B.Tech
3rd Semester (*Regular*)
SAS-2022

AUTUMN MID SEMETER EXAMINATION-2022

Subject: Discrete Mathematics

Code: MA-2013

Full Marks: 20 **Time: 1.5 Hrs**

Answer any FOUR QUESTIONS including question No. 1 which is compulsory. The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only

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1. Answer the following questions [5×1=5]

(a) Suppose x is a particular real number and p, q, r are the statements "$1 < x$", "$x < 5$", "$x = 5$" respectively. Write the inequality $1 < x \leq 5$ by using propositional logic.

SE: Writing $1 < x \leq 5$ as $1 < x$ and $x < 5$ or $x = 5$…………(0.5)

Then as $p \wedge (q \vee r)$…or $(p \wedge q) \vee r$…………(0.5)

(b) Write the negation of the statement "This computer program has a logical error or it is being run with an incomplete data set" using De Morgan's law.

SE: writing De Morgan's law $\neg(p \vee q) \equiv \neg p \wedge \neg q$………..(0.5)

This computer program has no logical error and it is not being run with an incomplete data set. ………………(0.5)

(c) Write the inverse of the conditional statement "An integer is divisible by 5 only if it is divisible by 15" in the form of "if…then…"

SE: Inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$. …….(0.5)

If an integer is not divisible by 5 then, it is not divisible by 15. ………….(0.5)

(d) Test whether the argument "If it rains, the prices of vegetables go up", "The prices of vegetables go up", therefore "It rains" is valid.

SE: writing the argument ……………….(0.5)

$$\begin{array}{l} p \rightarrow q \\ q \\ \therefore p \end{array}$$

Argument is not valid ……………(0.5)

(e) Let $P(x)$: x is an odd integer and $Q(x)$: x is a prime integer. Write the statement "Some odd integers are prime" by using predicates and quantifier.

SE: writing the statement as Some integers are odd and prime. ………….(0.5)

$\exists x\,(P(x) \wedge q(x))$ ……………(0.5)

2. (a) Determine whether the statements $p \wedge (q \vee r)$ and $(p \wedge q) \vee r$ are logically equivalent. [2.5]

SE. For correct truth table ……………..(2.0)

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q)$	$(p \wedge q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	F	F	T
F	T	F	T	F	F	F

F	F	T	T	F	F	T
F	F	F	F	F	F	F

(In Truth Table each correct row 0.25 mark, 8 correct row = 0.25 x 8= 2.0 mark)

Conclusion: From the table it is observed that when p is F, q is T and r is T both the given statements have different truth value. Thus, they are not logically equivalent. ………(0.5)

(b) Are these system specifications consistent? [2.5]

"Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

SE: Writing using propositional variables …………(1)

$$p \to \neg q$$
$$q \to r$$
$$\neg r \to \neg p$$

For truth table……..(1)

p	q	r	$P \to \sim q$	$q \to r$	$\sim r \to \sim p$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Conclusion: From the table it is observed that in four cases the given statements have same truth values T, thus these system specifications are consistent…………(0.5)

3. (a) Use truth table to determine whether the following argument form is valid. [2.5]

$$p \to q$$
$$\neg(p \vee r)$$
$$\therefore \neg p$$

SE. For correct truth table ……………..(2.0)

p	q	r	$p \to q$	$p \vee r$	$\sim(p \vee r)$	$\sim p$
T	T	T	T	T	F	F
T	T	F	T	T	F	F
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	F	T	T

(In Truth Table each correct row 0.25 mark, 8 correct row = 0.25 x 8= 2.0 mark)

Conclusion: From the table it is observed that in two cases $p \to q$ and $\sim(p \vee r)$ are true and in those cases $\sim p$ also true, thus the given argument is valid . …………(0.5)

(b) Write the proposition "Not all real numbers are rational numbers" using predicate and quantifier. [2.5]

SE: Define the domain all numbers……..(0.5)

Defining the predicate P(x): x is real …….(0.5)

Defining the predicate Q(x): x is rational …….(0.5)

Writing all real numbers are rational numbers using predicate and quantifier:

$$\forall x(P(x) \rightarrow Q(x)) \text{ ………(0.5)}$$

Writing not all real numbers are rational numbers as

$$\neg\left(\forall x\big(P(x) \rightarrow Q(x)\big)\right) \text{ ………(0.5)}$$

OR

Define the domain all real numbers……..(0.5)

Defining the predicate Q(x): x is rational …….(0.5)

Writing all real numbers are rational numbers using predicate and quantifier:

$$\forall x\, Q(x) \text{ ………(0.5)}$$

Writing not all real numbers are rational numbers as

$$\neg(\forall x\, Q(x)) \text{ ………(0.5)}$$

Writing equivalent

$$\exists x\, \neg Q(x) \text{……..(0.5)}$$

4. You are about to leave for your class and discover that you don't have your glasses. You know the following statements are true. [5]

"If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table." "If my glasses are on the kitchen table, then I saw them at breakfast." "I did not see my glasses at breakfast." "I was reading the newspaper in the living room or in the kitchen." "If I was reading the newspaper in the living room then my glasses are on the coffee table."
Where are the glasses?

SE: Writing propositional variable: ……………(0.5)

p: I was reading the newspaper in the kitchen
q: My glasses are on the kitchen table
r: I saw my glasses at breakfast
s: I was reading the newspaper in the living room
t: My glasses are on the coffee table

writing statements using propositional variable: …………(2.0)

(1) $p \rightarrow q$
(2) $q \rightarrow r$
(3) $\neg r$
(4) $s \vee p$
(5) $s \rightarrow t$

Step 1 …………..(0.5)
(1) $p \rightarrow q$
(2) $q \rightarrow r$
(6) $\therefore p \rightarrow r$ (by Hypothetical Syllogism)

Step 2 …………..(0.5)
(6) $p \rightarrow r$
(3) $\neg r$
(7) $\therefore \neg p$ (by modus tollons)

Step 3 …………..(0.5)
(4) $s \vee p$
(7) $\neg p$
(8) $\therefore s$ (by disjunctive Syllogism)

Step 4 …………..(0.5)
(5) $s \rightarrow t$
(8) s
(9) $\therefore t$ (by Modus Ponons)

Hence, my glasses are on the coffee table. …………..(0.5)

5. What is principle of strong induction? Use strong induction to show that every positive integer can be written as a sum of distinct powers of two. [5]

SE: Writing principle of strong induction: …………(1)

$$P(b)$$
$$(P(b+1) \wedge P(b+2) \wedge \cdots \wedge P(k)) \rightarrow P(k+1)$$
$$\therefore \forall n \geq P(n)$$

Writing the domain as all positive integer and predicate

$P(n)$: n can be written as a sum of distinct powers of two. ………..(0.5)

Basis step:

$$1 = 2^0$$

So, $P(1)$ is true ………..(0.5)

Inductive step:

Assume that $P(2), P(3), \ldots, P(k)$ are true for some $k > 1$ ………..(0.5)

To prove $P(k+1)$ is true

Case1: $k+1$ is odd : ………..(1.0)

Case1: $k+1$ is even: ………..(1.0)

Conclusion: ………..(0.5)

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