LECTURE - 1 WEEK - 1.

Vectors

Magnitude & Direction

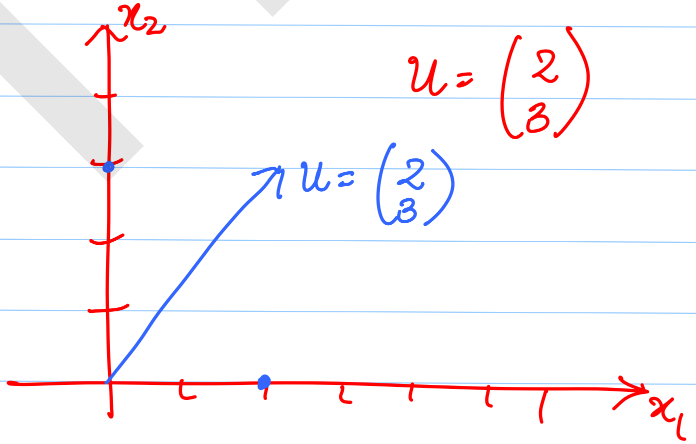
Vectors, Vector Spaces & Subspaces

— Focus of Week 1.

Vector :

$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ - 2 component vector

with components being u_1 & u_2.



u is a REAL vector → Components are real numbers

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \rightarrow \text{n-component Vector}$$

→ Ordered n-tuple.

Suppose we measure the biological parameters

n - components - n parameters we measure'

for ex: u_1 → Blood pressure
u_2 → Blood Glucose level
u_3 → Pulse rate
u_4 : Heart rate
u_5 : SPO_2 levels.

$$u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_5 \end{pmatrix} \rightarrow \text{Collection of 5 parameters.}$$

Collect these 5 parameters from 100 different patients

$$\begin{array}{c|ccccc} & u_1 & u_2 & u_3 & u_4 & u_5 \\ \hline P_1 & & & & & \\ \vdots & & & & & \\ P_{100} & & & & & \end{array}$$

Diff parameter we measure

Patients

Data.

Matrix

100x5.

Field $\mathbb{F}$:

Non empty collection of elements with operations $(+, \cdot)$

$+$: Field addition &

$\cdot$: Field Multiplication

Such that the following properties hold.

(i) There exists $0 \in \mathbb{F}$ s.t for any element $a \in \mathbb{F}$, $a+0 = 0+a = a$

Additive identity

(ii) For any two elements $a, b \in \mathbb{F}$,

$a+b \in \mathbb{F}$

(iii) There exists for every element $a \in \mathbb{F}$, the additive inverse $(-a) \in \mathbb{F}$ s.t

$a+(-a) = 0$

(iv) There exists the element $1 \in \mathbb{F}$ s.t

$a \cdot 1 = 1 \cdot a = a$.

1 : Multiplicative Identity

(v) For $a, b \in \mathbb{F}$,

$a \cdot b \in \mathbb{F}$

(vi) For every non zero element $a \in \mathbb{F}$, there exists a unique element $a^{-1} \in \mathbb{F}$ s.t

$a \cdot a^{-1} = a^{-1} \cdot a = 1$

a^{-1} : Multiplicative inverse of a.

Example 1:

Consider the set of real numbers $\mathbb{R}$.

(i) $0 \in \mathbb{R}$.

(ii) For any 2 real nos $a, b \in \mathbb{R}$, $a+b \in \mathbb{R}$.

(iii) For every $a \in \mathbb{R}$, $(-a) \in \mathbb{R}$ s.t $a+(-a) = -a+a = 0$

(iv) $1 \in \mathbb{R}$

(v) $a, b \in \mathbb{R}$, $a.b \in \mathbb{R}$

(vi) For every $a \neq 0$, $a \in \mathbb{R}$, a^{-1} s.t $a^{-1}.a = a.a^{-1} = 1$.

$\mathbb{R}$ is a field with operations $+, \cdot$

Ex:2 Set of integers. $\mathbb{N}$

(i) $0 \in \mathbb{N}$

(ii) $a+b \in \mathbb{N}$, for $a, b \in \mathbb{N}$

(iii) For every $a \in \mathbb{N}$, $(-a) \in \mathbb{N}$ s.t $a+(-a) = 0$

(iv) $1 \in \mathbb{N}$.

$1 \cdot a = a \cdot 1 = a$ for every $a \in \mathbb{N}$

(v) $a \cdot b \in \mathbb{N}$ for any two integers a, b.

(vi) Suppose $a \in \mathbb{N}$, (for ex $a = 2$), then $a^{-1} = 1/2 \notin \mathbb{N}$.

Set of integers is $\boxed{\text{Not}}$ a field.

Ex: 3: Consider the Set of integers modulo 5.

→ Set of remainders when divided by 5. → R_5.

$R_5 : \{0, 1, 2, 3, 4\}$.

$+ = \oplus_5 \Rightarrow$ Add modulo 5.

$\cdot = \odot_5 \Rightarrow$ Multiplicn mod 5.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

R_5 : Closed under addition mod 5.

Element of R_5	0	1	2	3	4
{Additive Inverse mod 5}	0	4	3	2	1

For every $a \in R_5$, we have $(-a)$ in R_5. → Additive Inverse mod 5 exists for every element in R_5.

$\odot_5$	1	$\boxed{2}$	3	4
1	1	2	3	4
2	2	4	1	3
$\boxed{3}$	3	1	4	2
4	4	3	2	1

R_5 is closed under multiplication.

(iv) $1 \in R_5 \rightarrow$ Multiplicative Identity mod 5 exists in R_5

(v) For $a, b \in R_5$, $a \odot_5 b \in R_5$.

(vi) For every $a \in R_5$, there exists a unique integer a^{-1} mod 5 s.t

$$a \odot_5 a^{-1} \equiv 1 \bmod 5.$$

$\downarrow$

Congruence.

$a =$	1	2	3	4
$(a^{-1})_5 =$	1	3	2	4

Set R_5 is a field with operations $\oplus_5$ & $\odot_5$.

Set of integers mod any integer

$\Rightarrow$ R_n $\rightarrow$ Is R_n for any n a field.?

n: Integer