CE 361A : Engineering Hydrology

Precipitation

Lecture -7

Revision: Analysis of Rainfall Data

Data collected from rain gauges

1. Present rainfall data
2. Check consistency
3. Estimate missing values
4. Areal average rainfall
5. Adequacy of raingauge
6. Depth-area-duration relationships (DAD)
7. Frequency analysis: Intensity-duration-frequency (IDF) curves
8. Probable maximum precipitation
9. Variability, Periodicity and Trends

special topic

Design Storm : DAD & IDF curves

Design Storm: DAD & IDF curves

Characteristics of a storm

1. Intensity (or depth)
2. Duration
3. Frequency
   – How often (or frequently) the storm occurs?
4. Areal distribution
   – How average depth during a storm event changes with area?

Frequency of storm

Terminology

Annual series

- Composed of annual values
- Example: total, daily maximum, 12-h maximum

Year	Annual Rainfall (cm)	Year	Annual Rainfall (cm)
1960	130	1971	90
1961	84	1972	102
1962	76	1973	108
1963	89	1974	60
1964	112	1975	75
1965	96	1976	120
1966	80	1977	160
1967	125	1978	85
1968	143	1979	106
1969	89	1980	83
1970	78	1981	95

Frequency of storm

Terminology

Exceedance probability (p)

- Magnitude of rainfall in an event is considered a random variable (X)
- x represents a value that X can assume
- $p = P(X \geq x)$

Year	Annual Rainfall (cm)
1960	130
1961	84
1962	76
1963	89
1964	112
1965	96
1966	80
1967	125
1968	143
1969	89
1970	78

X – Annual rainfall

$x = 100$ cm

$p = P(X \geq 100)$

$P(X \geq 100)$? $P(X \geq 50)$

Frequency of storm

Terminology

Return period or recurrence interval (T)

- $T = 1/p$
- Represents average interval between occurrence of an event

Example

- $p = P(X \geq 100) = 0.1$
- Return period = $T = 1/p = 10$ years
- It represents average interval between occurrences of an event
- It DOES NOT imply that the event will occur once in 10 years
- Which of the following two events will have larger return period?

$$T(X \geq 100) \; ? \; T(X \geq 50)$$

Frequency of storm

Terminology

Non-exceedance probability (q)

- $q = P(X < x) = 1 - p$

Example

- $p = P(X \geq 100) = 0.1$
- $q = P(X<100) = 1 – 0.1 = 0.9$

$P(X <100)$? $P(X < 50)$

Estimation of frequency of an event

1. Empirical method

   – Plotting position formulae

Table 2.4 Plotting Position Formulae

Method	***P***
California	m/N
Hazen	$(m-0.5)/N$
Weibull	$m/(N+1)$
Chegodayev	$(m-0.3)/(N+0.4)$
Blom	$(m-0.44)/(N+0.12)$
Gringorten	$(m-3/8)/(N+1/4)$

2. Theoretical distributions (Chapter 9)

   – Extreme value distribution

   – Pearson distribution

   – Transformal distribution (log-normal)

Estimation of frequency of an event: Empirical Method

Step 1. Create an annual time series

Year	Annual Rainfall (cm)	Year	Annual Rainfall (cm)
1960	130	1971	90
1961	84	1972	102
1962	76	1973	108
1963	89	1974	60
1964	112	1975	75
1965	96	1976	120
1966	80	1977	160
1967	125	1978	85
1968	143	1979	106
1969	89	1980	83
1970	78	1981	95

Estimation of frequency of an event: Empirical Method

Step 2. Arrange the series in descending order of values

Rank (m)	Annual Rainfall (cm)	Rank (m)	Annual Rainfall (cm)
1	160	12	90
2	143	13	89
3	130	14	89
4	125	15	85
5	120	16	84
6	112	17	83
7	108	18	80
8	106	19	78
9	102	20	76
10	96	21	75
11	95	22	60

Estimation of frequency of an event: Empirical Method

Step 3. Estimate exceedance probability (p) based plotting position formula

$$\text{Weibull Equation} \quad p = \frac{m}{N+1}$$

Rank (m)	Annual Rainfall (cm)	p	Rank (m)	Annual Rainfall (cm)	p
1	160	0.043	12	90	0.522
2	143	0.087	13	89	0.565
3	130	0.130	14	89	0.609
4	125	0.174	15	85	0.652
5	120	0.217	16	84	0.696
6	112	0.261	17	83	0.739
7	108	0.304	18	80	0.783
8	106	0.348	19	78	0.826
9	102	0.391	20	76	0.870
10	96	0.435	21	75	0.913
11	95	0.478	22	60	0.957

