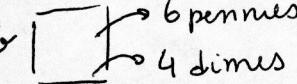
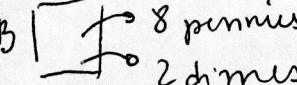


PROBLEM 1

6)  6 pennies
4 dimes

$$P(\text{Green}) = \frac{3}{4}$$

$$P(\text{Black}) = \frac{1}{4}$$

7)  8 pennies
2 dimes

Probability that the coins were picked from the green wallet:

$$P(G | D, P, P) = P(G) \cdot P(D, P, P | G)$$

$$= \frac{3/4 \cdot 4/10 \cdot 6/9 \cdot 5/8}{\underbrace{3/4 \cdot 4/10 \cdot 6/9 \cdot 5/8}_{\text{from green wallet}} + \underbrace{1/4 \cdot 2/10 \cdot 8/9 \cdot 7/8}_{\text{from black wallet}}} = \frac{\left(\frac{3 \cdot 4 \cdot 5 \cdot 6}{4 \cdot 10 \cdot 9 \cdot 8} \right)}{\left(\frac{3 \cdot 4 \cdot 5 \cdot 6}{4 \cdot 10 \cdot 9 \cdot 8} + \frac{1 \cdot 2 \cdot 8 \cdot 7}{4 \cdot 10 \cdot 9 \cdot 8} \right)} = \frac{360}{472} = 0.76$$

thus, $P(B | D, P, P) = 1 - 0.76 = 0.23$.

So is more likely that I picked the green wallet.

$$\begin{aligned} P(\text{error} | x) &= \min [P(w_1 | x), P(w_2 | x)] \\ &= \min [0.76, 0.23] \\ &= 0.23. \end{aligned}$$

PROBLEM 2

$$\lambda(x_i|w_j) = \begin{cases} 0 & i=j \\ \lambda_n & i=c+1 \\ \lambda_s & \text{otherwise} \end{cases}$$

Overall risk for $i=1, \dots, c$.

$$R(x_i|x) = \sum_{j=1}^{j=c} \lambda(x_i|w_j) P(w_j|x)$$

We know $\lambda(x_i|w_i)$, so we can write

$$R(x_i|x) = \lambda(x_i|w_i) P(w_i|x) + \sum_{j=1}^{j=c} \lambda(x_i|w_j) P(w_j|x)$$

$$R(x_i|x) = \lambda_s \left(\sum_{\substack{j=1 \\ j \neq i}}^{j=c} P(w_j|x) \right)$$

λ_s for $i \neq j$

$$R(x_i|x) = \lambda_s (1 - P(w_i|x))$$

$R(x_i|x)$ is the minimum risk if it's less than the cost of rejection, thus:

$$R(x_i|x) < \lambda_r$$

$$\lambda_s (1 - P(w_i|x)) < \lambda_r$$

$$\lambda_s - \lambda_s P(w_i|x) < \lambda_r$$

$$-P(w_i|x) < \frac{\lambda_r - \lambda_s}{\lambda_s}$$

$$P(w_i|x) \leq 1 - \frac{\lambda_r - \lambda_s}{\lambda_s}$$

If $\lambda_r = 0$, the cost of rejection would be minimum, and we would reject everything.

If $\lambda_r > \lambda_s$, we would always prefer to substitute and we would never reject.

$$p(x|\omega_1) \propto N(\mathbf{0}, \mathbf{I})$$

PROBLEM 3

$$p(x|\omega_2) \propto N\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{I}\right)$$

$$P(\omega_1) = P(\omega_2) = 1/2.$$

$$(a) \quad g_1(x) = g_2(x)$$

Let's find the two discriminant functions

$$g_1(x) = \left(\frac{\mu_1}{\sigma^2}\right)^t x - \frac{1}{2\sigma^2} \mu_1^t \mu_1 + \ln P(\omega_1)$$

$$\boxed{g_1(x) = \ln 1/2}$$

$$\begin{aligned} g_2(x) &= \left(\frac{\mu_2}{\sigma^2}\right)^t x - \frac{1}{2\sigma^2} \mu_2^t \mu_2 + \ln P(\omega_2) \\ &= [2 \ 1]^t x - \frac{1}{2} [2 \ 1] \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \ln 1/2 \end{aligned}$$

$$\boxed{g_2(x) = [2 \ 1]^t x - 5/2 + \ln 1/2}$$

thus,

$$g_1(x) = g_2(x)$$

$$\ln 1/2 = [2 \ 1]^t x - 5/2 + \ln 1/2$$

$$[2 \ 1]^t x = 5/2$$

$$[2 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 5/2 \quad \rightarrow \boxed{2x_1 + x_2 = 5/2}.$$

