Lect 3 Week 1.

Linear combination of vectors.

Suppose $u_1, u_2 \dots u_k$ are k vectors with n-components each

$$\Rightarrow \quad u_i = \begin{pmatrix} u_{i1} \\ u_{i2} \\ \vdots \\ u_{in} \end{pmatrix} \quad \text{for } i = 1, 2 \dots k$$

Let $\alpha_1, \alpha_2 \dots \alpha_k$ be scalars.
u_i for $i = 1, \dots k$ are real vectors and α_i $i = 1 \dots k$ are real

The vector

$$v = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_k u_k$$

is called the linear combination of the k-vectors $u_1, u_2 \dots u_k$.

Suppose $u_1, u_2 \dots u_k$ are the Standard unit vectors

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad u_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \quad \dots \quad u_k = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

k^{th} component.

Suppose $u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad u_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \alpha_1 u_1 + \alpha_2 u_2$$

$$= v_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + v_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Ex: $\begin{pmatrix} 3 \\ 4 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Suppose $\alpha_1 = \alpha_2 = \dots = \alpha_k = 1.$

$$\alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_k u_k \Rightarrow$$

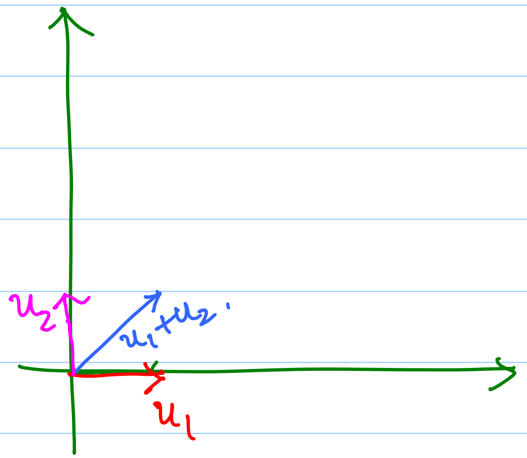
$$v = 1 u_1 + 1 u_2 + \dots + 1 u_k$$

$\Rightarrow v = u_1 + u_2 + \dots + u_k.$

v: Sum of the vectors.

Ex: $u_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad u_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$v = 1 u_1 + 1 u_2 = 1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}.$$



Suppose we have $u_1, u_2 \dots u_n$ all n-component vectors

Let $\alpha_1, \alpha_2 \dots \alpha_n$ be Scalars Such that $\alpha_1 = \alpha_2 = \dots = \alpha_n = \frac{1}{n}$

$$v = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$$

$$= \frac{1}{n}(u_1) + \frac{1}{n}(u_2) + \dots + \frac{1}{n} u_n$$

$$\Rightarrow \frac{1}{n}\left(u_1 + u_2 + \dots + u_n\right)$$

$\Rightarrow$ Average of the vectors.

If the coefficients or the scalars add upto 1, we call such combinations as the affine combination

Suppose the coefficients in an affine combination are all non-negative, we call this combination as

(i) CONVEX Combination

(ii) WEIGHTED AVERAGE.

Suppose $\alpha_1 = \alpha_2 = \cdots \alpha_{i-1} = 0$, $\alpha_i = 1$, $\alpha_{i+1} = \cdots = \alpha_k = 0$

the l.c.

$$\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_i u_i + \cdots + \alpha_k u_k$$

results in the vector u_i

Suppose we want to study the effect of a "transformation" on a vector space, what strategy do we adopt to do this?

Consider $u_1, u_2 \ldots u_n$ to be n-component vectors.

Look at that linear combination of the u-vectors that results in the n-comp zero vector.

Let $\alpha_1, \alpha_2 \ldots, \alpha_n$ be real scalars

The linear combination of u-vectors is given by

$$\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_n u_n = \emptyset_n$$

$$\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_n u_n = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_n$$

Ex: $u_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $u_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $u_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

$$\begin{pmatrix}0\\0\end{pmatrix} = \alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3$$

$$\begin{pmatrix}0\\0\end{pmatrix} = \alpha_1 \begin{pmatrix}2\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}1\\2\end{pmatrix} + \alpha_3 \begin{pmatrix}3\\3\end{pmatrix}$$

$$\begin{pmatrix}0\\0\end{pmatrix} = 1 \begin{pmatrix}2\\1\end{pmatrix} + 1 \begin{pmatrix}1\\2\end{pmatrix} - 1 \begin{pmatrix}3\\3\end{pmatrix}$$

There are multiple possibilities to get the 0 vector by choosing different scalars

$$\begin{pmatrix}0\\0\end{pmatrix} = 0 \begin{pmatrix}2\\1\end{pmatrix} + 0 \begin{pmatrix}1\\2\end{pmatrix} + 0 \begin{pmatrix}3\\3\end{pmatrix}$$

Linear dependence / linearly dep. vectors

Suppose $u_1, u_2 \cdots u_n$ are vectors and $\alpha_1, \alpha_2 \cdots \alpha_n$ are scalars then

$$\alpha_1 u_1 + \alpha_2 u_2 + \cdots + \alpha_n u_n = 0_n$$

such that not all α_i's are 0, we say that $u_1, u_2 \cdots u_n$ are linearly dependent

A set of vectors $u_1, u_2 \dots u_n$ is said to be linearly dep if for scalars $\alpha_1, \alpha_2 \dots \alpha_n$,

there exists a l.c

$$\alpha_1 u_1 + \dots + \alpha_n u_n$$

such that not all α_i's are zero

Any set of vectors that contains the zero vector is a linearly dependent set.

$$S = \{\underbrace{u_1, u_2, u_3 \dots, u_k}, 0\}$$

then

$$\underline{\underline{\vec{0}}} = \underline{\alpha_1} u_1 + \underline{\alpha_2} u_2 + \dots + \underline{\alpha_k} u_k + \underline{\underline{\alpha_r}} 0$$

$\alpha_1 = 0, \alpha_2 = 0 \dots \alpha_k = 0, \alpha_r \Rightarrow$ Arbitrary

Linearly Dep Set $\Rightarrow$ REDUNDANCY.