

06/12/16

DEMAND

Demand means desire for commodity or service backed by sufficient purchasing power and willingness to pay. There involves an element of time with the demand.

Factors affecting demand =>

3) Write any 4 imp. factors determining the demand for a commodity. (2 m)

- 1) Price of the commodity (Price $\propto \frac{1}{\text{Demand}}$)
- 2) Prices of the related goods
 ↳ (competitive goods)
 ↳ (substitutes and complementary goods)
 ↳ either
 ↳ consumed together
- 3) Income of the consumer
 ↳ (income rises demand rises)
- 4) Taste, choice and preference of the consumer
- 5) Size of population (Population \propto Demand)
- 6) Composition of population
- 7) Advertisements
- 8) Weather and climatic conditions
- 9) Price Expectation of the consumer

LAW OF DEMAND

Other things remaining constant, with the rise in the price quantity demanded goes down and vice versa.

(Other things - except the price of the commodity, all the other factors are other things)

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DEMAND SCHEDULE

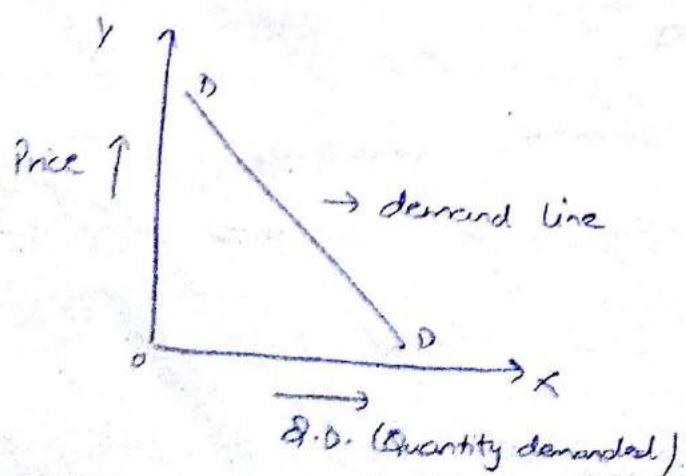
Demand schedule is a list or a table showing different quantity demanded volumes corresponding to different / various prices.

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Price	10	11	15	18	19	20
Quantity Demanded	100	90	67	50	43	34

DEMAND CURVE

$D = f(p)$ ceteris paribus \rightarrow other things remaining constant.



The demand curve shows that the relationship between price and demand to be inversely proportional i.e. when the price rises demand goes down and vice versa.

- Movement along the Demand Curve
- Shifting of the demand curve

Demand equation:

$$Q_x = a - bP_x$$

Q_x - demand of 'x' commodity

a - quantity intercept (when price is zero, what will

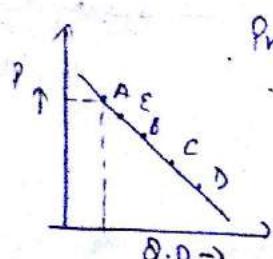
P_x - price of 'x' be the quantity)

b = slope

'-' → minus sign indicates that slope is negative

i.e. the demand line / curve is downwards sloping in nature.

Movement along the demand line - At different points on the line, we get different Price with respect to the quantity demanded.



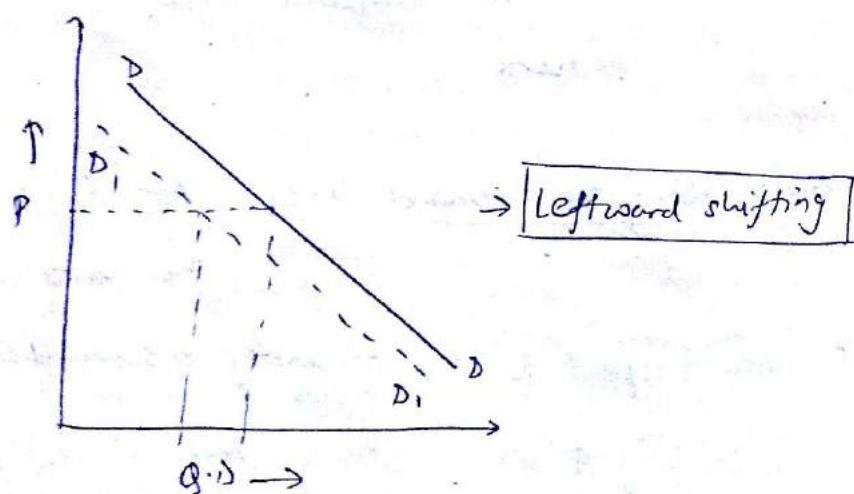
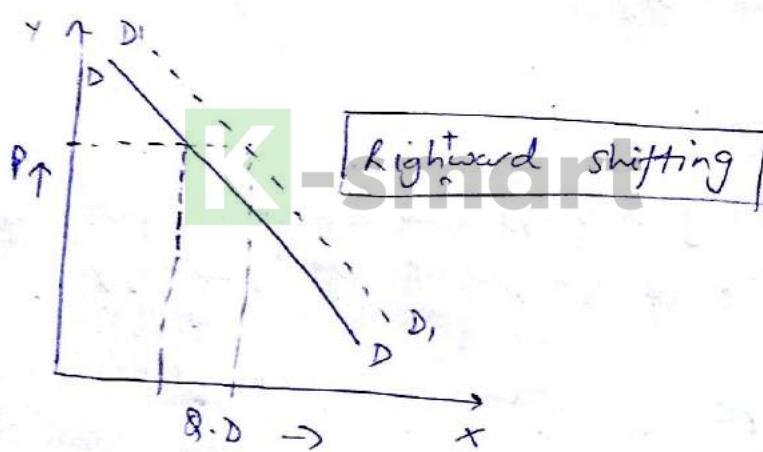
Price changes leads to the movement along the demand line

Movement along the demand curve is possible because of a change in the price only, other factors remaining constant.

Shifting of the Demand line :

It maybe of two types -

- a) Leftward shifting
- b) Rightward shifting



Shifting of the demand curve is possible because of
the change in the other factors like currency
constant.

SUPPLY

Quantity of commodity offered for sale in the market
at a given point of time given the prices is
understood as supply.

Stock is the potential supply.

$$S = f(P) \rightarrow \text{ceteris paribus}$$

K-smart (other things remaining
constant)

The relationship between price and supply is positive and
direct i.e. when the price increases supply increases.

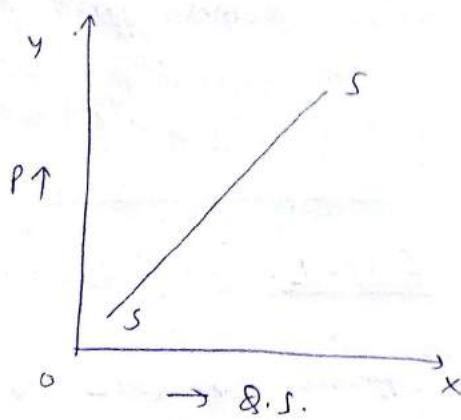
$$Q_s = C + dP$$

$C \rightarrow$ intercept

$d \rightarrow$ slope (positive)

: $P \rightarrow$ price

Supply Line:



② Nature
Supply

Supply schedule :-

Price	50	52	55	60	61	62
Q.S(kg)	100	120	135	155	180	200

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Factors affecting supply =>

- 1) Price of the commodity
- 2) Prices of the related goods
- 3) Cost of production
- 4) State of technology
- 5) Goal of the producer → (monopoly or competitive)
- 6) Means of transportation, communication, banking and insurance facilities

Shifting
change

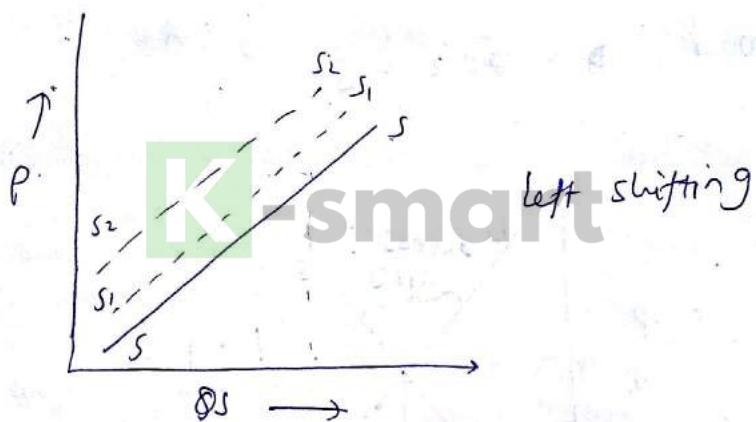
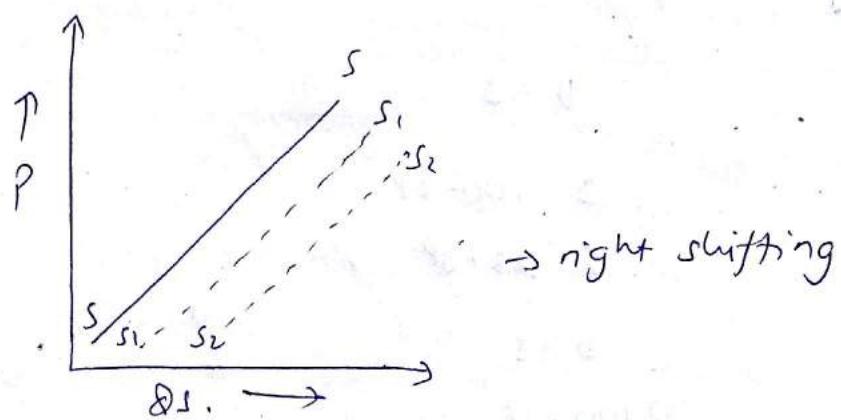
Movement

change

constant

?) Natural factors

Supply curve may shift to the left or to the right.



Shifting of the supply curve is possible because of a change in the other factors; price remaining constant.

Movement along the supply curve is possible because of a change in the price only; other factors remaining constant.

Demand-Supply Equilibrium \Rightarrow

Equilibrium in the market is attained ^{here}, where demand & supply are equal.

$$D = S$$

$$D = 100 - 2P$$

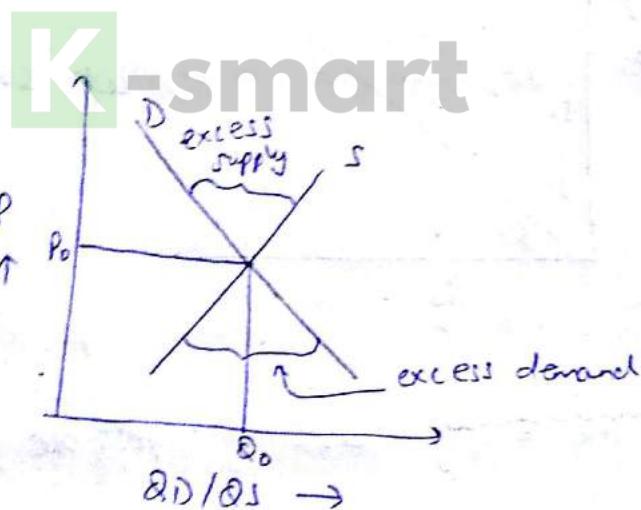
$$S = 28 + 3P$$

$$D = S$$

$$\Rightarrow 100 - 2P = 28 + 3P$$

$$\Rightarrow 5P = 72 \Rightarrow P = 14.4$$

Now, $D = 71.2 = S$



$$D = a - bp$$

$$S = c + dp$$

$$D = S$$

$$a - bp = c + dp$$

$$P = \frac{a - c}{d + b}$$

~~now putting P~~

Substituting P in D;

$$D = a - b \left[\frac{a-c}{b+d} \right]$$

$$D = \frac{ab + ad - ab + bc}{b+d}$$

$$D = \frac{ad + bc}{b+d}$$

P in S;

$$S = c + d \left[\frac{a-c}{b+d} \right]$$

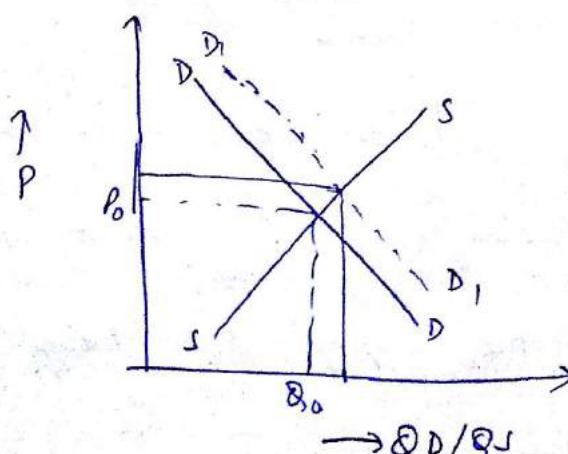
$$= \frac{cb + cd + ad - cd}{b+d}$$

$$S = \frac{bc + ad}{b+d}$$

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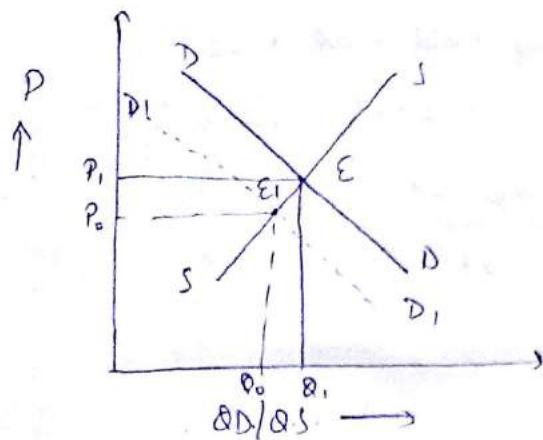
In the market, equilibrium condition is automatically restored because of either excess supply or excess demand.

If the supply condition remaining constant, there is a rightward shift in the demand condition

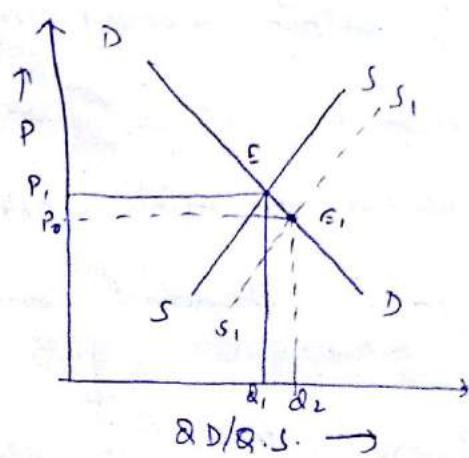


Supply will be adjusted to demand, this will result in an increase in the price.

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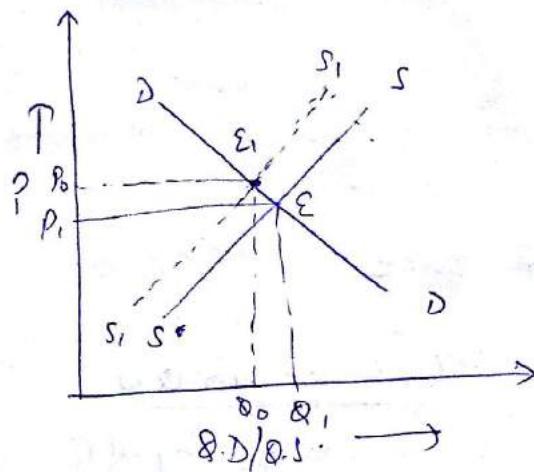


When the supply condition remaining constant, the demand condition shifts to the left, equilibrium will be to the left side and as a result, there will be a decrease in the price



When the demand condition remaining unchanged, the supply condition shifts to the right, demand will be adjusted to the supply but there will be a

fall in the price.



When the supply condition shifts to the left the demand condition remaining constant, this will result in an increase in the price.

Elasticity of Demand

Elasticity means degree of responsiveness.

Mainly speaking, there are 3 factors, determining the demand for a commodity. They are:-

- 1) Price of the commodity
- 2) Income of the consumer
- 3) Prices of the related goods.

Accordingly, we get 3 elasticity concept;

- i) Price elasticity of demand ~~(ed)~~ (e_p)
- ii) Income elasticity of demand (e_y)
- iii) Cross elasticity of demand (e_c)

PRICE ELASTICITY OF DEMAND :-

In this case, we measure the percentage change in the demand because of a certain change in the price.

$$e_p = \frac{\% \text{ change in } Q}{\% \text{ change in price}}$$

$$= \frac{\% \Delta Q}{\% \Delta P}$$

$$P = 10 \quad Q = 80$$

$$P_1 = 12 \quad Q_1 = 61$$

$$\% \Delta Q = \frac{Q - Q_1}{Q} = \frac{\Delta Q}{Q}$$

$$\% \Delta P = \frac{P - P_1}{P} = \frac{\Delta P}{P}$$

$$e_p = \frac{\Delta Q}{Q} / \frac{\Delta P}{P}$$

$$= \frac{\Delta Q}{Q} \times \frac{P}{\Delta P}$$

$$= \boxed{- \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}}$$

More elastic (ex- chocolates) - more response

Less elastic - less response to change in price or any other necessary goods
(ex - salt consumption remains unchanged)

Govt. will keep control over utility services.

* If we want to calculate e_p at a particular price,

$$\lim_{\Delta p \rightarrow 0} \left\{ \frac{-\Delta Q}{\Delta P} \times \frac{P}{Q} \right\} \Rightarrow e_p = \frac{\partial Q}{\partial P} \times \frac{P}{Q}$$

Q) Studies in USA indicates that the price elasticity for cigarettes is 0.4. If a pack of cigarette currently costs 2 dollar and the govt. wants to reduce smoking by 20%. By how much should the govt. increase the price and what is the new price?

