Mathematical Foundations for ML

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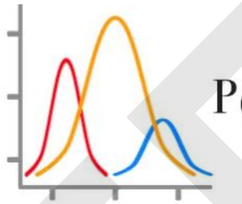
The instructors acknowledge all the authors, known and unknown, for making their articles available on the www

Machine Learning!

Linear Algebra

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{bmatrix} \text{matrix} \end{bmatrix}$$

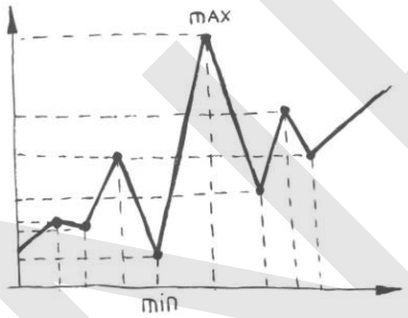


$P(A \mid B)$

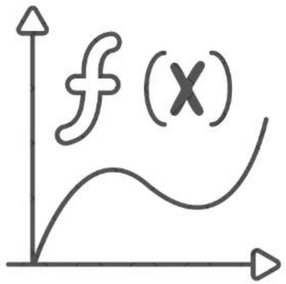
Statistics

Machine Learning

Graphs



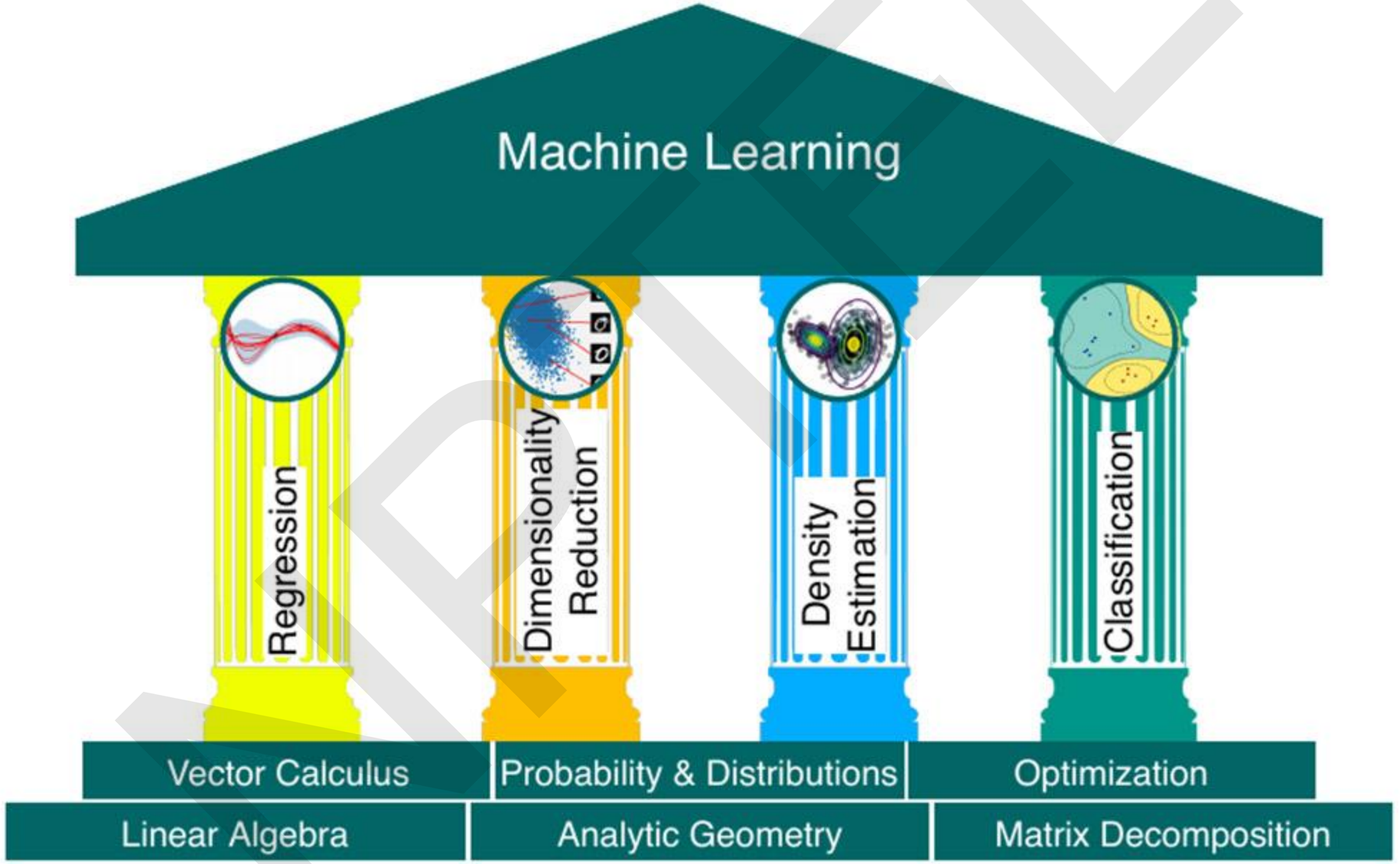
Calculus



enjoyalgorithms.com

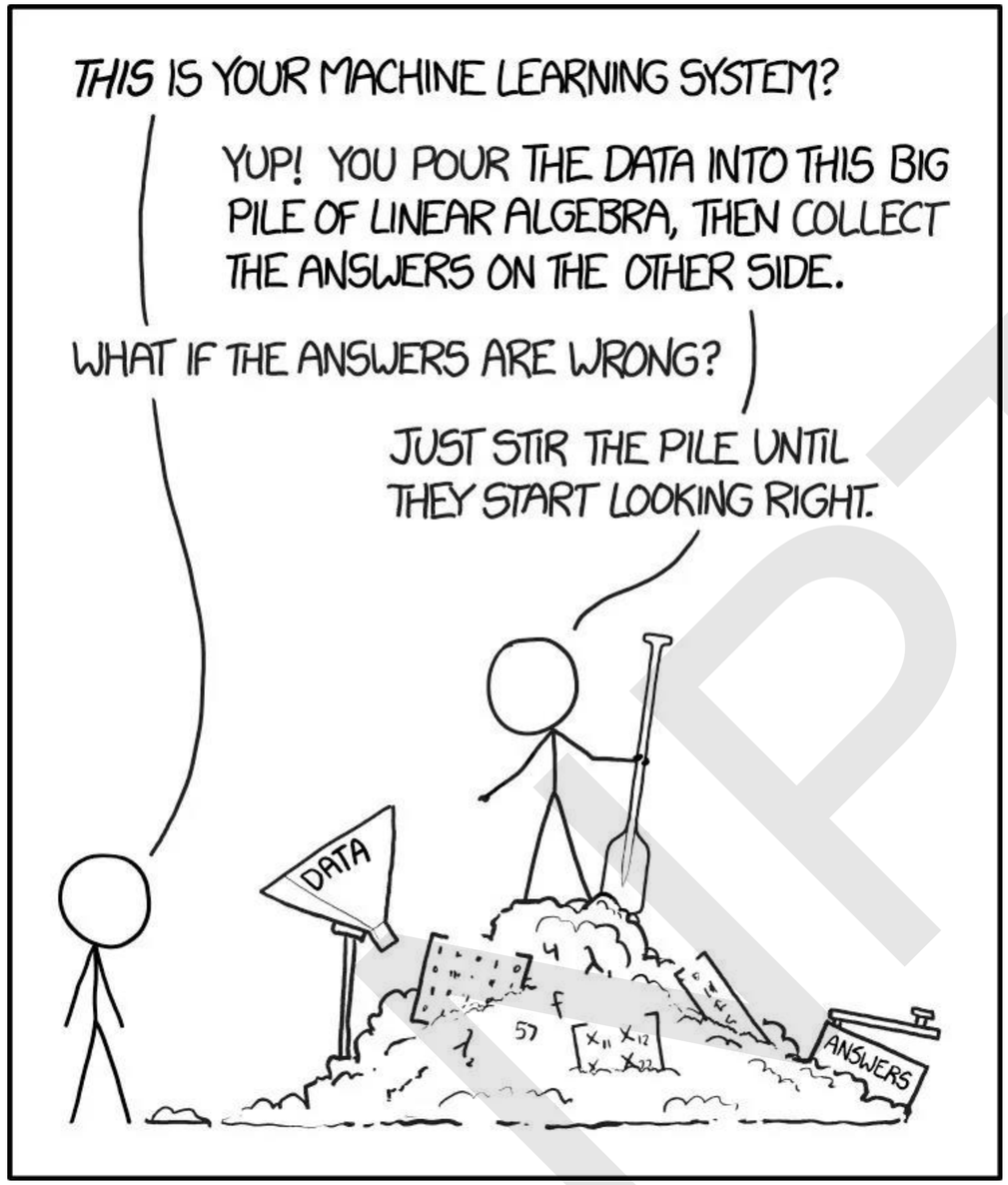
https://medium.com/enjoy-algorithm/detailed-maths-topics-in-machine-learning-ca55cd537709

