Topic: Gauss-Jordan elimination

Question: Use Gauss-Jordan elimination to solve the system with a rref matrix.

$$x + 3y = 13$$

$$2x + 4y = 16$$

Answer choices:

A $(x, y) = (5,2)$

B $(x, y) = (3, -1)$

C $(x, y) = (-1,3)$

D $(x, y) = (-2,5)$

Solution: D

The augmented matrix is

$$\begin{bmatrix} 1 & 3 & | & 13 \\ 2 & 4 & | & 16 \end{bmatrix}$$

The first row already has a leading 1. After $2R_1 - R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 3 & | & 13 \\ 0 & 2 & | & 10 \end{bmatrix}$$

The first column is done. After $(1/2)R_2 \to R_2$, the matrix is

$$\begin{bmatrix} 1 & 3 & | & 13 \\ 0 & 1 & | & 5 \end{bmatrix}$$

After $R_1 - 3R_2 \to R_1$, the matrix is

$$\begin{bmatrix} 1 & 0 & | & -2 \\ 0 & 1 & | & 5 \end{bmatrix}$$

The second column is done, and we get the solution set $(x, y) = (-2,5)$.

Topic: Gauss-Jordan elimination

Question: Use Gauss-Jordan elimination to solve the system with a rref matrix.

$$x + 4z = 11$$

$$x - y + 4z = 6$$

$$2x + 9z = 25$$

Answer choices:

A $(x, y, z) = (-1,5,3)$

B $(x, y, z) = (11,6,25)$

C $(x, y, z) = (1,0,12)$

D $(x, y, z) = (-3,8,3)$

Solution: A

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 11 \\ 1 & -1 & 4 & 6 \\ 2 & 0 & 9 & 25 \end{array}\right]$$

The first row already has a leading 1. After $R_1 - R_2 \to R_2$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 11 \\ 0 & 1 & 0 & 5 \\ 2 & 0 & 9 & 25 \end{array}\right]$$

After $2R_1 - R_3 \to R_3$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 11 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

The first and second columns are done. After $(-1)R_3 \to R_3$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 11 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

After $R_1 - 4R_3 \to R_1$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The third column is done, and we get the solution set $(x, y, z) = (-1,5,3)$.

Topic: Gauss-Jordan elimination

Question: Use Gauss-Jordan elimination to solve the system with a rref matrix.

$$2x + 4y + 10z = 30$$

$$x + y + 3z = 10$$

$$2x + y + 2z = 9$$

Answer choices:

A $(x, y, z) = (7, -3, 5)$

B $(x, y, z) = (-4, 1, 0)$

C $(x, y, z) = (2, -1, 3)$

D $(x, y, z) = (30, 10, 9)$

Solution: C

The augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 4 & 10 & 30 \\ 1 & 1 & 3 & 10 \\ 2 & 1 & 2 & 9 \end{array}\right]$$

After $(1/2)R_1 \to R_1$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 15 \\ 1 & 1 & 3 & 10 \\ 2 & 1 & 2 & 9 \end{array}\right]$$

After $R_1 - R_2 \to R_2$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 15 \\ 0 & 1 & 2 & 5 \\ 2 & 1 & 2 & 9 \end{array}\right]$$

After $2R_1 - R_3 \to R_3$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 15 \\ 0 & 1 & 2 & 5 \\ 0 & 3 & 8 & 21 \end{array}\right]$$

The first column is done. After $R_1 - 2R_2 \to R_1$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 2 & 5 \\ 0 & 3 & 8 & 21 \end{array}\right]$$

After $R_3 - 3R_2 \to R_3$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

The second column is done. After $(1/2)R_3 \to R_3$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

After $R_1 - R_3 \to R_1$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

After $R_2 - 2R_3 \to R_2$, the matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

The third column is done, and we get the solution set $(x, y, z) = (2, -1, 3)$.