Ans $e_p = 0.4$ $P = 2$ dollars

$$e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

New price = $\frac{50}{100} \times 2 + 2$
= 3 dollars

$$e_p = \frac{\% \Delta Q}{\% \Delta P}$$

$$\Rightarrow 0.4 = \frac{20}{\% \Delta P}$$

$$\text{Increase} = 1 \text{ dollar}$$

$$\Rightarrow \% \Delta P = \frac{20}{0.4}$$

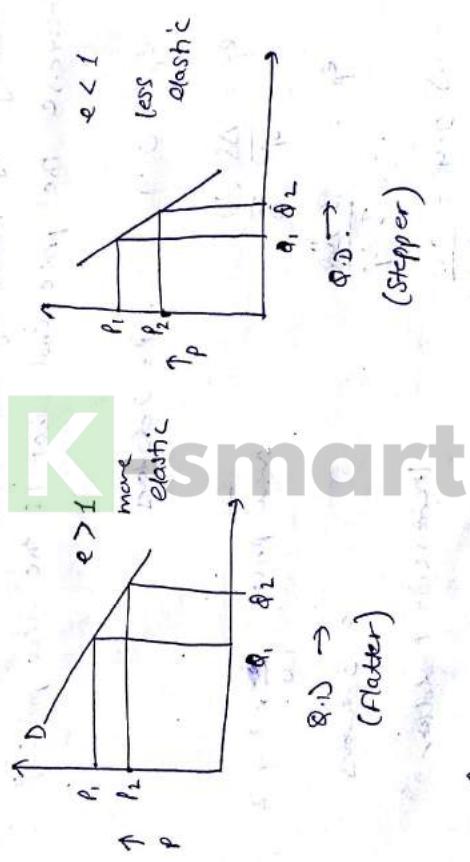
$$\Rightarrow \% \Delta P = 50$$

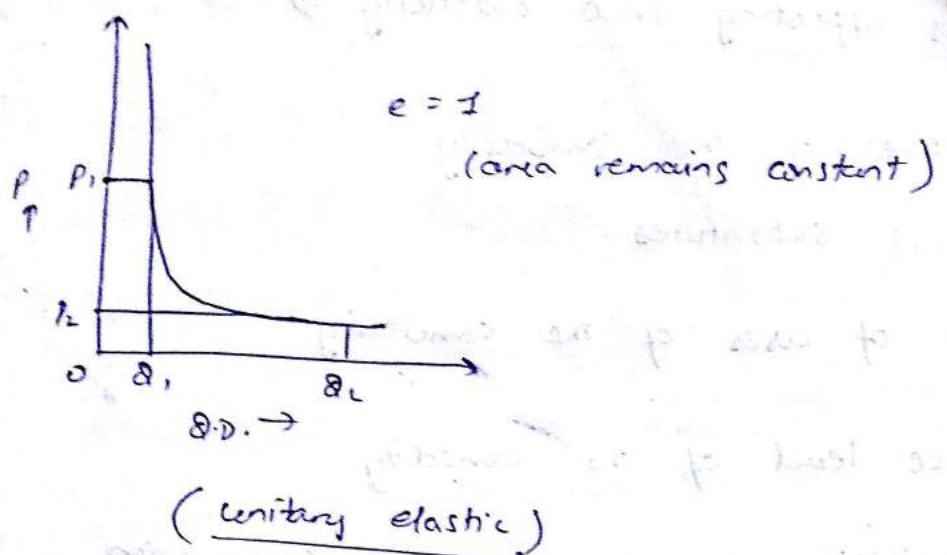
Q) Anna owns a secret sweet chocolate store. Anna charges 10 dollar each for his hand-made sweet chocolates. You being an engineering economist have calculated the elasticity of demand for the chocolate in that town to be 2.3. If Anna wants

to increase the total revenue what advice will you give him and why?

Ans ep is less than one with no increase in price people will buy more so with decrease in price people will buy.

Types of demand curves and the degree of elasticity





10/01/17

- a) Sona sells cookies for 4\$ a dozen and she sells 50 dozens. Sona decides to charge more and increases the price to 6\$ a dozen. Now she sells 40 dozens. (i) What is the elasticity of demand (ii) Assuming that the elasticity of demand remains constant, how many would she sell if price were 10\$ a dozen.

$$(i) \quad e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$$= \frac{50 - 40}{4 - 6} \times \frac{4}{50}$$

$$= -\frac{10}{2} \times \frac{4}{50} = -0.4$$

$$(ii) \quad 0.4 = \frac{\Delta Q}{-6} \times \frac{4}{50} \Rightarrow 30 = \Delta Q$$

$$\text{Ans: } 50 - 30 = 20 \text{ dozens}$$

Factors affecting Price Elasticity \Rightarrow

- 1) Nature of the commodity
- 2) No. of substitutes
- 3) No. of uses of the commodity
- 4) Price level of the commodity
- 5) Possibility of postponement of consumption
- 6) Joint demand
- 7) Consumer behaviour

INCOME ELASTICITY OF DEMAND \Rightarrow

In case of income elasticity of demand we measure the percentage change in demand because of a certain change in the income.

- for inferior goods, income elasticity, e_y is negative
- Income elasticity of demand for normal goods is positive but greater than 1. The goods are normal superiors.
- If income elasticity is in between 0 & 1, the goods are normal necessities.

11/01/17

$$(1) e_y = \frac{\% \Delta Q}{\% \Delta Y}$$

$$2) e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$$3) e_y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

CROSS ELASTICITY OF DEMAND \Rightarrow

In case of cross elasticity we measure % change in quantity of commodity x with respect to a certain change in the price of commodity y .

→ Cross elasticity between the substitutes is +ve.

→ Cross elasticity between the complementaries is -ve.

Higher the value of cross elasticity, stronger the degree of substitutability and complimentarity.

* In case the goods are perfect substitutes the cross elasticity between them will be +ve infinity.

* In case the goods are perfect complements the cross elasticity between them will be tending towards -ve infinity.

$$1) e_c = \frac{\% \Delta Q_x}{\% \Delta P_y}$$

$$(3) e_c = \frac{dQ_x}{dP_y} \times \frac{P_y}{Q_x}$$

$$2) e_c = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x}$$

(Q) In a year no. of cars sold decreased by 20%. During that year the price of car increased by 5%, per capita income declined by 2% and prices of petrol increased by 10%.

Income elasticity (e_y) for cars is estimated to be +1.5 and cross price elasticity (e_c) of petrol and cars is estimated to be -0.30.

i) What is the impact of decline in per capita income on the demand for cars?

$$\text{Ans) } e_y = +1.5 \quad \% \Delta Y = -2\%$$

$$1.5 = \frac{\% \Delta Q}{2} \rightarrow \% \Delta Q = 3\% \text{ (decline)}$$

ii) What is the impact of increase in the price of petrol on the demand for cars?

$$\text{Ans) } e_p = -0.30 \quad \% \Delta P_y = 10\%$$

$$\% \Delta Q_x = -3\%$$

iii) What is the price elasticity of demand?

$$\text{Ans) Total decline} = 20\%$$

6% for petrol and per capita

$$\% \Delta Q = (20-6)\% = 14\%$$

$$e_p = \frac{\% \Delta Q}{\% \Delta P} = \frac{14}{5} = 2.8$$

Q) Empirical studies about the automobile demand in a country have indicated that the price elasticity for cars is 1.2 and income elasticity is 2.8 in the year 2014. The present sales of car in the country is 8 million. If the price increases by 10% and income will increase by 5% next year.

- i) What will be its effect on the sales of car?
 ii) Find the expected sales of car in the next year.

Ans) i) $e_p = 1.2 \quad e_y = 2.8 \quad Q = 8 \text{ million}$

$\% \Delta P = 10\%$  $\% \Delta Y = 5\%$

$$1.2 = \frac{\% \Delta Q}{\% \Delta P}$$

$$\% \Delta Q = 12\% \text{ (decrease)}$$

ii) $2.8 = \frac{\% \Delta Q}{\% \Delta Y} \Rightarrow \% \Delta Q = 14\%$

$$\text{Net increase} = 2\%$$

$$\begin{aligned} \text{Expected sales} &= 8 \text{ million} + 2\% \text{ of } 8 \text{ millions} \\ &= 8.16 \text{ million.} \end{aligned}$$

13/01/17

- Q) Heartbeat headphones are retailed at \$200 per unit following a \$20 increase in price. of heartbeat headphones, there is an increase in the demand for a rival brand of headphones by 7.5%. What is the cross price elasticity of this price change?

$$e_c = 7.5 \times \frac{20}{200} = 0.75$$

↑ 9% increase

- Q) Consider the following goods and their cross elasticity.

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Goods	Cross elasticity
Air conditioning units and kW of electricity	- 0.34 - Complimentary
Coke and Pepsi	0.63 - substitutes
High fuel consuming Sports utility vehicles and gasoline	- 0.18 - complimentary
Mc Donald's Burger and Burger King Burgers	0.28 - substitutes
Butter & Margarine	1.54 - "

What are the signs of each of cross price elasticities imply about the relationship between goods?

ii) Use the information in the table, to calculate how a 5% increase in the price of Pepsi affects the demand for Coke.

$$\% \Delta p_y = 5\% \quad e_c = 0.63$$

$$\% \Delta Q_x = 0.63 \times 5 = 3.15\%$$

Q) Suppose the seller of textile cloths wants to lower the price of the cloth from Rs. 150 per meter to Rs 142.5 per meter. If its present sales are 2000m per month and further it is estimated that price elasticity of demand for the product is 0.7. Find whether or not the TR will increase as a result of his decision to lower the price level.

Given: $e_p = 0.7$, $P_1 = 150$

$$P_2 = 142.5$$

$$\Delta P = 7.5$$

$$e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$$0.7 = \frac{\Delta Q}{7.5} \times \frac{150}{2000}$$

$$\Delta Q = 70$$

$$\text{New } Q = 2070 \text{ m}$$

$$\text{Old } TR = 300000 \quad (150 \times 2000)$$

$$\text{New } TR = 294975$$

$\hookrightarrow TR \text{ will decrease}$

Q) Demand for a book is given by the eqn

$Q = 20000 - 60P$. Calculate the point price elasticity at the price 200

$$e_p = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$Q = 20000 - 60(200)$$

$$Q = 8000$$

$$e_p = -60 \times \frac{200}{8000} = -1.5$$

Relationship between e_p , AR and MR \Rightarrow

AR - Average Revenue

MR - Marginal Revenue (incremental revenue)

$$TR = P \cdot Q$$

17/01/17

$$AR = \frac{TR}{Q} = \frac{P \cdot Q}{Q} = P$$

$$\boxed{AR = P}$$

$$MR = \frac{dTR}{dQ}$$

$$MR = \frac{d(P \cdot Q)}{dQ}$$

$$MR = P \cdot \frac{dQ}{dQ} + Q \cdot \frac{dP}{dQ}$$

$$MR = P + \frac{dP}{dQ} \cdot Q$$

$$MR = P \left(1 + \frac{dP}{dQ} \cdot \frac{Q}{P} \right)$$

$$\left\{ e_p = \frac{dQ}{dP} \cdot \frac{P}{Q} \right\}$$

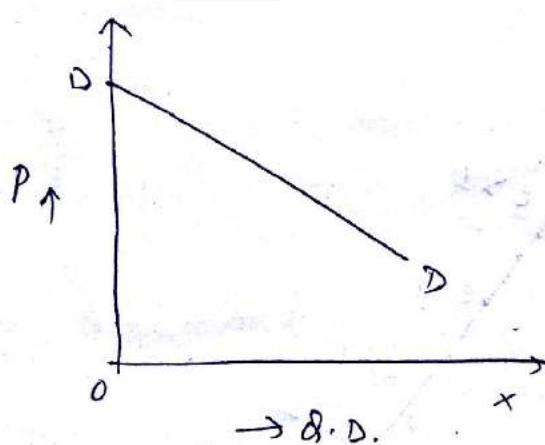
$$MR = P \left(1 + \frac{1}{e_p} \right)$$

$$MR = P \left(1 - \frac{1}{|e_p|} \right)$$

$$MR = AR \left(1 - \frac{1}{e_p} \right)$$

let the monopolist face a downward sloping demand line.

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$$x = a_0 - a_1 p$$

$$\Rightarrow p = \frac{a_0}{a_1} - \frac{1}{a_1} x$$

$$TR = p \cdot x$$

$$= a_0^* x - a_1^* x^2$$

$$AR = \frac{TR}{x}$$

$$\Rightarrow AR = a_0^* - a_1^* x$$

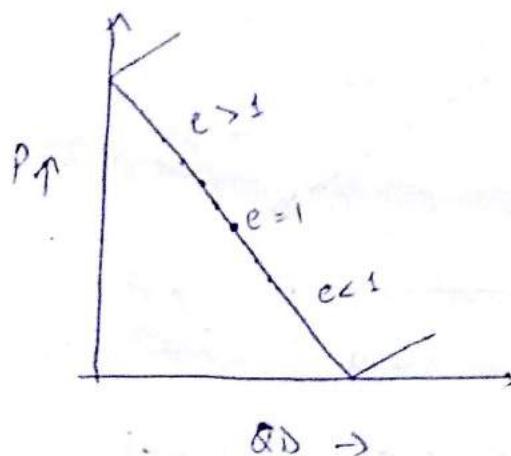
$$\Rightarrow \boxed{AR = P}$$

$$MR = \frac{dTR}{dx}$$

$$MR = a_0^* - 2a_1^* x$$

MR line is a straight line like AR line, the slope of MR is twice of the slope of AR, so MR line has to lie below the AR line.

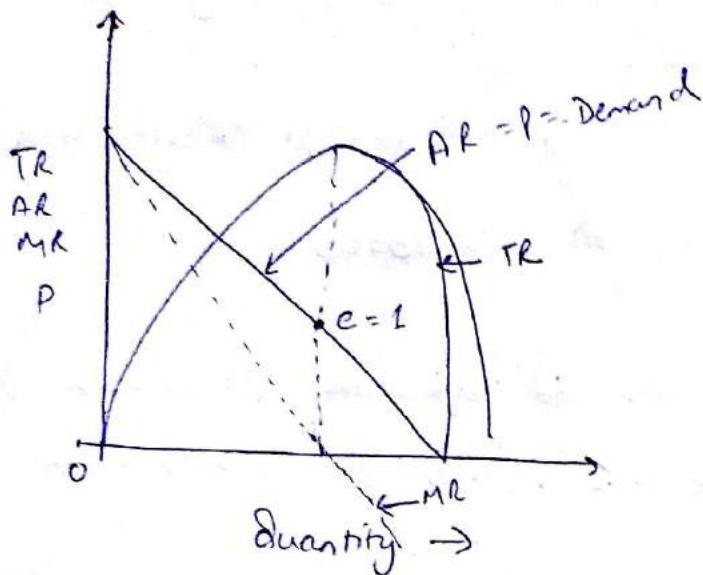
Measuring the price elasticity on a straight line demand curve at different points



when $e = 1$; $MR = 0$

$e > 1$; $MR = +ve$

$e < 1$; $MR = -ve$



when $MR = 0$; TR will be maximum

Q) The demand function faced by a company for its personal computers is $P = 50000 - 4Q$.

i) Write the MR function

$$P = 50000 - 4Q$$

$$R PQ = 50000Q - 4Q^2$$

$$\frac{dP}{dQ} = 50000 - 8Q \rightarrow MR$$

ii) At what price and quantity $MR = 0$

$$50000 - 8Q = 0$$

$$Q = \frac{50000}{8} = 6250$$

$$P = 50000 - 4(6250)$$

$$P = 25000$$

iii) At what price and quantity TR is maximum
at MR = 0, TR is max

$$\Rightarrow P = 25000$$

A) Given the demand equation $P = 50000 - 4Q$. find that elasticity $e = 1$, when TR is maximum.

$$MR = 0$$

$$\frac{dP}{dQ} = 50000 - 8Q$$

$$P = 25000$$

$$Q = 6250$$

$$Q = 12500 - \frac{1}{4}P$$

$$\frac{dQ}{dP} = -\frac{1}{4}$$

$$\frac{P}{Q} = \frac{25000}{6250} = 4$$

$$\frac{dQ}{dP} \cdot \frac{P}{Q} = e = 1 \rightarrow \text{Proved } e = 1$$

18/01/17

Given the demand function for coffee $Q = 150 - 10P$. ~~and~~

Find the quantity and price where TR is max.

$$P \cdot Q = 150P - 10P^2$$

$$10P = 150 - Q$$

$$\frac{dP}{dQ} =$$

$$10PQ = 150Q - Q^2$$

$$\frac{10dP}{dQ} = 150 - 2Q$$

$$15 - 2Q = 0$$

$$Q = 7.5$$

$$P = 7.5$$

ii) Find the value of MR when TR is max.

$$MR = 15 - \frac{Q}{5}$$

$$= 15 - \frac{7.5}{5} = 0$$

$$MR = 0$$

i) Find the elasticity when TR is maximum.

$$EP = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$= -10 \times \frac{7.5}{75} = -1$$

CONSUMER EQUILIBRIUM \Rightarrow

Consumer equilibrium point is a point where the consumer gets maximum satisfaction.