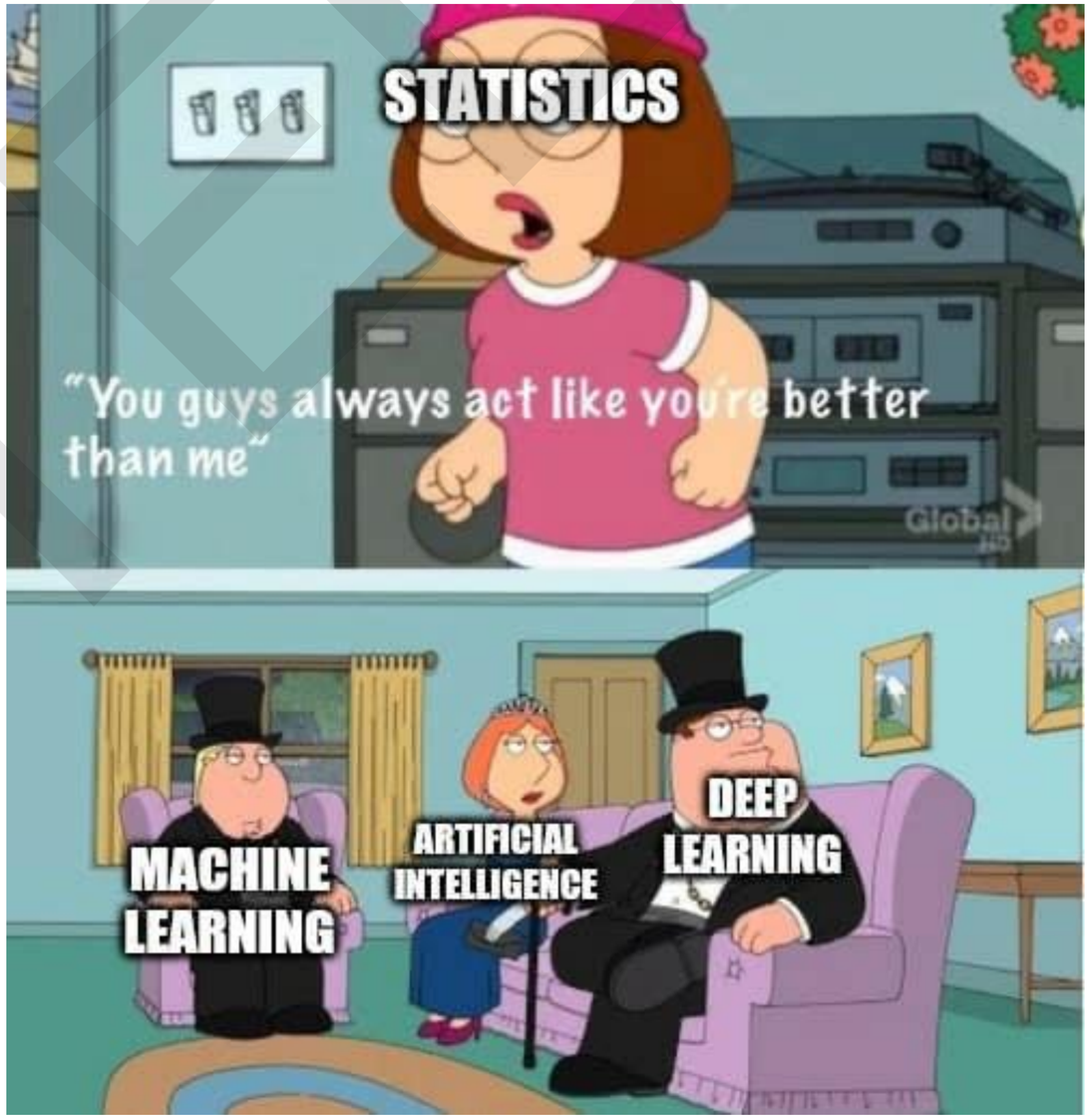
Machine Learning!



