MODULE - 1

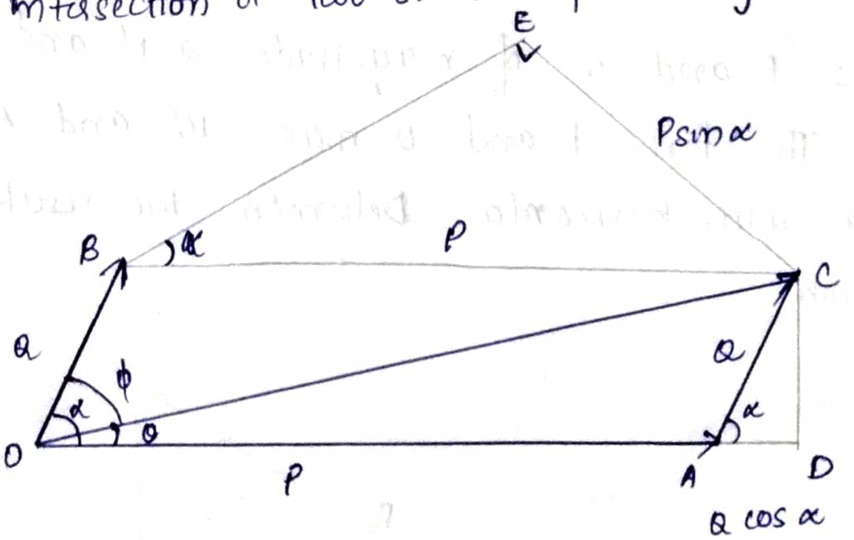
Introduction To Mechanics

- Mechanics
  - Statics
    - study of particle and rigid bodies at rest
  - Dynamics
    - Kinematics
    - Kinetics

STATICS

PARALLELOGRAM LAW

If two forces acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of llgm which passes through the point of intersection of two sides representing the forces.



$$OC^2 = OD^2 + DC^2$$

$$= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2 + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha \Rightarrow \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

$$\boxed{\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P + Q\cos\alpha} = \frac{\sin\alpha}{\frac{P}{Q} + \cos\alpha}}$$

case (i) $\alpha = 0^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{P^2 + Q^2 + 2PQ}$$

$$R = (P + Q)$$

case (ii) $\alpha = 90^\circ$

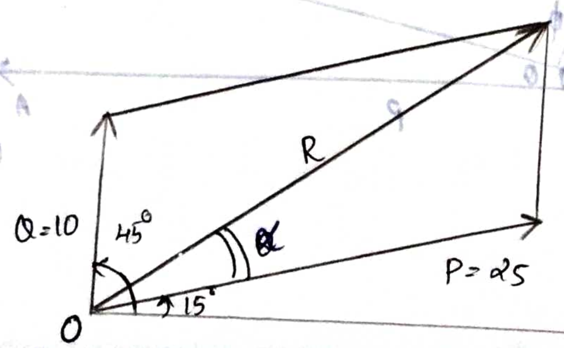
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ} = \sqrt{P^2 + Q^2}$$

case (iii) $\alpha = 180^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ} = \sqrt{P^2 + Q^2 - 2PQ}$$

$$R = P - Q$$

1) Two forces P and Q of magnitude 25N and 10N are acting at a point. The forces P and Q make 15° and 45°, measured anticlockwise with horizontal. Determine the resultant in magnitude and direction.



$P = 25N$ $\quad Q = 10N$ $\quad \alpha = 45 - 15 = 30^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} = \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \times \cos 30^\circ}$$

$$= \sqrt{725 + 500\cos 30^\circ} = \underline{34.03N}$$

The inclination of resultant force with direction of force P

$$\theta = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}} = \tan^{-1}\frac{\sin 30^\circ}{\cos 30^\circ + 2.5}$$

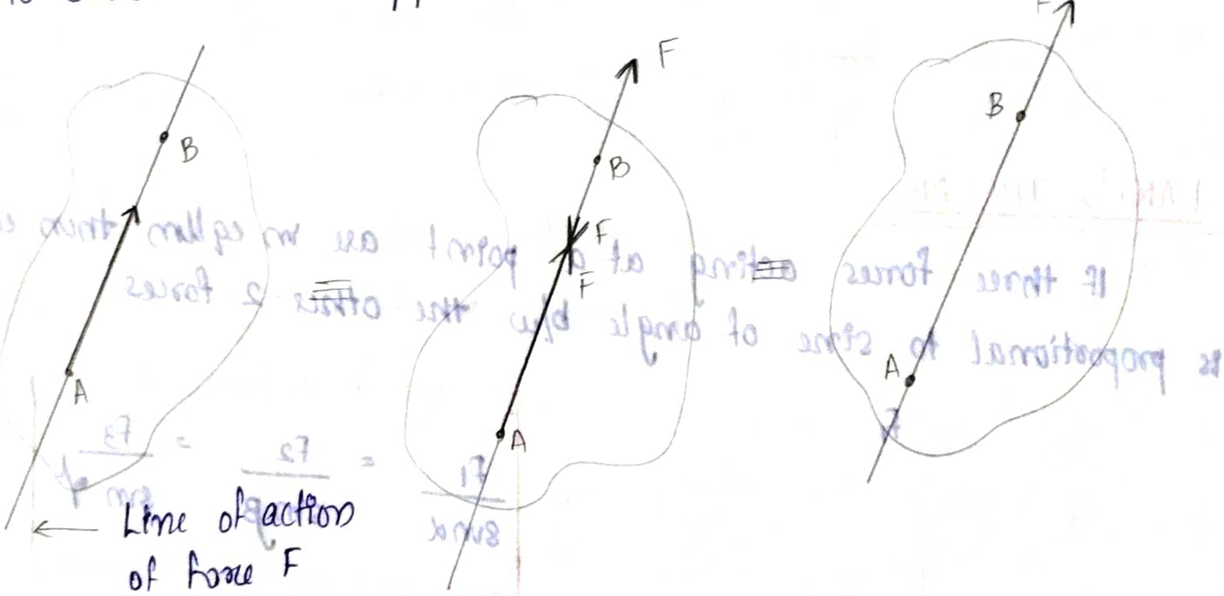
$$\theta = \underline{8.45^\circ}$$

$\therefore$ Inclination with horizontal is $15^\circ + \theta$

$$= 15 + 8.45 = \underline{23.45^\circ}$$

PRINCIPLE OF TRANSMISSIBILTY

The point of application of force can be transmitted along its line of action without changing the effect of the force on any rigid body to which its is applied.



EQUILIBRIUM LAWS

Equilibrium :- Condition in which the resultant of all forces & moments acting on the body is zero. $F = 0$, $M = 0$

For a two body force system

Forces must be

→ Equal in magnitude

→ opp. in direction

→ collinear

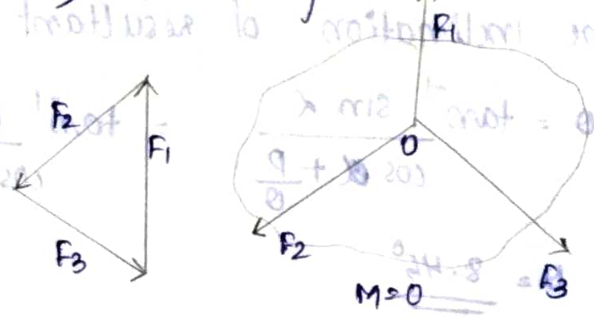
• For a 3 force body system

Forces must be

→ concurrent

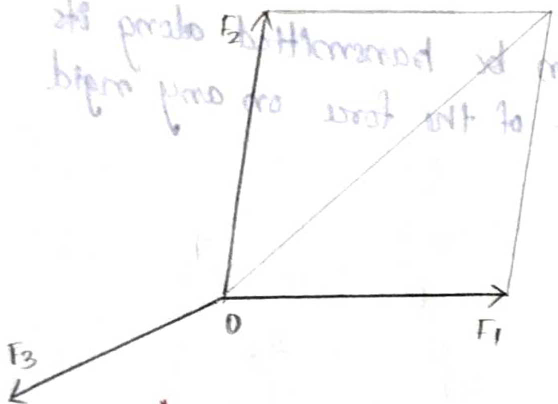
→ line of action should meet at a point

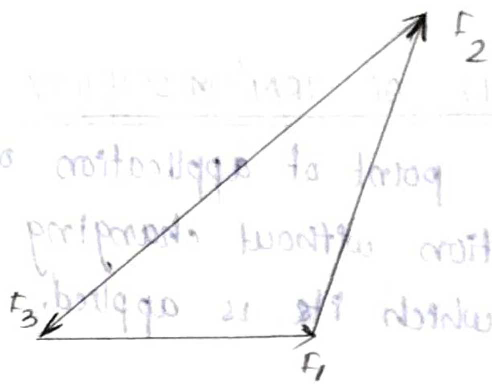
→ sum of any 2 force = 3rd force



LAW OF TRIANGLE OF FORCES

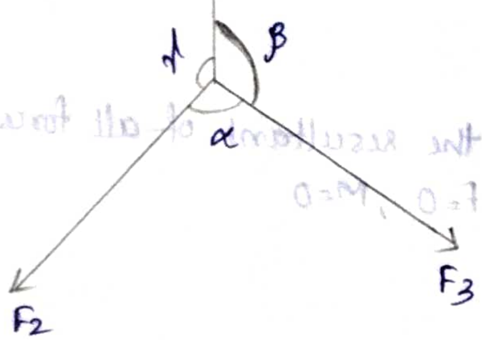
If three coplanar forces acting at a point are in eqllm, then they can be represented in magnitude and direction by the sides of a Δ taken in same order.





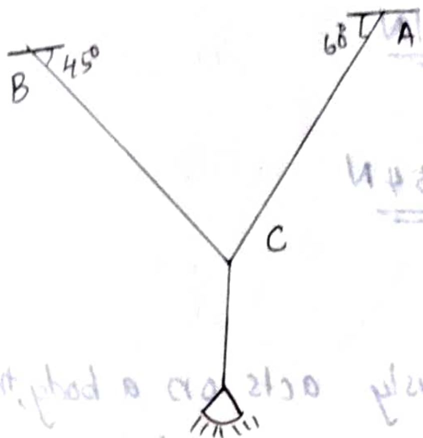
LAMI'S THEORM

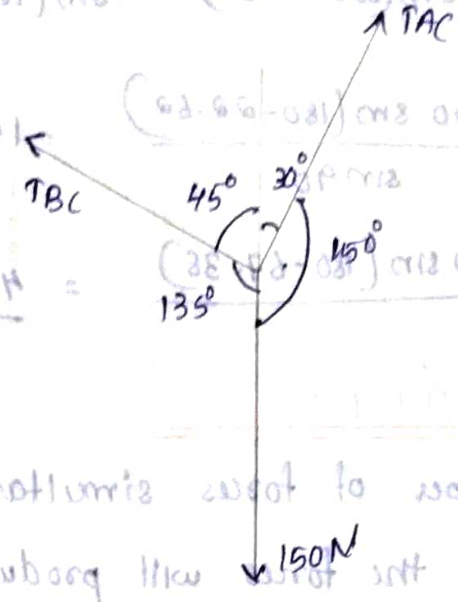
If three forces acting at a point are in eqllm then each force is proportional to sine of angle b/w the other 2 forces



$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

(2) An electric light fixture weighing 150N hangs from a point C by two stay wires AC and BC as shown. Determine tensions in AC and BC using Lami's



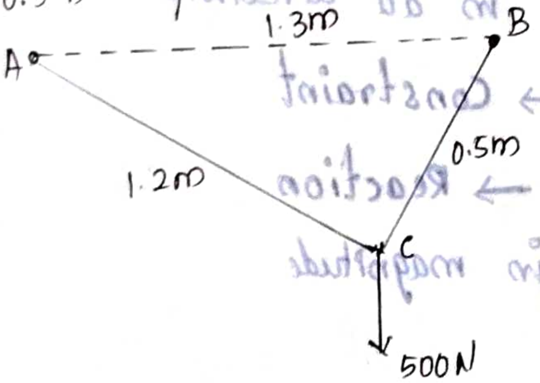


Using Lami's theorm,

$$\frac{150}{\sin 75} = \frac{T_{BC}}{\sin 150°} = \frac{T_{AC}}{\sin 135°}$$

$T_{BC} = \frac{150 \times \sin 150}{\sin 75} = 77.65N$ $\qquad$ $T_{AC} = \frac{150 \times \sin 135}{\sin 75} = 109.81N$

(2) Two cables AC and BC are tied together at the point C to support a load of 500N at C. A and B are at a distance of 1.3m and are on the same horizontal plane. AC and BC are 1.2m and 0.5m respectively. Find the tensions in cable AC and BC.



$AC^2 + BC^2 = AB^2$
$= 1.2^2 + 0.5^2 = 1.3^2$

$\because AC^2 + BC^2 = AB^2$, angle $ACB = 90°$ $[\theta + \phi = 90°]$

$\theta = \cos^{-1} \frac{1.2}{1.3} = 22.62°$ $\qquad$ $\phi = 90 - \theta = 90 - 22.62 = 67.38°$

Applying Lami's theorem

$$\frac{500}{\sin(\phi+\theta)} = \frac{T_{AC}}{\sin(180-\theta)} = \frac{T_{BC}}{\sin(180-\phi)}$$

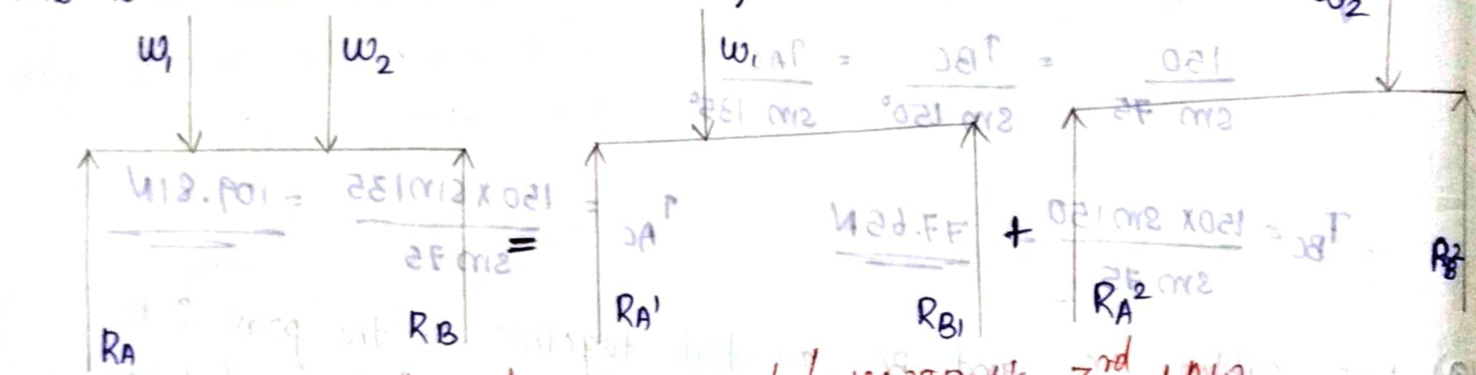
$$\frac{500}{\sin 90} = \frac{T_{AC}}{\sin(180-22.62)} = \frac{T_{BC}}{\sin(180-67.38)}$$

$$T_{AC} = \frac{500 \sin(180-22.62)}{\sin 90} = 192.31N$$

$$T_{BC} = \frac{500 \sin(180-67.38)}{} = 461.54N$$

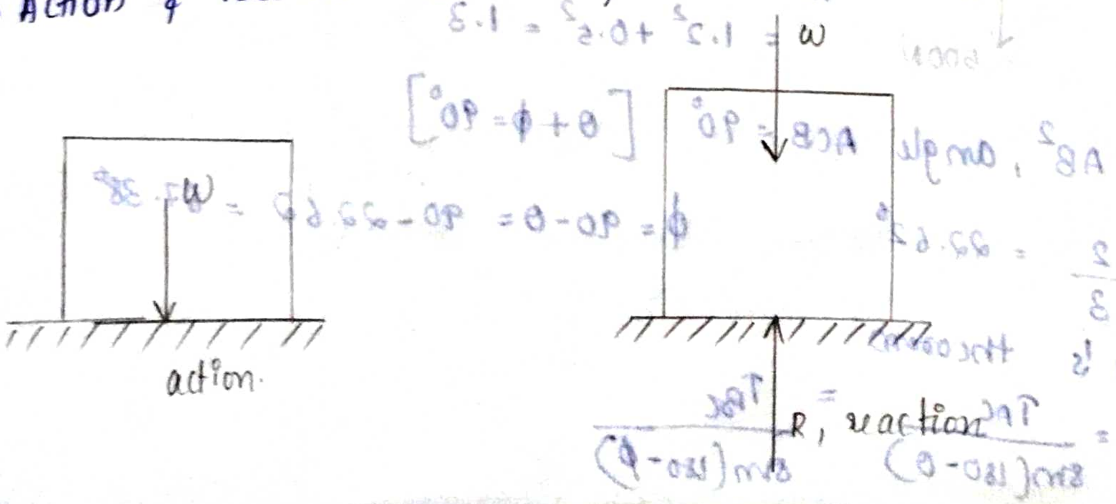
PRINCIPLE OF SUPERPOSITION

If a number of forces simultaneously acts on a body, then each ~~other~~ one of the forces will produce the same effect, when this force acts alone in the body.

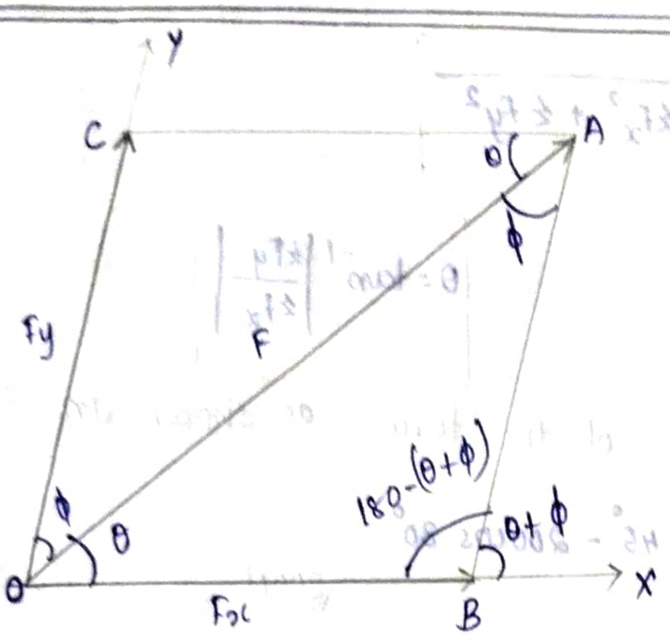


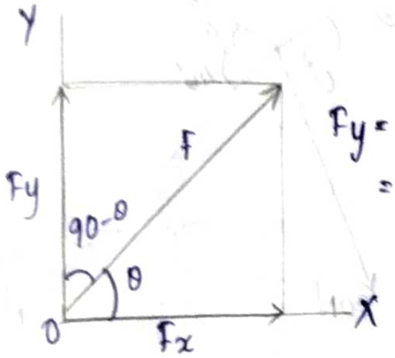
LAWS OF ACTION AND REACTION / NEWTON'S 3rd LAW

- Newton's third law :- To every action, there is an equal & opp reaction.
- A body is not always free to move in all directions
- Restriction to free motion of a body → Constraint
- Force applied by a surface on the body → Reaction
- Action & reaction are always equal in magnitude



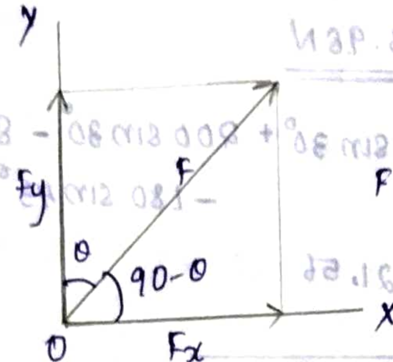
COMPOSITION & RESOLUTION OF FORCES





$F_y = F\cos(90-\theta)$

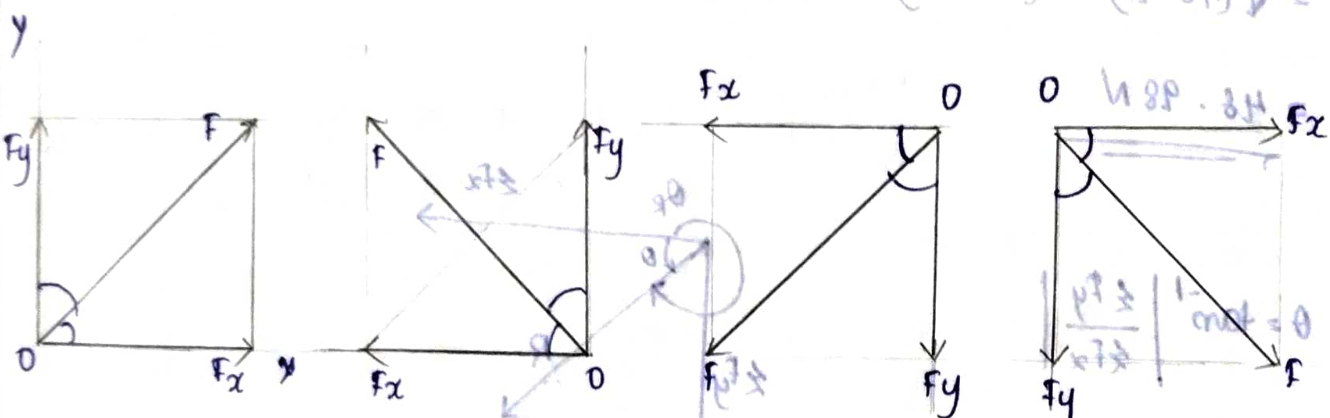
$= F\sin\theta$



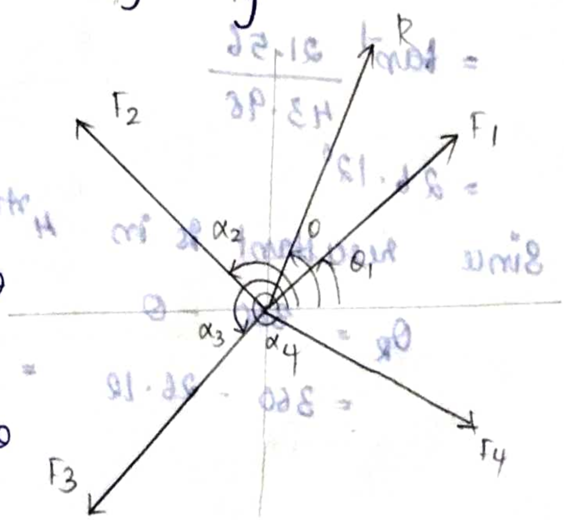
$F_y = F\cos\theta$

$F_x = F\,\cancel{\sin}\cos(90-\theta)$

$= F\sin\theta$



RESULTANT OF CO-PLANAR CONCURRENT FORCES



$F_1\cos\alpha_1 + F_2\cos\alpha_2 + F_3\cos\alpha_3 + F_4\cos\alpha_4 = R\cos\theta$

$\Sigma F_x = R\cos\theta$

$F_1\sin\alpha_1 + F_2\sin\alpha_2 + F_3\sin\alpha_3 + F_4\sin\alpha_4 = R\sin\theta$

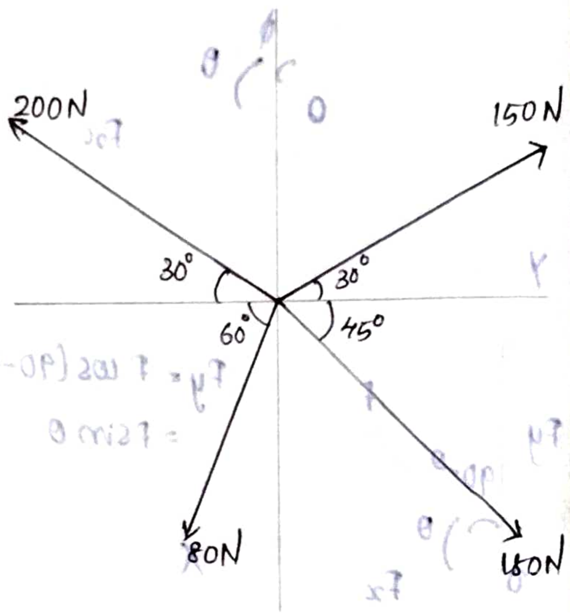
$\Sigma F_y = R\sin\theta$

$$\Sigma F_x^2 + \Sigma F_y^2 = R^2\cos^2\theta + R^2\sin^2\theta = R^2(\sin^2\theta + \cos^2\theta)$$

$$\boxed{R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}}$$

$$\tan\theta = \frac{|\Sigma F_y|}{|\Sigma F_x|} \qquad \boxed{\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|}$$

(3) Find the resultant of the forces as shown in fig.



$$\Sigma F_x = 15\cos 30^\circ + 180\cos 45^\circ - 200\cos 30^\circ - 80\cos 60^\circ$$

$$= \underline{43.98\,N}$$

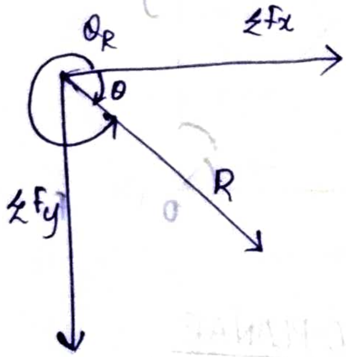
$$\Sigma F_y = 150\sin 30^\circ + 200\sin 30^\circ - 80\sin 60^\circ - 180\sin 45^\circ$$

$$= -21.56$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{(43.98)^2 + (-21.56)^2}$$

$$= \underline{48.98\,N}$$



$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{21.56}{43.98}$$

$$= 26.12^\circ$$

Since resultant is in 4th quadrant, the inclination of resultant

$$\theta_R = 360 - \theta$$

$$= 360 - 26.12 = \underline{333.88^\circ}$$

06/05/21

EQUILIBRIUM EQUATIONS

- A particle / body is in equilibrium if the resultant of no of forces acting on it is zero.
- If $R \neq 0$, Force required to bring the body to rest

  :- EQUILIBRIUM

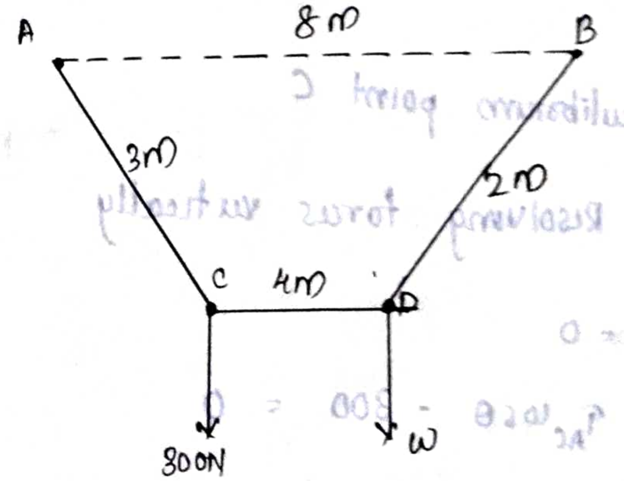
- Resultant & Equilibrium are in equal in magnitude and opposite in direction

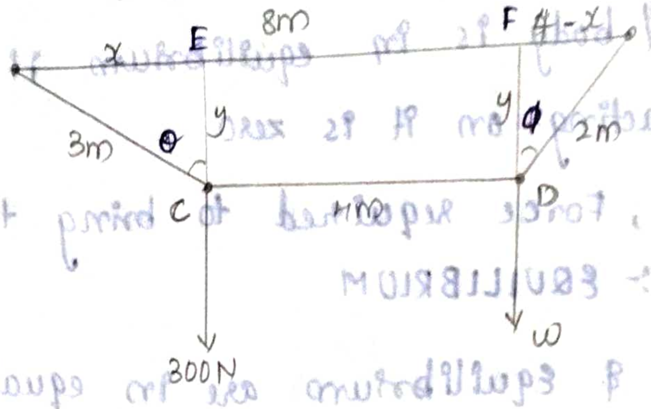
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$, where ΣF_x and ΣF_y are the sum of components of all the forces along two mutually $\perp^r$ x and y directions.

