Tutorial Variance propagation & Adjustment computations

1) An angle is measured eight times and the following observations are obtained:

52°38′7.2″,
10.1″
10.0″
6.4″
5.1″
6.1″
10.3″
9.9″

Compute the most probable value, standard error of single observation and variance of means. How many more observations of the same precision should be taken to reduce the standard error of mean to ±0.50″?

[Ans: MPV = 52°38′8.1″; σ_o = ±2.13″; σ_m= 0.753″; $n = \left(\frac{\pm_0}{0.50}\right)^2$; $n = 18$, another 10 observations]

2) An angle is measured by an observer A, 12 times and other observer B, 7 times with following results.

S. No.	A′s observation	B′s observation
1	146°48′7.5″	146°48′6.3″
2	8.7″	6.8″
3	3.2″	5.4″
4	9.6″	8.0″
5	8.0″	7.6″
6	2.4″	10.4″
7	5.0″	2.8″
8	1.4″	-
9	8.8″	-
10	2.2″	-
11	3.6″	-
12	0.8″	-

Find out:

i) Which set of observations is more precise and why?
ii) The MPV of angle
iii) Variance of the MPV of angle.

[Ans: i) Set B; ii) 146°48′6.39″; iii) σ_m^2 = 3.73, σ_m = +-1.93″]

$$\bar{X}_A = 5.1; \bar{X}_B = 7.11$$

$$\pm_A^2 = \frac{114.81}{11} = 10.44; \pm_B^2 = \frac{40.71}{7} = 5.81$$

$$\text{Weight} = \text{P}_\text{A}:\text{P}_\text{B}::\frac{1}{\pm_A^2}:\frac{1}{\pm_B^2} = 1:1.8$$

$$MPV = \frac{5.1x1 + 7.11x1.8}{2.8} = 6.39$$

$$\pm_m^2 = \left(\frac{P_A}{P_A + P_B}\right)^2 \pm_A^2 + \left(\frac{P_B}{P_A + P_B}\right)^2 \pm_B^2 = 3.73$$

3) A 30 m steel tape was found to be 0.050 m too short. This tape was used to measure the area of a square plot having 60 m sides. Find out:

i) True error in the area of the plot

ii) Error if the function is linearised and higher terms of error neglected.

[Ans: True error = 11.99m^2; Approx. error = 11.98m^2]

4) For the measurement of the height of the tower, following observations are taken from the station:
Distance of tower D = 350m ± 0.35m
Angle of elevation α = 8°35′20″ ±10″
Compute the height of the tower and its variance.

[Ans: $h = D\tan\sim\, = f(D,\sim);\Sigma_{hh} = J\pm^{2}_{D\sim}J^{T}$; h = 52.863m; σ_h= ±0.055%]

5) Interior angles A and B of a plane triangle are known and fixed. Side b (opposite B) is computed from the fixed values of A and B and the measured value of side a (opposite A). If the error is 0.015m, evaluate the resulting error in *b* for the following cases:

1) for A = 120° and B = 15°

2) for A = 15° and B = 120°

[Ans: db = 0.004m and 0.050m respectively]

6) In a **stadia leveling**, the difference in elevation *V* is computed (above axis) from the rod intercept *s* and vertical angle ‘α using the function *V* = (1/2) *KS* x sin2α where *K* is the stadia constant. If *K* = 100 (assumed errorless) and the errors in *S* and *a* are 0.005m and 60seconds of arc, respectively, evaluate *V* and the error in *V* for:

a) *S* = 1.500m α =10

b) *S* = 1.500m, α =15

[Ans: a)V = 0m; dv = 0.043m; b) V = 37.50m, dv = 0.163m]

7) If the sides of a water tank are 30m ± 5mm, 40m ± 5mm with height as 20m ± 8mm, what are the volume and its standard error? Give the Jacobean matrix.

[Ans: σ_v = ±10.8m^3]

8) Angles A, B and C were measured on the field and found to be 20°30′40″ ± 2″, 30°40′50″ ± 3″ and 40°50′10″ ± 4″ respectively. The covariance σ_{AB} , σ_{BC} and σ_{AC} were known to be the same having a value of 1.0 sec^2. If two random variables X and Y are represented by X = A + 2B and Y = B + 2C. Find Σ_{XY} .

[Ans: $\Sigma_{XY} = \begin{bmatrix} 44 & 25 \\ 25 & 77 \end{bmatrix}$]

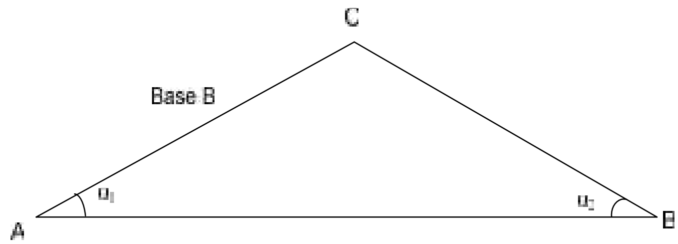
9) Two interior angles of a plane triangle are measured and the third angle is computed from them. If the two angle measurements are uncorrelated, with standard deviation 4.5″ and 6.0″, respectively evaluate the standard deviation of the computed angle.

