UNIT - 3
Geometrical Transformation

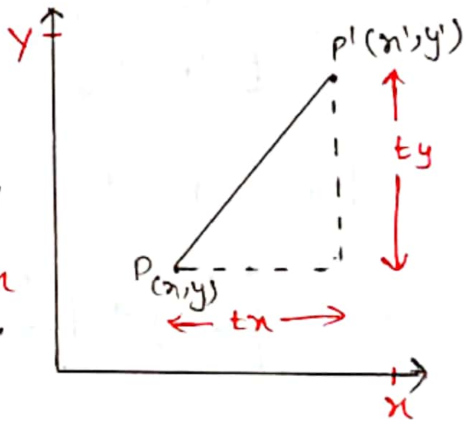
*** Transformation :-** Transformation means changes in orientation, size & shape of the object. It is the Geometric changes of an object from a current state to modified state.

→ Basic Geometric transformation are :-

(i) Translation (ii) Rotation (iii) Scaling

(i) Translation :- It is repositioning of an object from one ~~place~~ position to another is called Translation.

To translate a point from coordinate position $P(x, y)$ to another $P'(x', y')$, we add the algebraically translation distance t_x and t_y to original co-ordinat.



$$\boxed{\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}}$$

t_x and t_y called as shift vector. or Translation vector.

The matrix representation will be :-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Q. Translate the triangle [A(10,10), B(15,15), C(20,10)] 2 unit in x - direction and 1 unit in y direction.

Soln :- we know that $P' = P + T$

$$P' = [P] + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

for point A(10, 10) (x = 10, y = 10)

$$A' = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 10+2 \\ 10+1 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \end{bmatrix}$$

for point B (15, 15)

$$B' = \begin{bmatrix} 15 \\ 15 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 15+2 \\ 15+1 \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \end{bmatrix}$$

for point C (20, 10)

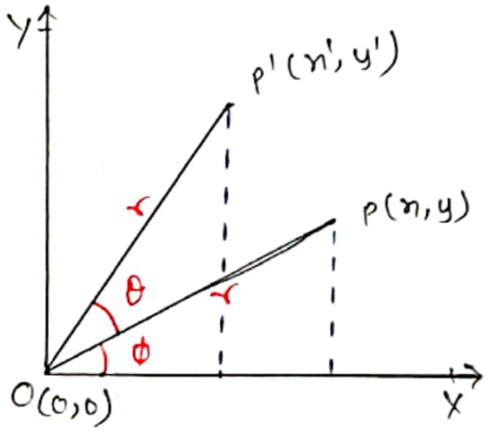
$$C' = \begin{bmatrix} 20 \\ 20 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C' = \begin{bmatrix} 20+2 \\ 20+1 \end{bmatrix} = \begin{bmatrix} 22 \\ 21 \end{bmatrix}$$

Final coordinate after translation are A'(12, 11), B'(17, 16), C'(22, 11)

(ii) **Rotation:-** It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise.

For Rotation we have to specify the angle of rotation and rotation point.



$$\boxed{\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}}$$

$x' = r\cos(\theta + \phi)$

$= r(\cos\theta\cos\phi - \sin\theta\sin\phi)$

$\Rightarrow r\cos\theta\cdot\cos\phi - r\sin\theta\cdot\sin\phi$

$\Rightarrow x\cos\theta - y\sin\theta$

$\qquad \downarrow \qquad\qquad \downarrow$

$\quad r\cos\phi \qquad r\sin\phi$

$y' = \cancel{r(\sin} \; r\sin(\theta + \phi)$

$\Rightarrow r(\sin\theta\cdot\cos\phi + \cos\theta\cdot\sin\phi)$

$\Rightarrow r\sin\theta\cdot\cos\phi + r\cos\theta\cdot\sin\phi$

$\Rightarrow x\sin\theta + y\cos\theta$

$\qquad \downarrow \qquad\qquad \downarrow$

$\quad r\cdot\cos\phi \qquad r\cdot\sin\phi$

$$\boxed{\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}}$$

→ **Matrix Representation:-** $P' = RP$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q. A point P (4,3) is rotated clockwise direction by the angle of 45°. find the rotation matrix R and the resultant points.

