

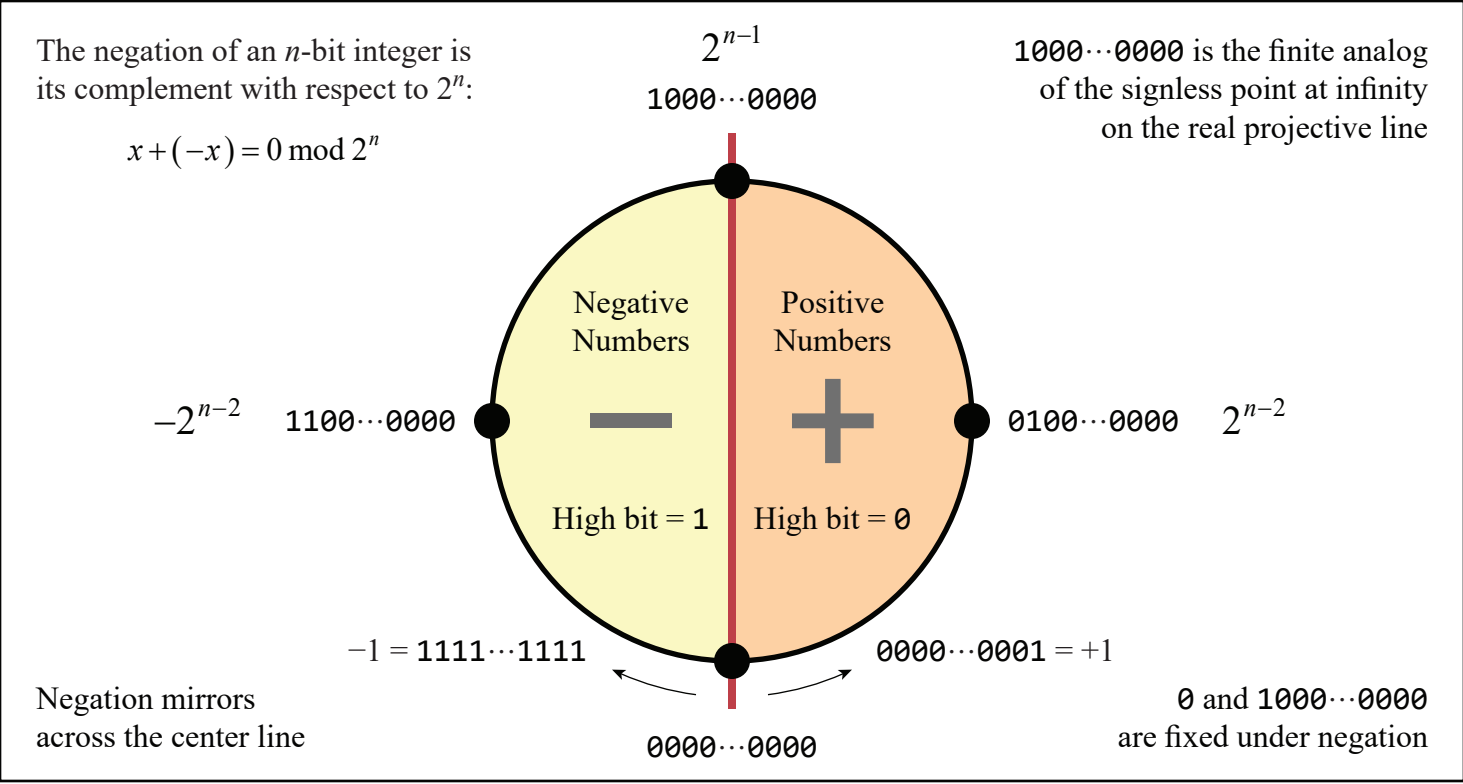
Binary Fundamentals

Powers of Two

Power	Decimal	Hexadecimal
2^1	2	0x00000002
2^2	4	0x00000004
2^3	8	0x00000008
2^4	16	0x00000010
2^5	32	0x00000020
2^6	64	0x00000040
2^7	128	0x00000080
2^8	256	0x00000100
2^9	512	0x00000200
2^{10}	1024	0x00000400
2^{11}	2048	0x00000800
2^{12}	4096	0x00001000

Power	Decimal	Hexadecimal
2^{13}	8192	0x00002000
2^{14}	16,384	0x00004000
2^{15}	32,768	0x00008000
2^{16}	65,536	0x00010000
2^{17}	131,072	0x00020000
2^{18}	262,144	0x00040000
2^{19}	524,288	0x00080000
2^{20}	1,048,576	0x00100000
2^{21}	2,097,152	0x00200000
2^{22}	4,194,304	0x00400000
2^{23}	8,388,608	0x00800000
2^{24}	16,777,216	0x01000000

Two's Complement



Logical Complement

NOT Bitwise NOT $\sim x$	x	$\sim x$
	0	1
	1	0

Logical Identities

Unary	Binary
$\sim x = -x - 1$	$\sim(x \& y) = \sim x \mid \sim y$
$-x = \sim x + 1$	$\sim(x \mid y) = \sim x \& \sim y$
$\sim \sim x = x + 1$	$\sim(x \wedge y) = \begin{cases} \sim x \wedge y \\ x \wedge \sim y \end{cases}$
$\sim \sim x = x - 1$	

