실습-DTFT

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실습 소개

- 랜덤신호와 음성신호 등의 이산시간푸리에변환(Discrete-Time Fourier Transform, DTFT)을 구해본다. 이를 통해, DTFT가 신호의 주파수 성분 분석에 유용하게 사용됨을 알 수 있다.
- DTFT의 시간이동(time shifting)과 주파수이동(frequency shifting) 특성들을 이해한다.
- 주의할 점: MATLAB 프로그램에서 행렬(벡터)의 차원

실습 소개

지정된 구간 내의 난수

구간 (-5,5)에 균일하게 분포된 난수로 구성된 10×1 열 벡터를 생성합니다.

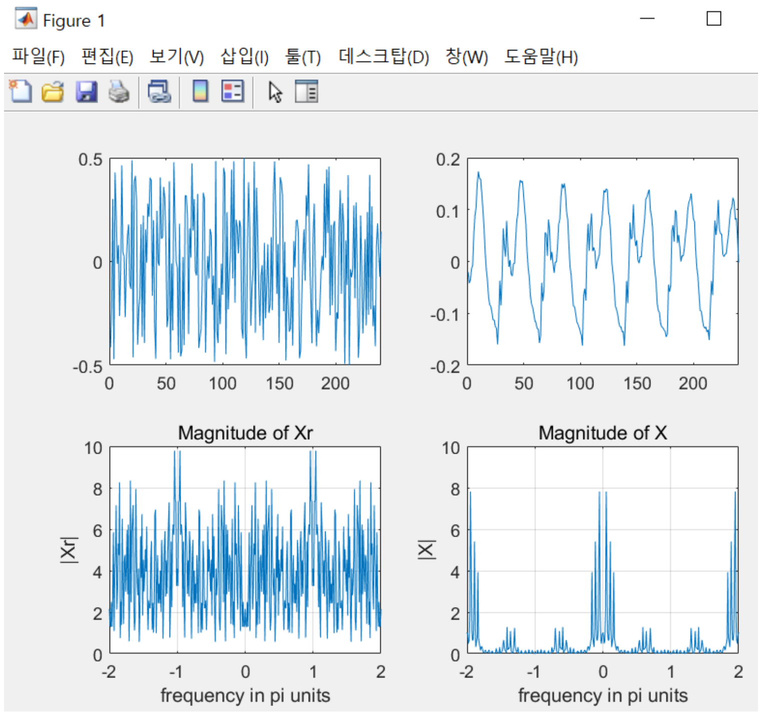
```
r = -5 + (5+5)*rand(10,1)
```

```
r = 10×1

    3.1472
    4.0579
   -3.7301
    4.1338
    1.3236
   -4.0246
   -2.2150
    0.4688
    4.5751
    4.6489
```

일반적으로 식 `r = a + (b-a).*rand(N,1)`을 사용하여 구간 (a,b)에 N개의 난수를 생성할 수 있습니다.

