Permeability

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Permeability

- Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy

Permeability

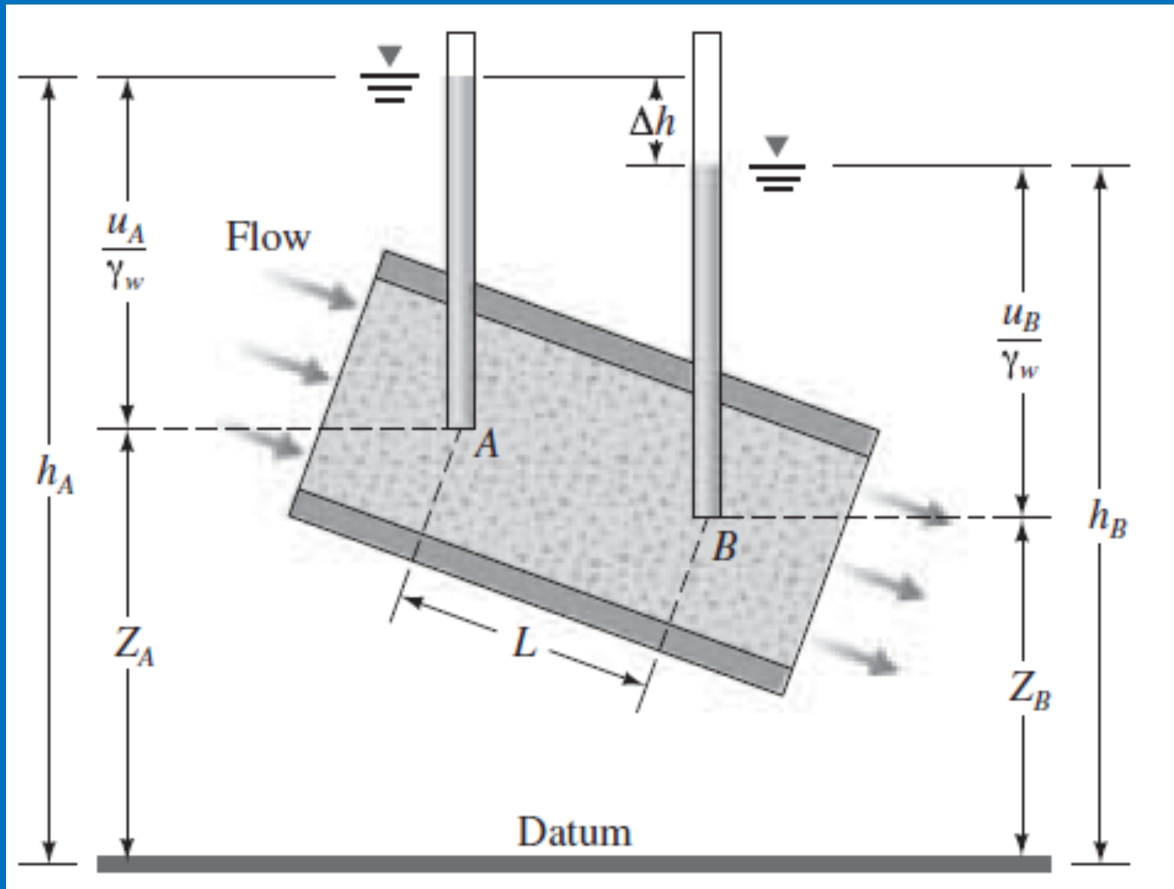

Fig. Pressure, elevation, and total heads for flow of water through soil

Permeability

- The head loss, Δh can be expressed as,

$$i = \Delta h / L \qquad (1.1)$$

Where,

i = hydraulic gradient

L = distance between points A and B

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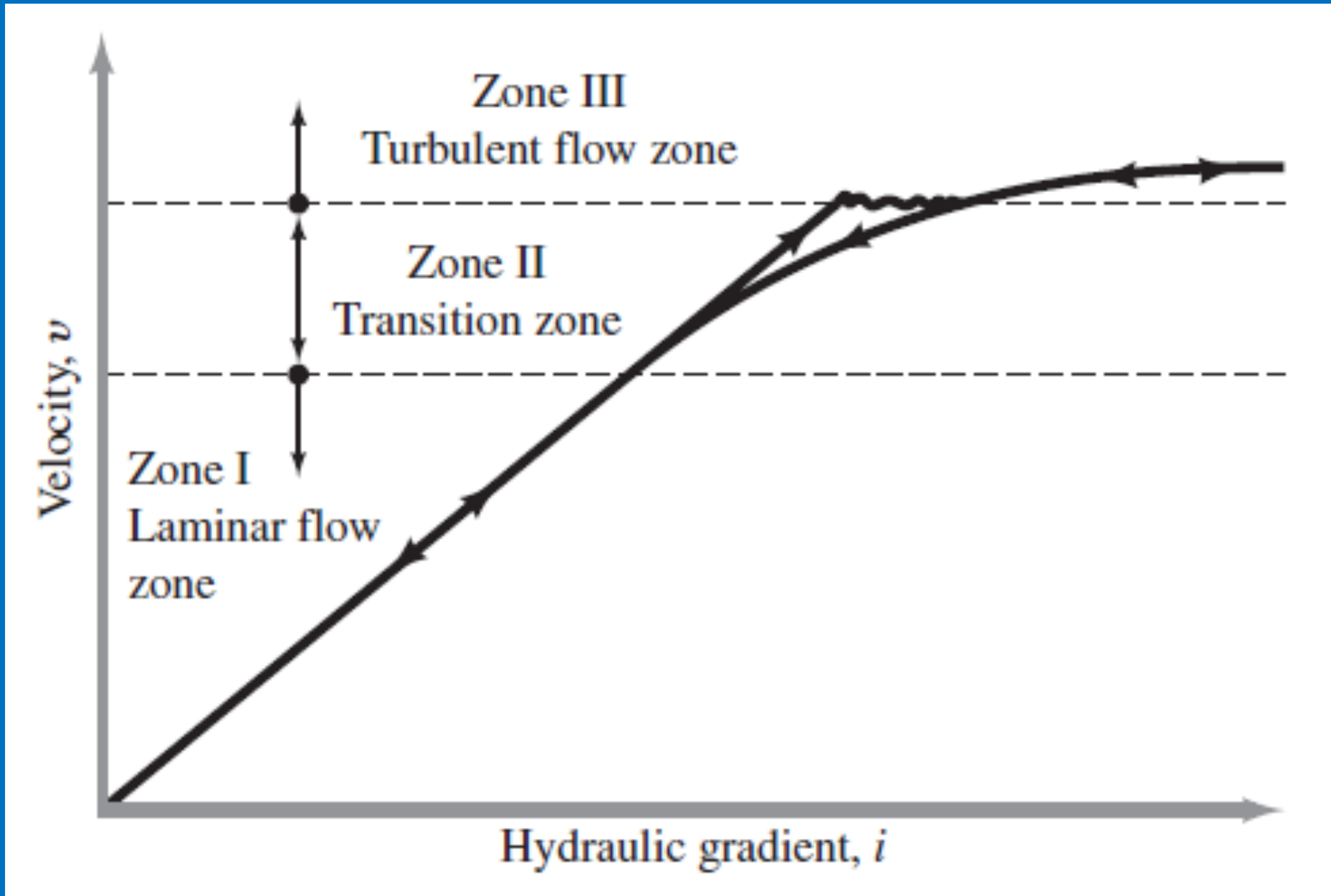

Fig. Variation of velocity, v with i

Permeability

- In most of the soil, the flow of water through the void spaces can be considered laminar, thus

$$v \propto i \qquad (1.2)$$