For R to be zero, both ΣF_x and ΣF_y must be zero. therefore the equation of equilibrium are,

1) $\Sigma F_x = 0$ 2) $\Sigma F_y = 0$

4 A rope 9m long is connected at A and B, two points on same level 8m apart. A load of 300N is suspended from a point fra C on rope 3m from A. What load connected to a point D, on the rope, 2m from B is necessary to keep CD parallel to AB.





From ΔAEC,

$$y^2 = 3^2 - x^2$$

From ΔBDF

$$y^2 = 2^2 - (4-x)^2$$

$$3^2 - x^2 = 2^2 - (16 + x^2 - 8x)$$

$$9 - x^2 = 4 - 16 - x^2 + 8x \Rightarrow 8x = 21 \Rightarrow x = 2.625m$$

$$\sin\theta = \frac{x}{3} = \frac{2.625}{3} = 0.875$$

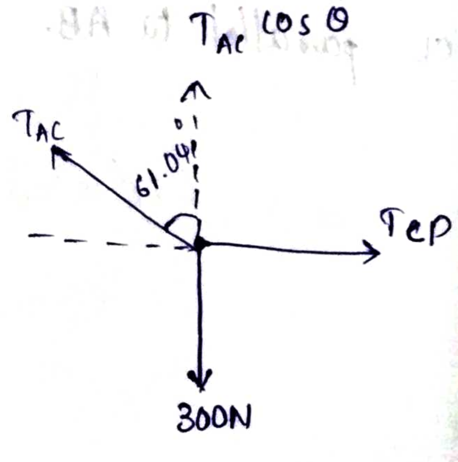
$$\theta = \sin^{-1} 0.875 = 61.04^\circ$$

$$\sin\phi = \frac{4-x}{2} = \frac{4-2.625}{2} = \frac{1.375}{2} = 0.6875$$

$$\phi = \sin^{-1} 0.6875 = 43.43^\circ$$

At equilibrium point C

Resolving forces vertically



$$\Sigma F_v = 0$$

$$T_{AC}\cos\theta - 300 = 0$$

$$T_{AC} = \frac{300}{\cos\theta} = \frac{300}{\cos 61.04} = \frac{300}{0.484} = \underline{619.83\,N}$$

Resolving forces horizontally,

$$\Sigma F_H = 0$$

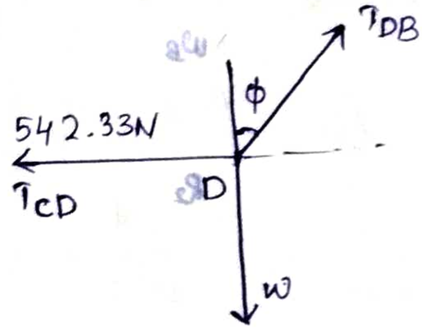
$$T_{CD} - T_{AC}\sin\theta = 0$$

$$T_{CD} = T_{AC}\sin\theta = 619.83 \sin 61.04$$

$$= 619.88 \times 0.875$$

$$= \underline{542.33\,N}$$

Consider equilibrium point at D



$$\Sigma F_H = 0$$

$$T_{DB}\sin\phi - T_{CD} = 0$$

$$T_{DB} = \frac{T_{CD}}{\sin\phi} = \frac{542.33}{\sin 43.43}$$

$$= \frac{542.33}{0.687} = \underline{788.56\,N}$$

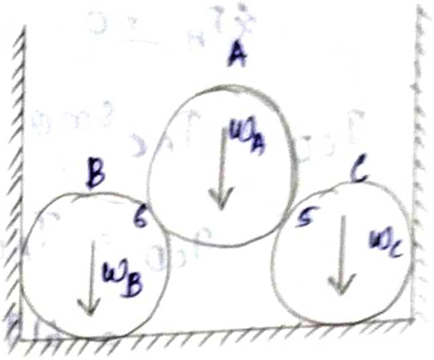
$$\Sigma F_V = 0$$

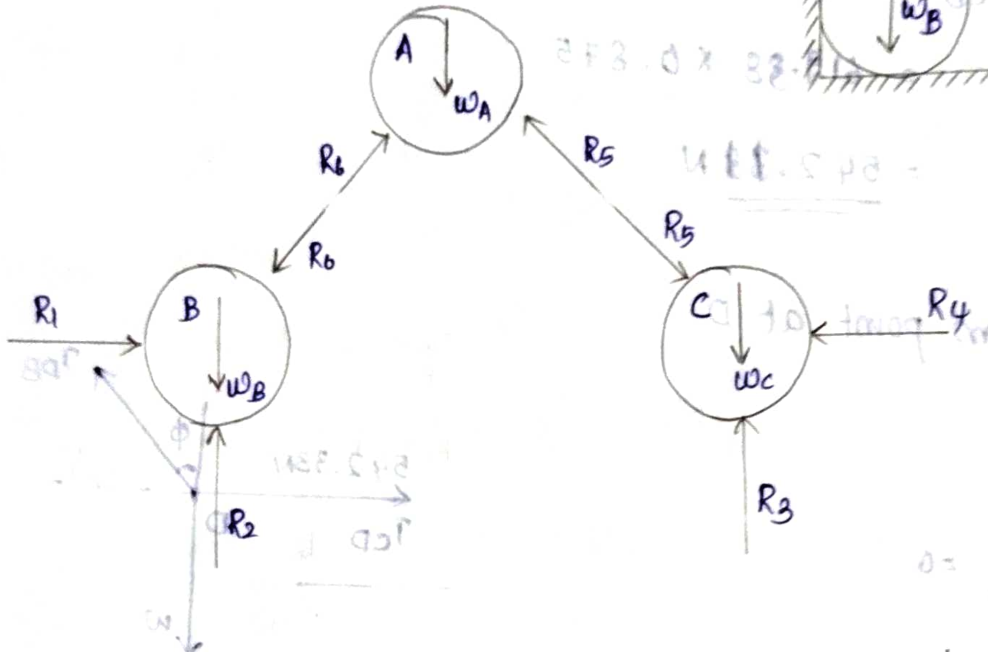
$$T_{DB}\cos\phi - W = 0$$

$$W = T_{DB}\cos\phi = 788.56 \cos 43.43$$

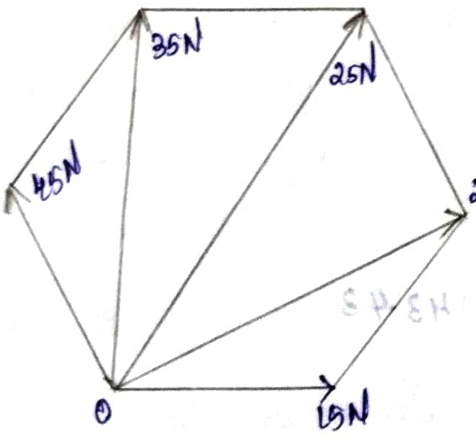
$$= \underline{572.66N}$$

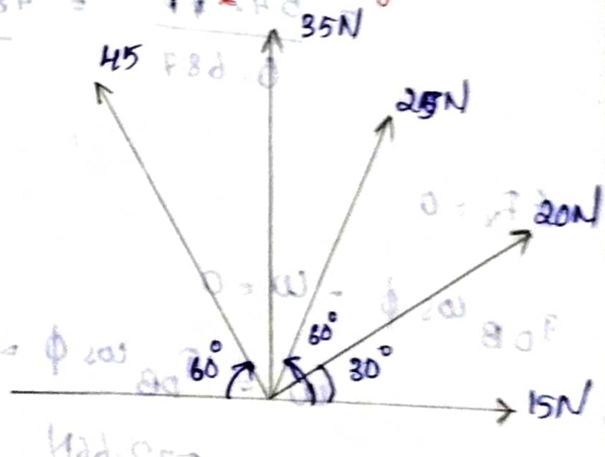
(5) Three smooth identical spheres A, B, C are placed in a rectangular channel as shown in fig. Draw the free body diagram of each sphere (KTU June 2016)





(6) Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of regular hexagon towards the other angular points as shown in Fig. Calculate the magnitude and direction of the resultant force (KTU Aug 2016)





Resolving forces along x-axis

$\Sigma F_x = 15 + 20 \cos 30^\circ + 25 \cos 60^\circ + 0 - 45 \cos 60^\circ$

$= \underline{22.32N}$

Resolving the forces along y-axis

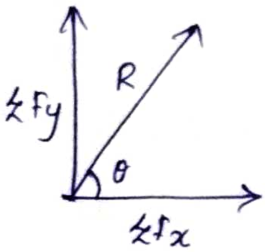
$\Sigma F_y = 0 + 20\sin 30 + 25\sin 60 + 35 + 45\sin 60 = \underline{105.62 N}$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{22.32^2 + 105.62^2} = \underline{107.95 N}$

Inclination of resultant with horizontal

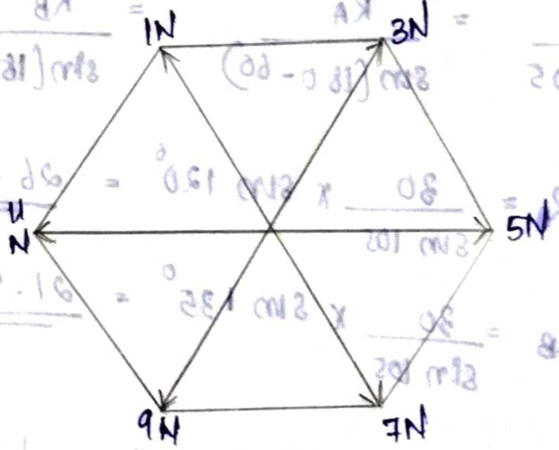
$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{105.62}{22.32}\right| = \underline{78.07^\circ}$



Inclination of resultant $\theta_R = \theta = \underline{78.07^\circ}$

(7) Concurrent forces 1, 3, 5, 7, 9, 11 are applied to the center of a regular hexagon acting towards its vertices as shown in fig. Determine the magnitude and direction of resultant (KTU, July 2016)

In a regular hexagon, the angle substended by each side at the center is 60°



Resolving the forces horizontally,

$\Sigma F_H = 5 + 3\cos 60^\circ - 1\cos 60^\circ - 11 - 9\cos 60^\circ + 7\cos 60^\circ$

$= -6N$

$= 6N$ towards left

ΣF_V [Resolving the forces vertically]

$\Sigma F_V = 0 + 3\sin 60^\circ + 1\sin 60^\circ + 0 - 9\sin 60^\circ - 7\sin 60^\circ$

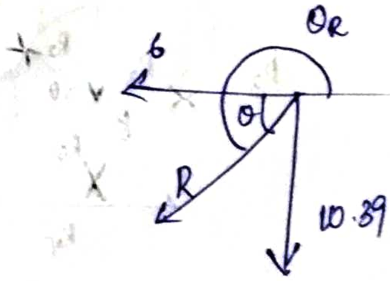
$= -10.39N$

Resultant $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{6^2 + (10.39)^2} = \underline{12N}$

Inclination of resultant with horizontal

$\tan\theta = \left|\frac{\Sigma F_V}{\Sigma F_H}\right| = \frac{10.39}{6} = 1.732$

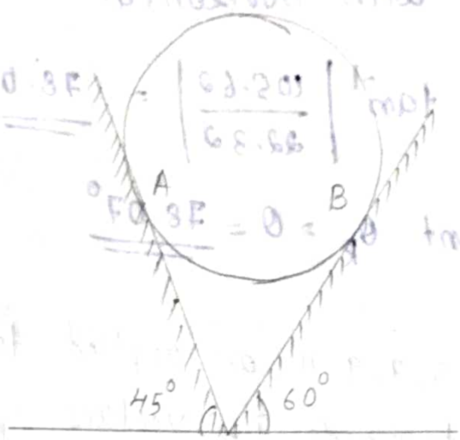
$\theta = \tan^{-1} 1.732 = \underline{60^\circ}$

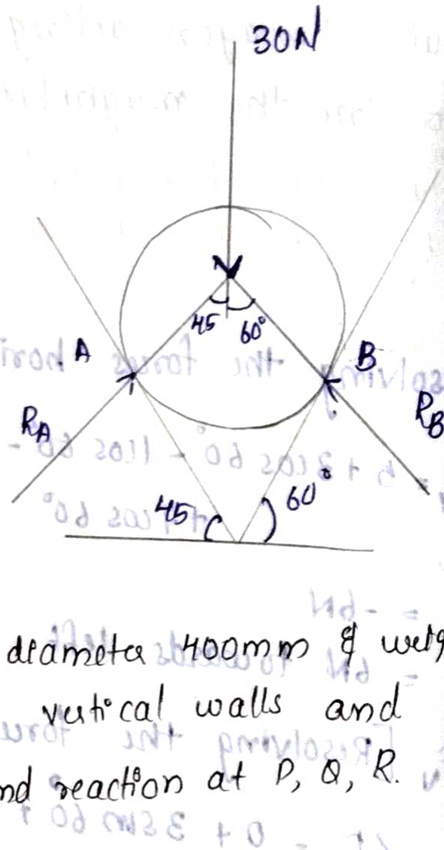


Since the resultant is in third quadrant,

$\theta_R = 180^\circ + \theta = 180 + 60 = 240^\circ$

(8) A solid cylinder 30mm diameter and weighing 300N is placed in a triangular channel as shown. Neglecting the friction b/w the contact surfaces, calculate the normal reaction on the sides of B channel. (KTU May 2019)





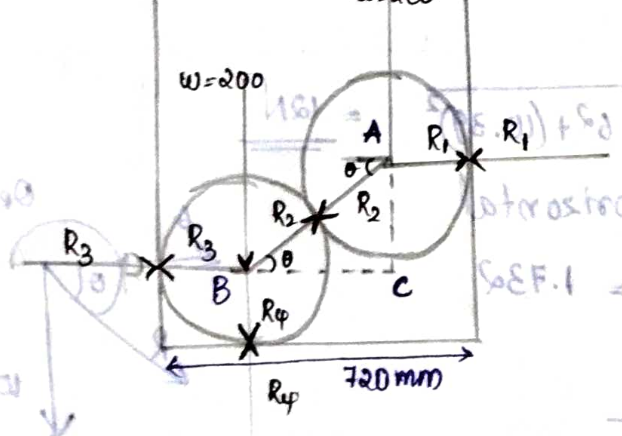
Applying Lami's theorem,

$$\frac{30}{\sin 105} = \frac{R_A}{\sin(180-60)} = \frac{R_B}{\sin(180-45)}$$

$$R_A = \frac{30}{\sin 105} \times \sin 120^\circ = \underline{26.89\,N}$$

$$R_B = \frac{30}{\sin 105} \times \sin 135^\circ = \underline{21.96\,N}$$

(9) Two smooth cylinders A and B each of diameter 400mm & weight 200N rest in a horizontal channel having vertical walls and base width of 720mm as shown in fig. Find reaction at P, Q, R.



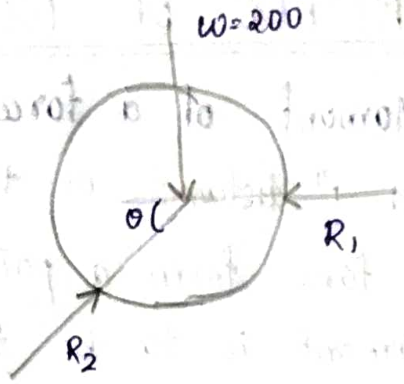
AB = 400mm

BC = 720 − 400 = 320mm

$$\cos\theta = \frac{BC}{AB} = \frac{320}{400}$$

$$\underline{\theta = 36.87^\circ}$$

Consider equilibrium of upper cylinder



Resolving the force vertically

For $\sum F_V = 0$

$R_2 \sin\theta - 200 = 0$

$R_2 = \frac{200}{\sin 36.87} = \underline{333.33 N}$

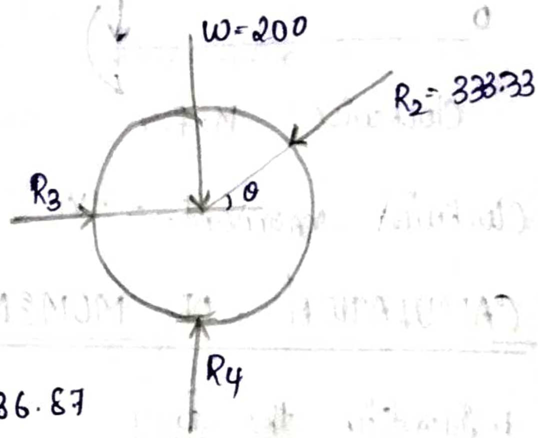
Resolving forces horizontally

$R_2 \cos\theta - R_1 = 0$

$R_1 = R_2 \cos\theta$

$= 333.33 \times \cos 36.87 = \underline{266.67 N}$

Consider equilibrium of lower cylinder



Resolving the force vertically,

$\sum F_V = 0$

$R_4 - 200 - 333.33 \sin\theta = 0$

$R_4 = 200 + 333.33 \sin 36.87$

$= \underline{400 N}$

Resolving the force horizontally,

$\sum F_H = 0$

$R_3 - 333.33 \cos\theta = 0$

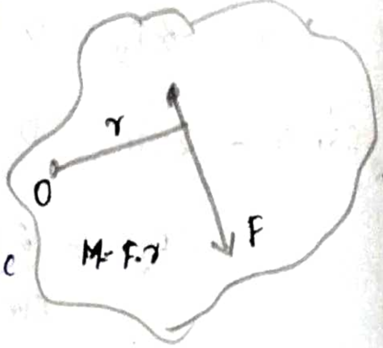
$R_3 = 333.33 \cos 36.87$

$= \underline{266.67 N}$

Reaction at P = 266.67N
Reaction at Q = 400 N
Reaction at R = 266.67 N

METHOD OF MOMENTS

- **Moment of a force about a point** : Product of a force and ⊥ʳ distance of the line of action of the force from a point about which moment is to be taken.



- Moment :- Rotating effect produced by the force on the body about that point.
- Perpendicular distance = Arm of the force / Moment arm
- Point about which moment is taken = Moment centre

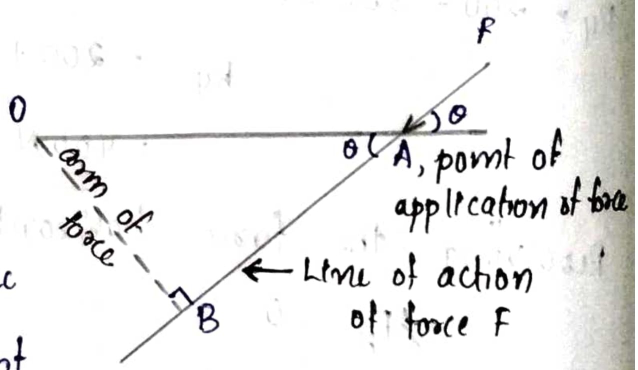
CLOCKWISE & ANTICLOCKWISE MOMENT

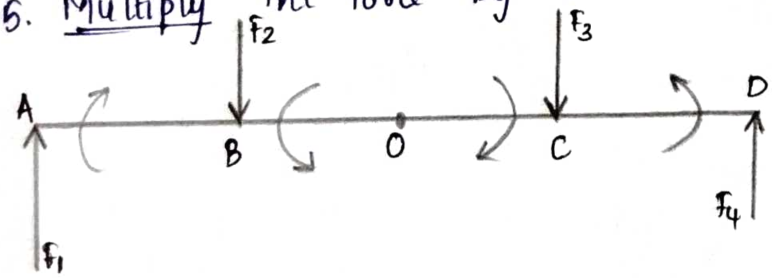
Clockwise moment = +ve

Anticlockwise moment = -ve

CALCULATION OF MOMENTS

1. Identify the force
2. Identify the moment centre
3. Identify the line of action of force
4. Calculate the ⊥ʳ distance of the line of action of the force from moment centre.
5. Multiply the force by the calculated ⊥ʳ distance





- Moment of force F_1 about O
  $= F_1 \times OA$; C.W
- Moment of force F_2 about O
  $= F_2 \times OB$, C.C.W
- Moment of force F_3 about O
  $= F_3 \times OC$, C.W
- Moment of force F_4 about O
  $= F_4 \times OD$, C.C.W

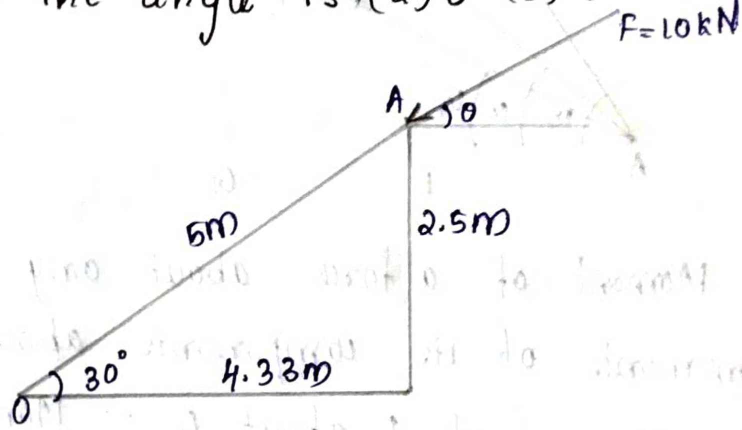
MAX & MIN MOMENTS

- Maximum moment :- Line of action of force is ⊥ to the line joining the moment centre & point of application of force
- Minimum moment :- (M=0) when,

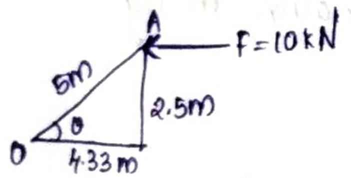
  (i) The force acts at the moment centre itself

  (ii) When the line of action of the force passes through the moment centre.

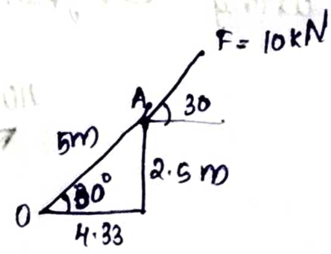
(10) Calculate the moment of force F = 10 kN acting at point A as shown in the fig when the angle is (a) 0° (b) 30° (c) 90°



(a) 0°



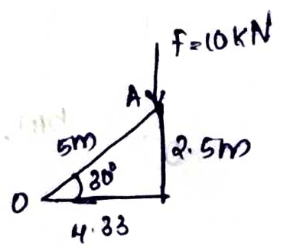
Moment M = 10×2.5 = 25 kN-m (anticlockwise)



(b) 30°

Moment M = 10 × 0 = 0 [same line of action]

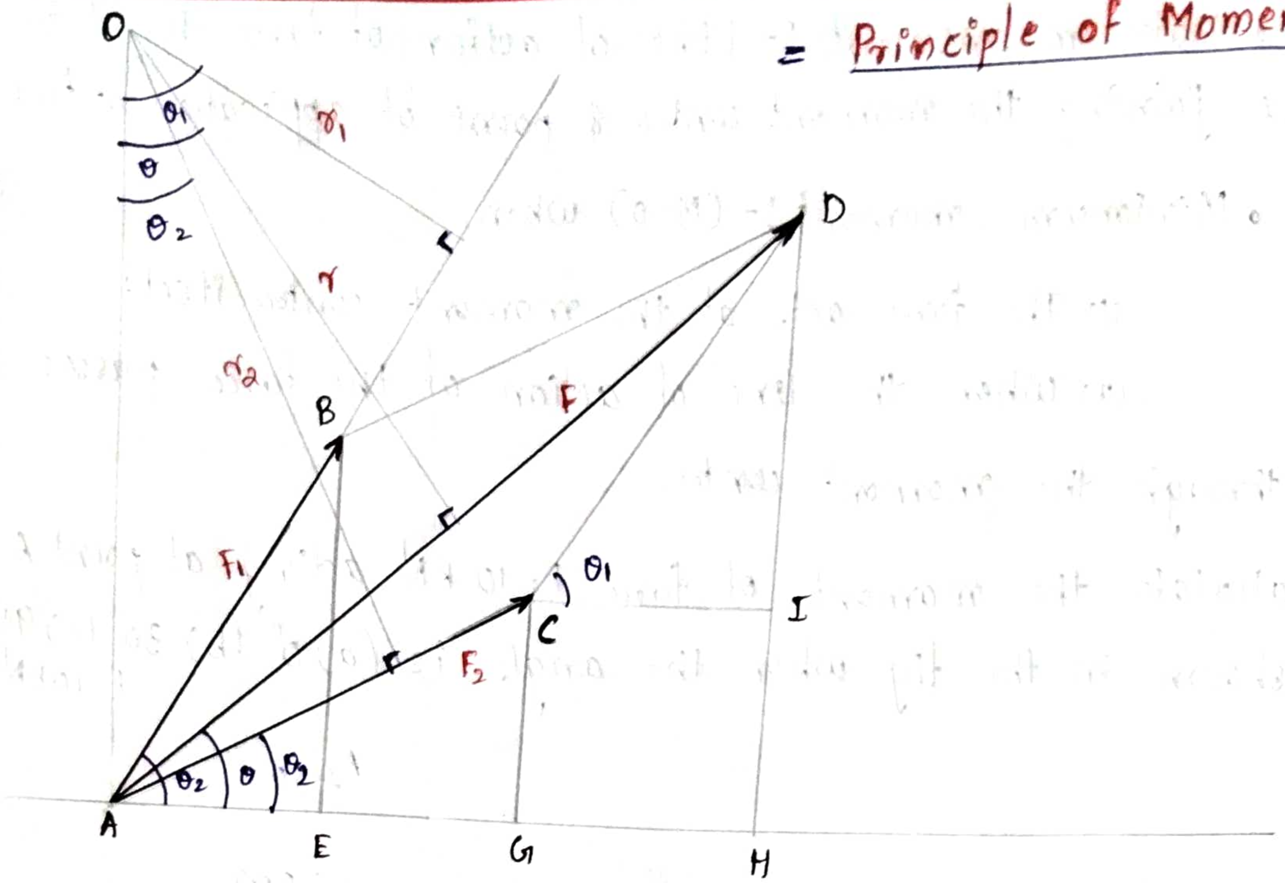
(c) 90°



Moment M = 10×4.33 = 43.3 kN-m (clockwise)

VARIGNON'S THEOREM OF MOMENTS

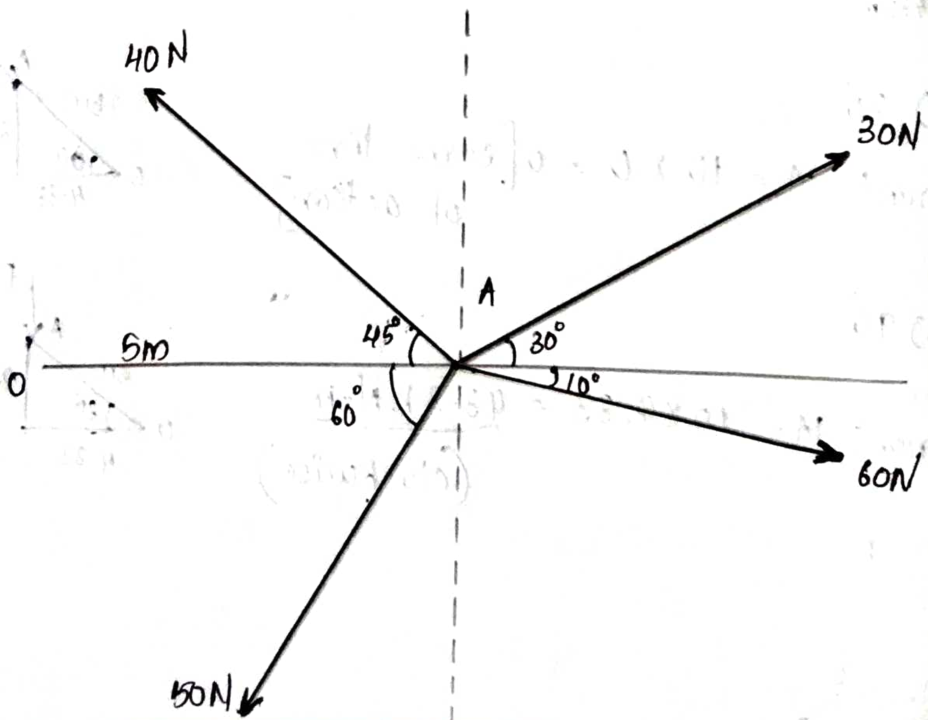
= Principle of Moments



Moment of a force about any axis is equal to the sum of moments of its components about that axis

Moment of F about O = Moment of F_1 about O + Moment of F_2 about O

(11) Calculate the moment of the force system shown in fig about O, using varignon's principle

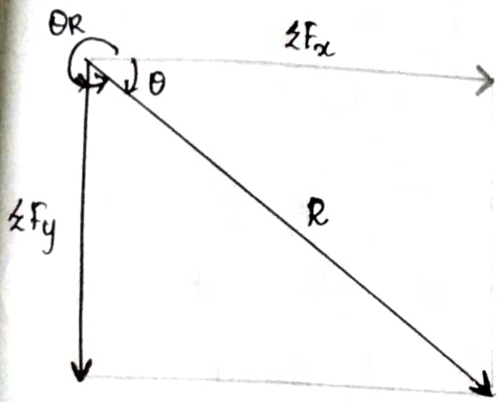


$$\Sigma F_x = 30\cos 30^\circ + 60\cos 10^\circ - 40\cos 45^\circ - 50\cos 60$$

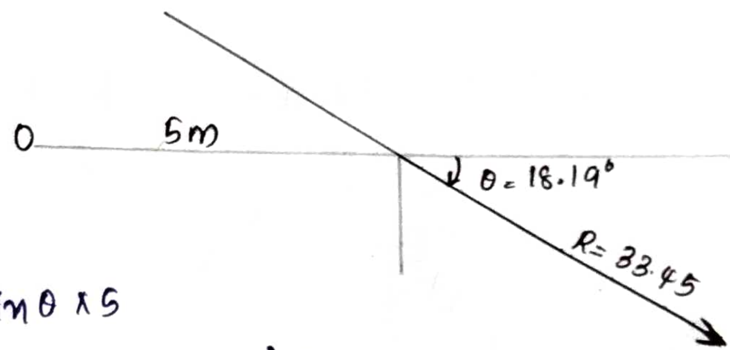
$$= \underline{31.78\ N}$$

$$\Sigma F_y = 30\sin 30^\circ + 40\sin 45^\circ - 50\sin 60^\circ - 60\sin 10^\circ$$

$$= \underline{-10.44\ N}$$



$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \tan^{-1}\left|\frac{10.44}{31.78}\right| = \underline{18.19^\circ}$$



$$M_o = R\sin\theta \times 5$$

$$= 33.45 \times \sin(18.19) \times 5$$

$$= 33.45 \times 0.312 \times 5 = \underline{52.21\ Nm}$$

MODULE : 2.

