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Lamb wave based automatic damage detection using matching pursuit and machine learning

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Abstract
In this study, matching pursuit (MP) has been tested with machine learning algorithms such as artificial neural networks (ANNs) and support vector machines (SVMs) to automate the process of damage detection in metallic plates. Here, damage detection is done using the Lamb wave response in a thin aluminium plate simulated using a finite element (FE) method. To reduce the complexity of the Lamb wave response, only the A_0 mode is excited and sensed. The procedure adopted for damage detection consists of three major steps, involving signal processing and machine learning (ML). In the first step, MP is used for de-noising and enhancing the sparsity of the database. In the existing literature, MP is used to decompose any signal into a linear combination of waveforms that are selected from a redundant dictionary. In this work, MP is deployed in two stages to make the database sparse as well as to de-noise it. After using MP on the database, it is then passed as input data for ML classifiers. ANN and SVM are used to detect the location of the potential damage from the reduced data. The study demonstrates that the SVM is a robust classifier in the presence of noise and is more efficient than the ANN. Out-of-sample data are used for the validation of the trained and tested classifier. Trained classifiers are found to be successful in the detection of damage with a detection rate of more than 95%.

Keywords: Lamb wave, matching pursuit, SVM, ANN, damage detection

(Some figures may appear in colour only in the online journal)

1. Introduction

Structural health monitoring (SHM) has in recent times attracted a lot of attention due to its significance in mechanical, aerospace and civil infrastructure. The existing damage detection techniques utilize different physical responses of a structure to extract information about existence, location and extent of damage. Further, a considerable effort has been directed towards in-service real-time damage detection involving minimum computational effort. Hence, to address the challenge of automatic damage detection in real time, two different approaches have been developed: model-based and data-driven (or model-free). In the former, a precise mathematical model of the structure is developed in advance, usually with certain assumptions such as system linearity, time-invariance etc. This approach is based on known structural parameters. On the other hand, a data driven based approach does not require a prior knowledge of the structure's parameters. Instead it directly relates the sensor data to the existence and location of damage patterns through statistical modelling [1].

Guided waves are of enormous interest with regard to non-destructive testing, especially for thin-walled structures. Lamb waves are guided waves which can exist in plate-like thin bodies with parallel free boundaries [1, 2]. In general, a Lamb wave based damage detection approach features the ability to inspect a large area, and excellent sensitivity to multiple defects with high precision, low energy consumption and the potential for online damage detection. However, the propagation of Lamb waves is highly complicated [1] and challenging, primarily due to dispersion, wave splitting into multiple modes, and waves reflected from boundaries/

discontinuities which may easily conceal damage-scattered components in the signal, making the extraction of damage features challenging. Several signal processing schemes have been implemented for this purpose.

In the field of SHM signal processing, the ideal approach involves extracting features that are sensitive to damage, but not too sensitive to operational and environmental variations. As in real world structures, such an approach is difficult, and intelligent feature extraction procedures are required [3]. In the existing literature, various feature extraction techniques have been presented based on time-series analysis, frequency-domain analysis and time-frequency analysis [4].

The objective of this study is to develop a signal processing technique effective for Lamb wave signals and also to devise an automatic damage detection technique which can be used for online health monitoring using the minimum possible processing power. In the existing literature MP has been mainly developed for signal compression and de-noising [5] e.g. for video size compression [6] and electrocardiography signal compression [7]. Uses such as image classification using MP have also been illustrated in the literature [8]. In the field of non-destructive testing, its use for signal processing of guided waves is being explored [9]. Attempts have been made to overcome challenges due to multiple modes and dispersion in guided waves for reliable damage detection [10, 11]. Li *et al* have demonstrated the application of correlation filtering-based MP for analyzing Lamb wave response in wavelet domain [12]. Das *et al* in their study have used Monte Carlo matching pursuit decomposition to analyse damages using a smaller dictionary [13]. Also, MP has been utilized to identify the modes in Lamb waves. Xu *et al* have used MP for Lamb wave decomposition and mode identification [14]. However, not much research has been done to explore the possibility of using MP along with machine learning algorithms for structural health monitoring [15]. As mentioned earlier, here, in addition to de-noising MP is primarily used to increase the sparsity of the Lamb wave response for efficient implementation of ML tools.

ML theories are bodies of knowledge that attempt to create computational relationships between quantities on the basis of observed data. Here, the computational rules are learned or inferred on the basis of observational evidence. ML theories are broadly defined to address the problems of (1) classification, (2) regression and (3) density estimation [16]. Automatic pattern recognition/classification of damage-sensitive features is an integral part of data driven SHM. ML algorithms can also be divided into *supervised* and *unsupervised* learning. The former requires a training data set consisting of input and output data for a postulated relationship so that associations may be learnt and errors corrected. On the contrary, the latter involves characterization of a set on the basis of measurements and determining the underlying structure. For this study, supervised learning is used as it is possible to generate a training database by creating damages in the structure. ANNs and SVMs are used for pattern recognition to classify the damage-sensitive data. In the existing literature, Worden and Manson [17] have illustrated that ML theories offer a natural framework to address damage detection problems. Ko and Ni [18] have explored developments such as the application of ML in the SHM of large bridges. Su and Ye [19] have used ANNs for the identification of de-lamination in composite structures. Additionally, Dworakowski *et al* have demonstrated the application of ANNs and Lamb waves for damage detection in aerospace structures. Also, Rho *et al* have used SVMs for pattern classification to aid the online health monitoring of jointed steel plates [20].

In the presented technique, MP is initially deployed to increase the sparsity or reduce the total number of non-zero data points by more than 40%, which is significant for ML algorithms to work efficiently. However, some relevant features from the database are lost. To address the posed challenge, MP is deployed in two stages. Initially, in the first step, it is used to increase the sparsity while retaining the important damage sensitive features. The processed sparse signal is then passed to the SVM classifier to ascertain the location of damage. Furthermore, in the second step, to increase the accuracy of the technique, part of the original signal in the vicinity of the estimated location of damage is extracted and MP is used again on that extracted signal. This way, MP is used to de-noise the signal while retaining almost all the major features, and it is then analyzed to improve the estimation of the location of the damage. Hence, by using MP in two stages along with SVM, good accuracy for damage detection is achieved. Figure 1 shows a flowchart of the damage detection technique developed in this study.