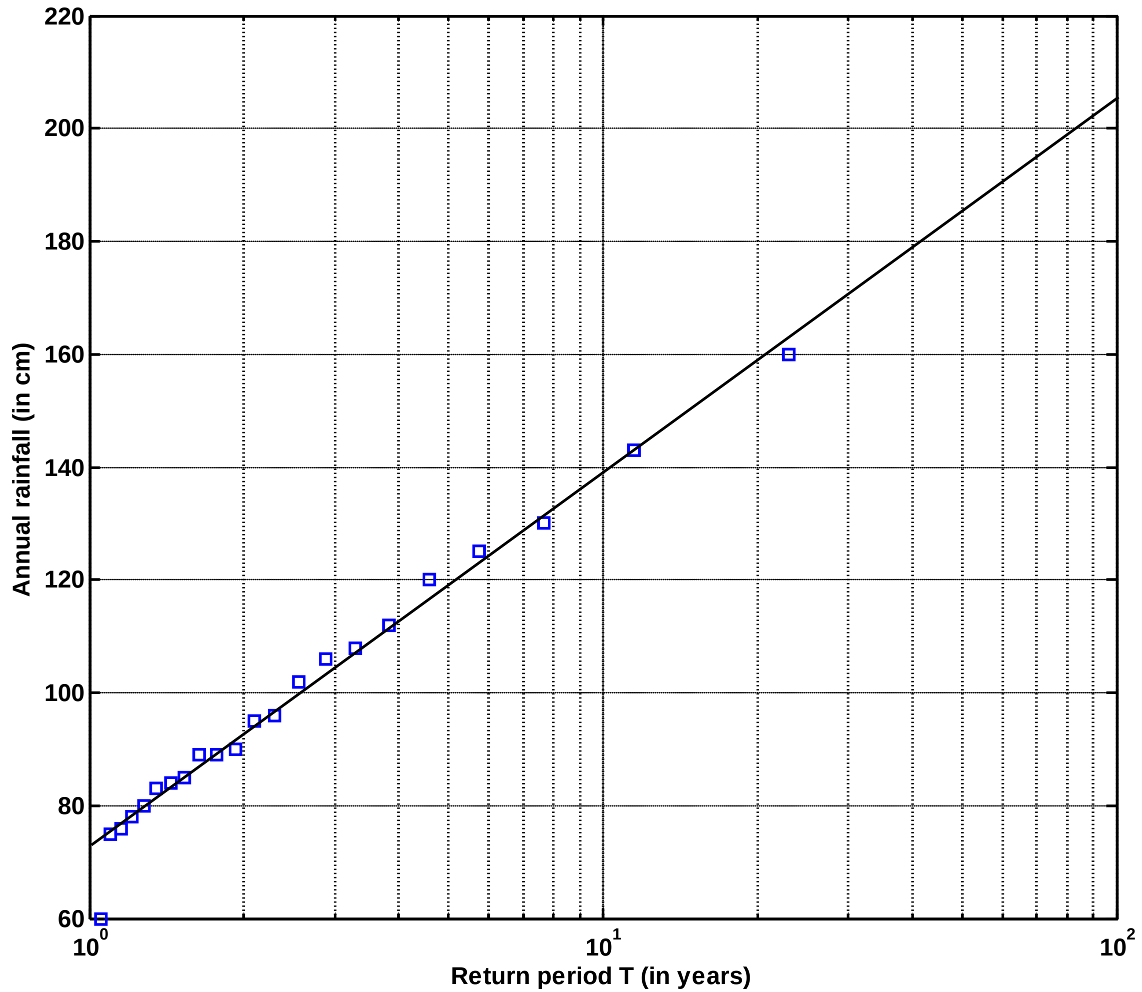
Estimation of frequency of an event: Empirical Method

Step 4. Estimate return period T = 1/p

Rank (m)	Annual Rainfall (cm)	p	T (years)	Rank (m)	Annual Rainfall (cm)	p	T (years)
1	160	0.043	23.00	12	90	0.522	1.92
2	143	0.087	11.50	13	89	0.565	1.77
3	130	0.130	7.67	14	89	0.609	1.64
4	125	0.174	5.75	15	85	0.652	1.53
5	120	0.217	4.60	16	84	0.696	1.44
6	112	0.261	3.83	17	83	0.739	1.35
7	108	0.304	3.29	18	80	0.783	1.28
8	106	0.348	2.88	19	78	0.826	1.21
9	102	0.391	2.56	20	76	0.870	1.15
10	96	0.435	2.30	21	75	0.913	1.10
11	95	0.478	2.09	22	60	0.957	1.05

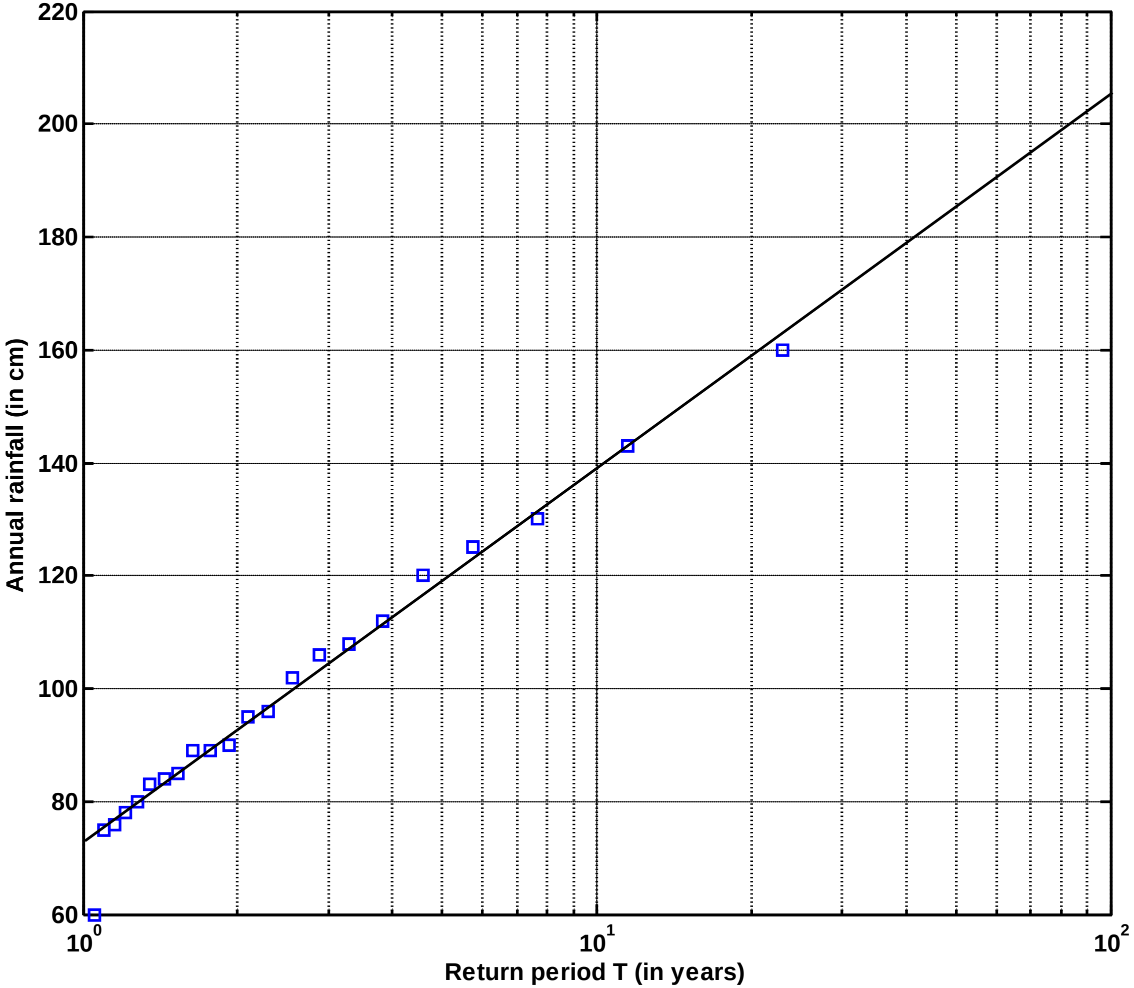
Estimation of frequency of an event: Empirical Method