Friction & Parallel Coplanar Forces.

Friction or Frictional force.

Whenever a body moves or tends to move over other body; a force opposite to the direction in which the body moves or tends to move is developed at the contact surface.

cause : Surface roughness.

perfectly smooth surface, = $F_{fr} = 0$

Friction
- static friction
  - No relative motion
  - $\boxed{F_S = \mu_s F_n}$
- kinetic friction
  - Relative motion
  - Independent of velocity
  - $\boxed{F_k = \mu_k F_n}$

As force increases, friction increases.

Max value of friction : Limiting friction.

→ Value of frictional force when motion is about to start.

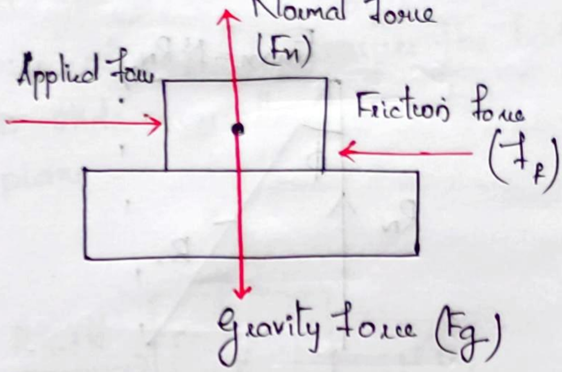
* Applied force >>> Limiting friction

→ Body moves in the direction of applied force

* Applied force <<< Limiting friction

→ Body remains at rest.

Free body diagram.

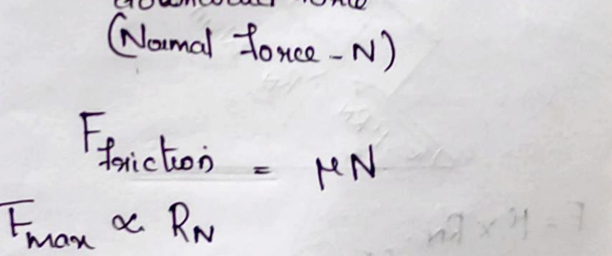


$F_g = mg$

$F_N = F_g$ (No motion)

$F = f_f$.

Coefficient of friction (μ)



$F_{friction} = \mu N$

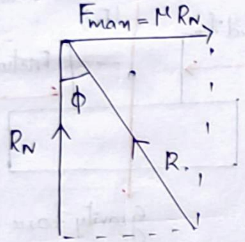
$F_{max} \propto R_N$

$= \mu R_N$

$\mu = \dfrac{F_{max}}{R_N}$.

Angle of friction (ϕ)

→ Angle b/w contact surface & normal reaction

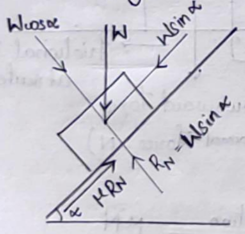


$$\tan\phi = \frac{\mu R_N}{R_N} = \mu.$$

Angle of friction $\phi = \tan^{-1}\mu$

Angle of repose (α)

Maximum inclination of a plane on which a body remains at rest, without applying external force.



$F = \mu \times R_N.$

$\Sigma F_x = 0$

$\mu R_n - W\sin\alpha = 0$

$\mu R_N = W\sin\alpha.$

$\Sigma F_y = 0$

$R_N - W\cos\alpha = 0$

$R_N = W\cos\alpha$

$\mu W\cos\alpha = W\sin\alpha$

$\tan\alpha = \mu = \tan\phi$

$\therefore \alpha = \phi$

Angle of repose = Angle of friction.

Coulomb's Laws of friction

1. The maximum friction that can be developed is independent of the area of contact.
2. At low velocity, the frictional force is independent of velocity of the contact surface.
3. The maximum frictional force is proportional to the normal reaction at the contact surface.

Other laws of friction

1. The force of friction always acts in a direction opposite to the direction in which the body moves or tends to move.
2. Till the limiting value is reached, the magnitude of friction is equal to the external force which tends to move the body.
3. The force of friction depends on the roughness of the surface in contact. When two perfectly smooth surfaces are in contact, frictional force is zero.

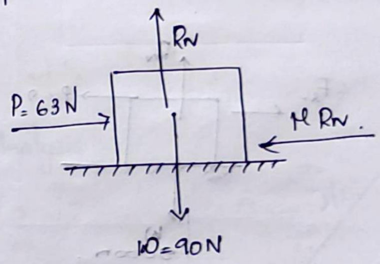
Analysis of single bodies:—

ⓐ consider single body

ⓑ various forces resolve horizontally & vertically

ⓒ apply conditions of equilibrium.

Q: A body of weight 90N is placed on a rough horizontal plane. Determine the coefficient of friction if the horizontal force of 63N just causes the body to slide over the horizontal plane.



$\Sigma F_y = 0$

$R_N - W = 0$

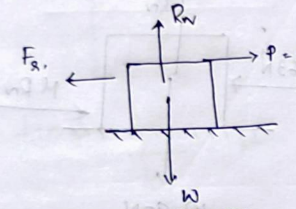
$R_N = W = 90\ N$

$\mu R_N = P = 63\ N$

$\mu R_N = 63\ N$

$$\mu = \frac{63}{R_N} = \frac{63}{90} = 0.7$$

Q. A uniform wooden cube of side 1m and weighing 500N rests on its side on a horizontal plane. Find the max horizontal force can be applied at the top edge of the cube to just make it slide without overturning. The coefficient of friction is 0.25



$\Sigma M = 0$ (overturning)

$F_R = .25 \times R_N$

$\Sigma F_v = 0 \quad R_N - W = 0$

$R_N = W = 5000\,N$

$\Sigma F_h = 0 \quad P - \mu R_N = 0$

$P = \mu R_N = 0.25 \times 5000$

$= \underline{1250\,N}$

$\Sigma M = 0$ abt A.

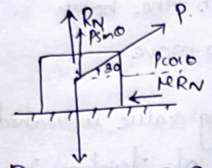
$P \times 1 - W \times 0.5 = 0.$

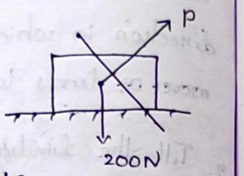
$P = W \times 0.5$

$= 5000 \times .5$

$= \underline{2500\,N}$

Q. A block weight 200N is placed on a rough horizontal floor. If $\mu = .25$ find the pull P required to move the block if P is inclined upwards at 30° to the horizontal.





$R_N + P\sin\theta - 200 = 0$

$R_N = 200 - P\sin\theta$

$P\cos\theta - \mu R_N = 0$

$P\cos\theta = \mu R_N$

$= 0.25(200 - P\sin 30)$

$P = \underline{50.45}$

Wedge friction

Analysis:—

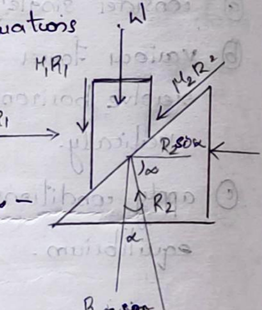
(a) fig

(b) main forces (W & P_{ext}), wedge angle

(c) consider body

(d) consider wedge

(e) solve equations

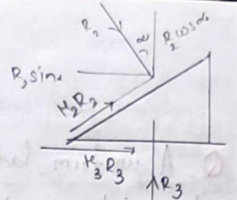


$\Sigma F_v = 0$

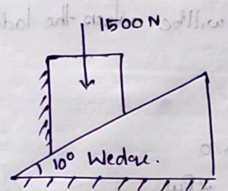
$R_2\cos\alpha - \mu_2 R_2 \sin\alpha - \mu_1 R_1 - W = 0$

$\Sigma F_H = 0$

$R_1 - \mu_2 R_2 \cos\alpha - R_2 \sin\alpha = 0$

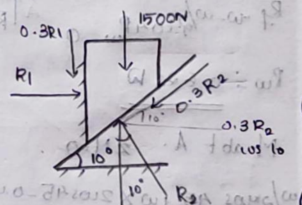


Q. Find the horizontal force P on the 10° wedge shown in the fig to raise the 1500 N load. The coefficient of friction is 0.3 at all contact surfaces.



Soln:

consider equilibrium of Load,



$R_1 - 0.3R_2\cos 10 - R_2\sin 10 = 0.$

$R_1 = R_2(0.3\cos 10 + \sin 10)$

$= 0.47\,R_2 \longrightarrow (1)$

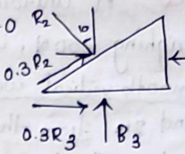
$R_2\cos 10 - 0.3R_2\sin 10 - 1500 - 0.3R_1 = 0$

$R_2\cos 10 - 0.3R_2\sin 10 - 0.3(0.47R_2) = 1500$

$R_2(\cos 10 - 0.3\sin 10 - 0.3 \times 0.47) = 1500$

$R_2 = 1894.63N$

consider equilibrium of wedge,



$R_3 + 0.3R_2\sin 10 - R_2\cos 10 = 0$

$R_3 = R_2(\cos 10 - 0.3\sin 10)$

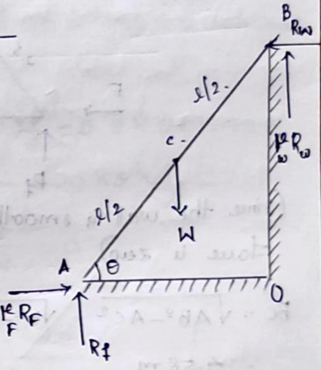
$= \underline{1767.15\,N}$

$0.3R_2\cos 10 + R_2\sin 10 + 0.3R_3 - P = 0$

$P = \underline{1418.90\,N}$

Ladder friction

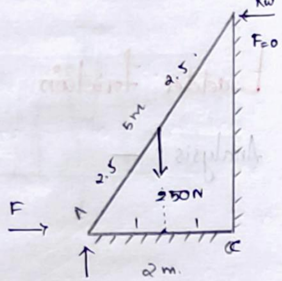
Analysis:—



(i) upper end B slips down → friction is upwards.

(ii) lower end moves away from the wall → direction of friction force is towards the wall.

Q. A uniform ladder 5m long weighing 250N, is placed against a smooth vertical wall with its lower end 2m from the wall. The coefficient between the ladder and floor is 0.25. show that ladder will remain in equilibrium in this position.



(Since the wall is smooth frictional force is zero)

$BC = \sqrt{AB^2 - AC^2} = \sqrt{5^2 - 2^2}$

$= 4.58$ m

$\Sigma F_V = 0$

$R_f - 250 = 0$

$R_f = 250$

Limiting frictional force $= 0.25 \times 250$

$= 62.5$N

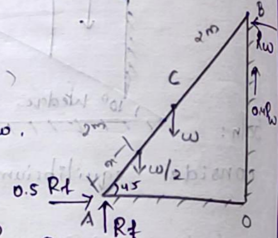
Moment abt B,

$\Sigma H = 0$

$R_f \times AC - 250 \times 1 - F \times BC = 0$

$F = 54.59$N

Q. A uniform ladder of 4m length rests against a wall which it makes an angle 45° as shown in fig. The coefficient of friction b/w the ladder and the wall is 0.4 & that between the ladder and floor is 0.5. If a man whose weight is half of that of a ladder ascends it, how high will be when the ladder slips?



Soln:

$\Sigma F_H = 0$

$0.5R_f - R_w = 0$

$R_f = 2R_w$.

$\Sigma F_V = 0$.

$R_f - w - w/2 + 0.4R_w = 0$

$R_w = 0.625\,W$

M abt A, $\Sigma H = 0$

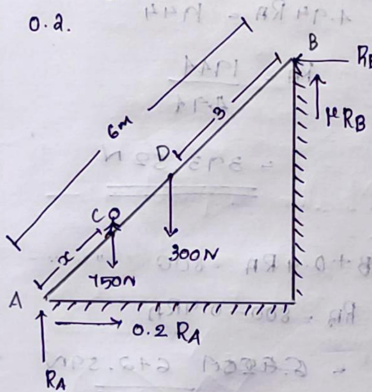
$w/2\,x\cos 45 + w \times 2\cos 45 - 0.4R_w \times 4\cos 45 - R_w\,4\sin 45 = 0$

$x/2 = 1.5$

$x = 3$ m

Tutorial

Q1. The ladder 6m long weighing 300N is resting against a wall at an angle of 60° to the horizontal. A man weighing 750N climbs the ladder. At what position along the ladder from the bottom does he induce slipping? The coefficient of friction for both the wall and the ground with the ladder is 0.2.



$\Sigma F_x = 0$

$0.2R_A - R_B = 0$

$0.2R_A = R_B$

$R_A = 5R_B$

$\Sigma F_y = 0$

$R_A + 0.2R_B - 300 - 750 = 0$

$5R_B + 0.2R_B = 1050$

$5.2R_B = 1050$

$R_B = \frac{1050}{5.2} = 201.9$ N

$R_A = 5R_B = 5 \times 201.9$N

$= 1009.62$N

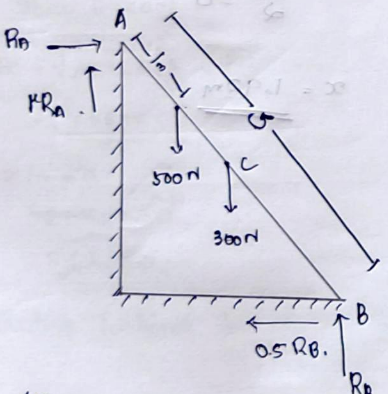
Moment about A

$R_B \cdot 6\sin 60 + 0.2R_B \times 6\cos 60 - 300 \times 3\cos 60 - 750 \times x\cos 60 = 0$

$201.92 \times 6 \times \frac{\sqrt{3}}{2} - 0.2 \times 201.92 \times 6 \times \frac{1}{2} - 300 \times 3 \times \frac{1}{2} - 750 \times \frac{x}{2} = 0$

$x = 1.92$m

Q.2. A ladder of length 5 m and weight 300N is placed against a vertical wall with which it makes an angle of 45°. The coefficient of friction between the floor and the ladder is 0.5 and that between the ladder & 0.4. In addition to its own weight, the ladder has to support a man of weight 500N at 1m from the top along the ladder. Determine the minimum horizontal force 'P' to be applied at the floor level to prevent the ladder from slipping.



$\Sigma F_y = 0$

$R_B + 0.4R_A - 300 - 500 = 0$

$R_B + 0.4R_A = 800 \rightarrow (1)$

$\Sigma F_x = 0$

$R_A - 0.5R_B - P = 0$

$R_A - 0.5R_B = P \rightarrow (2)$

Moments about B = 0

$\Sigma M = 0$

$R_A \cdot 5\cos 45 + 0.4R_A \cdot 5\cos 45 - 300 \times 2.5\cos 45 - 500 \times 4\cos 45 = 0$

$R_A(5\cos 45 + .4\cos 45) - 530 - 1414 = 0$

$4.94R_A = 1414 + 530$

$4.94R_A = 1944$

$R_A = \frac{1944}{4.94}$

$= 393.52\,N$

$R_B + 0.4R_A = 800$

$R_B = 800 - 0.4R_A$

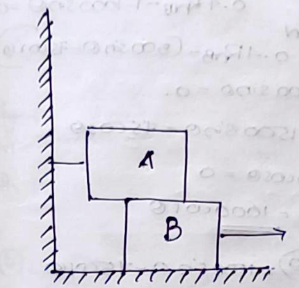
$= 642.59\,N$

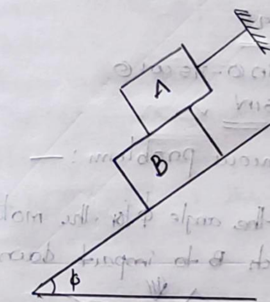
$P = R_A - 0.5R_B$

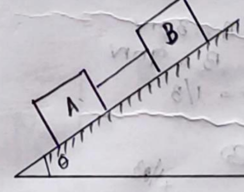
$= 393.52 - 0.5 \times 642.59$

$= 72.225\,N$

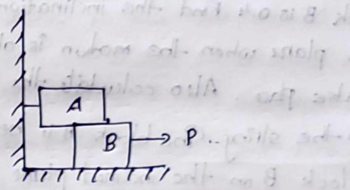
Analysis of friction in connected bodies.



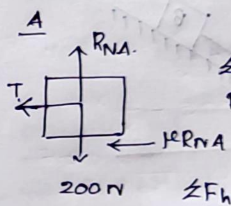




Q. Block A shown in figure weights 200N and block B weight 300N. Find the force P required to move block B. Assume the coefficient of friction for all surface as 0.3.



A



$\Sigma F_v = 0$

$R_{NA} - W = 0$

$R_{NA} = W = 200N$

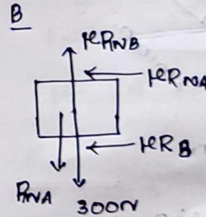
$\Sigma F_h = 0$

$\mu R_{NA} - T = 0$

$.3 \times 200 = T$

$T = 60N$

B



$\Sigma F_v = 0$

$R_{NB} - W - R_{NA} = 0$

$R_{NB} = W + R_{NA}$

$= 300 + 200 = 500N$

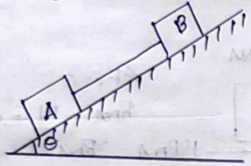
$\Sigma F_h = 0$

$P - \mu R_{NB} - \mu R_{NA} = 0$

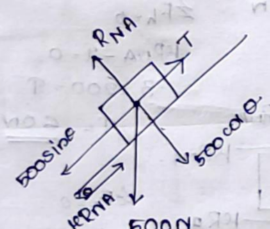
$P = \mu R_{NB} + \mu R_{NA}$

$.3 \times 500 + 0.3 \times 200 = 210N$

Q: Two blocks A & B of weights 500N & 1000N are placed on an inclined plane. The blocks are connected by a string parallel to the inclined plane. The coefficient of friction between the inclined plane and block A is 0.15 and that for block B is 0.4. Find the inclination of the plane when the motion is abt to take place. Also calculate the tension in the string. The block A is below Block B on the inclined plane



A



$T + 0.15 R_{NA} - 500\sin\theta = 0$.

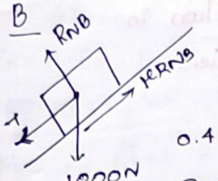
$R_{NA} - 500\cos\theta = 0$

$R_{NA} = 500\cos\theta$.

$T + 0.15 \times 500\cos\theta - 500\sin\theta = 0$.

$T = 500\sin\theta - 75\cos\theta$.

B



$0.4 R_{NB} - T - 1000\sin\theta = 0$

$0.4 R_{NB} - (500\sin\theta - 75\cos\theta) - 1000\sin\theta = 0$.

$0.4 R_{NB} = 1500\sin\theta - 75\cos\theta$

$R_{NB} - 1000\cos\theta = 0$

$R_{NB} = 1000\cos\theta$

$0.4(1000\cos\theta) = 1500\sin\theta - 75\cos\theta$

$400\cos\theta = 1500\sin\theta - 75\cos\theta$

$475\cos\theta = 1500\sin\theta$

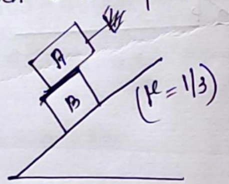
$\theta = 17.57°$

$T = 500\sin\theta - 75\cos\theta$

$= 79.61N$ ✓

Miscellaneous problem :—

Q. Find the angle ϕ for the motion of block B to impart down the plane.

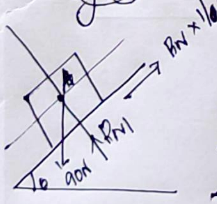


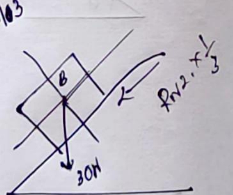
A = 30N

B = 90N

μ = 1/3

$\sum F_y = 0$





$\sum F_y = 0$

$R_{N2} - 30\cos\theta = 0$

$R_{N2} = 30\cos 30 \rightarrow (1)$

$\sum F_x = 0$

$T - \frac{R_{N2}}{3} - 30\sin 30 = 0$

$T = \frac{R_{N2}}{3} + 30\sin\theta \rightarrow (2)$

$T = \frac{\cancel{30}^{10}\cos 30}{\cancel{3}} + 30\sin\theta \longrightarrow (2)$

$\sum F_y = 0$

$R_{N1} - R_{N2} - 90\cos\theta = 0$.

$R_{N1} = 120\cos\theta \rightarrow (3)$

$\sum F_x = 0$

$\frac{R_{N1}}{3} + \frac{R_{N2}}{3} - 90\sin\theta = 0$

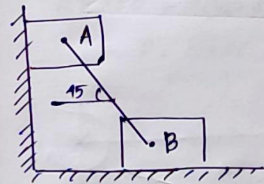
$R_{N1} + R_{N2} = 270\sin\theta$

$R_N = 270\sin\theta - 30\cos\theta \rightarrow (4)$

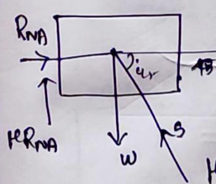
$\Rightarrow (4) \rightarrow 120\cos\theta = 270\sin\theta - 30\cos\theta$

$\theta = 29.055°$

Q. Two identical Blocks A & B of weight W are supported by a rigid bar inclined 45° with the horizontal as shown in fig. If the blocks are in limiting equilibrium, find the coefficient of friction, assuming it to be the same at the floor and the wall.



A



$\sum F_h = 0$

$R_{NA} - S\cos 45 = 0$

$R_{NA} = 0.707S$

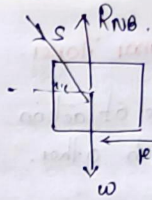
$\sum F_v = 0$

$\mu R_{NA} + S\sin 45 - W = 0$

$\mu \times .707S + 0.707S = W$

$0.707S(1+\mu) = W \rightarrow (1)$

B



$\sum F_V = 0$

$R_{NB} - W - S\sin 45 = 0$

$R_{NB} = W + .707S$.

$\sum F_h = 0$

$S\cos 45 - \mu R_{NB} = 0$

$S\cos 45 - \mu(W + .707S) = 0$

$.707S - \mu \cdot 0.707S = \mu W$

$.707S(1-\mu) = \mu W \rightarrow (2)$

$\frac{0.707S(1-\mu)}{.707S(1+\mu)} = \frac{\mu W}{W}$

$1 - \mu = \mu(1+\mu)$

$1 - \mu = \mu + \mu^2$

$\mu^2 + 2\mu - 1 = 0$

$\mu = \frac{-2+\sqrt{8}}{2} = \frac{-2+2\sqrt{2}}{2}$

$= 0.414$

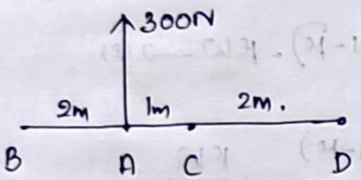
Parallel Coplanar forces.

Force whose line of action are parallel to each other.

Q1. Resolve the force of 300N as shown in fig. into two parallel components

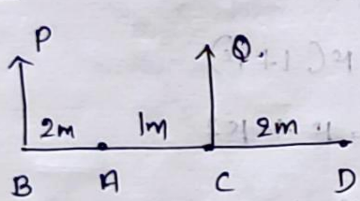
(i) at B & C

(ii) at C & D.



soln:

(i)



Moment at B due to 300N

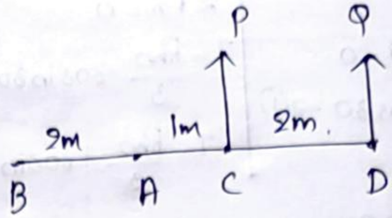
$= 300 \times 2 = \underline{600 Nm}$

$P \times 0 + Q \times 3 = 600$

$Q = \underline{\underline{200N}}$

$P + Q = 300$

$P = \underline{\underline{100N}}$



P+Q=300

Moment at D due to 300N

$= 300 \times 3 = 900.$

$P \times 2 + Q \times 0 = 900$

$P = \underline{\underline{450}}$

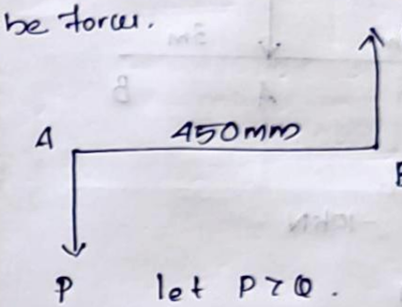
$P + Q = 300$

$450 + Q = 300$

$Q = 300 - 450$

$= \underline{\underline{-150N}}$

Q. Two unlike parallel forces are acting at a distance of 450 mm from each other. The forces are equivalent to a single force of 900N, which act a distance of 200mm from the greater of the two forces. Find the magnitude of the forces.



let $P > Q$.

$P - Q = 900 \rightarrow (1)$

Moment of B P abt B must be equal to moment of 900N abt B.