https://uhurasolutions.com/2020/10/14/why-is-mathematics-important-for-machine-learning-engineers-and-data-scientists/

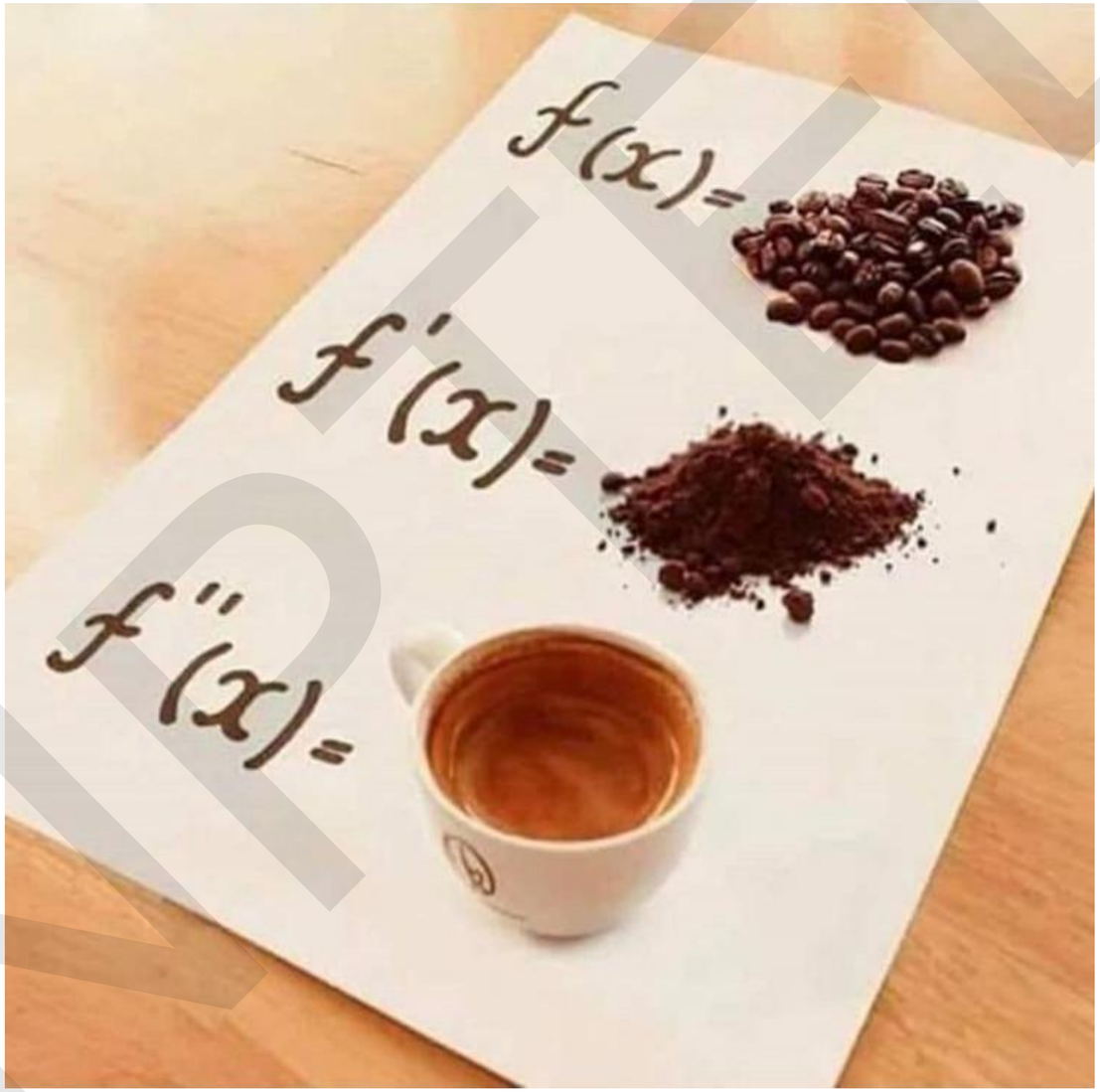
From the www



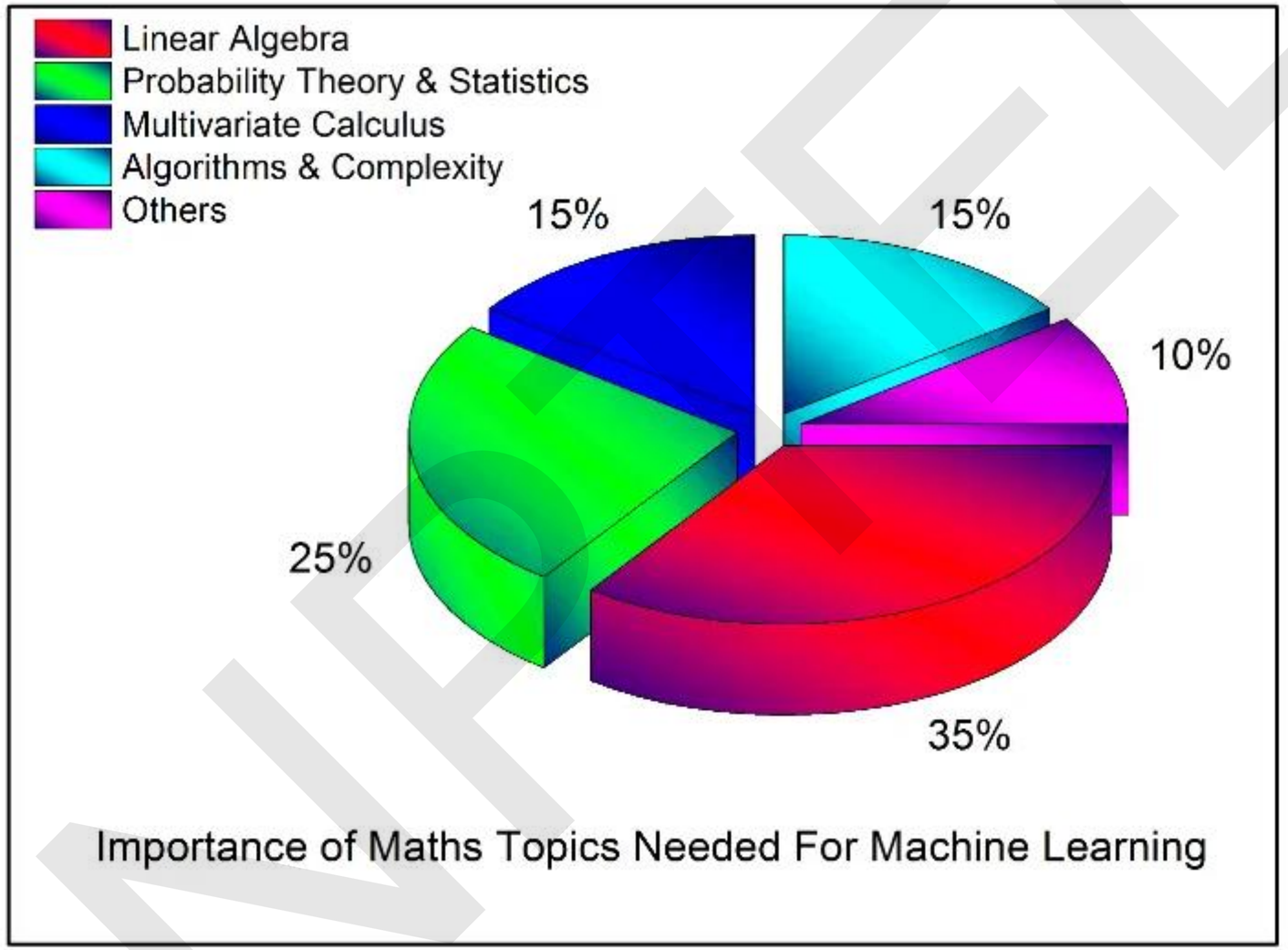