2. Finite element (FE) simulations

2.1. Setup for the simulation

A two-dimensional thin aluminium plate of length 1.2 m and thickness 1.6 mm was used and only the thickness plane in the direction of wave propagation was modelled for generation of the Lamb wave in the specimen, using plane strain assumption. To account for the damage in the plate, a rectangular notch of dimension 10 mm × 1 mm was considered as shown in the schematic of the plate in figure 2. Generation of the database for Lamb waves was done using FE simulations in ANSYS. To model the structure, a four noded element PLANE182 was selected. The material properties of aluminum were considered as follows: density 2712kg m^{-3}, Poisson's ratio 0.3 and Young's modulus 70 GPa. To model the plate, the element edge length was set to 0.25 mm and the total number of elements in the plate was around 5000. A 3.5 cycle sinusoidal modulated tone burst of frequency 200 kHz was used for excitation of the signal, as shown in figure 3(a). The frequency response for the initial disturbance is shown in figure 3(b). White Gaussian noise was added to the velocity data and the signal to noise ratio (SNR) was set to 130. The created database of each Lamb wave response had nearly 2000 data-points.

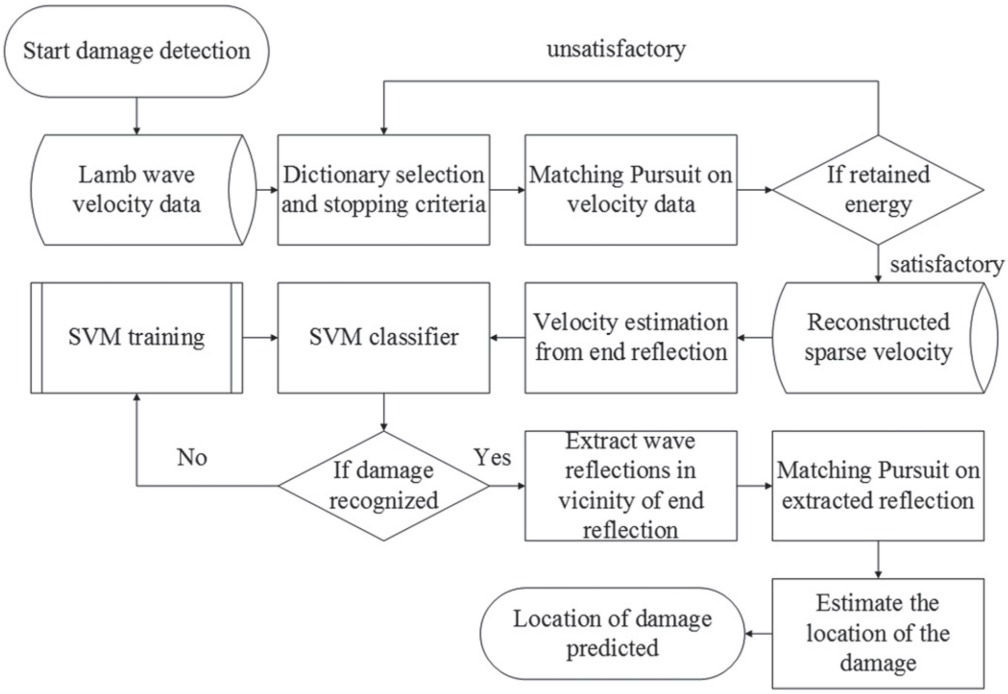

Figure 1. Schematic of damage detection technique.

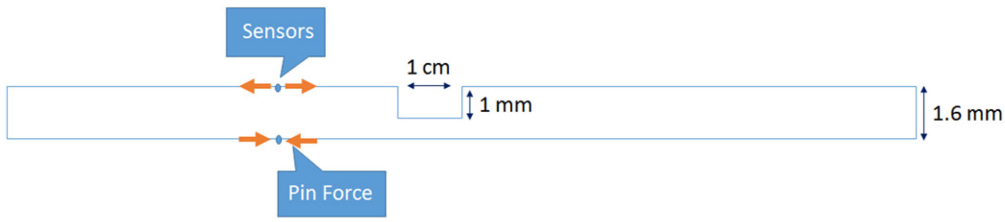

Figure 2. Schematic of the plate with notch.

2.2. Tuning of the Lamb wave

As discussed earlier, Lamb wave analysis is a challenging task due to its dispersive nature. The presence of multiple modes and reflections from the edges and discontinuities increase the complexity of analysis. These also often camouflage the damage response. In order to reduce the complexities, A_0 and S_0 modes can be selectively generated using two opposite poled lead zirconate titanate (PZT) patches placed exactly one below the other. In addition since Lamb wave propagation is particular to thin-walled structures, it is usually possible to place patches on either side of the structure to achieve single mode generation. For other cases, a piezoelectric wedge transducer with a certain angle of incidence can be used for selective generation of the Lamb wave mode as reported in reference [21]. Using pin forces in the configuration shown in figure 2 this PZT configuration is achieved. Here, two point forces 5 mm apart are applied on top and on the bottom surface in the opposite direction. Only A_0 mode is generated using this configuration. Similarly, S_0 mode can be selectively generated by placing PZT patches poled in the same direction and used for detection of delamination type damages. However, in this study A_0 mode is used as it is more sensitive to damage. Frequency range selection is also quite relevant as it affects the sensitivity to damage. The recommendation is to have the maximum possible frequency attainable with high SNR. In addition, to avoid problems, for a given thickness of plate, the frequency should be less than the cut-off frequency of the higher Lamb wave mode. To identify the correct frequency, the dispersion curve was plotted, using 'The Lamb Toolbox' in Matlab [22]. Figure 4 shows the dispersion curve obtained using the toolbox. Velocity for A_0 from the plot came out at around 2677 m s^{-1} and for S_0 around 5475 m s^{-1} at a frequency of 200 kHz.