(b)

$$p(x|\omega_1) \propto N(\mathbf{0}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix})$$

$$p(x|\omega_2) \propto N\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix}\right)$$

$$g_1(x) = \left(\frac{\mu_1}{\sigma^2}\right)^t x - \frac{1}{2\sigma^2} \mu_1^t \mu_1 + \ln P(\omega_1)$$

$$= \ln P(\omega_1) = \boxed{\ln 1/2}$$



$$g_2(x) = x^T w_2 x + w_2^T x + w_{20}$$

$$w_2 = -\frac{1}{2} \Sigma_2^{-1}$$

$$w_2 = \Sigma_2^{-1} \mu_2$$

$$w_{20} = -\frac{1}{2} \mu_2^T \Sigma_2^{-1} \mu_2 - \frac{1}{2} \ln |\Sigma_2| + \ln P(w_2)$$

$$\boxed{w_2 = -\frac{1}{20} \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix}} \quad \boxed{w_2 = \frac{1}{10} \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}}$$

$$w_{20} = -\frac{1}{2} [2 \ 1] \frac{1}{10} \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \frac{1}{2} \ln 10 + \ln \frac{1}{2}$$

$$\boxed{w_{20} = -\frac{1}{2} - \frac{1}{2} \ln 10 + \ln \frac{1}{2}}$$

$$g_2(x) = x^T \left(-\frac{1}{20}\right) \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} x + \begin{bmatrix} 1/2 & 0 \end{bmatrix} x - \frac{1}{2} - \frac{1}{2} \ln 10 + \ln \frac{1}{2}$$

so, the decision boundary is found when

$$g_1(x) = g_2(x)$$

$$w_{12} = x^T \left(-\frac{1}{20}\right) \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} x + \begin{bmatrix} 1/2 & 0 \end{bmatrix} x - \frac{1}{2} - \frac{1}{2} \ln 10 + \cancel{\ln \frac{1}{2}}$$

$$x^T \left(-\frac{1}{20}\right) \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} x + \begin{bmatrix} 1/2 & 0 \end{bmatrix} x = \frac{1}{2} + \frac{1}{2} \ln 10$$

$$[x_1 \ x_2] \left(-\frac{1}{20}\right) \begin{bmatrix} 4 & -3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{2} + \frac{1}{2} \ln 10$$

$$\left(-\frac{1}{20}\right) [x_1 \ x_2] \begin{bmatrix} 4x_1 - 3x_2 \\ -2x_1 + 4x_2 \end{bmatrix} + \frac{1}{2} x_1 = \frac{1}{2} + \frac{1}{2} \ln 10$$

$$\left(\frac{-1}{20}\right) (4x_1^2 - 3x_1x_2 - 2x_1x_2 + 4x_2^2) + \frac{1}{2} x_1 = \frac{1}{2} + \frac{1}{2} \ln 10$$

$$\boxed{-\frac{1}{5}x_1^2 + \frac{1}{4}x_2x_1 - \frac{1}{5}x_2^2 + \frac{1}{2}x_1 = \frac{1}{2} + \frac{1}{2} \ln 10} \quad \cancel{\text{.}}$$

PROBLEM 4

PART 1

```
function data = getRndn(N, mean, var)
    data = mean + sqrt(var).*randn(N,1);
end
```

PART 2

$$\begin{array}{lll} N_1 = 1000 & N_2 = 2000 & N_3 = 3000 \\ \mu_1 = 2 & \mu_2 = 4 & \mu_3 = ? \\ \sigma_1^2 = 4 & \sigma_2^2 = 9 & \sigma_3^2 = ? \end{array}$$

$$\text{mean}_1 = \frac{\text{sum of elts } N_1}{1000} \rightarrow \text{sum of elts } N_1 = 1000 \cdot \text{mean}_1$$

$$\text{mean}_2 = \frac{\text{sum of elts } N_2}{2000} \rightarrow \text{sum of elts } N_2 = 2000 \cdot \text{mean}_2$$

$$\text{mean}_3 = \frac{\text{sum of elts } N_3}{3000} = \frac{\text{sum of elts } N_1 + \text{sum of elts } N_2}{3000}$$

$$= \frac{1000 \cdot \text{mean}_1 + 2000 \cdot \text{mean}_2}{3000} = \frac{2 + 2.4}{3} = 3.33$$

$$\text{Var}_3 = \sum_{i=1}^{3000} (x_i - \text{mean}_3)^2$$

$$= \sum_{i=1}^{3000} x_i^2 - 6.66 \sum_{i=1}^{3000} x_i + 3000 \cdot 3.33^2$$

$$= \sum_{i=1}^{3000} x_i^2 - 6.66 \sum_{i=1}^{3000} x_i + 3000 \cdot 3.33^2$$