

**Topic:** Pivot entries and row-echelon forms**Question:** Which matrix is in row-echelon form?**Answer choices:**

A 
$$\begin{bmatrix} 1 & -5 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B 
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C 
$$\begin{bmatrix} 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

D 
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



**Solution: A**

In the matrix in answer choice A, the first non-zero entry in each row is a 1, the row consisting of only 0s is at the bottom, and the pivots follow a staircase pattern that moves down and to the right. The second column includes a non-zero entry, but it's not a pivot column. Therefore, the matrix is in row-echelon form.

The matrix in answer choice B is not in row-echelon form, because the first non-zero entry in  $R_3$  is not 1.

The matrix in answer choice C is not in row-echelon form, because the first non-zero entry in  $R_1$  and  $R_2$  appear in the same column.

The matrix in answer choice D is not in row-echelon form, because  $R_2$  is a row of 0s that would need to appear at the bottom of the matrix.



**Topic:** Pivot entries and row-echelon forms**Question:** Which matrix is in reduced row-echelon form?**Answer choices:**

A 
$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B 
$$\begin{bmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

C 
$$\begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

D 
$$\begin{bmatrix} 1 & -9 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



**Solution: C**

In the matrix in answer choice C, the first non-zero entry in each row is 1, the row consisting of only 0s is at the bottom, and the pivots follow a staircase pattern that moves down and to the right. The two pivot columns include only the pivot entry, and otherwise only zero entries. The second and the fourth columns include a non-zero entry, but they're not a pivot columns. Therefore, the matrix is in reduced row-echelon form.

In the matrix in answer choice A, the leading 1 in  $R_3$  is not the only non-zero entry in its column.

In the matrix in answer choice B, the leading 1 in  $R_3$  is not the only non-zero entry in its column.

In the matrix in answer choice D,  $R_2$  is a row of only 0s, but it has a non-zero row below it.



**Topic:** Pivot entries and row-echelon forms**Question:** What is the reduced row-echelon form of the matrix?

$$\begin{bmatrix} 3 & 9 & 15 & 96 \\ -4 & -12 & -18 & -118 \end{bmatrix}$$

**Answer choices:**

A  $\begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

B  $\begin{bmatrix} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

C  $\begin{bmatrix} 1 & 3 & 5 & 32 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

D  $\begin{bmatrix} 1 & 3 & 5 & 32 \\ 0 & 0 & 2 & 10 \end{bmatrix}$



**Solution: B**

Start with  $(1/3)R_1 \rightarrow R_1$  to get the pivot in the first column.

$$\begin{bmatrix} 3 & 9 & 15 & 96 \\ -4 & -12 & -18 & -118 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 32 \\ -4 & -12 & -18 & -118 \end{bmatrix}$$

Zero out the rest of the first column with  $4R_1 + R_2 \rightarrow R_2$ .

$$\begin{bmatrix} 1 & 3 & 5 & 32 \\ 0 & 0 & 2 & 10 \end{bmatrix}$$

Get the pivot in the second row with  $(1/2)R_2 \rightarrow R_2$ .

$$\begin{bmatrix} 1 & 3 & 5 & 32 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Zero out the rest of the third column with  $-5R_2 + R_1 \rightarrow R_1$ .

$$\begin{bmatrix} 1 & 3 & 0 & 7 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

In this matrix, the first non-zero entry in each row is a 1, and there are no rows consisting of only 0s. The pivots follow a staircase pattern, and the two pivot columns include only the pivot entry, and otherwise only zero entries. The second and fourth columns include a non-zero entry, but they are not pivot columns. Therefore, this matrix is in reduced row-echelon form.

