Quiz #6 Solutions

Math 55 with Professor Stankova
Discussion Section #102 with GSI James Moody

Wednesday, the 5th of October 2016
Write your name at the top!

Question 1 [12 points] Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and so on. [Hint: For the inductive step, separately consider the case where $k+1$ is even and where it is odd. When it is even, note that $(k+1)/2$ is an integer.]

Base Case: For $n = 1$, we can write $1 = 2^0$.

Inductive Step: Suppose that for $1 \leq k \leq n$ we can write k as a sum of distinct powers of 2. We want to show that n+1 can be written as a sum of distinct powers of two.

If $n+1$ is even, then $\frac{n+1}{2}$ is an integer, and $1 \leq \frac{n+1}{2} \leq n$, so by the inductive hypothesis we can write $\frac{n+1}{2} = 2^{a_1} + 2^{a_2} + ... + 2^{a_j}$, where $a_1, ..., a_j$ are all distinct. But then $n+1 = 2 * (2^{a_1} + 2^{a_2} + ... + 2^{a_j}) = 2^{a_1+1} + 2^{a_2+1} + ... + 2^{a_j+1}$. Notice that since $a_1, a_2, ..., a_j$ were distinct, $a_1+1, a_2+1, ..., a_j+1$ are distinct.

If $n+1$ is odd, then by by inductive hypothesis we can write $n = 2^{a_1} + ... + 2^{a_j}$, where $a_1, ..., a_j$ are distinct powers. Taking both sides mod 2, we notice that all non-zero powers of 2 drop out on the right hand side, and the left hand side is 0 (because n is even). This tells us that 2^0 cannot appear in the sum on the right hand side. But then we can write $n+1 = 2^0 + 2^{a_1} + 2^{a_2} + ... + 2^{a_j}$. Notice that $0, a_1, a_2, ..., a_j$ are still all distinct because, 0 was not one of $a_1, ..., a_j$.

Question 2 [± 1 **point**] Strong induction is equivalent to induction.

True —or— False?

Question 3 [± 1 **point**] The well-ordering property says that every non-empty set of nonnegative integers has a greatest element.

True —or— **False**?

Question 4 [± 1 **point**] Every non-empty set of nonnegative real numbers has a least element.

True —or— **False**?