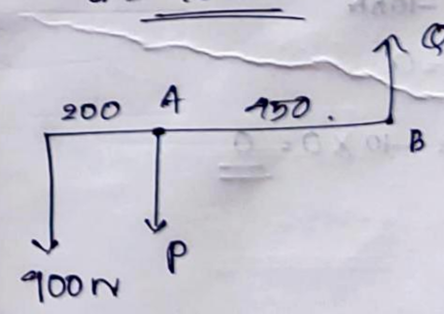
$P \times 450 = 900 \times 450$

$P = \underline{\underline{1200N}}$

$1300 - Q = 900$

$1300 - 900 = Q$

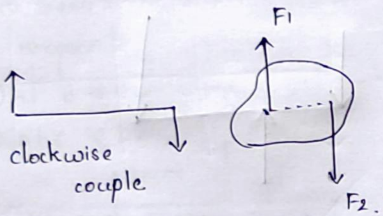
$Q = \underline{400N}$



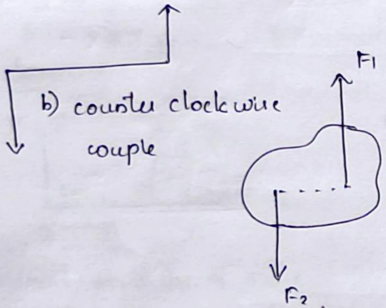
Couple.

Two forces having the same magnitude, parallel line of action and opposite ~~d~~ in direction forms a couple.

Resultant of these forces is zero.



a) clockwise couple

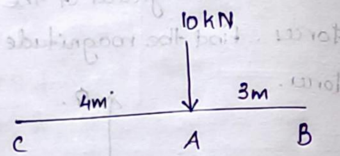


b) counter clockwise couple

Q. Replace the force acting at A by a force and couple

(i) at B

(ii) at C



A

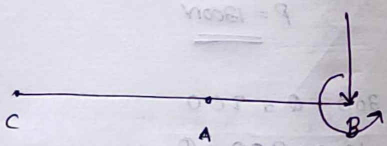
$\Sigma F_V = -10 kN$

$\Sigma F_H = 0$

$\Sigma M_B = -10 \times 3 = -30 kNm$

$= 30 kNm$

$=$ counter clock wise



B

$\Sigma F_V = -10 kN$

$\Sigma F_H = 0$

$\Sigma M_B = -10 \times 0 = \underline{0}$

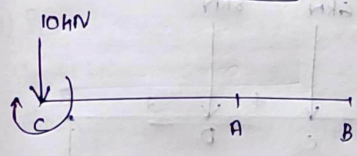
$\therefore$ a counter clock wise moment of magnitude 30 kNm should be applied at B.

A

$\Sigma F_V = -10 kN$

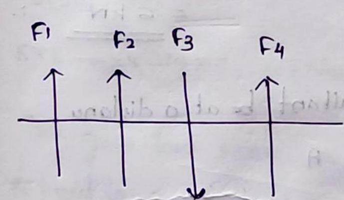
$\Sigma F_H = 0$

$\Sigma M_C = 10 \times 4 = 40 kNm$ (c.w)



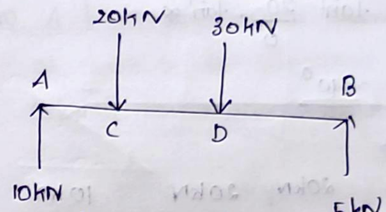
Resultant of Parallel forces.

Since all forces are parallel, resultant will be the algebraic sum of the forces.



$R = F_1 + F_2 - F_3 + F_4$

Q. Calculate the resultant of the force system of parallel forces as shown in figure.

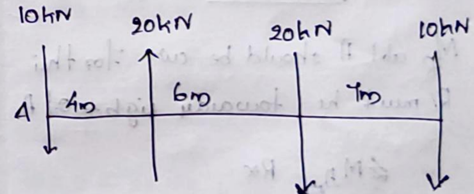


$R = 10 - 20 - 30 + 5$

$= -35 kN$

$= 35 kN \downarrow$

Q. Determine the resultant of the system of forces shown in fig



$\Sigma F_H = 0$

$\Sigma F_V = -10 + 20 - 20 - 10 = -20 kN$

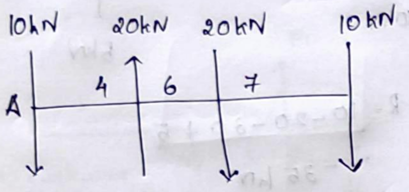
$R = \sqrt{(F_H)^2 + (F_V)^2} = \sqrt{0 + (20)^2}$

$= \underline{20 kN}$

$$\theta = \tan^{-1}\frac{\Sigma F_V}{\Sigma F_H}$$

$$= \tan^{-1}\frac{20}{0} = \tan^{-1}\infty$$

$$\theta = 90^\circ$$



$$\Sigma M_A = 10\times0 - 20\times4 + 20\times10 + 10\times17$$

$$= -80 + 200 + 170$$

$$= 290\ \text{kNm}\ (cw)$$

M_R abt A should be cw. for this R must be towards right of A

$$\Sigma M_A = Rx$$

$$x = \frac{\Sigma M_A}{R} = \frac{29\cancel{0}}{2\cancel{0}} = 14.5$$

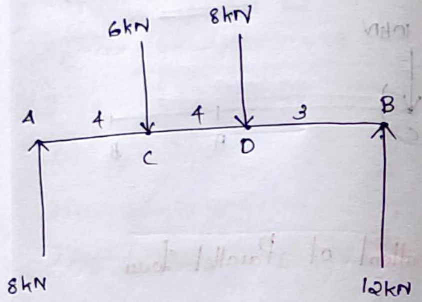
$$x = 14.5$$

Q. A rigid bar AB is acted upon by forces as shown in fig. Reduce the force system to

(i) single force

(ii) force moment system at A &

(iii) force moment system at D.



$$\Sigma F_y = 8 - 6 - 8 + 12 = \cancel{8}\ 6\ \text{kN}$$

$$R = \sqrt{(\Sigma F_h)^2 + (\Sigma F_v)^2} = \sqrt{0+36}$$

$$= 6\ \text{kN}$$

Let resultant be at a distance x from A

$$\Sigma M_B = 6\times4 + 8\times8 + -12\times11$$

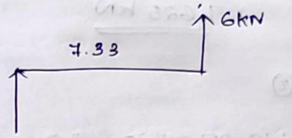
$$= -44\ \text{kNm}$$

$$= 44\ \text{kNm}\ \text{ccw}$$

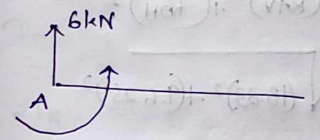
M of resultant abt A = $6\times x$

$$6\times x = 44$$

$$x = \frac{44}{6} = 7.33$$



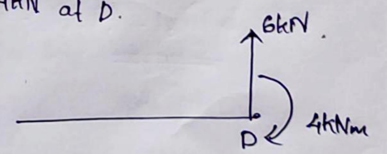
(ii) Sum of moments of forces abt A is 44 kNm. The resultant force is 6kN upwards ∴ the system can be reduced to a force moment system at A as



(iii) The sum of moments of forces about D is

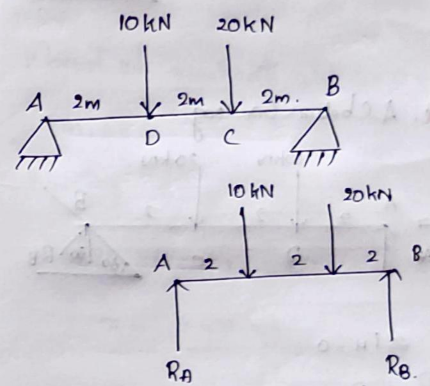
$$8\times8 - 6\times4 - 12\times3 = 4\ \text{kNm}\ cw$$

The given force system can be reduced to a single force of 6kN along with a cw moment of 4kN at D.



Simple beams subjected to concentrated vertical loads.

Q. A beam 6m long is loaded as shown in the figure. Calculate the reactions at A & B.



$$\Sigma F_V = 0$$

$$R_A - 10 - 20 + R_B = 0$$

$$R_A + R_B = 30 \longrightarrow (1)$$

$$\Sigma M_A = 0$$

$$10\times2 + 20\times4 + -R_B\times6 = 0$$

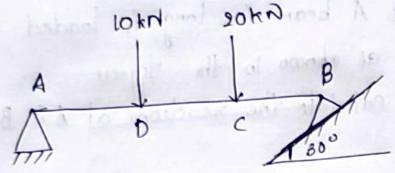
$$R_B = \frac{100}{6} = 16.67\ \text{kN}$$

$$R_A + R_B = 30$$

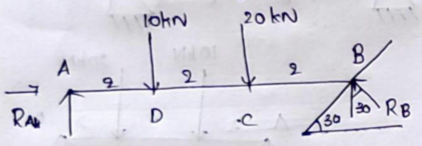
$$R_A = 30 - R_B = 30 - 16.67$$

$$= 13.33\ \text{kN}$$

Q. A beam 6m long is located as shown in fig. calculate the reactions at A & B.



Q. A beam 6m long



$\Sigma F_H = 0$

$R_{AH} - R_B \sin 30 = 0.$

$R_{AH} = R_B \sin\theta \rightarrow (1)$

$\Sigma F_V = 0$

$R_{AV} - 10 - 20 + R_B \cos 30 = 0$

$R_{AV} + R_B \cos\theta = 30 \quad \rightarrow (2)$

$\Sigma M_A = 0$

$10\times2 + 20\times4 - (R_B \cos 30)\times 6 = 0$

$20 + 80 = 6 R_B \cos 30$

$R_B = \dfrac{100}{6\cos 30}$

$= \underline{19.25 \text{ kN}}$

From (1)

$R_{AH} = R_B \sin 30$

$= 19.25 \times 0.5$

$= \underline{\underline{9.625 \text{ kN}}}$

From (2)

$R_{AV} + 19.25 \cos 30 = 30$

$R_{AV} = 80 - 19.25 \cos 30$

$= \underline{\underline{13.33 \text{ kN}}}$

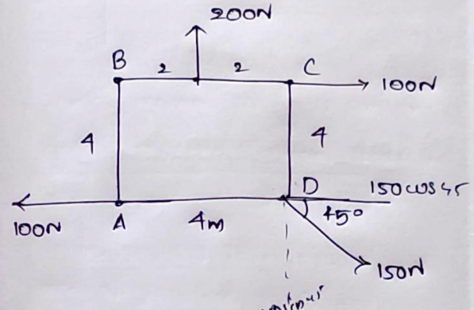
$R_A = \sqrt{(R_{AV})^2 + (R_{AH})^2}$

$= \sqrt{(13.33)^2 + (9.625)^2}$

$= \underline{\underline{16.44 \text{ kN}}}$

Miscellaneous Questions

Q. For the system of forces shown in figure, determine the magnitude, direction and position of the resultant force wrt A



Resolving forces horizontally

$\Sigma F_H = 100 - 100 + 150 \cos 45$

$= \underline{\underline{106.07 \text{ N}}}$

$\Sigma F_V = 200 - 150 \sin 45$

$= \underline{\underline{93.93 \text{ N}}}$

$R = \sqrt{(106.07)^2 + (93.93)^2}$

$= \underline{\underline{141.68 \text{ N}}}$

$\theta = \tan^{-1} \dfrac{93.93}{106.07}$

$= \underline{\underline{41.53°}}$

Sum of moments of all forces abt A

$\Sigma M_A = 150 \sin 45 \times 4 + 100\times4 - 200\times2$

$= \underline{424.26 \text{ Nm}}$ c.w.

Moment of resultant abt A $= R \times x$

$= 141.68x$

Equating ΣM_A & Rx

$424.26 = 141.68x$

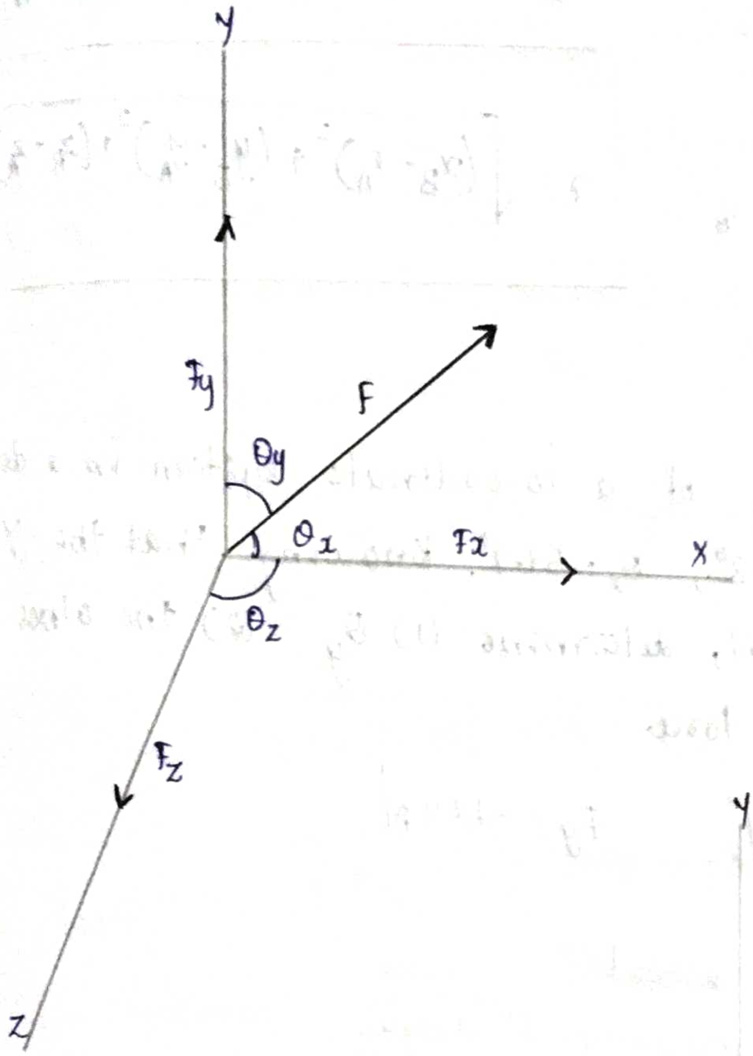
$x = \dfrac{424.26}{141.68}$

$= \underline{\underline{3m}}$

29/09/2021
wednesday

MODULE : 03

FORCES IN SPACE

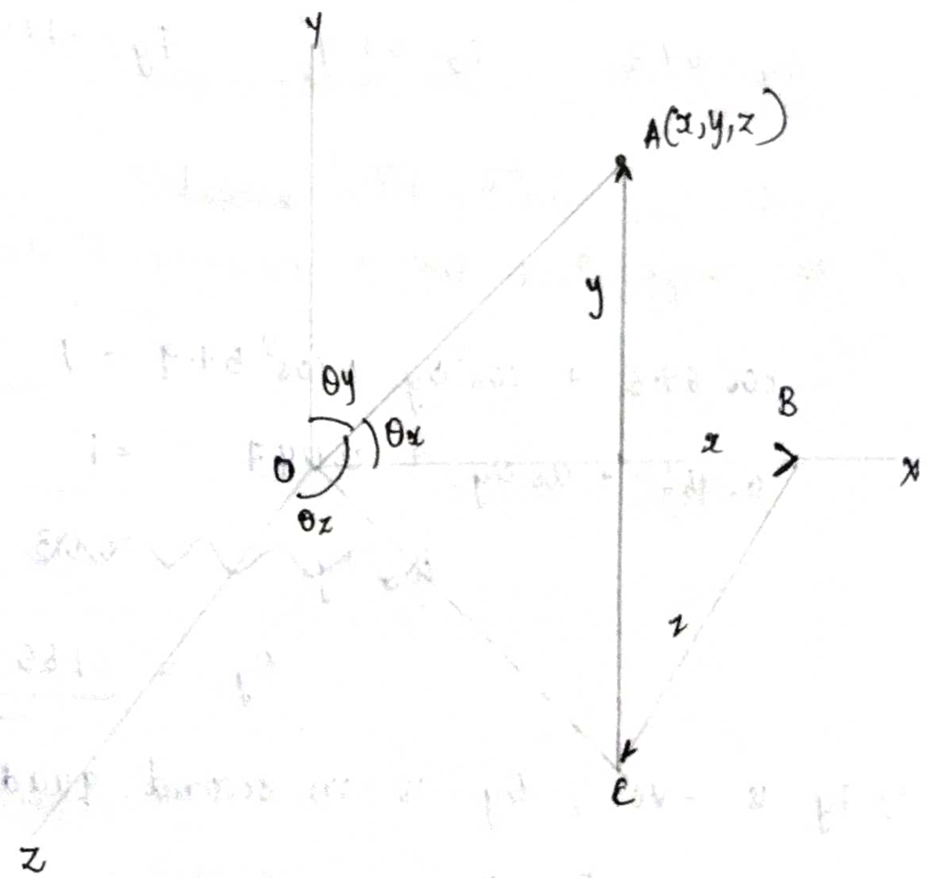


$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$

$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

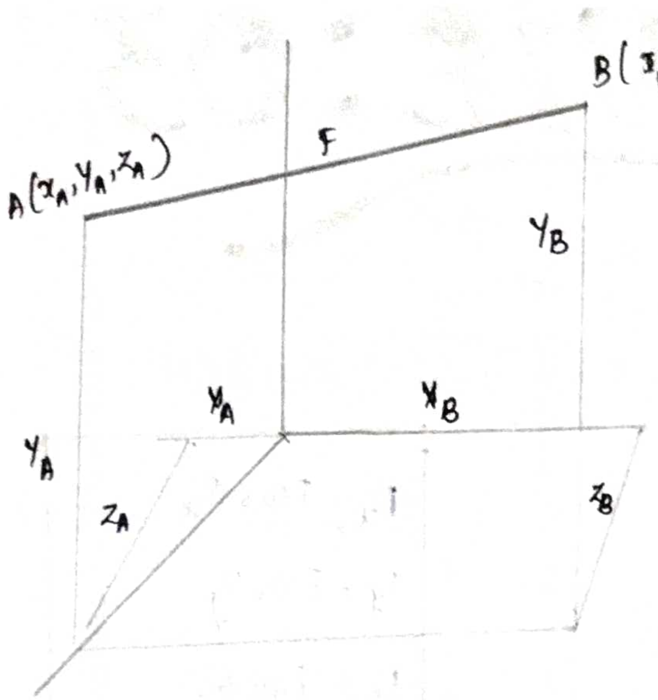


$OA^2 = OC^2 + CA^2$

$= OB^2 + BC^2 + CA^2$

$r^2 = x^2 + y^2 + z^2$

$r = \sqrt{x^2 + y^2 + z^2}$



$\vec{AB} = X_B - X_A$

$\vec{BA} = X_A - X_B$

$X = X_B - X_A$

$Y = Y_B - Y_A$

$Z = Z_B - Z_A$

$$\boxed{r = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

(1) A force acts at the origin of a co-ordinate system in a directi defined by the angle $\theta_x = 69.3^\circ$, $\theta_z = 57.9^\circ$. Knowing that the y component of force is -174 N, determine (1) θ_y (2) the other components & magnitude of force.

$\theta_x = 69.3^\circ$ $\quad\theta_z = 57.9^\circ$ $\quad F_y = -174$ N

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

The angle eq. b/w 3 mutually ⊥ angles

1.013

1.584

$$\cos^2 69.3 + \cos^2\theta_y + \cos^2 57.9 = 1$$

$$0.966 + \cos^2\theta_y + 0.047 = 1$$

$\cancel{\cos^2\theta_y \; 0.013}$

180 − θ

$$\theta_y = 39.65 \text{ or } 140.35$$

$\because F_y$ is $-ve$, θ_y is in second quadrant & thus $\theta_y = 140.3$

$$F_y = F\cos\theta_y$$

$$-174 = F\cos(140.35) \Rightarrow F = \frac{-174}{\cos(140.35)} = 225.98$$

$F_x = F\cos\theta_x = 225.95 \times \cos 69.3 = 225.95 \times 0.983 = \underline{\underline{79.87}}$

$F_z = F\cos\theta_z = 225.95 \times \cos 57.9 = 225.95 \times 0.218$
$= \underline{\underline{120N}}$

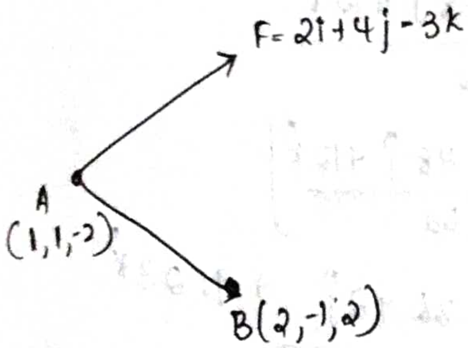
MOMENT

$M = F \times d$

$\underline{M = Fr\sin\theta} \rightarrow \boxed{M = \vec{r} \times \vec{F}}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = i(yF_z - zF_y) - j(xF_z - zF_x) + k(xF_y - yF_x)$$

(2) A Force $F = 2i + 4j - 3k$ is applied at a point $A(1,1,-2)$ Find the moment of force F about $B(2,-1,2)$



$x_A = 1 \quad x_y = 1 \quad x_z = -2 \quad x_B = 2 \quad y_B = -1 \quad z_B = 2$

$r = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$

$= (1-2)\hat{i} + (1--1)\hat{j} + (-2-2)\hat{k}$

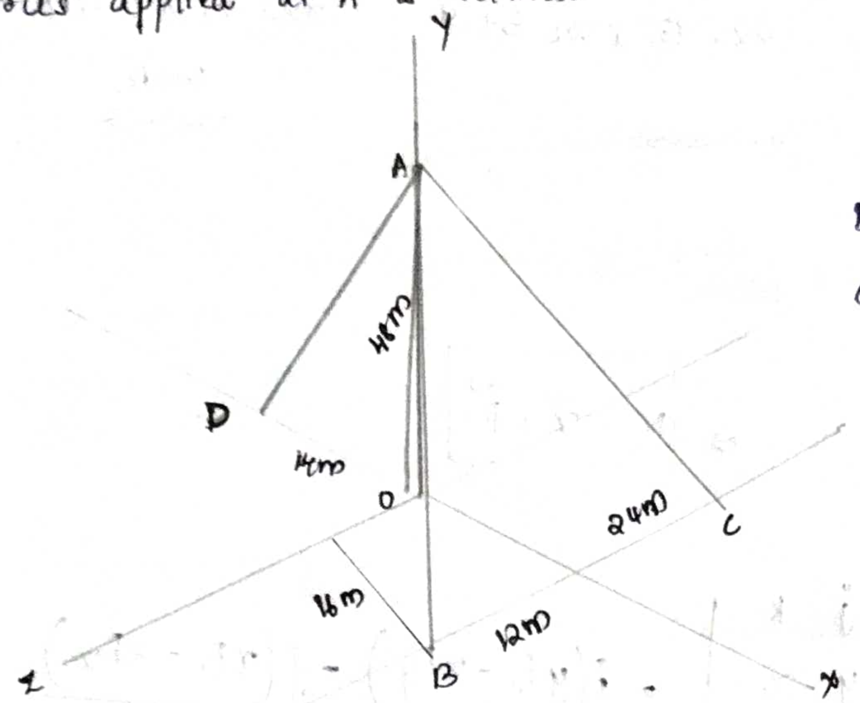
$= -1\hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -1 & 2 & -4 \\ 2 & 4 & -3 \end{vmatrix} = \hat{i}(2\times -3 - -4\times 4) - \hat{j}(3 - -8) + \hat{k}(-4 - 4)$$

$= \hat{i}(-6 + 16) - \hat{j}(3 + 8) + \hat{k}(-4 - 4)$

$= \underline{10\hat{i} - 11\hat{j} - 8\hat{k}}$

30/09/2021 (3) A post is held vertical position by 3 cables AB, AC & AC. If T of cable AB = 40N, Calculate T of AC & AD so that resultant of 3 forces applied at A is vertical.



A (0, 48, 0)

B (16, 0, 12)

C (16, 0, 24)

D (-14, 0, 0)

$r_{AB} = \sqrt{(16-0)^2 + (0-48)^2 + (12-0)^2} = \sqrt{16^2 + 48^2 + 12^2} = \sqrt{2704} = \underline{\underline{52\,m}}$

$r_{AC} = \sqrt{(16-0)^2 + (0-48)^2 + (24-0)^2} = \sqrt{3136} = \underline{\underline{56\,m}}$

$r_{AD} = \sqrt{(-14)^2 + (-48)^2 + 0^2} = \sqrt{2500} = \underline{\underline{50m}}$

Unit vector in direction of AB $= \dfrac{16\hat{i} - 48\hat{j} + 12\hat{k}}{52}$

Force vector in direction of AB $= 40\left[\dfrac{16\hat{i} - 48\hat{j} + 12\hat{k}}{52}\right]$

$= 12.31\hat{i} - \underline{36.92\hat{j}} + 9.23\hat{k}$

Unit vector in direction of AC $= \dfrac{16\hat{i} - 48\hat{j} + 24\hat{k}}{56}$

Force vector $= F_{AC}\left[\dfrac{16\hat{i} - 48\hat{j} + 24\hat{k}}{56}\right]$

$= 0.29F_{AC}\hat{i} - 0.86F_{AC}\hat{j} - 0.43F_{AC}k$

Unit vector in direction of AD = $-14\hat{i} - 48\hat{j} / 50$

Force vector $= F_{AD}\left[\dfrac{-14\hat{i} - 48\hat{j}}{50}\right] = -0.28 F_{AD}\hat{i} - 0.96 F_{AD}\hat{j}$

Resultant force at A = $F_{AB} + F_{AC} + F_{AD}$

$= (12.31\hat{i} - 36.92\hat{j} + 9.23\hat{k}) + (0.29 F_{AC}\hat{i} - 0.89 F_{AC}\hat{j} - 0.43 F_{AC}\hat{k})$
$+ (-0.28 F_{AD}\hat{i} - 0.96 F_{AD}\hat{j})$

$= (12.31\hat{i} + 0.29 F_{AC}\hat{i} - 0.28 F_{AD}\hat{i}) + (-36.92 - 0.89 F_{AC} - 0.96 F_{AD})\hat{j}$
$+ (9.23\hat{k} - 0.43 F_{AC}\hat{k})$.

For resultant to vertical, x & z comp. = 0.

$F_z = 9.23 - 0.43 F_{AC} = 0$

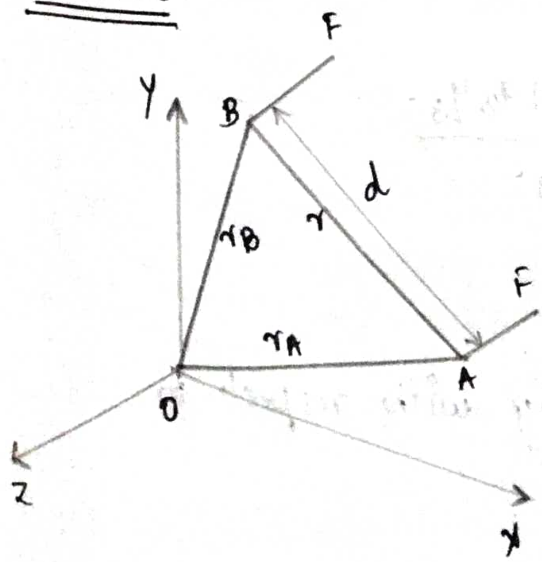
$F_{AC} = \dfrac{9.23}{0.43} = \underline{\underline{21.47\,N}}$ $\underline{\underline{F_{AC} = 21.47\,N}}$

$F_x = 12.31 + 0.29 F_{AC} - 0.28 F_{AD} = 0$

$12.31 + 0.29 \times 21.47 - 0.28 F_{AD} = 0$

$F_{AD} = \dfrac{18.5363}{0.28} = \underline{66.20\,N}$ $\underline{\underline{F_{AD} = 66.20\,N}}$

COUPLE



$\boxed{M = \vec{r} \times \vec{F}}$

r = dis. b/w the points.

(4) Two forces $\vec{F_1} = 50i + 80j + 100k$ & $F_2 = -50i - 80j - 100k$ acts at point A(0.7, 1.5, 1) & B(1, 0.9, -1) respectively. Calculate the moment of the force & ⊥r distance b/w the forces.