https://thunderdungeon.com/2024/05/17/ai-memes-3/

How I wish Calculus was!



Machine Learning!



https://medium.com/data-science/the-mathematics-of-machine-learning-894f046c568

Math in Machine Learning

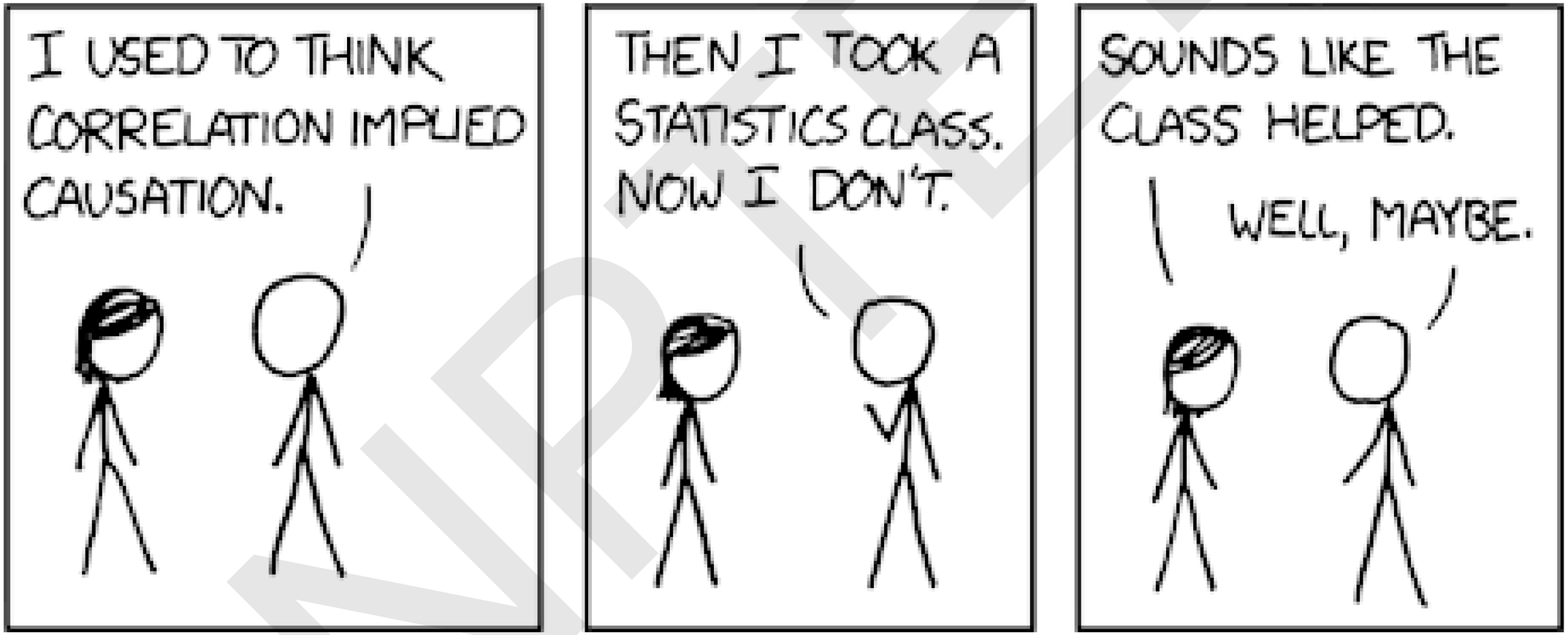
- Data for each variable - vector
- Table of such vectors - Matrix
- Playing around with matrices - Linear Algebra and Matrix Theory
- Dot product of two vectors - Similarity
- Combination of vectors - Feature in ML
- Idea of Projection - Dimensionality reduction