- In the fractured rock, stones, gravels and very coarse sands, turbulent flow conditions may exist and equation (1.2) may not be valid

Permeability

Darcy's Law

- The law says,

$$v = ki \qquad (1.3)$$

Where,

v = discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross sectional area of soil at right angles to the direction of flow

k = hydraulic conductivity or co-efficient of permeability

Permeability

Darcy's Law

- In equation (1.3), v is the discharge velocity based on the gross cross sectional area of the soil, However, the actual velocity of water, i.e., seepage velocity through the void space is greater than v

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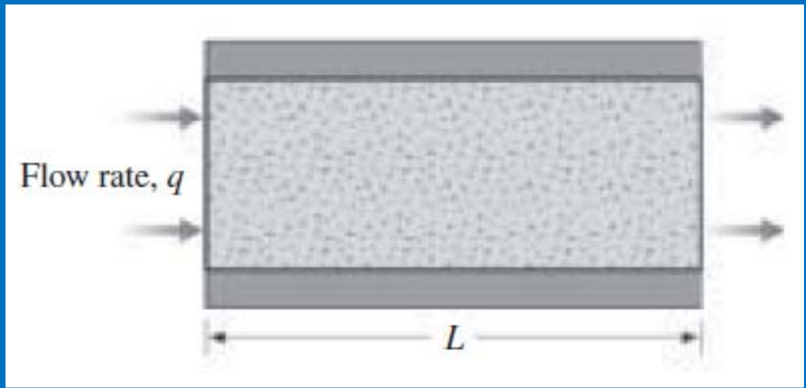
Darcy's Law