$\vec{F_1} = 50\hat{i} + 80j + 100k$

$\vec{F_2} = -50\hat{i} + -80j - 100k$

$x_A = 0.7 \quad x_B = 1.5 \quad x_C = 1$

$x_A = 1 \quad x_B = 0.9 \quad x_C = -1$

$|\vec{r}| = \sqrt{(1-0.7)^2 + (0.9-1.5)^2 + (-1-1)^2}$

$= \sqrt{(0.3)^2 + (-0.6)^2 + (-2)^2} = \sqrt{0.09 + 0.36 + 4}$

$= \sqrt{4.45} = \underline{\underline{2.11}}$

$\vec{r} = -0.3\hat{i} + 0.6\hat{j} + 2\hat{k}$

$$M = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -0.3 & 0.6 & 2 \\ 50 & 80 & 100 \end{vmatrix} = (60-160)\hat{i} + (100+30)\hat{j} + (-24-30)$$

$= \underline{100\hat{i} + 130\hat{j} - 54\hat{k}}$

$|\vec{M}| = \sqrt{100^2 + 130^2 + 54^2} = 172.67$

Magnitude of force $F = \sqrt{50^2 + 80^2 + 100^2} = 137.48$

Moment of couple $= F \times d \quad \Rightarrow d = \frac{M}{F} = \frac{172.67}{137.48} = \underline{1.26m}$

CENTROID OF COMPOSITE AREAS

$$\bar{x} = \frac{\int x a}{\int a} = \frac{\Sigma(xa)}{\Sigma a} = \frac{a_1x_1 + a_2x_2 + a_3x_3 \ldots}{a_1 + a_2 + a_3 \ldots}$$

$$\bar{y} = \frac{\int ya}{\int a} = \frac{\Sigma(ya)}{\Sigma a} = \frac{a_1y_1 + a_2y_2 + a_3y_3 \ldots}{a_1 + a_2 + a_3 \ldots}$$

(5) Locate the centroid of 'T' section

Since the section is symmetrically with respect to y-axis, $\bar{x} = 0$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

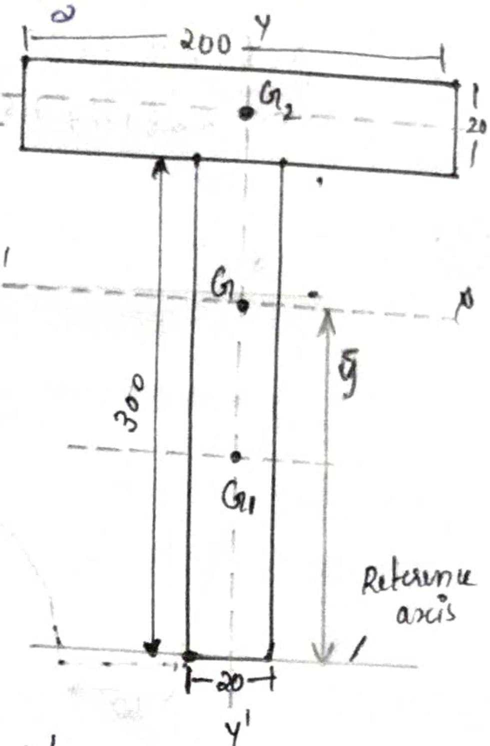
$a_1 = 300 \times 20 = 6000\,mm^2$

$a_2 = 200 \times 20 = 4000\,mm^2$

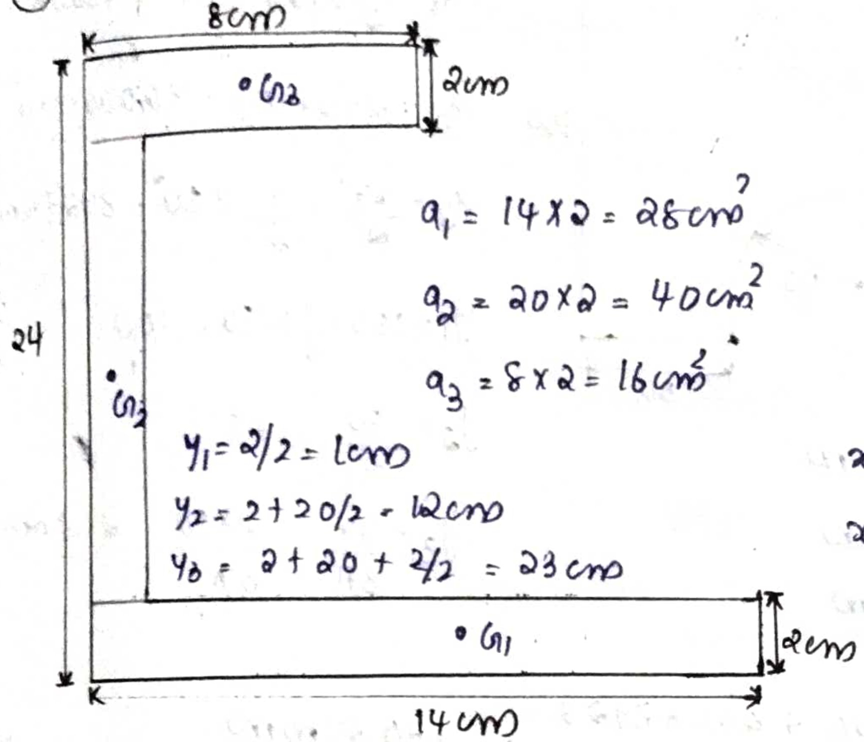
$y_1 = \frac{300}{2} = 150\,mm$

$y_2 = 300 + \frac{20}{2} = 310\,mm$

$\bar{y} = \frac{6000 \times 150 + 4000 \times 310}{6000 + 400} = \underline{214\,mm}$



(6) Locate the centroid of the area



$a_1 = 14 \times 2 = 28\,cm^2$

$a_2 = 20 \times 2 = 40\,cm^2$

$a_3 = 8 \times 2 = 16\,cm^2$

$y_1 = 2/2 = 1\,cm$

$y_2 = 2 + 20/2 = 12\,cm$

$y_3 = 2 + 20 + 2/2 = 23\,cm$

$x_1 = 14/2 = 7\,cm$

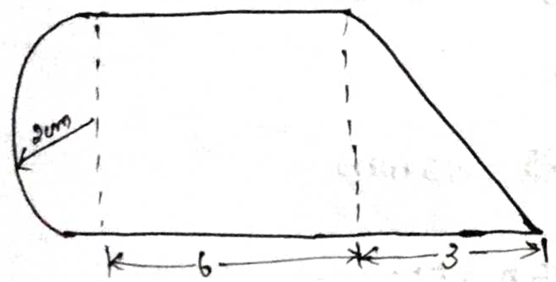
$x_2 = 2/2 = 1\,cm$

$x_3 = 8/2 = 4\,cm$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{28 \times 7 + 40 \times 1 + 16 \times 4}{28 + 40 + 16} = \underline{3.57\,cm}$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{28 \times 1 + 40 \times 12 \times 23 \times 4}{28 + 40 + 16} = \underline{10.43\,cm}$

(7)



$a_1 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 2^2 = 6.28\,cm^2$

$a_2 = 6 \times 4 = 24\,cm^2$

$a_3 = \frac{1}{2} \times 3 \times 4 = 6\,cm^2$

$x_1 = r - \frac{4r}{3\pi} = 2 - \frac{4 \times 2}{3\pi} = 1.15\,cm$

$x_2 = 2 + 6/2 = 5\,cm$

$x_3 = 2 + 6 + \frac{1}{3} \times 3 = 9\,cm$

$y_1 = 2\,cm$

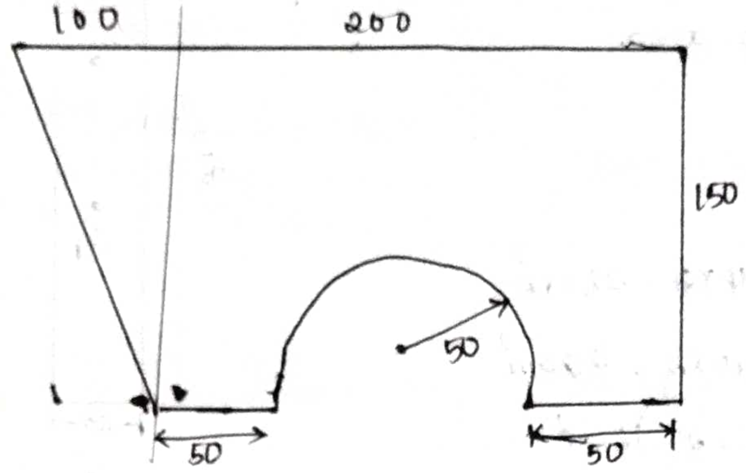
$y_2 = 2\,cm$

$y_3 = \frac{1}{3} \times 4 = 1.33\,cm$

$$\bar{x} = \frac{6.28 \times 1.15 + 24 \times 5 + 6 \times 9}{6.28 + 24 + 6} = \underline{5 cm}$$

$$\bar{y} = \frac{6.28 \times 2 + 24 \times 2 + 6 \times 1.33}{6.28 + 24 + 6} = \underline{1.89 cm}$$

(8)



$a_1 = \frac{1}{2} \times 100 \times 150 = 7500 mm^2$

$a_2 = 200 \times 150 = 30000 mm^2$

$a_3 = \frac{\pi r^2}{2} = \frac{\pi}{2} \times 50^2 = 3927 mm^2$

$y_1 = 150 - \frac{1}{3} \times 150 = 100$

$y_2 = \frac{150}{2} = 75$

$y_3 = \frac{4r}{3\pi} - \frac{4 \times 50}{3\pi} = 21.2 mm$

$x_1 = 100 - \frac{1}{3} \times 100 = 66.67 mm$

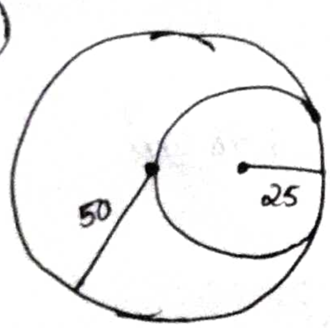
$x_2 = 100 + 200/2 = 200 mm$

$x_3 = 100 + 50 + 50 = 200 mm$

$$\bar{x} = \frac{66.7 \times 7500 + 200 \times 30000 + 200 \times 3927}{7500 + 30000 + 3927} = \underline{170.21 mm}$$

$$\bar{y} = \frac{100 \times 7500 + 75 \times 30000 + 21.2 \times 3927}{7600 + 30000 + 3927} = \underline{86.88 mm}$$

(9)



$a_1 = \pi R^2 = \pi \times 50 \times 50 = 2500\pi \ mm^2$

$a_2 = \pi r^2 = \pi \times 25^2 = 625\pi \ mm^2$

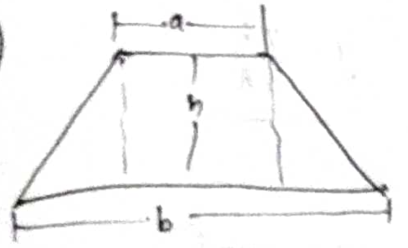
$x_1 = R = 50 mm$

$x_2 = R + r = 50 + 25 = 75 mm$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{2500\pi \times 50 - 625\pi \times 75}{2500\pi - 625\pi} = 41.67 mm$$

$\bar{y} = 0$

(10)



$a_1 = ah$

$a_2 = a_3 = \frac{1}{2}\frac{(b-a)}{2} h$

$a_1 + a_2 + a_3 = \frac{a+b}{2} h$

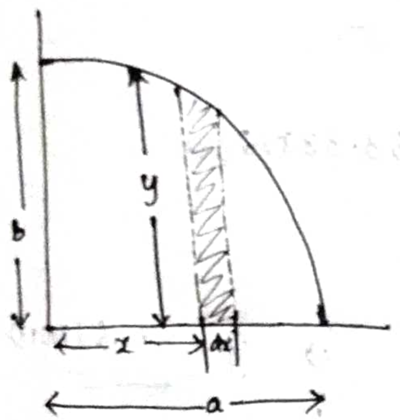
$y_1 = h/2$

$y_2 = y_3 = \frac{1}{3} h$

$\bar{x} = 0$

$$\bar{y} = \frac{ah \times \frac{h}{2} + 2\left[\frac{1}{2}\left(\frac{b-a}{2}\right)h\right]}{\left(\frac{a+b}{2}\right)h} = \frac{2a+b}{a+b} \times \frac{h}{3}$$

(11) Quadrent eclipse



Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$$

$$y = b/a \sqrt{a^2 - x^2}$$

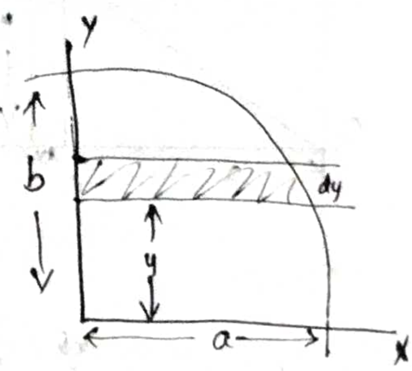
$$x = \frac{a}{b}\sqrt{b^2 - y^2}$$

$$\bar{x} = \frac{\int x \, dA}{\int dA}$$

$$\int dA = \int_0^a y \, dx = \int_0^a \frac{b}{a}\sqrt{a^2 - x^2} \, dx = \frac{b}{a}\frac{\pi a^2}{4} = \frac{\pi ab}{4}$$

$$\int x \, dA = \int_0^a xy \, dx = \int_0^a x \frac{b}{a}\sqrt{a^2 - x^2} \, dx = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2} \, x \, dx$$

Let $a^2 - x^2 = t^2$



Differentiating both sides $-2x \, dx = 2t \, dt$

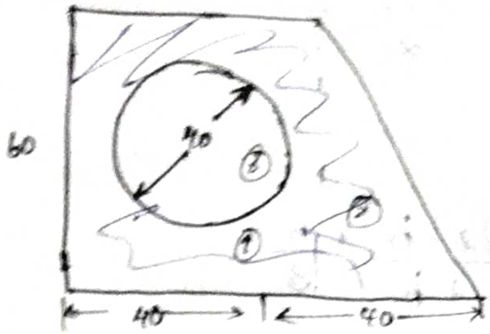
$x \, dx = -t \, dt$

$$\int x \, dA = \frac{b}{a}\int_0^a t[-t \, dt] = -\frac{b}{a}\left[\frac{t^3}{3}\right]_0^a$$

$$= -\frac{b}{3a}\left[(a^2 - x^2)^{3/2}\right]_0^a = \frac{b}{3a}\left[(a^2 - a^2)^{3/2} - (a^2 - 0)^{3/2}\right] = \frac{b}{3a}\left[0 - a^3\right] = \frac{ba^3}{3}$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{ba^3}{3}}{\frac{\pi ab}{4}} \rightarrow \bar{x} = \frac{4a}{3\pi} \qquad \bar{y} = \frac{4b}{3\pi}$$

(12)



$a_1 = 40 \times 60 = 2400\,mm^2$

$a_2 = \frac{1}{2} \times 40 \times 60 = 1200\,mm^2$

$a_3 = \pi r^2 = \pi\, 20^2 = 1256.64\,mm^2$

$y_1 = 60/2 = 30 \qquad y_2 = \frac{1}{3} \times 60 = 20\,mm$

$x_1 = 40/2 = 20\,mm$

$x_2 = 40 + \frac{1}{3} \times 40 = 53.33\,mm \qquad y_3 = \bar{y}$

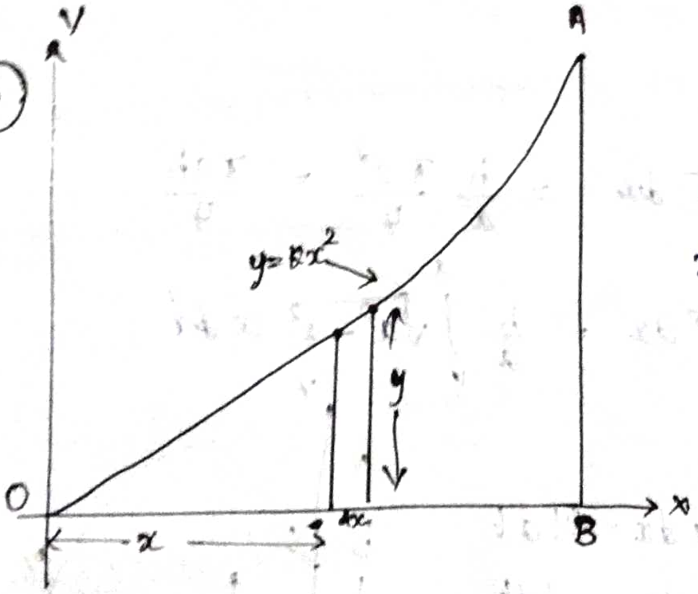
$$\bar{x} = \frac{2400 \times 20 + 1200 \times 53.33 - 1256.64\bar{x}}{2400 + 1200 - 1256.64}$$

$$2343.36\,\bar{x} + 1256.64\bar{x} = 48000 + 53.33 \times 1200$$

$$\bar{x} = 31.11\,mm$$

$$\bar{y} = \frac{2400 \times 30 + 1200 \times 20 - 1256.64\bar{y}}{2400 + 1200 - 1256.64} \Rightarrow \bar{y} = 26.67\,mm$$

(13)



$d_A = y\,dx = kx^2\,dx$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^{25} x\,kx^2\,dx}{\int_0^{25} kx^2\,dx}$$

$$= \frac{k\left[x^4/4\right]_0^{25}}{k\left[x^3/3\right]_0^{25}} = \frac{25^4/4}{25^3/3}$$

$$= \frac{3}{4} \times 25 = 18.75\,cm$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\int_0^{25} \frac{kx^2}{2}\,kx^2\,dx}{\int_0^{25} kx^2\,dx} = \frac{\frac{k^2}{2}\left[x^5/5\right]_0^{25}}{k\left[x^3/3\right]_0^{25}} = \frac{3}{10}k\,25^2 = 187.5k$$

At A, $x = 25$, $y = 15$

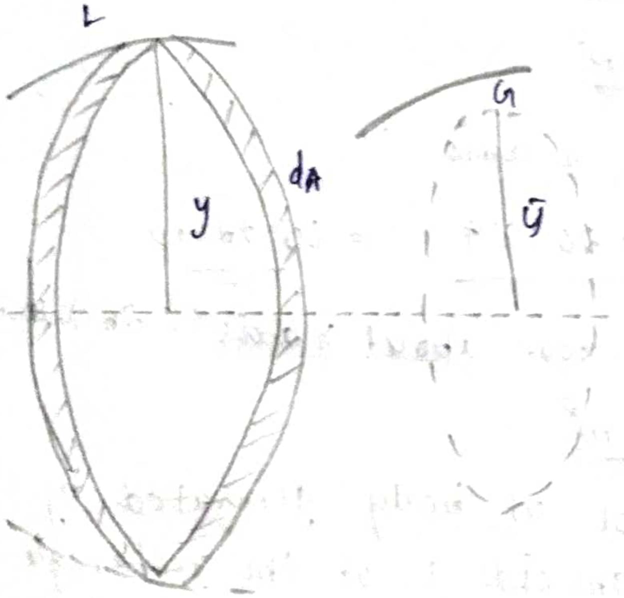
$y = kx^2$ $\therefore K = \frac{15}{25^2}$ $\therefore \bar{y} = \frac{187.5 \times 15}{25^2} = 4.5 cm$

$15 = K \times 25^2$

THEOREM OF PAPPUS - GULDINUS

Theorem :- 01

The area of surface generated by revolving a plane curve about a non intersecting axes in the plane of curve = prdt of length of curve & distance travelled by the centroid of the curve while surface is being generated.



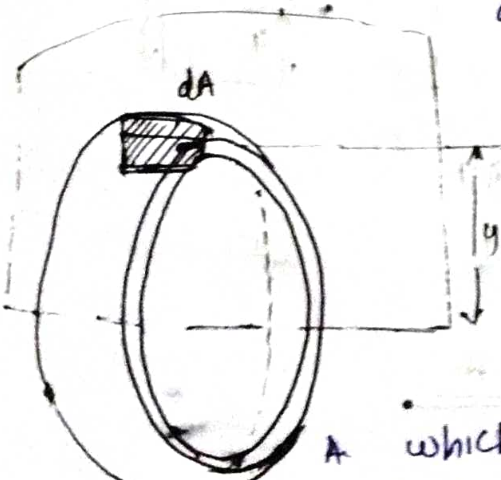
Consider an element of length dL. The area generated by the element is equal to $2\pi y\, dL$.

$$A = \int 2\pi y\, dL = 2\pi \int y\, dL$$

$$= 2\pi \bar{y} L$$

$2\pi\bar{y}$ is the distance travelled by the centroid of curve of length L.

Theorem :- 02



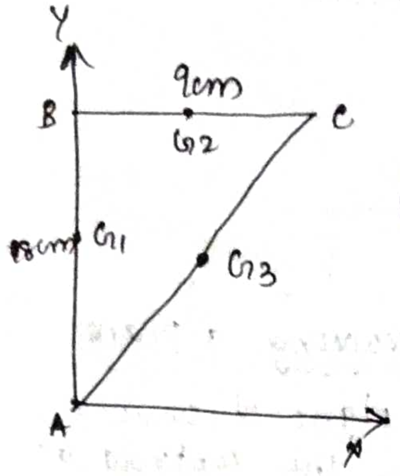
The volume of a body generated by revolving a plane area about a non-intersecting axis in the plane of area equal to the prdt of area = prdt of area & distance travelled by centroid of the plane area while the body is being generated.

Consider an element dA of the area which is revolved about x axis. The volume dv generated by element dA in one revolution = $2\pi y\, dA$

$$V = \int 2\pi y\, dA = 2\pi \int y\, dA = \underbrace{2\pi \bar{y}}\, A$$

distance travelled by centroid of area A.

(14) Calculate the S.A obtained by revolving the line ABC as shown in the figure about (i) X axis (ii) Y-axis



Length of line $= 18 + 9 + \sqrt{18^2 + 9^2}$

$= 18 + 9 + 20.1 = 47.1$cm

Distance of centroid from Y axis,

$$\bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3}{L_1 + L_2 + L_3}$$

$$= \frac{18 \times 0 + 9 \times 4.5 + 20.1 \times 4.5}{47.1} = \underline{2.78\text{cm}}$$

Distance travelled by centroid in one revolution about Y axis

$= 2 \times \pi \times 2.78$cm

Area $= 47.1 \times 2\pi \times 2.78 = \underline{822.7\text{1cm}^2}$

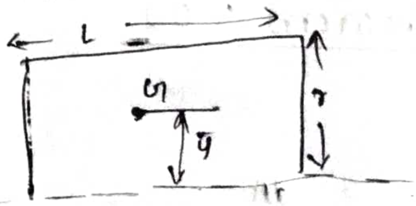
Distance of centroid from x axis

$$\bar{y} = \frac{9 \times 18 + 18 \times 9 + 20.1 \times 9}{47.1} = \underline{10.72\text{cm}}$$

Dist. travelled by centroid in one rev. about X axis $= 2\pi \times 10.72$cm

Area $= 47.1 \times 2\pi \times 10.72 = \underline{3172.46\text{cm}^2}$

(15) Obtain an expression for the vol of body generated by revol. of an ~~reg~~ rectangular area. The side L of the rectangle is in tough with axis of rotation & the other side is of length r.



Area of rectangle $= L \times r$

Distance of centroid from axis, $\bar{y} = r/2$

Distance travelled by centroid in one revolution $= 2\pi\bar{y} = 2\pi \frac{r}{2} = \pi r$

Volume of body generated $= Lr\pi r = \pi r^2 L$

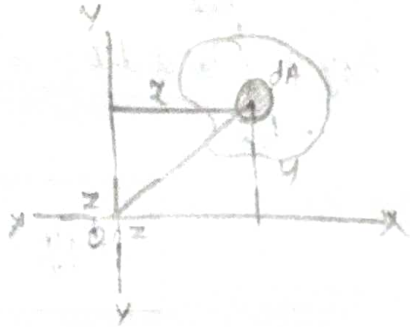
MOMENT OF INERTIA

$$\boxed{\begin{array}{l} I_{AB} = \sum r^2 dA = \int r^2 dA \\ \text{Radius of gyration } K = \sqrt{\frac{I}{A}} \end{array}}$$

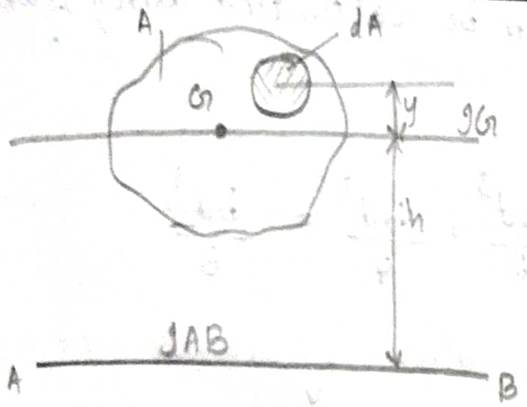
PERPENDICULAR AXIS THEOREM

If I_{xx} and I_{yy} are the moment of inertia of an area A about mutually ⊥ axes XX & YY, in the plane of area, then the moment of inertia of the area about the z axis which is ⊥ to XX & YY axis & passing through the point of intersection of XX & YY axis is given by,

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$



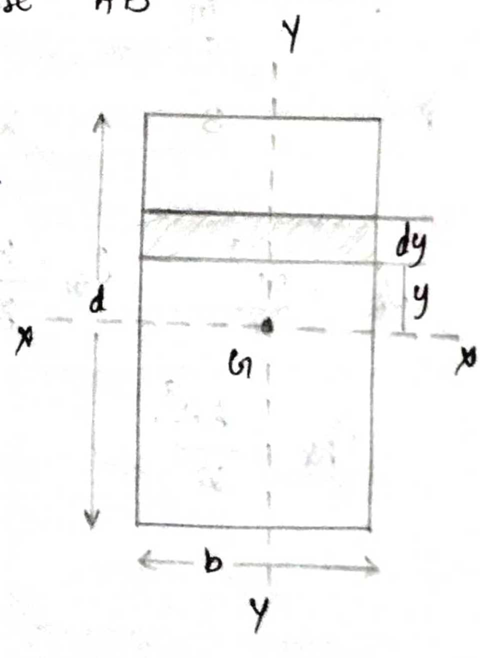
PARALLEL AXIS THEOREM



If I_G is the moment of inertia of a plane lamina of area A, about its centroidal axis in the plane of lamina, then the moment of inertia about any axis AB which is ∥ to the centroidal axis and at a distance h from centroidal axis is given by,

$$\boxed{I_{AB} = I_G + Ah^2}$$

(16) Calculate the moment of inertia of a rectangular cross section about the centroidal axes and about its base AB



Area of element, $dA = b.dy$

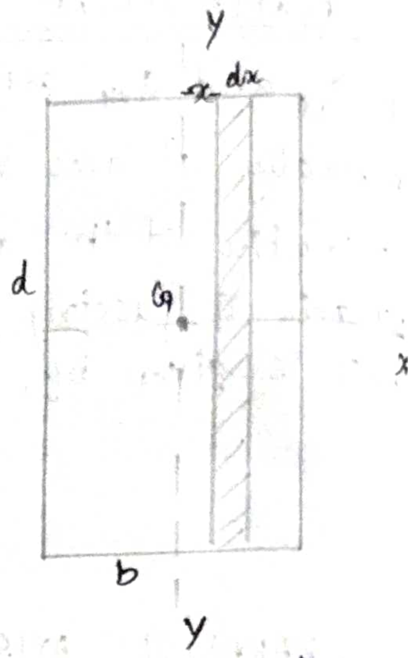
Second moment of this elemental area about X axis =

$$dI_{xx} = y^2 dA = y^2 b.dy$$

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\, dy = b\left[\frac{y^3}{3}\right]_{-d/2}^{d/2} = \frac{b}{3}\left[\frac{d^3}{8} + \frac{d^3}{8}\right]$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Area of element, $dA = d * dx$



Second moment of elemental area about Y axis

$$dI_{yy} = x^2 dA = x^2 d \cdot dx$$

$$I_{yy} = \int_{-b/2}^{b/2} x^2 d\,dx = d\left[\frac{x^3}{3}\right]_{-b/2}^{b/2} = \frac{d}{3}\left[\frac{b^3}{8} + \frac{b^3}{8}\right]$$