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$
$$x_1\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} + x_2\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Math in Machine Learning

- ML - Prediction - Classification and Regression
- Prediction to make informed decisions and effective risk management
- Confidence quantification - Requires Probability
- Ideas of Conditional Probability, Bayes Theorem, random variables, Conditioning random variables, Correlation, Covariance etc…

Math in Machine Learning



Math in Machine Learning

- ML - Training on historical data
- Training ML models - Finding out and adjusting the model parameters to optimise (maximise or minimise) some performance measure
- Maximize or minimise - Multivariable calculus and optimisation
- Gradient descent algorithm to find parameters that optimise - multivariable calculus
- Optimisation - Constrained or unconstrained!

Course Flow

Week 1 to Week 4 - Linear Algebra

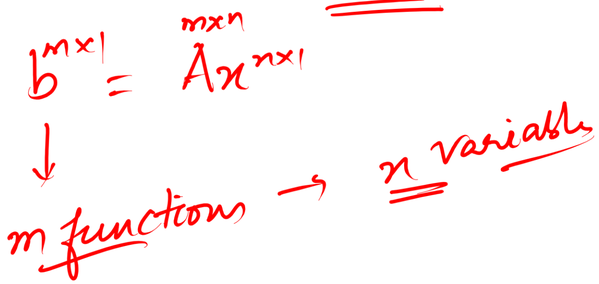
- Week 1: Vectors, Vector Spaces and Subspaces
- Week 2: Linear Transformations, eigenvalues and eigenvectors
- Week 3: Orthogonality, Projection and Real symmetric matrices
- Week 4: Singular value decomposition, Principal Component Analysis, Support Vector Machines and Applications

Week 5 to Week 8 - Probability and Random Variables

- Week 5: Probability Foundations - From Events to Bayes' Theorem
- Week 6: Random Variables, Moments of Random Variables
- Week 7: Jointly Distributed Random Variables, Conditioning of Random variables
- Week 8: Limit Theorems, Sample Geometry, Covariance Matrices and Properties

Week 9 to Week 12 -Multivariate Calculus, Optimization and Applications

- Week 9: Taylor's series, Partial Derivatives, Chain rule, Gradient, Jacobian, Hessian
- Week 10: Matrix Derivatives, Gradient Descent and Stochastic Gradient Descent, Constrained and Unconstrained optimization, Lagrangian, Least Squares and PCA
- Week 11: Neural Nets, Perceptron, Back Propagation Algorithm
- Week 12: Algorithms for ML - Classification, Clustering and Regression



From the www

https://www.facebook.com/convolutionalmemes/posts/so-thats-what-an-eigenvalue-is-debojeet/751177288670107/

Namaste!