LECT. 3 WEEK 1

Some examples of vector spaces.

1) Field $\mathbb{F}$ itself is a vector space

2) $\mathbb{R}^n$ for any n is a vector space over $\mathbb{R}$.

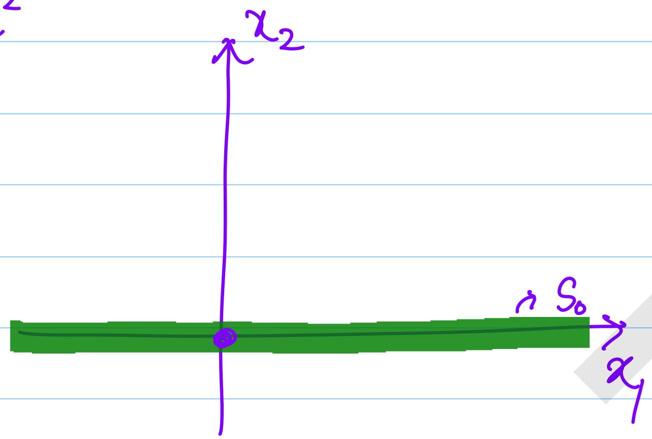
3) Set of all polynomials of degree $\leq n$ and with coeffts real is a vector space over $\mathbb{R}$.

4) Set of square matrices over $\mathbb{R}$

5) Set of all real symmetric matrices

6) Set of all continuous functions of time t for t in $(-\infty, \infty)$ defined over $\mathbb{R}$.

Consider $\mathbb{R}^2$



Subset of $\mathbb{R}^2$, $S_k = \left\{ \begin{pmatrix} x_1 \\ kx_1 \end{pmatrix}, \; x_1 \in \mathbb{R}, \; k \in \mathbb{R} \right\}$.

$k = 0$: $S_0 = \left\{ \begin{pmatrix} x_1 \\ 0 \end{pmatrix}, \; x_1 \in \mathbb{R} \right\}$.

Let $u, v \in S_0$

$\Rightarrow \underline{u} = \begin{pmatrix} u_1 \\ 0 \end{pmatrix}, \quad \underline{v} = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$

$\underline{u} + \underline{v} = \begin{pmatrix} u_1 \\ 0 \end{pmatrix} + \begin{pmatrix} v_1 \\ 0 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ 0 + 0 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ 0 \end{pmatrix} \in S_0$

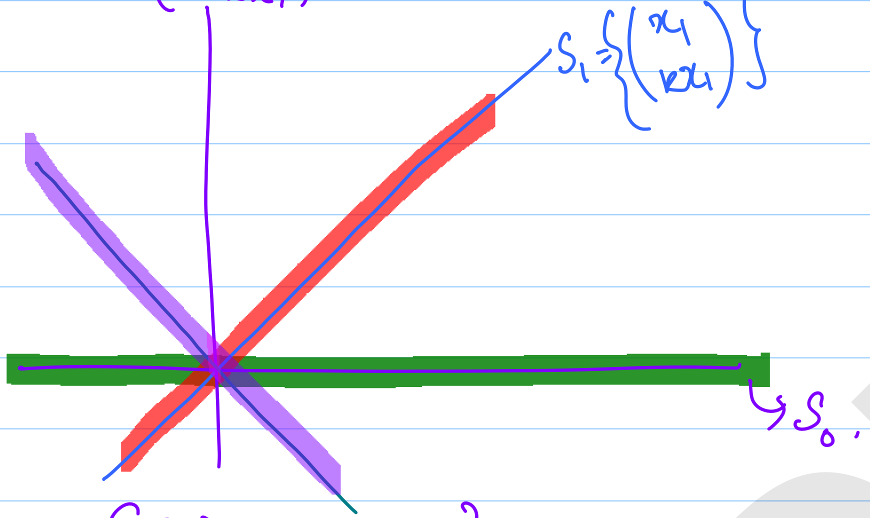
S_0 is closed under vector addn.

For $\alpha \in \mathbb{R}$ $\alpha \cdot \begin{pmatrix} u_1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha u_1 \\ 0 \end{pmatrix} \in S_0$

S_0 is closed under Scalar Multiplicn

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S_0 \rightarrow S_0$ is a VS over $\mathbb{R}$.

$S_k = \left\{ \begin{pmatrix} x_1 \\ kx_1 \end{pmatrix}, \; x_1 \in \mathbb{R}, \; k \in \mathbb{R} \right\}.$



$S_1 = \left\{ \begin{pmatrix} x_1 \\ x_1 \end{pmatrix}, \; x_1 \in \mathbb{R} \right\}.$

$\underline{u} = \begin{pmatrix} u_1 \\ u_1 \end{pmatrix} \quad v = \begin{pmatrix} v_1 \\ v_1 \end{pmatrix} \Rightarrow u+v = \begin{pmatrix} u_1+v_1 \\ u_1+v_1 \end{pmatrix}$

S_1 is closed under VA.

$\alpha \in \mathbb{R}, \quad \alpha \begin{pmatrix} u_1 \\ u_1 \end{pmatrix} = \begin{pmatrix} \alpha u_1 \\ \alpha u_1 \end{pmatrix} \in S_1$

S_1 is closed under SM.

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S_1 \quad \Rightarrow \quad S_1$ is a VS over $\mathbb{R}$.

$S_{-1} = \left\{ \begin{pmatrix} x_1 \\ -x_1 \end{pmatrix}, \; x_1 \in \mathbb{R} \right\}$

Try; S_{-1} is also a vector space over $\mathbb{R}$.

Any line passing thro' the origin is a vector space over $\mathbb{R}$.

Any subset of a vector space V, which by itself is a vector space with operations as defined in V is called a vector subspace of V.

(i) Every set is a subset of itself

$\therefore$ Any $\mathbb{R}^n$ is a trivial subspace of $\mathbb{R}^n$

(ii) $\emptyset_0 = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} \Rightarrow$ Subspace of $\mathbb{R}^2$.

Set containing only the zero vector is a vector subspace $\to$ TRIVIAL Subspace.

(iii) In any $\mathbb{R}^n$, a plane passing thro' the origin is a vector subspace.

Some questions:

① What happens if we add multiple vectors in a vector space?

⇒ What is it to say combining vectors?

② Suppose we want to transform an entire vector space, what is the strategy to study the effect of the transformation on the vector space?