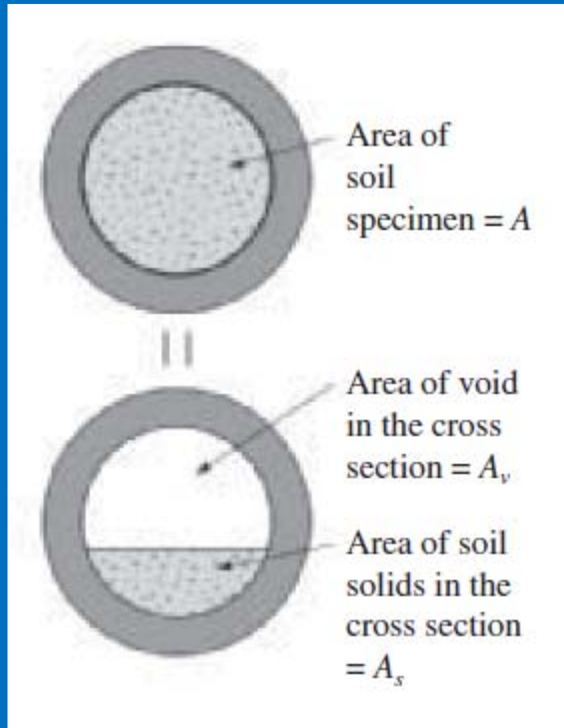


Fig. Sectional view of soil specimen

Permeability

Darcy's Law

- Now, if the quantity of water flowing through the soil in unit time is q then,

$$q = vA = v_s A_v \qquad (1.4)$$

Where,

v_s = seepage velocity

However,

$$A = A_v + A_s$$

Permeability

Darcy's Law

$$q = v(A_v + A_s) = v_s A_v$$

$$v_s = [v(A_v + A_s)]/A_v = [v(A_v + A_s)L]/(A_v L)$$

or, $$v_s = [v(V_v + V_s)]/V_v \qquad (1.5)$$

Where,

V_v = volume of voids

V_s = volume of solids

$$v_s = v[1 + V_v/V_s]/(V_v/V_s) = v(1+e)/e = v/n \qquad (1.6)$$

Permeability

Factors affecting hydraulic conductivity

Hydraulic conductivity depends on several factors

- **Fluid viscosity**
- **Pore size distribution**
- **Grain size distribution**
- **Void ratio**
- **Roughness of mineral particles**
- **Degree of saturation**

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Factors affecting hydraulic conductivity

It is conventional to express k at a temperature of 20°C.

So,

$$k_{20°C} = (\eta_{T°C}/\eta_{20°C})k_{T°C} \qquad (1.7)$$

Where, η = viscosity

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Laboratory determination of hydraulic conductivity