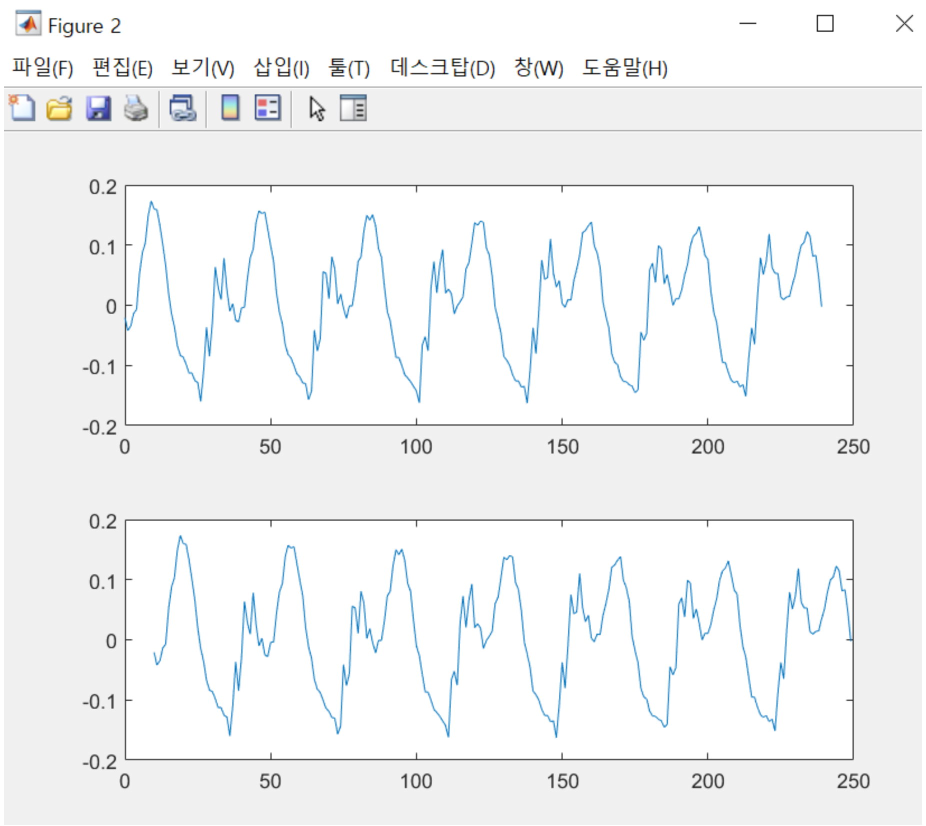
랜덤신호와 음성신호의 DTFT

```
clear; close all; clc

N = 240;
xr =                 ; % rand() 함수 사용. (-0.5, 0.5) uniform 분포

[input_x, Fs] = audioread('8k16bit.wav');
bp = 11001;
x = input_x(bp:bp+N-1); % a part of speech. 240 samples (30 msec)

M = 100;
n = 0:N-1;
k =           ;  w =           ; % frequency: -2*pi ~ 2*pi

W = exp(-1i*pi/M*(k'*n)); % 변환 행렬

figure(1)
Xr =          ;    % 랜덤신호 xr의 DTFT
X =           ;    % 음성신호 x의 DTFT
subplot(2,2,1); plot(xr); % random signal
subplot(2,2,2); plot(x); % speech signal
subplot(2,2,3); plot(w/   ,abs(Xr)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|Xr|')
title('Magnitude of Xr')
subplot(2,2,4); plot(w/   ,abs(X)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|X|')
title('Magnitude of X')
```



Time Shifting

```
%
% Time shifting
%
y = x; m =        ;   % time shift: 10 samples
W =                      ; % 변환 행렬
Y =              ;    % DTFT of y

% Graphical verification
figure(2)
subplot(2,1,1); plot(n, x)
subplot(2,1,2); plot(m, y)
```



The properties of the DTFT

- Time shifting
  - A shift in the time domain corresponds to the phase shifting.

$$x(n-n_0) \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})\, e^{-j\omega n_0}$$

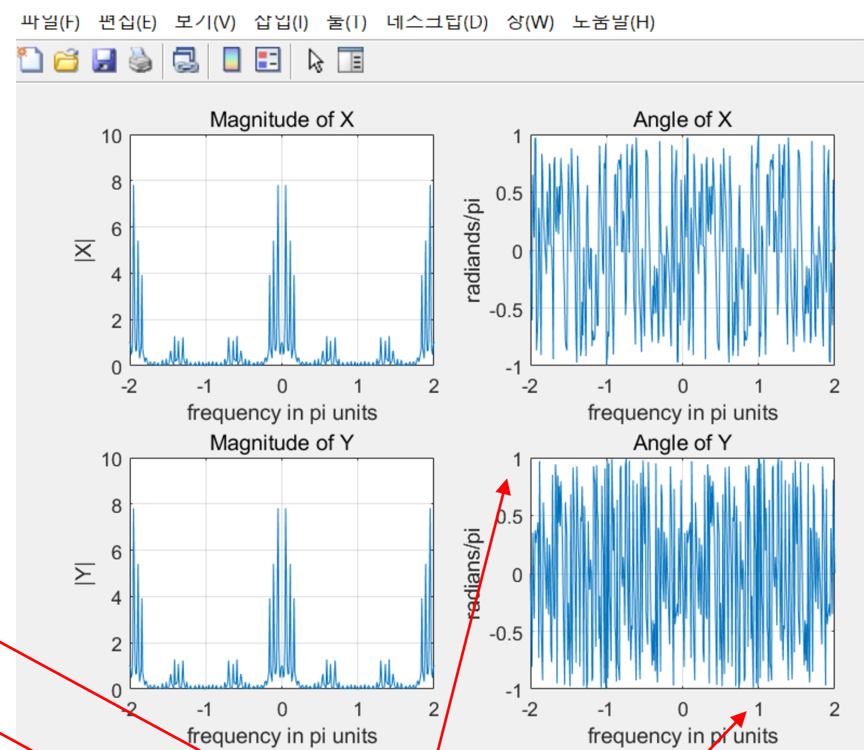
Proof :

$$\sum_{n=-\infty}^{\infty} x(\underbrace{n-n_0}_{m})e^{-j\omega n} = \sum_{m=-\infty}^{\infty} x(m)e^{-j\omega(m+n_0)} = e^{-j\omega n_0}\sum_{m=-\infty}^{\infty} x(m)e^{-j\omega m}$$