2.3. Velocity response of the Lamb wave

As illustrated in figure 2, the velocity response is recorded using two sensors measuring the in-plane response. The response from one of the sensors is shown in figure 5, showing both the A_0 and the S_0 modes. When the Lamb modes interact with the edges or any discontinuity, a mode conversation is observed. An A_0 mode converts into both A_0 and S_0 modes, referred to as A_0A_0 and A_0S_0, representing A_0 mode generated from an A_0 mode and S_0 mode generated from an A_0 mode respectively. These modes are generated as

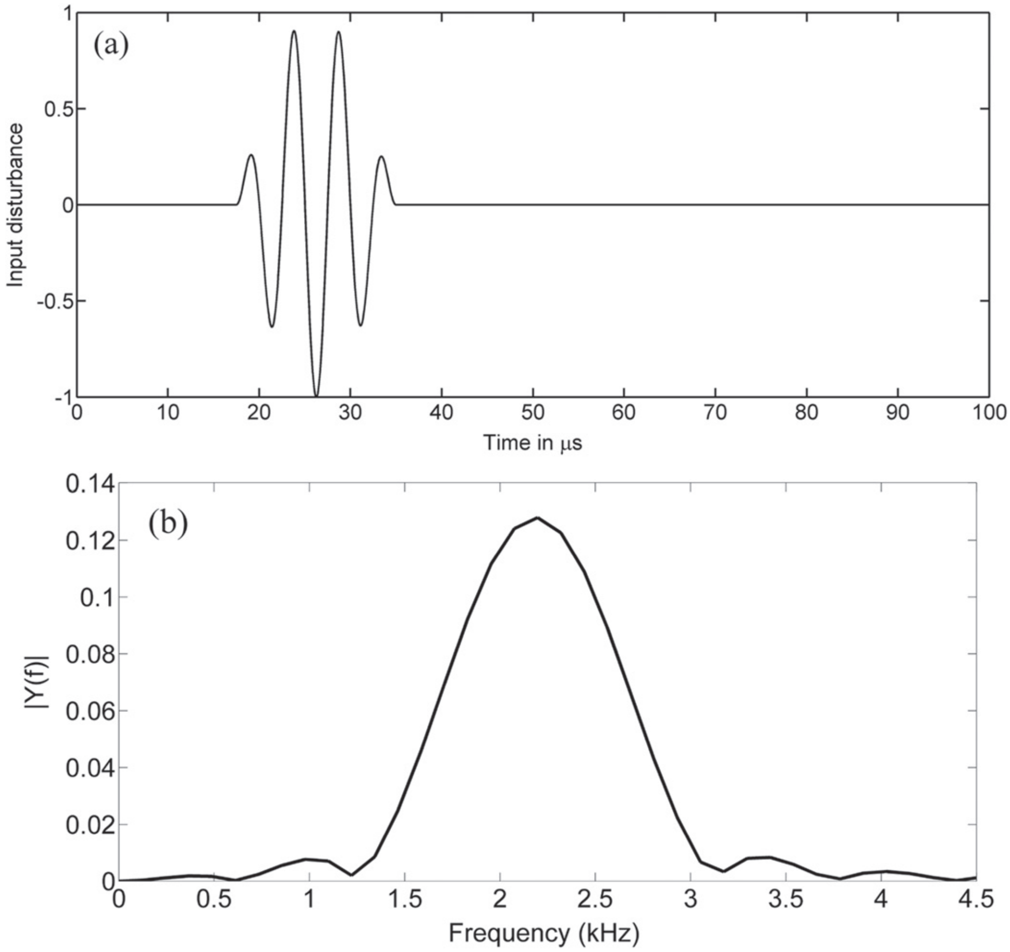

Figure 3. (a) Sinusoidal modulated tone burst—excitation signal; (b) Frequency response of excitation signal.

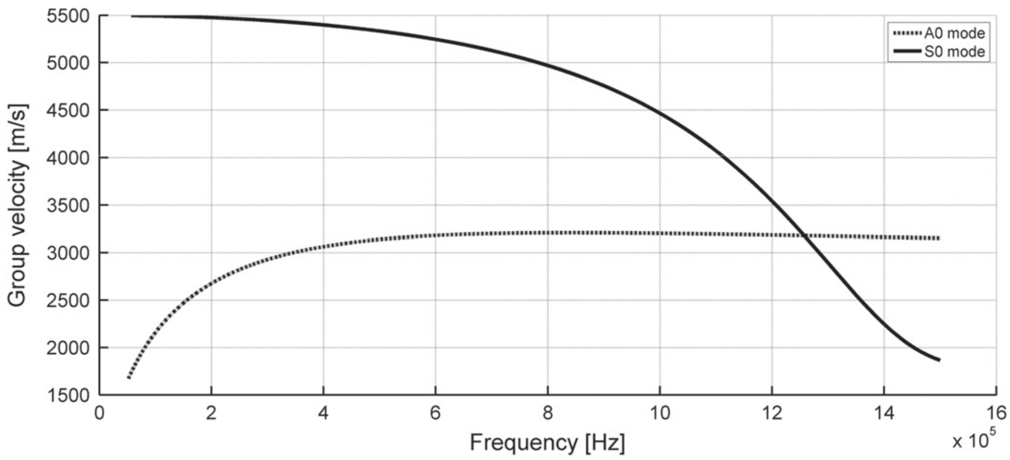

Figure 4. Dispersion plot for thickeness of plate = 1.6 mm.

the A_0 mode interacts with the damage. These modes become further split into the respective A_0 and S_0 modes, named as $A_0A_0A_0$, $A_0A_0S_0$, $A_0S_0A_0$ and $A_0S_0S_0$, following the same nomenclature, on the second interaction with the boundary/discontinuity. The nomenclature of converted modes is based on work by Ramadas *et al* [21]. The splitting of modes is illustrated in figure 6. Symmetric inplane signals, namely A_0S_0 and $A_0A_0S_0$, are eliminated by subtracting the response of the top sensor from the bottom sensor [23]. Hence, A_0A_0 and $A_0A_0A_0$ are the major residual modes after subtraction. Figure 7 shows the output along with the labels for the reflection from the edge and damage.

