GAbreu_Assignment3

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Data 605 - Assignment 3

Problem Set 1

1. What is the rank of the matrix A?

```r
#Building matrix A
A <- matrix(cbind(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), byrow = T, ncol = 4)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```r
#Going to use the function created in HW 2 to see how many pivot entries are in the matrix

LU_Decomp <- function(mat){

#Get the rows of the matrix
mat_rows <- nrow(mat)
U <- mat
L <- diag(mat_rows)

#a is rows, b is columns
# Create loop to go through every row, column by column
for(a in 1:(nrow(mat) - 1)){
    for(b in ((a + 1):nrow(mat))){
    #Solve for L first, if not, the other values won't populate...
    L[a,b] <- (U[b,a]/U[a,a])
    U[b,] <- U[b,]-(U[a,] * (U[b,a]/U[a,a]))
    }
 }
return(list("L" = L, "U" = U))
}


LU_Decomp(A)
```

```
## $L
##      [,1] [,2] [,3]  [,4]
## [1,]    1   -1  0.0  5.00
## [2,]    0    1  0.5 -3.00
## [3,]    0    0  1.0  1.25
## [4,]    0    0  0.0  1.00
##
## $U
##      [,1] [,2] [,3]   [,4]
## [1,]    1    2    3  4.000
## [2,]    0    2    4  7.000
## [3,]    0    0   -4 -2.500
## [4,]    0    0    0  1.125
```

There are 4 pivot entries, therefore the rank should be 4. Let's check with the default rankMatrix function from the Matrix library.

```r
rankMatrix(A)[1]
```

```
## [1] 4
```

The rank function also confirms the rank is 4.

2. Given an mxn matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

Answer: If m>n and the n column is independent, then the rank of an mxn matrix is n.

If the matrix is non-zero, then the minimum rank is 1.

3. What is the rank of matrix B?

```r
B <- matrix(c(1,2,1,3,6,3,2,4,2), byrow = T, nrow = 3)
B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```r
rankMatrix(B)[1]
```

```
## [1] 1
```

The rank of the matrix is 1.

Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

Please show your work using an R-markdown document. Please name your assignment submission with your first initial and last name.

Here is the problem done by hand:

```r
knitr::include_graphics(path = "HW_PS2_1.pdf")
```

PROBLEM SET 2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \qquad E_\lambda = \lambda I_n - A \rightarrow \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\left(\begin{bmatrix} \lambda-1 & -2 & -3 \\ 0 & \lambda-4 & -5 \\ 0 & 0 & \lambda-6 \end{bmatrix} \right) = 0 \rightarrow \cancel{0 \left| \quad \right|} + \lambda-6 \begin{vmatrix} \lambda-1 & -2 \\ 0 & \lambda-4 \end{vmatrix} - \cancel{0 \left| \quad \right|}$$

$$\rightarrow \lambda-6\left[(\lambda-1)(\lambda-4) - (0)\right]$$

$$\rightarrow \lambda-6\left[\lambda^2 - 4\lambda - \lambda + 4\right]$$

$$\rightarrow \lambda-6\left[\lambda^2 - 5\lambda + 4\right]$$

$$(\lambda-6)(\lambda-4)(\lambda-1)$$

Characteristic polynomial:

$$\lambda^3 - 5\lambda^2 + 4\lambda - 6\lambda^2 + 30\lambda - 24$$

$$\boxed{\lambda^3 - 11\lambda^2 + 34\lambda - 24}$$

Eigen values: $\boxed{\lambda = 6, 4, 1}$

For $\lambda = 6$:

$$\begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2/5 & -3/5 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2/5 & -3/5 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{2/5 R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & -8/5 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

assume $V_3 = 1$

$V_1 - 8/5 V_3 = 0$

$\boxed{V_1 = 8/5 V_3}$

$V_2 - 5/2 V_3 = 0$

$\boxed{V_2 = 5/2 V_3}$

$$E_{\lambda_6} = \text{span} \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix}$$

```r
knitr::include_graphics(path = "HW3_PS2_PT2.pdf")
```

For $\lambda = 4 \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$V_1 - 2/3 V_2 = 0$ $\quad \underbrace{V_2 = 1}_{\text{assume}}$ $\quad V_3 = 0$

$V_1 = 2/3 V_2$

$E_{\lambda_4} = \text{span} \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$

For $\lambda = 1 \rightarrow \begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & -2 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{array}{c} \begin{array}{ccc} V_1 & V_2 & V_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$0 + V_2 + 0 = 0$ $\quad V_3 = 0$ $\quad$ assume $V_1 = 1$

$V_2 = 0$

$E_{\lambda_1} = \text{span} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

```
A_2 <- matrix(c(1,2,3,0,4,5,0,0,6), byrow = T, nrow = 3)

#function to compute polynomial
charpoly(A_2)
```

```
## [1]   1 -11  34 -24
```

```
e_values <- eigen(A_2)

e_values$values
```

```
## [1] 6 4 1
```

```
e_values$vectors
```

```
##           [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```

```
#Eigen vector calculation

lambda_6_dist = sqrt((5/2)^2 + (8/5)^2 + 1)
lambda_6_dist
```

```
## [1] 3.132092
```

```r
lambda_4_dist = sqrt((2/3)^2 + 1)
lambda_4_dist
```

```
## [1] 1.20185
```

```r
lambda6_v1 = (8/5)/lambda_6_dist
lambda6_v2 = (5/2)/lambda_6_dist
lambda6_v3 = 1/lambda_6_dist

lambda6_evectors = matrix(c(lambda6_v1, lambda6_v2, lambda6_v3))
lambda6_evectors
```

```
##           [,1]
## [1,] 0.5108407
## [2,] 0.7981886
## [3,] 0.3192754
```

```r
lambda4_v1 = (2/3)/lambda_4_dist
lambda4_v2 = 1/lambda_4_dist


lambda4_evectors = matrix(c(lambda4_v1,lambda4_v2))
lambda4_evectors
```

```
##           [,1]
## [1,] 0.5547002
## [2,] 0.8320503
```