Step 5. Plot the graph between event magnitude and T in a log-log or semi-log axes and fit a straight line



Estimation of frequency of an event: Empirical Method

Questions: Estimate annual rainfall with return period of 10 and 40 years



T = 10 years

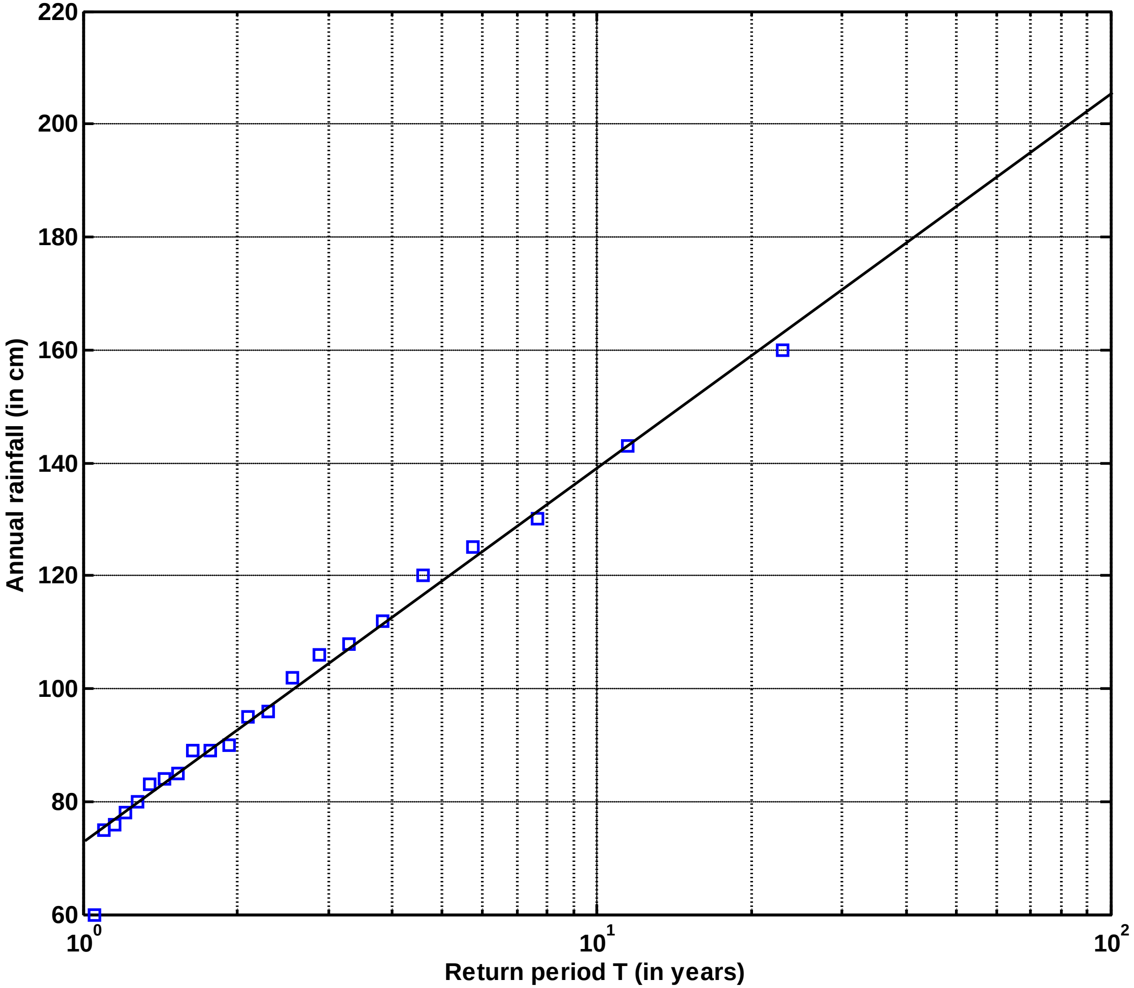
x = 138 cm

T = 40 years

x = 178 cm

Estimation of frequency of an event: Empirical Method

Questions: What is the return period for rainfall exceeding 200 cm



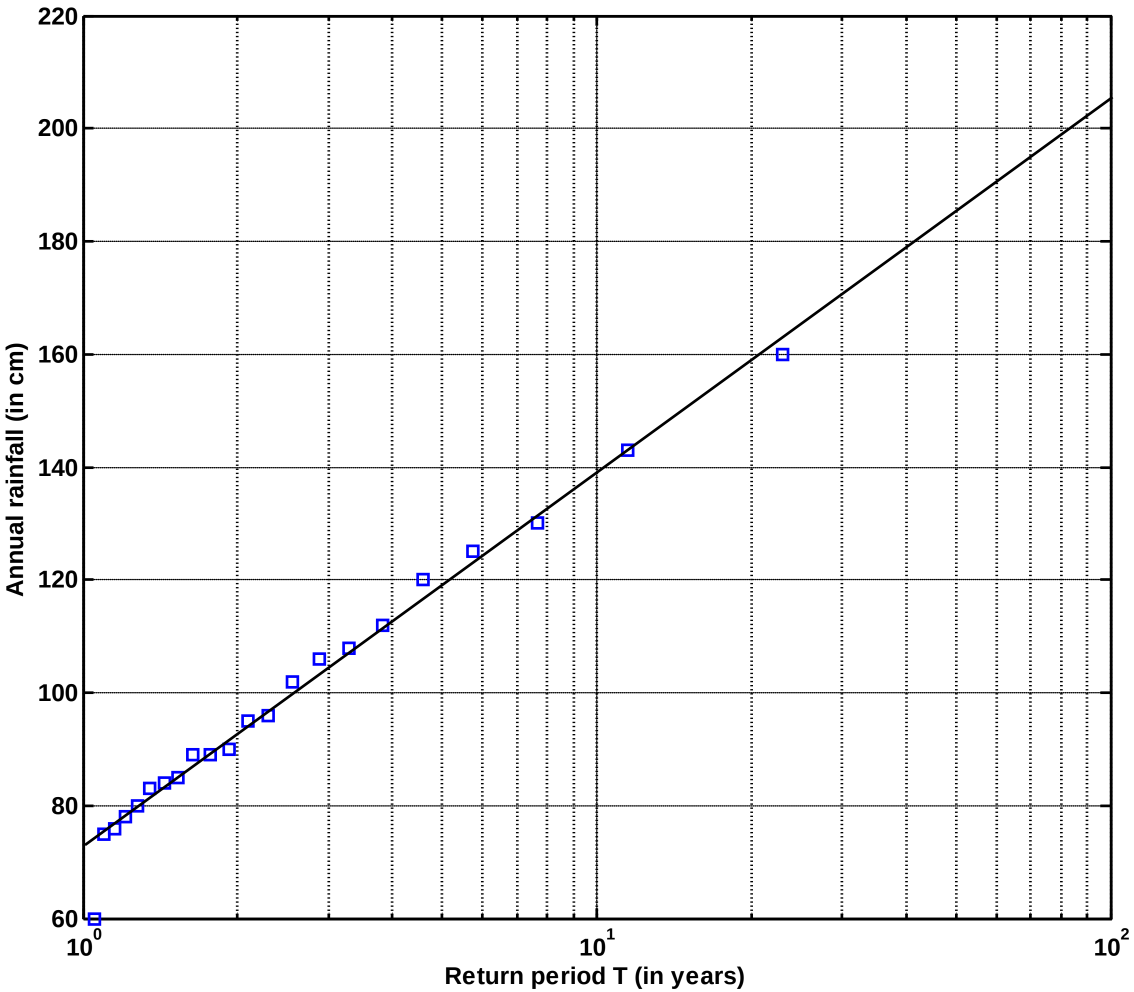
x = 200 cm

T = 82 years

p = 1/T = .0122

Estimation of frequency of an event: Empirical Method

Questions: What is the 50 % dependable annual rainfall



p = 0.5

T = 2 years

x = 92 cm

Working with probabilities

If the probability of an occurrence of an event is *p*, what is the probability that the event will occur *r* times in *n* successive years ?

Binomial distribution

$$P_{r,n} = {}^{n}C_r p^r q^{n-r} = \frac{n!}{(n-r)!r!} p^r q^{n-r}$$

Working with probabilities

Example: Return period of 205 cm annual rainfall depth is 100 years. Determine

- Probability that annual rainfall will be greater than 205 cm in a year
- Probability that such an event will occur once in 10 years
- Probability that such an event will occur twice in 10 years