Sol^n . $P' = R \cdot P$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{2} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}$$

Q. obtain the final Coordinate after two rotation on point P(6, 9) with rotation angles are 30° and 60° respectively.

$P' = R(\theta_1 + \theta_2) \cdot P$

$$P' = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} \cos(30+60) & -\sin(30+60) & 0 \\ \sin(30+60) & \cos(30+60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ 1 \end{bmatrix}$$

Final co-ordinates after rotations are $(-9, 6)$.

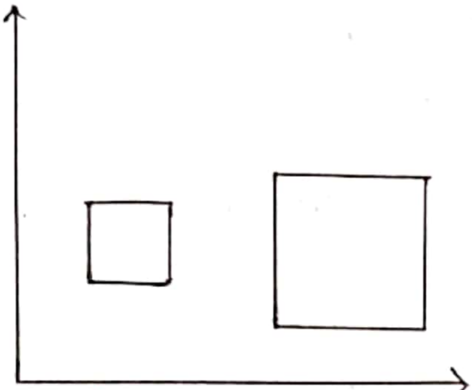
* matrix for Homogeneous Co-ordinate Rotation Clockwise:-

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

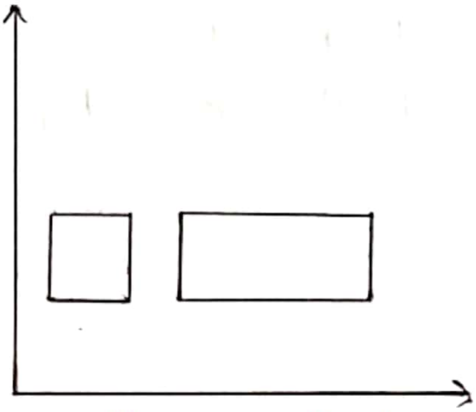
* matrix for Homogeneous Co-ordinate rotation Anticlockwise:-

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) **Scaling :-** Scaling transformation is used to alter the size of given object. The alteration of the size of the object is defined by scaling factor.



Uniform Scaling



Differential Scaling

There are two scaling factors i.e. S_x in x direction, S_y in y direction.

→ **matrix for scaling :-**

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

S_x and S_y is scaling matrix. $\boxed{P' = S_x S_y \cdot P}$

Q. Obtain the final co-ordinates after two scaling on line pq [P(2,2), q(8,8)] with scaling factors are (2,2) and (3,3) respectively.

$$P' = S(S_{x_1} \cdot S_{x_2}, S_{y_1} \cdot S_{y_2}) \cdot P$$

$$p' = \begin{bmatrix} Sx_1 \cdot Sx_2 & 0 & 0 \\ 0 & Sy_1 \cdot Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 2 \cdot 3 & 0 & 0 \\ 0 & 2 \cdot 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 12+0+0 & 48+0+0 \\ 0+12+0 & 0+48+0 \\ 0+0+1 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 12 & 48 \\ 12 & 48 \\ 1 & 1 \end{bmatrix}$$

final co-ordinates are p(12, 12) , q(48, 48)

Q. Scale a polygon with co-ordinate A(2,5) , B(7,10) , C(10,2) by 2 unit in x-direction and 3 unit in y-direction.

$$R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad P = \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

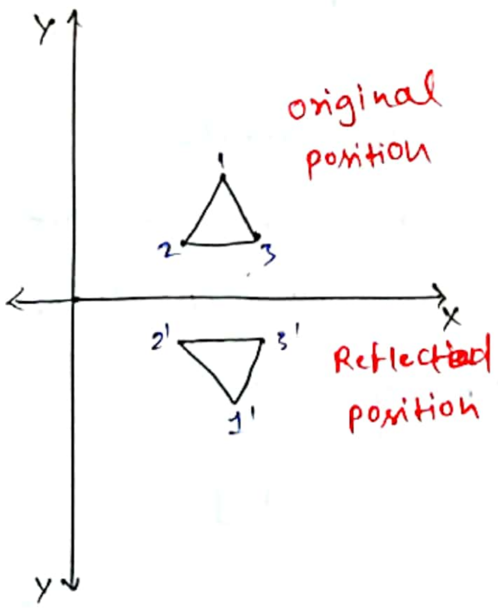
$$p' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 4+0+0 & 14+0+0 & 20+0+0 \\ 0+15+0 & 0+30+0 & 0+6+0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 20 \\ 15 & 30 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

final co-ordinates are A(4,15), B(14,30), C(20,6)

(iv) Reflection :- Reflection is a transformation which produces mirror image of a given object. This translation can be produce Rela to x-axis or y-axis. The axis around which reflection takes place is called angle of Reflection.

