Math Battles **Name:** ______________________

2017

Team Ramanujan's Math Paper

June 21, 2017

Time Limit: 48 Hours Teaching Assistant ______________________

This exam contains 2 pages (including this cover page) and 7 questions.
Total of points is 100.

Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction.

Grade Table (for teacher use only)

Question	Points	Score
1	10	
2	25	
3	10	
4	10	
5	25	
6	20	
7	0	
Total:	100	

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1. (10 points) Find all integers n for which the equation

$$(x+y+z)^2 = nxyz$$

is solvable in positive integers.

2. (25 points) The area of a convex pentagon $ABCDE$ is S, and the circumradii of the triangles ABC, BCD, CDE, DEA, EAB are R_1, R_2, R_3, R_4, R_5. Prove the inequality

$$R_1^4 + R_2^4 + R_3^4 + R_4^4 + R_5^4 \geq \frac{4}{5\sin^2 108^\circ} S^2$$

3. (10 points) A number of robots are placed on the squares of a finite, rectangular grid of squares. A square can hold any number of robots. Every edge of each square of the grid is classified as either passable or impassable. All edges on the boundary of the grid are impassable. You can give any of the commands up, down, left, or right.

All of the robots then simultaneously try to move in the specified direction. If the edge adjacent to a robot in that direction is passable, the robot moves across the edg3e and into the next square. Otherwise, the robot remains on its current square. You can then give another command of up, down, left, or right, then another, for as long as you want. Suppose that for any individual robot, and any square on the grid, there is a finite sequence of commands that will move that robot to that square.

Prove that you can also give a finite sequence of commands such that all of the robots end up on the same square at the same time.

4. (10 points) Find the degree of the polynomial

$$1^a + 2^a + 3^a + ... + x^a$$

where a is a positive integer.

5. (25 points) If x, y, z, r are read numbers, prove that

$$\sum x^4 + (3r^2 - 1) \sum x^2y^2 + 3r(1-r)xyz \sum x \geq 3r \sum x^3 y$$

6. (20 points) Solve

$$y^2 = x^3 - 2$$

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7. (0 points) Mark box if true.

   ☐ 2+2=4

   ☐ $\frac{d}{dx}(x^2 + 1) = 2x + 1$

   ☐ The Moon is made of cheese.