Unit - 5

Measures of Central tendency :-

Average :-

- Mathematical
  - Arithematic Mean
  - Geometric mean
  - Harmonic Mean
- Positional
  - Median
  - Mode

* Arithematic Mean or Mean - Direct method
Assume mean method
Step deviation method

* Direct method :-

E.g - Individual Series :-

X
3
2
1
5
4

$n = 5$

$\sum X = 3+2+1+5+4 = 15$

$$\bar{X} = \frac{\Sigma x}{n} = \frac{\cancel{15}^{3}}{\cancel{5}} = 3 \text{ Ans}$$

E.g - Discreate Series :-

X	f	f(x)
3	4	12
2	1	2
5	3	15
7	2	14
	$\Sigma f = 10$	$\Sigma f(x) = 43$

$$\bar{X} = \frac{\Sigma f(x)}{\Sigma f} = \frac{43}{10} = 4.3 \text{ Ans}$$

E.g - Continuous Series :-

C.I.	f	x	f(x)
0-10	3	5	15
10-20	1	15	15
20-30	2	25	50
30-40	4	35	140
	$\Sigma f = 10$		$\Sigma f(x) = 220$

$$\bar{X} = \frac{\Sigma f(x)}{\Sigma f} = \frac{22\cancel{0}}{1\cancel{0}} = 22 \text{ Ans}$$

* Assume Mean method :-

Step-1 Assume any no as assume mean.
Step-2 Deviation from assume mean.

Deviation - When a particular no is subtracted from all the observations of the series

Step-3. find Sum of deviation from assume mean.

Eg - Individual Series :-

X	$X-A$.
3	$3-2=1$
2 $=A$	$2-2=0$
1	$1-2=-1$
5	$5-2=3$
4	$4-2=2$
	$\Sigma(X-A)=5$

$$\bar{X} = A + \frac{\Sigma(X-A)}{n}$$

$$= 2 + \frac{\cancel{5}\,1}{\cancel{5}}$$

$$= 2+1$$

$$= 3 \text{ Ans}$$

* Discoreate Series :-

X	f	(X-A)	f(X-A)
3 = A	4	0	0
2	1	-1	-1
5	3	2	6
7	2	4	8
	$\Sigma f = 10$		$\Sigma f(X-A) = 13$

$$\bar{X} = A + \frac{\Sigma f(X-A)}{\Sigma f}$$

$$= 3 + \frac{13}{10}$$

$$= 3 + 1.3$$

$$= 4.3$$ Ans

* Continuous Series :-

C.I.	f	x	x-A	f(x-A)
0-10	3	5	-10	-30
10-20	1	15 = A	0	0
20-30	2	25	10	20
30-40	4	35	20	80
	$\Sigma f = 10$			$\Sigma f(x-A) = 70$

$$\bar{X} = A + \frac{\Sigma f(x-A)}{\Sigma f}$$

$$= 15 + \frac{7\cancel{0}}{1\cancel{0}}$$

$$= 15 + 7$$

$$= 22 \text{ Ans}$$

* Step Deviation method :-

Continuous Series :-

C.I.	f	x	d (x-A)	d' = d/i	d'f
0-10	3	5	-10	-10/10 = -1	-3
10-20	1	15 = A	0	0/10 = 0	0
20-30	2	25	10	10/10 = 1	2
30-40	4	35	20	20/10 = 2	8
	$\Sigma f = 10$				$\Sigma d'f = 7$

$$\bar{X} = A + \frac{\Sigma d'f}{\Sigma f} \times i$$

$$= 15 + \frac{7}{\cancel{10}} \times \cancel{10}$$

$$= 22 \text{ Ans}$$

* Median (Middle Most Value) :-

E.g-(1) Individual Series :-

x	Arrange => x
8	2
7	3
3	7
2	8
15	15
17	17
20	20

$n = 7$

$\left(\frac{n+1}{2}\right)$ th term

$\frac{7+1}{2} = \frac{\cancel{8}^{4}}{\cancel{2}} = 4$th term

Median = 8 Ans

E.g-(2)

x	x'
3	3
9	7
11	8
8	9
15	11
7	15

$n = 6$

$\left(\frac{n+1}{2}\right)$ th term => $\frac{6+1}{2} = 3.5$th term

= 3rd term + ·5 (4th term − 3th term)
= 8 + ·5 (9−8)
= 8 + ·5
= 8·5 Ans

* Discreate Series :-

E.g - (1)

x	f	d	F	C.F.
3	4	2	7	7
8	3	3	4	11
7	2	$\boxed{7}$	$\boxed{2}$	$\boxed{13}$
9	5	8	3	16
2	7	9	5	21
11	3	11	3	24

$n = \Sigma f = 24$

$= \left(\frac{n+1}{2}\right)$ th term

$= \frac{24+1}{2} = 12.5$th term

Median = 7 Ans

E.g - (2)

x	f	x'	f'	C.F.
8	3	4	4	4
7	2	7	2	6
9	5	$\boxed{8}$	$\boxed{3}$	$\boxed{9}$
11	6	9	5	14
4	4	11	6	20

$n = \Sigma f = 20$

$= \left(\frac{n+1}{2}\right)$ th term

$= \frac{21}{2}$

$=$ 10.5 th term

$=$ 8 Ans

* Continuous Series :-

C.I.	f	C.F
0-10	3	3 = Cfp
10-20	2	5
20-30	4	9
30-40	1	10

$n = 10$

$\left(\frac{n}{2}\right)$ th term

$\left(\frac{10}{2}\right)$ th term

5 th term

$$\text{Median} = l + \frac{\frac{n}{2} - Cfp}{f} \times i$$

$$= 10 + \frac{5-3}{10} \times 10$$

$$= 10 + 2$$

$$= 12 \text{ Ans}$$

* Mode / Model :-

No with highest frequency

Eg - (1). 5, 2, 4, 5, 6, 5

Mode = 5 Ans

Eg- Discrete Series :-

x	f
10	3
9	2
7	5
11	9
15	11

Highest frequency is 11 and the mode is 15 Ans

Continuous Series :-

C.I.	F
0-10	4
10-20	7 → f_0
$\boxed{20-30}$	$\boxed{9}$ → f_1
30-40	2 → f_2
40-50	1

