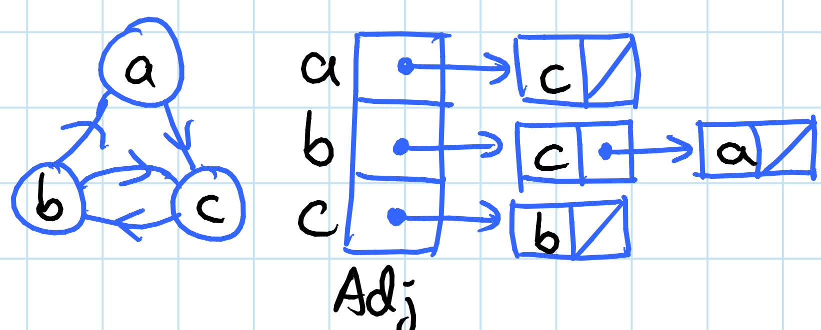
6.006	Lecture 14	Nov. 1, 2011

Outline: Search II: DFS (II of 2)
- depth-first search
- edge classification
- cycle testing
- topological sort

Recall:
- graph search: explore a graph
  e.g. find a path from start vertex s to a desired vertex
- adjacency lists: array Adj of $|V|$ linked lists
  - for each vertex $u \in V$, Adj[u] stores u's neighbors, i.e. $\{v \in V \mid (u,v) \in E\}$
    just outgoing edges if directed

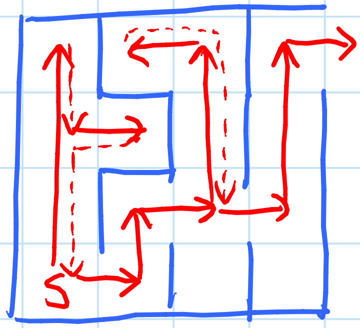
e.g.



- BFS: explore level-by-level from s
  - find shortest paths

Depth-first search (DFS): like exploring a maze

- follow path until you get stuck
- backtrack along breadcrumbs until reach unexplored neighbor
- recursively explore
- careful not to repeat a vertex




```
parent = {s: None}

DFS-visit(s, Adj):
    for v in Adj[s]:
        if v not in parent:
            parent[v] = s
            DFS-visit(v, Adj)
```

start

finish

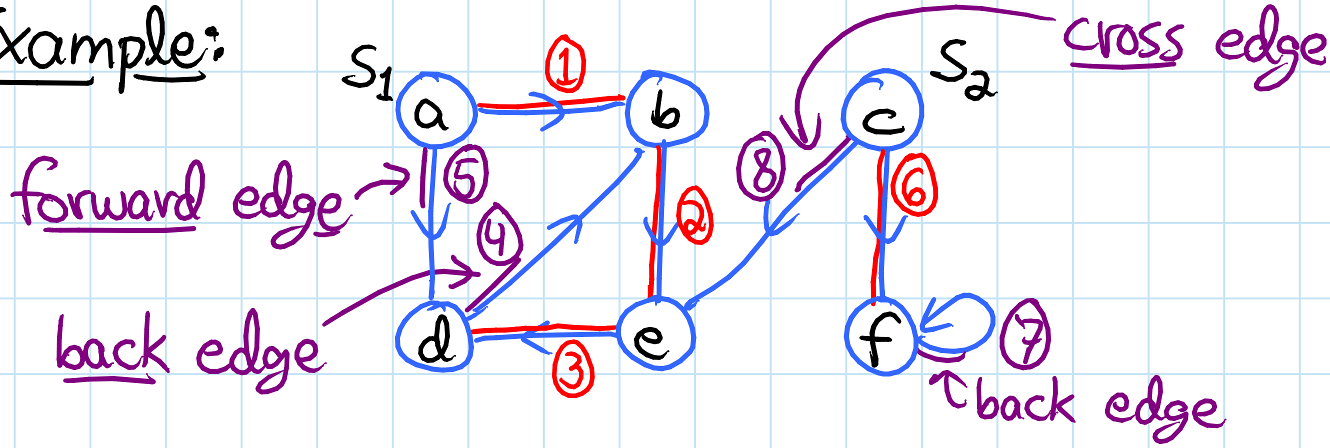
search from start vertex s (only see stuff reachable from s)

```
DFS(V, Adj):
    parent = {}
    for s in V:
        if s not in parent:
            parent[s] = None
            DFS-visit(s, Adj)
```

explore entire graph

(could do same to extend BFS)

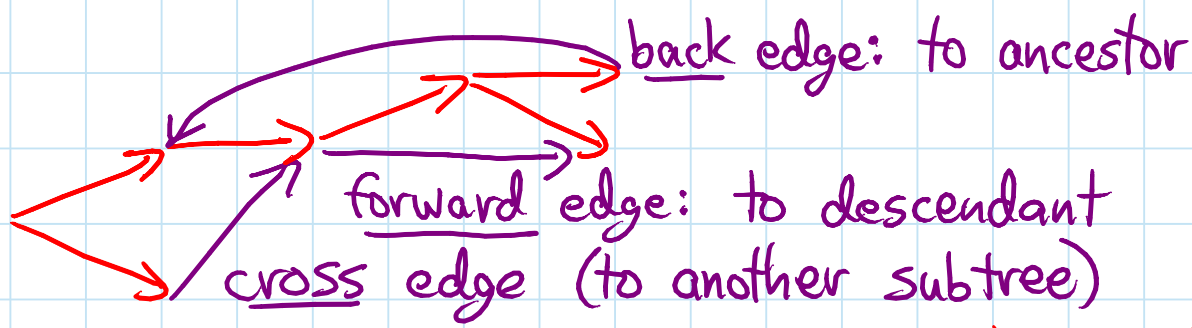
Example:



Edge classification:

tree edges (formed by parent)

nontree edges



- to compute this classification, mark nodes for duration they are "on the stack" → back or not
- only tree & back edges in undir. graph

Analysis:

- DFS-visit gets called with a vertex s only once (because then parent[s] set)

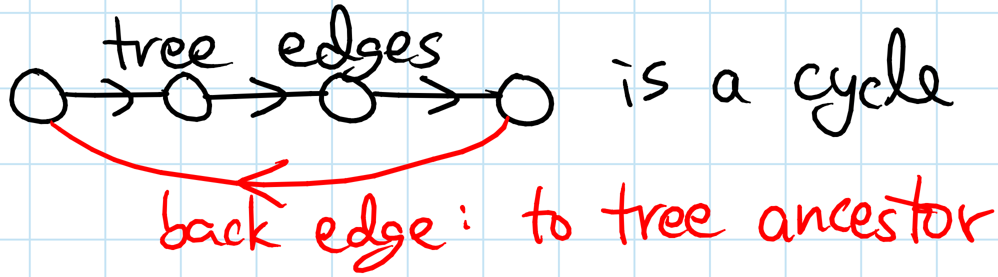
$\Rightarrow$ time in DFS-visit $= \sum_{s \in V} |Adj[s]| = O(E)$

- DFS outer loop adds just $O(V)$

$\Rightarrow O(V+E)$ time (linear time)

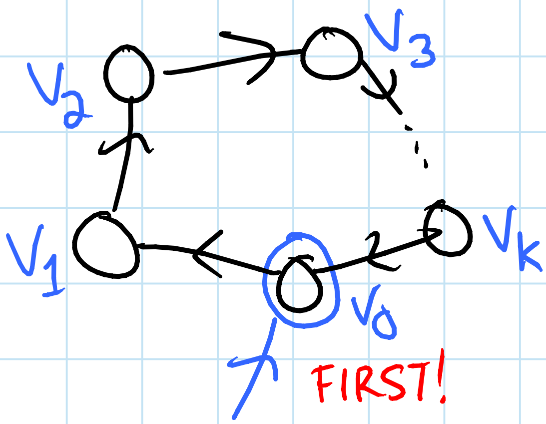
Cycle detection: graph G has a cycle
$\Leftrightarrow$ DFS has a back edge

Proof: $(\Leftarrow)$



is a cycle

$(\Rightarrow)$ consider first visit to cycle:



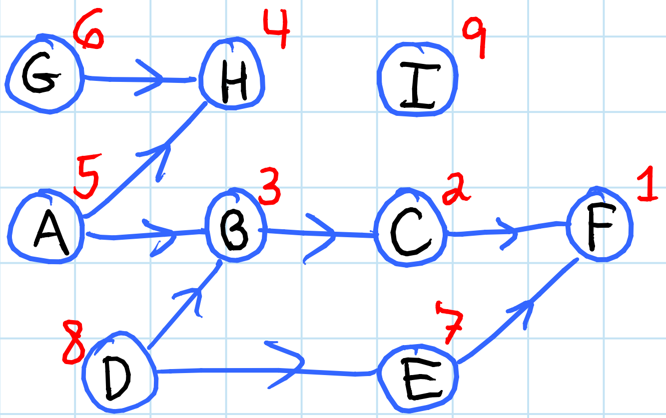
- before visit to v_i finishes,
  will visit v_{i+1} (& finish):
  will consider edge (v_i, v_{i+1})
  $\Rightarrow$ visit v_{i+1} now or already did

$\Rightarrow$ before visit to v_0 finishes,
will visit v_k (& didn't before)

$\Rightarrow$ before visit to v_k (or v_0) finishes,
will see (v_k, v_0) as back edge.

$\square$

Job scheduling: given directed acyclic graph (DAG),
where vertices represent tasks
& edges represent dependencies,
order tasks without violating dependencies



DFS finishing times

Source = vertex with no incoming edges
= schedulable at beginning (A, G, I)

Attempt: BFS from each source:

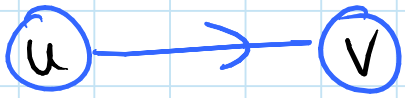
- from A finds A, BH, C, F
- from D finds D, BE, CF ←
- from G finds G, H
- from I finds I

slow... and wrong!

Topological sort: reverse of DFS finishing times
(time at which DFS-Visit(v) finishes)

```
DFS-Visit(v)
    ...
    order.append(v)
order.reverse()
```

Correctness: for any edge (u,v),
u ordered before v
i.e. v finished before u



- if u visited before v:
  - before visit to u finishes, will visit v (via (u,v) or otherwise)
  - $\Rightarrow$ v finishes before u
- if v visited before u:
  - graph is acyclic
  - $\Rightarrow$ u can't be reached from v
  - $\Rightarrow$ visit to v finishes before visiting u $\square$