

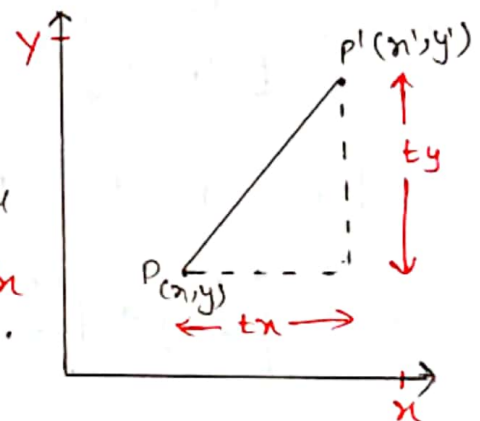
* Transformation:- Transformation means changes in orientation, size & shape of the object. It is the geometric changes of an object from a current state to modified state.

→ Basic Geometric transformation are:-

(i) Translation (ii) Rotation (iii) Scaling

(i) Translation:- It is repositioning of an object from one ~~place~~ ^{position} to another is called Translation.

To translate a point from coordinate position $P(x, y)$ to another $P'(x', y')$, we add the algebraically translation distance t_x and t_y to original co-ordinates.



$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}$$

t_x and t_y called as shift vector. or Translation vector.

The matrix representation will be:-

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Q. Translate the triangle $[A(10,10), B(15,15), C(20,10)]$ 2 unit in x -direction and 1 unit in y direction.

Solⁿ:- we know that $P' = P + T$

$$P' = [P] + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

for point $A(10,10)$ ($x=10, y=10$)

$$A' = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 10+2 \\ 10+1 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \end{bmatrix}$$

for point $B(15,15)$

$$B' = \begin{bmatrix} 15 \\ 15 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 15+2 \\ 15+1 \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \end{bmatrix}$$

for point $C(20,10)$

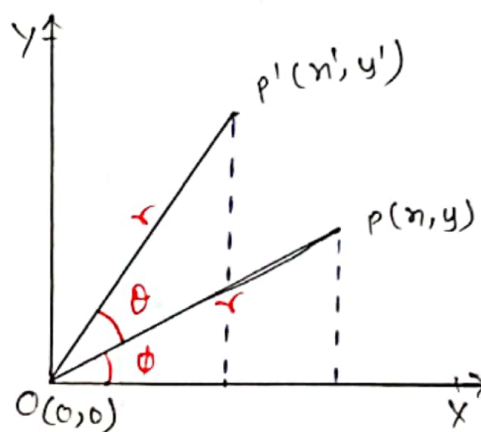
$$C' = \begin{bmatrix} 20 \\ 20 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C' = \begin{bmatrix} 20+2 \\ 20+1 \end{bmatrix} = \begin{bmatrix} 22 \\ 21 \end{bmatrix}$$

Final coordinate after translation are $A'(12,11)$, $B'(17,16)$, $C'(22,21)$

(ii) Rotation:- It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise. For Rotation we have to specify the angle of rotation and rotation point.

$$\begin{aligned}x' &= r \cos(\theta + \phi) \\&= r(\cos\theta \cos\phi - \sin\theta \sin\phi) \\&\Rightarrow r \cos\theta \cdot \cos\phi - r \sin\theta \cdot \sin\phi \\&\Rightarrow x \cos\theta - y \sin\theta \\&\quad \downarrow \quad \quad \downarrow \\&\quad r \cos\phi \quad \quad r \sin\phi\end{aligned}$$



$$\begin{aligned}x &= r \cos\theta \\y &= r \sin\theta\end{aligned}$$

$$\begin{aligned}y' &= r \sin(\theta + \phi) \\&\Rightarrow r(\sin\theta \cos\phi + \cos\theta \sin\phi) \\&\Rightarrow r \sin\theta \cdot \cos\phi + r \cos\theta \cdot \sin\phi \\&\Rightarrow x \sin\theta + y \cos\theta \\&\quad \downarrow \quad \quad \downarrow \\&\quad r \cdot \cos\phi \quad \quad r \cdot \sin\phi\end{aligned}$$

$$\begin{aligned}x' &= x \cos\theta - y \sin\theta \\y' &= x \sin\theta + y \cos\theta\end{aligned}$$

→ Matrix Representation:-

$$P' = R P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q. A point $P(4,3)$ is rotated clockwise direction by the angle of 45° . Find the rotation matrix R and the resultant points.