- Probability that the event will occur at least once in 10 years

Adapted from the solved example 2.7 of Subramanya (2008)

Working with probabilities

- Probability that annual rainfall will be greater than 205 cm in a year

$$p = P(X \geq 205) = 1/T = 0.01$$

- Probability that such an event will occur once in 10 years

$$P_{r,n} = P_{1,10} = \frac{n!}{(n-r)!r!}\, p^r q^{n-r}$$
$$= \frac{10!}{9!1!}\, 0.01^1(1-0.01)^9 = 10\ \times 0.01 \times 0.9135 = 0.091$$

- Probability that such an event will occur twice in 10 years

$$P_{r,n} = P_{2,10} = \frac{n!}{(n-r)!r!}\, p^r q^{n-r}$$
$$= \frac{10!}{8!2!}\, 0.01^2(1-0.01)^8 = 45\ \times 0.0001 \times 0.923 = 0.0042$$

Working with probabilities

- Probability that the event will occur at least once in 10 years

$$
\begin{aligned}
P_1 &= 1 - q^n \\
&= 1 - (1 - 0.01)^{10} = 0.0956
\end{aligned}
$$

Maximum Intensity Duration Frequency Curve (IDF)

- Curves showing relationship between maximum intensity, duration and frequency at a station
- Similar to IDF, maximum depth duration frequency (DDF) curves can be obtained
- In India, IDF curves are provide by India Meteorological Department (IMD)
- Wherever possible IDF curves should be prepared for hydrologic design
- A minimum of 20 years hourly dataset is desirable to estimate IDF curves

