Demand Foresting/ Business Forecasting

(Least Square Method)

1. Method of Least Squares- Fitting Linear Trend (For Odd Years)

Given the following data, forecast the estimated value of sales for the year 2017 using the least square method.

Year	Sales (Y) ('000 units)
2010	125
2011	128
2012	133
2013	135
2014	140
2015	141
2016	143

Find estimated sales for the year 2017?

For the year 2017 or X=4, how much will be value of Y?

Answer:

Year	Sales (Y) ('000 units)	X	XY	X^2
2010	125	-3	-375	9
2011	128	-2	-256	4
2012	133	-1	-133	1
2013	135	0	0	0
2014	140	1	140	1
2015	141	2	282	4
2016	143	3	429	9
	$\sum Y$ =945	$\sum X = 0$	$\sum XY = 87$	$\sum X^2 = 28$

Here 'N' is number of observations = 7.

Here we need to convert "Year" into "X" variable and "X" is considered as the independent variable for our analysis.

$$X = \frac{Year - Origin}{Interval}$$

Where, origin is the mid-year i.e. 2013

Interval is the year gap i.e. 1.

So for the year 2010, X will be

$$X = \frac{Year - Origin}{Interval} = \frac{2010 - 2013}{1} = -3$$

Similarly, for the years 2011, 2012, 2013, 2014, 2015, and 2016, the X values will be -2, -1, 0, 1, 2, and 3, respectively.

Here estimated Y i.e. $\hat{Y} = a + bX$

Actual Y i.e. $Y = a + bX$

In order to get the estimated Y values, first we have to find out the values of "a" and "b". For this we need to solve two normal equations as follows:

$$\sum Y = Na + b\sum X \quad (1)$$

$$\sum XY = a\sum X + b\sum X^2 \quad (2)$$

Solving the above two normal equations with the help of above table, we can find out the values of "a" and "b" as follows:

From equation (1), we can get,

$$a = \frac{\Sigma Y}{N} = \frac{945}{7} = 135$$

Rom equation (2),

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{87}{28} = 3.11$$

Now, substituting the values of 'a' and 'b' we can get,

$$\hat{Y} = 135 + 3.11X$$

For the year 2017 or $X = \frac{Year - Origin}{Interval} = \frac{2017-2013}{1} = 4$, $\hat{Y}_{2017} = 135 + (3.11 \times 4) = 147.44$ ***(ANS)***

Similarly, you can calculate the estimated values for all the years.

2. **Method of Least Squares- Fitting Linear Trend (For Even Years)**

Given the following data, forecast the estimated value of sales for the year 2017 using the least square method.

Year	Sales (Y) ('000 units)
2009	80
2010	90
2011	92
2012	83
2013	94
2014	99
2015	92
2016	104

Find estimated sales for the year 2017?

Sol:

Year	Sales (Y) ('000 units)	X	XY	X^2
2009	80	-7	-560	49
2010	90	-5	-450	25
2011	92	-3	-276	9
2012	83	-1	-83	1
2013	94	1	94	1
2014	99	3	297	9
2015	92	5	460	25
2016	104	7	728	49
	$\sum Y = 734$	$\sum X = 0$	$\sum XY = 210$	$\sum X^2 = 168$

Here, origin can be found as follows:

$$Origin = \frac{2012+2013}{2} = 2012.5$$

Now the values of 'X' can be found as follows:

$$X = \frac{Year - Origin}{\frac{1}{2}\ of\ interval}$$

$$\text{For}\quad X_{2009} = \frac{2009-2012.5}{0.5} = \frac{-3.5}{0.5} = -7$$

Similarly, for 2010,.....,2016, the values are inserted in the above table.

In order to get the estimated Y values, first we have to find out the values of "a" and "b". For this we need to solve two normal equations as follows:

$$\sum Y = Na + b\sum X \quad (1)$$

$$\sum XY = a\sum X + b\sum X^2 \quad (2)$$

Solving the above two normal equations with the help of above table, we can find out the values of "a" and "b" as follows:

From equation (1), we can get,

$$a = \frac{\Sigma Y}{N} = \frac{734}{8} = 91.75$$

Rom equation (2),

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{210}{168} = 1.25$$

Now, substituting the values of ‘a’ and ‘b’ we can get,

$$\hat{Y} = 91.75 + 1.25X$$

For $X_{2017} = \frac{2017-2012.5}{0.5} = \frac{4.5}{0.5} = 9$

, $\hat{Y} = 91.75 + (1.25 \times 9) = 103$ ***(ANS)***

Similarly, you can calculate the estimated values for all the years.