

Data605_Assignment7_GA

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Data 605 Assignment # 7

1. Let X_1, X_2, \dots, X_n be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k . Let Y denote the minimum of the X_i 's. Find the distribution of Y .

Answer:

Since Y is equivalent to the minimum of the X_i 's, there is an “ k ” amount of possibilities that X_i can take: 1, 2, 3, ..., k . The total possible number of assignments for the collection of random variables is k^n .

When $Y = 1$, the way to get 1 is $k^n - (k-1)^n$, with k^n representing the total possibilities and $(k-1)^n$ are the options where none of the random variables equal 1.

$$P(X=1) = (k^n - (k-1)^n) / k^n$$

$$P(X=2) = ((k^n - (k-2)^n) - [k^n - (k-1)^n]) / k^n \rightarrow (k-1)^n - (k-2)^n / k^n$$

$$P(X=i) = ((k-i+1)^n - (k-i)^n) / k^n$$

Part 2

2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors).

This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.).

- a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

```
fail_p <- 1/10

#probability of the machine not failing
success_p <- 1 - fail_p

#use p_geom formula to model geometric
geometric <- 1 - pgeom(8-1, fail_p)
geometric
```

```
## [1] 0.4304672
```

```
#Expected value
value_geomtric <- 1/fail_p
value_geomtric
```

```
## [1] 10
```

```
#Standard Deviation
sd_geo <- sqrt(success_p/(fail_p)^2)

sd_geo
```

```
## [1] 9.486833
```

- b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential.

```
lambda <- 1/10

exponential_p <- pexp(8, lambda, lower.tail = FALSE)
exponential_p
```

```
## [1] 0.449329
```

```
#Expected Value
expected_exp_val <- 1/lambda
expected_exp_val
```

```
## [1] 10
```

```
#STD Value

expo_sd <- sqrt(1/(lambda^2))
expo_sd
```

```
## [1] 10
```

- c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

```
p <- .1
complement <- 1 - p
success <- 0
n <- 8

binomial_p <- dbinom(success, n, p)
binomial_p
```

```
## [1] 0.4304672
```

```
#Expected value
binomial_exp_value <- n * p
binomial_exp_value
```

```
## [1] 0.8
```

```
# STD Value
binomial_sd <- sqrt(n * p * complement)
binomial_sd
```

```
## [1] 0.8485281
```

- d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson.

```
lambda <- 8/10
ppois(0, lambda = lambda)
```

```
## [1] 0.449329
```

```
#Expected Value
Expected_pois_val <- lambda
Expected_pois_val
```

```
## [1] 0.8
```

```
#Standard Deviation
standard_pois_dev <- sqrt(lambda)
standard_pois_dev
```

```
## [1] 0.8944272
```