Constant head test

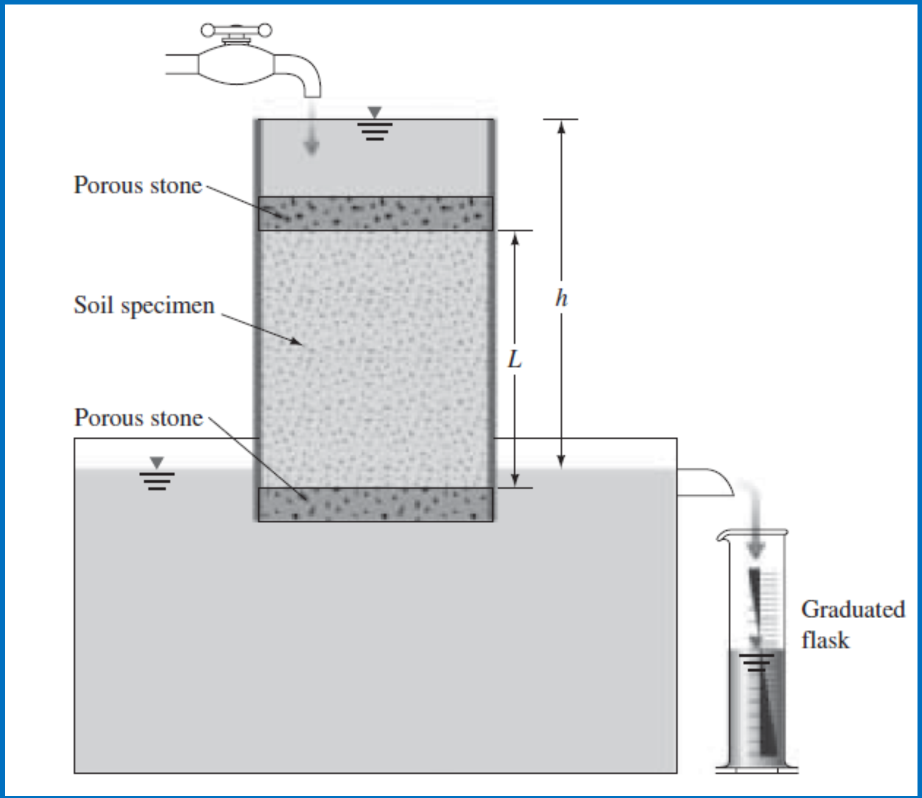

Fig. Constant head test set-up

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Laboratory determination of hydraulic conductivity

Constant head test

Total volume of water collected

$$Q = A*v*t = A*(ki)*t \qquad (1.8)$$

Where,

Q = volume of water collected

A = Area of cross section of soil specimen

t = duration of water collection

$i = h/L$, L = Length of specimen

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Laboratory determination of hydraulic conductivity

Constant head test

Therefore, $Q = A*(kh/L)*t$ (1.9)

Or, $k = QL/Aht$ (1.10)

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Laboratory determination of hydraulic conductivity

Falling head test

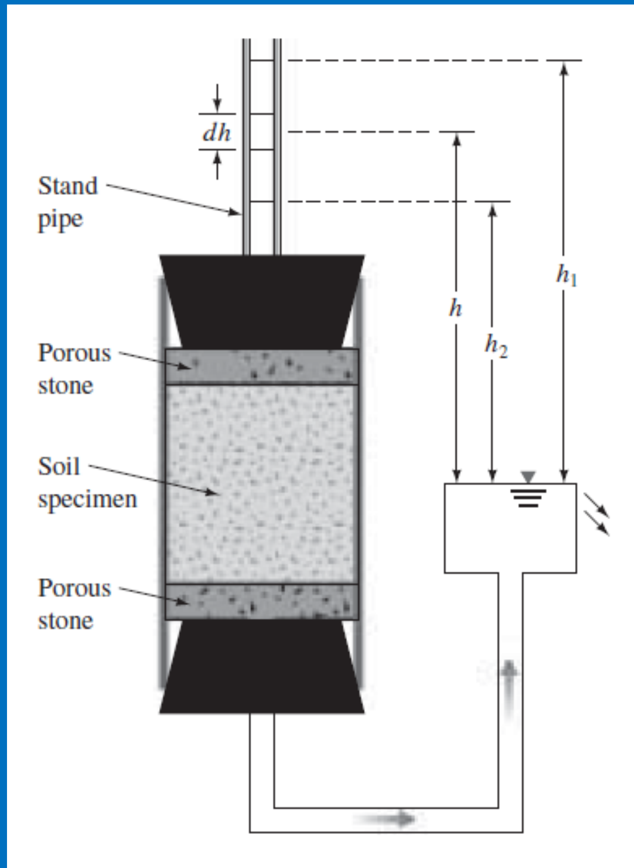

Fig. Falling head test setup

Permeability

Laboratory determination of hydraulic conductivity

Falling head test

- The initial head difference h_1 at time $t = 0$ is recorded and water is allowed to flow through the soil specimen such that the final head difference at the time $t = t_2$ is h_2

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Laboratory determination of hydraulic conductivity

Falling head test

- The rate of flow at any time t is

$$q = k(h/L)A = -a(dh/dt) \qquad (1.11)$$

Where,

q = flow rate

a = cross sectional area of stand pipe

A = cross sectional area of soil specimen

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Laboratory determination of hydraulic conductivity

Falling head test

- Rearranging equation (1.11),

$$dt = (aL/AK)*(-dh/h) \qquad (1.12)$$

Or, $\int_0^t dt = (aL/AK)*\int_{h_1}^{h_2}(-dh/h)$

Or, $t = (aL/AK)*\ln(h_1/h_2)$

Or, $$k = (aL/At)*\ln(h_1/h_2) \qquad (1.13)$$

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Empirical relations for hydraulic conductivity