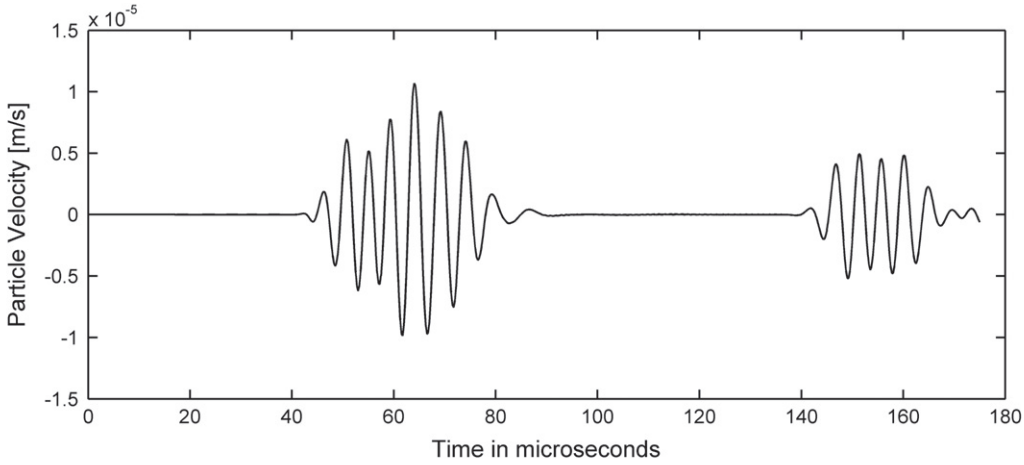

Figure 5. Dispersed Lamb wave, A_0 and S_0 modes.

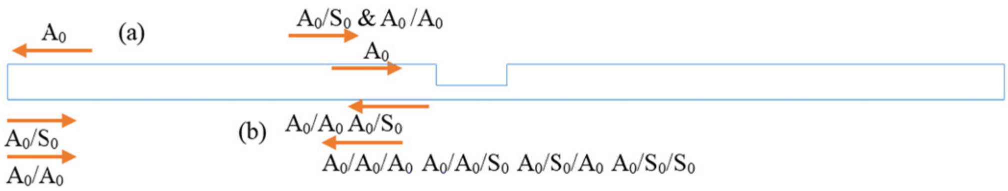

Figure 6. Mode conversion (a) at boundary, (b) at damage.

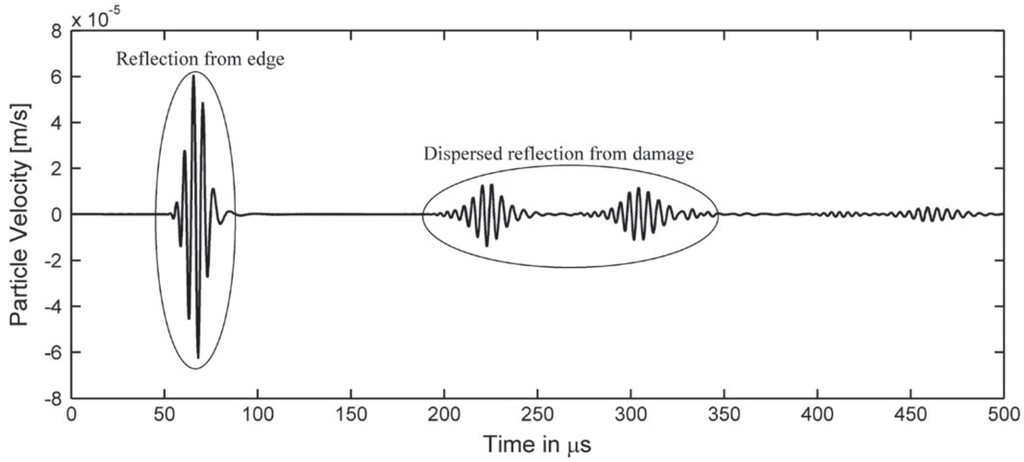

Figure 7. A_0 mode after subtraction of the velocity response.

3. Matching pursuit

As said earlier, MP is a signal processing technique which iteratively projects a target signal into a linear combination of defined basis functions in the dictionary. In the first iteration, parameters of the first basis function, such as scale, amplitude and arrival time, are optimized so that the target signal can be projected on to it. Next, the remaining component of the passed target signal, after being projected on to the first basis function (referred to as residual signal), is projected on to the second basis function. Finally, these steps are repeated until all of the wave components of interest have been identified and decomposed. The stopping criteria can be based on fixing the total number of iterations or fixing the maximum relative allowable error in the signal.

The algorithm iteratively projects a signal **f** onto a given over-complete dictionary **D** and chooses functions from dictionary-called atoms (g_γ) that best match the signal in each iteration. The formula is illustrated in equation (1) where n indexes the atoms and a_n is the weighting factor for each of the chosen atoms.

$$f(t) = \sum_{n=0}^{\infty} a_n g_{\gamma_n} \tag{1}$$

The algorithm for MP is [5]:

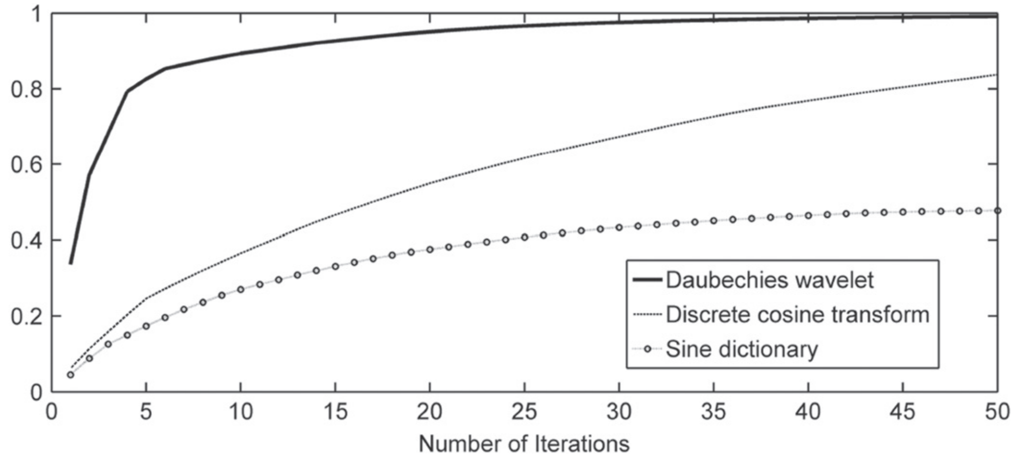

Figure 8. Retained energy using different functions as the dictionary.

