LECTURE 2 WEEK 1

R_6: Set of integers mod 6.

$R_6 = \{0, 1, 2, 3, 4, 5\}$

$\oplus_6$ & $\odot_6$.

$\oplus_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

R_6 is closed under addition

$\odot_6$	1	2	3	4	5
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	4	3	2	1

R_6 is not a field because it does not have multiplicative inverses for some elements.

R_p (where 'p is a prime), the set of integers mod p is always a field.

R_2 : $R_2 = \{0, 1\}$

$\oplus_2$	0	1
0	0	1
1	1	0

$\odot_2$	1	0
1	1	0
0	0	0

R_2 : Is a field.

$\oplus_2 \rightarrow$ XOR gate / operation

$\odot_2 \rightarrow$ AND operation

Vector Space V.

Let $\mathbb{F}$ be any field. Let V be a non-empty collection of objects called the "vectors"

V is a vector space over $\mathbb{F}$ if rules for adding two vectors and Scalar multiplication exists such that V is closed under vector addition & Scalar Multiplicⁿ

Mathematically:

(i) For $u, v \in V$, $u+v \in V$

(ii) For $\alpha \in \mathbb{F}$, $u \in V$, $\alpha u \in V$

and the following properties hold.

(i) Commutativity: For all $u, v \in V$,

$$u+v = v+u$$

(ii) Associativity: $u, v, w \in V$,

$$(u+v)+w = u+(v+w)$$

For scalars $\alpha, \beta \in \mathbb{F}$, $u \in V$

$$(\alpha\beta)u = \alpha(\beta u)$$

(iii) Distributivity:

$\alpha \in \mathbb{F}, \quad u, v \in V$

$\alpha(u+v) = \alpha u + \alpha v.$

$\alpha, \beta \in \mathbb{F}, \quad u \in V$

$(\alpha+\beta)u = \alpha u + \beta u.$

(iv) Additive Identity:

$\underline{0} \in V$ s.t $\underline{0} + \underline{u} = \underline{u}$ for all $\underline{u} \in V$

(v) Additive Inverse:

For every $\underline{u} \in V$, there exits $-\underline{u} \in V$ s.t $\underline{u} + (-\underline{u}) = \underline{0}$

(vi) Multiplicative Identity

$1\underline{u} = \underline{u}, \quad u \in V$

Construct a 2 component vector over the field $\mathbb{F}$.

Let $\mathbb{F} = \mathbb{R}$.

2 copies of $\mathbb{R}$.

$$\underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \Rightarrow u \in \mathbb{R}^2 \quad \mathbb{R} \times \mathbb{R}$$

u is a 2component real vector

n- copies of $\mathbb{R}$

$$\Rightarrow u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \rightarrow \underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{n}$$

$u \in \mathbb{R}^n$

How do we do vector addition?

$u, v \in \mathbb{R}^2$ $\quad$ V is defined over $\mathbb{R}$.

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad u_1, u_2 \in \mathbb{R}$$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad v_1, v_2 \in \mathbb{R}.$$

$$\underline{u + v} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix}$$

Vector Addn.

Addn as defined in the field $\mathbb{R}$.

Scalar Multiplicn:

$\alpha \in \mathbb{R}, \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad u_1, u_2 \in \mathbb{R}.$

$\underline{\underline{\alpha}} \cdot \underline{\underline{u}} = \alpha \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \alpha \underline{\underline{\cdot}} u_1 \\ \alpha \underline{\underline{\cdot}} u_2 \end{pmatrix}$

Scalar Multiplicn of a vector.

↳ Multiplicn as defined in the field $\mathbb{R}$.

① Examples for vector spaces.