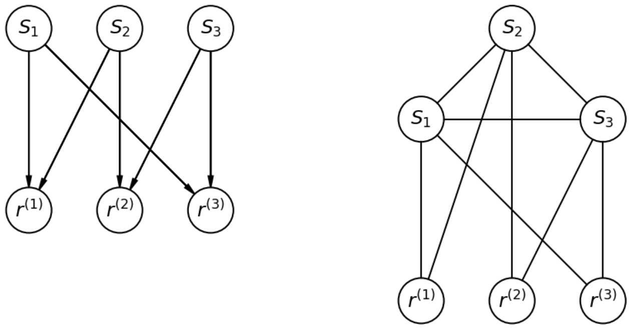
Probabilistic Modelling and Reasoning: Assignment

UNN: S1796157

Question 1

a-c)



These graphs show the solutions to question a and question c. On the left side we have the DGM and on the right side we have the undirected minimal I-map generatet from the DGM.

b)

- $s_1 \perp s_2 \mid r^{(2)}$. This independence assertion holds since all trails from s_1 and s_2 are blocked;
- $s_1 \perp s_2 \mid r^{(3)}, r^{(2)}$. This independence assertion does not hold since there is a trail between s_1 and s_2 which is not blocked, namely $s_1 \to r^{(3)} \to s_3 \to r^{(2)} \to s_2$.

d)

Independence	DGM	UGM	Justification
$s_1 \perp\!\!\!\perp r^2 \mid \{s_2, s_3\}$	✓	✓	DGM: cond. on s_2 and s_3 blocks all path from s_1. UGM: cond. on s_2 and s_3 blocks all path between s_1 and $r^{(2)}$.
$s_1 \perp\!\!\!\perp r^2 \mid \{r^{(1)}, s_3\}$	✗	✗	DGM: path not blocked $s_1 \to r^{(1)} \to s_2 \to r^{(2)}$. UGM: path not blocked $s_1 \to s_3 \to r^2$.
$s_1 \perp\!\!\!\perp s_2$	✓	✗	DGM: connected head-to-head with no evidence on $r^{(1)}$, hence trail blocked. UGM: hence the independence relation does not hold..
$s_1 \perp\!\!\!\perp s_2 \mid \{r^{(1)}\}$	✗	✗	DGM: path not blocked $s_1 \to r^{(1)} \to s_2$. UGM: s_1 and s_2 are connected directly, hence the independence relation does not hold.
$r^{(1)} \perp\!\!\!\perp r^{(2)}$	✗	✗	DGM: connected tail-to-tail with no evidence on s_2. UGM: there are many open paths between $r_{(1)}$ and $r^{(2)}$.
$r^{(1)} \perp\!\!\!\perp r^{(2)} \mid \{s_2\}$	✓	✗	DGM: all path between $r^{(1)}$ and $r^{(2)}$ are blocked. UGM: cond. on s_2 leaves many open paths between $r_{(1)}$ and $r^{(2)}$.
$r^{(1)} \perp\!\!\!\perp r^{(2)} \mid \{s_1, s_2\}$	✓	✓	DGM: all path between $r^{(1)}$ and $r^{(2)}$ are blocked. UGM: cond. on s_1 and s_2 blocks all paths.
$r^{(1)} \perp\!\!\!\perp r^{(2)} \mid \{s_2, r^3\}$	✗	✗	DGM: open path between $r^{(1)}$ and $r^{(2)}$, namely $r^{(1)} \to s_1 \to r^{(3)} \to s_3 \to r^{(2)}$. UGM: cond. on s_2 and $r^{\{}(3)\}$ leaves many open paths.

Question 2

a)

The factorization of the pmf over the graph showed in the answer a of question1 is given by the formula showed below, in which pa_i represents the parents of the variable x_i in the graph:

$$\mathbb{P}(x_1, \ldots, x_n) = \prod_{i=1}^{n} \mathbb{P}[x_i \mid pa_i] \tag{1}$$

Therefore, the pmf for $\mathbb{P}[s_1, s_2, s_3, r^{(1)}, r^{(2)}, r^{(3)}]$ becomes:

$$\mathbb{P}[s_1, s_2, s_3, r^{(1)}, r^{(2)}, r^{(3)}] = \mathbb{P}[r^1 \mid s_1, s_2] \cdot \mathbb{P}[r^2 \mid s_2, s_3] \cdot \mathbb{P}[r^3 \mid s_1, s_3] \cdot \mathbb{P}[s_1] \cdot \mathbb{P}[s_2] \cdot \mathbb{P}[s_3] \tag{2}$$

b)

- $\phi_1(s_1)$
- $\phi_2(s_2)$
- $\phi_3(s_3)$
- $\phi_A(s_1, s_2, r^{(1)})$
- $\phi_B(s_2, s_3, r^{(2)})$
- $\phi_C(s_1, s_3, r^{(3)})$

c