- **For fairly uniform sand, Hazen (1931) proposed an empirical relationship for k**

$$k \text{ (cm/sec)} = c(D_{10})^2 \qquad (1.14)$$

Where,

c = a constant, 1 - 1.5

D_{10} = effective size in mm

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Empirical relations for hydraulic conductivity

- Casagrande proposed a simple relationship for k for fine to medium clean sand

$$k = 1.4e^2 k_{0.85} \qquad (1.15)$$

Where,

k = hydraulic conductivity at a void ratio e

$k_{0.85}$ = hydraulic conductivity at $e = 0.85$

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Empirical relations for hydraulic conductivity

- **Kozeny-Carman proposed that,**

$$k \propto e^3/(1+e) \qquad (1.16)$$

Or,

$$k = C_1 * [e^3/(1+e)] \qquad (1.17)$$

Where, C_1 is a constant which can be obtained by getting k at two different e values

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Equivalent hydraulic conductivity in stratified soil

- In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations

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Equivalent hydraulic conductivity in stratified soil

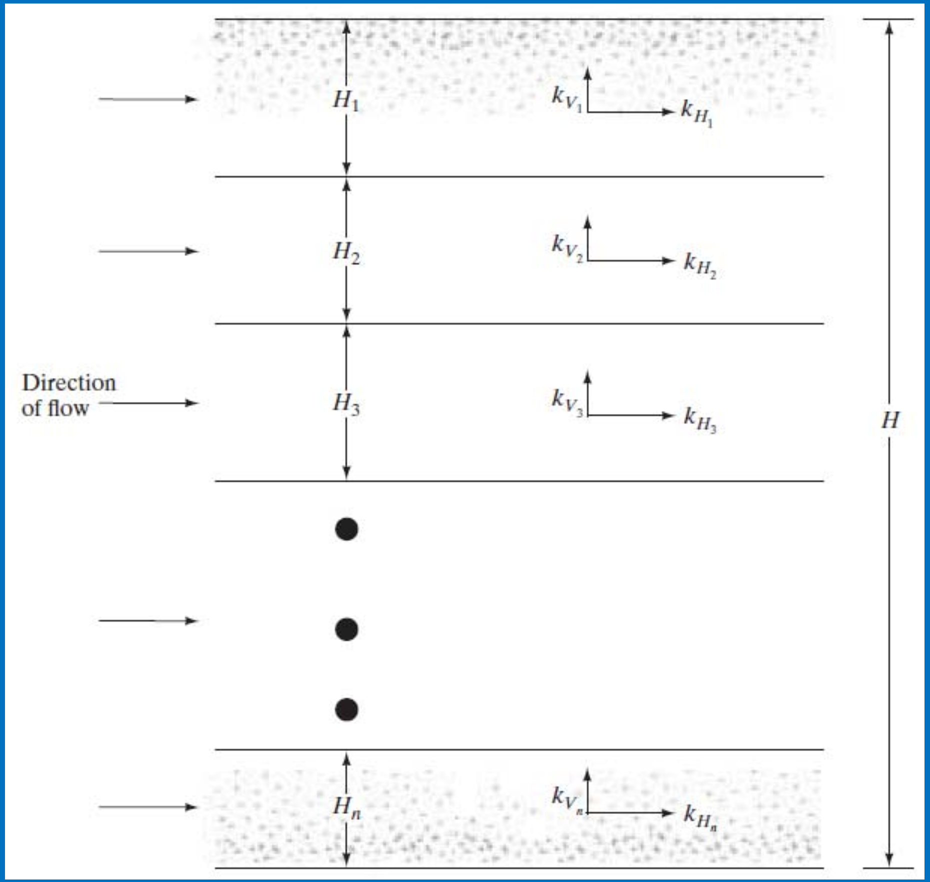

Fig. n layers of soil with flow in the horizontal direction

Permeability

Equivalent hydraulic conductivity in stratified soil

- Total flow through the cross section in unit time can be written as,

$$q = v*1*H$$

$$q = v_1H_1 + v_2H_2 + \ldots\ldots + v_nH_n \qquad (1.18)$$

Where, v = average discharge velocity

$v_1, v_2, \ldots\ldots, v_n$ = discharge velocity of flow in respective layers

Permeability

Equivalent hydraulic conductivity in stratified soil

- Let k_{H1}, k_{H2}, ... , k_{Hn} are the hydraulic conductivity of individual layer and $k_{H(eq)}$ is the equivalent hydraulic conductivity in the horizontal direction

