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Question-1

(1) Soln

$$\left(\begin{array}{ccccc:c} 7 & 2 & -2 & -4 & 3 & 8 \\ -3 & -3 & 0 & 2 & 1 & -1 \\ 4 & -1 & -8 & 0 & 20 & 1 \end{array}\right)$$

Since number of unknowns are less than rank of Matrix.

∴ There are only two possibilites.

→ no solution

→ Infinitely Many solution.

$R_2 = R_2 + \frac{3}{7} R_1$

$R_3 = R_3 - \frac{4}{7} R_1$

$$\left(\begin{array}{ccccc:c} 7 & 2 & -2 & -4 & 3 & 8 \\ 0 & -\frac{15}{7} & -\frac{6}{7} & \frac{2}{7} & \frac{16}{7} & \frac{20}{7} \\ 0 & -\frac{15}{7} & -\frac{48}{7} & \frac{16}{7} & \frac{128}{7} & -\frac{25}{7} \end{array}\right)$$

$R_3 = R_3 * 7 \quad R_2 = R_2 * 7 \quad R_3 = R_3 - R_2$

$$\left(\begin{array}{ccccc:c} 7 & 2 & -2 & -4 & 3 & 8 \\ 0 & -15 & -6 & 2 & 16 & 20 \\ 0 & 0 & -42 & 14 & 112 & -45 \end{array}\right)$$

$\rho(A) = \rho(A:B)$

∴ Infinitely Many solutions.

$\cancel{7x + 2y - 2z - 4}$

$$
\begin{aligned}
7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 &= 8 \\
-15x_2 - 6x_3 + 2x_4 + 16x_5 &= 20 \\
-42x_3 + 14x_4 + 112x_5 &= -45
\end{aligned}
$$

$x_4 = a \quad x_5 = b$ (∵ free variables)

$$
\boxed{
\begin{aligned}
7x_1 + 2x_2 - 2x_3 &= 8 + 4a - 3b. \\
15x_2 + 6x_3 &= -20 + 2a + 16b \\
42x_3 &= 45 + 14a + 112b
\end{aligned}
}
$$

$$
\left(\begin{array}{ccc:c}
7 & 2 & -2 & 8 + 4a - 3b \\
0 & 15 & 6 & -20 + 2a + 16b \\
0 & 0 & 42 & 45 + 14a + 112b
\end{array}\right).
$$

converting to row canonical form.

$R_2 = \cancel{6R_3}\ R_2 - 6R_3$ $\qquad R_3/42$

$$
\left(\begin{array}{ccc:c}
7 & 2 & -2 & 8 + 4a - 3b \\
0 & 15 & 0 & -20 + 2a + 16b \\
0 & 0 & 1 & \frac{45 + 14a + 11b}{42}
\end{array}\right).
$$

$R_1 = R_1 + 2R_3$

$$
\left(\begin{array}{ccc:c}
7 & 2 & 0 & 8 + 4a - 3b + \frac{45 + 14a + 11b}{21} \\
0 & 15 & 0 & -20 + 2a + 16b \\
6 & 0 & 1 & 45 + 14a + 11b.
\end{array}\right)
$$

$R_2/15 \qquad R_1 = R_1 - 2R_2$

$$
\left(\begin{array}{ccc:c}
1 & 0 & 0 & \frac{1}{7}\left(\frac{182 + 88a - 52b}{21} - \frac{4a + 16b - 40}{15}\right) \\
0 & 1 & 0 & -20 + 2a + 16b \\
0 & 0 & 1 & 45 + 14a + 11b.
\end{array}\right)
$$

$$
\Rightarrow \quad
\begin{aligned}
x_1 &= \frac{1}{7}\left(\frac{182 + 88a - 52b}{21} - \frac{4a + 16b - 40}{15}\right) \\
x_2 &= -20 + 2a + 16b \\
x_3 &= 45 + 14a + 11b.
\end{aligned}
$$

Question-2

(2) $$\left(\begin{array}{ccc:c} 1 & -1 & 5 & 14 \\ -1 & 5 & -1 & -18 \\ -2 & -3 & -3 & 13 \end{array}\right)$$

$R_2 = R_1 + R_2 \quad R_3 = 2R_1 + R_3$

$10 - 3$

$$\left(\begin{array}{ccc:c} 1 & -1 & 5 & 14 \\ 0 & 4 & 4 & -4 \\ 0 & -5 & 7 & +1 \end{array}\right)$$

$R_2/4 \quad R_3 + 5R_1 \quad R_1 = R_1 + R_2$

$7+5$

$49-5.$

$$\left(\begin{array}{ccc:c} 1 & 0 & 6 & 13 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 12 & 36 \end{array}\right)$$

$R_3/12 \quad R_2 = R_2 - R_3 \quad R_1 = R_1 - 6R_3$

$$\left(\begin{array}{ccc:c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$$\boxed{x = -5 \quad y = -4 \quad z = 3}$$

Question-3

(3) $A = \begin{pmatrix} 1 & -2 & 0 & -4 \\ 3 & 2 & 1 & 4 \\ 2 & 3 & 7 & 2 \\ -1 & 2 & 0 & 3 \end{pmatrix}$

$R_2 = R_2 - 3R_1$

$R_3 = R_3 - 2R_1$

$R_4 = R_4 + R_1$

$A = \begin{pmatrix} 1 & -2 & 0 & -4 \\ 0 & 8 & 1 & 16 \\ 0 & 7 & 7 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$R_2 = R_2 - R_3$

$\begin{pmatrix} 1 & -2 & 0 & -4 \\ 0 & 1 & -6 & 6 \\ 0 & 7 & 7 & 10 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$R_3 = R_3 - 7R_2$

$\begin{pmatrix} 1 & -2 & 0 & -4 \\ 0 & 1 & -6 & 6 \\ 0 & 0 & 49 & -32 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$-R_4$, $\frac{R_3}{49}$

$\begin{pmatrix} 1 & -2 & 0 & -4 \\ 0 & 1 & -6 & 6 \\ 0 & 0 & 1 & -\frac{32}{49} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$R_3 = R_3 + \frac{32}{49} R_4$, $R_2 = R_2 - 6R_4$, $R_1 = R_1 + 4R_3$

$$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_2 = R_2 + 6R_3$

$$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_4 \Leftrightarrow R_2$ $R_2 \Leftrightarrow R_3$

$$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

canoical forom

$\cancel{C_1 \to C_1 + 2C_2}$

$\boxed{\text{Rank of Matrix} = 3}$

Question-4

(4) $x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$

for system to be consistent

$\rho(A) = \rho(B)$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 6 & -11 & b \\ 1 & -2 & 7 & c \end{array}\right)$$

$R_2 = R_2 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & b-2a \\ 1 & -2 & 7 & c \end{array}\right)$$

$R_3 = R_3 - R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & -4 & 10 & c-a \end{array}\right)$$

$R_3 = R_3 + 2R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 2 & -5 & b-2a \\ 0 & 0 & 0 & c-5a+2b \end{array}\right)$$

for $\rho(A) = \rho(B)$

$\boxed{c - 5a + 2b = 0}$