(max_iters):`
   $\rightarrow$ `dw = grad(J,w,x,y)`
   $\rightarrow$ $m1 = \beta_1 \cdot m1 + (1 - \beta_1) \cdot dw$
   $\rightarrow$ $m2 = \beta_2 \cdot m2 + (1 - \beta_2) \cdot dw^2$
   $\rightarrow$ $w- = \eta \cdot m1/(\mathtt{sqrt}(m2) + \epsilon)$

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2. Bias correction is performed (since the estimates start from 0)

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   $\rightarrow w- = \eta \cdot m1/(\texttt{sqrt}(m2) + \epsilon)$
2. Bias correction is performed (since the estimates start from 0)
3. Adam works well in practice (mostly with a fixed set of values for the hyper-params)

Many more variants exist

1. AMSGrad
2. Nadam
3. AdaMax
4. AdaDelta