What can we say about diagonalizability of A in each of those cases?

(i) CASE 3: eigenvalues are distinct

$\Rightarrow$ eigenvectors with each eigenval are in different directions/ linearly indep.

$\Rightarrow P = [\text{evector 1} \quad \text{eigvec2} \quad \text{eigvec3}]$

P is invertible & $A = PDP^{-1}$ exists!

Case 2: $AM(\underline{\lambda_1}) = \underline{\underline{2}}$ $\xrightarrow{}$ Real $\qquad AM(\lambda_2) = 1$

$\Downarrow \qquad\qquad \Downarrow$ GM=1.

evector.

$\underbrace{(A^{3\times3} - \lambda_1 I)}\vec{x} = \vec{0}$

Null space of $(A - \lambda_1 I)$

Recall: Rank Nullity Theorem.

Rank(A) + Nullity(A) = No. of cols of A.

We can find 2 l.i eigenvectors for each λ_1

$\Rightarrow$ P is inv & A is diagonali

Case(1) $\rightarrow$ $A^{3\times 3}$ with one eigenval. repeated thrice.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\det(A) = 1$

Trace of $A = 3$.

$\lambda_1 = 1$ thrice.

AM of $\lambda_1 = 3$.

Prod of eigval. of $A = \det(A)$

Sum of eigval of $A = \text{Trace}(A)$

$=$ Sum of diagonal elements of A.

$$(A - \lambda I)\vec{x} = \vec{0}$$

$$= \begin{bmatrix} 1-1 & 1 & 1 \\ 0 & 1-1 & 1 \\ 0 & 0 & 1-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 \leftarrow R_1 - R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$; $x_3 = 0$

$0 x_1 = 0 \Rightarrow x_1 = k$, arbitrary

$\therefore$ The eigenvector corresp. to $\lambda = 1$ is $\begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix} = \underline{\underline{k \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}}$.

$\lambda = 1$ is deficient & A is therefore not diagonalizable.

(1) eigenvectors corresp. to distinct eigenvalues are linearly indep. Hence these can be used as a basis for Col.Sp(A).

(2) A is an $n \times n$ real matrix and diagonalizable.

Let λ_1 be an eigenval of $A^{n \times n}$ & let u be the associated eigenvector.

$\Rightarrow A\underline{u} = \underline{\lambda_1} u.$

Consider $c_1 u$ where c_1 is a real scalar.

$$A(c_1 u) = c_1 (Au) = c_1(\lambda_1 u) = \lambda_1 (c_1 u)$$

$+$

$$A(c_2 u) = \lambda_1 (c_2 u)$$

$$A(c_1 u + c_2 u) = A((c_1+c_2)u) = (c_1+c_2)(Au) = (c_1+c_2)\lambda_1 u$$
$$= \lambda_1 (c_1+c_2) u = \lambda_1 (c_1 u + c_2 u)$$

$$A\vec{0} = \lambda_1 \vec{0}$$

$\Rightarrow$ Set of eigenvectors associated with an eigenval λ_1 is closed under

a) Vector Addition

b) Scalar Multip

and has the zero vector.

Hence the set of eigenvectors associated with a specific eigenval λ_1 forms a vector subspace.

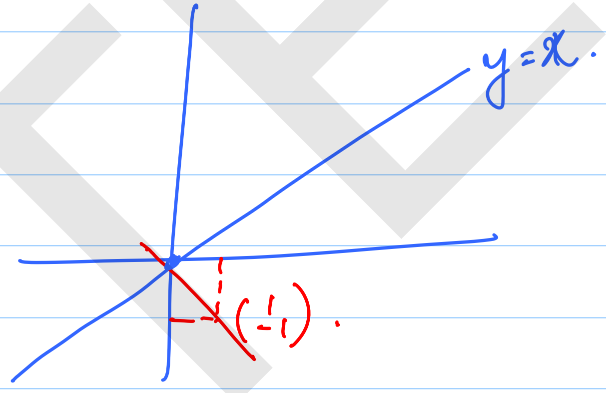
This is called the eigensubspace or the invariant subspace associated with λ_1

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\lambda_1 = 3, \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 1, \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$A\left(k\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = A\begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} 3k \\ 3k \end{pmatrix} = 3\begin{pmatrix} k \\ k \end{pmatrix}.$$



$$A\left(k\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) = A\begin{pmatrix} k \\ -k \end{pmatrix} = \begin{pmatrix} k \\ -k \end{pmatrix} = \left\{k\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$$