Input: Signal: $\mathbf{f(t)}$, dictionary $\mathbf{D}$.

Output: List of coefficients: $\left(a_n, g_{\gamma_n}\right)$.

Initialization:

$R_1 \leftarrow f(t)$;

$n \leftarrow 1$;

Repeat:

find $g_{\gamma_n} \in D$ with maximum inner product $\left| \left\langle R_n, g_{\gamma_n} \right\rangle \right|$

$a_n \leftarrow \left\langle R_n, g_{\gamma_n} \right\rangle$;

$R_{n+1} \leftarrow R_n - a_n g_{\gamma_n}$

$n \leftarrow n + 1$;

Until stop condition

The dictionary is designed such that it accurately represents characteristics of a given signal. Hence, selection of an optimum dictionary is a crucial task. In this study the dictionary was selected on the basis of retained signal energy per iteration. Initially, dictionaries such as sin, dct and wavelet dictionaries were tested and from them the symlet wavelet dictionary was selected as it retains high energy in signifcantly fewer iterations. Retained energy graphs are plotted in figure 8 for various functions and in figure 9 for vanishing moments and levels of the symlet wavelet dictionary. On analyzing these figures, a symlet wave of level four with five vanishing moments was selected for this study. The symlet wavelet dictionary used is shown in figure 10.

In addition, as the main motivation for signal processing using MP is to increase the sparsity of the database, it is controlled by setting the stopping criteria for MP (number of iterations). If the number of iterations is too few, then the required features will be lost. On the other hand, if the number of iterations is very high, then noise will not be eliminated from the signal. Figure 11 shows the response after MP for different numbers of iterations. For 500 iterations, noise can be observed, but in the second case, due to fewer iterations (only five), some of the features are lost. The number of iterations or the stopping criteria can be based on the retained energy or simply on the total number of iterations. Here, the total number of iterations have been fixed to nine based on analyzing the retained energy shown in figure 9.

4. Classification of damage

4.1. Artificial neural networks

These networks are computational models inspired by the central nervous system that are capable of pattern recognition. Here, the ANN scheme is implemented using the Matlab Neural Network Toolbox. A two-layer feed-forward network space, with sigmoid hidden and output neurons, has been used. The network is trained with scaled conjugate gradient back-propagation. Here, ten hidden layers have been considered, as shown in figure 12, which depicts the implemented ANN schematic. Figure 13 shows damage classification; output of the network is plotted along with the wave using a dotted line to denote the probability of damage.

4.2. Support vector machine

SVM is an automated ML technique which uses a hypothesis space of functions in a high dimensional feature space. The simplest known SVM model is the linear SVM and it works for data that are linearly separable in original space. After the introduction of non-linear kernel functions for SVM, the technique is also applicable for non-linear classification. The effectiveness of SVM depends on selection of the kernel and its parameters for training. Different kernel functions, namely linear, quadratic, polynomial, multilayer perceptron and Gaussian radial basis function, were tested for training data. Polynomial function was the exception as the rest failed to capture the required characteristic of the signal. The third degree polynomial kernel function was used for this study, and the output is illustrated in figure 14 where the dotted lines show the damage identified or classfied by the SVM classifier.

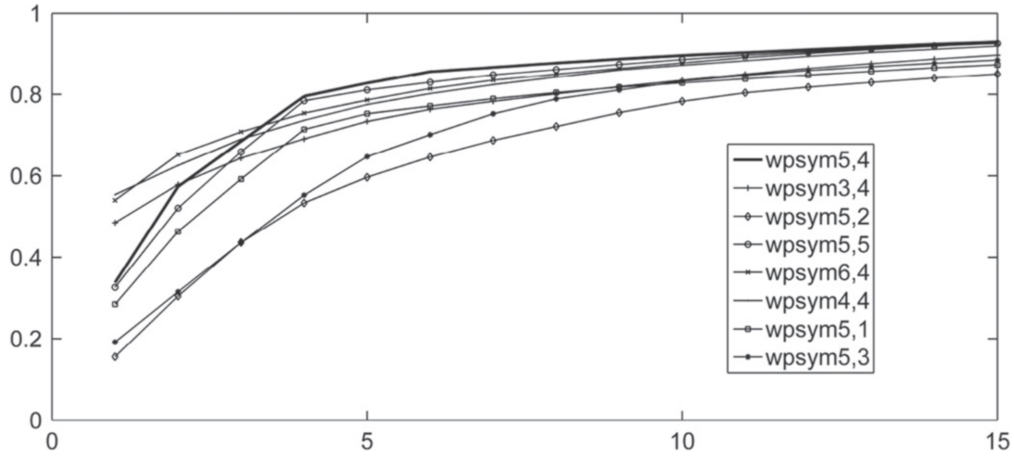

Figure 9. Retained energy using different symlets as the dictionary.