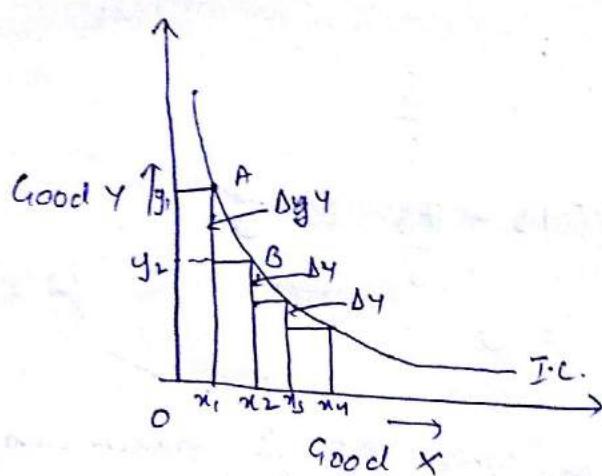
There are two goods X & Y.

$$\left. \begin{array}{l} M = \text{Money income} \\ P_x = \text{Price of } X \\ P_y = \text{Price of } Y \end{array} \right\} - \text{constraints}$$

INDIFFERENCE CURVE \Rightarrow

Indifference curve is a locus of combination of two commodities / goods giving the consumer the same level of satisfaction.

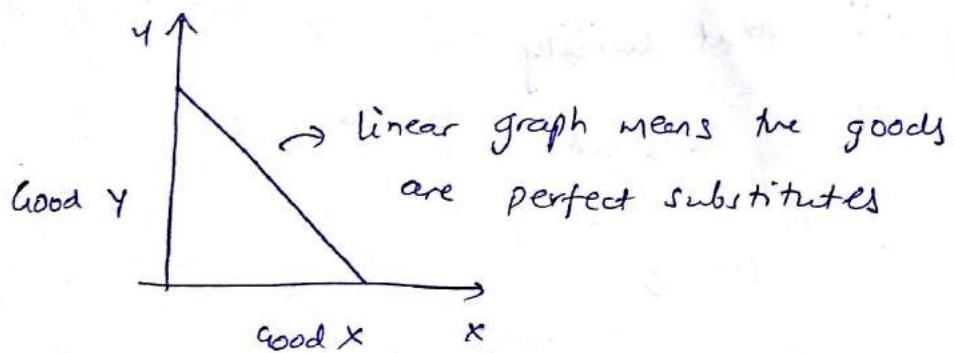
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I.C. is convex to the origin because two goods are normally not perfect substitutes. Therefore, the consumer has to sacrifice less and less of Y for every additional unit of X to maintain this same level of satisfaction.

20/01/17

ΔY is falling for every additional value of X .



Marginal Rate of Substitution (MRS) ;→

MRS indicates the rate at which the commodities can be substituted at the margin in such a manner that the total utility remains constant

MRS X for Y 

MRS_{XY} indicates the amt. of Y the consumer is willing to sacrifice for an additional unit of X .

$$-\frac{\Delta Y}{\Delta X} = -\frac{dy}{dx} = \text{slope of IC}$$

↓

$$= MRS_{XY} \quad (\text{slope of indifference curve})$$

Loss in utility = Gain in utility

$$-\Delta Y \cdot MU_Y = \Delta X \cdot MU_X$$

$$\Rightarrow -\frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y} = \text{slope of IC} = MRS_{XY}$$

$$MRS_{XY} = \frac{MU_X}{MU_Y}$$

⇒ NO two IC
other.

I - Indifference

II - Indifference

$$\frac{\partial U}{\partial x} = 0$$

$$\frac{\partial U}{\partial x} \text{ or } \frac{\partial U}{\partial y} \cdot dy = 0$$

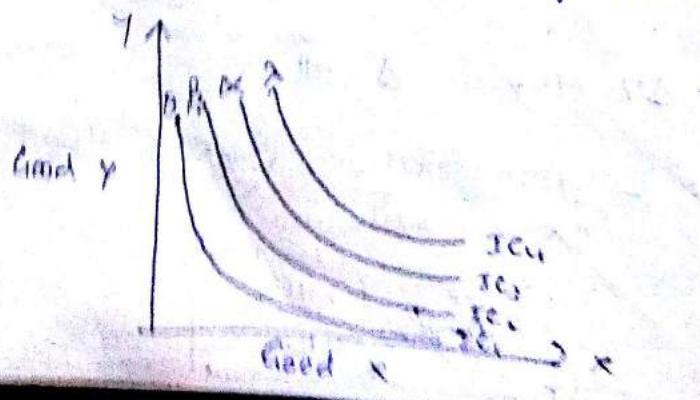
$$\Rightarrow MU_x dx + MU_y dy = 0$$

$$\Rightarrow -\frac{dy}{dx} = \frac{MU_x}{MU_y}$$

MU_y is falling i.e. slope is falling, so it is known as DMRs (decreasing marginal rate of substitution).

⇒ IC is convex to the origin because of the operation of DMRs.

• I → A high IC gives higher level of satisfaction.



at point
below A
above A

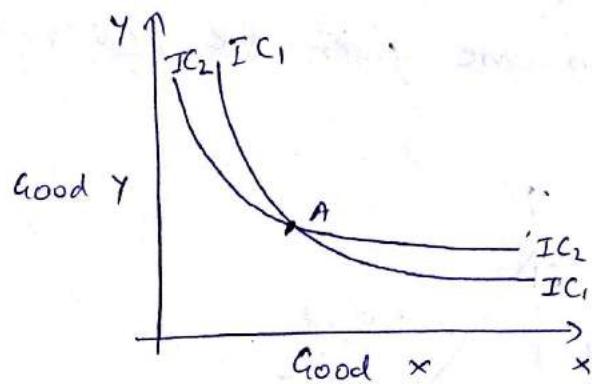
in between

because no

→ A family

$$IC_4 > IC_3 > IC_2 > IC_1$$

→ No two IC can either touch or intersect each other.

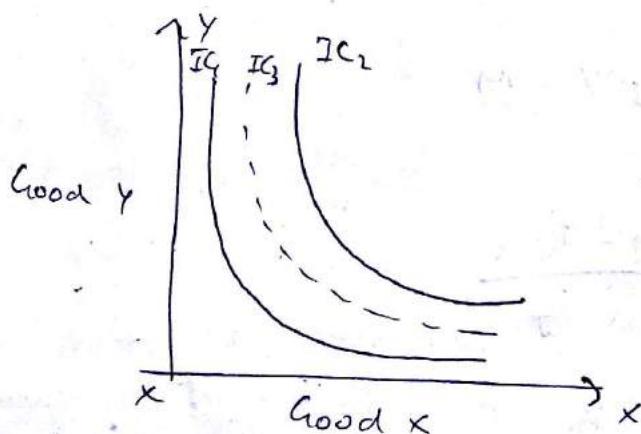


at point A : $IC_1 = IC_2$

below A : $IC_2 > IC_1$

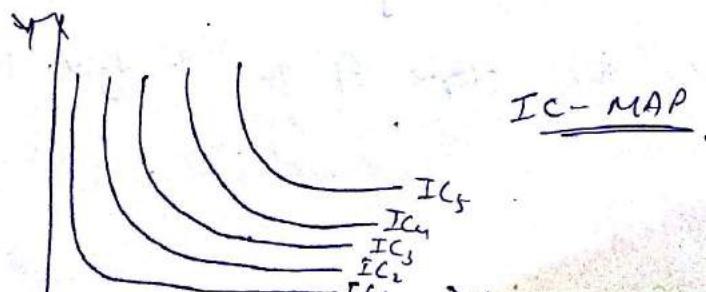
above A : $IC_1 > IC_2$

→ In between two IC a third IC can pass through,



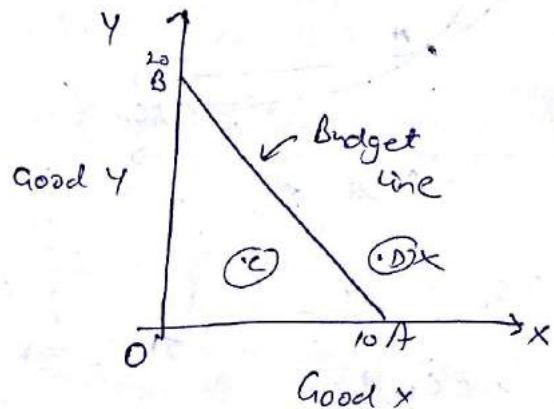
because no two ICs can intersect each other.

→ A family of ICs is considered as an IC Map.



Budget Line :-

Budget line shows all combinations of two goods that the consumer can buy by spending his entire money income given the prices of the good.



Budget equation:-

$$\frac{X}{P_x} + \frac{Y}{P_y} = M$$

$$P_x \cdot X + P_y \cdot Y = M$$

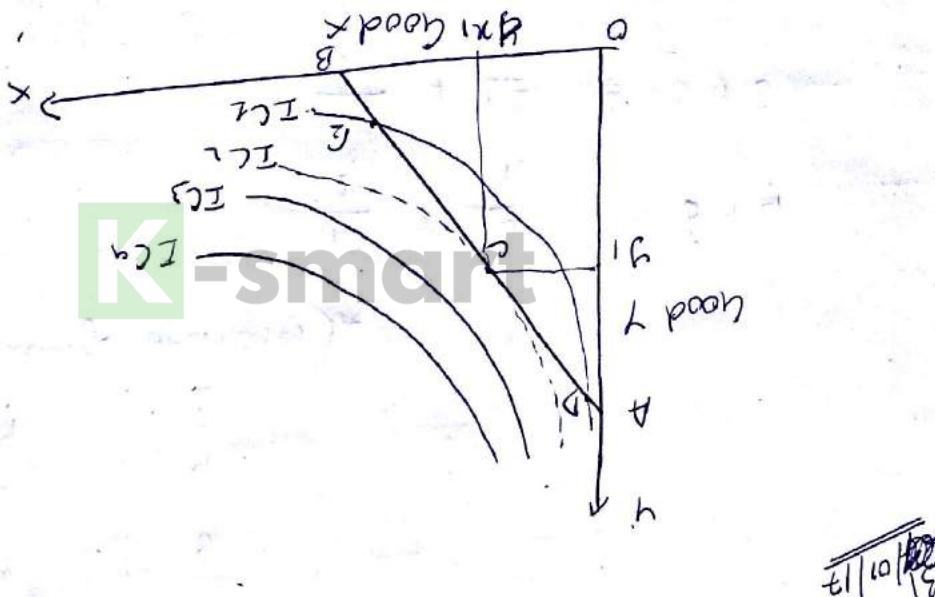
$$Y = \frac{M - P_x \cdot X}{P_y}$$

$$\frac{dY}{dX} = -\frac{P_x}{P_y}$$

$$\Rightarrow -\frac{dY}{dX} = \frac{P_x}{P_y} \rightarrow \text{slope of the budget line.}$$

Geometrically the slope of the budget line is $\frac{OB}{OA}$

- because it is a lower indifference curve
at point C. The consumer should not come to ICL
and the consumer derives maximum satisfaction at
 $Ox_1 + Oy_1$ in the best
sum for the consumer.
2) At the point of equilibrium, the
slope of the budget line is equal to the slope of the indifference
curve at the consumer equilibrium;



slope of the budget line in the case of price.

$$\frac{P_y}{P_x} = \frac{Oy}{Ox}$$

$$\frac{P_y}{P_x} = 40$$

$$\frac{P_y}{P_x} = 10$$

$$\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

→ Equilibrium condition

$$\Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

d) Quantity

	<u>MU_X</u>	<u>MU_y</u>
1	12	16
2	10	12
3	8	8
4	6	4
5	4	2

(marginal utility always decreases)

$$\text{Income} = 10\text{\$}$$

$$P_x = 2\text{\$}$$

$$P_y = 4\text{\$}$$

$$\frac{12}{2} + \frac{16}{4} = 6 + 4$$

$$\frac{10}{2} + \frac{12}{4} = 5 + 3$$

X
3 units Y
1 unit

$$\frac{8}{2} = \frac{16}{4}$$

$$\rightarrow 3x + 1y$$

$$(3 \times 2) + (1 \times 4) = 10\text{\$} \rightarrow \text{income}$$

$$\rightarrow \frac{4x}{2} = \frac{12}{4}$$

$$4x + 2y$$

$$(4 \times 2) + (2 \times 4) = \$ 16$$

→ (beyond budget income)

Q) $P_x = 2$, $P_y = 1$

Budget = 5\$

<u>Quantity</u>	<u>MU_X</u>	<u>MU_Y</u>
1	24	9
2	18	8
3	12	6
4	6	1

$$2x + 1y = 4+1 = 5\text{\$}$$

$$(2 \times 2) + (1 \times 1) = 5\text{\$} \quad (\text{equilibrium})$$

K-smart

24/01/17

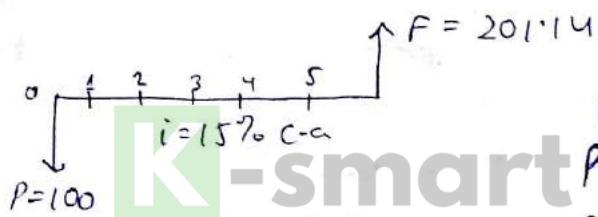
Time Value of Money \Rightarrow (Growth of capital)

Growth of capital per unit of time. Unit of time maybe annually, semi-annually, quarterly or monthly. Interest rate is responsible for the growth of capital.

$$i = 15\%$$

$$\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ 100 & 115 & 132.25 & 152.09 & 174.90 & 201.14 \end{array}$$

\rightarrow growth of capital



Cash flow diagram

i - rate of interest compounded periodically
P - initial investment

F - Future amount/
(Compound amt.)

n - no. of interest period

Interest formula \rightarrow

1) Single payment compound amount

$$F = ?$$

given P, i, n

$$F_1 = P + Pi = P(1+i)$$

$$F_2 = F_1(1+i) = P(1+i)^2$$

$$F_3 = P(1+i)^3$$

$$F_n = P(1+i)^n$$

$$F = P(F/P, i, n)$$

2) Single payment present worth

$$P = \frac{F}{(1+i)^n} = F \times \frac{1}{(1+i)^n}$$

$$P = F (P/F i n)$$

Q) A person wishes to have a sum of ₹ 1000000 for his son's education after 10 years from now. What is the single payment that he should deposit now to get the desired amount? The bank gives 4% rate of interest compounded annually.

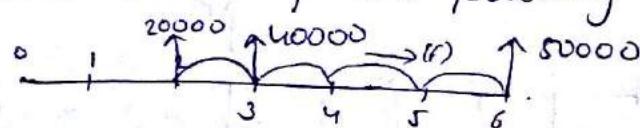
$$P = \frac{F}{(1+i)^n} = \frac{1000000}{(1+0.04)^{10}} = ₹ 67556.42$$

Q) You deposit ₹ 50000 now in your bank account. What will be the maturity value after 10 years if the money is growing at 15% annually?

$$\begin{aligned} F &= P(1+i)^n \\ &= 50000 (1+0.15)^{10} \end{aligned}$$

$$P = ₹ 2,02,277$$

Q) Find the present worth of the following cash diagram



$$i = 10\%$$

$$n = 6$$

$$P = \frac{F}{(1+i)^n} = ₹ 1322 + \frac{40000}{(1+0.1)^4} + \frac{60000}{(1+0.1)^5} + \frac{80000}{(1+0.1)^6}$$

$$P = \frac{20000}{(1+1)^2} + \frac{40000}{(1+1)^3} + \frac{50000}{(1+1)^6}$$

$$P = \text{£ } 74805$$

$$F = 20000(1+1)^4 + 40000(1+1)^3 + 50000$$

25/01/17

3) Equal payment series sinking fund

Sinking fund is a fund ~~collected~~ established to accumulate a future sum of money (F) through the collection of an uniform series of payments.

Here, all payments are a constant value (A)

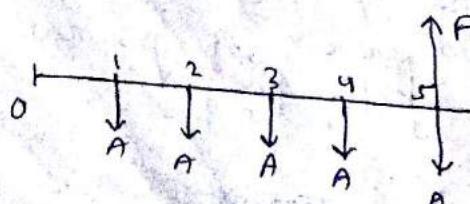
and payments are deposited at the end of the interest period

i) A = annual equivalent amount

F = compound amount

Objective - To find A

Given - F , i , n



<u>EOY</u> <u>(end of year)</u>	<u>Amount of A:</u>	<u>Future ^{value}</u>
1	1000	$1000(1.08)^4$
2	1000	$1000(1.08)^3$
3	1000	$1000(1.08)^2$
i = 8%	4	$1000(1.08)^1$
	5	<u>1000</u>
		<u>$F = 5866.601$</u>

Using general notations;

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \dots + A(1+i)^{n-n}$$

$$F = A \left[(1+i)^{n-1} + (1+i)^{n-2} + \dots + (1+i)^{n-(n-1)} + (1+i)^{n-n} \right] \quad \text{--- (1)}$$

Multiplying both sides of eqⁿ (1) by $(1+i)$

$$F(1+i) = A \left[(1+i)^n + (1+i)^{n-1} + \dots + (1+i)^2 + (1+i) \right] \quad \text{--- (2)}$$

Subtracting eqⁿ (1) from eqⁿ (2)

$$Fi = A \left[(1+i)^n - 1 \right]$$

$$\Rightarrow \boxed{A = \frac{F \times i}{[(1+i)^n - 1]}} = \boxed{F \times (A/F, i, n)}$$

4) Equal payment series compound amount

$$F = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 1000 \left[\frac{(1.08)^5 - 1}{0.08} \right] \quad (\text{from prev. question})$$

$$F = 5866.601$$

$$F = A \times [F/A \text{ in}]$$

8) A person who is now 35 years old is planning for his retire life. He plans to invest an equal

sum of £ 10,000 at the end of every year

for the next 25 years at the rate of interest 8%

compounded annually. Find the maturity value when the person is 60 years.

$$A = 10000$$

$$n = 25$$

$$i = 8\%$$

$$F = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

$$= 10000 \left[\frac{1.08^{25} - 1}{0.08} \right]$$

$$F = 731059.39$$

8) You spend £12000 every year on games. How much would be the amount if you decided not to game for upcoming 40 years. Given the interest rate at 6%.

$$A = 12000$$

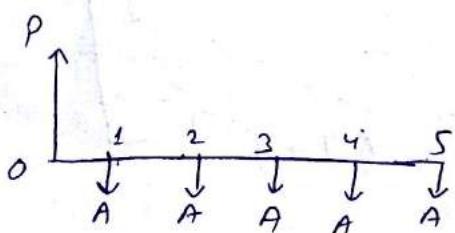
$$F = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

$$= 12000 \left[\frac{1.06^{40} - 1}{0.06} \right]$$

$$F = £18,57,143.59$$

27/01/17

3) Equal payment series capital recovery factor



i%
?