Solⁿ . $P' = R \cdot P$

$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} \end{aligned}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}$$

Q. Obtain the final coordinate after two rotations on point $P(6,9)$ with rotation angles are 30° and 60° respectively.

$$P' = R(\theta_1 + \theta_2) \cdot P$$

$$P' = \begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} \cos(30+60) & -\sin(30+60) & 0 \\ \sin(30+60) & \cos(30+60) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ 1 \end{bmatrix}$$

final Co-ordinates after rotations are $(-9, 6)$.

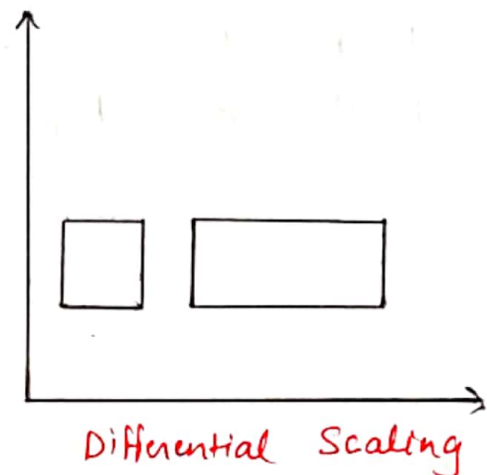
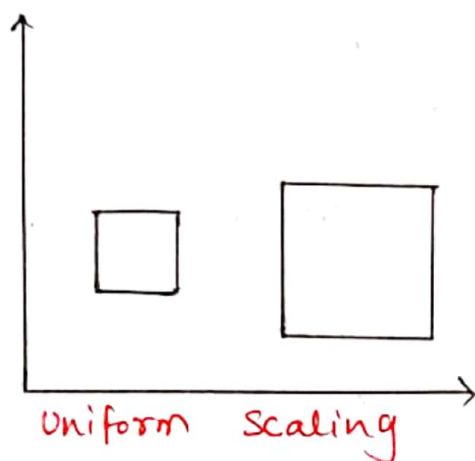
* matrix for Homogeneous Co-ordinate Rotation Clockwise:-

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* matrix for Homogeneous Co-ordinate rotation Anticlockwise:-

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) Scaling:- Scaling transformation is used to alter the size of given object. The alteration of the size of the object is defined by Scaling factor.



There are two scaling factors i.e. S_x in x direction, S_y in y direction.

→ matrix for scaling:-

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

S_x and S_y is scaling matrix, $P' = S_x S_y \cdot P$

Q. obtain the final co-ordinates after two scaling on line pq [$P(2,2)$, $Q(8,8)$] with scaling factors are $(2,2)$ and $(3,3)$ respectively.

$$P' = S(S_{x1} \cdot S_{x2}, S_{y1} \cdot S_{y2}) \cdot P$$

$$P^1 = \begin{bmatrix} s_{x1} \cdot s_{x2} & 0 & 0 \\ 0 & s_{y1} \cdot s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$P^1 = \begin{bmatrix} 2 \cdot 3 & 0 & 0 \\ 0 & 2 \cdot 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$P^1 = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 2 & 8 \\ 1 & 1 \end{bmatrix}$$

$$P^1 = \begin{bmatrix} 12+0+0 & 48+0+0 \\ 0+12+0 & 0+48+0 \\ 0+0+1 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 12 & 48 \\ 12 & 48 \\ 1 & 1 \end{bmatrix}$$

final co-ordinates are $p(12, 12)$, $q(48, 48)$

Q. Scale a polygon with co-ordinate $A(2, 5)$, $B(7, 10)$, $C(10, 2)$ by 2 unit in x-direction and 3 unit in y-direction.

$$R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

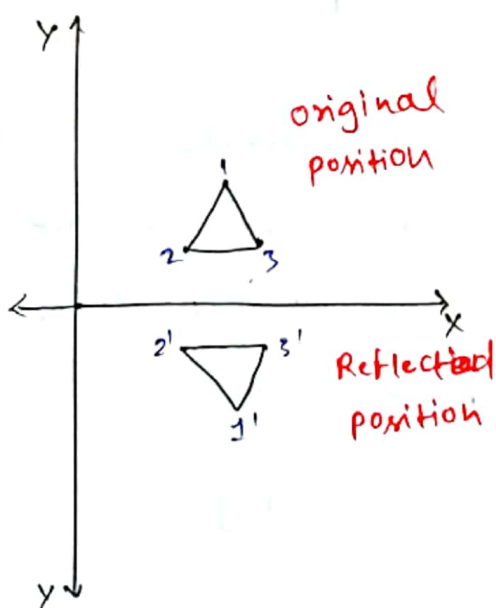
$$P^1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 7 & 10 \\ 5 & 10 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 4+0+0 & 14+0+0 & 20+0+0 \\ 0+15+0 & 0+30+0 & 0+6+0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 20 \\ 15 & 30 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinates are A(4,15), B(14,30), C(20,6)

(iv) Reflection:- Reflection is a transformation which produces mirror image of a given object. This translation can be produce Relate to x-axis or y-axis. The axis around which reflection takes place is called angle of reflection.