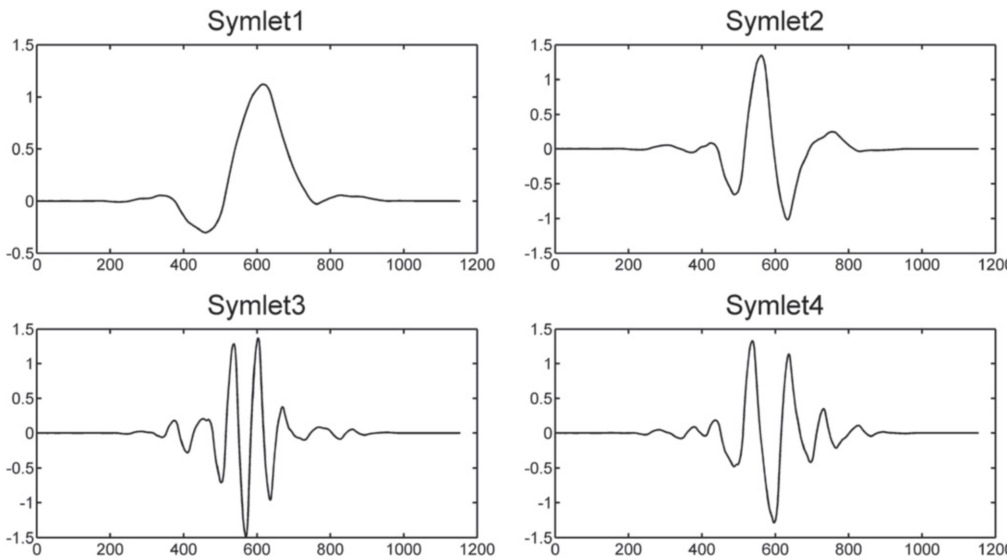

Figure 10. Symlet5,4—The dictionary used for MP.

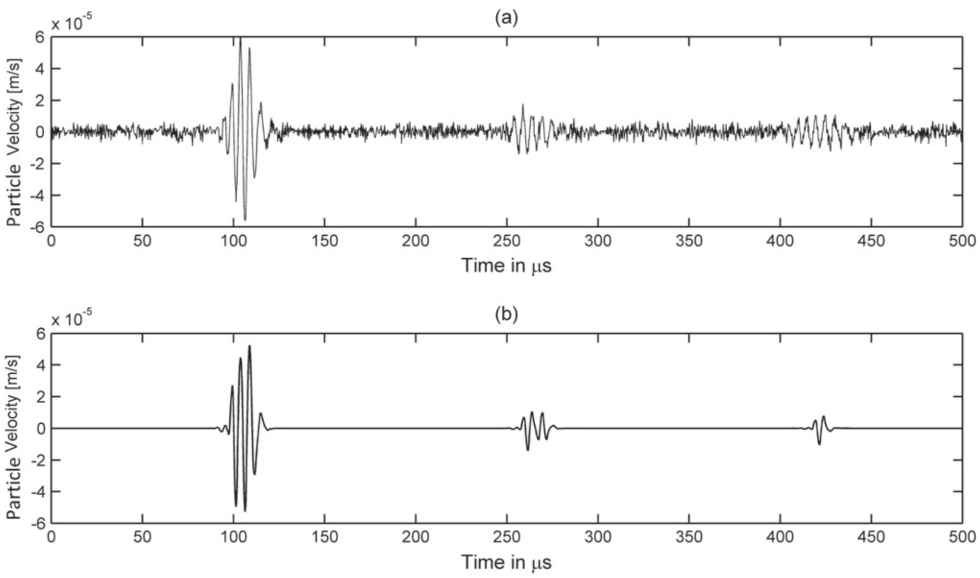

Figure 11. Impact of number of iterations in MP (a) 500 iterations, (b) 5 iterations.

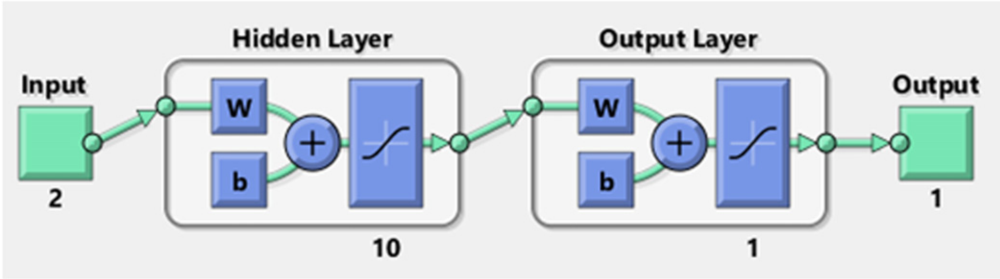

Figure 12. ANN scheme implemented.

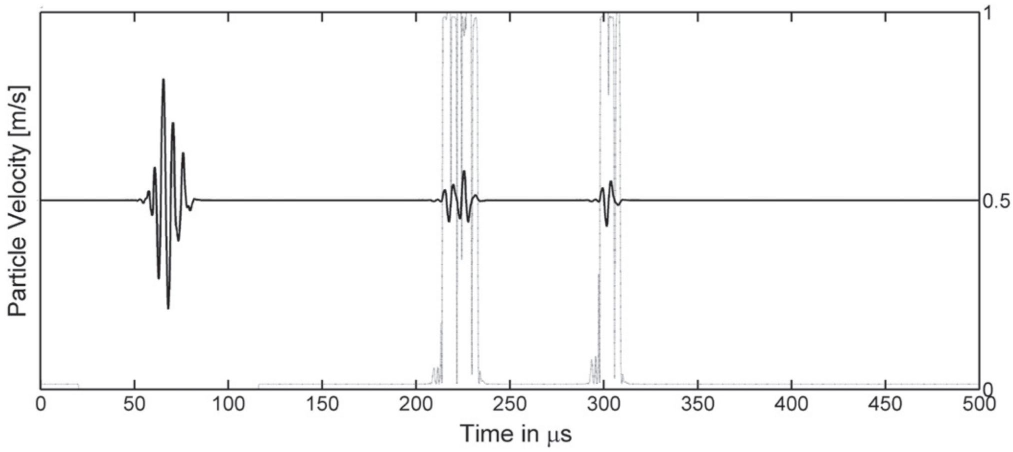

Figure 13. Two layer feed-forward ANN for classification.