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in minutes)	Cumulative rainfall (in mm)
0	0
30	6
60	18
90	21
120	36
150	43
180	49
210	52
240	53
270	54

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in min)	Cumulative rainfall (in mm)	Incremental depth (mm)
		30 minutes
0	0	
30	6	6
60	18	12
90	21	3
120	36	15
150	43	7
180	49	6
210	52	3
240	53	1
270	54	1

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in min)	Cumulative rainfall (in mm)	Incremental depth (mm)	
		30 min	60 min
0	0		
30	6	6	
60	18	12	18
90	21	3	15
120	36	15	18
150	43	7	22
180	49	6	13
210	52	3	9
240	53	1	4
270	54	1	2

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in min)	Cumulative rainfall (in mm)	Incremental depth (mm)								
		30 min	60 min	90 min	120 min	150 min	180 min	210 min	240 min	270 min
0	0									
30	6	6								
60	18	12	18							
90	21	3	15	21						
120	36	15	18	30	36					
150	43	7	22	25	37	43				
180	49	6	13	28	31	43	49			
210	52	3	9	16	31	34	46	52		
240	53	1	4	10	17	32	35	47	53	
270	54	1	2	5	11	18	33	36	48	54

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in min)	Cumulative rainfall (in mm)	Incremental depth (mm)								
		30 min	60 min	90 min	120 min	150 min	180 min	210 min	240 min	270 min
0	0									
30	6	6								
60	18	12	18							
90	21	3	15	21						
120	36	15	18	30	36					
150	43	7	22	25	37	43				
180	49	6	13	28	31	43	49			
210	52	3	9	16	31	34	46	52		
240	53	1	4	10	17	32	35	47	53	
270	54	1	2	5	11	18	33	36	48	54

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Time since start (in min)	Cumulative rainfall (in mm)	Incremental depth (mm)								
		30 min	60 min	90 min	120 min	150 min	180 min	210 min	240 min	270 min
0	0									
30	6	6								
60	18	12	18							
90	21	3	15	21						
120	36	15	18	30	36					
150	43	7	22	25	37	43				
180	49	6	13	28	31	43	49			
210	52	3	9	16	31	34	46	52		
240	53	1	4	10	17	32	35	47	53	
270	54	1	2	5	11	18	33	36	48	54

Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Step 3. Develop a relation between maximum intensity and duration

Duration	(in min)								
	30	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**270**
Maximum Depth (mm)	15	22	30	37	43	49	52	53	54

Steps in preparing IDF curves

Step 1. For each year select most intense storms

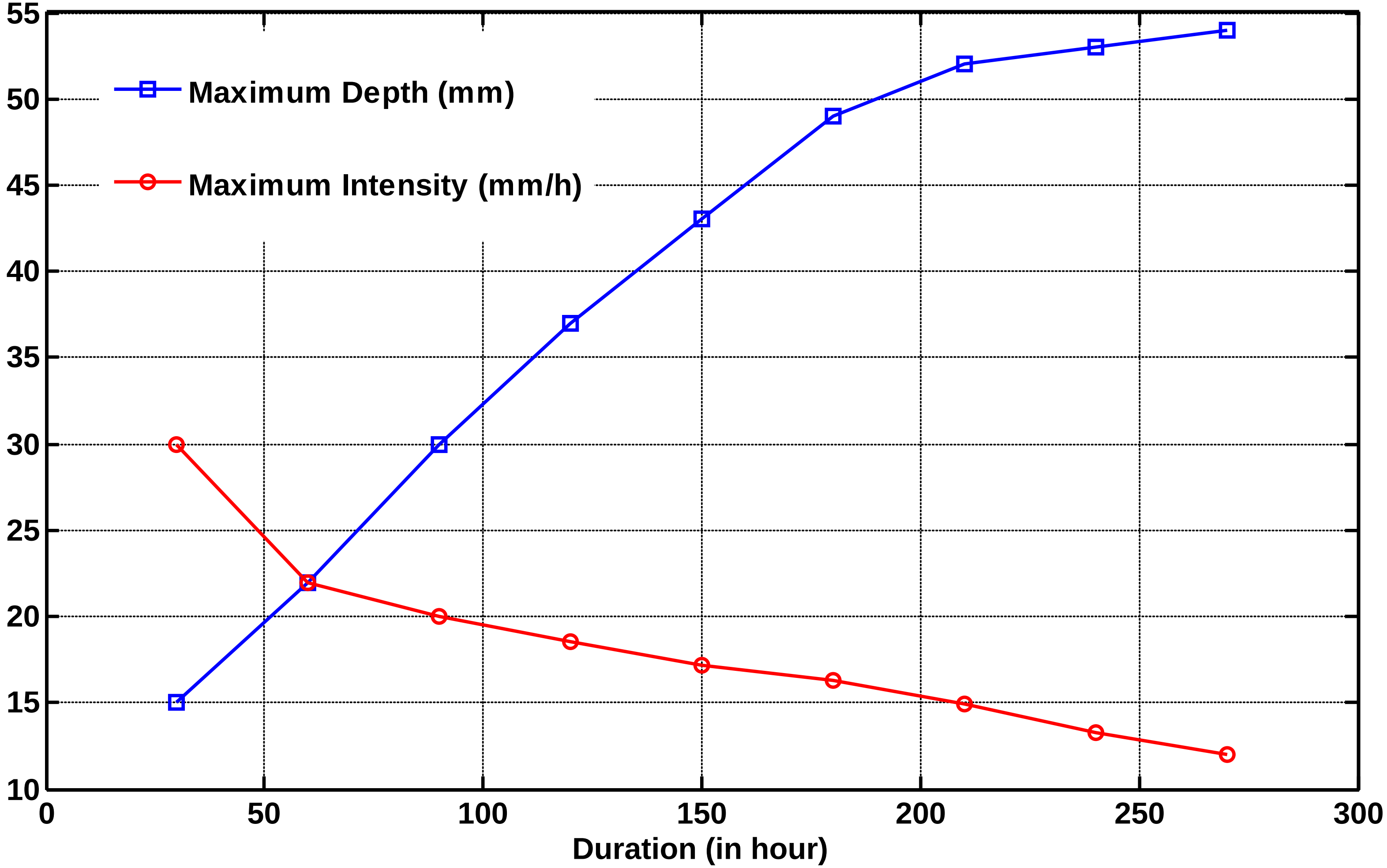
Step 2. For each storm estimate the maximum intensity for a selected durations