[Ans: 7.5″]

10) The following data refer to a triangulation network as shown in the figure (not to the scale):
Base b = 20372.35±0.20m ; α_1= 25°0′0″ ; α_1 = 75°0′0″

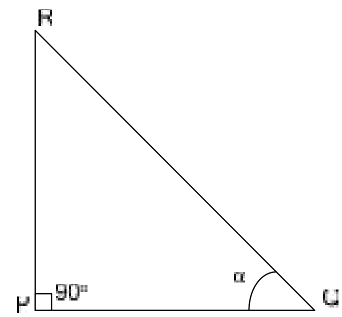
Standard error on all the angle measurements is ±3″. Compute the side BC and its standard error.

[Ans: 8913.445±0.290m]



11) In a right angled triangle, sides PR and QR were measured independently and found to be 101.059m and 208.025m respectively. The standard error for the measurement of PR was ±0.012m and that of QR was ±0.02m. Evaluate the side PQ and angle PQR and their variances. Also find out the corelation, if any, between side PQ and angle PQR.

[Ans: PQ = 181.83±0.0242 ; $\alpha = \pm 17.5''$; $\rho_{\alpha,PQ} = -0.816$]



12) The interior angles of n-sided closed traverse are measured. All measurements are uncorrelated. If the standard deviation of each measurement is 4.0″, evaluate the standard deviation of the sum of the measured angles for (a) n = 5; (b) n = 10; (c) n = 20.

13) A distance is measured in three segments. Each segment is measured more than once and in each case a mean segment length (simple average) is obtained. The distance is then computed as the sum of mean segment lengths. Following are the measurements all uncorrelated in meters:

Segment-1	Segment-2	Segment-3
234.615	316.642	887.177
234.633	319.617	887.151
	319.649	887.186
	319.626	

Compute the distance and determine the standard deviation of the computed distance if the standard deviation of a 100m measurement is 0.010m and the variance of each measurement is directly proportional to its magnitude.

14) A distance is measured using three different methods. The observed values and their standard deviation (all in meters) are shown below:

Observation	Std Dev
352.095	0.020
352.147	0.030
352.062	0.060

Determine the weighted mean of the observed distances and the standard deviation of this weighted mean assuming the three observations are independent.

15) Six independent determinations of the elevation of a point are made. These values and their corresponding weights are shown below.

Elevation (m)	Weight
214.151	2
214.213	1
214.114	2
214.167	3
214.130	5
214.189	3

Compute the weighted mean of the six elevations and evaluate the standard deviation of this weighted mean if a weight of 2 corresponds to a standard deviation of 0.030m.

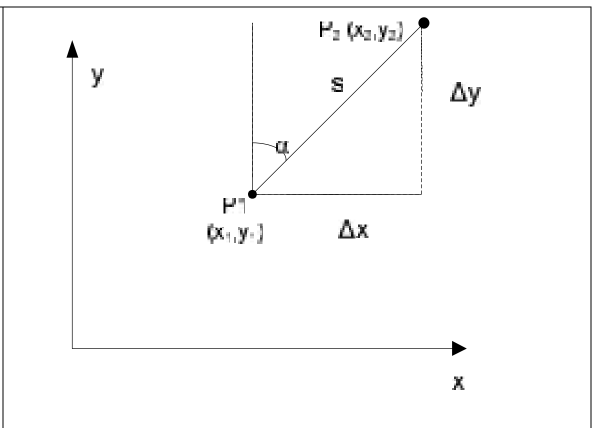
16) The area of trapezoidal parcel of land is computed as follows:

$$Area = \left(\frac{a_1 + a_2}{2} \right) b$$

Where a_1, a_2 and b are independently measured dimensions. If measured values for a_1, a_2 and b are 319.414 m, 481.112 m and 502.317 m, respectively, and their standard deviations are 0.030 m, and 0.042 m, and 0.020 m respectively, compute the area of the parcel and the standard deviation of this computed area.

17) The coordinates of P_1 are x_1 = 1000.00m, y_1 = 1000.00m; the coordinates of P_2 are x_2 = 1800.00m, y_2 = 1600 m. The covariance matrix for the vector $[x_1, y_1, x_2, y_2]^T$ is

$$\begin{bmatrix} 4 & 2 & 2 & 0 \\ 2 & 8 & -2 & 6 \\ 2 & -2 & 8 & -2 \\ 0 & 6 & -2 & 12 \end{bmatrix} \text{cm}^2$$



Evaluate the covariance matrix for Δx and Δy, where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$. Then evaluate the covariance matrix for the length s and azimuth a of P_1P_2 and determine their standard deviation and correlation coefficient.