Objective : To find 'A' (recovery amount).

given : P, i, n

$$P = \frac{F}{(1+i)^n}$$

i=6% £04

Amt. of 'A'

Present worth

$$1 \quad 1000 \quad 1000(1.06)^{-1}$$

$$2 \quad 1000 \quad 1000(1.06)^{-2}$$

$$3 \quad 1000 \quad 1000(1.06)^{-3}$$

$$4 \quad 1000 \quad 1000(1.06)^{-4}$$

$$5 \quad 1000 \quad 1000(1.06)^{-5} \boxed{P = 3992.71}$$

$$P = 3992 \cdot 71 \text{ (loan amount)}$$

Using general notations, we can write the conversion table:

$$P = A(1+i)^{-1} + A(1+i)^{-2} + \dots + A(1+i)^{-n+1} \\ + A(1+i)^{-n}$$

$$P = A \left[(1+i)^{-1} + (1+i)^{-2} + \dots + (1+i)^{-n+1} + (1+i)^{-n} \right] \quad (1)$$

Multiplying both sides of eqⁿ (1) by $(1+i)^{-1}$

$$P(1+i)^{-1} = A \left[(1+i)^{-2} + (1+i)^{-3} + \dots + (1+i)^{-n} + (1+i)^{-n-1} \right] \quad (2)$$

Subtracting eqⁿ (1) from eqⁿ (2).

$$P \left[(1+i)^{-1} - 1 \right] = A \left[(1+i)^{-n-1} - (1+i)^{-1} \right] \quad (3)$$

$$P \left[\frac{-i}{1+i} \right] = A \left[(1+i)^{-n-1} - (1+i)^{-1} \right]$$

Multiplying both sides of eqⁿ (3) by $-(1+i)$

$$\therefore P_i = A \left[\frac{(1+i)^n - 1}{(1+i)^n} \right]$$

$$\Rightarrow A = \frac{P \times \left[i(1+i)^n \right]}{(1+i)^n - 1}$$

$$\therefore A = P(A/P_i)^n$$

6) Equal payment series Present worth

$$P = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$
$$P = A(P/A, i, n)$$

8) You spend £ 12000

8) A judge punished you to pay an amount of £ 9000 every year for 15 years. to an old man who got badly hurt by an accident you caused. You are a rich man and wish to pay at once now. Market rate of interest is 5%, how much money you'll pay to the old man now.

$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$P = 9000 \left[\frac{1.05^{15} - 1}{0.05 \times 1.05^{15}} \right]$$

$$P = £ 93416.92$$

8) You are soon going to be a graduate and want to know how high your annual salary should be if you wish to be a 10 millionaire within 10 years. $i = 6\%$

$$A = \frac{F \times i}{(1+i)^n - 1} = 758679.58$$

8) Demand and supply schedule of a commodity are given as:

P	10	12	14	16	18	20
D =	500	450	400	350	300	250
S =	320	360	400	440	480	520

find the equilibrium price

$$P = 14 \text{ (equilibrium price, because } D = S)$$

8) Given the supply schedule as:

P	50	100	150
Q_s	25	50	75

find the expected supply function.

$$B = \frac{\Delta Q}{\Delta P} = \frac{50}{100} = 0.5$$

$$Q = A + BP$$

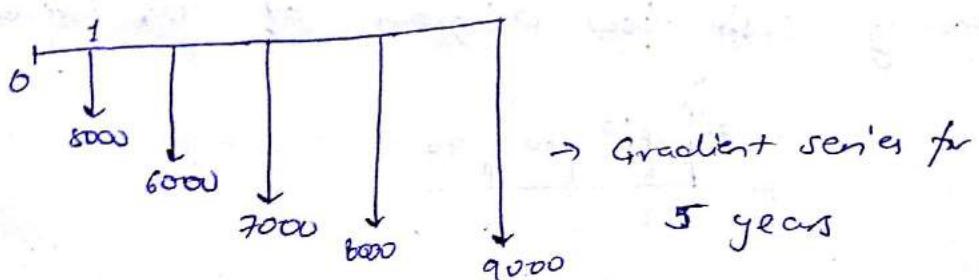
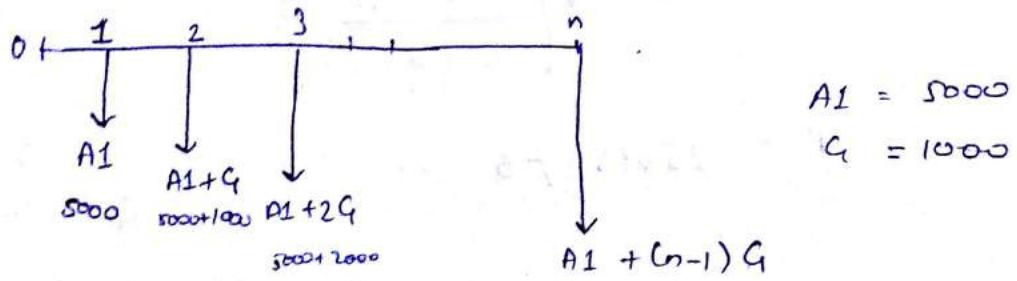
$$25 = A + 0.5 \times 50$$

$$A = 0$$

$$\Rightarrow \boxed{Q = 0.5P}$$

14/02/17

⑦ Uniform Gradient Series Annual Equivalent Amount



$PW(10\%) \rightarrow$ Present worth

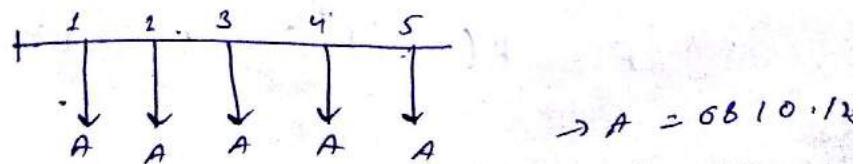
$$PW = \frac{8000}{(1+i)^1} + \frac{6000}{(1+i)^2} + \frac{7000}{(1+i)^3} + \frac{8000}{(1+i)^4} + \frac{9000}{(1+i)^5}$$

$$\Rightarrow PW = 25815.735$$

$$A = A_1 \pm G \left[\frac{(1+i)^n - i^n - 1}{i((1+i)^n - 1)} \right]$$

$$A = 5000 + 1000 \left[\frac{(1.1)^5 - 0.1 \times 5 - 1}{0.1(1.1)^5 - 0.1} \right]$$

$$A = 6810.125 \text{ (at the end of every year)}$$

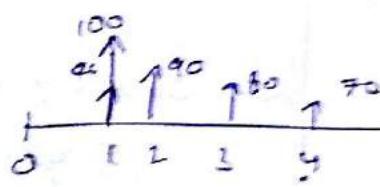


$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$P = 6810.125 \left[\frac{1.1^5 - 1}{0.1 \times 1.1^5} \right]$$

$$P = 25815.73$$

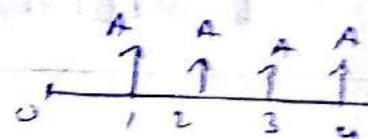
8) find the annual equivalent amount of the following cash flow diagram at 10% interest rate



$$A = A_1 - G \left[\frac{(1+i)^n - i^n - 1}{i(1+i)^n - i^n} \right]$$

$$A = 100 - 10 \left[\frac{1.1^{10} - (0.1 \times 1)^{10} - 1}{0.1 \times 1.1^{10} - 0.1} \right]$$

$$A = 86.188$$



$$P = 86.188 \left[\frac{1.1^4 - 1}{0.1 \times 1.1^4} \right]$$

$$P = 273.204$$

$$F = P(1+i)^n = 399.99$$

15/02/17

Net Present Value Method of Comparison \Rightarrow (NPV Method)

$$NPV = GPV - C_0$$

GPV \rightarrow Gross Present Value

$C_0 \rightarrow$ cost at zero time

MARR - Minimum attractive rate of return

Discount rate

Interest rate

$NPV > 0 \rightarrow$ proposal is accepted

$NPV < 0 \rightarrow$ proposal is rejected

$NPV = 0 \rightarrow$ situation of indifference

Only positive will be accepted

Q) An investment of 136000 \$ generates the following cash inflow. Determine the NPV at the discount rate 10%.

Year	1	2	3	4	5
Cash inflow (\$)	30000	40000	60000	30000	20000

$$\begin{aligned}
 NPV(10\%) &= \frac{30000}{(1.1)^1} + \frac{40000}{(1.1)^2} + \frac{60000}{(1.1)^3} + \frac{30000}{(1.1)^4} + \\
 &\quad \frac{20000}{(1.1)^5} - 136000 = 2318.296 \text{ $}
 \end{aligned}$$

\downarrow
NPV > 0; accepted

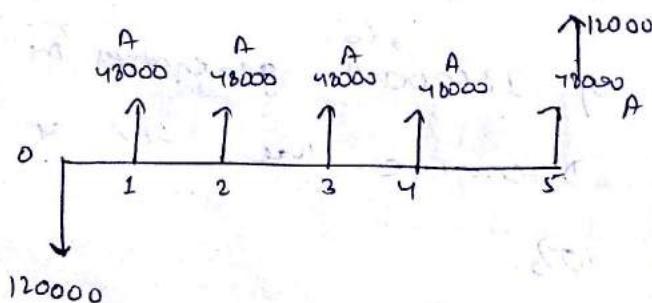
8) Your company is looking for purchasing a front-end loader at a cost of \$120000. The loader would have a useful life of 5 years with a salvage value of \$12000 at the end of 5th year. The loader can earn \$95 per hour. It costs \$30 per hour to operate and \$25 per hour is paid to the operator. Using 1200 billable hours per year determine the NPV for the purchase of at a MARR of 20%. Should your company purchase the loader?

17/02/17

Loader

$$\text{Net income per hour} = \$95 - \$30 - \$25 = \$40$$

$$1200 \text{ billable hours} = 1200 \times 40 = \$48000 \text{ per year.}$$



$$NPV(20\%) = -120000 + 48000(P/A_{in}) + 12000(P/F_{in})$$

$$= -120000 + 48000 \left[\frac{1.2^5 - 1}{1.2^5(0.2)} \right] + \frac{12000}{(1.2)^5}$$

$$NPV = \$26371.91$$

accepted ($NPV > 0$)

INTERNAL RATE OF RETURN (IRR) \Rightarrow

IRR speaks about the efficiency of the investment, quality of the investment and the yield of the investment. IRR is the discount rate where $NPV = 0$

- Q) find the IRR of the following cash flows of a project and state whether the project should be accepted if the company requires a minimum return of 17%. (MARR)

Time	0	1	2	3	4
Cash flow	4000	1200	1410	1875	1150



$$NPV = \frac{1200}{1.1} + \frac{1410}{(1.1)^2} + \frac{1875}{(1.1)^3} + \frac{1150}{(1.1)^4} - 4000$$

$$NPV = 2450.38 - 450.379 \quad (\text{at } 10\%)$$

$$NPV = -81.28 \quad (\text{at } 16\%)$$

$$\boxed{IRR = \text{Lower rate} + \frac{NPV_{lower}}{NPV_{lower} - NPV_{higher}} \times (High - Low)}$$

$$= 0.1 + \frac{450.379}{450.379 + 81.28} \times (0.06)$$

$$IRR = 15.08\% \quad (20.0)$$

21/01/17

- Q) An investment of \$136000 generates the following cash inflows. Determine the IRR.

Year	1	2	3	4	5
Cash inflows	\$30000	\$40000	\$60000	\$30000	\$20000

$$NPV = -\frac{30000}{n^1} + \frac{40000}{n^2} + \frac{60000}{n^3} + \frac{30000}{n^4} + \frac{20000}{n^5}$$

$$NPV(10\%) = 2318.29$$

$$NPV(12\%) = -4205.63$$

$$NPV(11\%) = -1005.63$$

K-smart

$$r = 0.1 + \frac{2318.29}{2318.29 + 4205.63} \times (0.02)$$

$$r = 10.7\%$$

- Q) Find the IRR of the following cash flows by trial & error method.

Year	0	1	2	3	4	5
Cash (\$) flows	\$1000	\$500	\$400	\$200	\$200	\$100

$$NPV(15\%) = \cancel{15\%} 32.811$$

$$NPV(20\%) = -53.178$$

$$r = 0.15 + \frac{32.811}{32.811 + 53.178} (0.05)$$

$$r = 16.9\%$$

(Answers)
Present

21/01/17

1) P/A

2) P/A

3) A/0

Q) Instal

is es

expected

value

windows

for 1st

3 year

used

(unconstrained)
Present Worth Method of Comparison (PWC method) \Rightarrow

+ve \rightarrow benefit / income / revenue / profit / salvage value

-ve \rightarrow cost / investment

22/02/17

1) P/F in

$$P = \frac{F}{(1+i)^n}$$

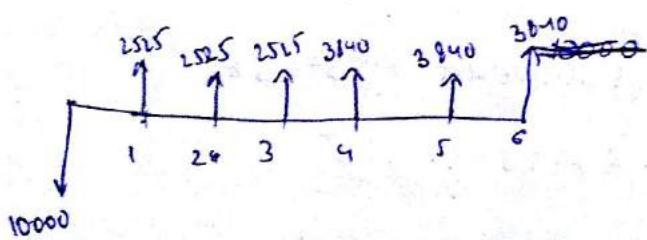
2) P/A in

$$P = A \left[\frac{(1+i)^n - 1}{i^n (1+i)} \right]$$

3) A/G in

$$A = A1 + G \left[\frac{(1+i)^n - i^n - 1}{i(1+i)^n - i} \right]$$

- Q) Installing thermal windows on a small office building is estimated to cost \$10000. The windows are expected to last for 6 years and have no salvage value at that time. The energy savings from the windows are expected to be \$2525 each year for 1st 3 years and \$3840 for each of the remaining 3 years. If the MARR is 15% and PW method is used; is this an attractive investment?



$$P = A \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= 2525 \left[\frac{1.15^3 - 1}{0.15(1.15)^3} \right] = 2525 \left[\frac{1.15^3 - 1}{0.15(1.15)^2} \right]$$

~~$\rightarrow 1319.66$~~

$$PV(15\%) = -10000 + 5765.14$$

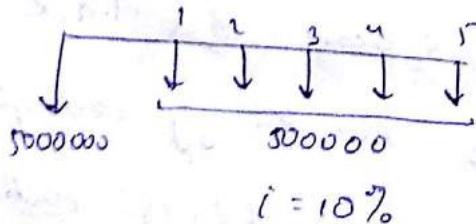
$$PV(15\%) = -10000 + \frac{2525}{1.15} + \frac{2525}{1.15^2} + \frac{2525}{1.15^3} + \frac{3840}{1.15^4} +$$

K-smart

$$\frac{3840}{1.15^5} + \frac{3840}{1.15^6}$$

$$PV(15\%) = 1529.972$$

Q)



$$P = 5000000 \left[\frac{1.1^5 - 1}{0.1(1.10)^5} \right]$$

$$P = \cancel{1517656.093} - 1895393.385$$

$$PV(10\%) = -5000000 - 1895393.385$$

$$= -6895393.385$$

8) Compare the PW of the following two mechanical plant auctions. Assuming an interest rate of 12% compounded annually. Particulars are given below:

Particulars	Auction A	Auction B
Initial cost	\$ 50000	\$ 70000
Annual maintenance cost	\$ 6000	\$ 4000
Residual value (Salvage)	\$ 8000	\$ 5000
Life	15 years	15 years

Auction A :

$$\begin{aligned}
 PW(12\%) &= -50000 - 6000 \left[\frac{1.12^{15} - 1}{0.12(1.12^{15})} \right] + \frac{8000}{1.12^{15}} \\
 &= -50000 - 40865.18 + 1461.57 \\
 &= -89403.61
 \end{aligned}$$

Auction B :

$$\begin{aligned}
 PW(12\%) &= -70000 - 4000 \left[\frac{1.12^{15} - 1}{0.12 \times 1.12^{15}} \right] + \frac{5000}{1.12^{15}} \\
 &= -70000 - 27243.457 + 913.48 \\
 PW(12\%) &= -96329.97
 \end{aligned}$$

28/02/17

Annual Worth Method of Comparison (AW method) \Rightarrow

In this method, we express all the cash flows in terms of annual worth i.e. in terms of

1) $A/P \text{ in}$

$$A = P \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

2) $A/F \text{ in}$

$$A = F \frac{i}{[(1+i)^n - 1]}$$

3) $A/G \text{ in}$



$$A = A_1 \pm G \left[\frac{(1+i)^n - 1}{i((1+i)^n - 1)} \right]$$

Q) A public utility company is trying to decide between 2 sizes of pipes for a new waterline. A 250 mm pipe will have an initial cost of \$40000 whereas a 300mm line will cost \$46000.

