Week 2

Linear Transformations, Eigenvalues and eigenvectors

Linear Transformations.

Mappings b/w vector spaces.

They obey the rules of vector addition & Scalar multiplication

Consider $y = x^2$

$y = f(x) = x^2$

Eg: $x = -3$ $\quad y = f(x) = (-3)^2 = 9.$

Let A be a matrix

A 2×2 matrix

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \longrightarrow \boxed{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} \longrightarrow b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\overset{A}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}} \underline{\underline{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}} = \underline{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

Examples:

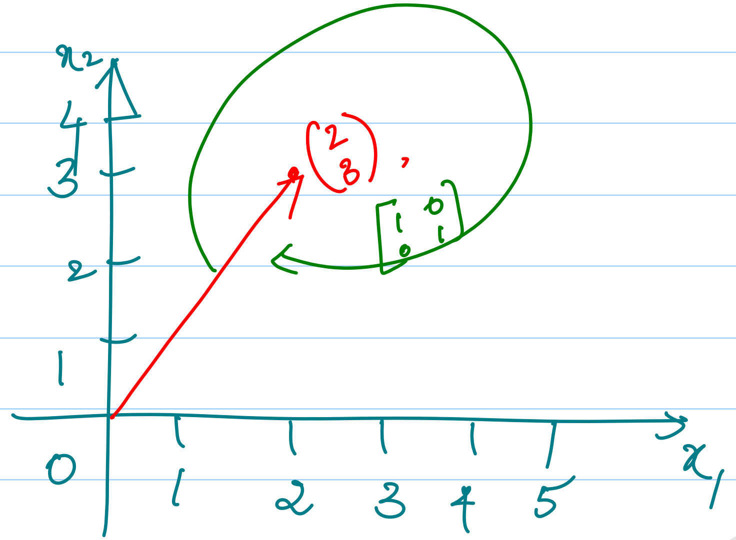
1. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$Ax = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longrightarrow \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Identity transformation.



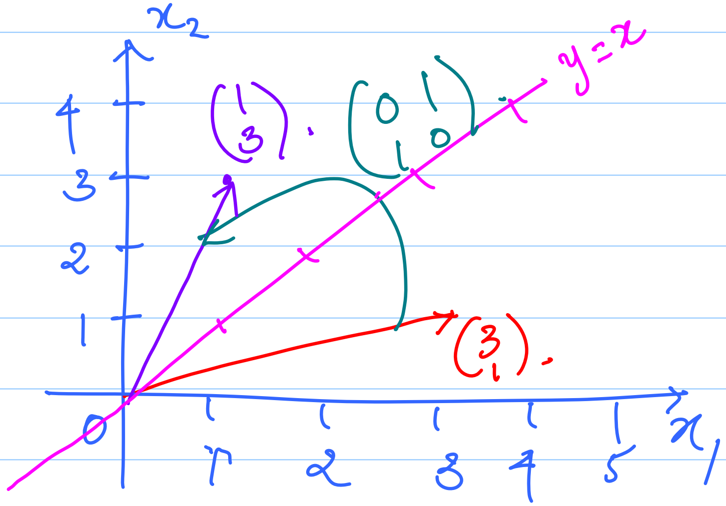
$$x = \begin{bmatrix}2\\3\end{bmatrix} \longrightarrow \boxed{\begin{bmatrix}1&0\\0&1\end{bmatrix}} \longrightarrow \begin{bmatrix}2\\3\end{bmatrix}.$$

Example 2:

$$A = \begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$x = \begin{bmatrix}x_1\\x_2\end{bmatrix} \longrightarrow \boxed{\begin{bmatrix}0&1\\1&0\end{bmatrix}} \longrightarrow \begin{bmatrix}x_2\\x_1\end{bmatrix}$$

Swaps the two rows of a matrix



$$\begin{bmatrix}3\\1\end{bmatrix} \longrightarrow \begin{bmatrix}0&1\\1&0\end{bmatrix} \longrightarrow \begin{bmatrix}1\\3\end{bmatrix}$$

$$\begin{bmatrix}0&1\\1&0\end{bmatrix} \rightarrow \text{Reflection Tranformation.}$$

Example 3:

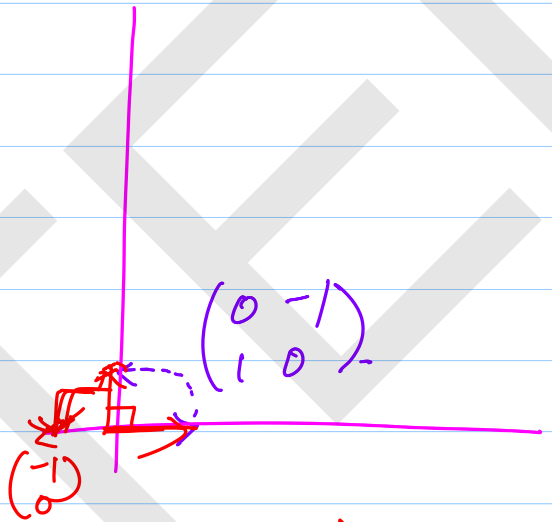
$$A = \begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

$$\begin{pmatrix}x_1\\x_2\end{pmatrix} \longrightarrow \boxed{\begin{bmatrix}0&-1\\1&0\end{bmatrix}} \longrightarrow \begin{bmatrix}-x_2\\x_1\end{bmatrix}$$

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

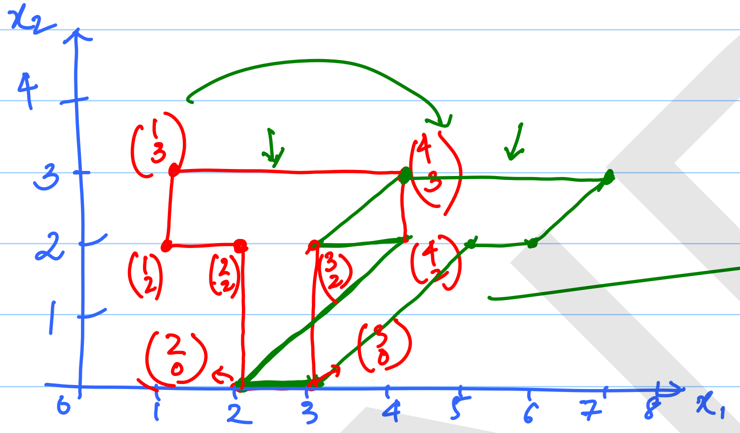
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$



$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \rightarrow \text{Rotation by } 90^\circ.$$

Example: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ Shear transf. $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ x_2 x_1

$$\underset{x}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \longrightarrow \underset{A}{\boxed{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}} \longrightarrow \underset{Ax.}{\begin{bmatrix} x_1 + x_2 \\ x_2 \end{bmatrix}}$$



$$\underset{A}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 2 & 1 & 1 & 4 & 4 & 3 & 3 \\ 0 & 2 & 2 & 3 & 3 & 2 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 3 & 4 & 7 & 6 & 5 & 3 \\ 0 & 2 & 2 & 3 & 3 & 2 & 2 & 0 \end{bmatrix}$$

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