Time Shifting

$$x(n - n_0) \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j\omega})\, e^{-j\omega n_0}$$

```
figure(3)
subplot(2,2,1); plot(w/pi,abs(X)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|X|')
title('Magnitude of X')
subplot(2,2,2); plot(w/pi,angle(X)/pi); grid; axis([-2,2,-1,1])
xlabel('frequency in pi units'); ylabel('radiands/pi')
title('Angle of X')
subplot(2,2,3); plot(w/pi,abs(Y)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|Y|')
title('Magnitude of Y')
subplot(2,2,4); plot(w/pi,angle(Y)/pi); grid; axis([-2,2,-1,1])
xlabel('frequency in pi units'); ylabel('radians/pi')
title('Angle of Y')
```



여기가 pi에 해당함

The properties of the DTFT

- Frequency shifting
  - Multiplication by a complex exponential corresponds to a shift in the frequency domain.

$$x(n)\, e^{j\omega_0 n} \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)})$$

Proof :

$$\sum_{n=-\infty}^{\infty} x(n) e^{j\omega_0 n} e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(n) e^{-j(\omega-\omega_0)n} = X(e^{j(w-\omega_0)})$$

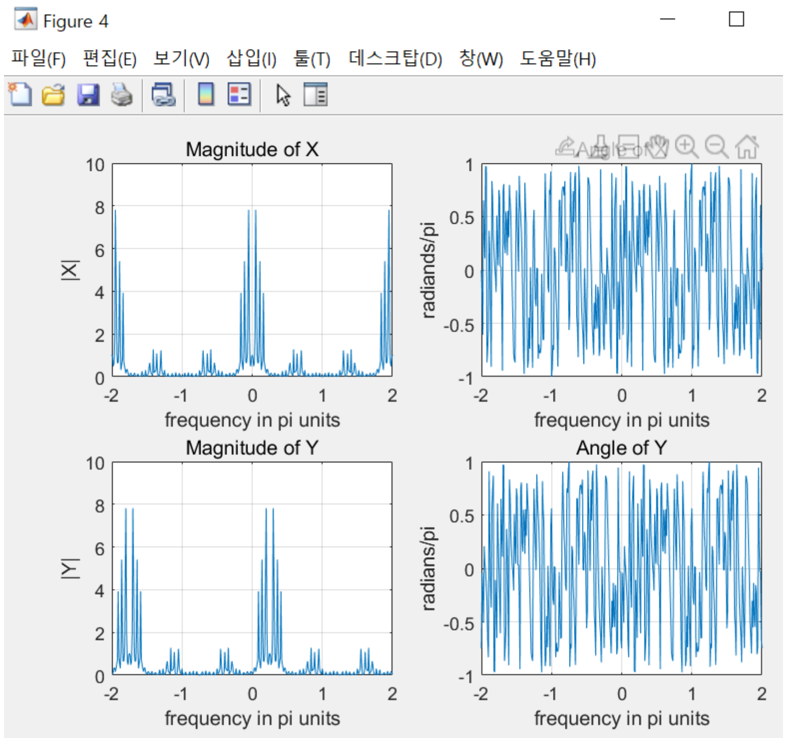
$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\omega n}$$

Frequency Shifting

element by element 연산

$$x(n)\, e^{j\omega_0 n} \overset{\text{DTFT}}{\longleftrightarrow} X(e^{j(\omega-\omega_0)})$$

```
%
% Frequency shifting
%
y =            .*x; % multiplied by a complex exponential signal
W =                  ; % 변환 행렬
Y =        ;    % DTFT of y

% Graphical verification
figure(4)
subplot(2,2,1); plot(w/pi,abs(X)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|X|')
title('Magnitude of X')
subplot(2,2,2); plot(w/pi,angle(X)/pi); grid; axis([-2,2,-1,1])
xlabel('frequency in pi units'); ylabel('radiands/pi')
title('Angle of X')
subplot(2,2,3); plot(w/pi,abs(Y)); grid; axis([-2,2,0,10])
xlabel('frequency in pi units'); ylabel('|Y|')
title('Magnitude of Y')
subplot(2,2,4); plot(w/pi,angle(Y)/pi); grid; axis([-2,2,-1,1])
xlabel('frequency in pi units'); ylabel('radians/pi')
title('Angle of Y')
```



$\frac{\pi}{4}$ 만큼 오른쪽으로 이동함