Since there is more head loss through the 250mm pipe, the pumping cost for the smaller line is expected to be \$2500 per year more than for the 300mm line. If the pipes are expected to last for 15 yrs?

which size should be selected at the rate of interest
12% per year, use an annual worth analysis?

$$AW(12\%) = \text{initial cost} = -40000$$

$$280\text{mm} \quad = -40000(A/P 12\% 15) = -21800$$

$$AW(12\%) = -46000 [A/P 12\% 15] = -6783.91$$

$$300\text{mm} \quad AW(12\%) = -5872.96 = +2500$$

$$250\text{mm} \quad = -3372.96 - 8372.96$$

Q) Find the AW of the following two methods and decide which method should be selected. Life of both the methods is 6 years and interest rate is 8% compounded annually.

Particulars	Amt of A	Amt. of B
Initial cost	160000	240000
Annual operating cost	60000	16000
Salvage value	30000	80000

$$AW(8\%) = -160000[A/P 8\% 6] - 60000 + 30000 \left[\frac{0.06}{1.06^6 - 1} \right]$$

$$(A) = -34610.46 - 64610.46 - 90520.99$$

$$AW(8\%) = -240000 \left[A/P \text{ in } \right] + 16000 + 80000 \left\{ \begin{array}{l} 0.08 \\ 1.08^6 \\ 1.08^7 \end{array} \right\}$$

(B)

$$= -12004.307 - 57010.4618$$

01/03/17

- Q) In the above question, life of A is 6 years and life of B is 7 years.

$$AW(8\%) = -90521.01$$

(A)

$$AW(8\%) = -240000 \left[\frac{0.08 \times 1.08^6}{1.08^7 - 1} \right] - 16000 +$$

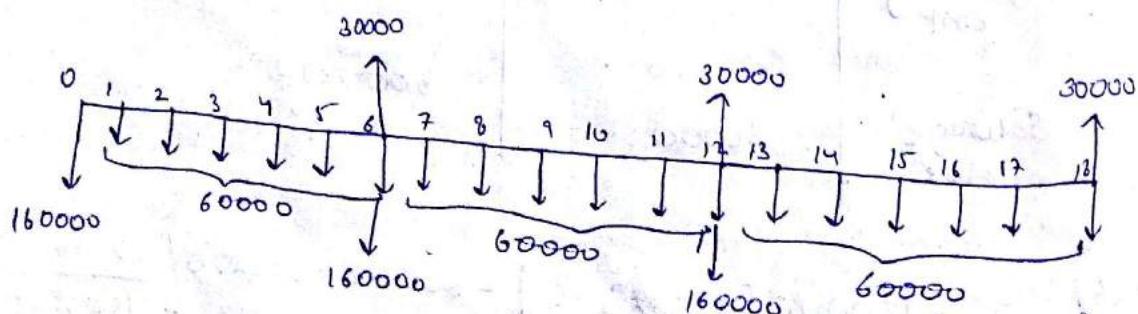
(S)

$80000 \left[\frac{0.08}{1.08^7 - 1} \right]$

$$= -48012.75$$

In case of 2 unequal years for both the methods, find the LCM of both years and calculate it accordingly.

Method A :

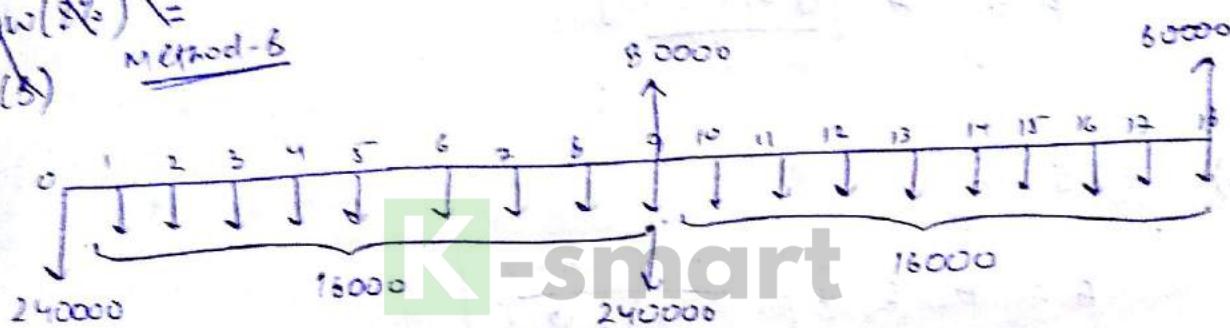


$$\begin{aligned}
 Aw(5\%) &= -60000 - 160000 \left[\frac{0.05 \times 1.05^{15}}{1.05^5 - 1} \right] - \frac{160000}{1.05^5} \left[\frac{0.05 \times 1.05^5}{1.05^5 - 1} \right] \\
 (a) &\quad - 160000 \left[\frac{0.05 \times 1.05^{15}}{1.05^{15} - 1} \right] + 30000 \left[\frac{0.05}{1.05^{15} - 1} \right]
 \end{aligned}$$

$\Delta = -171125.861$

$$\begin{aligned}
 &= -22022.335 - 8741.254 - 5506.423 + 301.062 \\
 &= -90521
 \end{aligned}$$

$$\begin{aligned}
 Aw(\%) &= \\
 (b) &\quad \text{Method-B}
 \end{aligned}$$



$$\begin{aligned}
 Aw(5\%) &= -16000 - 240000 \left[\frac{0.05 \times 1.05^{15}}{1.05^5 - 1} \right] - \frac{160000}{1.05^5} \left[\frac{0.05 \times 1.05^5}{1.05^{15} - 1} \right] \\
 (b) &\quad - \frac{160000}{1.05^9} \left[\frac{0.05 \times 1.05^{15}}{1.05^{15} - 1} \right] + 60000 \left[\frac{0.05}{1.05^{15} - 1} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= -41608.503 - 6540.418 - 8540.418 + 2136.168 \\
 &= -48012
 \end{aligned}$$

03/03/17

Future Worth Method (FW Method) :-

In this method we convert the cash flows to future time i.e. to F.

1) F/P in

$$F = P(1+i)^n$$

2) F/A in

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

3) A/G in

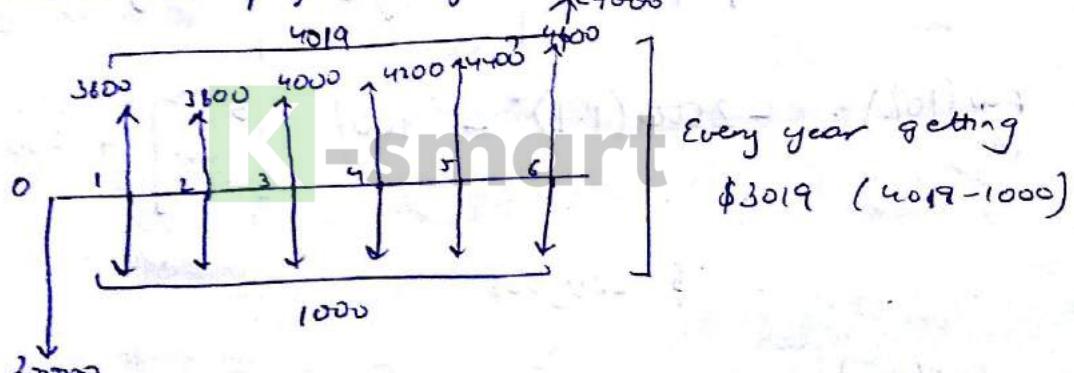
$$A = A_i \pm G \left[\frac{(1+i)^n - 1}{i} \right]$$

Q) A high efficiency lighting project for a company ~~will~~ saves \$10000 a year, in energy cost. If mat \$10000 a year is deposited into an energy management savings account that gives 10% interest. How much money will be available in 20 years to replace the old model with a new model?

$$F = A \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F = 10000 \left[\frac{1.1^{20} - 1}{0.1} \right] = \$572749.99$$

Q) You have agreed to make investment in your friend's agricultural farm. This would require an amount of \$ 20000 as initial investment on your part. Your friend promises you a revenue of \$ 3600 per year the 1st year and thereafter the revenue increases by \$ 200 per year. Your share of the estimated annual expenses is \$ 1000. You are planning to invest for 6 years. Your friend has made the commitment to buy out your share of the business at that time for \$ 24000. You have decided to set a personal MARR of 15% per year. Judge the profitability of the investment project by using FW method.



$$A = A_1 + G \left[\frac{(1+i)^n - 1}{i(1+i)^n - i} \right]$$

$$= 3600 + 200 \left[\frac{1.15^6 - 1}{0.15(1.15)^6 - 0.15} \right]$$

$$A = \$4019$$

$$FW(15\%) = -20000(F/P, 15\%, n) + 3019(F/A, 15\%, n) + 24000$$

$$FW(15\%) = \$4166.32$$

(more benefits than the cost, proposal will be accepted) \rightarrow since FW is +ve, it indicates that benefits are more than the cost paid.

Q) Consider the following information about machine A and machine B and decide which machine should be selected on the basis of the FAW method, given

$$i = 10\%$$

<u>Particulars</u>	Machine A	Machine B
<u>Op. Cost</u>	\$2500	\$3500
Annual operating G&T	\$700	\$700
<u>Saleage value</u>	\$200	\$350
Life period	5 yrs	5 yrs

$$\begin{aligned}
 \text{FAW}_{(A)} &= -2500(1+1)^5 - 700 \left[\frac{(1.1^5 - 1)}{0.1} \right] + 200 \\
 &= -\$9320.865
 \end{aligned}$$

$$\begin{aligned}
 \text{FAW}_{(B)} &= -3500(1+1)^5 - 700 \left[\frac{(1.1^5 - 1)}{0.1} \right] + 350 \\
 &= -\$9560.355
 \end{aligned}$$

Finding FAW, when the life of machine A and the life of machine B is 6 years.

Benefit - Cost Analysis

(10 million = 1 crore)

$$\text{B/C ratio} = \frac{\text{Equivalent Benefits}}{\text{Equivalent costs}}$$

(M)

if $B/C > 1$; accepted

$B/C < 1$; rejected

07/03/17

Q) A govt. bridge project requires an initial investment of £10 million, and operating maintenance cost of £250000 per year for 20 years life of the bridge. The annual user benefits of £2000000 per year are estimated to arise from savings in travel distance, time and fuel. If the interest rate is 7%, determine whether the needs to be implemented by using B/C ratio method.

$$AW = -10000000 \left[\frac{0.07 \times 1.07^{20}}{1.07^{20} - 1} \right] - 250000$$

(7%)
(cost)

$$= -1193929.257$$

$$AW \\ (7\%) = 2000000$$

Benefits

$$B/C \text{ ratio} = \frac{2000000}{1193929.257} = 1.675 > 1 \text{ (Accepted)}$$

$$\frac{P_{02}}{(1+i)} = 1000000 \left[\frac{(1+i)^{12}}{1-(1+i)^{-12}} \right] = 1000000 \left[\frac{1.07^{12}}{0.07} \right]$$

~~$\therefore P_0 = 148915717.71$~~

$$\frac{P_{02}}{(1+i)} = 6000000 \left[\frac{(1+i)^{12}}{1-(1+i)^{-12}} \right]$$

~~$\therefore P_0 = 819909846.71$~~

$i/c\ ratio = 1.675 > 1$ (accepted)

- D) Good Tyre company is considering the purchase of new tire balancing equipment. The machine cost \$12679 and have an annual savings of \$2000 with a salvage value of \$250 at the end of 12 years. If the MARR is 6%, decide the desired ability of purchasing the equipment by the ratio method.

$$\frac{P_{02}}{(1+i)} = 2000 \left[\frac{0.06 \times 1.06^{12}}{1.06^{12} - 1} \right] + \frac{250}{1.06^{12}} \left[\frac{1.06^{12} - 1}{0.06 \times 1.06^{12}} \right]$$

~~$\therefore P_0 = 287.6502$~~

~~$$2000 \left[\frac{1.06^{12}}{0.06 \times 1.06^{12}} \right] + 250 \left[\frac{1.06^{12} - 1}{0.06 \times 1.06^{12}} \right]$$~~

DEPRECIATION CALCULATION

15/03/17

Depreciation Calculation

Depreciation means fall in the value of the ^{fixed} asset, or, it means spreading the cost of the asset over its lifetime.

Methods of Depreciation Calculation :-

- ① Straight line method
- ② Sum-of-years digit method
- ③ Declining Balance method

Straight Line Method → **K-smart**

In this method, we charge a constant amount as depreciation at the end of every year.

$$d_k = \frac{C - S_{N_0}}{N} \quad (1 \leq k \leq N)$$

d_k → annual depreciation in the year k

C → cost basis

S_{N_0} → Salvage value ~~at the~~

N → life period

Q) The cost of a vehicle is ₹100000, the salvage value at the end of its life of 10 years is ₹20000, calculate annual depreciation and book value at the end of each year by straight line method.

$$\text{dR} = \frac{100000 - 20000}{10} = 20000$$

<u>Y</u>	<u>dR</u>	<u>Book value</u>
0	0	100000
1	20000	82000
2	20000	64000
3	20000	46000
4	20000	28000
5	20000	10000
6	20000	-12000
7	20000	-34000
8	20000	-56000
9	20000	-78000
10	20000	-100000

Sum of years digit method \rightarrow (SYD)

(1 ques.)

<u>Year</u>	<u>Years in reverse order</u>	<u>Depreciation factor (d.f.)</u>
1	5	5/15
2	4	4/15
3	3	3/15
4	2	2/15
5	1	1/15
		$\frac{1}{15}$

$$d = (P - F) \times d.f.$$

P \rightarrow Cost



F \rightarrow Salvage value

$$d.f. = \frac{n-t+1}{n(n+1)/2} \rightarrow \text{factor for any year}$$

n \rightarrow life period

t \rightarrow particular year for which d.f. has to be calculated

Q10/17

- Q) Cost of a machine is ₹100000. Salvage value at the end of 5 years is ₹20000. Find the depreciation for each year by SYD method.

Year Depreciation

$$1^{\text{st}} \quad (100000 - 20000) \times \frac{5}{15} \\ = 26,666$$

$$2^{\text{nd}} \quad (100000 - 20000) \times \frac{4}{15} \\ = 21333$$

$$3^{\text{rd}} \quad 80000 \times \frac{3}{15} = 16000$$

$$4^{\text{th}} \quad 80000 \times \frac{2}{15} = 10666$$

$$5^{\text{th}} \quad 5333$$

Total depreciation = 80000

Declining balance method :-

Q) An automobile company has purchased a wheel alignment device for ₹ 10 lakh. The device can be used for 10 years. The salvage value at the end of the life of the device is 10% of the purchase value. Calculate depreciation and book value of the device by declining balance method at depreciation rate of 20% per year.

<u>Year</u>	<u>Depreciation</u>	<u>Book value</u>
0	0	10 00000
1	$0.2 \times 100 = 20$	80 lakh
2	$0.2 \times 80 = 160$	640k
3	$0.2 \times 640 = 128$	512000
4	$0.2 \times 512000 = 102400$	409600
5	81920	327680
6	65536	262144
7	52428.6	209715.2
8	41943.04	167772.16
9	33554.432	134217.728
10	26843.5456	107374.16

Book value cannot be less than salvage value,

For a specific period, book value,

$$\boxed{D_t = k(1-k)^{t-1} \times P}$$

$$\boxed{B_t = k(1-k)^t \times P}$$

for 7th year

$$D_t = 52428.6$$

$$B_t = 209715.2$$

k = dep. rate

P = purchase val.

t = period.

o) A machine was purchased at the cost of £28000, and £1000 was spent on its insulation. Calculate the depreciation at 20% p.a. by declining balance method. The salvage value after a period of 5 years is £10000

$$k = 0.2$$

$$\frac{28000 + 1000}{28000} = 30000$$

<u>Years</u>	<u>Depreciation</u>	<u>Book value</u>
0	0	30000
1	6000	24000
2	4800	19200
3	3840	15360
4	3072	12288
5	2457.6	9830.4 less than the salvage value

change of calculation;

Now strike out 2457.6 and 9830.4 and subtract 5 values from previous years book value (2288), then find out depreciation accordingly.

Payback Period =

Payback period = initial investment

Annual benefit/annual cash inflows

Payback period reflects the time period by which the initial investment is recovered.

- (i) The initial investment in a project from ~~is~~
 & cash flow in \$ million for the first year
 (ii) In second year, there is 3% growth -
 in cash flow and revenue which is the 1st year cash
 flow projected growth in project

Year Initial Investment Projected Growth

1 1000 10%

2 1100 10%

3 1210 10%

4 1331 10%

5 1464 10%

K-smart
 K-Smart is a supermarket chain in India
 (located in ~~Delhi~~ ~~Mumbai~~)
 located in ~~Delhi~~ ~~Mumbai~~

- (i) The initial investment in a project in ~~Supermarket~~
 The annual cash inflows are summarized below
 for a period of 8 years.

Year	1	2	3	4	5	6	7	8
Annual	1000	1000	1000	1000	1000	1000	1000	1000
Recovery	1000	1000	1000	1000	1000	1000	1000	1000
Net Cash Flow	0	0	0	0	0	0	0	0

Final cash flow
 1000
 1000
 1000
 1000

<u>Year</u>	<u>cash inflow actual</u>	<u>needed</u>	<u>PBP</u>
1	400000	400000	1
2	500000	500000	1
3	300000	300000	1
4	400000	400000	1
5	600000	400000	$\frac{1}{12}$
6	700000	0	$\frac{1}{12} \times 8$ months
7	800000	0	$4 \text{ years } 8 \text{ months}$

Nominal v/s Effective Interest Rate \Rightarrow

Q) A person invests a sum of ₹5000 in a bank at a nominal rate of interest of 12%. The compounding is done quarterly. Find the maturity amount of the deposit after 10 years.

$$P = ₹5000$$

$$n = 10$$

$$i = 12\%$$

$$r = ?$$

F/P in \rightarrow required

Soln: no. of interest period per year = 4

revised no. of interest period = 40

interest rate per quarter = 3%

$$F = P(1+i)^n$$
$$= 5000 (1.03)^{10}$$

$$F = 16310.19$$

$$\text{Effective Rate of Interest, } R = \left(1 + \frac{i}{c}\right)^c - 1$$

c - no. of interest period per year.

$$R = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$R = 0.1235 = 12.35\% = 12.350881\% \text{ (exact)}$$

$$F = 5000 (1+R)^n$$

$$= 5000 (1.1235)^{10}$$

$$= 16308.91$$

$$F = 5000 \cdot (1 + 12.350881)^{10}$$

$$F = 16310.19$$

22/03/17

PRODUCTION

Production means change or transformation of physical inputs into physical output.