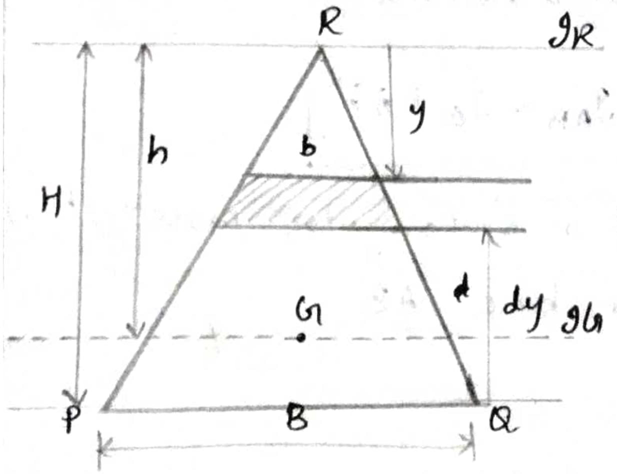
$$\boxed{I_{yy} = \frac{db^3}{12}}$$

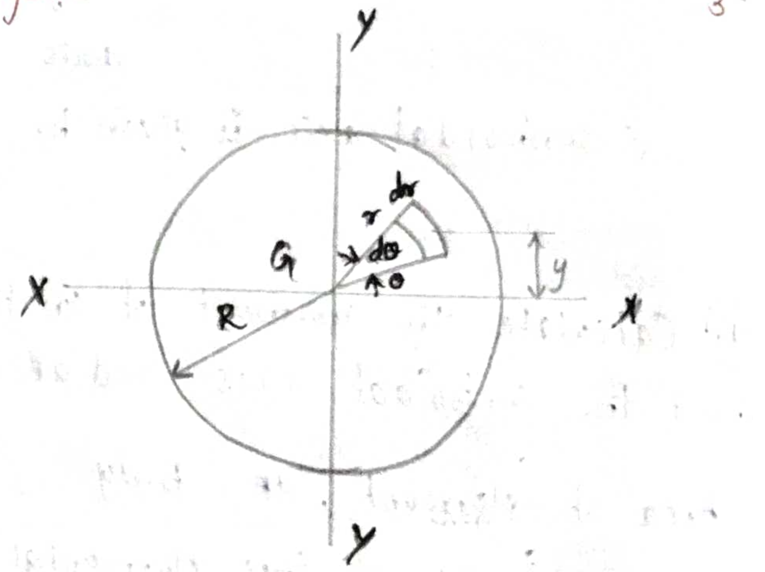
Moment of inertia about the base can be calculated using ‖ axis

$$I_{AB} = I_G + Ah^2$$

$$= \frac{bd^3}{12} + bd \times \left(\frac{d}{2}\right)^2 = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

Moment of inertia of a rectangular lamina about its base is $\frac{bd^3}{3}$



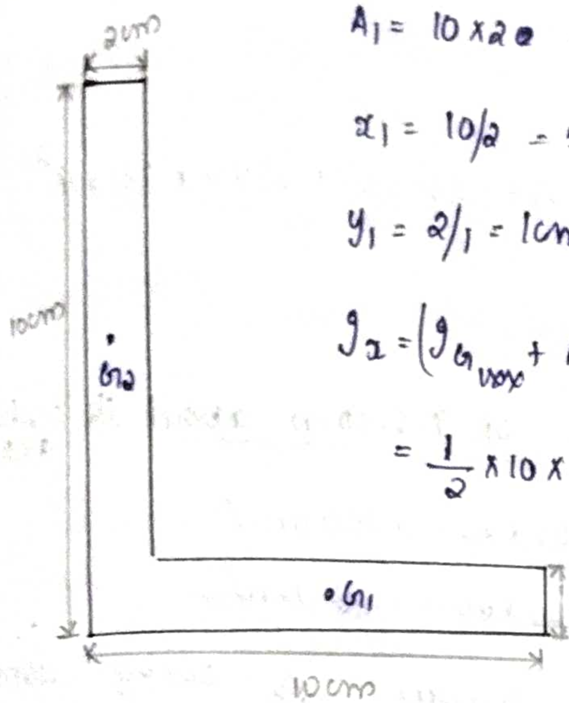


$$I_R = \frac{BH^3}{4} \qquad I_G = \frac{BH^3}{36}$$

$$I_{PQ} = \frac{BH^3}{12}$$

$$I_{yy} = I_{xx} = \frac{\pi R^4}{4}$$

(17) Calculate the M.I of angle section having the dimensions as



$A_1 = 10 \times 2 = 20cm^2$ $\qquad$ $A_2 = 8 \times 2 = 16cm^2$

$x_1 = 10/2 = 5cm$ $\qquad$ $x_2 = 2/2 = 1cm$

$y_1 = 2/1 = 1cm$ $\qquad$ $y_2 = 2 + 8/2 = 6cm$

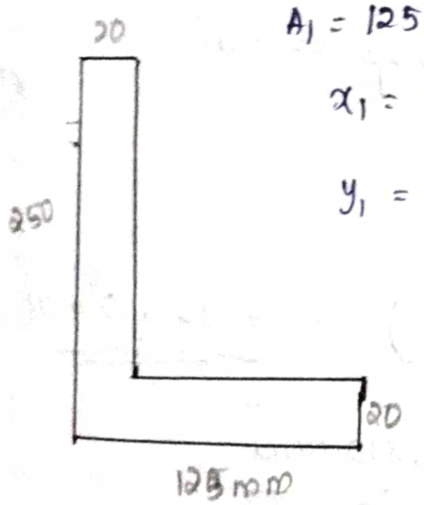
$$I_x = (I_{G_1xx} + A_1 y_1^2) + (I_{G_2xx} + A_2 y_2^2)$$

$$= \frac{1}{2} \times 10 \times 2^3 + 20 \times 1^2 + \frac{1}{12} \times 2 \times 8^3 + 16 \times 6^2 = 688cm^4$$

$$I_y = I_{G_1yy} + A_1 X_1^2 + I_{G_2yy} + A_2 X_2^2$$

$$= \frac{1}{12} \times 2 \times 10^3 + 20 \times 5^2 + \frac{1}{12} \times 8 \times 2^3 + 16 \times 1^3 = \underline{\underline{688cm^4}}$$

(18) Calculate the M.I of an unequal angle iron section of 250 mm × 20mm about its centroid axes



$A_1 = 125 \times 20 = 2500 mm^2$ $\qquad$ $A_2 = 230 \times 20 = 4600 mm^2$

$x_1 = 125/2 = 62.5mm$ $\qquad$ $x_2 = 20/2 = 10mm$

$y_1 = 20/2 = 10mm$ $\qquad$ $y_2 = 20 + \frac{230}{2} = 135mm$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = 28.49mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = 91mm.$$

$I_{G_1xx} = \frac{1}{12} \times 125 \times 20^3 = 83333.33 mm^4$ $\qquad$ $I_{G_2xx} = \frac{1}{12} \times 120 \times 230^2 = 20278333.33 mm^4$

$I_{G_1yy} = \frac{1}{12} \times 20 \times 125^3 = 3255208.33 mm^4$ $\qquad$ $I_{G_2yy} = \frac{1}{12} \times 230 \times 20^3 = 153333.33mm^4$

$h_1 = GG_1 = \bar{y} - y_1 = 91 - 10 = 81mm$ $\qquad$ $h_2 = GG_2 = y_2 - \bar{y} = 135 - 91 = 44mm$

$$I_{G_{xx}} = I_{G_1xx} + A_1 h_1^2 + I_{G_2xx} + A_2 h_2^2$$

$$= 83333.33 + (2500 \times 81^2) + 20278333.33 + 4600 \times 44^2$$

$$= 45669766.66 mm^4$$

h_1 is the horizontal distance GG_1 & h_2 is the horizontal distance GG_2

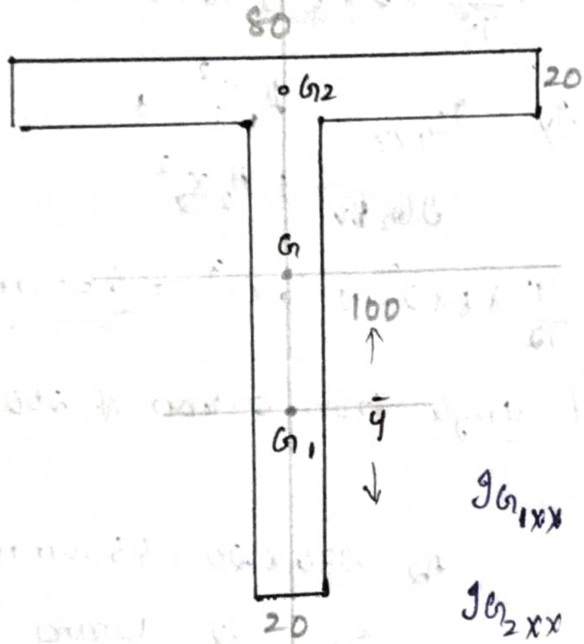
$h_1 = x_1 - \bar{x} = 62.5 - 28.49 = 34.01mm$

$h_2 = \bar{x} - x_2 = 28.49 - 10 = 18.49 mm$

$I_{G_{yy}} = 8255208.33 + 2500 \times 34.01^2 + 153333.33 + 4600 \times 18.49^2$

$= 787288.7.37 mm^4$

(19) Determine the moment of inertia of T-section about its centroid axis



$A_1 = 100 \times 20 = 2000 mm^2$

$A_2 = 80 \times 20 = 1600 mm^2$

$y_1 = \frac{100}{2} = 50mm \qquad y_2 = 100 + \frac{20}{2} = 110mm$

$\bar{y} = \frac{2000 \times 50 + 1600 \times 110}{2000 + 1600} = 76.67$

$I_{G_{1xx}} = \frac{20 \times 100^3}{12} = 1666666.67 mm^4$

$I_{G_{2xx}} = \frac{80 \times 20^3}{12} = 53333.33 mm^4$

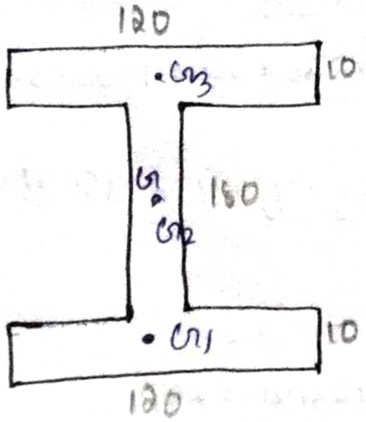
$h_1 = \bar{y} - y = 76.67 - 50 = 26.67$

$h_2 = y_2 - \bar{y} = 110 - 76.76 = 33.33$

$I_{G_{xx}} = (I_{G_{1xx}} + A_1 h_1^2) + (I_{G_{2xx}} + A_2 h_2^2) = 4.92 \times 10^6 mm^4$

$I_{G_{yy}} = \frac{100 \times 20^3}{12} + \frac{20 \times 80^3}{12} = 9.2 \times 10^5 mm^4$

(20)



$I_{GG} = I_{G_1yy} + I_{G_2yy} + I_{G_3yy}$

$= \frac{1}{12}\left[10 \times 120^3 + 180 \times 10^3 + 10 \times 120^3\right]$

$= 2895000 mm^4$

$I_{G_{1xx}} = I_{G_{3xx}} = \frac{1}{12} \times 120 \times 10^3 = 10^4 mm^4$

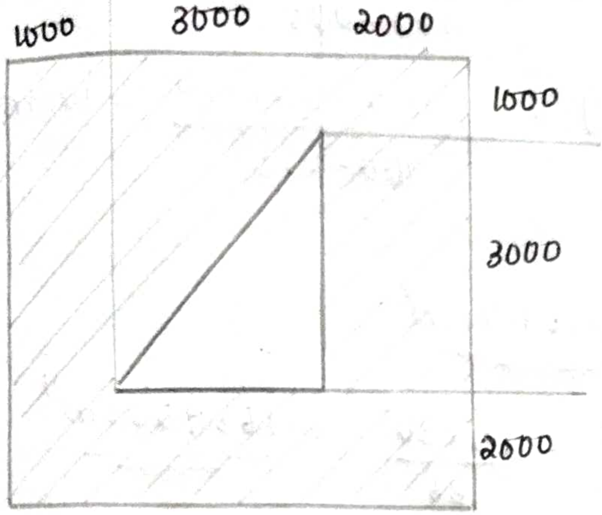
$I_{G_2xx} = \frac{1}{12} \times 10 \times 180^3 = 4860000\,mm^4$

$\cancel{I_{G_3xx} = \frac{1}{12} \times 10 \times 180^3 =}$ $\quad h_1 = h_3 = 100 - 5 = 95\,mm, \quad h_2 = 0$

$$I_{Gxx} = (10000 + 1200 \times 95^2) \times 2 + (4860000 + 180 \times 10 \times 0)$$

$$= \underline{26540000\,mm^4}$$

(21) M.I of shaded area w.r.t the centrodial axes.



$a_1 = 6 \times 10^3 \times 6 \times 10^3 = 36 \times 10^6\,mm^2$

$a_2 = \frac{1}{2} \times 3 \times 3 \times 10^6 = 4.5 \times 10^6\,mm^2$

$x_1 = 3 \times 10^3\,mm \qquad y_1 = 3 \times 10^3\,mm$

$x_2 = \frac{2}{3} \times 3000 + 1000 = 3 \times 10^3\,mm$

$y_2 = \frac{1}{3} \times 3000 + 2000 = 3 \times 10^3\,mm$

$$\bar{x} = \frac{36 \times 10^6 \times 3 \times 10^3 - 4.5 \times 10^6 \times 3 \times 10^3}{36 \times 10^6 - 4.5 \times 10^6} = 3 \times 10^3\,mm$$

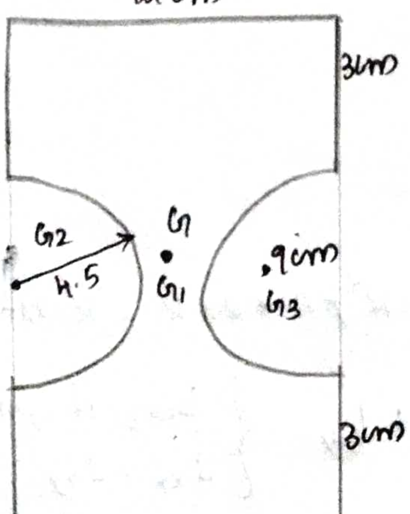
$$I_{Gxx} = I_{G_1xx} - I_{G_2xx} = \frac{1}{12} \times 6 \times 10^3 \times (6 \times 10^3)^3 - \frac{3 \times 10^3 \times (3 \times 10^3)^3}{36}$$

$$= \underline{105.75 \times 10^{12}\,mm^4}$$

$$I_{Gyy} = I_{G_1yy} - I_{G_2yy} = \frac{1}{12} \times 6 \times 10^3 \times (6 \times 10^3)^3 - \frac{3 \times 10^3 \times (3 \times 10^3)^3}{36}$$

$$= \underline{105.75 \times 10^{12}\,mm^4}$$

(22)



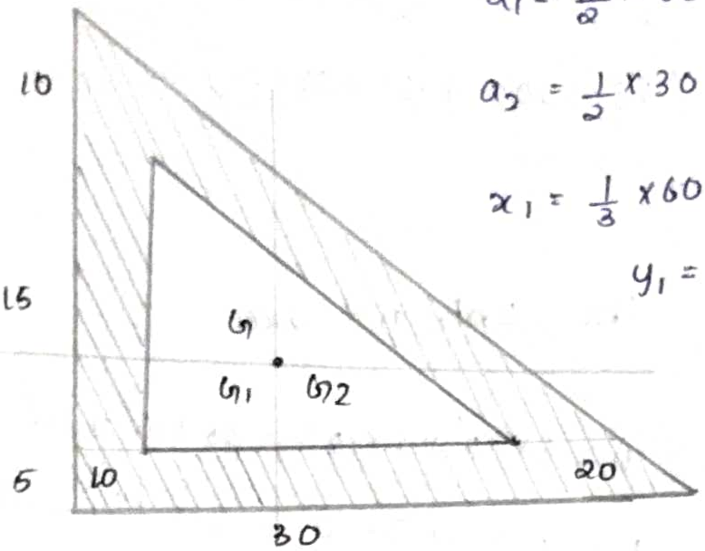
$$I_{Gxx} = I_{G_1xx} - I_{G_2xx} - I_{G_3xx}$$

$$= I_{G_1xx} - 2 \times I_{G_2xx}$$

$$I_{Gyy} = I_{G_1yy} - (I_{G_2yy} + A_2 h_2^2) - (I_{G_3yy} + A_3 h_3^2)$$

$$= I_{G_1yy} - 2(I_{G_2yy} + A_2 h_2^2)$$

(23) Determine the M.I of shaded area w.r.t centroidal axes



$a_1 = \frac{1}{2} \times 60 \times 30 = 900\,cm^2$

$a_2 = \frac{1}{2} \times 30 \times 15 = 225\,cm^2$

$x_1 = \frac{1}{3} \times 60 = 20 \qquad x_2 = \frac{1}{3} \times 30 + 10 = 20$

$y_1 = \frac{1}{3} \times 30 = 10 \qquad y_2 = \frac{1}{3} \times 15 + 5 = 10$

$\bar{x} = \frac{900 \times 20 - 225 \times 20}{900 - 225} = 20\,cm.$

$\bar{y} = \frac{900 \times 10 - 225 \times 10}{900 - 225} = 10\,cm$

$I_{Gxx} = I_{G_1xx} - I_{G_2xx}$

$= \frac{60 \times 30^3}{36} - \frac{30 \times 15^3}{36} = \underline{42187.5\,cm^4}$

$I_{Gyy} = I_{G_1yy} - I_{G_2yy} = \frac{30 \times 60^3}{36} - \frac{15 \times 30^3}{36} = \underline{168750\,cm^4}$

Note

Translatory inertia is defined as mass & rotational inertia is known as moment of inertia.

MASS MOMENT OF INERTIA

unit - kgm^2

The moment of inertia of a body about an axis at a distance d and // to centroidal axis is = sum of M.I about centroidal axis and produce of mass & sq. of distance b/w parallel axis

$$\boxed{I = I_G + md^2}$$

Mass moment of inertia of ring

M.I of ring about zz axis

$I_{zz} = \int_0^{2\pi R} \rho A\, dl \times R^2 = R^2 \rho A [l]_0^{2\pi R} = R^2 \rho A \times 2\pi R = R^2\, 2\pi R A \rho$

$\boxed{I_{zz} = mR^2}$ $\qquad I_{zz} = I_{xx} + I_{yy}$ $\left\{ \begin{array}{l} \text{b/c of sym.} \\ I_{xx} = I_{yy} \end{array} \right.$

$$I_{zz} = I_{xx} + I_{yy} = 2 I_{xx}$$

$$\boxed{I_{xx} = I_{yy} = \frac{MR^2}{2}}$$

Mass moment of inertia of a disc

$$I_{zz} = \int_0^R 2\pi r \, dr \, t \rho r^2 = 2\pi t \rho \int_0^R r^3 dr = 2\pi t \rho \left[\frac{r^4}{4}\right]_0^R = (\pi R^2 t \rho) \frac{R^2}{2} = \frac{mR^2}{2}$$

polar moment of mass, $I_{zz} = I_{xx} + I_{yy} = \frac{mR^2}{2}$

$$\boxed{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{mR^2}{4}}$$

Mass moment of inertia of a cylinder

Mass of element, $dm = \pi R^2 dy \, \rho$

Moment of inertia, thin circular disc about its centroidal xx axis

$$dI = \frac{dm R^2}{4} \Rightarrow dI_{xx} = dI + (dm) y^2$$

$$dI_{xx} = \left[\frac{dm R^2}{4} + dm y^2\right] dm = \pi R^2 dy \frac{\rho R^2}{4} + \pi R^2 dy \times \rho y^2$$

$$I_{xx} = \int_{-b/2}^{b/2} \left(\frac{\pi R^2}{4} \rho\right) dy + \pi R^2 \rho \int_{-b/2}^{b/2} y^2 dy = 2\pi \frac{R^4}{4} \rho [y]_0^{b/2} + 2\pi R^2 \rho \left[\frac{y^3}{3}\right]_0^{b/2}$$

$$= \frac{M}{4}\left(\frac{3R^2 + b^2}{3}\right) = \frac{M}{12}(3R^2 + b^2)$$

$$\boxed{I_{zz} = I_{xx} = \frac{M}{12}(3R^2 + b^2)}$$

04/08/2021
Wednesday

MODULE-4

RECTILINEAR TRANSLATION

- Velocity $v = \frac{dx}{dt}$
- accelaration $a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$

EQUATION OF KINEMATICS

$$\boxed{\begin{array}{l} \bullet\ v = u + at \\ \bullet\ v^2 = u^2 + 2as \\ \bullet\ s = ut + \frac{1}{2}at^2 \end{array}}$$

a = accelaration (m/s^2)
t = time
v = final velocity (m/s)
u = initial velocity (m/s)
s = distance

(a) For a freely falling body

$v = u + gt$

$v^2 = u^2 + 2gh$

$h = ut + \frac{1}{2}gt^2$

(b) When a particle moves upward

$v = u - gt$

$v^2 = u^2 - 2gh$

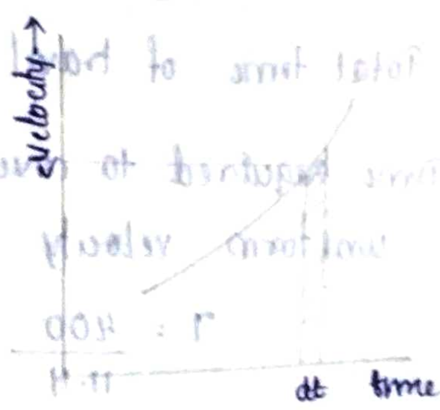
$h = ut - \frac{1}{2}gt^2$

VELOCITY-TIME CURVE

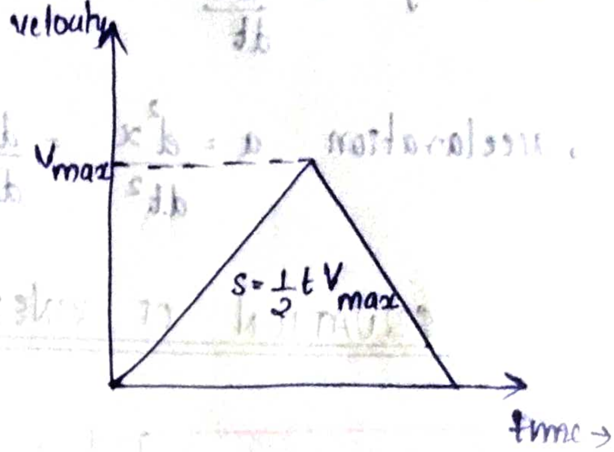
x axis → time   y-axis → velocity

area under v-t curve → displacement.

slope of v-t curve → accelaration.



① A train travels b/w 2 stopping stations, 7 m apart in 14 min. Assuming that its motion is one of uniform accelaration for part of the journey and uniform retardation for the rest, prove that the greatest speed on the journey is 60 km/hr.



$S = 7 km \qquad u = 0$

$t = 14 min = \left(\frac{14}{60}\right) hr$

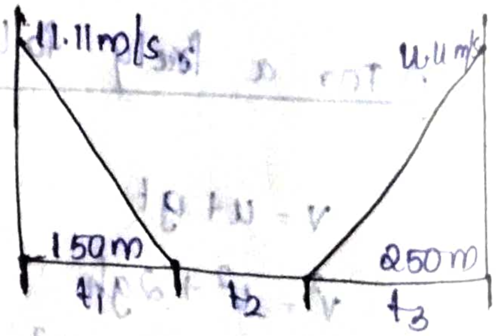
$S = \frac{1}{2} t V_{max}$

$V_{max} = \frac{S \times 2}{t} = \frac{7 \times 2 \times 60}{14} = \underline{60 \ km/hr}$

05/08

② A car travelling at 40 kmph sights a distant signal at 150m and comes uniformly to rest at the signal. It remains at rest for 20s. As allowed by the signal, it uniformly accelarate and attain 40 kmph in 250m. Calculate the time lost due to signal.

From velocity-time graph



$150 = \frac{1}{2} \times t_1 \times 11.11$

$t_1 = \frac{300}{11.11} = 27s$

$t_2 = 20s$

$t_3 \quad 250 = \frac{1}{2} \times t_3 \times 11.11 \Rightarrow t_3 = \frac{500}{11.11} = 45s$

Total time of travel $= t_1 + t_2 + t_3 = 27 + 20 + 45 = \underline{92s}$

Time required to cover a distance of (150+250m) = 400m with a uniform velocity of 11.11 m/s

$T = \frac{400}{11.11} = \underline{36s}$

Time lost due to signal

$(t_1+t_2+t_2) - T = 92 - 36 = \underline{56 s}$

(3) The motion of a particle along a straight line is defined as $S = 25t + 5t^2 - 2t^3$, where S is in metres and t in second.

Find (i) velocity and accelaration at the start.

(ii) the time the particle reaches maximum velocity

(iii) the maximum velocity of the particle

$S = 25t + 5t^2 - 2t^3$

Velocity $V = 25 + 10t - 6t^2$

Accelaration $a = 10 - 12t$

(i) At $t = 0$,

$V = 25 + 0 - 0 = \underline{25 \text{ m/s}}$

$a = 10 - 0 = \underline{10 \text{ m/s}^2}$

(ii) At maximum velocity, $\frac{dV}{dt} = 0$ i.e, $a = 0$

$0 = 10 - 2t$

$12t = 10 \Rightarrow t = \frac{10}{12} = \underline{0.83 s}$

(iii) The maximum velocity of the particle at $t = 0.83 s$

$\therefore V_{max} = 25 + 10 \times 0.83 - 6 \times 0.83 \times 0.83$

$= 25 + 8.3 - 4.13 = 25 + 4.17$

$= \underline{29.17 \text{ m/s}}$

H.W

(4) The displacement of a particle is given by $s = t^3 - 3t^2 + 2t + 5$. Find the time at which the accelaration is zero and the time at which velocity is 2m/s.

$$s = t^3 - 3t^2 + 2t + 5$$

(i) $\frac{dv}{dt} = a = 0$;

$$a = \frac{d^2 s}{dt^2} = \frac{d}{dt}(3t^2 - 6t + 2) = 6t - 6$$

$a = 0$

$0 = 6t - 6 \Rightarrow 6t = 6 \Rightarrow \underline{\underline{t = 1s}}$

(ii) Time at which $v = 2m/s$

$$v = 3t^2 - 6t + 2$$

$2 = 3t^2 - 6t + 2 \Rightarrow 3t^2 - 6t = 0$

$3t^2 = 6t \Rightarrow \underline{\underline{t = 2s}}$

H.W

(5) A point is moving in a straight line with accelaration given by $a = 15t - 20$. It passes through a reference point at $t = 0$ and another point 30m away after an interval of 5 seconds. Calculate the displacement, velocity and accelaration of the point after a futher interval of 5 seconds.