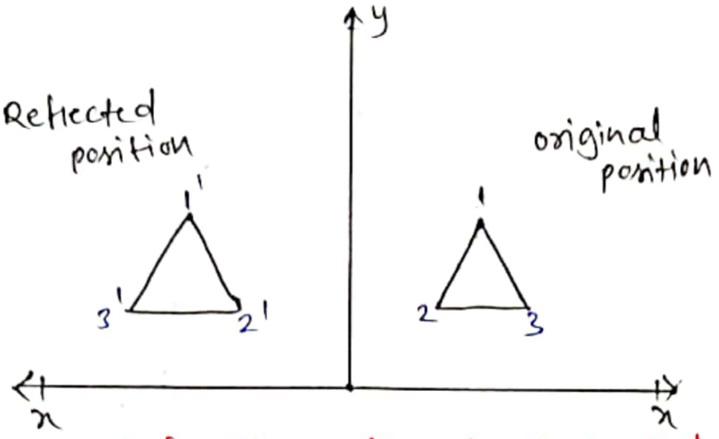


Reflection about x-axis

This transformation keeps x-val are same, but flips (change the sign) y value of coordinate positions.

Transformation Matrix is :-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection About y-axis

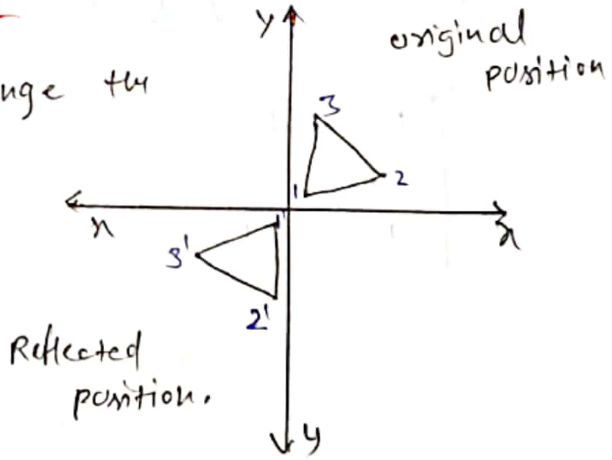
This transformation keeps y values are same, but flips (change the sign) x value of coordinate position.

Transformation Matrix :-

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Reflection About origin :-

This transformation flips (change the sign) x and y both values or co-ordinate position.

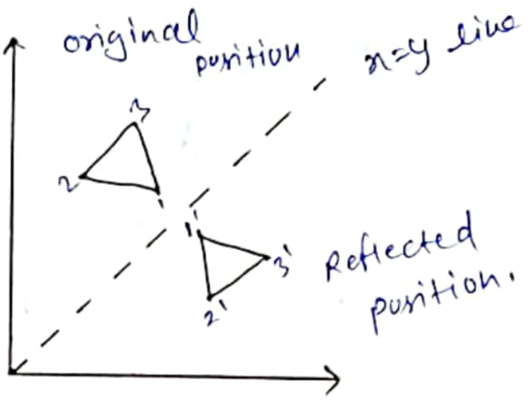


~~Ref~~ Transformation Matrix :-

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Transformation matrix for reflection abute the line.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Q. find the co-ordinate after reflection of the triangle [A(10,10), B(15,15), C(20,10)] about x-axis.

$$P' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 20 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 10 & 15 & 20 \\ -10 & -15 & -10 \\ 1 & 1 & 1 \end{bmatrix}$$

final co-ordinate after reflection are A(10,-10), B(15,-15), C(20,-10)

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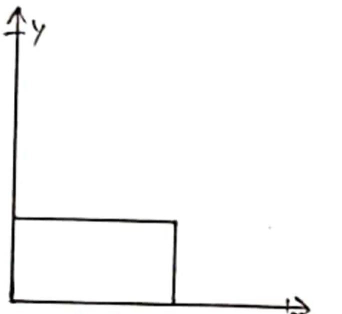
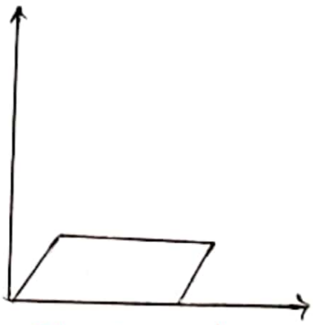
(v) **Shear Transformation** :- It is transformation which changes the shape of the object. The shear can be in one direction or two directions.