Step 3. Develop a relation between maximum intensity and duration

Duration	(in min)								
	30	**60**	**90**	**120**	**150**	**180**	**210**	**240**	**270**
Maximum Depth (mm)	15	22	30	37	43	49	52	53	54
Maximum Intensity (mm/h)	30	22	20	18.5	17.2	16.3	14.9	13.3	12

Steps in preparing IDF curves

Relationship between Intensity (Depth) and duration



Steps in preparing IDF curves

Step 1. For each year select most intense storms

Step 2. For each storm estimate the maximum intensity for a selected durations

Step 3. Develop a relation between maximum intensity and duration

Step 4. For each year identify maximum intensity for a given duration

Step 5. Prepare an annual series of maximum intensity and duration and do the frequency analysis as discussed earlier

Steps in preparing IDF curves

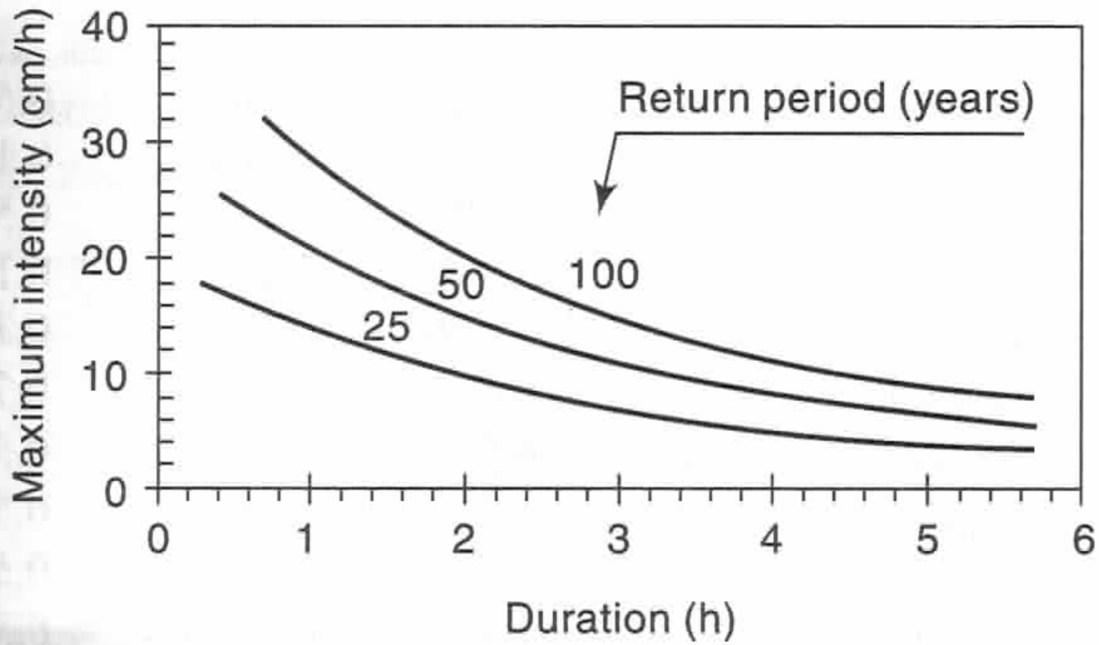


Fig. 2.19 Maximum Intensity-Duration-Frequency Curves

Source: Subramanya (2008)

Steps in preparing IDF curves

Step 6. Analytically the relationships are expressed as

$$\boxed{i = \frac{KT^x}{(D+a)^n}}$$

i = maximum intensity (cm/h)

T = return period (years)

D = duration (hours)

$K, x,$ and n coefficeints for the area or station

Table 2.6 Typical values of Coefficients K, x, a and n in Eq. (2.15)

[Ref. 10]

Zone	Place	K	x	a	n
Northern Zone	Allahabad	4.911	0.1667	0.25	0.6293
	Amritsar	14.41	0.1304	1.40	1.2963
	Dehradun	6.00	0.22	0.50	0.8000
	Jodhpur	4.098	0.1677	0.50	1.0369
	Srinagar	1.503	0.2730	0.25	1.0636
	Average for the zone	5.914	0.1623	0.50	1.0127
Central Zone	Bhopal	6.9296	0.1892	0.50	0.8767
	Nagpur	11.45	0.1560	1.25	1.0324
	Raipur	4.683	0. 1389	0.15	0.9284
	Average for the zone	7.4645	0.1712	0.75	0.9599