$a = 15t - 20$; at $t = 0$, $s = 0$, at $t = 5$, $s = 30m$

$$a = \frac{dv}{dt} = 15t - 20$$

$$v = \int \frac{dv}{dt} = \frac{15t^2}{2} - 20t$$

$$v = \frac{ds}{dt} = 7.5t^2 - 20t + C_1$$

$$S = \int (7.5t^2 - 20t + C_1) \cdot dt$$

$$= \frac{7.5t^3}{3} - \frac{20t^2}{2} + C_1 t + C_2$$

$$= 2.5t^3 - 10t^2 + C_1 t + C_2$$

At $t=0$, $S=0$,

$$0 = 0 - 0 + 0 + C_2$$

$$\therefore C_2 = 0$$

At $t=5$, $S = 30m$ (1)

$$30 = 2.5 \times 5^3 - 10 \times 5^2 + C_1 \times 5 + 0$$

$$30 = 2.5 \times 125 - 10 \times 25 + C_1 \times 5$$

$$30 = 312.5 - 250 + 5C_1$$

$$30 = 62.5 + 5C_1$$

$$5C_1 = 32.5$$

$$C_1 = -6.5$$

Displacement, velocity and accelaration at the end of 10s

$$S = 2.5t^3 - 10t^2 - 6.5t$$

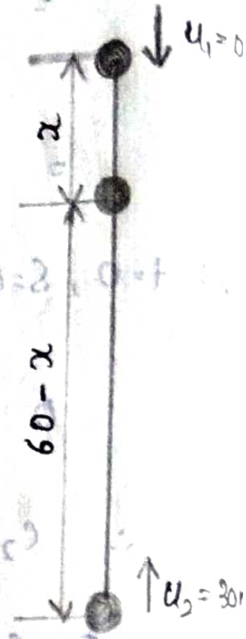
$$= 2.5 \times 10^3 - 10 \times 10^2 - 6.5 \times 10$$

$$= 2500 - 1000 - 65 = \underline{1435\,m}$$

Velocity $V = 7.5t^2 - 20t - 6.5 = 7.5 \times 10^2 - 20 \times 10 - 6.5 = \underline{543.5\,m/s}$

Accelaration $a = 15t - 20 = 15 \times 10 - 20 = \underline{130\,m/s^2}$

⑥ A stone is dropped from the top of a tower, 60 m high. At the same time another stone is thrown upwards from the foot of the tower with a velocity of 30 m/s. When and where does the two stones cross each other?



Height of tower $h = 60\,m$

$u_1 = 0, \quad u_2 = 30\,m/s$

$t_1 = t_2 = t$

Let x be the distance from the top of the tower where the two stone cross each other.

$$x = u_1 t + \frac{1}{2} g t^2 = 0 + \frac{1}{2} g t^2 \quad \text{——— ①}$$

$$60 - x = u_2 t - \frac{1}{2} g t^2 \quad \text{——— ②}$$

Adding equations ① and ②

$$60 = u_2 \times t$$

$$t = \frac{60}{30} = 2\,s$$

$$x = u_1 t + \frac{1}{2} g t^2$$

$$= 0 + \frac{1}{2} \times 9.81 \times 2^2 = 9.81 \times 2 = 19.62\,m$$

The two stones will cross each other at a distance of 19.62 m from the top of the tower after 2 seconds.

KINETICS

Three approaches to solution of problems in kinetics

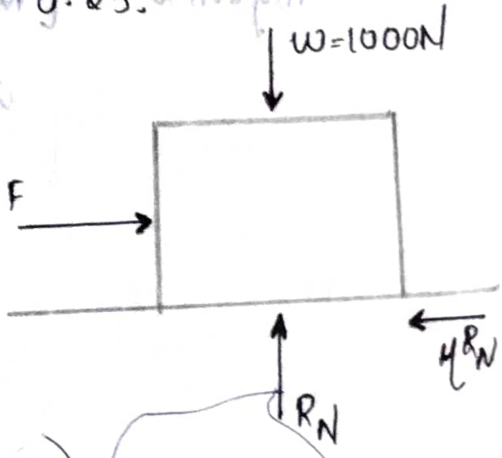
1. Direct application of Newton's Second Law.
2. Use of Work-energy Principle.
3. Solution by Impulse and Momentum.

DIRECT APPLICATION OF NEWTON'S SECOND LAW

- Force is directly proportional to the product of mass and accelaration.
- Newton's law reduces to $F = m \times a$.
- When a system of force acts on a body.

Resultant/Net force = (mass) x (Accelaration in the direction of resultant of force)

1) A block weighing 1000N rest on a horizontal plane. Find the magnitude of the force required to give the block an accelaration of 2.5 m/s² to the right. The coefficient of kinetic friction b/w the block and the plane is 0.25.



$W = 1000N$, $a = 2.5\ m/s^2$, $\mu = 0.25$

Since there is no motion in the vertical direction,

Net force in the vertical direction = 0

$$R_N - W = 0$$

$$R_N = W = 1000\ N$$

Net force in the horizontal direction = $m \times a$

$$F - \mu R_N = m \times a$$

$$F - 0.25 \times 1000 = \frac{1000}{9.81} \times 2.5$$

$$F - 250 = 254.84$$

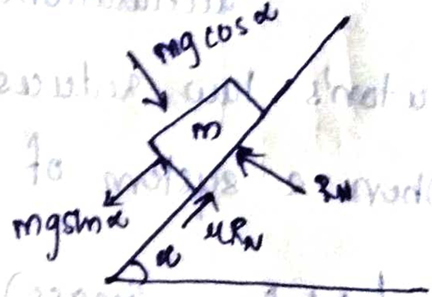
$$F = 254.84 + 250$$

$$\underline{F = 504.84}$$

11/08/21

8 A body of mass 50 kg slides down a rough inclined plane whose inclination to the horizontal is 30°. If $\mu = 0.4$, find a?

$m = 50\,kg \quad \alpha = 30° \quad \mu = 0.4$



No motion in ⊥ direction of inclined plane

$$R_N - mg\cos\alpha = 0$$

$$R_N = mg\cos\alpha$$

Net force along inclined plane

$$mg\sin\alpha - \mu R_N = ma \Rightarrow mg\sin\alpha - \mu mg\cos\alpha = ma$$

$$a = g\sin\alpha - \mu g\cos\alpha$$

$$= 9.81 \sin 30 - 0.4 \times 9.81 \times \cos 30$$

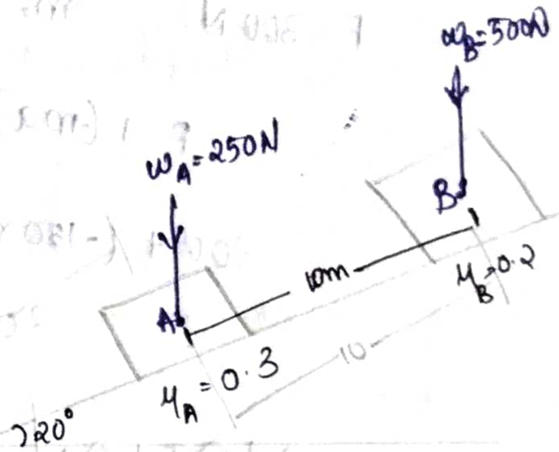
$$\underline{= 1.51\ m/s^2}$$

(9) Two blocks A and B are held stationary 10m apart on a 20° incline as shown. The coefficient of friction μ_A = 0.3 while it is 0.2 b/w plane B $\mu_B = 0.2$. If blocks are released simultaneously, calculate the time taken & distance travelled by each block before they are at verge of collision.

Consider motion of block A

$$\text{Net force} = m \times a$$

$$m_A g \sin\theta - \mu R_{NA} = m_A a_A$$



$$m_A g \sin\theta - \mu m_A g \cos\theta = m_A a_A$$

$$250 \sin 20 - 0.3 R_{NA} = \frac{250}{9.81} a_A$$

$$250 \sin 20 - 0.3 \times 250 \times \cos 20 = \frac{250}{9.81} a_A$$

$$a_A = 0.59 \ m/s^2$$

Consider the motion of block B,

Net force = mass x accelaration

$$m_B g \sin\theta - \mu R_{NB} = m_B \times a_B \Rightarrow m_B g \sin\theta - \mu m_B g \cos\theta = m_B \times a_B$$

$$500 \sin 20 - 0.2 \times R_{NB} = \frac{500}{9.81} \times a_B \Rightarrow a_B = 1.51 \ m/s^2$$

Let x be the distance travelled by A in t seconds then the distance travelled by B in the same t second will be $(10+x)$

$$S_A = x = u_A t + \frac{1}{2} a_A t^2 \rightarrow x = 0 + \frac{1}{2} \times 0.59 \times t^2$$

$$S_B = 10 + x = 0 + \frac{1}{2} \times a_B t^2 = 0 + \frac{1}{2} \times 1.51 \times t^2$$

$$\therefore t = 4.66 s$$

$$x = \frac{1}{2} \times 0.59 \times (4.66)^2 = 6.41 m$$

D' ALEMBERT'S PRINCIPLE :- application of Newton's law

The resultant of a system of force acting on a body in motion is in dynamic equilibrium with the inertia force.

(10) A force of 300 N acts on a body of mass 150 kg. Calculate the accelaration of the body using D Alembert's Principle.

$F = 300\ N \qquad m = 150\ kg$

$$\boxed{\begin{array}{l} F + (-ma) = 0 \\ (-ma) = \text{inertia force} \end{array}}$$

$$F + (-ma) = 0$$

$$300 + (-150 \times a) = 0$$

$$300 = 150a \Rightarrow \underline{a = 2\ m/s^2}$$

MOTION OF LIFT

- application of newton's second law.

(a) Lift moving downwards

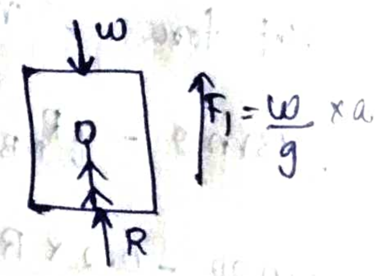
- a → downwards ; inertia of force → upwards

$$R + F_1 - W = 0$$

$$R = W - F_1$$

$$= W - \frac{W}{g} a$$

$$\boxed{R = W\left[1 - \frac{a}{g}\right]}$$

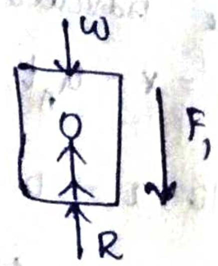


(b) Lift moving upwards

$$R - W - F_1 = 0$$

$$R = W + F_1$$

$$\boxed{R = W\left[1 + \frac{a}{g}\right]}$$



NOTES :-

- A lift moves with uniform velocity, the accelaration of lift is 0.
- when lift moves down with accelaration, man exerts less force on the floor of lift, and when lift moves up with accelaration, man exerts more force on the floor of lift.
- Direction of inertia force is opposite to direction of accelaration
  → when lift accelarates
- Direction of inertia force is same to direction of accelaration
  → when lift decelarates.

(11) A lift has an upward a. is $1.2 m/s^2$, what a force will a man weighing 750 N exert on floor of lift? what force would be exert if the lift had an accelaration of $1.2 m/s^2$ downward what upward a would cause his weight to exert a force of 900N on the floor?

[KTU Jan 2016, June 2016, May 2019]

case (i)

When the lift moves upward $a = 1.2 m/s^2$ $W = 750 N$

$$R = W\left[1 + \frac{a}{g}\right] = 750\left[1 + \frac{1.2}{9.81}\right] = \underline{841.74 N}$$

case (ii)

when lift moving downwards

$$R = W\left[1 - \frac{a}{g}\right] = 750\left[1 - \frac{1.2}{9.81}\right] = \underline{658.26 N}$$

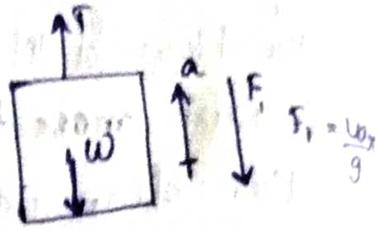
case (iii)

$W = 750N$ $R = 900N$

when lift moves up $R = W\left[1 + \frac{a}{g}\right] \Rightarrow 900 = 750\left[1 + \frac{a}{9.8}\right]$

$$\frac{900}{750} = 1 + \frac{a}{9.8} \Rightarrow \underline{a = 1.96 m/s^2}$$

(12) An elevator of total weight 5000N starts to move upwards with a constant acc of $1 m/s^2$. Find the force in the cable during the accelaration motion. Also find the force at the accelaration elevator under the feet of a man weighing 600N when the elevator moves up with a uniform retardation of $1 m/s^2$.



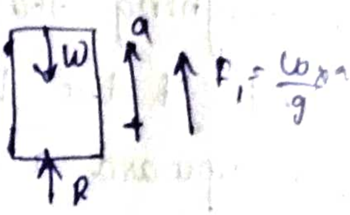
case (i) Elevator moves upwards with accelaration

$$T - W - F_1 = 0$$

$$T = W + \frac{W}{g} a$$

$$T = W\left[1 + \frac{a}{g}\right] = 5000\left[1 + \frac{1}{9.81}\right] = \underline{5509.68N}$$

case (ii) Elevator moves up with uniform decelaration; mutha of force is upwards $W = 600N$



$$R + F_1 - W = 0$$

$$R = W - F_1 = W - \frac{W}{g} a$$

$$= W\left[1 - \frac{a}{g}\right] = 600\left[1 - \frac{1}{9.81}\right] = \underline{538.84N}$$

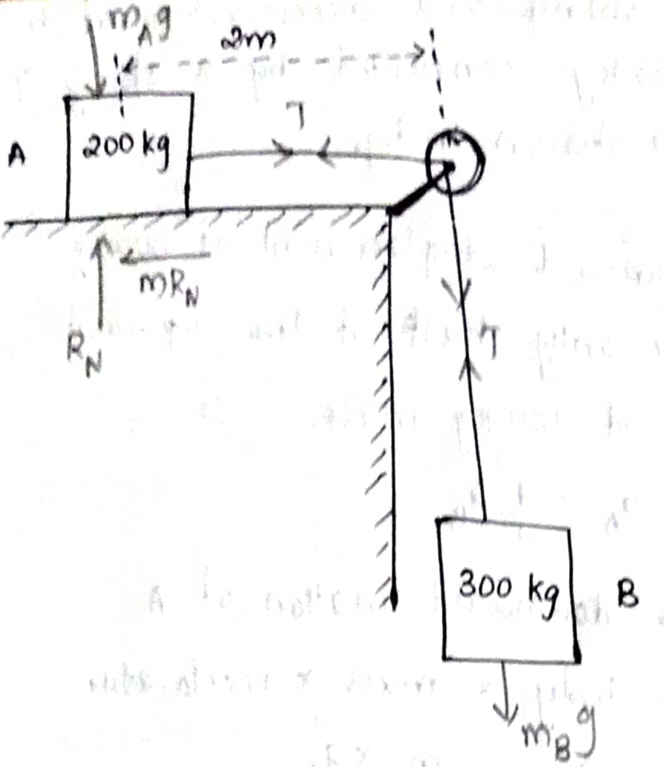
13/08/2021

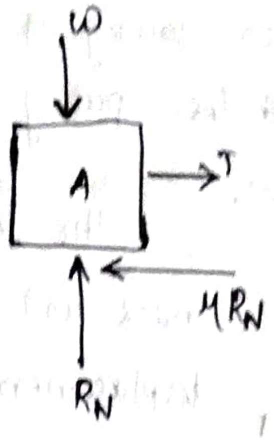
MOTION OF CONNECTED BODIES

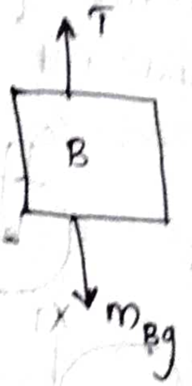
Consider motion of each body seperately an apply Newton's Law of motion and find accelaration of the body and tension in the string.

- Force in the direction of motion = +ve
- Force in the opp. direction of motion = -ve.

(13) Two blocks are joined by an inextensible string as shown in the fig. If the system is released from rest, determine the velocity of block after it has moved 2m. Assume the coefficient of friction b/w block and plane is 0.25. The pulley is weightless and frictionless.







$m_A = 200\ kg$. $m_B = 300\ kg$

$\mu = 0.25$

Let T be the tension in the string, since $x_A = x_B$

Consider the motion of block A,

Net force = mass x accelaration

$$T - \mu R_N = m_A \times a$$

$$T - 0.25 \times 200 \times 9.81 = 200 \times a \quad \text{——— (1)}$$

Consider the vertical motion of block B,

Net force = mass x accelaration

$$m_B g - T = m_B \times a$$

$$300 \times 9.81 - T = 300 \times a \quad \text{——— (ii)}$$

Adding (i) + (ii)

$$300 \times 9.81 - T + T - 0.25 \times 200 \times 9.81 = 200a + 300a$$

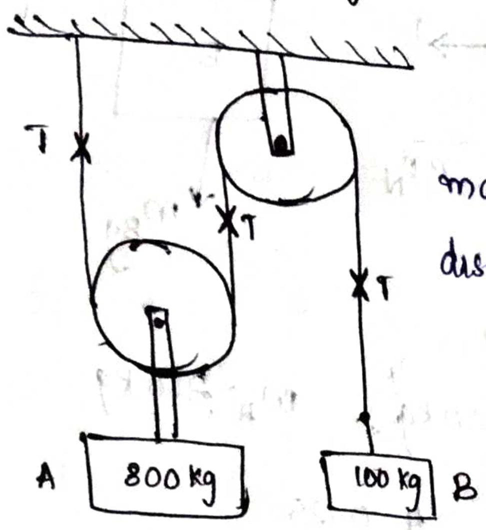
$$a = 4.905\ m/s^2$$

$$v^2 = u^2 + 2aS$$

$$v^2 = 0 + 2 \times 4.905 \times 2$$

$$v = 4.43\ m/s$$

(14) Determine the tension in the string and acceleration of the two bodies of mass 300 kg & 100 kg connected by a string & frictionless and weightless pulley as shown in fig.



The downward displacement of 300 kg mass will be only half of the upward displacement of 100 kg mass.

$$a_A = \frac{1}{2} a_B.$$

Consider downward motion of A

Net body = mass × acceleration

$$m_A g - 2T = m_A \times a_A$$

$$300 \times 9.81 - 2T = 300 \times a_A$$

$$1471.5 - T = 150 a_A \quad \text{—— (i)}$$

Consider upward motion of B

Net force = mass × acceleration

$$T - m_B g = m_B a_B$$

$$T - 100 \times 9.81 = 100 \times 2 \times a_A$$

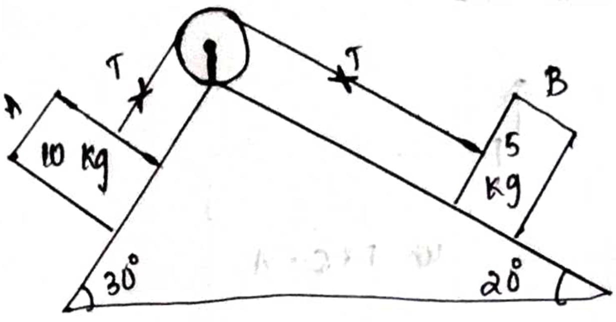
$$T - 981 = 200 a_A \quad \text{—— (ii)}$$

(i) + (ii)

$$1471.5 - T + T - 981 = 350 a_A$$

$$490.5 = 350 a_A \Rightarrow \underline{a_A = 1.40 \, m/s^2}$$

(15) Two smooth inclined planes whose inclinations with horizontal are 30° and 20° are placed back to back. Two bodies of mass 10 kg and 5kg are placed on them & are connected by a string as shown in fig. Calculate the T in the string and accelaration of the bodies.



The downward displacement of body A will be equal to upward placement of B

$a_A = a_B = a$

Consider motion of A

Net force $= m_A \times a_A$

$m_A g \sin\theta - T = m_A \times a_A$

$10 \times 9.81 \times \sin 30° - T = 10 \times a \Rightarrow 9.81 \times 5 - T = 10a$ ——— (i)

Consider motion of B

$T - m_B g \sin\theta = m_B \times a_B$

$T - 5 \times 9.81 \times 0.34 = 5 a_B$ ——— (ii)

(i) + (ii)

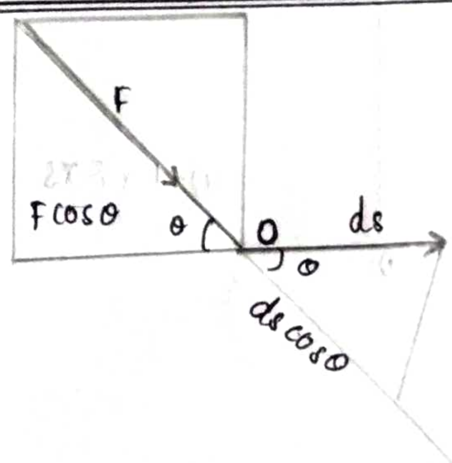
$9.81 \times 5 - T + T - 5 \times 9.81 \times 0.34 = 15a$

$32.37 = 15a \Rightarrow \underline{a = 2.16 \text{ m/s}^2}$

18/08/2021

WORK-ENERGY EQ. IN RECTILINEAR TRANSLATION

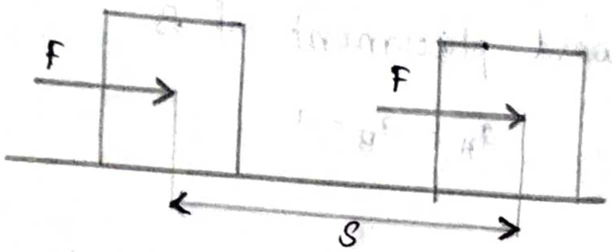
<u>WORK</u>

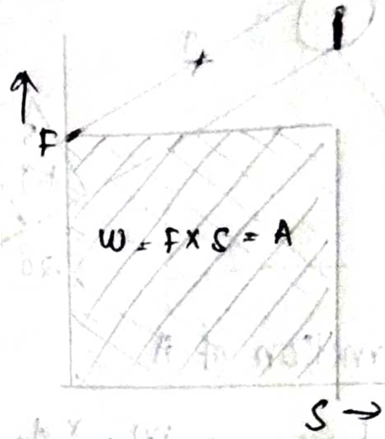


- Force → body moved along the line of action.
- work done, $dW = F\,ds\cos\theta$

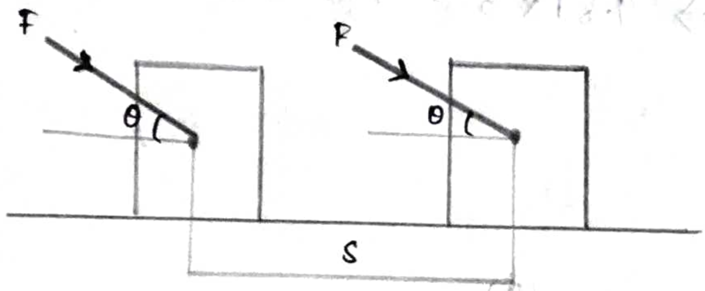
- Work is positive :- Force or component of force in same direction of displacement
- Work is negative :- opp. direction
- Unit :- Joule [Newton-metre (N.m)]

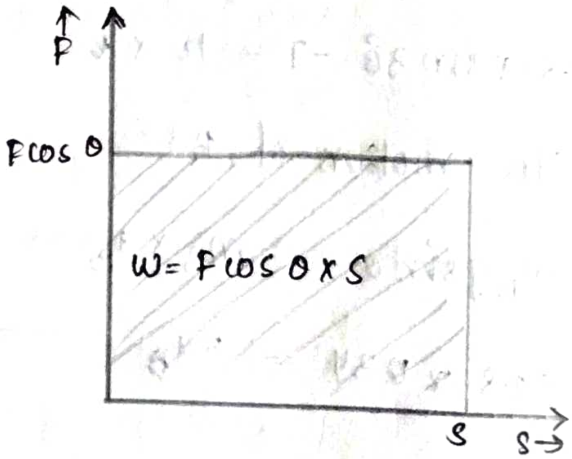
Case (i) When force F is constant and $\theta = 0$



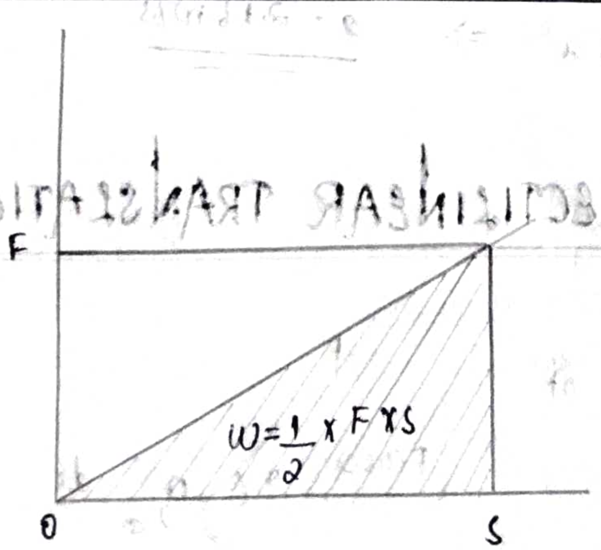


(ii) When F is constant and is inclined at θ with direction of motion.





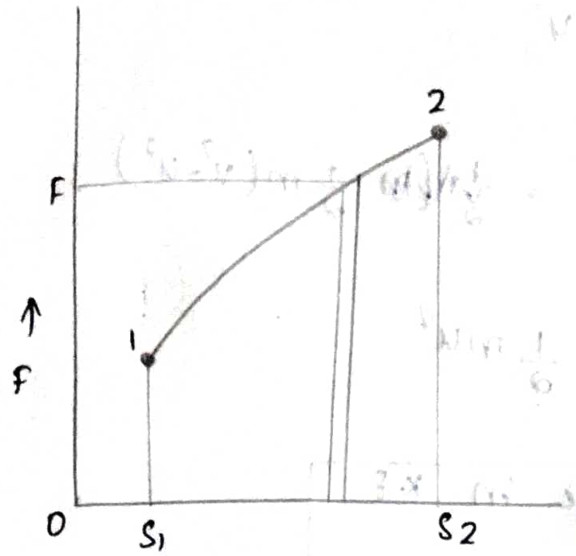
(iii) When the force F varies linearly & $\theta = 0$



work done W = (average force during the displacement) × displacement

$$W = \frac{(0+F)}{2} \times S = \frac{1}{2} \times F \times S$$

case (IV): Work done by a variable force, $F = f(s)$



Work done W = area under the curve 1-2 of force displacement diagram

$$W = \int_1^2 F \times ds$$

ENERGY

- capacity to do work
- unit = N-m or J
- K.E = $\frac{1}{2} m v^2$
- P.E = mgh

WORK - ENERGY PRINCIPLE

The work done by a system of force acting on a body during a displacement is equal to the change in kinetic energy of the body during the same displacement.