→ **Shearing in the x-direction** :-



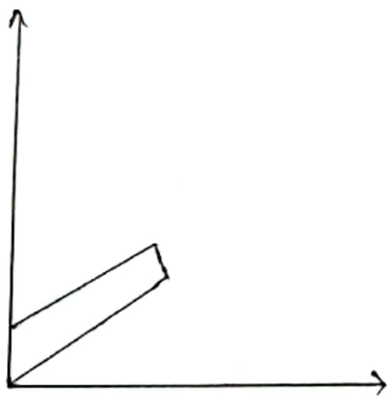
original object



Shear in x-direction

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

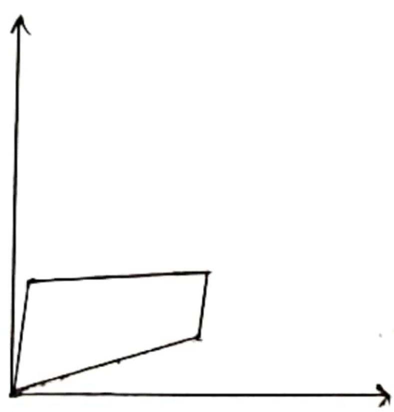
→ **Shearing in y-direction** :-



$$\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ **Shearing in x-y directions** :-

$$\begin{bmatrix} 1 & sh_y & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



* **Homogeneous Co-ordinates:-** To perform a sequence of transformation such as translation followed by rotation and scaling. we need to follow a sequential process like:-

→ First translate the coordinates.

→ Rotate the translated coordinate.

→ Then scale the rotated coordinates to complete the composite transformation.

To shorten this process, we have to use 3x3 transformation matrix instead of 2x2.

* **Composite Transformation:-** A number of transformation or sequence of transformation can be combined into single one called as composition. The resultant matrix called as composite matrix.

Q. Find composite matrix?

(i) Apply translation by $v(1,1)$

(ii) Rotate the translation by $45°$ in anticlockwise direction.

(iii) Scaling the rotated object by $(1, 1/2)$.

→ for translation:-

$$T_1 = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

→ for Rotation:-

$$T_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 \Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ for Scaling:-

$$T_3 = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$T = T_3 \cdot T_2 \cdot T_1$

or

$T = T_1 \cdot T_2 \cdot T_3$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} + (-1/\sqrt{2}) + 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}\ (+1/\sqrt{2}) + 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 * 1/\sqrt{2} & 1/2 * 1/\sqrt{2} & \frac{1}{2} \times \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2\sqrt{2} & 1/2\sqrt{2} & \sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2\sqrt{2} & -1/2\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

* IMQs :-

Q. Find the transformation at triangle A(1,0), B(0,1) C(1,1) by translating 1 unit in x and y direction and then rotate 45' about origin.

Soln ⇒ Translation by 1 unit in x,y direction.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 1 & 1+1 \\ 1 & 1+1 & 1+1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Coordinate after transformation A(2,1), B(1,2), C(2,2)

→ Now Rotation about 45' anticlockwise.
~~about the origin (anticlockwise)~~

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 * 1/\sqrt{2} + \frac{1}{\sqrt{2}} & 1/\sqrt{2} + 2 * \frac{1}{\sqrt{2}} & 2 * 1/\sqrt{2} + 2 * 1/\sqrt{2} \\ -2 * 1/\sqrt{2} + 1/\sqrt{2} & -1/\sqrt{2} + 2 * 1/\sqrt{2} & -2 * 1/\sqrt{2} + 2 * 1/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ \frac{-2}{\sqrt{2}} + \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} + \frac{2}{\sqrt{2}} & \cancel{\frac{-2}{\sqrt{2}}} + \cancel{\frac{2}{\sqrt{2}}} \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{4}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\cancel{\frac{4}{\sqrt{2}}} \Rightarrow \cancel{\frac{2 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}}}$

final coordinates are :- $A\left(\frac{3}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, $B\left(\frac{3}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$,

$C(2\sqrt{2}, 0)$

Q. perform 45' rotation of triangle $A(0,0)$, $B(1,1)$, $C(5,2)$.

(a) About the origin

(b) About the point $(-1,-1)$

Sol^n (a) Rotation About the origin.