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$$= 20 + \frac{9-7}{2\times 9 - 7 - 2} \times 10$$

$$= 20 + \frac{2}{9} \times 10$$

$$= 20 + \frac{20}{9}$$

$$= 20 + 2.2$$

$$= 22.2 \text{ Ans}$$

Relationship Between Mean, Median & mode :-

$$\boxed{\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}}$$

* Standard Deviation :-

$$\boxed{\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\overline{x})^2}} \text{ OR } \boxed{\sigma = \sqrt{\frac{(x-\overline{x})^2}{n}}}$$

* Variance :-

$$\boxed{\sigma^2 = \frac{(x-\overline{x})^2}{n}}$$

E.g - ① Find the Standard deviation of the data 11, 14, 15, 17, 18. and find the variance.
Imp

Sol- $\overline{x} = \frac{\Sigma x}{n} = \frac{11+14+15+17+18}{5}$

$= \frac{\cancel{75}\ 15}{\cancel{5}}$

$\overline{x} = 15$

$(x-\overline{x})^2 \Rightarrow (11-15)^2 + (14-15)^2 + (15-15)^2 + (17-15)^2 + (18-15)^2$

$(x-\overline{x})^2 = 16 + 1 + 0 + 4 + 9$
$= 30$

$$S.D = \sigma = \sqrt{\frac{(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{30 6}{\cancel{5}}}$$

$$= \sqrt{6}$$

$$= 2.44 \text{ Ans}$$

Variance $\sigma^2 = \frac{(x-\bar{x})^2}{n}$

$$\sigma^2 = \frac{30 6}{\cancel{5}}$$

$$\sigma^2 = 6 \text{ Ans}$$

* Karl Pearson's Coefficient of Correlation:

$$\boxed{r = \frac{\Sigma XY}{\sqrt{(\Sigma X^2)(\Sigma Y^2)}}}$$

E.g - (1) Calculate the correlation coefficient
Imp between the following data:

Scanned with CamScanner

X	5	9	13	17	21
Y	12	20	25	33	35

Sol- $\bar{X} = \frac{5+9+13+17+21}{5}$

$= \frac{65}{5} = 13$

$\bar{Y} = \frac{12+20+25+33+35}{5}$

$= \frac{125}{5} = 25$

Let $x = (X-\bar{X})$ and $y = (Y-\bar{Y})$.

X	$x=(X-13)$	$x^2=(X-13)^2$	Y	$y=(Y-25)$	$y^2=(Y-25)^2$	xy
5	-8	64	12	-13	169	104
9	-4	16	20	-5	25	20
13	0	0	25	0	0	0
17	4	16	33	8	64	32
21	8	64	35	10	100	80
	$\Sigma x=0$	$\Sigma x^2=160$		$\Sigma y=0$	$\Sigma y^2=358$	$\Sigma xy=236$

$$r = \frac{\Sigma XY}{\sqrt{(\Sigma X^2)(\Sigma Y^2)}}$$

$$= \frac{236}{\sqrt{160 \times 358}}$$

$$= \frac{236}{239.33}$$

$$= 0.986 \text{ Ans}$$

* Spearman's Rank Correlation :-

$$\boxed{r = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}}$$

E.g. - ② Obtain the rank correlation coefficient for the following :-

Imp

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

Sol-

x	y	$x = x'$	$y = y'$	$d = x' - y'$	d^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16

$\Sigma d^2 = 72$

$$\cancel{x = \frac{\Sigma xy}{\sqrt{(\Sigma x^2)(\Sigma y^2)}}}$$

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 72}{10(100-1)}$$

$$\begin{array}{r} 72 \\ \times\ 6 \\ \hline 432 \end{array}$$

10×99

$$= 1 - \frac{432}{990}$$

$$= \frac{990 - 432}{990}$$

$$= \frac{558}{990} = 0.56 \text{ Ans}$$

* Regression :-

E.g. - ②.
Imp

x	y	x^2	y^2	xy
1	15	1	225	15
3	18	9	324	54
5	21	25	441	105
7	23	49	529	161
9	22	81	484	198

$\Sigma x = 25$ $\Sigma y = 99$ $\Sigma x^2 = 165$ $\Sigma y^2 = 2003$ $\Sigma xy = 533$

$$\bar{x} = \frac{\cancel{25}^{5}}{\cancel{5}} = 5$$

$$\bar{y} = \frac{99}{5} = 19.8$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x . \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5 \times 533 - 25 \times 99}{5 \times 165 - (25)^2}$$

$$= \frac{2665 - 2475}{825 - 625}$$

$$= \frac{19\cancel{0}}{20\cancel{0}} = 0.95$$

Regression line y on x :-

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

$$(y-19.8) = 0.95(x-5)$$
$$(y-19.8) = 0.95x - 4.75$$
$$y - 0.95x = -4.75 + 19.8$$
$$y - 0.95x = 15.05$$
$$y = 0.95x + 15.05$$

$$y = 0.95x + 15.05$$
$$= 0.95 \times 4 + 15.05$$
$$= 3.8 + 15.05$$
$$= 18.85 \text{ Ans}$$

[$x = 4$ According the Question]