Production function is the technical or mathematical relationship between inputs and output.

$$\frac{TP}{Q} = \text{output}$$

(Total product)

L & K = inputs; where L is the variable factor
i.e. labour and K is the fixed factor i.e. capital (produced means
of production)

Two time periods are →

- 1) Short period / short run
- 2) Long period / long run

In the short period, the variable factors are made variable only but the fixed factors are constant.

In the long period, all factors are variable.

- 1) Short run production function
- 2) Long run production function

GPP (Short run production function) $\Rightarrow Q = f(L, K)$

LPP (Long run production function) $\Rightarrow Q = f(K, L)$

Short Run Production Function:-

It means the study of the technical relationship between inputs and output for the short run i.e. when one is fixed and other is variable.

Law of Variable Proportion: Variable proportion indicates the change in the factor proportions i.e. when K is constant in the short run and L is made variable. Law of variable proportion explains the return to the change in the factor proportions or, this law explains the return to **K-mart**

Typical Production Function:-

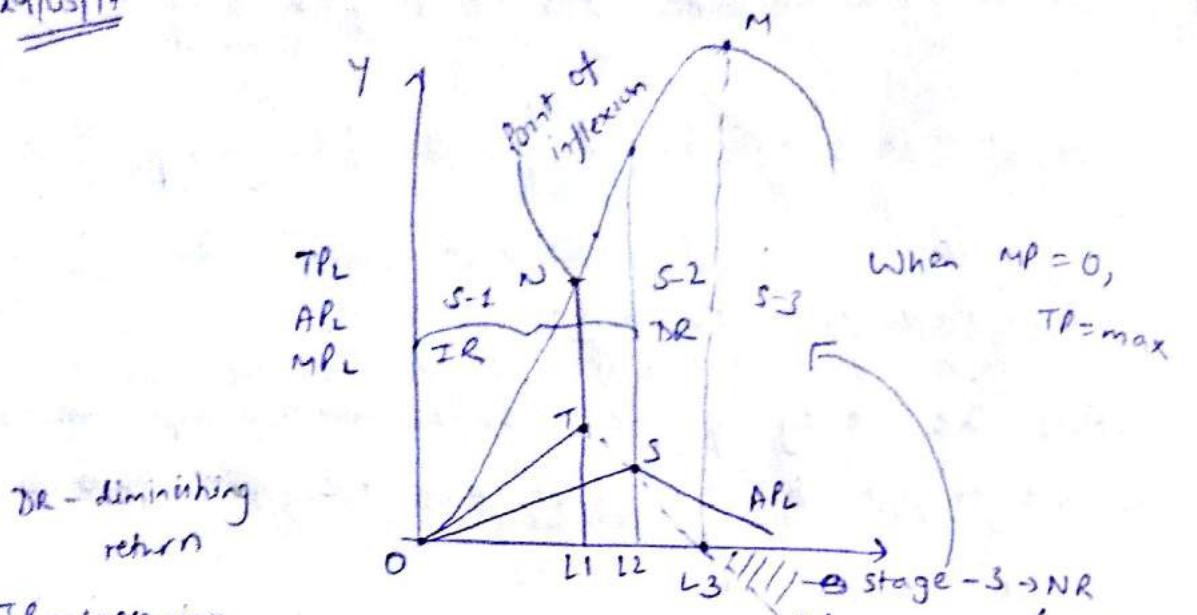
$$1) TP_L (\text{Total product}) / \partial L = AP_L \propto L$$

$$2) AP_L (\text{Average product}) = \frac{Q}{L}$$

$$3) MP_L (\text{Marginal product}) = TP_L - TP_{L-1} = \frac{\partial Q}{\partial L} \quad [MP_L = \frac{\partial Q}{\partial K}]$$

As equal increments of one input are added, the inputs of other productive services being held constant beyond a certain point the resulting increase in output of the product will decrease i.e.

24/03/17

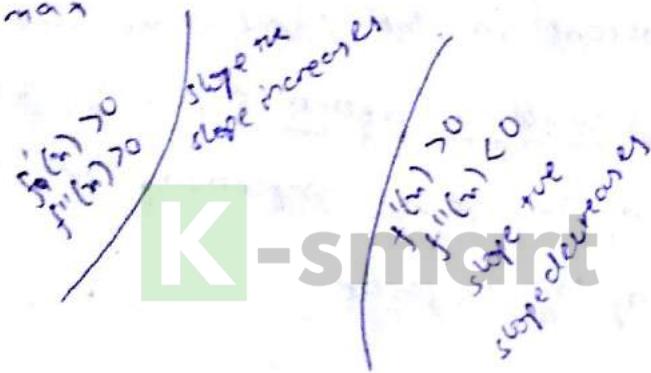


At T, MP \rightarrow max

At S \rightarrow AP_L \rightarrow max

$$y = f(x)$$

(VF - variable factor)



<u>Q</u>	<u>MP</u>	<u>Q</u>	<u>MP</u>
20	—	20	—
28	8	25	5
35	7	31	6
40	5	36	7
44	4	46	9
46	2	55	9
47	1		

As we go on applying more and more columns of the variable factors keeping the fixed factor constant after sometime, first the marginal product and then the

Average product of the factor will diminish.

Relationship between AP_L and MP_L \Rightarrow

When AP_L rises $\rightarrow MP_L > AP_L$

When AP_L falling $\rightarrow MP_L < AP_L$

When AP_L is max $\rightarrow MP_L = AP_L$

$$Q = AP_L \times L$$

$$MP_L = \frac{dQ}{dL}$$

$$\Rightarrow MP_L = \frac{L \cdot dAP_L}{dL} + AP_L$$

(i) When AP_L is rising $\Rightarrow \frac{dAP_L}{dL} > 0 ; MP_L > AP_L$

ii) When AP_L is falling $\Rightarrow -\frac{dAP_L}{dL} < 0 ; MP_L < AP_L$

iii) When AP_L is max $\Rightarrow \frac{dAP_L}{dL} = 0 ; MP_L = AP_L$

Q) Complete the following table:

No. of factor employment	TP	AP	MP
0	0	0	0
1	4	4	4
2	10	5	6
3	18	6	8
4	24	6	6
5	25	5	1

Q) No. of factor employment

	TP	MP	AP
0	0	0	0
1	225	225	225
2	525	262.5	300
3	900	300	375
4	1140	285	240
5	1125	225	225
6			225

Q) No. of factor employment

	TP	MP	AP
0	0	0	0
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	-15	225

Q) A ce
L and
of 72
follow

Assume

(i) find the no.

(ii) find the a

b) A certain production process employs 2 inputs L and R (raw material). Output (Q) is a function of these 2 inputs and is given by the following relationship $Q = 6L^2K^2 - 0.10L^3R^3$.

Assume that the raw materials are fixed at 10 units.

- (i) find the no. of units of input L that maximizes the total product function

$$Q = 600L^2 - 100L^3 \quad (R=10)$$

$$\frac{dQ}{dL} = 0 = Q_{\max}$$

$$\frac{dQ}{dL} = 1200L - 300L^2 = 0$$

$$\Rightarrow L = 4$$

- (ii) Find the no. of units of input L that maximizes the average product function.

$$Q = 600L^2 - 100L^3$$

$$\frac{Q}{L} = 600L - 100L^2$$

$\therefore AP_L$ is max

$$\frac{dAP_L}{dL} = 0$$

$$600 - 200L = 0 \Rightarrow L = 3$$

$$1200L - 300L^2 = 600L - 100L^2 \quad [AP_L = MP_L]$$

$$600L - 200L^2 = 0$$

$$L = 3$$

(iii) find the units of output & value added

$$val = \frac{dQ}{dL} = 200L + 5000$$

$$\frac{dQ}{dL} = 200 \text{ (value added)}$$

$$val = 200L \Rightarrow$$

$$\Rightarrow L = 2$$

8) Given the short run production function as $TR = 10L - L^2$,
find the no. of laborers beyond which $MP = 0$

$$\frac{dTR}{dL} = 10 - 2L = 0 \quad (MP = 0)$$

$$L = 5$$

Cobb-Douglas Production Function =

Functional form :
$$Y = A L^\alpha K^\beta$$

23/3/17 Q = Output

L and K = Labor and Capital respectively.

α and β = Output elasticities of labor and capital.

A is a constant and is considered as the total factor productivity.

$\rightarrow \alpha + \beta > 1$; so it is increasing return to scale

$\rightarrow \alpha + \beta < 1$; so it is diminishing return to scale

$\alpha + \beta = 1$; so it is constant return to scale

Q) Verify the return to scale from the following production function; $Q = 5K^{0.3}L^{0.7}$

$$L = 2, K = 3$$

$$\Rightarrow Q = 11.293$$

$$L = 4, K = 6$$

$$\Rightarrow Q = 22.586$$

On doubling L & K, Q gets doubled

Q) $Q = 5K + 7L + KL$, what is the return to scale?

$$\text{let } K = 2, L = 4$$

$$Q = 46$$

$$\text{let } K = 4, L = 8$$

$$Q = 108$$

more than double so it is increasing return to scale.

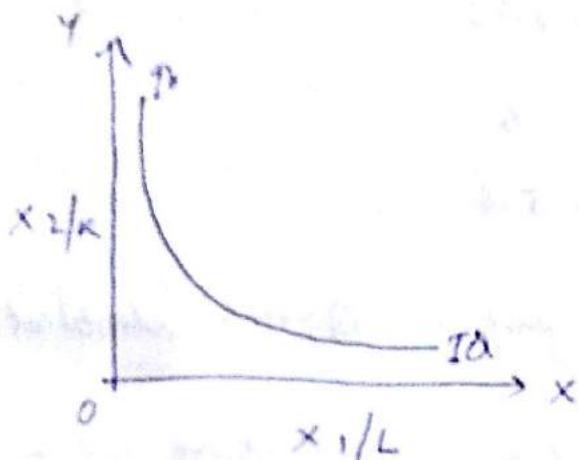
PRODUCER'S EQUILIBRIUM =)

ISO-QUANT / EQUAL PRODUCT CURVE

- Indifference curve is used in case of consumer equilibrium. While iso-quant is used in case of producer equilibrium.

- In case of indifference curve, we take 2 goods but in case of iso-quant we take two factors of production i.e. labour and capital.

Iso-quants are loci of combination of two factors of production producing the same level of output, so a particular iso-quant is meant for a particular level of output.



$$q_0 = f(x_1, x_2)$$

Properties:

K-smart

i) Iso-quant is negatively sloped.

$$dq_0 = 0$$

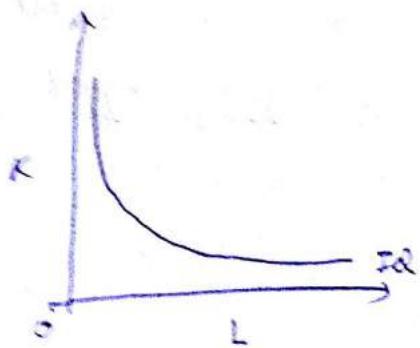
$$\left(\frac{\partial q_0}{\partial x_1} \right)_{x_2} dx_1 + \left(\frac{\partial q_0}{\partial x_2} \right)_{x_1} dx_2 = 0$$

MRS

$$\Rightarrow M_{R_1} dx_1 + M_{R_2} dx_2 = 0$$

$$\therefore \frac{dx_2}{dx_1} = - \left(\frac{M_{R_1}}{M_{R_2}} \right)$$

$\frac{dx_2}{dx_1}$ represents the slope of the iso-quant and is known as MRS (Marginal Rate of Substitution). Technical



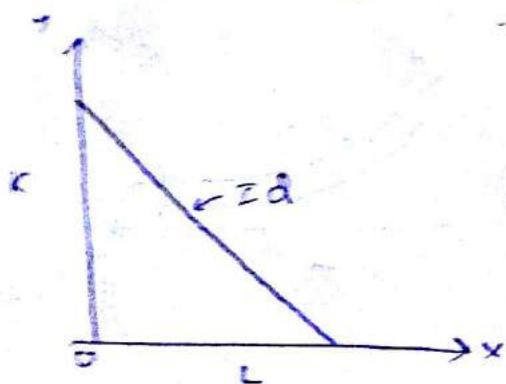
$$MRTS_{LK} = \frac{\Delta K}{\Delta L} = \frac{dK}{dL}$$

Ans 12

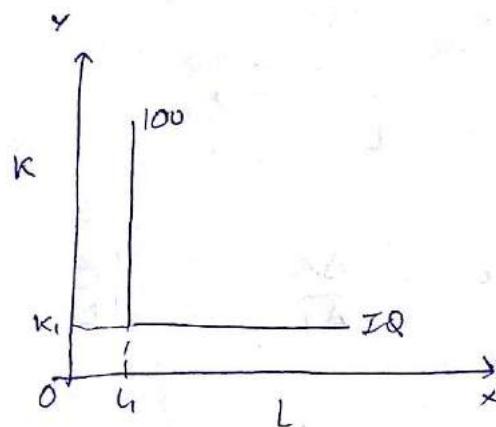
iso-quant is convex to the origin.



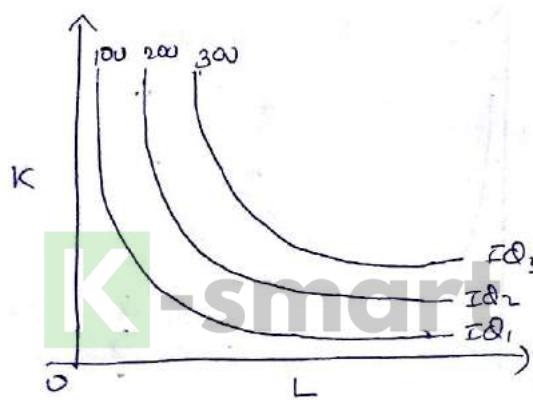
- Iso-quant is convex to the origin because of DMTS (Diminishing Marginal Rate of Technical Substitution) i.e. slope is decreasing. This is so because two factors of production are not perfect substitutes. If the factors of production are perfect substitutes then the shape of the iso-quant would be straight line.



If the 2 factors of production are perfect complements, the iso-quant would be right-angled.

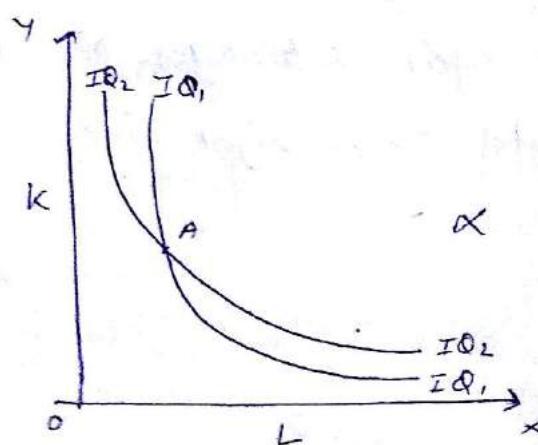


3) A higher iso-quant gives a higher level of output

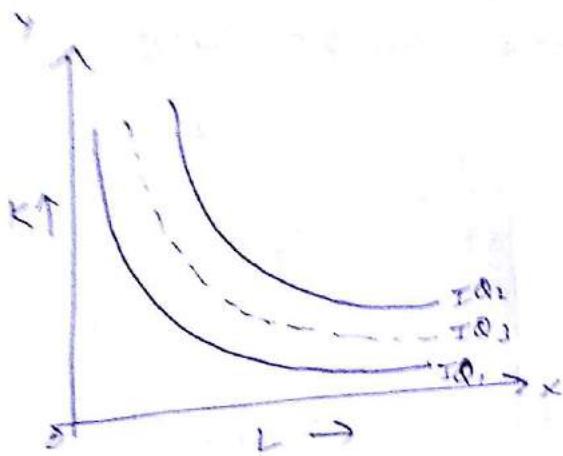


$$IQ_3 > IQ_2 > IQ_1$$

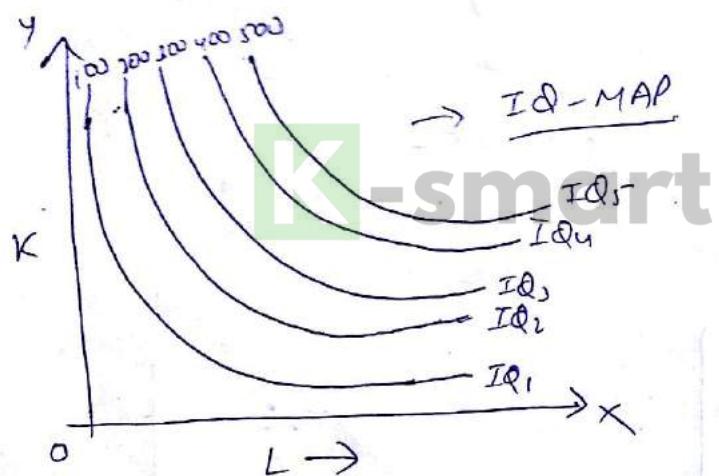
4) No two iso-quant can either touch or intersect each other.