$$\begin{bmatrix} x_1' & x_2' & x_3' \\ y_1 & y_2' & y_3' \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x' & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1' & x_2' & x_3' \\ y_1' & y_2' & y_3' \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45' & -\sin 45' & 0 \\ \sin 45' & \cos 45' & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1/\sqrt{2} - 1/\sqrt{2} & 5/\sqrt{2} - 2/\sqrt{2} \\ 0 & 1/\sqrt{2} + 1/\sqrt{2} & 5/\sqrt{2} + 2/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & \sqrt{2} & 7/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

coordinates after Rotation about the origin are:-

$A(0,0)$, $B(0,\sqrt{2})$, $C(3/\sqrt{2}, 7/\sqrt{2})$

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(b) Rotation About the point D(−1,−1)

Step 1:- Take point D to origin. → T_1

Step 2:- perform Rotation About the origin. → T_2

Step 3:- Take point D(−1,−1) back to its position. → T_3

$M = T_3 \cdot T_2 \cdot T_1$

$$T_1 = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0-(-1) \\ 0 & 1 & 0-(-1) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} \cancel{1} & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & \cancel{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \overset{0}{-} 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} + 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now transforming the points A, B, C.

for this we do $M \cdot [ABC]$

$$[A'B'C'] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & \cancel{1/\sqrt{2}} - \cancel{1/\sqrt{2}} - 1 & 5/\sqrt{2} - 2/\sqrt{2} - 1 \\ \sqrt{2}-1 & 1/\sqrt{2} + 1/\sqrt{2} + \sqrt{2}-1 & 5/\sqrt{2} + 2/\sqrt{2} + \sqrt{2}-1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & \frac{\sqrt{3}}{2} - 1 \\ \sqrt{2}-1 & 2\sqrt{2}-1 & 7/\sqrt{2} + \sqrt{2} - 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & \frac{\sqrt{3}-2}{2} \\ \sqrt{2}-1 & 2\sqrt{2}-1 & \sqrt{2}-1 \\ 1 & 1 & 1 \end{bmatrix}$$

final coordinates after rotation About point $(-1, -1)$ are.

$A(-1, \sqrt{2}-1)$, $B(-1, 2\sqrt{2}-1)$, $C\left(\frac{\sqrt{3}-2}{2}, \sqrt{2}-1\right)$

Q. Magnified triangle with the vertices A(0,0), B(1,1) and C(5,2) to twice its size while clipping C(5,2) fixed.

New triangle co-ordinates = positive translation matrix * scaling matrix * Negative translation matrix

for point C(5,2)

for point C(5,2)

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & -10 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -10+5 \\ 0 & 2 & -4+2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now multiply the resultant matrix with triangle matrix.

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

New Coordinate of Triangle $\Rightarrow$ A(-5,-2), B(-3,0), C(5,2)

Q. Translate the polygon with coordinate A(3,6), B(8,11), C(11,3) by 2 unit in x direction and 3 unit in y-direction.

Sol^n $\Rightarrow$ $t_x = 2$, $t_y = 3$

$P' = T \cdot P$

$$P' = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 8 & 11 \\ 6 & 11 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+2 & 8+2 & 11+2 \\ 6+3 & 11+3 & 3+3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 13 \\ 9 & 14 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinate after Translation are :-

A(5,9), B(10,14), ~~C(14,6)~~ ~~C(14,6)~~
C(13,6)

Q. write down the transformation matrix for x & y shear.

transformation matrix for x-shear:-

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ sh_x & 1 \end{bmatrix}$$

Transformation matrix for y-shear:-

$$\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & sh_y \\ 0 & 1 \end{bmatrix}$$

Q. Find the Composite matrix:-

(a) Apply translation by point (1,1) rotate the translation by 45° anticlockwise.

(b) Scale the rotated object by (1, 1/2)

Solⁿ (a) Translating by point (1,1) = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Rotating by 45° anticlockwise = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite matrix $= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

rotated object $\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Ans.

(b) Scale the rotated object by $(1, 1/2)$

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{2} & 1 \\ & & \\ & & \end{bmatrix}$$

Composite matrix :- $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} + 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} + 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Rotated object $= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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(b) Scale the rotated object by (1, 1/2)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/2\sqrt{2} & 1/2\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$