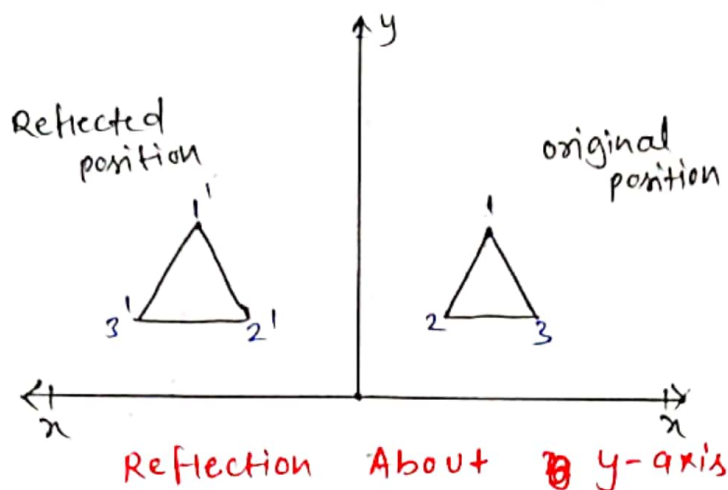


Reflection about x-axis

This transformation keeps x-values same, but flips (change the sign) y value of coordinate positions.

Transformation Matrix is:-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



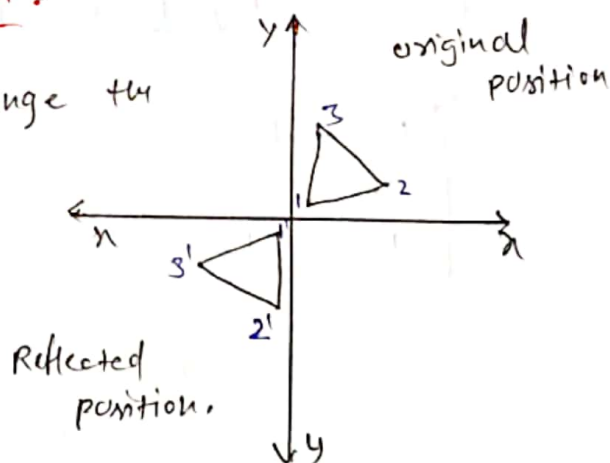
This transformation keeps y values are same, but flips (change the sign) x value of coordinate position.

Transformation Matrix :-

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Reflection About origin :-

This transformation flips (change the sign) x and y both values or co-ordinate position.



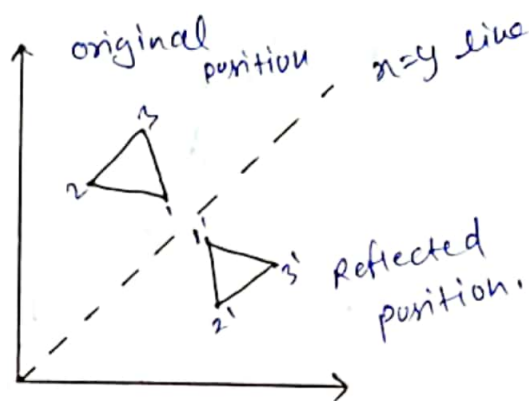
~~Ref~~

Transformation Matrix :-

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Transformation matrix for reflection about the line.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Q. Find the co-ordinate after reflection of the triangle $[A(10, 10), B(15, 15), C(20, 10)]$ about x -axis.

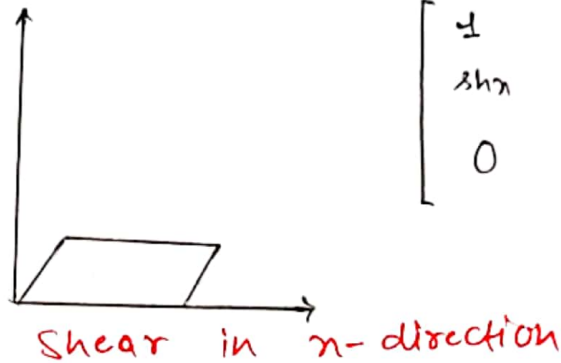
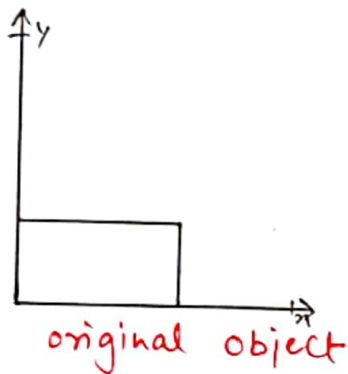
$$P' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 10 & 15 & 20 \\ 10 & 15 & 20 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 10 & 15 & 20 \\ -10 & -15 & -10 \\ 1 & 1 & 1 \end{bmatrix}$$