3) In between ~~they~~ drawn, because they cannot touch or intersect each other.



4) A family of iso-quants is known as an iso-quant map.



④ Producer equilibrium (to be taught)

Cost of Production \Rightarrow

Cost functions are the derived functions and they are derived from the production functions.

$$C = f(Q)$$

C/TC - total cost

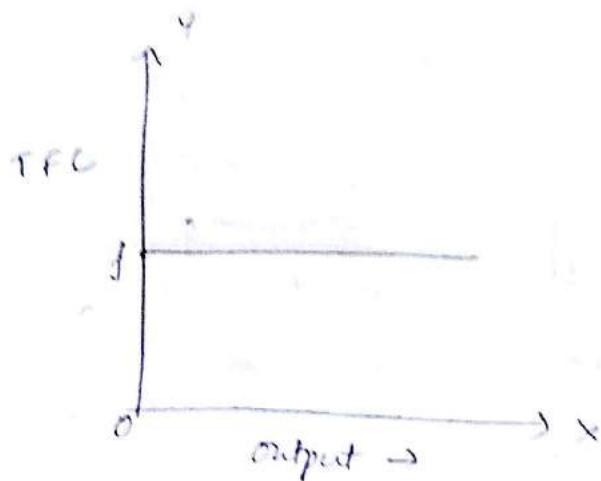
①

$$TC = TFC + TVC$$

TFC = Total fixed cost

TVC = Total variable cost

i) TFC

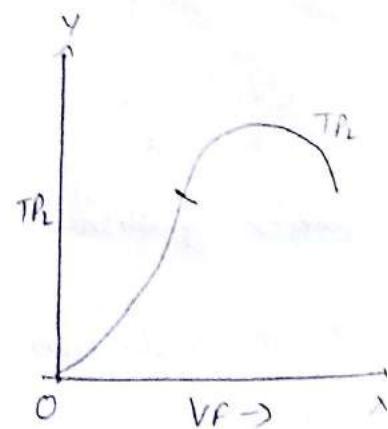
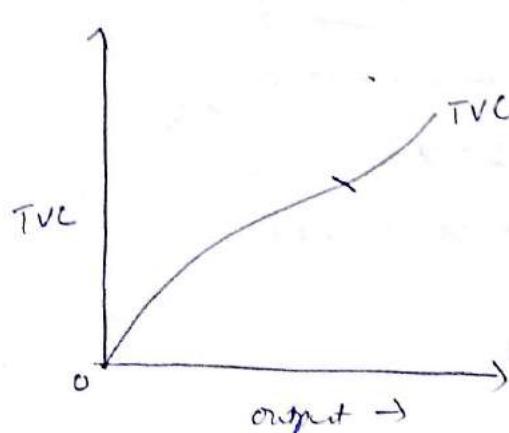


$$\delta = 0; TFC = 0f$$

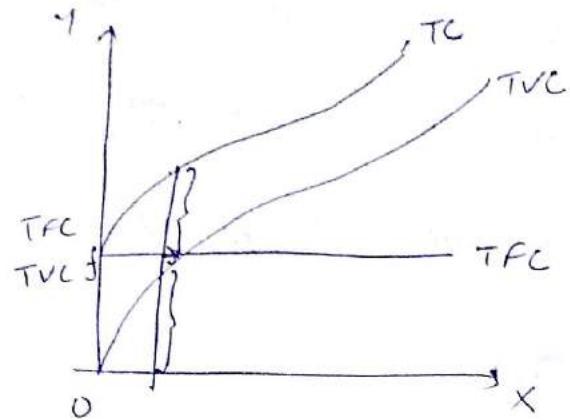
$$TVC = 0$$

K-smart

ii) TVC



When $\delta = 0$ then $TVC = 0$ (proved above) so the TVC curves start from origin.



TC and TVC are parallel to each other at all levels of output because ~~their~~ vertical distance represents $\begin{bmatrix} TC = TFC + TVC \\ TC - TVC = TFC \end{bmatrix}$ TFC, which is constant throughout.

② AC (Average cost) or (Average Total cost (ATC))

K-smart

$$AC = \frac{TC}{Q} = \frac{TVC + TFC}{Q}$$

$$AC = \frac{TVC}{Q} + \frac{TFC}{Q}$$

$$AC = AVC + AFC$$

i) $AVC = \frac{TVC}{Q}$

$$TVC = L \cdot w$$

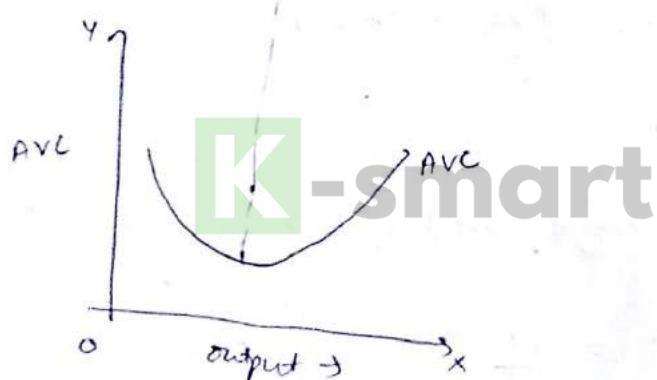
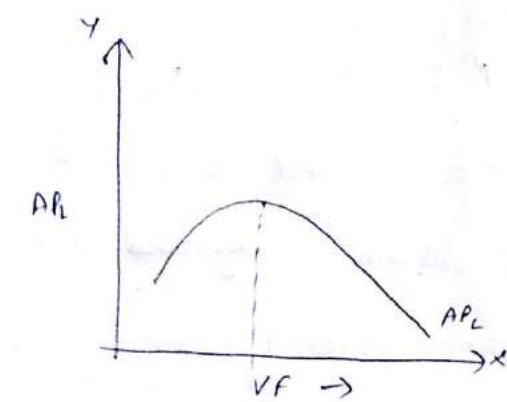
L = variable factor labour used

w = factor price

$$Q = AP_L \times L$$

$$AVC = \frac{L \cdot w}{AP_L \times L} = w \left(\frac{1}{AP_L} \right)$$

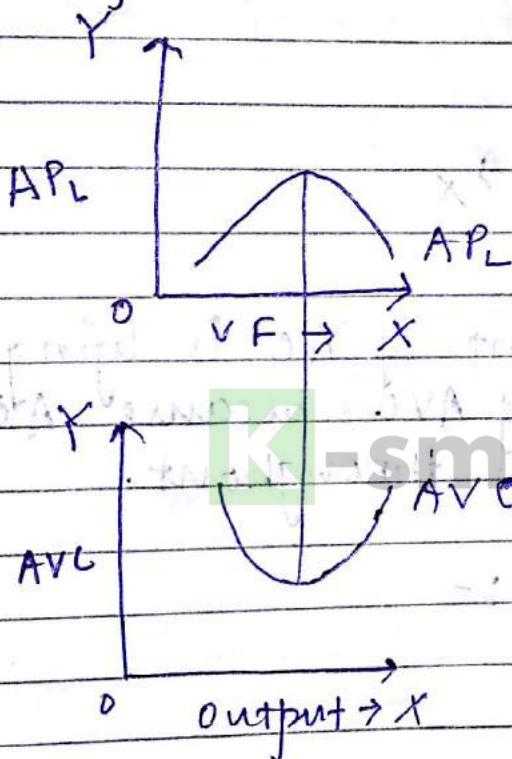
- 2) Therefore, AVC curve looks like the AP curve,
turned upside down with a minimum point of AVC ,
corresponding to maximum point of AP .



$$Q = AP_L \times L$$

$$AVC = \frac{L \times w}{AP_L \times L} = w \left(\frac{1}{AP_L} \right).$$

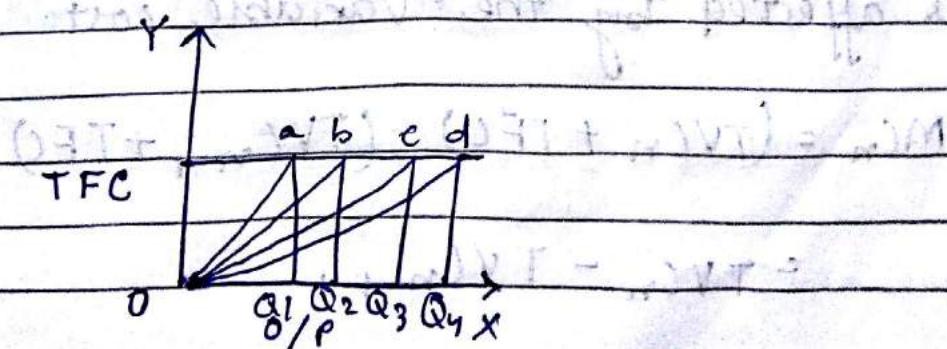
AVC curve looks like the AP_L curve when turned upside down with the minimum point of AVC corresponding to the max. point of AP.

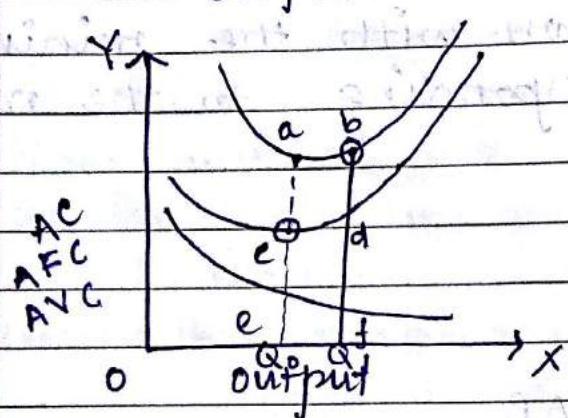
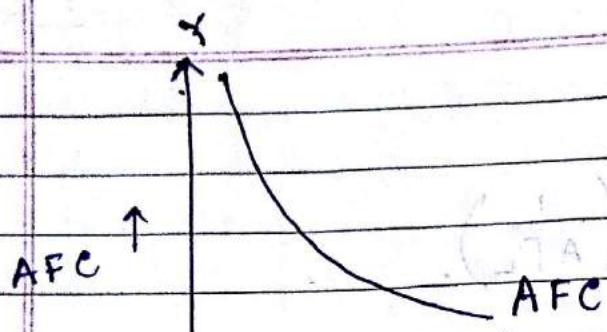


AFC :-

$$AFC = \frac{TFC}{Q}$$

AFC can be drawn from the slope of the lines drawn from the origin to TFC.





The minimum point of AC is lying to the right of minimum point of AVC because AC include AFC that decreases throughout.

③ MC (marginal cost) :-

$$MC = \frac{\Delta C}{\Delta Q}$$

$$MC = \frac{dC}{dQ}$$

$$MC_n = TC_n - TC_{n-1}$$

MC is not affected by the fixed cost rather it is affected by the variable cost.

$$\begin{aligned} MC_n &= (TVC_n + TFC) - (TVC_{n-1} + TFC) \\ &= TVC_n - TVC_{n-1} \end{aligned}$$

$$MC = \frac{\Delta TVC}{\Delta Q}$$

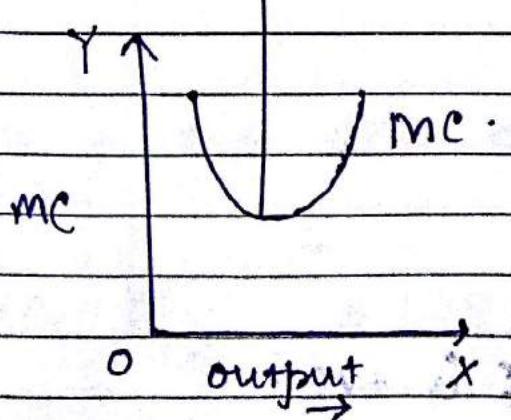
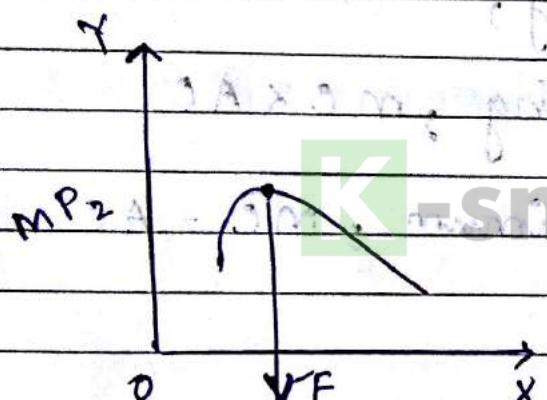
$$MC = \frac{d TVC}{d Q}$$

$$MC = \frac{\Delta TVC}{\Delta Q}$$

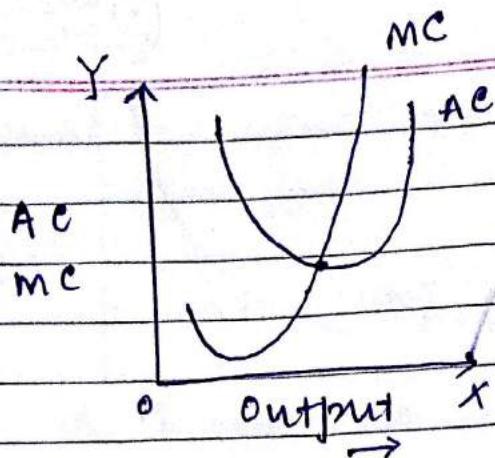
$$MC = \frac{\Delta L \cdot w}{\Delta Q}$$

$$MC = w \left(\frac{\Delta L}{\Delta Q} \right)$$

$$MC = w \left(\frac{1}{MP_L} \right) \quad \{ MP_L = \frac{\Delta Q}{\Delta L} \}$$



* Relationship between AC and MC :-



MC curve cuts the AC curve at its minimum point because MP curve cuts the AP curve at its maximum point.

- when AC is rising, $MC > AC$
- when AC is falling, $MC < AC$
- when AC is minimum, $MC = AC$

$$AC = \frac{C}{Q}$$

$$\therefore C = AC \cdot Q$$

$$MC = \frac{dC}{dQ}$$

$$MC = Q \cdot \frac{dAC}{dQ} + AC$$

i) slope is rising, $\frac{dAC}{dQ}$ is +ve. $\therefore 0$

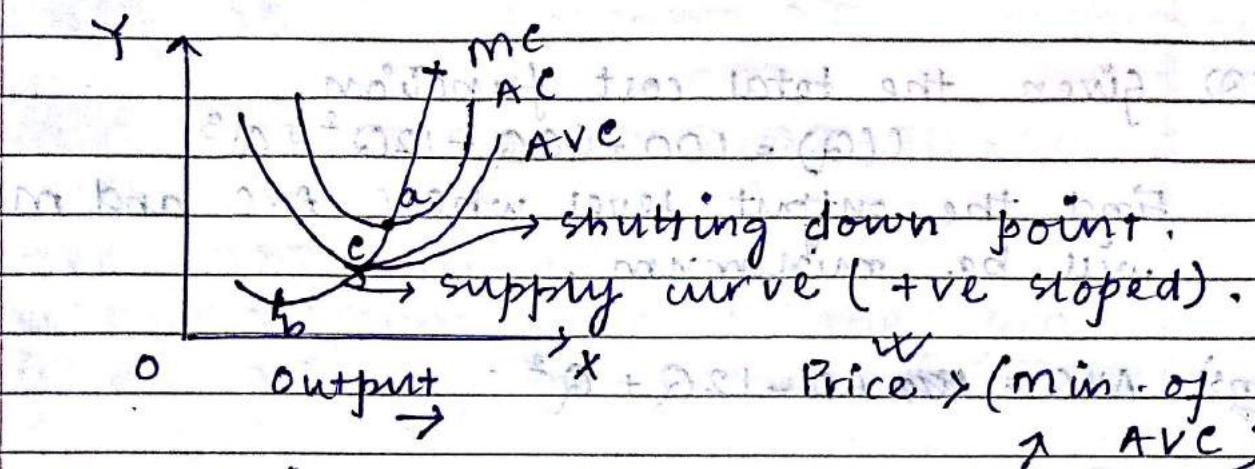
$$MC > AC$$

ii) AC is falling, $\frac{dAC}{dQ}$ is -ve.

$$MC < AC$$

iii) AC is minimum; $\frac{dAC}{dQ} = 0 \therefore MC = AC$

Derivation of short-run supply curve from the mc curve:



b to c → isn't considered as supply curve

(Q) Given the cost function $C = 100 + 10Q + Q^2$

a) What is the AFC of producing 2 units of O/P?

$$(A) a) AFC = \frac{100}{2} = 50.$$

b) what is the AVC of producing 2 units of O/P?

$$b) AVC = \frac{\partial TC}{\partial Q} = 10Q + Q^2 = 10 + Q = 12.$$

c) what is MC of producing 2 unit of O/P?

$$c) MC = \frac{\partial VC}{\partial Q} = 10 + 2Q = 14.$$

d) what is the AC?

Ans) $AC = \frac{C}{Q} = \frac{100 + 10Q + Q^2}{Q}$

$$= 50 + 10 + 2 = 62$$

(Q) Given the total cost function,

$$C(Q) = 100 + 60Q - 12Q^2 + Q^3$$

Find the output level where AVC and MC will be minimum.

Ans) $AVC = 60 - 12Q + Q^2$

$$\frac{d(AVC)}{dQ} = 0$$

$$\Rightarrow -12 + 2Q = 0$$

$$\Rightarrow Q = 6$$

$MC = \frac{dTVC}{dQ}$

$$dQ$$

$$= 60 - 24Q + 3Q^2$$

$$\frac{d(MC)}{dQ} = 0$$

$$\Rightarrow -24 + 6Q = 0$$

$$\Rightarrow Q = 4$$

(Q) Given $TVC = 150Q - 20Q^2 + Q^3$,

Determine below what price the firm would shut down production.