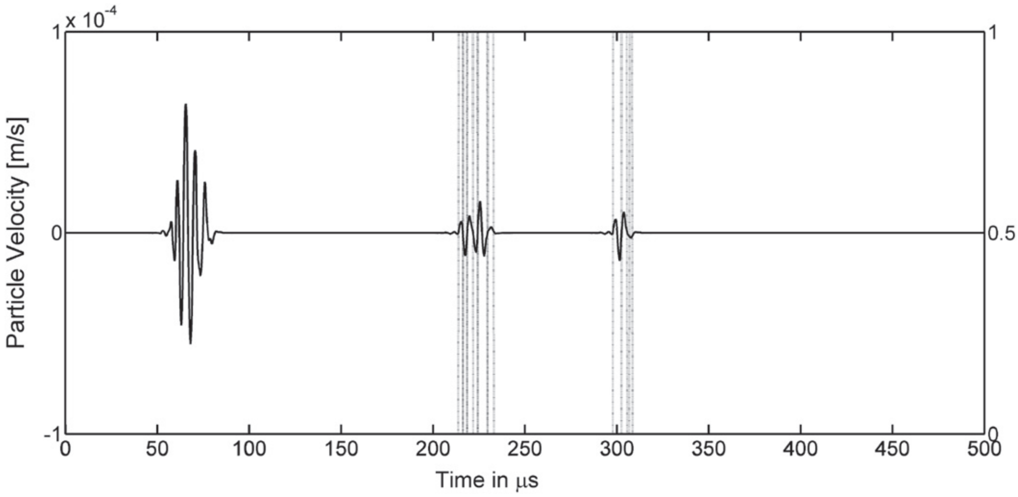

Figure 14. SVM classifier trained with a third degree polynomial kernel.

4.3. Training for SVM and ANN

In this study, for training of both the ANN network and the the SVM classifier, a database consisting of velocity response was used. The input database had around 12,800 rows of data. It consisted of two columns, with one column having the velocity response and the other column having binary values (i.e. 0 for reflection from the edge and 1 otherwise). A second column was added to improve efficiency and to give a hint to the algorithm that it should ignore the reflection from the edges. The output/target dataset also consisted of binary values with 1 for reflection from damage and 0 for the remaining points.

5. Damage detection

The automated damage detection technique consists of three major steps. The first step involves increasing the sparsity of the signal using MP with the aim of reducing the number of non-zero data-points to improve ML classification. The

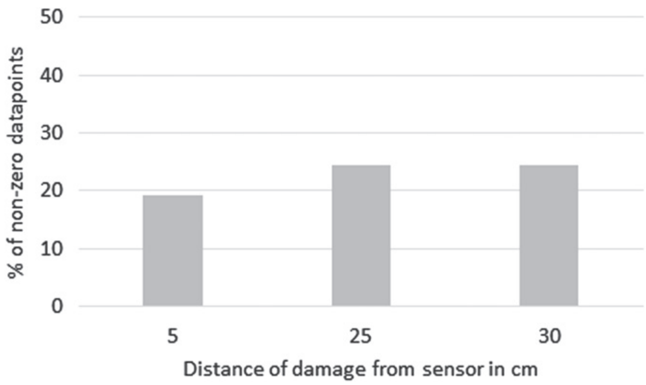

Figure 15. Sparsity achieved using MP.

second step is the ML classification where ANN and SVM have both been tested and used. Lastly, the third step involves using MP again for de-noising the detected damage reflection classified by ML classifiers. As mentioned earlier, ANSYS simulations are used to obtain the velocity response, as shown in figure 7, which is used as input for this damage detection technique. For a single damage in the structure, the aim is to firstly ascertain the reflection from the end, find out the exact velocity through that and then select the right mode for determining the damage location. In general, the magnitude of reflection from a damage is small compared to the edge reflection (e.g., figure 7 shows the magnitude of the reflection from the edge to be much higher than the reflections from the damage). Here, velocity is calculated by ascertaining the reflection of the highest magnitude and then using the initial set of data available, i.e. the distance of the sensor from the edge. In the cases studied, since the sensor was nearer to the left boundary, reflection from the left boundary was considered for obtaining the time of flight and thereby the velocity.

5.1. Increasing sparsity using MP and velocity estimation

As discussed earlier, here MP was used to reduce the non-zero datapoints (see figure 11). The *temporal MP* function was used to perform MP [24]. After using MP, a fair idea of reflections in the wave was obtained. The largest magnitude of reflection was from the end or the boundary. As the distance between the sensor and the boundary was known, velocity was estimated and compared with the theoretical value for validation. Figure 15 shows the sparsity achieved using eight iterations of MP as discussed in section 3. Out of 2000 total data-points after using MP only around 500 non-zero data points remained. All the unnecessary features such as noise were discarded from the signal. Three different examples with different damage locations were considered to emphasize the efficiency and genericness of MP in achieving sparsity. It can be observed from figure 15 that in all three cases, MP successfully reduced the non-zero data points by about 50%.

Table 1. Confusion matrix-ANN.

		Truth Data		
	ANN	No damage	Damage	Accuracy
Classifier Results	No damage	2999	149	95.27%
	Damage	8	42	84.00%
	Reliability	99.73%	21.99%	95.09%

Table 2. Confusion matrix-SVM.

		Truth Data		
	SVM	No damage	Damage	Accuracy
Classifier Results	No damage	3012	6	99.80%
	Damage	138	42	23.33%
	Reliability	95.62%	87.50%	**95.50%**

5.2. Classification using ML

Following MP and increasing the sparsity, ML is used to classify in order to determine the different modes of the reflected wave. For a single damage, the reflection with the higher speed is selected for analysis as it has the same mode as the reflection from the end. For this study, SVM was found to be a better classifier than ANN. Comparing figures 13 and 14 gives a rough idea that SVM spans the damage reflection more efficiently as compared to ANN as the lines spanning the damage are denser for SVM and show consistency for different damage locations. In the case of ANN, the initial features were not captured consistently, leading to higher errors in location estimation. Figure 17 shows the comparison of ANN and SVM for damage detection considering different damage and sensor locations.

5.3. Confusion matrix to test training

To validate training and to compare ANN and SVM, confusion matrices have been used. 75% of the data is used for training and the remaining 25% is used to test the effectiveness of training. Tables 1 and 2 show the matrices for ANN and SVM respectively. The overall accuracy of SVM (95.50%) is slightly higher than ANN (95.09%). Also, the user accuracy or simply the reliability for data-points with damage is observed to be much higher for SVM as compared to ANN for this given case. κ, a single value metric to compare two confusion matrices, is calculated and comes out at 0.33 for ANN and 0.35 for SVM. According to Koch and Landis [25], a κ value anywhere between 0.2 and 0.4 indicates that the classifier is in a *fair* agreement.