final co-ordinate after reflection are $A(10, -10)$,
 $B(15, -15)$, $C(20, -10)$

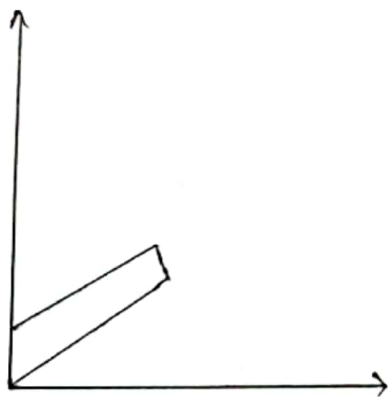
(v) Shear Transformation:- It is transformation which changes the shape of the object. The shear can be in one direction or two directions.

→ Shearing in the x-direction:-



$$\begin{bmatrix} 1 & 0 & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

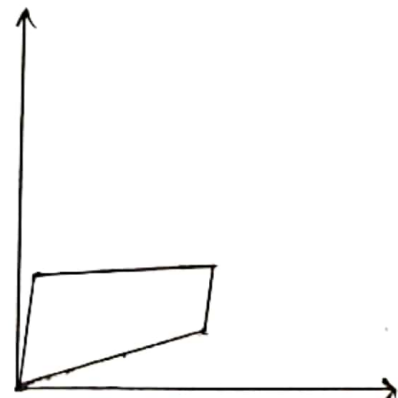
→ Shearing in y-direction:-



$$\begin{bmatrix} 1 & sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Shearing in x-y directions:-

$$\begin{bmatrix} 1 & sh_y & 0 \\ sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



* Homogeneous Co-ordinates:- To perform a sequence of transformation such as translation followed by rotation and scaling. we need to follow a sequential process like:-

- First translate the coordinates.
- Rotate the translated coordinate.
- Then scale the rotated coordinates to complete the composite transformation.

To shorten this process, we have to use 3×3 transformation matrix instead of 2×2 .

* Composite Transformation:- A number of transformation or sequence of transformation can be combined into single one called as Composition. The resultant matrix called as Composite matrix.

Q. Find Composite matrix?

- (i) Apply translation by $T(1,1)$
- (ii) Rotate the translation by 45° in anticlockwise direction.
- (iii) Scaling the rotated object by $(1, 1/2)$.

→ for translation:-

$$T_1 = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

→ for Rotation:-

$$T_2 = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 \Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ for scaling:-

$$T_3 = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = T_3 \cdot T_2 \cdot T_1$$

$$\text{or } T = T_1 \cdot T_2 \cdot T_3$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} + (-1/\sqrt{2}) + 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} + (1/\sqrt{2}) + 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 \times 1/\sqrt{2} & 1/2 \times 1/\sqrt{2} & \frac{1}{2} \times \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2\sqrt{2} & 1/2\sqrt{2} & \sqrt{2}/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2\sqrt{2} & 1/2\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ 1/2 & 1/2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

* ImQ₁ :-

Q. find the transformation of triangle A(1,0), B(0,1), C(1,1) by translating 1 unit in x and y direction and then rotate 45° about origin.

Solⁿ ⇒ Translation by 1 unit in x, y direction.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 1 & 1+1 \\ 1 & 1+1 & 1+1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Coordinate after transformation A(2,1), B(1,2), C(2,2)

→ Now Rotation about 45° anticlockwise.

~~about the origin (anticlockwise)~~

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \times 1/\sqrt{2} + 1/\sqrt{2} & 1/\sqrt{2} + 2 \times 1/\sqrt{2} & 2 \times 1/\sqrt{2} + 2 \times 1/\sqrt{2} \\ -2 \times 1/\sqrt{2} + 1/\sqrt{2} & -1/\sqrt{2} + 2 \times 1/\sqrt{2} & -2 \times 1/\sqrt{2} + 2 \times 1/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ -\frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} & -\frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{4}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\begin{array}{|l} \frac{4}{\sqrt{2}} \\ \frac{2 \times 1/\sqrt{2}}{\sqrt{2}} \end{array}$$

Final coordinates are:- $A(\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$, $B(\frac{3}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$,
 $C(2\sqrt{2}, 0)$