Depth Duration Frequency curves

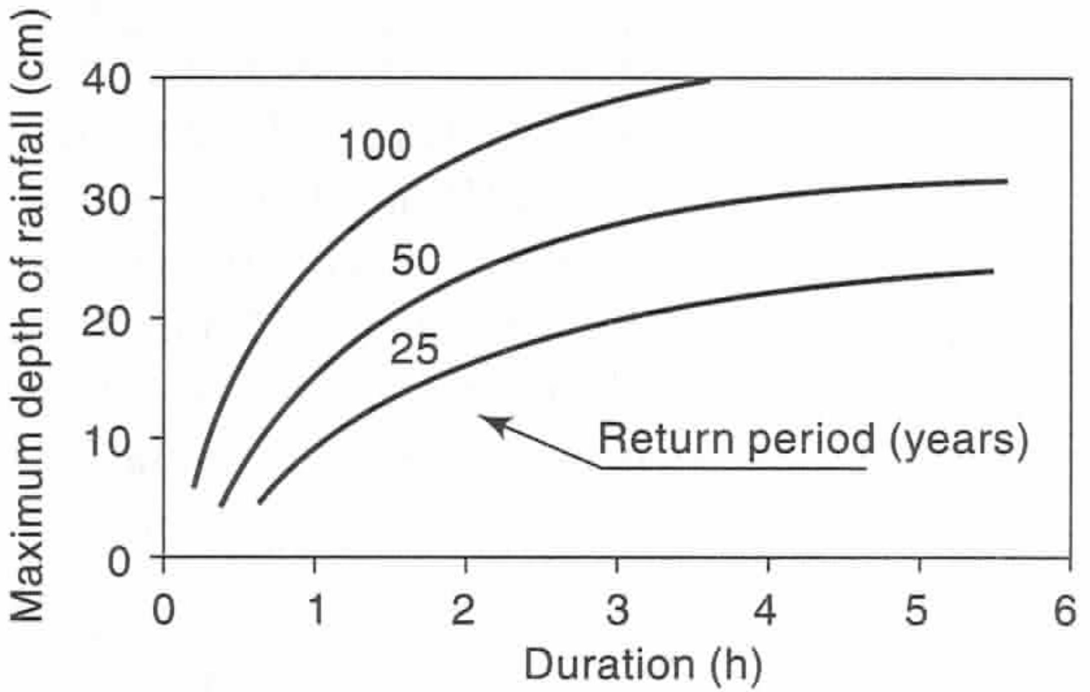


Fig. 2.20 Maximum Depth-Duration-Frequency Curves

Source: Subramanya (2008)

Alternate way of presenting IDF curves

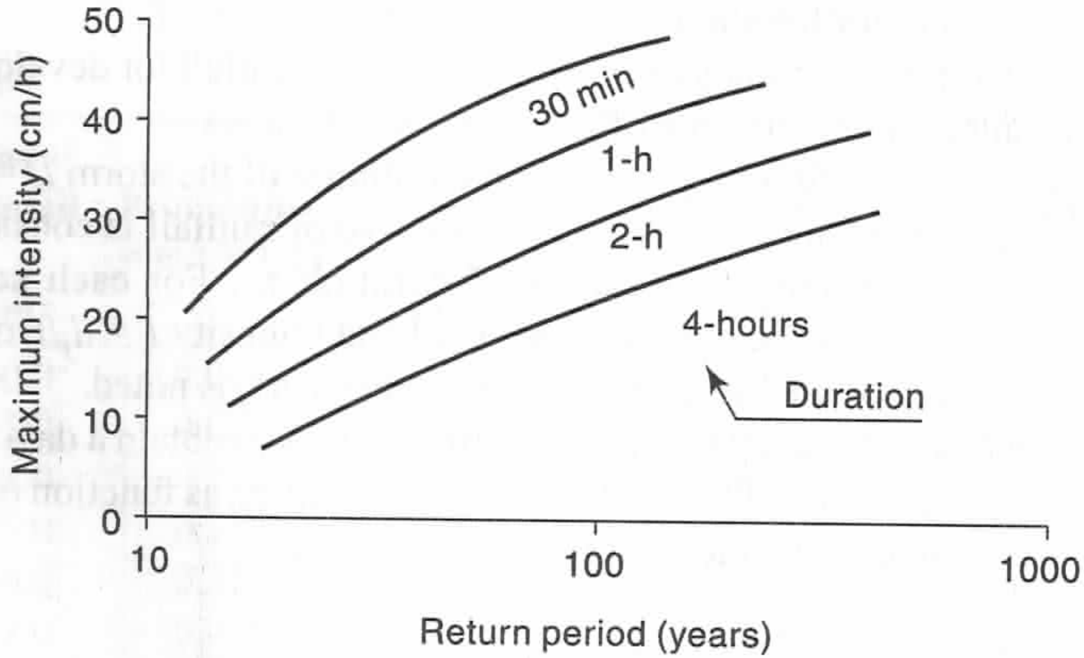


Fig. 2.18 Maximum Intensity-Return Period-Duration Curves

Source: Subramanya (2008)