Resultant Force = $m \times a$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

$$a = v \frac{dv}{ds}$$

$$F = m \times a$$
$$= m \times v \frac{dv}{ds}$$

$$\boxed{F \times ds = m\, v\, dv}$$

Integrating on both sides

$$\int_0^s F.ds = \int_u^v m\,v\,dv$$

$$F \times S = m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}m(v^2-u^2)$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\boxed{\text{workdone} = \text{change in K.E}}$$

IMPULSE - MOMENTUM PRINCIPLE

- derived from Newton's Second Law
- momentum = product of mass & velocity. $(m \times v)$
- Impulsive force = large force acts over a short period of time
- The impulsive of a force F acting over a time interval t_1 to t_2 is defined by the integral. $\int_{t_1}^{t_2} F\,dt$

- If F is the resultant force acting on a body of mass m, then Newton's second law

$$F = ma$$

$$a = \frac{dv}{dt}$$

$$F = m\frac{dv}{dt} = \frac{d}{dt}(mv)$$

$$F\,dt = d(mv) \Rightarrow \int F\,dt = \int d\,mv = m\int dv$$

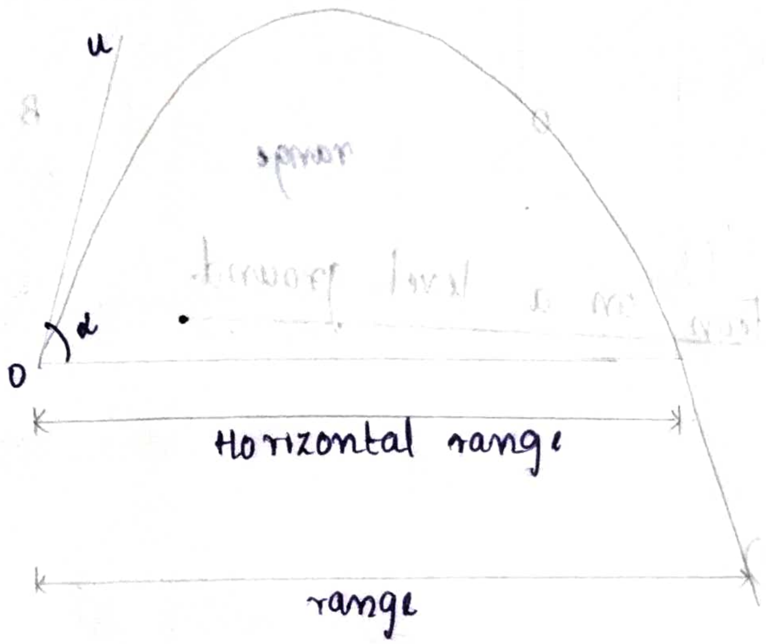
$$\int_{t_1}^{t_2} F.dt = m[v]_{v_1}^{v_2} = m(v_2 - v_1)$$

$$F(t_2 - t_1) = m(v_2 - v_1) \rightarrow \boxed{Ft = m(v_2 - v_1)}$$

$$\boxed{\text{Impulse} = \text{final momentum} - \text{initial momentum}}$$

01/09

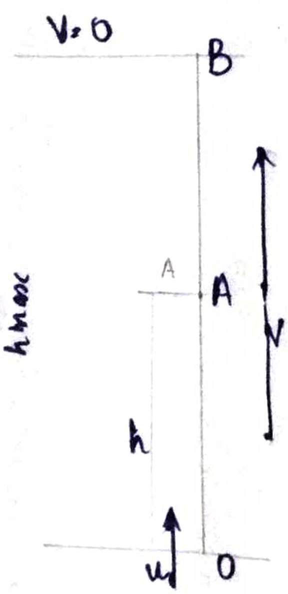
MOTION OF PROJECTILE



Case 1 :- Motion of a particle projected vertically into space

$\alpha = 90^\circ$

when $h = h_{max}$, $V = 0$



$$\boxed{h_{max} = \frac{u^2}{2g}}$$

Time to attain max height $t_1 = \frac{u}{g}$

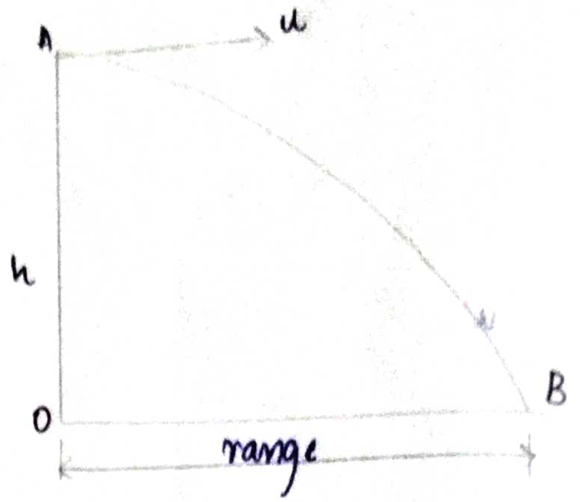
time of flight $\boxed{2t_1 = \frac{2u}{g} = T}$

$\boxed{\text{Range} = 0}$

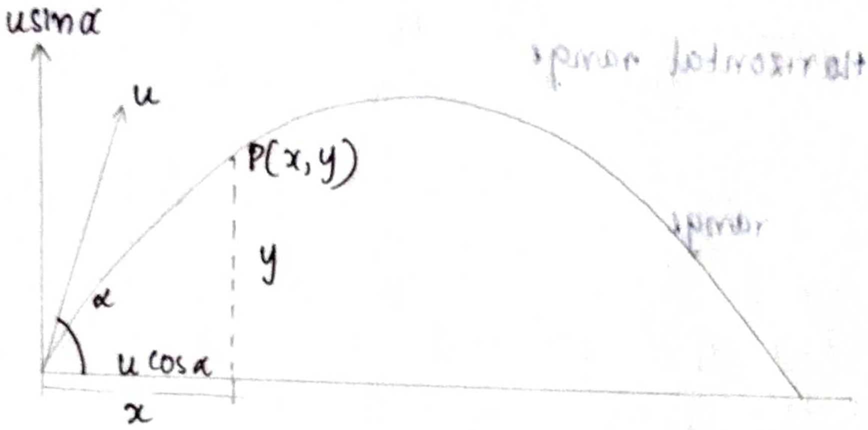
Case 2 :- motion of a plane thrown horizontally into space

Time of flight $\boxed{T = t = \sqrt{\frac{2h}{g}}}$

$\boxed{\text{range} = u \times t = u\sqrt{\frac{2h}{g}}}$



Case 3 :- Inclined projection on a level ground.



Prove that the trajectory of an inclined projection on a level ground is a parabola

$$x = (u\cos\alpha) \times t$$

$$t = \frac{x}{u\cos\alpha} \quad \text{——————} \quad (1)$$

$$h = (u\sin\alpha) \times t - \frac{1}{2}gt^2$$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2} g \left(\frac{x}{u\cos\alpha}\right)^2$$

$$y = x \frac{\sin\alpha}{\cos\alpha} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\alpha}$$

$$\boxed{y = x\tan\alpha - \frac{1}{2} \frac{gx^2}{u^2\cos^2\alpha}}$$

Equation of a parabola

EQUATIONS OF A PROJECTILE MOTION

Maximum height $\boxed{h_{max} = \frac{u^2\sin^2\alpha}{2g}}$

Time to attain h_{max} $\boxed{T = \frac{2u\sin\alpha}{g}}$

Horizontal range $\boxed{R = \frac{u^2}{g}\sin 2\alpha}$

(16) $u = 100\, m/s$ $\alpha = 30^\circ$

Find the horizontal range, h_{max} attained by bullet & time of flight.

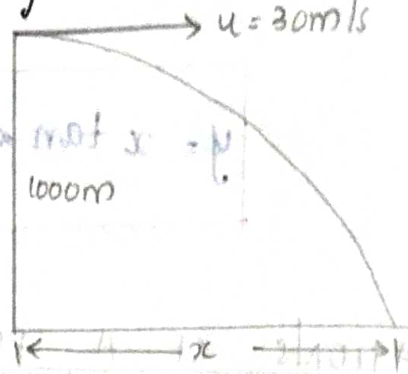
$$R = \frac{u^2}{g}\sin 2\alpha = \frac{100 \times 100}{9.8} \sin 60^\circ$$

$$= \frac{100 \times 100}{9.8} \times \frac{\sqrt{3}}{2} = 883.77\, m$$

$$H_{max} = \frac{u^2\sin^2\alpha}{2g} = \frac{100 \times 100}{2 \times 10} \times \frac{1}{2} \times \frac{1}{2} = 127.55\, m$$

$$T = \frac{2u\sin\alpha}{g} = \frac{2\times100\times 1/2}{9.8} = \underline{10.20s}$$

(17) A pilot flying his bomber at a height of 1000m with uniform horizontal velocity of 30 m/s wants to strike a target on the ground. At what distance from the target, he should release the bomb?



$u=0$, $h = ut + \frac{1}{2}gt^2$

$$1000 = 0 + \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2000}{9}} = \underline{14.29s}$$

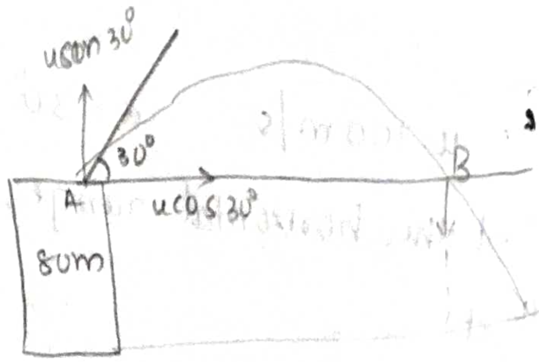
∵ the horizontal velocity remains constant, the horizontal distance moved in 14.29 seconds is $v \times t$

$$x = 30\times14.29 = \underline{428.7m}$$

PRACTISE PROBLEMS

(18) A stone is thrown upwards at an angle of 30° to the horizontal from a point P on a tower of height 80 m and it strikes the ground. The initial velocity of stone = 100 m/s. Calculate (a) time of flight of stone (b) The greatest elevation above the ground reached by the stone.

$u = 100m/s$ $\quad h = 80m$ $\quad \alpha = 30°$



(a) $\underline{t}$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$-80 = 100\sin30t - \frac{1}{2}\times9.8t^2$$

$$4.905t^2 - 50t - 80 = 0 \Rightarrow \underline{t = 11.59s}$$

(b)

$$H = \frac{v^2\sin^2\alpha}{2g} = \frac{100\times100\times\sin^2 30}{2\times9.8} = 127.42$$

Greatest elevation $= 127.42 + 80 = \underline{207.42m}$

(19) A cricket ball thrown by a fielder from a height of 2m at an angle 45° to the horizontal with an initial velocity 25 m/s hit the wickets at the height of 0.6m from ground. How far was the fielder from wickets?

$u_y = u\sin 45^\circ = 25 \times 0.707 = 17.68\ m/s$

$u_x = u\cos 45^\circ = 25 \times 0.707 = 17.68\ m/s$

$s_y = u_y t - \frac{1}{2} g t^2$

$(0.6 - 2) = 17.68 \times t - \frac{1}{2} \times 9.8 \times t^2$

$-1.4 = 17.68t - 4.9t^2 \Rightarrow 4.9t^2 - 17.68t - 1.4 = 0$

$\underline{t = 3.68\,s}$

Range $= 25\cos 45 \times 3.68 = \underline{65.05m}$

03/09/2021 Friday

MODULE : 05

ROTATION

KINEMATICS OF ROTATION

- Angular displacement, θ in radians
- Angular velocity, ω
- Angular accelaration, α

$$\omega = \frac{d\theta}{dt} \text{ rad/s} \qquad \alpha = \frac{d\omega}{dt} \text{ rad/s}^2$$

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$

N = angular velocity in rpm (revolutions per minute)

(a) For uniformly accelarated angular motion

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

(b) For uniform angular retardation

$$\omega = \omega_0 - \alpha t$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$\theta = \omega_0 t - \frac{1}{2}\alpha t^2$$

Relation between linear ~~accelaration~~ velocity and angular ~~accelaration~~ velocity

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega \qquad v = r\omega$$

Relation between linear accelaration and angular accelaration

$$a = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha \qquad a = r\alpha$$

(1) The armature of an electric motor, has angular speed of 1800 rpm at the instant when power is cut off. If it comes to rest in 6 seconds, calculate the angular deceleration assuming it is constant. How many revolutions does the armature make during this period?

$N_1 = 1800\,rpm$

$\omega_1 = \frac{2\pi N_1}{60} = 2\pi \times 30\ rad/s$

$\omega_2 = \omega_1 + \alpha t$

$0 = 2\pi \times 30 - \alpha \times 6$

$\alpha = \frac{2\pi \times 30}{6} = 31.4\ rad$

08/09/2021 Wednesday

(2) A body accelarates uniformly at $5\ rad/s^2$ & is found to be rotating at $90\ rad/s$ at the end of 12 seconds. Determine the initial velocity and angle turned during this interval.

$\alpha = 5\ rad/s^2$ $\quad \omega_2 = 90\ rad/s$ $\quad t = 12\,s$

$\omega_2 = \omega_1 + \alpha t$

$90 = \omega_1 + 5 \times 12$

$\omega_1 = 90 - 60 = 30\ rad/s$

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$= 30 \times 12 + \frac{1}{2} \times 5 \times 12 \times 12$

$= 360 + 360 = 720\ rad$

(3) During the starting phase of computer it is observed that a storage disc which was initially at rest executed 2.5 revolution in 0.5 s. Assuming that the angular accelaration of motion was uniform. Determine

(a) α and (b) velocity of disc at $t = 0.5\,s$

$\omega_1 = 0$

$\theta = 2.5\ rev = 2.5 \times 2\pi = 5\pi\ rad$

$t = 0.5\,s$

(i) angular accelaration

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

$5\pi = 0 + \frac{1}{2}\alpha \times 0.5 \times 0.5$

$\alpha = 125.66\ rad/s^2$

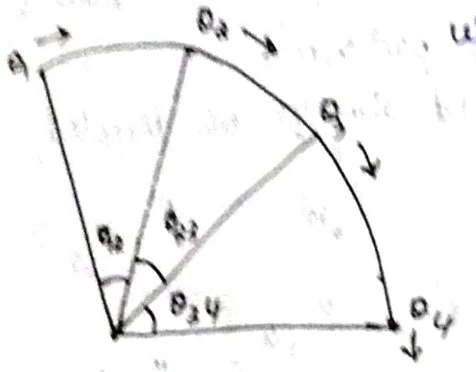
(ii) velocity of disc

$\omega_2 = \omega_1 + \alpha t$

$= 0 + 125.66 \times 0.5$

$= 62.83\ rad/s$

(6) A wheel accelarates from rest to a speed of 180 rpm uniformly in 0.4 s. It then rotates at that speed for 2 s before decelarating to rest in 0.3 s. Determine the total revolutions made by the wheel.



$\omega_0 = 0 \quad N_2 = 180\,rpm$

$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 180}{60} = 6\pi\ rad/s$

$t_{12} = 0.4s \quad t_{2-3} = 2s \quad t_{3-4} = 0.3s$

$\omega_2 = \omega_1 + \alpha t_{1-2}$

$6\pi = 0 + \alpha \times 0.4 \Rightarrow \underline{\alpha = 47.1\ rad/s^2}$

$\theta_{1-2} = \omega_1 t_{1-2} + \frac{1}{2}\alpha t_{1-2}^2 = 0 + \frac{1}{2} \times 47.1 \times 0.4 \times 0.4 = \underline{3.77\ rad/s}$

During θ_{2-3} rotation of wheel, angular accelaration = 0

$\therefore \omega_3 = \omega_2$

$\theta_{2-3} = \omega_2 \times t_{2-3} = 6\pi \times 2 = 12\pi\ rad$

$\omega_4 = \omega_3 + \alpha \times t_{3-4} \Rightarrow 0 = 6\pi + \alpha \times 0.3 \Rightarrow \underline{\alpha = -62.8\ rad/s^2}$

$\theta_{3-4} = \omega_2 t_{3-4} + \frac{1}{2}\alpha t_{3-4}^2 = 6\pi \times 0.3 - \frac{1}{2} \times 62.8 \times 0.3^2$

$= \underline{2.83\ rad}$

Total angular displacement

$\theta = \theta_{1-2} + \theta_{2-3} + \theta_{3-4}$

$= 3.77 + 12\pi + 2.83 = 44.28\ rad$

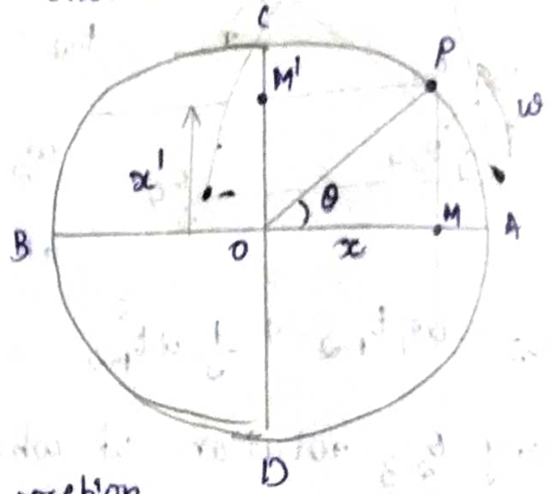
$= \underline{7.05\ revolutions}$

09/09/2021
Thursday

SIMPLE HARMONIC MOTION (SHM)

Conditions for a period motion to be simple harmonic.

- The accelaration of the body/particle performing periodic motion should be proportional to the distance of the body/particle from fixed point called the centre of SHM (mean position)
- The accelaration of the body/particle should always be directed towards the mean position.



$\theta = \omega t$; For one oscillation,

$t = t_p$ & $\theta = 2\pi$

$2\pi = \omega t_p$ $\boxed{t_p = \frac{2\pi}{\omega}}$

The displacement of M from mean position,

$OM = x = OP\cos\theta$ $\boxed{x = r\cos\omega t}$

$$V = \frac{dx}{dt} = -r\omega \sin\omega t$$

$\boxed{V = r\omega \sin\omega t} = r\omega\sin\theta = r\omega \frac{PM}{OP} = r\omega \frac{\sqrt{r^2 - x^2}}{r}$

$$\boxed{V = \omega\sqrt{r^2 - x^2}}$$

Accelaration of M,

$a = -r\omega^2 \cos\omega t$ $\boxed{a = -\omega^2 x}$

NOTES

- when the time of motion is measured from the mean position

$x = r\sin\omega t$

- when the time of motion is measured from the extreme position

$x = r\cos\omega t$

- In both cases, $\boxed{V = \omega\sqrt{r^2 - x^2}}$ $\boxed{a = -\omega^2 x}$

• Maximum velocity is at $x=0$ (at mean position)

• Maximum accelaration is at $x=r$ (at extreme position)

$V_{max} = \omega\sqrt{r^2-0}$ $\Rightarrow$ $\boxed{V_{max} = r\omega}$ $\boxed{a_{max} = -\omega^2 r}$

⑤ A body moving with SHM has velocities of 10m/s and 4m/s at 2 & 4m distance from mean position. Find the amplitude & time period of the body.

$V = \omega\sqrt{r^2-x^2}$

At $x=2$, $V=10$m/s

$10 = \omega\sqrt{r^2-2^2}$

At $x=4$m, $V=4$m/s

$4 = \omega\sqrt{r^2-4^2}$

$\frac{10}{4} = \frac{\sqrt{r^2-4^2}}{\sqrt{r^2-2^2}}$ $\Rightarrow$ $\frac{100}{16} = \frac{r^2-16}{r^2-4}$ $\Rightarrow$ $r=4.28$m

$10 = \omega\sqrt{4.62^2-4^2}$

$\omega = \frac{10}{3.78} = 2.64$ rad/s

$t_p = \frac{2\pi}{\omega} = \frac{2\times3.14}{2.64} = 2.38$s

10/09/2021
Friday

⑥ A body is vibrating with SHM of amplitude 150mm and frequency 3cps. Calculate maximum velocity & accelaration of the body

$r = 150$mm $= 0.15$m

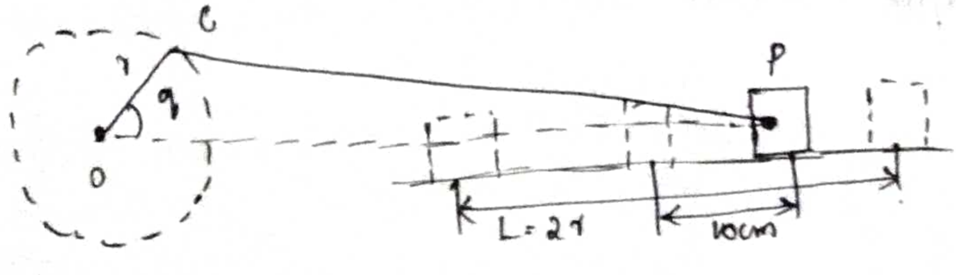
$f = 3$cps

$\omega = 2\pi f = 2\pi\times3 = 6\pi$ rad/s

$V_{max} = r\omega = 0.15\times6\pi = 2.83$ m/s

$a_{max} = \omega^2 r = 2.83\times6\pi = 53.3$ m/s^2

⑦ The piston of an IC engine move with SHM. The crank rotates at 420 rpm and stroke length is 40cm. Find the v and a of piston when it is at a distance of 10cm from mean position.



Speed of crack = 420 rpm $\quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 420}{60} = 43.98$ rad/s

stroke length $L = 2 \times$ crank radius

crank radius $r = \frac{L}{2} = \frac{40}{2} = 20$ cm

$x = 10$ cm $= 0.1$ m

$$v = \omega\sqrt{r^2 - x^2} = \underline{7.62 \text{ m/s}}$$

$$a = \omega^2 r = \underline{193.42 \text{ m/s}^2}$$

24/09/2021
Friday

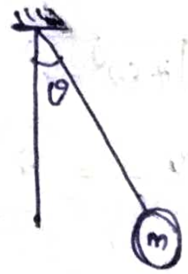
FREE VIBRATIONS

Free vibration :- If the disturbing force is applied just to start the motion and is then removed, leaving it to vibrate by itself

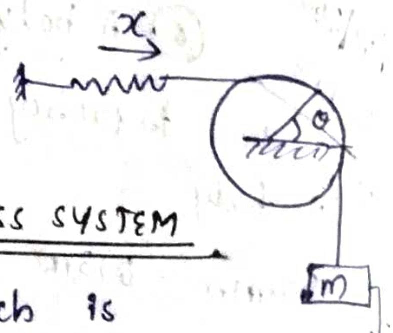
Force vibration :- If the disturbing force acts at periodic intervals on the system, the system is said to undergo forced vibration

DEGREE OF FREEDOM

- No. of independent co-ordinates required to defines the configuration of the system.



- Constraints to the motion reduce the degree of freedom



UNDAMPED FREE VIBRATIONS OF SPRING MASS SYSTEM

- The opposing force is called spring force which is proportional to the displacement of the spring.

- Spring force, $F \propto x$

$$\boxed{F = Kx}, \quad K = \text{stiffness of spring}$$

Unit = N/m

$$\therefore \frac{d^2x}{dt^2} + \frac{K}{m}x = 0$$

$$\boxed{\omega = \sqrt{K/m}}$$

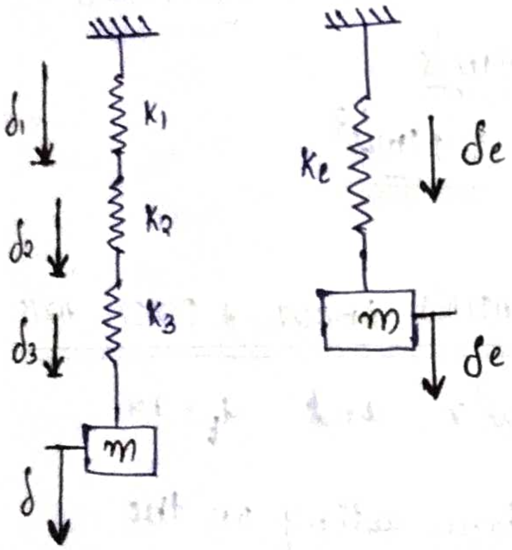
since the system vibrates freely, this frequency is called natural frequency & is denoted by ω.

$$\omega = 2\pi f \;,\; f = \frac{\omega}{2\pi}$$

$$\boxed{f_n = \frac{1}{2\pi} \times \sqrt{K/m}}$$

SPRINGS

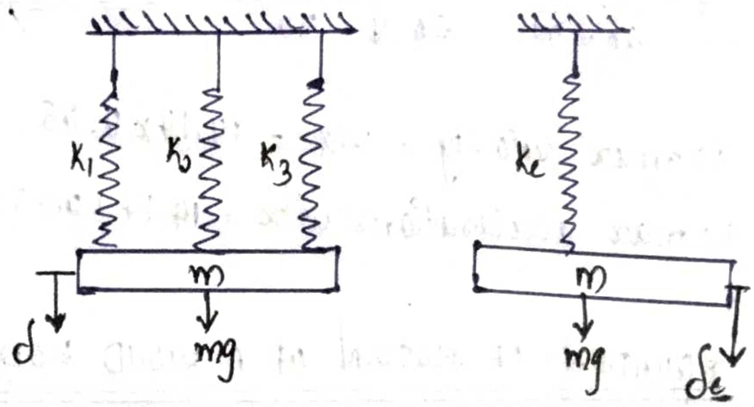
Springs in series



$$\delta_e = \delta = \delta_1 + \delta_2 + \delta_3$$

$$\boxed{\therefore \frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$$

Springs in Parallel

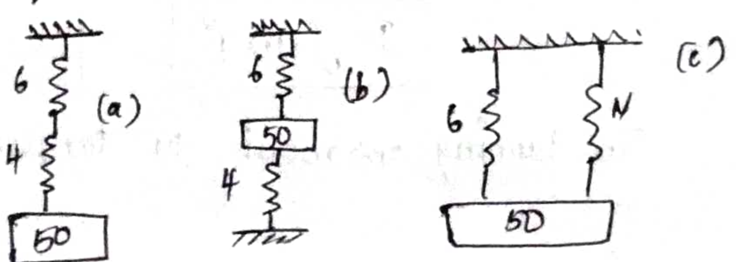


$$\delta e = \delta$$

$$\boxed{K_e = K_1 + K_2 + K_3}$$

(8) A body of mass 50kg is suspended by two springs of stiffness 4kN/m and 6kN/m as shown in fig(a), (b) and (c). The body is pulled 50mm down from its equilibrium position and then released. Calculate:-

(i) frequency of oscillation
(ii) max velocity
(iii) max acceleration



Case 1 : Fig (a)

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{6} + \frac{1}{4} = \frac{10}{24} \Rightarrow K_e = \frac{24}{10} = 2.4\ kN/m$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{2.4 \times 1000}{50}} = 1.10\ cps$$

$$\omega = 2\pi f = 2\pi \times 1.10 = 6.93$$

(ii) max velocity $= \omega X = 6.93 \times 0.05 = 0.35\ m/s$

(iii) max accelaration $= \omega^2 x = 6.93 \times 0.35 = 2.4\ m/s^2$

Case 2 :- Fig (b), Fig (c)

$$K_e = K_1 + K_2 = 4 + 6 = 10\ kN/m$$

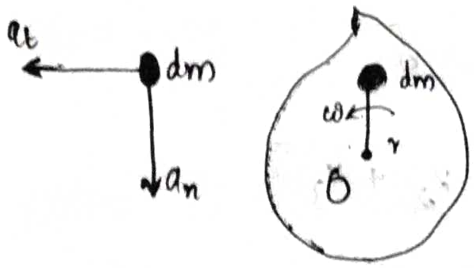
$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{10 \times 1000}{50}} = 2.25\ cps$$

$$\omega = 2\pi f = 2\pi \times 2.25 = 14.14\ rad/s$$

(ii) max velocity $= \omega X = 14.14 \times 0.05 = 0.707\ m/s$

(iii) max accelaration $= \omega^2 x = 14.14 \times 0.05 \times 14.14 = 9.99\ m/s^2$

EQUATION OF MOTION OF A RIGID BODY ROTATING ABOUT A FIXED AXIS



$a_n = a_c = \omega^2 r$ and $a_t = r\alpha$

The tangential force acting on the elementary mass

= mass x tangential accelaration

$= dm \times (r\alpha)$

Moment of this force about O $= dm \times (r\alpha) \times r = dm\, r^2 \alpha$

The turning moment or torque on whole body about O,

$T = \int dm\, r^2 \alpha = I\alpha$. I = mass moment of inertia of body

$$\boxed{I = \int dm\, r^2}$$

The turning moment or torque $\boxed{T = I\alpha}$

KINETIC ENERGY DUE TO ROTATION

Kinetic energy of elementary mass $= \frac{1}{2} dm\, v^2 = \frac{1}{2} dm\, \omega^2 r^2 = \frac{1}{2} dm\, r^2 \omega^2$

Kinetic energy of whole body,

$$K.E = \int \frac{1}{2} dm\, r^2 \omega^2 = \frac{1}{2} I \omega^2$$

$$\boxed{K.E = \frac{1}{2} I \omega^2}$$

WORKDONE IN ROTATION

$$\boxed{\text{Workdone} = T \times \theta}$$

WORK-ENERGY EQUATION FOR ROTATION

The workdone by a torque acting on a body during an angular displacement is equal to the change in K.E of the body during the same displacement.

(9) A string 5m long is wound around the axle of a wheel. The string is pulled with a constant force of 250N. The wheel rotates at 800 rpm when the string leaves the axle. Find the moment of inertia of the wheel

$r\theta = 5$

$\theta = \frac{5}{r}$

Force $= 250N$

Length of string $= 5m$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60}$$

$$= 10\pi \text{ rad/s}$$

Change in K.E = work done

$\left[\frac{1}{2} I \omega^2 - 0\right] = T\theta$

$= F \times r \times \frac{5}{r} = F \times 5 = 250 \times 5 = 1250 \text{ Nm}$

$\frac{1}{2} \times I \times (10\pi)^2 = 1250 \Rightarrow I = \frac{2500}{(10\pi)^2} = 2.53 \text{ kgm}^2$