Q. perform 45° rotation of triangle $A(0,0)$, $B(1,1)$, $C(5,2)$.

(a) About the origin

(b) About the point $(-1,-1)$

Solⁿ (a) Rotation About the origin.

$$\begin{bmatrix} x_1' & x_2' & x_3' \\ y_1' & y_2' & y_3' \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1' & x_2' & x_3' \\ y_1' & y_2' & y_3' \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1/\sqrt{2} - 1/\sqrt{2} & 5/\sqrt{2} - 2/\sqrt{2} \\ 0 & 1/\sqrt{2} + 1/\sqrt{2} & 5/\sqrt{2} + 2/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & \sqrt{2} & 7/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Coordinates after rotation about the origin are:-

$$A(0,0), B(0,\sqrt{2}), C(3/\sqrt{2}, 7/\sqrt{2})$$

⑥ Rotation About the point $D(-1, -1)$

Step 1:- Take point D to origin. $\rightarrow T_1$

Step 2:- perform Rotation About the origin. $\rightarrow T_2$

Step 3:- Take point $D(-1, -1)$ back to its position. $\rightarrow T_3$

$$M = T_3 \cdot T_2 \cdot T_1$$

$$T_1 = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0-(-1) \\ 0 & 1 & 0-(-1) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} - 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} + 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Magnified triangle with the vertices $A(0,0)$, $B(1,1)$ and $C(5,2)$ to twice its size while clipping $C(5,2)$ fixed.

New triangle co-ordinates = positive translation matrix \uparrow * scaling matrix * Negative translation matrix \downarrow
 for point $C(5,2)$
 for point $C(5,2)$

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & -10 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -10+5 \\ 0 & 2 & -4+2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now multiply the resultant matrix with triangle matrix.

$$M = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now transforming the points A, B, C.

for this we do $M \cdot [ABC]$

$$[A'B'C'] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1/\sqrt{2} - 1/\sqrt{2} - 1 & 5\sqrt{2} - 2\sqrt{2} - 1 \\ \sqrt{2}-1 & 1/\sqrt{2} + 1/\sqrt{2} + \sqrt{2}-1 & 5\sqrt{2} + 2\sqrt{2} + \sqrt{2}-1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & \frac{\sqrt{2}}{2} - 1 \\ \sqrt{2}-1 & 2\sqrt{2}-1 & 7\sqrt{2} + \sqrt{2} - 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & \frac{\sqrt{2}-2}{2} \\ \sqrt{2}-1 & 2\sqrt{2}-1 & 8\sqrt{2}-1 \\ 1 & 1 & 1 \end{bmatrix}$$

Final coordinates after rotation about point $(-1, -1)$ are.

$$A(-1, \sqrt{2}-1), B(-1, 2\sqrt{2}-1), C\left(\frac{\sqrt{2}-2}{2}, \sqrt{2}-1\right)$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

New Coordinate of Triangle $\Rightarrow A(-5, -2), B(-3, 0), C(5, 2)$

Q. Translate the polygon with coordinate $A(3, 6), B(8, 11), C(11, 3)$ by 2 unit in x direction and 3 unit in y-direction.

Solⁿ $\Rightarrow t_x = 2, t_y = 3$

$$P' = T \cdot P$$

$$P' = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 8 & 11 \\ 6 & 11 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3+2 & 8+2 & 11+2 \\ 6+3 & 11+3 & 3+3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 13 \\ 9 & 14 & 6 \\ 1 & 1 & 1 \end{bmatrix}$$

Final co-ordinate after translation are:-

$A(5, 9), B(10, 14), C(13, 6)$

Q. write down the transformation matrix for x & y shear.

Transformation matrix for x -shear:-

$$\begin{bmatrix} 1 & 0 & 0 \\ shx & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 \\ shx & 1 \end{bmatrix}$$

Transformation matrix for y -shear:-

$$\begin{bmatrix} 1 & shy & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & shy \\ 0 & 1 \end{bmatrix}$$

Q. find the Composite matrix:-

(a) Apply translation by point $(1,1)$ rotate the translation by 45° anticlockwise.

(b) Scale the rotated object by $(1, 1/2)$

Solⁿ (a) Translating by point $(1,1) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Rotating by 45° anticlockwise = $\begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

rotated object $\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Ans.

(b) Scale the rotated object by $(1, 1/2)$

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ -1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/2\sqrt{2} & 1 \\ -1/\sqrt{2} & 1/2\sqrt{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Composite matrix :- $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} + 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} + 1/\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$

Rotated object $\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⑥ Scale. the rotated object by $(1, 1/2)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \sqrt{2} \\ -1/2\sqrt{2} & 1/2\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$