Problem Set 2

$$A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix} \qquad E_\lambda = \lambda I_n - A \rightarrow \lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} - \begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix} \rightarrow \begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix} - \begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$$

$$\left(\begin{bmatrix}\lambda-1&-2&-3\\0&\lambda-4&-5\\0&0&\lambda-6\end{bmatrix}\right) = 0 \rightarrow 0\begin{vmatrix}\quad&\quad\\\quad&\quad\end{vmatrix} + \lambda-6\begin{vmatrix}\lambda-1&-2\\0&\lambda-4\end{vmatrix} - 0\begin{vmatrix}\quad&\quad\\\quad&\quad\end{vmatrix}$$

$$\rightarrow \lambda-6\left[(\lambda-1)(\lambda-4)-(0)\right]$$

$$\rightarrow \lambda-6\left[\lambda^2-4\lambda-\lambda+4\right]$$

$$\rightarrow \lambda-6\left[\lambda^2-5\lambda+4\right]$$

$$(\lambda-6)(\lambda-4)(\lambda-1)$$

Eigen values: $\boxed{\lambda = 6, 4, 1}$

characteristic polynomial:

$\lambda^3-5\lambda^2+4\lambda-6\lambda^2+30\lambda-24$

$\boxed{\lambda^3-11\lambda^2+34\lambda-24}$

For $\lambda=6$:

$$\begin{bmatrix}5&-2&-3\\0&2&-5\\0&0&0\end{bmatrix} \rightarrow \begin{bmatrix}1&-2/5&-3/5\\0&2&-5\\0&0&0\end{bmatrix} \rightarrow \begin{bmatrix}1&-2/5&-3/5\\0&1&-5/2\\0&0&0\end{bmatrix} \xrightarrow{2/5R_2+R_1\rightarrow R_1} \begin{bmatrix}1&0&-8/5\\0&1&-5/2\\0&0&0\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix}$$

assume $V_3 = 1$

$V_1 - 8/5 V_3 = 0$

$\boxed{V_1 = 8/5 V_3}$

$V_2 - 5/2 V_3 = 0$

$\boxed{V_2 = 5/2 V_3}$

$$E\lambda_6 = \text{span}\begin{bmatrix}8/5\\5/2\\1\end{bmatrix}$$

For $\lambda = 4 \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} V_1 & V_2 & V_3 \\ \end{matrix} \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$V_1 - 2/3 V_2 = 0$ $\quad$ $\underbrace{V_2 = 1}_{\text{assume}}$ $\quad$ $V_3 = 0$

$V_1 = 2/3 V_2$

$$E_{\lambda_4} = \text{span} \begin{bmatrix} 2/3 \\ 1 \\ 0 \end{bmatrix}$$

For $\lambda = 1 \rightarrow \begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & -2 \\ 0 & 0 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & 3/2 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{matrix} V_1 & V_2 & V_3 \\ \end{matrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$0 + V_2 + 0 = 0$ $\quad$ $V_3 = 0$ $\quad$ assume $V_1 = 1$

$V_2 = 0$

$$E_{\lambda_1} = \text{span} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$