

⇒ Time & Space Complexity

↳ Relation between Input size and Running Time

main() {

 Sys0 ("Hello"); $\underline{1}$

 ↳ constant time

 Sys0 ("Hello"); $\underline{2}$

}

main() {

$n=1, 2, 3, \dots, 10^6$

 for (int i=0; i<n; i++) {
 Sys0 ("Hi"); } $\underline{n \text{ operation}}$

}

Time :- how many operations we are performing
in a program.

↳ Types of operations

- | input | output operation |
|-------|------------------|
| n | n |
| n | n^2 |
| n | n^3 |
| n | $\log(n)$ |
| : | : |
| : | : |
- ↳ linear operation $\Rightarrow n \rightarrow n$
 - ↳ quadratic operation $\Rightarrow n \rightarrow n^2$
 - ↳ cubic operations $\Rightarrow n \rightarrow n^3$
 - ↳ logarithmic operation $\Rightarrow n \rightarrow \log(n)$
 - ↳ constant operations $\Rightarrow n \rightarrow 1$

Ex 1

```

public static void main() {
    Scanner scn = new Scanner();
    int n = scn.nextInt();
    for (int i=0; i<n; i++) {
        System.out.println("Hello");
    }
}

```

] n times

input	operation
$n = 1$	1
$n = 10$	10
$n = 90$	90
$n = 100000$	100000

relation

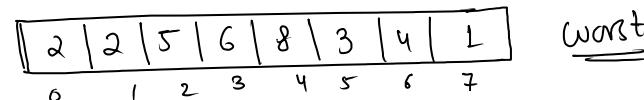
$$\boxed{\text{input} \propto n}$$

$O(n)$

→ Complexity (Time)

- 1) Best Case :- when least no. of operations have to be done
- 2) Average Case :- when average no. of operations have to be done
- 3) Worst Case :- when most no. of operations have to be done

Ques You have an array and you have to find "1" in it



worst

n operations

```

for (int i=0; i<n; i++) {
    if (arr[i] == 1) {
        System.out.println("1");
        break;
    }
}

```

$1 \text{ sec} = 10^9$
 $1 \rightarrow 7$

$n = 7 \times 10^9$

Ex 2

```

public static void main() {
    Scanner scn = new Scanner();
    int n = scn.nextInt();
    for (int i=0; i<n; i++) {
        for (int j=0; j<n; j++) {
            System.out.println("Hello");
        }
    }
}

```

input	input	operation
$n = 1$	\rightarrow	\perp
$n = 10$	\rightarrow	100
$n = 90$	\rightarrow	8100
$n = 100000$ (10^5)	\rightarrow	10^{10}

$n=1 \quad i=0, j=0 \} \perp \text{time}$
 $n=2 \quad i=0, j=0, 1 \} 4 \text{ time}$
 $i=1, j=0, 1 \}$
 $n=3 \quad i=0, \underbrace{j=0, 1, 2}_{\perp} \} 9 \text{ time}$
 $i=1, \underbrace{j=0, 1, 2}_{\perp}$
 $i=2, \underbrace{j=0, 1, 2}_{\perp}$

$n=4 \rightarrow 16 \text{ times}$

relation

$\text{input} \propto n^2$

$\hookrightarrow O(n^2)$

$\text{input} \propto n \times m$

$\hookrightarrow O(n \times m)$

Ex 3

```

    Public static void main() {
        Scanner scr = new Scanner();
        int n = scr.nextInt();
        for (int i=0; i<n; i++) {
            System.out.println("Hello");
        }
        for (int j=0; j<m; j++) {
            System.out.println("Hello");
        }
    }
  
```

n operations

m operations

Input $\propto n+m$

e.g., $n = 2$
 $m = 3$
(5)

Note:- when we add complexities (time)
then smaller value will get neglected
(bcz larger value have more significance)

e.g., $n = 5$ ✓
 $m = 500000$ ✓

$O(m)$

Capital O notation

$O(n+m)$

Compare

$n=1$

1

1

1

$n=2$

2

4

8

$n=3$

3

9

27

\vdots

\vdots

\vdots

\vdots

$n=10^5$

10^5

10^{10}

10^{15}

$\frac{1\text{ sec}}{10\text{ min}}$

$\frac{10\text{ min}}{1}$

1. Constant Time Complexity - $O(1)$ fast

2. Logarithmic Time Complexity - $O(\log n)$ fast

3. Linear Time Complexity - $O(n)$ average

4. $O(n \log n)$ Time Complexity bad

5. Quadratic Time Complexity - $O(n^2)$ very bad

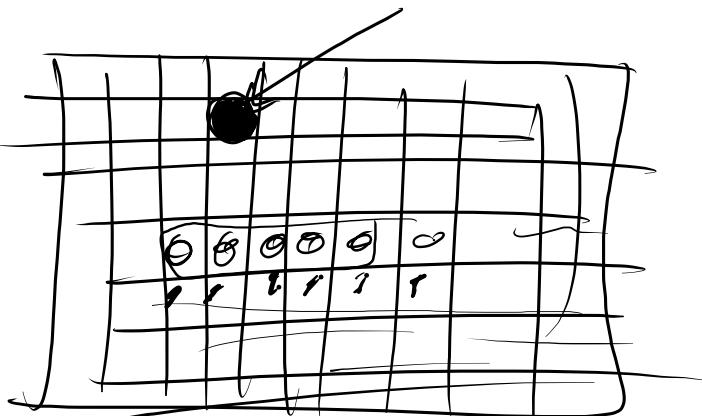
6) $O(n^3)$ → very very bad

```
public static void main () {  
    Scanner scn = new Scanner ();  
    int n = scn.nextInt(); // 1, 5, 50000  
    System.out.println ("Hello");  
}
```

$O(1) > O(\log n) > O(n) > O(n \log n) > O(n^2) > O(n^3)$

→ Space Complexity // no. of variables used

```
Public static void main () {  
    Scanner scn = new Scanner();  
    int n = scn.nextInt(); ← 1, 2, 1000, 105  
    for (int i=0; i<n; i++) { O(1) //constant  
        System.out.println("Hello");  
    }  
}
```



```
Public static void main () {  
    Scanner scn = new Scanner();  
    int n = scn.nextInt(); ← 1, 2, 1000, 105  
    int [] arr = new int [n]; ← O(n)  
    for (int i=0; i<n; i++) {  
        System.out.println(arr[i]);  
    }  
}
```

Space operation $\propto n$ (input)