Then, $v = k_{H(eq)} i_{(eq)}$; $v_1 = k_{H1} i_1$; $v_n = k_{Hn} i_n$

Here, $i_{(eq)} = i_1 = ... = i_n$

$$k_{H(eq)} = (k_{H1}H_1 + k_{H2}H_2 + ... + k_{Hn}H_n)/H \qquad (1.19)$$

Permeability

Equivalent hydraulic conductivity in stratified soil

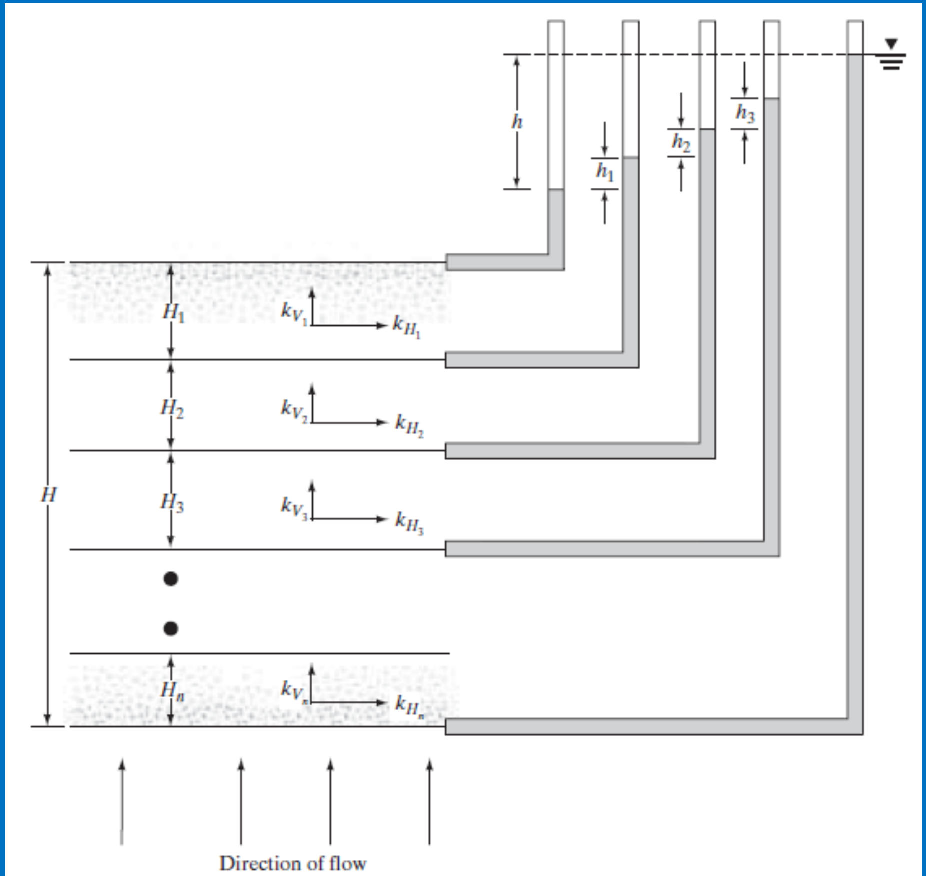

Fig. n layers of soil with flow in the vertical direction

Permeability

Equivalent hydraulic conductivity in stratified soil

- In this case the velocity of flow through all layers is same
- Total head loss h is equal to the sum of the head losses in all layers

Thus, $v = v_1 = v_2 = \ldots = v_n$ (1.20)

and, $h = h_1 + h_2 + \ldots + h_n$ (1.21)

So, $k_{V(eq)}(h/H) = k_{V1}\, i_1 = k_{V2}\, i_2 = \ldots = k_{Vn}\, i_n$ (1.22)

Permeability

Equivalent hydraulic conductivity in stratified soil

- Again from equation (1.21)

$$h = H_1 i_1 + H_2 i_2 + \ldots\ldots\ldots + H_n i_n \qquad (1.23)$$

- Solving equations (1.22) and (1.23), we get

$$k_{V(eq)} = H / [H_1/kV_1 + H_2/kV_2 + \ldots + H_n/kVn] \qquad (1.24)$$

Permeability

In-situ hydraulic conductivity of compacted soils

Porous probes

- Porous probes are pushed or driven into the soil
- Constant and falling head permeability test are performed
- For constant head hydraulic conductivity