DDF curves for India

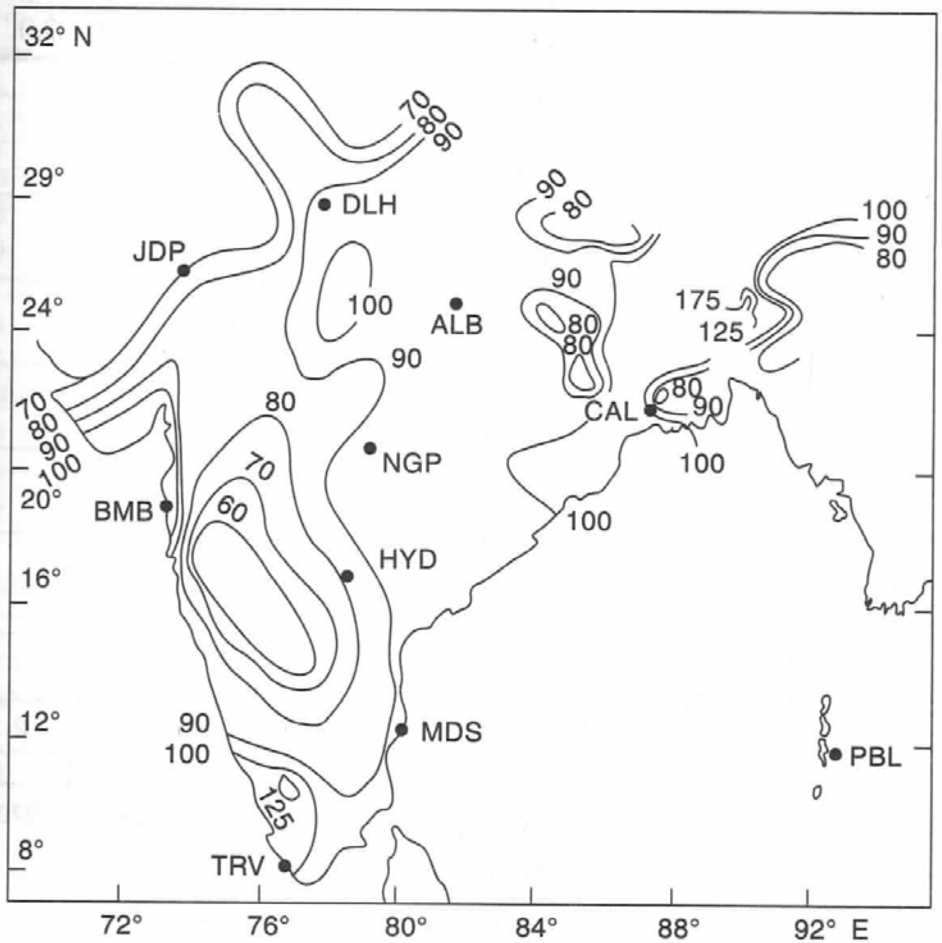


Fig. 2.22 Isopluvial Map of 50 yr-1 h Maximum Rainfall (mm) (Reproduced from *Natural Resources of Humid Tropical Asia—Natural Resources Research*, XII. © UNESCO, 1974, with permission of UNESCO)

Source: Subramanya (2008)

DDF curves for India

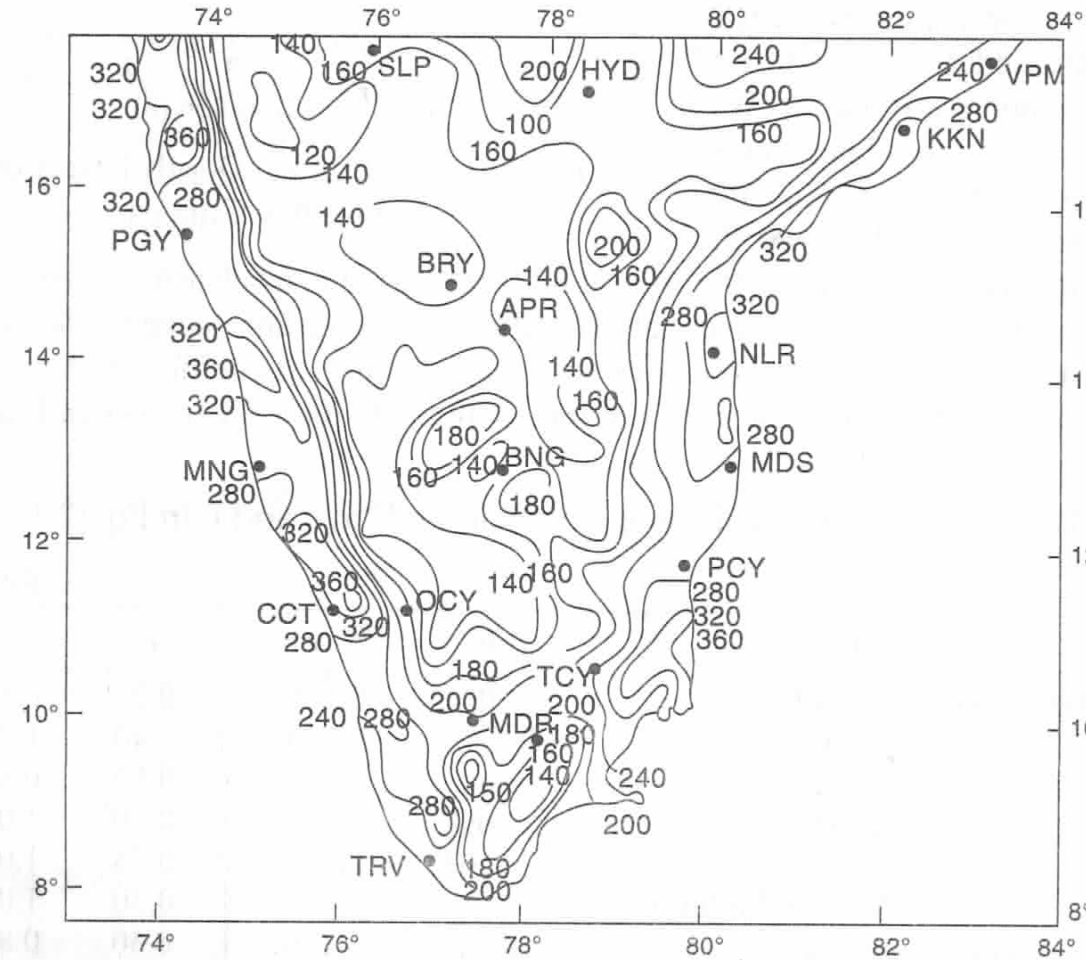


Fig. 2.21 Isopluvial Map of 50 yr-24 h Maximum Rainfall (mm)
(Reproduced with permission from India Meteorological Department)

Source: Subramanya (2008)