Floating-Point

Half precision 16-bit floating-point	Single precision 32-bit floating-point																												
 sign s value $= (-1)^s 2^{e-15} \left(1 + \frac{m}{2^{10}}\right)$	 sign s value $= (-1)^s 2^{e-127} \left(1 + \frac{m}{2^{23}}\right)$																												
Double precision 64-bit floating-point																													
 sign s value $= (-1)^s 2^{e-1023} \left(1 + \frac{m}{2^{52}}\right)$																													
<table><tr><th>Special Floating-Point Value</th><th>Half</th><th>Float</th><th>Double</th></tr><tr><td>+0.0</td><td>0x0000</td><td>0x00000000</td><td>0x00000000_00000000</td></tr><tr><td>+1.0</td><td>0x3C00</td><td>0x3F800000</td><td>0x3FF00000_00000000</td></tr><tr><td>Positive infinity</td><td>0x7C00</td><td>0x7F800000</td><td>0x7FF00000_00000000</td></tr><tr><td>Smallest positive normalized value</td><td>0x0400</td><td>0x00800000</td><td>0x00100000_00000000</td></tr><tr><td>Upper limit of non-integer values</td><td>0x6400</td><td>0x4B000000</td><td>0x43300000_00000000</td></tr><tr><td>Largest representable positive value</td><td>0x7BFF</td><td>0x7F7FFFFF</td><td>0x7FEFFFFFF_FFFFFFFF</td></tr></table>		Special Floating-Point Value	Half	Float	Double	+0.0	0x0000	0x00000000	0x00000000_00000000	+1.0	0x3C00	0x3F800000	0x3FF00000_00000000	Positive infinity	0x7C00	0x7F800000	0x7FF00000_00000000	Smallest positive normalized value	0x0400	0x00800000	0x00100000_00000000	Upper limit of non-integer values	0x6400	0x4B000000	0x43300000_00000000	Largest representable positive value	0x7BFF	0x7F7FFFFF	0x7FEFFFFFF_FFFFFFFF
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Binary Logical Operations

<div>AND</div> <div>Bitwise AND</div> <div>$x \& y$</div>	<table><tr><th>x</th><th>y</th><th>$x \& y$</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x	y	$x \& y$	0	0	0	0	1	0	1	0	0	1	1	1
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Mask Creation

Formula	Operation / Effect	Illustration																	
$\sim x \mid (x - 1)$	Create mask for all bits other than lowest 1 bit. $000\cdots000$ becomes $111\cdots111$.	<table><tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td></tr></table>	1	0	1	1	1	0	0	0	1	1	1	1	1	0	1	1	1
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$x \mid \sim(x + 1)$	Create mask for all bits other than lowest 0 bit. $111\cdots111$ is unchanged.	<table><tr><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td></tr></table>	0	1	1	0	0	1	1	1	1	1	1	1	1	0	1	1	1
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$\sim x \& (x - 1)$	Create mask for bits right of lowest 1 bit, exclusive. $000\cdots000$ becomes $111\cdots111$.	<table><tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td></tr></table>	1	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1
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Bit Manipulation

Formula	Operation / Effect	Illustration
$x \ \& \ (x - 1)$	Clear lowest 1 bit. If result is zero, then x is zero or 2^k . $000\cdots000$ is unchanged.	<div><div>1 0 1 1 1 0 0 0</div><div>↓</div><div>1 0 1 1 0 0 0 0</div></div>
$x \ \ (x + 1)$	Set lowest 0 bit. $111\cdots111$ is unchanged.	<div><div>0 1 1 0 0 1 1 1</div><div>↓</div><div>0 1 1 0 1 1 1 1</div></div>
$x \ \ (x - 1)$	Set all bits to right of lowest 1 bit. $000\cdots000$ becomes $111\cdots111$.	<div><div>1 0 1 1 1 0 0 0</div><div>↓</div><div>1 0 1 1 1 1 1 1</div></div>
$x \ \& \ (x + 1)$	Clear all bits to right of lowest 0 bit. If result is zero, then x is zero or $2^k - 1$. $111\cdots111$ becomes $000\cdots000$.	<div><div>0 1 1 0 0 1 1 1</div><div>↓</div><div>0 1 1 0 0 0 0 0</div></div>
$x \ \& \ -x$	Extract lowest 1 bit. $000\cdots000$ is unchanged.	<div><div>1 0 1 1 1 0 0 0</div><div>↓</div><div>0 0 0 0 0 1 0 0</div></div>
$\sim x \ \& \ (x + 1)$	Extract lowest 0 bit (as a 1 bit). $111\cdots111$ becomes $000\cdots000$.	<div><div>0 1 1 0 0 1 1 1</div><div>↓</div><div>0 0 0 0 0 1 0 0</div></div>