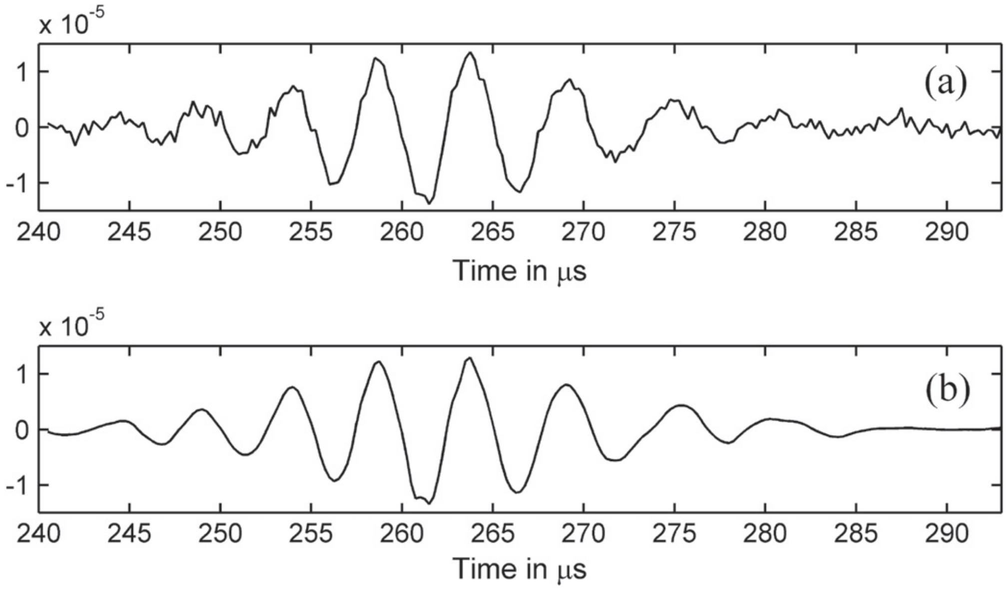

Figure 16. (a) Before MP, (b) De-noised reflection after MP.

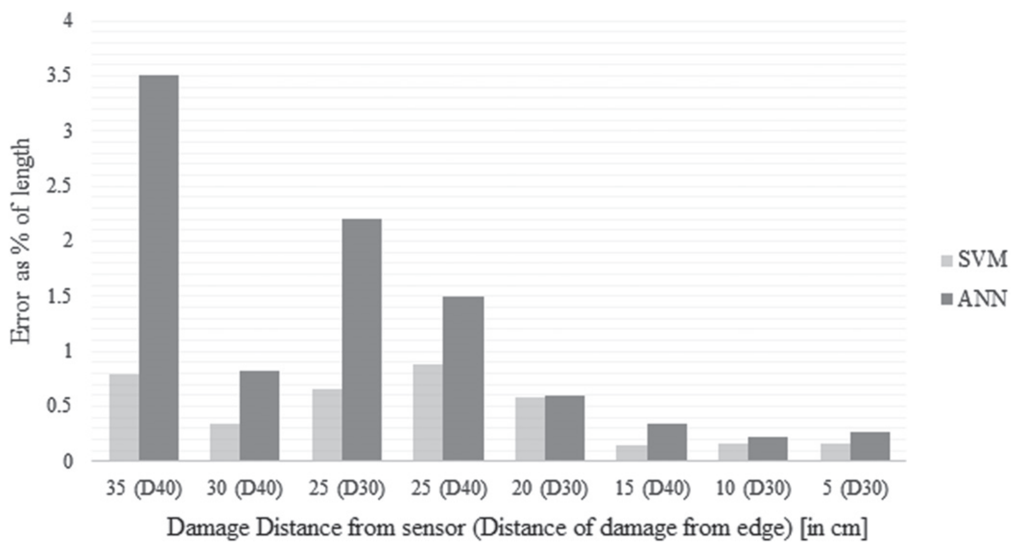

Figure 17. Comparison of ANN and SVM in damage detection.

5.4. De-noising using MP and damage location estimation

Finally, after using ML for classification, probable reflection from damage is extracted from the initial database because, as discussed, some features are lost after MP, which might lead to a slightly incorrect prediction of damage. Hence, only a small part of the velocity database in the vicinity of the damage reflection as predicted by ML classifiers is extracted. On this extracted signal, MP is used again with the sole purpose of de-noising it. The same dictionary is used here. After processing, a de-noised reflection is obtained, as shown in figure 16, which is used to predict the accurate distance of damage from the sensor. Here, from the extracted dataset the maximum group velocity is computed and then the starting point of reflection is considered at around 10% of the maximum value. Lastly, using the time obtained from this point, the distance is calculated by multiplying the time with velocity computed from the reflection from the edge (section 5.1). Thus, distance of the damage is computed automatically. Figure 17 shows that for SVM the error in estimation is less than 1% of the length of the plate.

6. Conclusion

The presented study attempts to develop a data driven damage detection technique in metallic plate using the Lamb wave response. A two-stage MP approach is proposed to estimate the location of damage from the Lamb wave signal measured in plate. The key idea of this study is to make the technique computationally efficient so that online health monitoring can be performed using the limited processing power available on-board. The proposed method successfully extracts meaningful pulses from simulated noisy signals. Very small echoes reflected from a crack, which would be difficult to detect by other methods, were captured using this approach. Using SVM/ANN and MP, the process is automated, and error in estimating the location of damage is less than 1% of the total length of plate. Hence, the technique possesses potential for SHM of real-life aerospace structures which would be explored in future work. Also, the proposed technique presently works for detection of a single damage, further investigation and study is required on dispersion of Lamb waves to make this technique implementable for multiple damages. Additionally, as only two sensors are used back to back, the technique predicts only the distance of the damage from the sensor and does not indicate the direction. Thus, this issue would be tackled in future work by using an array of sensors.

Acknowledgments

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