$$k = q/Fh \qquad (1.25)$$

Permeability

In-situ hydraulic conductivity of compacted soils

Porous probes

- For falling head hydraulic conductivity

$$k = \frac{[(\pi d^2)/4]}{[F(t_2 - t_1)]} * \ln(h_1/h_2) \qquad (1.26)$$

Permeability

In-situ hydraulic conductivity of compacted soils

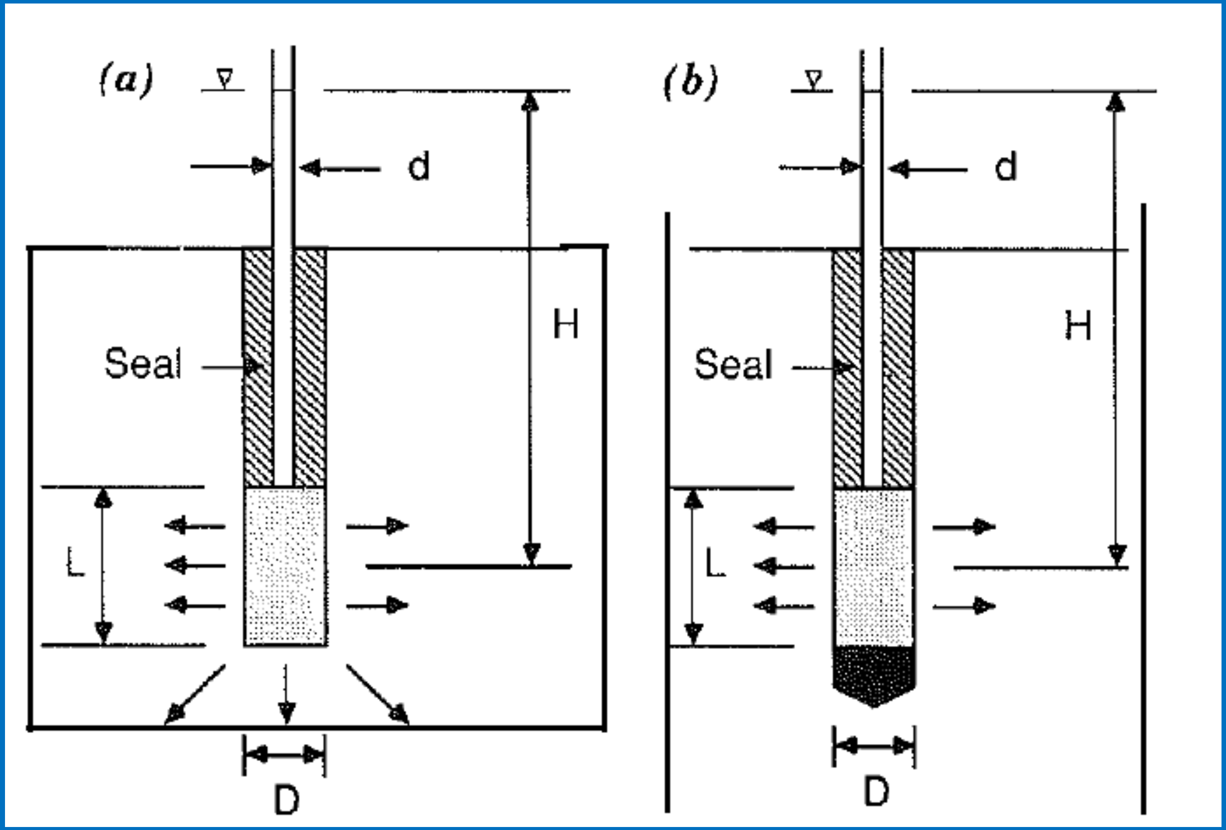

Fig. Porous probes (a) with permeable base (b) with impermeable base

Permeability

In-situ hydraulic conductivity of compacted soils

Porous Probes

- For probes with permeable base

$$F = \frac{(2\pi L)}{\ln[(L/D)+\sqrt{\{1+(L/D)^2\}}]} \qquad (1.27)$$

- For probes with impermeable base

$$F = \frac{(2\pi L)}{\ln[(L/D)+\sqrt{\{1+(L/D)^2\}}]} - 2.8D \qquad (1.28)$$

Permeability

Permeability test in the field by pumping from wells

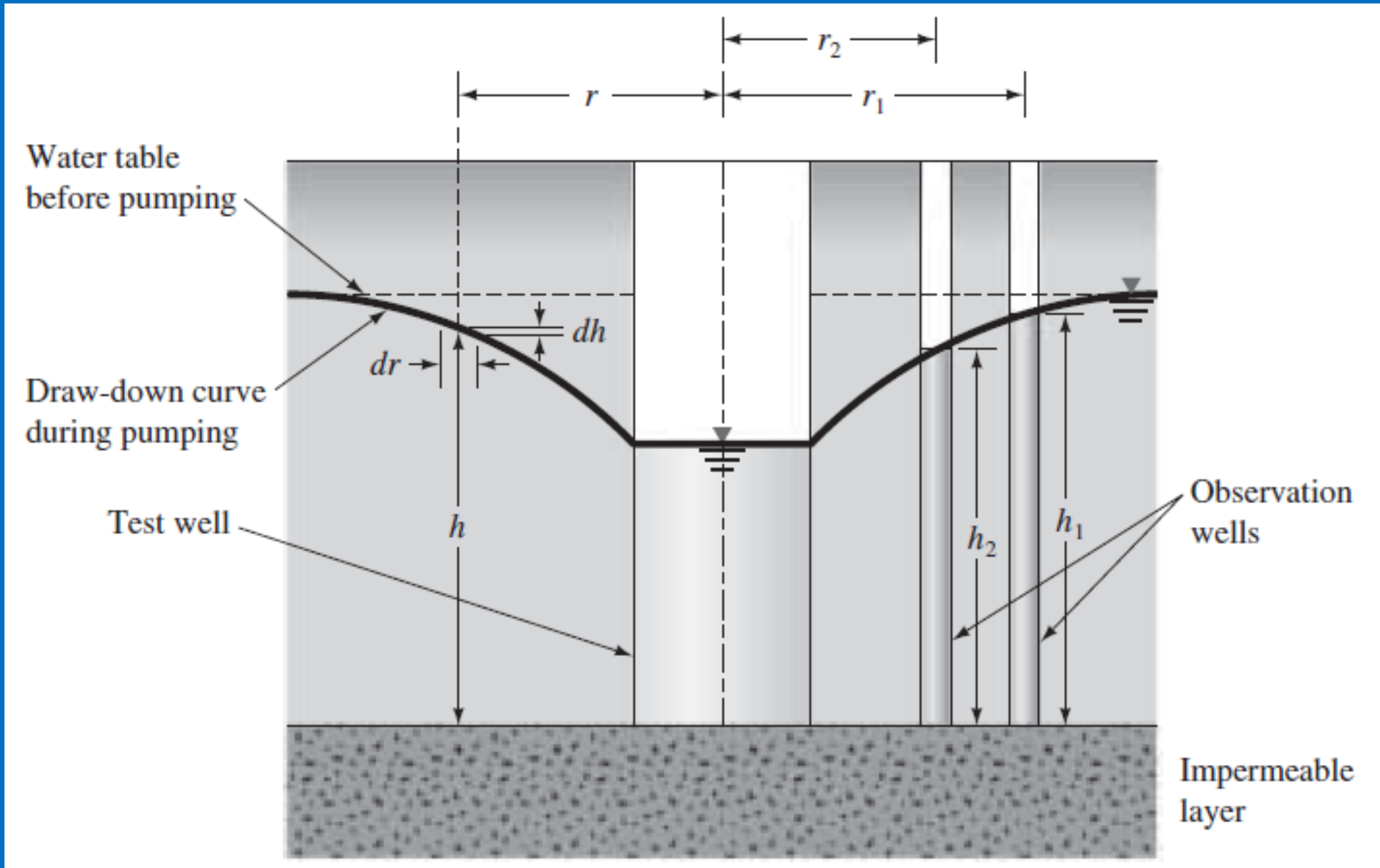

Fig. Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum

Permeability

Permeability test in the field by pumping from wells

- During the test water is pumped out at a constant rate from a test well
- Several observation wells are made at a various radial diameters
- The layer is unconfined and underlain by an impermeable layer

Permeability

Permeability test in the field by pumping from wells

- **After steady state is reached in test and observation wells**

$$q = k(dh/dr)(2\pi rh) \qquad (1.29)$$

Or, $$\int_{r_2}^{r_1} \left(\frac{dr}{r}\right) = (2\pi k/q)\int_{h_2}^{h_1} (h.dh) \qquad (1.30)$$

Or, $$k = [q*\ln(r_1/r_2)]/[\pi(h_1^2 - h_2^2)] \qquad (1.31)$$

Permeability

Permeability test in the field by pumping from wells

- From the field measurement, if q, r_1, r_2 , h_1 and h_2 are known then k can be obtained from equation (1.31)