Ans) $AVC = 150 - 20Q + Q^2$. two constraint (a)

$$\frac{d(AVC)}{dQ} = 0$$

$$\Rightarrow -20 + 2Q = 0$$

$$\Rightarrow Q = 10$$

$$AVC = 150 - 200 + 100 = 100 - 50 = 50$$

(G) $TVC = 75Q - 10Q^2 + Q^3$ acs

will the farm produce the product if the price of the product is £40?

Ans) $AVC = 75 - 10Q + 3Q^2$ DA

~~$$\frac{d(AVC)}{dQ} = 0$$~~

$$\frac{d(AVC)}{dQ} = 0$$

$$\Rightarrow -10 + 6Q = 0$$

$$\Rightarrow Q = \frac{10}{6}$$

$$\Rightarrow -10Q + 2Q = 0$$

$$\Rightarrow Q = 5$$

$$AVC = 75 - 100 + 3(100)$$

$$= £66.6667$$

$$AVC = 75 - 110(5) + 25 \times 5 = 25$$

$$= 50$$

No.

(Q) The total cost and the level of output of a company is given below. Find TVC , AVC , AFC , AC and MC if the TFC is ₹ 100.

<u>Quantity (Q)</u>	<u>TC (Total Cost)</u>	<u>$TFC = ₹ 100$</u>	<u>TVC</u>
1	130	100	30
2	160	100	60
3	190	100	90
4	220	100	120
5	250	100	150
6	280	100	180

<u>Ans</u>	<u>AVC</u>	<u>AFC</u>	<u>AC</u>	<u>MC</u>
	30	100	130	30
	30	50	80	30
	30	33.33	63.33	30
	30	25	55	30
	30	20	50	30
	30	16.67	46.67	30

(Q) When a farm produces 200 units the total cost of production is ₹ 4000. When the farm increases the output to 220 the total cost rises to ₹ 4900. When the farm produces 0 output the cost is ₹ 1000. What is the fixed cost per unit? when the farm produces 200 units?

Ans) $TFC = 1000$

$$AFC = \frac{1000}{200} = ₹ 5$$

(Q) Identify the different phases about the behaviour of output implied in the law of variable proportion from the following information.

<u>Units of variable inputs</u>	<u>marginal Product (MP_L)</u>	
1	20	
2	25	→ IR (Increasing Rate)
3	26	of Return
4	20	
5	17	→ Decreasing Return
6	-10	→ Negative Return

BREAK EVEN POINT :- (BEP).

It is a point where $TR = TC$.

$$SP - AVC = CM$$

SP = Selling Price

AVC = Average Variable Cost

cm = contribution margin

Fixed cost £ 100,000

$$\frac{100,000}{5} = 20,000$$

$$SP - AVC = CM$$

$$20 - 15 = 5$$

$$TC = TFC + TVC$$

$$= 100,000 + 15 \times 20,000$$

$$= 240,000$$

$$TR = P \cdot Q$$

$$= 20 \times 20,000$$

$$= 240,000$$

20,000

$$\text{Total contribution} = 5 \times 20,000$$

$$= ₹ 100,000.$$

At BEP,

$$TR = TC$$

Total contribution = fixed cost

BEP is a no profit no loss point. It is an equilibrium point.

It is a critical point.

It is a balancing point.

The producer wants to reach at the BEP as quick as possible so that he can earn profit.

PROFIT VOLUME RATIO (P/V ratio):

$$P/V = \frac{CM}{Sales} = \frac{\text{Total contribution}}{\text{Total sales.}}$$

$$\text{P/V ratio} = \frac{5}{20} = \frac{1}{4} = 25\%$$

(For every rupee of sales, the producer is getting 25 paise for coverage of fixed cost).

P/V ratio indicates the contribution of every rupee towards the coverage of fixed cost.

MARGINAL EQUATION :-

$$\boxed{\text{Sales} - \text{VC} = \text{FC} + \text{P}}$$

P = Profit

VC = Variable Cost

$$20000 \times 20$$

$$= 400000$$

$$400000 - 300000$$

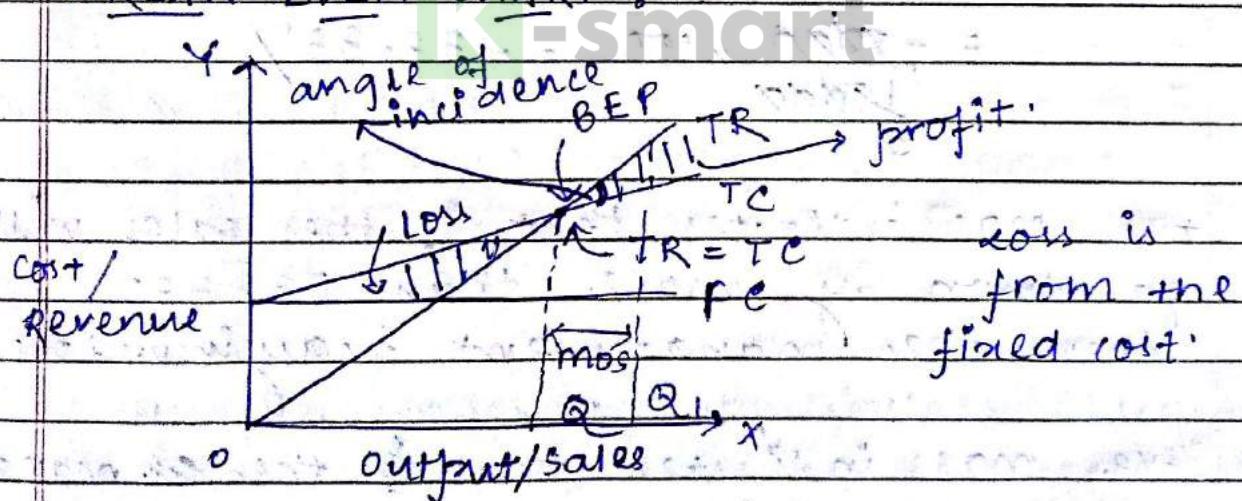
$$= 100000$$

$$15 \times 20000$$

$$= 300000$$

Before break even point, you lose (loss).
After BEP, you gain (profit).

BREAK EVEN CHART :-



Greater the angle of incidence, more is the profit.

Gap between BEP and new quantity is
mos

margin of safety (mos) is the excess of actual sales over BE sales (break even sales)

MOS = Actual sales - Break even sales

$$= 30,000 - 20,000 = 10,000 \text{ unit}$$

$$\text{MOS\%} = \frac{\text{As} - \text{BES}}{\text{As}} \times 100$$

$$= \frac{30,000 - 20,000}{30,000} \times 100$$

$$= 33.3\%$$

$$\text{MOS\%} = \frac{15,000 - 20,000}{15,000} \times 100$$

$$= \frac{-5000}{15000} \times 100 = -33.33\%$$

* +ve MOS indicates that if the sales will go down by more than 33.33% then the management may incur loss.

* -ve MOS indicates that the management should improve the sales by atleast 33.33% to avoid loss.

$$\text{Break even Quantity (Q}_B) = \frac{\text{FC}}{\text{CM per unit}}$$

$$\text{Break even sales (S}_B) = Q_B \times \text{SP. value}$$

$$\frac{TFC}{P/V \text{ ratio}} = S_B$$

$$\frac{P/V}{SP} = \frac{CM}{SP} = \frac{SP - AVC}{SP}$$

$$S_B = \frac{TFC}{SP - AVC} \times SP$$

Q_B

(Q) From the following information find out the amount of profit earned during the year.

$$FC = ₹ 2,50,000$$

$$VC = ₹ 10 \text{ per unit} = AVC$$

$$SP = ₹ 15 \text{ per unit}$$

$$Q = 75,000 \text{ units}$$

K-Smart

$$\text{Ans) } S - VC = FC \pm P$$

$$\Rightarrow 75,000 \times 15 - 10 \times 75,000 = 2,50,000 \pm P$$

$$\Rightarrow 3,75,000 = 2,50,000 \pm P$$

$$\Rightarrow P = ₹ 1,25,000$$

(Q) From the following particulars calculate

- ① BEP in terms of units and sales value.
- ② Sales required to earn profit of ₹ 90,000.

$$FC = ₹ 72,000$$

$$VC/\text{unit} = AVC = ₹ 15$$

$$SP \text{ per unit} = ₹ 24$$

$$\text{Ans) } Q_B = \frac{FC}{CM \text{ per unit}} = \frac{72,000}{24 - 15} = 8000$$

$$S_B = Q_B \times SP = ₹ 1,92,000$$

(2) $S - VC = FC + P$

$$\Rightarrow S - 15 \times 8000 = 72,000 + 90,000$$

$$\Rightarrow S = 22,82,000$$

since the profit will be zero for Q_B so we cannot use it to calculate VC and thus S .

$$\text{sales required} = \frac{FC + \text{desired profit}}{\text{Contribution per unit}}$$

$$= 18000$$

(Q) A farm has the volume of sales ₹ 40,0000000 and $S_B = ₹ 25,0000000$

$$\text{Ans) MOS} = \frac{15 \times 100}{40} = 37.5\%$$

If the sales goes down by more than 37.5%, then the management will incur loss.

(Q) Consider the following data of a company:

$$\text{Sales} = ₹ 1,20,000$$

$$FC = ₹ 25,000$$

$$VC = ₹ 45,000$$

i) Find contribution = sales - VC
 $= ₹ 75,000$

ii) Profit = ₹ 75,000 - ₹ 25000 (contribution - FC),
 $= ₹ 50,000$

After FC is covered, we will get the profit

iii) BEP :-

$\frac{P/V \text{ ratio}}{\text{Contribution}} = \frac{75,000}{\text{Sales}} \times 100 = 62.5\%$

BEP = $\frac{25000}{0.625} = ₹ 40,000$

$\left\{ \begin{array}{l} \text{BEP} = \frac{\text{FC}}{\text{P/V ratio}} \\ \end{array} \right\}$

iv) MOS :

$$\begin{aligned} \text{MOS} &= AS - BEP \\ &= 120000 - 40000 \\ &= 80000. \end{aligned}$$

MOS = $\frac{\text{Profit}}{\text{P/V ratio}} = \frac{₹ 50,000}{0.625} = ₹ 80,000.$

(Q) Consider the following data

Sales = ₹ 80,000

FC = ₹ 15,000

VC = ₹ 35,000

Find contribution, profit, BEP, MOS

Ans) contribution = ₹ 45,000

$$\text{ii) Profit} = \bar{x} - 30,000$$

$$\text{vii) BEP} = \frac{FC}{P/V \text{ ratio}} \quad P/V \text{ ratio} = \frac{\text{U}5,000}{8.0,000} = 0.625$$

$$= \frac{\$15,000}{0.5625}$$

$$= \text{£} 26666.67$$

$$w) \text{ MOS} = \frac{\text{Profit}}{\text{Cost}}$$

P/V ratio

$$= \frac{\$30,000}{0.5625}$$

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OPTIMUM INPUT COMBINATION :-

(AND PRODUCER'S EQUILIBRIUM).

- (Q) Explain the producer's equilibrium with the help of isoquants.

$$\text{Slope of isoquant} = MRTS_{LK} = \frac{MP_L}{MP_K}$$

An important problem faced by the producer is to decide about the particular combination of factors to be employed to produce a product. Although there are various combinations of factors (isoquants) which can yield a given level of output but the problem is the producer has to choose one combination of factor from these combinations.

A profit maximizing producer will want to minimize the cost or maximize the output for a given level of cost. However the choice of a particular combination of factors by the entrepreneur always depends on:

- (i) Technical possibilities of production i.e., isoquant map.
- (ii) Prices of the factors (labor and capital) used to produce a particular product

ISO-COST LINE:

iso-cost line shows various combinations of two factors that the firm can buy with a given level of outlay (cost). For example,

$$C = 300 \text{ (cost)} \quad \text{wage:}$$

$$\text{Labour price} = \frac{C}{L} \cdot (w = 4)$$

$$\text{Capital price} = \frac{C}{K} \cdot (r = 5) \quad \begin{matrix} \nearrow \\ \text{rent.} \end{matrix}$$

L = Labour Unit

w = wage

Then labour payment = $L \times w$.

capital ~~price~~ = K

r = Rent

Then capital payment = $K \times r$.

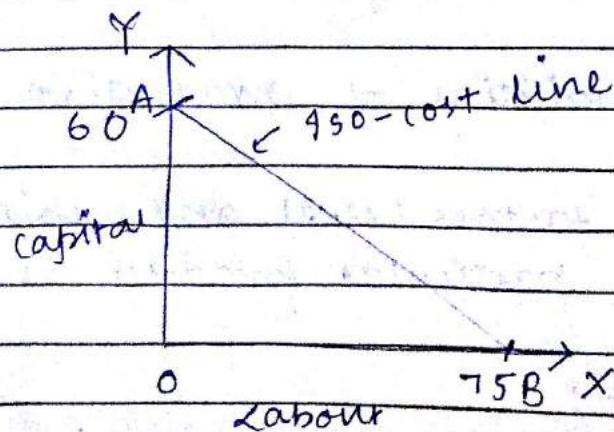
$$C = wL + rK$$



labour units used = $\frac{300}{4}$ (without capital)

$$= 75 \underset{r}{\cancel{1}} = C$$

capital units used = $\frac{300}{5} = 60 \underset{w}{\cancel{K}} = C$



$$OA = \frac{C}{r} \text{ (graphically)}$$

$$OB = \frac{C}{w}$$

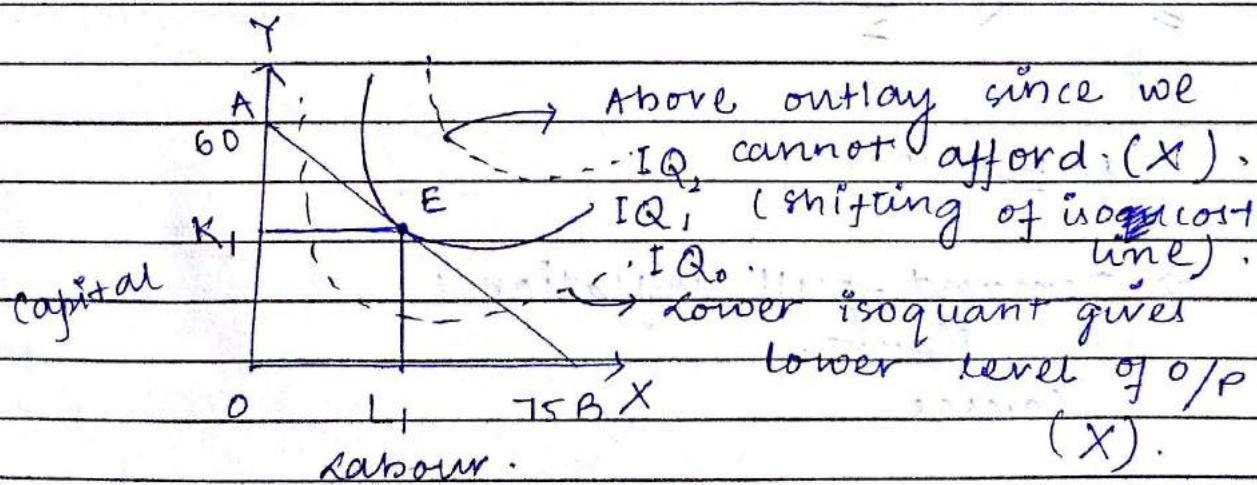
The slope of the iso-cost line is $\frac{w}{r}$.

For producer equilibrium,

- ① The slope of the iso-cost line should be equal to the slope of the isoquant (at the equilibrium point) i.e., at the point of equilibrium the iso-cost line should be tangential to the isoquant.
- ② At the point of equilibrium, the isoquant must be convex to the origin.

$$\text{So, } MRTS_{LK} = \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$



- (Q) A farm producing o/p L and K in such quantities that $MP_L = 15$, $MP_K = 18$, $w = 3$, $r = 2$. Is the farm using efficient factor combination for production? (No)

Ans) as $\frac{MP_L}{w} \neq \frac{MP_K}{r}$ [5 ≠ 4]

unmark (Q) Explain the producer's equilibrium condition. ($\frac{MPL}{w} > MP_K \text{ and } r$)

(Q) The wage rate of labour is ₹ 6 and the price of raw material is ₹ 2. The MPL is 16 and MP_K is 4. Can the farm operate under these conditions and maximize profit?

$$\text{Ans) } \frac{MPL}{w} = \frac{MP_K}{r}$$

$$\Rightarrow \frac{16}{6} \neq \frac{4}{2}$$

$\Rightarrow 2.67 > 2$. (x). [It is not an efficient condition]

(Q) A farm reports that MP of labour is 5 and $MRTSLK = 2$. What is MP_K ?

$$\text{Ans) } MP_K = \frac{5}{2} = 2.5$$

Inflation :-

→ Demand-pull inflation:

Figure.

Causes

→ Cost-push inflation:

Figure

Causes

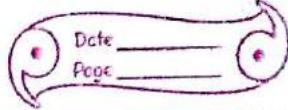
→ Control of inflation:

Monetary policy (Bank rate)

Fiscal policy. (Explain the points)

nt)

monetary policy :-



- ✓ Bank Rate
 - ✓ open market operation.
 - ✓ Cash Reserve ratio (CRR)
 - ✓ Statutory Liquidity ratio.
(SLR)
- } 10 lines on each.

(Q) what is national income (NI)?

(Q) what is Gross Domestic Product (GDP)?

(Q) Define Gross National Product (GNP).

(Q) what is net domestic product (NDP)?

(Q) what is meant by net national product (NNP)?